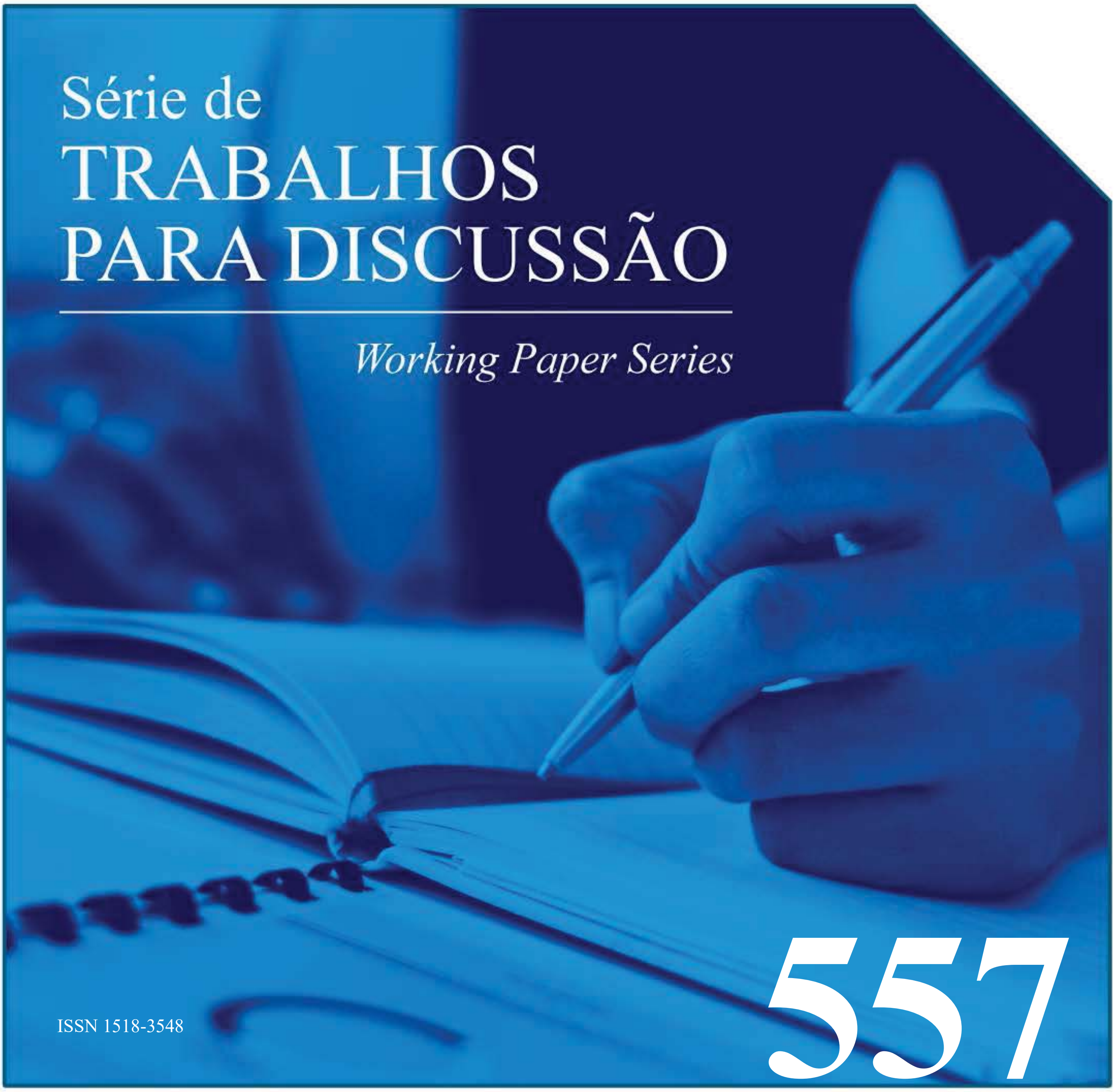

October 2021

Does Default Pecking Order Impact Systemic Risk? evidence from Brazilian data

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BANCO CENTRAL DO BRASIL

ISSN 1518-3548
CGC 00.038.166/0001-05

Working Paper Series	Brasília	no. 557	Outubro	2021	p. 1-27

Working Paper Series

Edited by the Research Department (Depep) – E-mail: workingpaper@bcb.gov.br

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Non-Technical Summary

Systemic risk is usually computed considering that, in case of default by any debtor, creditors will suffer a loss proportional to the loan granted to the defaulting debtor. For instance, if a given debtor fails to meet 10% of its total obligations, each of its creditors will lose the same fraction (10%) of the loan granted to that debtor.

Despite its simplicity, this assumption is unrealistic. Default leads to the impairment of the creditor-debtor relationship. Creditors are less likely to grant loans to debtors that defaulted on them in the past. Aware of this situation, debtors prefer to default first on creditors whose relationship is less valuable to them in the future.

In this study, we relax this assumption by assuming that defaulting debtors set a *default pecking order*. More specifically, defaulting debtors sort their creditors in ascending order according to some criterion (equity, out-degree, loan extended, or randomly) to rank the creditors they are willing to default on first. The idea is that they are willing to default first on the weaker creditors. For instance, suppose a debtor has a total amount of debt obligations equal to \$1,000 and decided not to meet 10% of this value (total default amount = \$100). The creditor on the top of the list granted \$80 to that debtor and the second one, \$50. Therefore, the debtor will default \$80 on the first creditor (as the default cannot be greater than the loan granted) and \$20 (the residual value, 100 - 80) on the second one.

Following the default pecking order hypothesis, we calculate the systemic risk using an extensive Brazilian data set. We found out the adoption of the default pecking order increases significantly the systemic risk. Moreover, the rise in the systemic risk brought on by the default pecking order over the proportional loss case decreases with the level of the initial shock and is higher for small-sized agents.

A possible explanation for our results is that, when the proportional loss (which maximizes the risk-sharing) is abandoned, the shock propagation effect becomes relatively more important and the systemic risk increases. We test this hypothesis by assessing the role of interconnectedness (as measured by the network density) as a determinant of systemic risk. The results corroborate this hypothesis. When the default pecking order is adopted, the density has a positive impact on the systemic risk. It suggests in this case the financial network acts mainly as a channel for shock propagation rather than for risk-sharing. Hence, a more interconnected network leads to a higher systemic risk.

Sumário Não Técnico

O risco sistêmico é geralmente calculado supondo-se que, em caso de inadimplência de algum devedor, os credores sofrerão uma perda proporcional ao empréstimo concedido ao devedor inadimplente. Por exemplo, se um determinado devedor inadimplir em 10% de suas dívidas totais, cada um de seus credores perderá a mesma fração (10%) do empréstimo concedido a esse devedor.

Apesar de simples, essa suposição é irrealista. A inadimplência leva ao comprometimento da relação credor-devedor. Os credores são menos propensos a conceder empréstimos a devedores que lhes inadimpliram no passado. Cientes disso, os devedores preferem inadimplir primeiro aos credores cujo relacionamento futuro lhes é menos valioso.

Neste estudo, relaxamos essa suposição assumindo que os devedores inadimplentes definem um *default pecking order*. Mais especificamente, os devedores inadimplentes classificam seus credores em ordem crescente de acordo com algum critério (patrimônio líquido, *out-degree*, empréstimo concedido ou aleatoriamente) para definir os credores que desejam inadimplir primeiro. A ideia é que eles estão dispostos a dar o calote primeiro nos credores mais fracos. Por exemplo, suponha que um devedor tenha uma dívida total de $ 1.000 e decide não honrar 10% desse valor (valor total do calote = $ 100). O credor no topo da lista concedeu $ 80 a esse devedor e o segundo, $ 50. Portanto, o devedor irá dar um calote de $ 80 no primeiro credor (já que o calote não pode ser maior do que o empréstimo concedido) e de $ 20 (o valor residual, 100 - 80) no segundo credor.

Supondo que devedores adotarão o *default pecking order*, calculamos o risco sistêmico usando um extenso conjunto de dados brasileiros. Constatamos que a adoção do *default pecking order* aumenta significativamente o risco sistêmico. Além disso, o aumento do risco sistêmico decorrente do *default pecking order* (em comparação com o risco sistêmico calculado conforme a abordagem tradicional) diminui com a magnitude do choque inicial e é maior para os agentes de pequeno porte.

Uma possível explicação para nossos resultados é que, quando a distribuição proporcional das perdas (que maximiza o compartilhamento de risco) é abandonada, o efeito de propagação do choque torna-se relativamente mais importante e o risco sistêmico aumenta. Testamos essa hipótese avaliando o papel da interconectividade (medida pela densidade da rede) na determinação do risco sistêmico. Os resultados corroboram essa hipótese. Quando o *default pecking order* é adotado, a densidade tem um impacto positivo no risco sistêmico. Isso sugere que, neste caso, a rede financeira atua principalmente como um canal para a propagação do choque, em vez de para o compartilhamento de risco. Consequentemente, uma rede mais interconectada leva a um risco sistêmico maior.

Does Default Pecking Order Impact Systemic Risk? evidence from Brazilian data

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Abstract

In network models of systemic risk, the loss distribution of a distressed debtor among its creditors follows a pro-rata fashion. It is proportional to the loan granted to the debtor. Despite its simplicity, this assumption is unrealistic. In this study, we create a framework for the computation of the systemic risk assuming a heterogeneous pattern of loss distribution, the *default pecking order*. Distressed debtors employ some criterion (equity, out-degree, or loan extended) to rank the creditors they are willing to default on first. Applying this framework to an extensive Brazilian data set, we found out the adoption of the default pecking order increases significantly the systemic risk. The rise in the systemic risk brought by the heterogeneous distribution over the homogeneous case decreases with the level of the initial shock and is higher for small-sized agents. This result can be interpreted in the light of the dual role of the financial network, which can be a channel for both risk-sharing and shock propagation. We test this hypothesis by assessing the role of interconnectedness (as measured by the network density) in driving the systemic risk. The results corroborate this hypothesis. When the homogeneous loss distribution (which maximizes risk-sharing) is abandoned, the density has a positive impact on the systemic risk. It suggests in this case the financial network acts mainly as a channel for shock propagation rather than for risk-sharing.

Keywords: systemic risk, default pecking order, complex networks

JEL Classification: D85, E44, G21, G23, G28

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1 Introduction

The aim of this paper is to shed some light on the following question: does the default pecking order – that is, a criterion for deciding which creditors to default on first – have some effect on the systemic risk? Network models have been extensively applied to the assessment of systemic risk in financial systems (Eisenberg and Noe (2001), Nier et al. (2007), Gai and Kapadia (2010), Upper (2011), Caccioli et al. (2012), Battiston et al. (2012b), Battiston et al. (2012a), Hałaj and Kok (2013), Roukny et al. (2013), Acemoglu et al. (2015)). Here, we build on network models in which agents are interconnected through contractual debt obligations (e.g., an interbank market or a bank-firm credit network). Agents are endowed with an economic value – equity. An outgoing link from agent i to agent j means that the former is the latter's creditor (or, alternatively, that the latter is the former's debtor). The weight of each link represents the value of the claim. The sum of the weights of the agent's outgoing (incoming) links corresponds to its assets (liabilities). Consider the stylized interbank market depicted in Figure 1. For instance, node C received loans from nodes B and G (of values 2 and 3, respectively), and extended loans to nodes E and F (both of value 3).

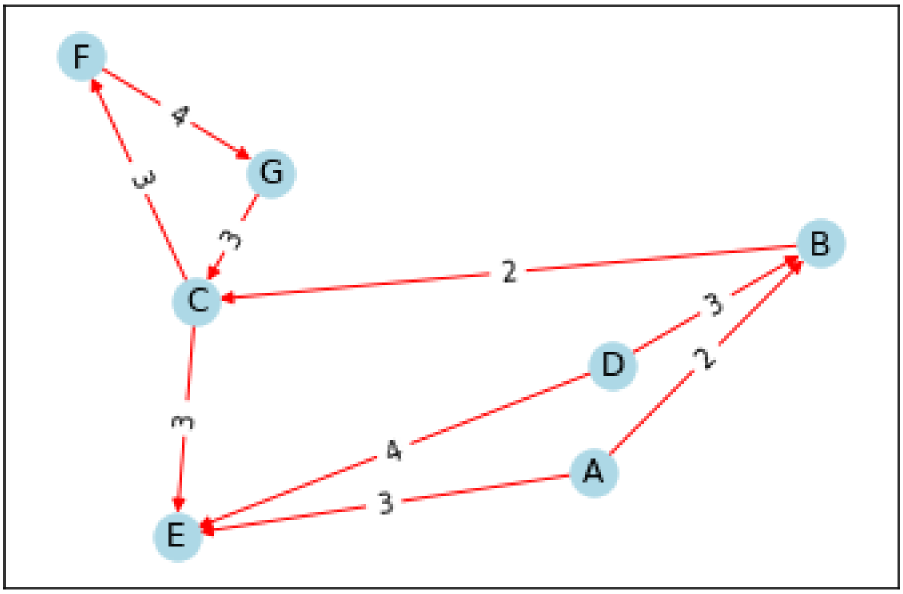


Figure 1: *Stylized representation of an interbank market.*

Negative shocks (which may be idiosyncratic or endogenous to the model) are at least partially absorbed by the agent's equity. However, part of this loss may be borne by their creditors, as this shock may lead the agent to not fully honor its debt obligations with its counterparties. Systemic risk is measured as the fraction of aggregate equity lost as a result of a shock spread throughout the network.

How shocks on a given agent affects its creditors varies according to the approach. In network models, there are two approaches of shock propagation. In the Eisenberg and Noe (2001) (E-N) approach, contagion is triggered by the default of the agent – i.e., by the complete depletion of its resources. The *DebtRank* (Bardoscia et al. (2015); Battiston et al. (2012a,b)) approach holds two main differences with respect to the E-N approach: the losses which trigger contagion are i) potential rather than real, and ii) may be partial rather than necessarily complete.

One strand of studies within the E-N approach considers that a negative shock is first absorbed by the agent's equity. Only the residual between the shock and the equity (if any) is transmitted to the agent's creditors. In the classical model developed by Eisenberg and Noe (2001), the residual loss is transmitted to creditors in proportion to their nominal claims on the assets of the debtor. Consider again the example depicted in Figure 1. Suppose node E has an equity e_E of 80. It is hit by a negative shock of size $s_E = 85$. Three other nodes (A, C, and D) have extended loans to E of amounts $l_{AE} = 3$, $l_{CE} = 3$, and $l_{DE} = 4$, adding up to $l_E = 10$. Only the residual loss $r_E = s_E - e_E = 5$ will be transmitted to its creditors proportionally to their claims on E. Therefore, D will bear 40% (l_{DE}/l_E) of the residual loss (2). Similarly, A and C will bear 30% (1.5) each. This mechanism of loss transmission is present in other studies. Elsinger et al. (2006) assumed that e_i is a random variable and that interbank loans are junior to other assets. Nier et al. (2007) developed a theoretical framework to assess the role of the parameters of the financial system (as the level of banks' capitalization and the banks' degree) on systemic risk and knock-on defaults.

Another strand of the literature relying on the E-N mechanism of shock propagation considers that the default of a given agent will imply that their creditors will lose a fraction – the *loss given default* (LGD) – of their exposures to the defaulted agent. The main drawback of this approach comes from the lack of an appropriate criterion to define the LGD value. Empirical evidence shows LGD has a bi-modal distribution (either relatively high or low) and is higher in recessions (Asarnow and Edwards (1995); Dermine and De Carvalho (2006); Hurt and Felsovalyi (1998); Schuermann (2003)). Moreover, the computation of LGD depends on some information, such as the availability of collateral or the seniority of the claims, which is usually not accessible. The seminal theoretical paper of Gai and Kapadia (2010) sets for simplicity an LGD of 100%. Empirical papers (e.g., Blavarg and Nimander (2001); Degryse et al. (2007); Upper and Worms (2004); van Lelyveld and Liedorp (2006)) compute systemic risk for a broad range of values of LGD, which is assumed to be exogenous and constant across creditors. Upper and Worms (2004), for instance, use the following values of LGD: 0.05, 0.1, 0.25, 0.4, 0.5, and 0.75.

The *DebtRank* methodology poses that losses in the assets extended to a given debtor are proportional to the fraction of equity lost by the debtor – the *distress*. In the original *DebtRank* formulation (Battiston et al. (2012a,b)), if a debtor i has lost a fraction $h_i(t) \leq 1$ of its equity, its creditor j will suffer a loss equal to $A_{ji}h_i(t)$, where A_{ji} is the exposure of j towards i. In order to ensure the systemic risk estimate will converge, each node propagates only the first shock it receives. However, due to the cyclicality of the network of exposures, the same node can be hit more than once. Hence, by ignoring these further shocks, the original *DebtRank* leads to an underestimation of the systemic risk. Bardoscia et al. (2015) solve both problems – non-convergence and underestimation of the systemic risk – by incorporating cyclical propagation of additional (rather than accumulated) losses. In their approach, the *differential DebtRank*, the formula above is updated to $A_{ji}[h_i(t) - h_i(t-1)] = A_{ji}\Delta h_i(t)$.

In spite of the differences among them, all models reviewed above share a common characteristic: a debtor affected by a negative shock distributes the loss among its creditors following a pro-rata fashion. The loss suffered by the creditor as a percentage of its exposure on the debtor hit by the shock is fixed. The LGD is assumed to be exogenous and constant across creditors, even when a range of LGD values is tested. In the residual loss approach, the LGD can be implicitly computed as the residual loss over the debtor's total liabilities. In the example discussed above, $20/100 = 0.2$. In the *DebtRank* approach, the loss transmitted by the stressed debtor as a fraction of their creditors' claims is $h_i(t)$ (or $\Delta h_i(t)$ in the updated formulation). That is, it is equal to the loss suffered by the debtor as a fraction of its equity.

However, the assumption that losses are imposed on creditors proportionally to the loan extended to the debtor under distress is unrealistic. It is more plausible to assume that debtors will choose to default on certain creditors first, before transmitting the residual loss to other creditors. Agents can be better off by setting a *default pecking order* (DPO) rather than adopting a pro-rata mechanism for loss distribution. Default implies the impairment of the relationship with the creditor. For instance, creditors will grant less loans to borrowers who have defaulted on them in the past. Therefore, agents will be more likely to default on weaker creditors due to the smaller expected value of continuing this relationship (Schiantarelli et al. (2020)). In the model developed by Bertschinger et al. (2019), agents maximize the resources that propagate through the network and return as additional internal assets. Under this criterion, the pro-rata mechanism is not always optimal. Thus, in a more realistic framework, each creditor would lose a fraction of its loans equal to $\beta_j = f(\mathbf{X})$, where $\mathbf{X}$ is a set of creditor's attributes used by debtors to set the DPO.

In this study, we compute the systemic risk assuming a heterogeneous distribution of losses by distressed debtors. In our framework, contagion is triggered by stress – i.e., a partial loss of equity – as in the *differential DebtRank* approach (Bardoscia et al. (2015)). The aggregate loss imposed by a given debtor on its counterparties is equal to the fraction of equity lost by the debtor times its liabilities. Although this mechanism resembles that of the *differential DebtRank* approach, there is a key difference between that framework and ours. In the *differential DebtRank* approach, losses are due to a mark-to-market adjustment made by the creditors of their exposures on the stressed debtor. On the other hand, we assume distressed debtors will perform a *strategic default*. This term was coined by bankers to define a default situation in which there is the ability, but not the willingness, to pay (Das and Meadows (2013)). This is not an uncommon phenomenon. For instance, assessing the U.S. housing mortgage market, Guiso et al. (2013) found out 26.4% (35.1%) of defaults appear to be strategic in March 2009 (September 2010).The few empirical studies addressing the determinants of strategic default on corporate loans (Asimakopoulos et al. (2016); Karthik et al. (2018)) bring evidences that a decrease in the debtor's profitability – i.e., a worsening in its financial health – increases its probability of a strategic default. This is coherent with our hypothesis that distressed debtors will perform a strategic default.

The debtor will choose a heuristic in order to rank their creditors, setting a DPO for the loss distribution. We will set heuristics based on the evidences brought by the literature. The first evidence is that creditor's weakness affects negatively its probability of default. Results presented by

Schiantarelli et al. (2020) show, everything else constant, debtors are more likely to default on weaker creditors. The expected value of continuing a relationship with a weaker creditor is smaller, as it has a smaller ability in fulfilling the debtor's needs (e.g., its demand for credit). An agent's strength can be proxied by its size. Some studies on the creditor-specific determinants of loan default (Salas and Saurina (2002), Rajan and Dhal (2003), Hu et al. (2004)) mention size as negatively correlated to non-performing loans. This can be explained by the fact that large banks are more involved in risk diversification (Salas and Saurina (2002), Rajan and Dhal (2003)) and have higher capabilities for loan evaluation (Hu et al. (2004)). However, another possible explanation is that debtors are more likely to default on loans extended by weaker creditors first for the reasons discussed above.[1]

The second evidence is that there are peer effects on debtor's default decisions. This is discussed in some theoretical frameworks using global games (Bond and Rai (2009), Carrasco and Salgado (2014), Drozd and Serrano-Padial (2018)). Moreover, there is evidence of peer effects on default decisions brought by empirical studies (Breza (2012), Li et al. (2009), Li et al. (2013)). Similar results were provided by the experimental study of Trautmann and Vlahu (2013). Default peer effects work through many channels. An increase in the number of defaulting borrowers may, for instance, i) threaten the creditor's future lending ability (Bond and Rai (2009)), ii) increase the lender's verification cost (Carrasco and Salgado (2014)), iii) convey information about the probability of being sued (Guiso et al. (2013)), and iv) reduce the lender's enforcement ability (Drozd and Serrano-Padial (2018); Vlahu (2008)). All these channels provide an incentive for other borrowers to default as well. We assume default incentives are driven by the creditor's out-degree (i.e., its number of debtors) relying on these default peer effects. As discussed above, the probability of a given borrower defaulting on a given creditor is negatively related to some attributes of the creditor (e.g., its future viability as a lender or its ability to enforce defaulting loans). Moreover, borrowers default negatively affects these attributes. Suppose the borrower will decide to default if these attributes fall below a given threshold. These attributes would not be threatened by few defaults if the creditor has a high out-degree. On the other hand, a debtor would be more likely to default on a creditor with a small out-degree, as in this case, a few defaults could be enough to reduce the attributes to a level below the threshold.

Based on these evidences, we set three heuristics. In the first heuristic, the distressed debtor ranks its creditors in ascending order according to their equity. The debtor's loss will be transmitted to the first creditor of the list, at the limit of the loan extended by this creditor to the debtor. The residual loss, if any, will be transmitted to the next creditor in the list, and so on, until all the debtor's loss has been transmitted to its creditors. In the second heuristic, the process is the same, but the creditors will be ranked in ascending order according to their out-degree. We also consider a heuristic in which debtors rank their creditors according to the loan granted by them. As the default leads to the impairment of the relationship with the creditor, it is reasonable to assume that debtors will be willing to default first on those creditors that granted them smaller loans. Finally, it is important to

[1]There are counterbalancing effects of creditors' size on its non-performing loans. The "too-big-to-fail" channel can lead large banks to engage in riskier activities, as they expect to be protected by the government in case of failure (Stern and Feldman (2004)). Hence, the overall effect of creditor's size on its non-performing loans is ambiguous. At any rate, it does not invalidate our hypothesis, as we are discussing to which creditors the debtor will transmit its losses *given that it has already decided to default*.

verify whether the impact on the systemic risk – if any – is due to the heterogeneous distribution of losses *per se* or to the heterogeneity in the loss distribution according to the heuristics we set. To address this point, we set a heuristic according to which creditors are randomly ranked by distressed debtors.

We apply this framework to a data set comprising quarterly information (from March 2012 through December 2015) on two Brazilian credit networks: the bank-bank (interbank) network and the firm-bank bipartite network. On the combination of these two layers, we compute the systemic risk for different levels of the initial shock. The shock is represented by an equity loss suffered by institution i in a given fraction ζ. The fraction of the overall system equity lost due to the propagation of this shock throughout the network is the systemic risk $s_{i,\zeta}$. The systemic risk is computed according to three rules of loss distribution: i) homogeneous distribution (the standard one), ii) heterogeneous distribution according to each of the three heuristics (equity, out-degree, and loans extended), and iii) heterogeneous distribution following a random sorting of the creditors.

Our results show the adoption of a DPO considerably increases the systemic risk *vis-à-vis* the standard approach. For an initial shock of 0.1, the random sorting entails a systemic risk 7.4 (15.5) times greater than the homogeneous distribution of losses in the case of banks (firms). The sorting according to the heuristics (equity, out-degree, and loan extended) entails even greater increase of systemic risk: 11.4, 14, and 27.5 times, respectively, for the banks; 32.2, 36.8, and 94.8 for the firms. It shows the heuristic also plays an important role in the increase of the systemic risk. When loss is transmitted preferentially to more fragile agents rather than randomly, the average systemic risk is higher. Moreover, the proportional increase in the systemic risk is higher for smaller values of the initial shock. For an initial shock of 0.5, in the case of banks, the ratio between the systemic risk entailed by the heterogeneous distribution of losses (random sorting, equity, out-degree, and loan extended) and that entailed by the standard approach is, respectively, 3, 3.7, 4.6, and 7.3. If the initial shock is equal to 1 (i.e., complete default), these ratios are even smaller: 2.4, 2.5, 3.2, and 4.6. A similar pattern can be observed for the firms. Finally, we also find the rise in the systemic risk brought by the heterogeneous distribution over the homogeneous case is higher when the shock is on small-sized agents.

Our findings corroborate those of Tran et al. (2018). In a stylized banking network, the authors show that sequential losses entail a smaller total loss to the system than a single larger loss of the same cumulative magnitude. In our study, we show that when the loss is concentrated in a few creditors, instead of being shared equally among all creditors, the systemic risk is greater.

Our results are also related to the dual role played by financial networks in its *robust-yet-fragile* nature (Chinazzi and Fagiolo (2015)). They are a channel for risk-sharing, but also propagate shocks. When a proportional distribution of losses is adopted, as in the standard approach, the risk-sharing is maximized. On the other hand, if loss is mainly transmitted to more fragile creditors, shock propagation prevails over risk-sharing and the systemic risk is higher.

We test this hypothesis by assessing the determinants of systemic risk. Among the potential explanatory variables, we include the interconnectedness, as measured by the density of the financial network. We perform this task employing machine learning techniques (random forest and XGBoost) and Shapley values analysis. The results corroborate this hypothesis. Under the homogeneous loss distribution, which maximizes risk-sharing, the impact of interconnectedness on systemic risk is meagre. However, when this assumption is abandoned, the density has a positive impact on the systemic risk. It suggests that, in the first case, the two financial network effects (risk-sharing and shock propagation) counterbalance. In the second case, the shock propagation effect gains relative importance over the risk-sharing effect. Under the DPO, weakest nodes bear a fraction of the loss greater than their share of the loan extended to distressed debtors. Thus, more interconnections would result in a heavier penalty on them and lead to a higher systemic risk.

Our contribution to the literature is twofold. First, we show the unrealistic assumption of homogeneous loss distribution leads to a non-negligible underestimation of the systemic risk. Therefore, by incorporating our methodology, systemic risk models can provide a more accurate measure of the systemic risk. Second, we shed some light on the role played by interconnectedness in driving systemic risk. We show this depends on the assumptions concerning the loss distribution by distressed debtors. Essentially, when losses are mainly transmitted to weaker creditors, a more interconnected network leads to a higher systemic risk.

This paper proceeds as follows. The data set and methodological issues are discussed in Sections 2 and 3, respectively. Section 4 brings the results. Finally, final considerations are presented in Section 5.

2 The data set

Our data set comprises several unique Brazilian databases with supervisory and accounting data. We extract quarterly information from March 2012 through December 2015 (16 periods) and build two networks: the bank-bank (interbank) network and bank-firm bipartite network.

In the interbank network, we consider all types of unsecured financial instruments registered in the Central Bank of Brazil. The main types of financial instruments are credit, capital, foreign exchange operations, and money markets. These operations are registered and controlled by different custodian institutions: Cetip[2] (private securities), the Central Bank of Brazil's Credit Risk Bureau System (SCR)[3] (credit-based operations), and the BM&FBOVESPA[4] (swaps and options operations).

[2]Cetip is a depositary of mainly private fixed income, state and city public securities, and other securities. As a central securities depositary, Cetip processes the issue, redemption, and custody of securities, as well as, when applicable, the payment of interest and other events related to them. The institutions eligible to participate in Cetip include commercial banks, multiple banks, savings banks, investment banks, development banks, brokerage companies, securities distribution companies, goods and future contracts brokerage companies, leasing companies, institutional investors, non-financial companies (including investment funds and private pension companies) and foreign investors.

[3]SCR is a very thorough data set that records every single credit operation within the Brazilian financial system worth 200BRL or above. Up to June 30th, 2016, this lower limit was 1,000BRL. Therefore, all the data we are assessing have been retrieved under this rule. SCR details, among other things, the identification of the bank, the client, the loan's time to maturity and the parcel that is overdue, modality of loan, credit origin (earmarked or non-earmarked), interest rate, and risk classification of the operation and the client.

[4]BM&FBOVESPA is a privately-owned company that was created in 2008 through the integration of the Sao Paulo

On March 30th, 2017, BM&FBOVESPA and Cetip merged into a new company named B3.

We consider net financial exposures among different financial conglomerates or individual financial institutions that do not belong to conglomerates (classified as "b1", "b2", or "b4" in the Central Bank of Brazil's classification system), removing intra-conglomerate exposures. Institutions with negative equity were excluded. Financial institutions' equity was retrieved from `https://www3.bcb.gov.br/ifdata`.

In the bank-firm network, we considered accounting and supervisory data from non-financial firms listed on the Brazilian stock exchange (BM&FBOVESPA). Information on firms' equity was retrieved from the *Economatica* database. For each of these firms, we identified the loans granted by financial institutions using the SCR information. The criteria to include a financial institution in the bank-firm network are the same of the interbank network – that is, financial institutions with positive equity and classified as "b1", "b2", or "b4".

Table 1 brings some statistics of the financial networks. They present some characteristics reported by other empirical studies on financial networks, such as disassortative behavior (e.g., Bottazzi et al. (2020)), sparseness (e.g., de Souza et al. (2016)), and a distribution of banks' degrees wider than that of firms in the bank-firm network (e.g., Luu and Lux (2019)). In each period t, we combine both networks to create the overall matrix of exposures $\mathbf{A}_t \in NB_t \times (NB_t + NF_t))$, where NB_t is the number of banks at t, NF_t is the number of firms at t, and A_{ijt} is the net exposure of i towards j at t. Recalling that creditors can be only banks, and debtors can be either firms or banks.

Table 1: *Topological features of the financial networks – average over periods.*

Variable	Interbank network	Bank-firm credit network
Number of banks	128.75	128.75
Number of firms	–	313.50
Banks' in/out-degree – average	10.17	22.23
Banks' in-degree – minimum	0	-
Banks' in-degree – maximum	49.81	-
Banks' out-degree – minimum	0	1
Banks' out-degree – maximum	80.38	233.94
Firms' in-degree – average	–	4.99
Firms' in-degree – minimum	–	1
Firms' in-degree – maximum	–	27.19
Assortativity	-0.3652	-0.3649
Density	0.0793	0.0581

Stock Exchange (Bolsa de Valores de Sao Paulo) and the Brazilian Mercantile & Futures Exchange (Bolsa de Mercadorias e Futuros). As Brazil's main intermediary for capital market transactions the company develops, implements and provides systems for trading equities, equity derivatives, fixed income securities, federal government bonds, financial derivatives, spot FX, and agricultural commodities.

3 Methodology

3.1 Systemic risk computation

We compute the systemic risk on the exposure network $\mathbf{A}$ following the *differential DebtRank* (Bardoscia et al. (2015)) methodology. The aggregate loan extended to j is A_j. At period 0, we impose an exogenous shock on agent j, reducing its equity by a fraction of ζ.[5] It will cause a subsequent loss on their creditors, indexed by i, whose aggregate value is equal to $\mathbf{A}_j\zeta$. At period 2, j's creditors will propagate this loss to their creditors in a similar fashion, and so on. We define $L_{ij}(t)$ as the accumulated loss transmitted by j to i up to period t. Moreover, $\Delta L_{ij}(t) = L_{ij}(t) - L_{ij}(t-1)$ is the new flow of loss transmitted by j to i and $L_i(t) = \sum_j L_{ij}(t)$ is the total loss transmitted to i by their debtors up to t. Finally, $\Delta L_i(t) = L_i(t) - L_i(t-1)$ is the variation in the total loss transmitted to i by their debtors up to t.

There are two mechanisms of loss distribution. In the first mechanism, which corresponds to the standard approach of systemic risk computation in network models, the loss distribution is homogeneous, proportional to the loan extended by each creditor to the distressed debtor. In the second mechanism, the distribution is heterogeneous. Creditors are ranked according to a given heuristic. The distressed debtor defaults first on the top creditor of the rank, transmitting only the residual loss to the remaining creditors. The rest of this section discusses each mechanism more formally.

Homogeneous loss distribution In the homogeneous case, the dynamics of loss propagation are represented by the following equations:

$$\Delta L_{ij}(t) = min\left(A_{ij} - L_{ij}(t-1), \mathbf{A}_{ij}\frac{[L_j(t-1) - L_j(t-2)]}{E_j}\right), \tag{1}$$

$$\Delta L_i(t) = min\left(E_i - L_i(t-1), \sum_j \Delta L_{ij}(t)\right), \tag{2}$$

in which $t \geq 0$ and E_j is agent j's equity. Thus, when an agent j suffers an additional loss equal to a fraction ζ of its equity, it will impose a loss to its creditors that corresponds to ζ times their exposures towards j. Observe that equity positions as well as the exposure network are time-invariant, i.e., they are taken as exogenous. The propagation considers stress differentials rather than stress absolute values (hence the methodology's name) to avoid double-counting.

Observe that, from Equation 1, L_{ij} cannot be greater than A_{ij}, i.e., j cannot impose to i a loss greater than i's exposures towards j. When $L_{ij} = A_{ij}$, j stops imposing losses on i. Moreover, as can be observed from Equation 2, L_i – the loss imposed on agent i – cannot be greater than E_i. That is, i's losses cannot be greater than its equity. When $L_i = E_i$, i stops propagating losses to other agents.

[5] In this paper, there are two period notations. The subscript notation t – present in the previous section and omitted in this subsection for simplicity – refers to the date (month-year). The notation between parenthesis refers to the number of iterations after the shock, where 0 refers to the period in which the shock is imposed.

Heterogeneous loss distribution In the second approach, the distressed debtor prefers to default on certain creditors first. Agent j will rank their creditors as $i = 1, 2, \ldots, J$, where J is j's number of creditors. It prefers to default on creditor 1 first, then on creditor 2, and so on. Debtor j transmits all the loss to creditor 1 at the limit of the loan extended by the creditor 1 to j. Only will the residual loss, if any, be transmitted to creditor 2. The process continues until j has transmitted the entire loss to their creditors. Therefore, Equation 1 is replaced by

$$\Delta L_{ij}(t) = min\left(A_{ij} - L_{ij}(t-1), \mathbf{A}_j\zeta - \sum_{k<i}\Delta L_{kj}(t)\right), \tag{3}$$

where $\zeta = \frac{L_j(t-1)-L_j(t-2)}{E_j}$. Therefore, the loss propagated to a certain creditor i is equal to the aggregate loss agent j will transmit to its creditors, $A_j\zeta$, minus the loss already transmitted to the other agents j prefers to default on rather than i. Equation 2 also holds in this case.

Figure 2 illustrates the differences between the two approaches. Suppose a distressed debtor has four creditors. They are ranked according to the debtor's default preference, meaning the debtor prefers to default on creditor 1 (C1) first. The loss the debtor will transmit to their creditors corresponds to 20% of its entire debt. In the figure on the left, which corresponds to the homogeneous loss distribution case, the debtor transmits to each creditor a loss which corresponds to 20% of the loan granted by the creditor. In the heterogeneous case (figure on the right), the debtor transmits the maximum loss it can to C1 (in this case, all loans granted by C1) and only the residual loss to C2.

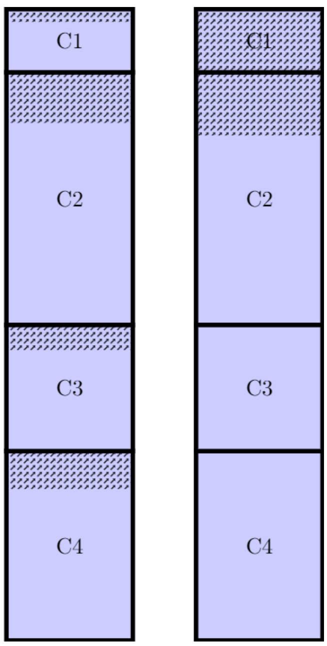


Figure 2: *Homogeneous (left) and heterogeneous (right) loss distribution. The loan extended by each creditor is proportional to the heights of the rectangles. The hatched area corresponds to the loss. In the homogeneous case, the fraction of the hatched area (0.2) is the same in all rectangles. The sum of the hatched area (i.e., the aggregate loss) is the same in both figures.*

Systemic risk After the initial shock on j, following one of the loss distribution mechanisms, the system converges after a sufficiently large number of periods $T \gg 1$. Then we have the final matrix of losses $\mathbf{L}^{j,\zeta} \in N \times 1$, where $L_i^{j,\zeta}$ is the total loss suffered by agent i after an initial shock of size ζ at agent j.

We repeat this process for the other agents. We define the systemic risk of agent i as

$$s_{i,\zeta} = 100 \times \frac{\sum_j [L_j^{i,\zeta} - L_j^{i,\zeta}(0)]}{\sum_j E_j}, \tag{4}$$

where $L_j^{i,\zeta}(0) = \zeta E_j$ if $j = i$ and 0 otherwise. Therefore, $s_{i,\zeta}$ measures the percentage of the aggregate agents' equity which is lost as a consequence of an initial shock of size ζ to agent i's equity. Note that we remove the initial shock to the agent i ($L_i^{i,\zeta}(0)$) from the computation of the systemic risk , as we are interested only in the losses caused by the contagion. Moreover, we compute $s_{i,\zeta}$ also for the agent that suffered the initial shock. Due to network cyclicality, a shock propagated by a given agent can hit it back.

3.2 Machine learning techniques

After computing the systemic risk for our data instances using the methodology presented in Section 3.1, we will employ two machine learning techniques – random forest (RF) and XGBoost (XB) – to assess the determinants of the systemic risk (Section 4.2). Shapley values will be used to give a better interpretability to our results. These techniques will be discussed briefly in this subsection.

Random forest and XGBoost RF (Breiman (2001)) and XB (Friedman et al. (2000)) are ensemble learning methods that can be used for both classification and regression. RF operates by constructing several decision trees.[6] For regression tasks, which is the case in this paper, it returns the average prediction of the individual decision trees. XB is an optimization algorithm that works with an ensemble of weak predictors (usually, decision trees) and creates a more efficient predictor model. At each boosting stage, the XB algorithm attempts to increase the performance of the predecessor model by including a new estimator. In both cases, the purpose is to estimate a predicted output $\hat{y}_i$ from an observed output y_i and a vector of explanatory variables X_i.

The models are trained and validated through a process known as *repeated k-fold cross-validation*. The data set, comprised of the output to be predicted and a set of potential explanatory variables, is split into k different parts (folds). $k-1$ folds are used in the development of the model. Then, the model is trained on the remaining fold: the predicted output $\hat{y}_i$ and the observed output y_i of the remaining fold are used to compute score measures, such as the root mean squared error (RMSE) and the R^2. Each fold is used as the testing data set. Hence, for instance, in a repeated k-fold cross-validation with $k = 5$ and 10 repetitions, a total of 50 regressions are run.

These score measures are used to tune the number of estimators of both methods. In the RF, the number of estimators is the number of decision trees in each forest. In the XB, this is the number of

[6]On decision trees, see, e.g., Breiman et al. (1984)

boosting stages to be performed. The number of estimators varies within a grid of ascending values. For each of these values, the regressions are run, and the average score is computed. The number of estimators is chosen so that increasing it does not improve the performance of the method.

Shapley values We go further on the interpretability of our results by resorting to the computation of Shapley values. This approach, originated from the coalition games theory (Shapley (1953); Shoham and Leyton-Brown (2008)), provides evidence on features' importance. Moreover, Shapley values can also inform whether a given feature is positively or negatively correlated to the output. We compute Shapley values through the SHAP (SHapley Additive exPlanation) framework proposed by Lundberg and Lee (2017). The authors propose an explainer model g aiming at predicting an output using a set of M features as inputs. The predicted value for a given data-instance is given by

$$g(z') = \phi_0 + \sum_{i=1}^{M} \phi_i z'_i, \tag{5}$$

where z' is a binary variable indicating whether feature i was included in the model or not. Therefore, the SHAP value ϕ_i indicates to what extent the feature i shifts the predicted value up or down from a given mean output ϕ_0. Lundberg and Lee (2017) showed, under certain properties (local accuracy, missingness, and consistency), ϕ_i corresponds to the Shapley value of the game theory. The SHAP value of feature i is given by

$$\phi_i = \sum_{S \subseteq M \setminus i} \frac{|S|!(|M| - |S| - 1)!}{M!} [F(S \cup \{i\}) - F(S)]. \tag{6}$$

Therefore, the SHAP value of feature i for a given data-instance computes the difference between the predicted value of the instance using all features in S plus feature i, $F(S \cup \{i\})$, and the prediction excluding feature i, $F(S)$. This is weighted and summed over all possible feature vector combinations of all possible subsets S.[7]

4 Results

4.1 General results

We compute the systemic risk (Equation 4) considering three values of ζ (0.1, 0.5, and 1.0) for 7,076 observations (2,060 date-bank data-instances and 5,016 date-firm data-instances). We consider five loss distribution mechanisms:

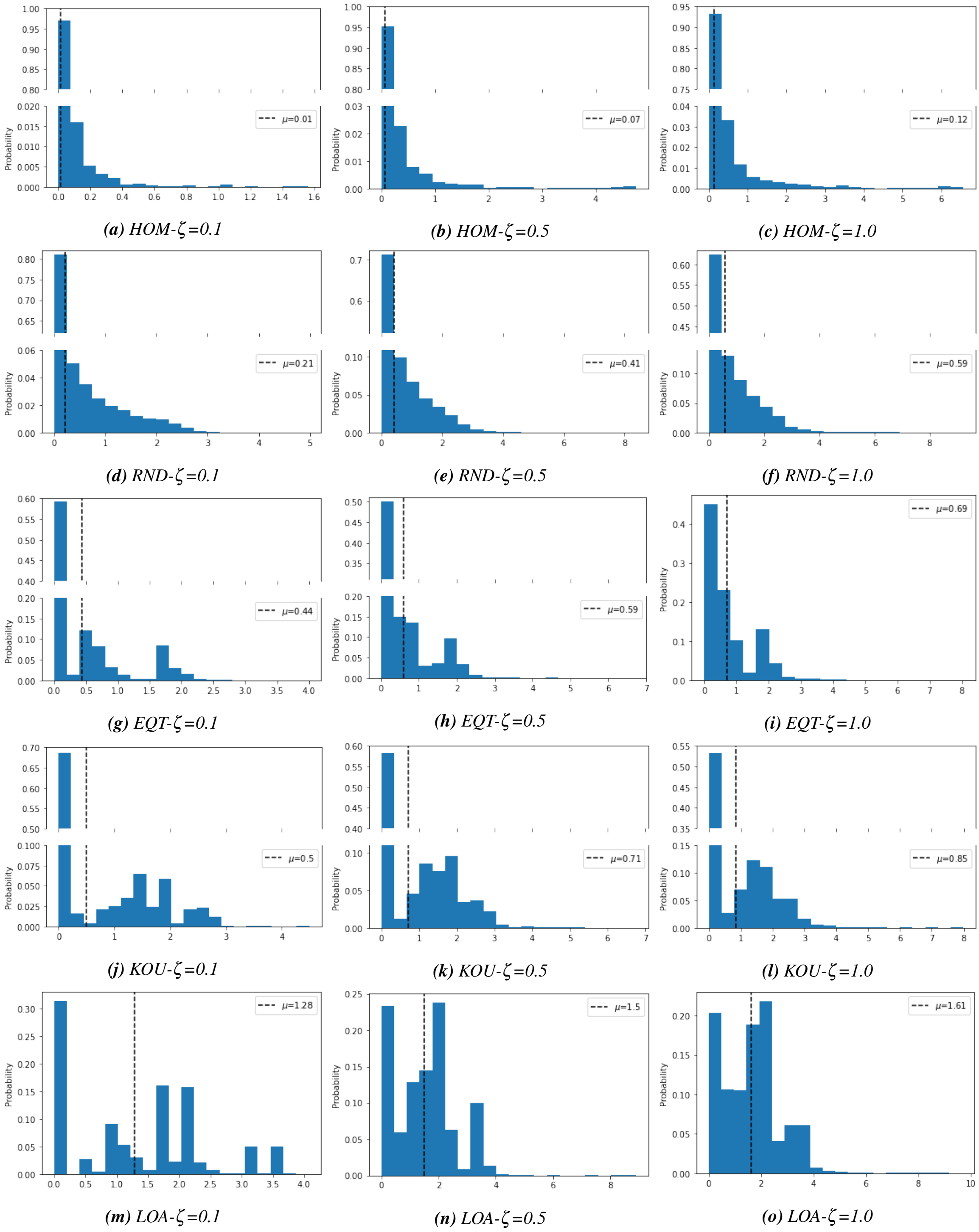
- The homogeneous loss distribution mechanism.
- The heterogeneous distribution mechanism according to the three heuristics discussed in Section 1: equity, out-degree, and loan extended. For instance, according to the equity heuristic,

[7]For details on the calculation of SHAP values, see, e.g., Lundberg and Lee (2017) and Kalair and Connaughton (2021).

the distressed debtor ranks their creditors in ascending order by their equities, preferring to default on those with smaller equity first. The other heuristics follow the same reasoning (i.e., creditors are ranked in ascending order according to their out-degree or their loan extended).

- The heterogeneous distribution mechanism in which the creditors are randomly sorted. This is to check whether the differences regarding the homogeneous case (if any) are due to the heterogeneous distribution *per se* or also due to the heuristic adopted. We run 10 different realizations considering this loss distribution mechanism.

The results are presented in Figures 3 and 4. It can be seen the systemic risk increases when a heterogeneous loss distribution is adopted. Moreover, the systemic risk of firms is smaller than that of banks. This is because firms are only debtors in the network. They cannot be hit back by shocks to themselves. By contrast, banks can be both debtors and creditors. Thus, shocks to banks are amplified by the cyclicality of the network.



(a) *HOM-ζ=0.1* ***(b)*** *HOM-ζ=0.5* ***(c)*** *HOM-ζ=1.0*

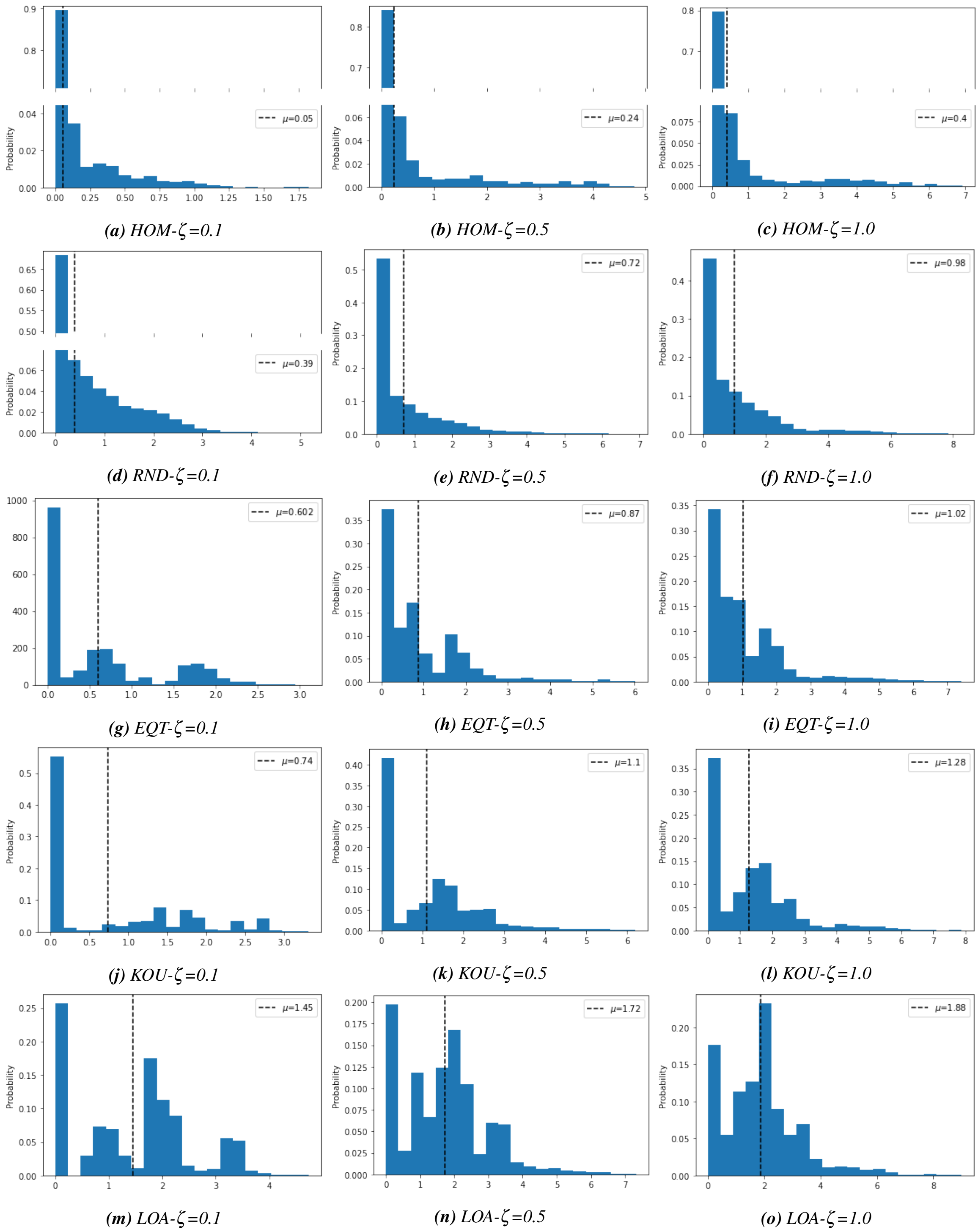
(d) *RND-ζ=0.1* ***(e)*** *RND-ζ=0.5* ***(f)*** *RND-ζ=1.0*

(g) *EQT-ζ=0.1* ***(h)*** *EQT-ζ=0.5* ***(i)*** *EQT-ζ=1.0*

(j) *KOU-ζ=0.1* ***(k)*** *KOU-ζ=0.5* ***(l)*** *KOU-ζ=1.0*

(m) *LOA-ζ=0.1* ***(n)*** *LOA-ζ=0.5* ***(o)*** *LOA-ζ=1.0*

Figure 3: *Density probability of the systemic risk – firms. Legend: HOM: homogeneous loss distribution; RND: random sorting; EQT: equity heuristic; KOU: out-degree heuristic; LOA: loan granted heuristic. The vertical lines indicate the average systemic risk.*



(a) *HOM-ζ=0.1* ***(b)*** *HOM-ζ=0.5* ***(c)*** *HOM-ζ=1.0*

(d) *RND-ζ=0.1* ***(e)*** *RND-ζ=0.5* ***(f)*** *RND-ζ=1.0*

(g) *EQT-ζ=0.1* ***(h)*** *EQT-ζ=0.5* ***(i)*** *EQT-ζ=1.0*

(j) *KOU-ζ=0.1* ***(k)*** *KOU-ζ=0.5* ***(l)*** *KOU-ζ=1.0*

(m) *LOA-ζ=0.1* ***(n)*** *LOA-ζ=0.5* ***(o)*** *LOA-ζ=1.0*

Figure 4: *Density probability of the systemic risk – banks. Legend: HOM: homogeneous loss distribution; RND: random sorting; EQT: equity heuristic; KOU: out-degree heuristic; LOA: loan granted heuristic. The vertical lines indicate the average systemic risk.*

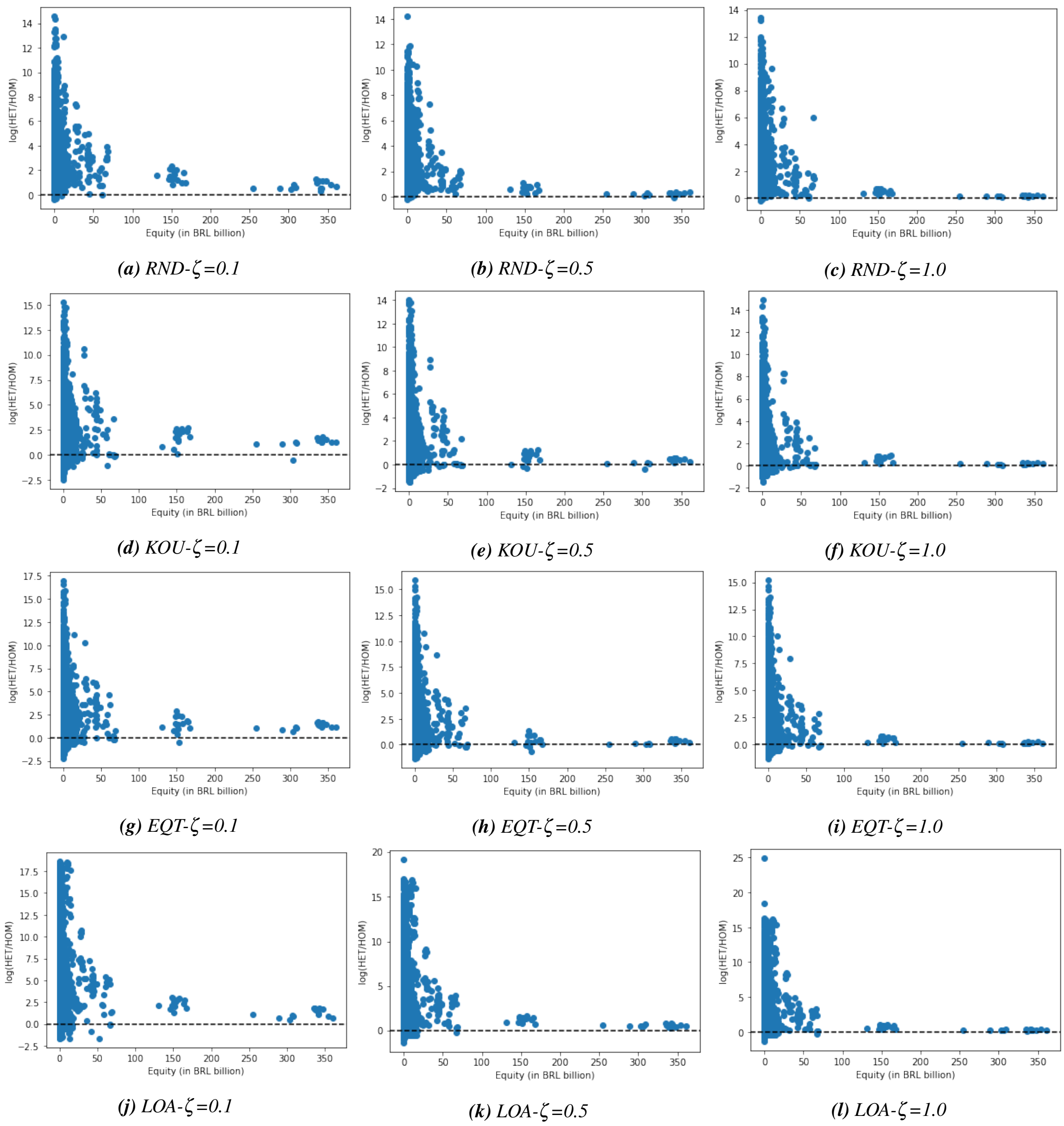
The heuristic also plays an important role in the increase of the systemic risk. When loss is transmitted preferentially to more fragile agents rather than randomly, the average systemic risk is higher. The loan granted heuristic entails the highest average systemic risk. Moreover, the rise in the systemic risk brought by the heterogeneous distribution over the homogeneous case decreases with the level of the initial shock. For an initial shock of 0.1, the ratio between the systemic risk entailed by the heterogeneous distribution of losses (random sorting, equity, out-degree, and loans granted) and that entailed by the standard approach is, respectively, 7.4, 11.4, 14.0, and 27.5 for the banks. If the initial shock is 0.5, these ratios decrease to 3.0, 3.7, 4.6, and 7.3. For an initial shock of 1.0 (complete default), these values are even smaller: 2.4, 2.5, 3.2, and 4.6. A similar decreasing pattern can be observed for firms. Finally, the heuristics also change the distribution of the systemic risk. The distribution of the systemic risk entailed by the homogeneous distribution and the random sorting heterogeneous distribution of losses is right-skewed. When some heuristic is introduced, the distribution of the systemic risk becomes multimodal.

Table 2 presents the weighted average systemic risk $\sum_i w_{it} s_{i\zeta_t}$, where w_{it} is the agent i's participation in total equity at period t. Thus, this is the loss caused by a shock in the agents weighted by their equities. In this case the differences between the systemic risk entailed by the different approaches are less remarkable. This suggests the increase in the systemic risk brought by the heterogeneous loss distribution depends on the agents' equity. We then investigate the relationship between the systemic risk entailed by the heterogeneous distributions of losses and that entailed by the homogeneous distribution HET/HOM at the agent level. Results are presented in Figures 5 and 6. For the sake of better visualization this ratio is presented on a natural logarithmic scale. The highest differences between the systemic risk entailed by the heterogeneous loss distributions and that entailed by the homogeneous approach are observed in the small-sized agents. In a few cases, this ratio is smaller than one.

Table 2: *Average systemic risk for different levels of ζ, weighted by the agent's equity. Legend: HOM: homogeneous loss distribution; RND: random sorting; EQT: equity heuristic; KOU: out-degree heuristic; LOA: loan granted heuristic.*

Type	Method	ζ=0.1	ζ=0.5	ζ=1.0
Firms	HOM	0.20	0.87	1.43
	RND*	0.59	1.34	2.00
	EQT	0.89	1.42	2.01
	KOU	1.03	1.55	2.16
	LOA	1.38	2.36	2.73
Banks	HOM	0.14	0.58	0.86
	RND*	0.42	0.80	1.11
	EQT	0.42	0.85	1.06
	KOU	0.47	0.94	1.15
	LOA	0.73	1.13	1.30

*: Average of 10 realizations.



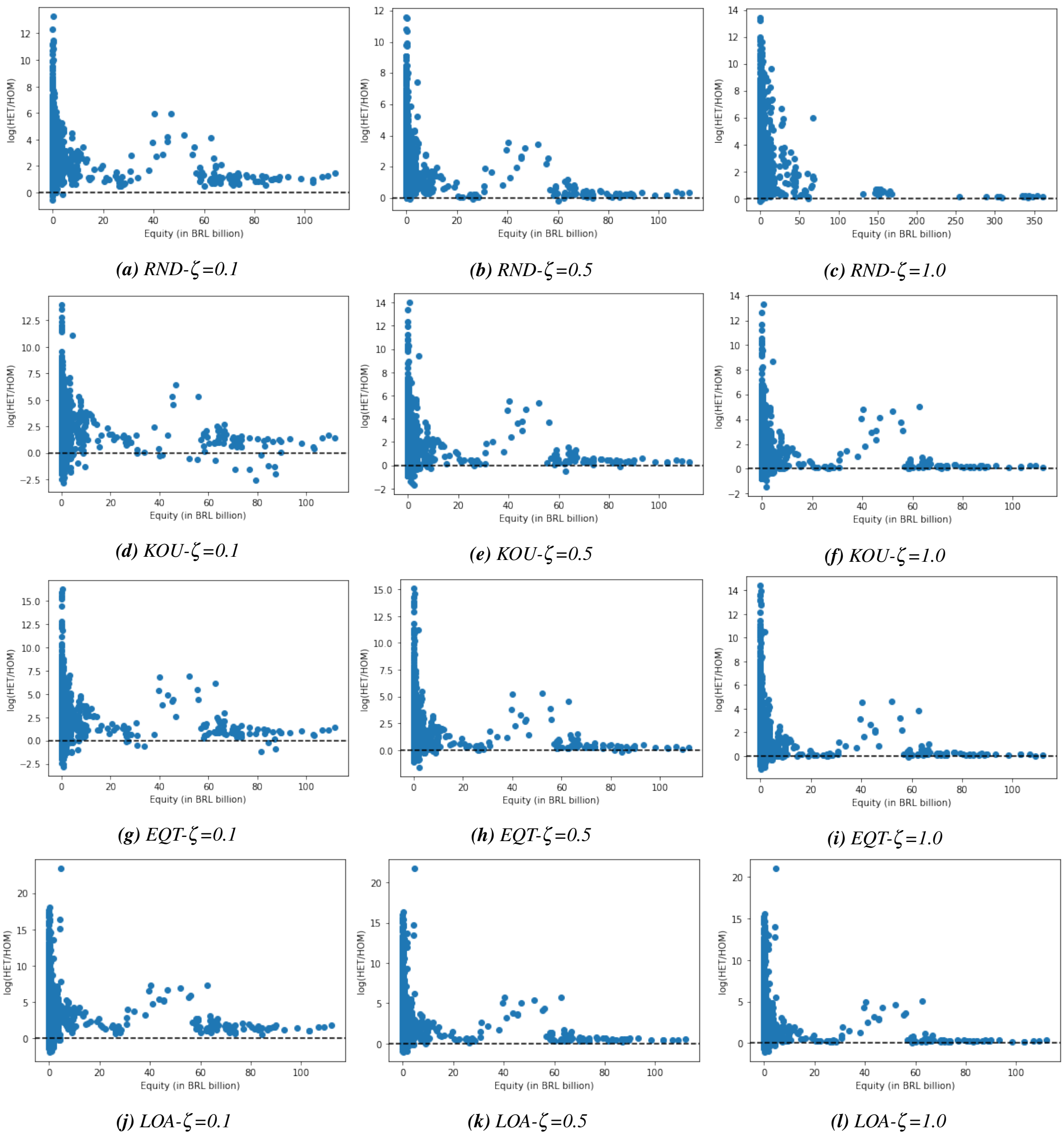
(a) RND-ζ=0.1 *(b) RND-ζ=0.5* *(c) RND-ζ=1.0*

(d) KOU-ζ=0.1 *(e) KOU-ζ=0.5* *(f) KOU-ζ=1.0*

(g) EQT-ζ=0.1 *(h) EQT-ζ=0.5* *(i) EQT-ζ=1.0*

(j) LOA-ζ=0.1 *(k) LOA-ζ=0.5* *(l) LOA-ζ=1.0*

Figure 5: *Systemic risk entailed by the heterogeneous loss distribution-to-systemic risk entailed by the homogeneous loss distribution ratio – firms. Legend: RND: random sorting; EQT: equity heuristic; KOU: out-degree heuristic; LOA: loan granted heuristic.*



(a) RND-ζ=0.1 *(b) RND-ζ=0.5* *(c) RND-ζ=1.0*

(d) KOU-ζ=0.1 *(e) KOU-ζ=0.5* *(f) KOU-ζ=1.0*

(g) EQT-ζ=0.1 *(h) EQT-ζ=0.5* *(i) EQT-ζ=1.0*

(j) LOA-ζ=0.1 *(k) LOA-ζ=0.5* *(l) LOA-ζ=1.0*

Figure 6: *Systemic risk entailed by the heterogeneous loss distribution-to-systemic risk entailed by the homogeneous loss distribution ratio – banks. Legend: RND: random sorting; EQT: equity heuristic; KOU: out-degree heuristic; LOA: loan granted heuristic.*

4.2 Risk-sharing versus shock propagation

Our results can be interpreted in the light of the dual role of financial networks. They can work as a channel for risk-sharing, but also for shock propagation (Chinazzi and Fagiolo (2015)). The role played by interconnectedness in the systemic risk depends on which effect prevails over the other. If the risk-sharing effect prevails over the shock propagation effect, a more interconnected network will be more robust. Hence, the systemic risk is smaller. Otherwise, more interconnections will increase the level of systemic risk. When the distribution of losses follows a pro-rata fashion, risk-sharing is maximized. On the other hand, when the loss is mostly transmitted to the more fragile creditors, the financial network acts mainly as a shock propagator. The shock propagation effect of the financial network gains relevance, while the risk-sharing effect is curtailed. This explains the increase in the systemic risk when the homogeneous loss distribution is abandoned.

We go further on this hypothesis by assessing the effect of interconnectedness on our measures of systemic risk. We employed two machine learning techniques – RF and XB – to predict the agents' systemic risk. Besides individual level variables (equity, total borrowing-to-equity ratio, PageRank, in-degree, and a dummy variable for firms), we use the density of the financial network as a potential explanatory variable. The potential explanatory variables are described in Table 3. For a given date, we calculate the density of the financial network as follows:

$$dens_t = \frac{k_t}{NB_t \times (NB_t + NF_t - 1)}, \tag{7}$$

where k_t is the total number of links of the network at period t, NB_t is the total number of different banks in the network at t, and NF_t is the total number of different firms. The denominator represents the maximum number of links, as only banks can be lenders (except for themselves), but both banks and firms can be borrowers. Thus, the density is the number of links over the number of possible links.

Table 3: *Potential determinants assessed in the study.*

Variable	Acronym
Equity (net worth)	NW
Total borrowing-to-equity ratio	CA
PageRank	PR
In-degree	Kin
Dummy variable for type of agent (firm: 1)	Type
Density of the financial network	dens

We implemented a repeated k-folds cross-validation with k = 5 folds and 10 repetitions. After tuning the number of estimators of both methods using the root mean squared error (RMSE) as the score measure within the grid [30, 50, 70, 100, 300, 500], we set the value of both parameters as 50. Finally, we apply both techniques to predict the systemic risk. The outputs to be predicted are the four measures of systemic risk, each one engendered by a different methodology of loss distribution (homogeneous, equity heuristic, out-degree heuristic, and loan granted heuristic). The potential explanatory variables are those presented in Table 3. Both methods have a similar performance in terms of the average R^2 (Figure 7), which generally increases with ζ.

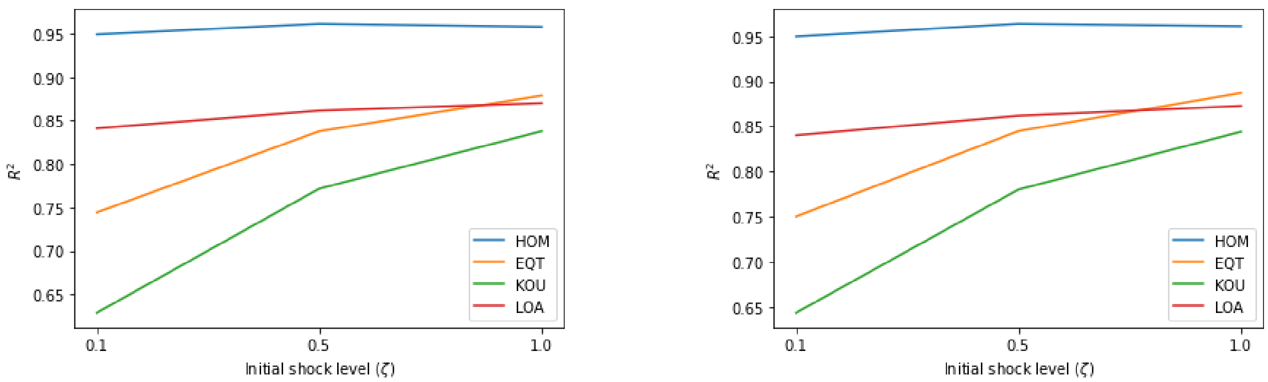
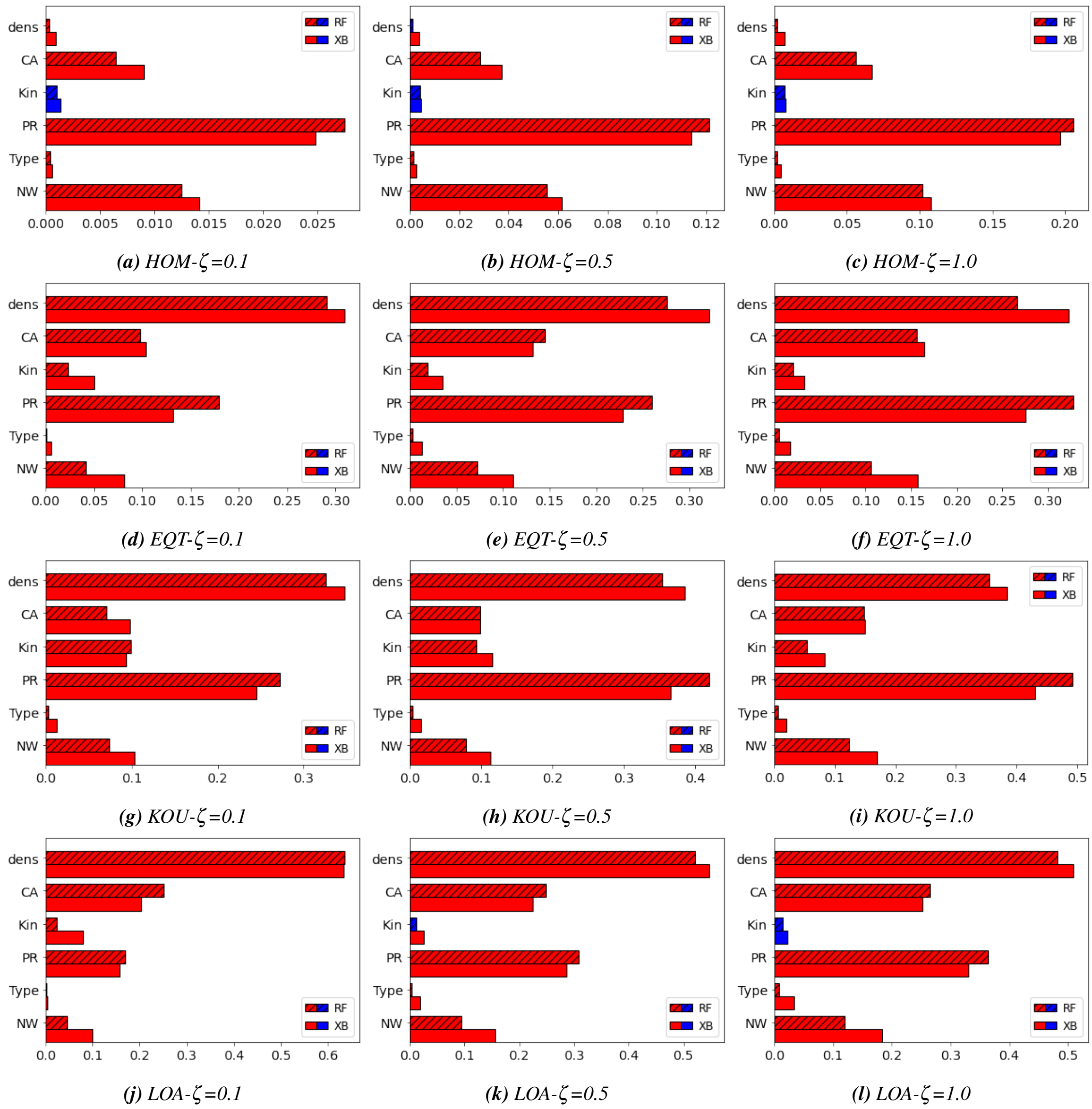


Figure 7: *Average R^2 of the regressions performed by RF (left) and XB (right). Legend: HOM: homogeneous loss distribution; RND: random sorting; EQT: equity heuristic; KOU: out-degree heuristic; LOA: loan granted heuristic.*

We assess the importance of each feature in driving the systemic risk by computing Shapley values through the SHAP (SHapley Additive exPlanation) framework (Lundberg and Lee (2017)). Once the two machine learning techniques are employed as explainer models for our systemic risk measures, we compute the SHAP values. Then we compute, for each feature presented in Table 3, the average absolute SHAP value over all data-instances. Finally, we multiply this value by the sign of the correlation between the feature value and the SHAP value. The final value gives us two pieces of information. Its absolute value shows the feature importance in driving the output. Its sign informs whether the feature is positively or negatively correlated to the output.

The results are presented in Figure 8. Density has a meagre effect in driving systemic risk under the homogeneous loss distribution rule. However, this is an important systemic risk driver when the heterogeneous distribution of loss is adopted, rivaling in importance with PageRank. Its impact is positive, which means that a more interconnected network leads to a higher systemic risk when losses are mainly transmitted to fragile creditors.

These results corroborate our hypothesis. When risk-sharing is maximized through the adoption of the homogeneous distribution of loss, interconnectedness (as measured by the network's density) has no significant impact on systemic risk. This suggests both effects (risk-sharing and shock propagation) almost counterbalance. However, when losses are transmitted mostly to fragile creditors, the shock propagation effect gains relative importance over the risk-sharing effect. In this case, interconnectedness is positively correlated to systemic risk, as the financial network now is acting mainly as a channel for shock propagation rather than for risk sharing.



(a) HOM-ζ=0.1 *(b) HOM-ζ=0.5* *(c) HOM-ζ=1.0*

(d) EQT-ζ=0.1 *(e) EQT-ζ=0.5* *(f) EQT-ζ=1.0*

(g) KOU-ζ=0.1 *(h) KOU-ζ=0.5* *(i) KOU-ζ=1.0*

(j) LOA-ζ=0.1 *(k) LOA-ζ=0.5* *(l) LOA-ζ=1.0*

Figure 8: *Average absolute SHAP values. Legend: HOM: homogeneous loss distribution; EQT: equity heuristic; KOU: out-degree heuristic; LOA: loan granted heuristic. Bars in red (blue) indicates the corresponding feature is positively (negatively) correlated to the SHAP value.*

5 Concluding remarks

In this paper, we computed the systemic risk relaxing the assumption of pro-rata distribution of losses by distressed debtors among their creditors. This assumption is adopted by traditional network models of systemic risk. However, this assumption is not realistic. Empirical studies show debtors are more likely to default on fragile creditors. Default implies the impairment of the debtor-creditor relationship. For this reason, debtors prefer to default first on those creditors whose relationships are less valuable to them in the future.

For a Brazilian multilayer credit network (interbank credit network and bank-firm credit network), we compared the systemic risk computed under the homogeneous and heterogeneous rule of loss distribution. In the homogeneous rule, distressed debtors transmit the loss to its creditors proportionally to the loan extended by these creditors. In the heterogeneous rule, creditors are ranked in ascending order according to some criterion (equity, out-degree, loan extended, or randomly). Once distressed debtors compute the aggregate loss to be transmitted to their creditors, they default on their creditors following this default pecking order. Therefore, under the heterogeneous rule (except in the random sorting case), losses are mostly transmitted to the more fragile creditors.

Our results show systemic risk increases substantially when the heterogeneous loss distribution is adopted. The rise in the systemic risk brought by the heterogeneous distribution over the homogeneous case decreases with the level of the initial shock and is higher for small-sized agents. These results are related to the dual role of financial networks. They can be a channel for both risk-sharing and shock transmission. Risk-sharing is maximized under the homogeneous loss distribution. Creditors experience a loss which is proportional to the loan they extended to the distressed debtor. When this assumption is relaxed, the shock transmission effect of the financial network gains relative importance over the risk-sharing effect, leading to an increase in the systemic risk.

We tested this hypothesis by assessing the determinants of systemic risk under different rules of loss distribution. Among the potential explanatory variables, we included the density of the financial network. We performed this task employing two machine learning techniques (random forest and XGBoost) and Shapely values to give more interpretability to our results. The results corroborated this hypothesis. Under the homogeneous rule (i.e., when risk-sharing is maximized), the two effects of the financial network (risk-sharing and shock transmission) counterbalance and the density has a meagre impact on systemic risk. On the other hand, when losses are mostly transmitted to more fragile creditors, the density has a positive impact on systemic risk, suggesting in this case the financial network acts mainly as a shock transmission channel.

Our study contributes to the literature on systemic risk by showing the magnitude of the systemic risk depends on the strategy of loss distribution adopted by the debtors. The unrealistic assumption that losses are evenly distributed among creditors leads to an underestimation of the systemic risk. Moreover, we shed a new light on the role played by interconnectedness on systemic risk. When the assumption of homogeneous loss distribution is relaxed, the shock transmission channel of the financial network becomes relatively more important *vis-à-vis* the risk-sharing channel. In this case, interconnectedness has a positive impact on systemic risk, as the financial network is working mainly as a shock transmission channel.

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