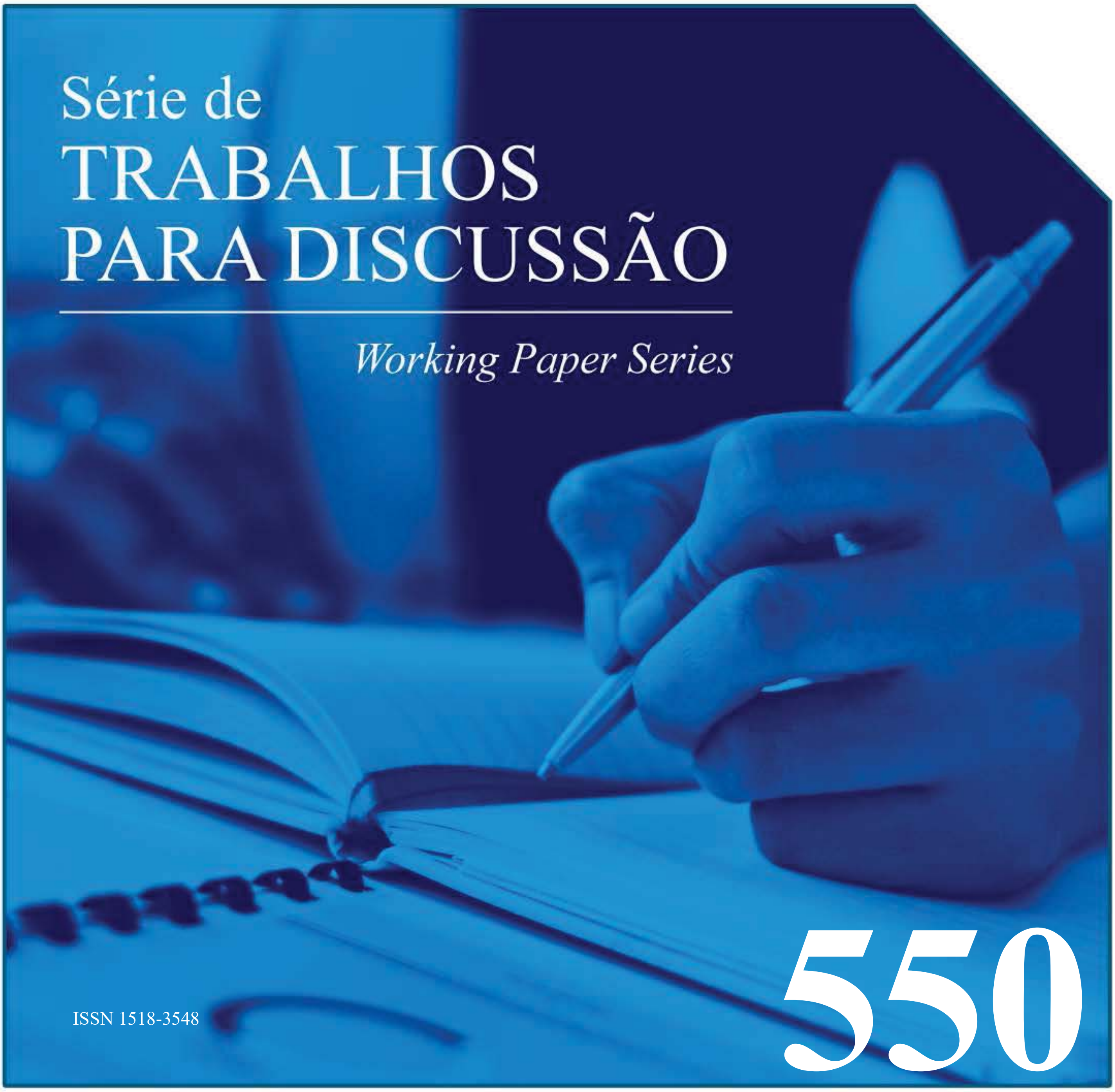

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High Lending Interest Rates in Brazil: cost or concentration?

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Sumário Não Técnico

As taxas de juros dos empréstimos bancários no Brasil são uma das mais altas do mundo. Essa particularidade brasileira tem sido objeto de vários estudos e também é discutida rotineiramente na mídia. Alguns argumentam que isso reflete a falta de competição bancária, já que o sistema financeiro é altamente concentrado no país. Outros argumentam que isso se deve aos custos mais elevados, dado, por exemplo, que os bancos perdem quase todo o dinheiro emprestado em caso de inadimplência no Brasil, enquanto em outros países eles conseguem recuperar uma parcela significativa. Afinal, é o custo o principal determinante ou é a concentração? É a essa pergunta que busco responder neste artigo, que analisa o mercado de crédito com recursos livres, em que as taxas de juros e o valor emprestado são livremente escolhidos pelos bancos. Para isso, inicialmente estimo um modelo econômico aplicado ao mercado bancário usando dados de março de 2011 a maio de 2019. Como esperado, eu encontro que (i) taxas de juro de empréstimo mais elevadas diminuem o valor que as empresas e famílias querem tomar emprestado e (ii) um sistema financeiro mais concentrado aumenta as taxas de empréstimo. Usando o modelo estimado, procuro responder à questão de pesquisa por meio de exercícios contrafactuais. Mais especificamente, calculo quais seriam as taxas de empréstimo se alguns de seus determinantes fossem alterados para os valores observados em outros países sul-americanos. Eu encontro que as taxas de empréstimo mais altas no Brasil frente a outros países da América do Sul entre 2012 e 2016 podem ser explicadas por (i) IOF, (ii) alto nível da taxa Selic, (iii) maior inadimplência, (iv) menor capacidade de recuperação de recursos perdidos por inadimplência (ou, em termos técnicos, menor taxa de recuperação) e (v) sistema financeiro mais concentrado. Esse resultado é determinado principalmente pelos componentes de custo, uma vez que as variações do IOF, da taxa Selic, da inadimplência e da taxa de recuperação respondem por 89% da redução da taxa de juros no cenário simulado. Em termos de *spread* bancário, que é essencialmente a diferença entre a taxa de empréstimo e a taxa Selic e é uma medida comum de margem dos bancos, os custos explicam 78% da queda. Portanto, a importância da concentração é apenas de segunda ordem: as taxas de empréstimo mais elevadas no Brasil são em grande parte uma história de custos mais altos dos bancos.

Non-technical Summary

Interest rates charged by banks on loans in Brazil are one of the highest in the world. This Brazilian idiosyncrasy has been subject of several studies and is also routinely discussed in the media. Some argue this reflects lack of competition among banks, as the financial system is highly concentrated in the country. Others argue this is due to higher costs, given, for instance, that banks lose almost all the money lent in the event of default in Brazil, while in other countries they can recover a significant portion. After all, is it cost the main determinant or is it concentration? This is the question I seek to answer in this paper, which analyses the nonearmarked market, where the interest rates and the amount lent are freely chosen by banks. For this purpose, I initially estimate an economic model applied to the banking market using data from March 2011 to May 2019. As expected, I find (i) higher lending interest rates decrease the value firms and households want to borrow and (ii) a more concentrated financial system increases the lending rates. Using the estimated model, I try to answer the research question, through counterfactual exercises. More specifically, I calculate what the lending rates would be if some of their determinants were changed to fit the observed in other American countries. I find the higher lending rates in Brazil compared to other South American countries between 2012 and 2016 can be explained by (i) IOF tax, (ii) high level of the Selic interest rate, (iii) higher delinquency, (iv) less capability to recover resources lost due to default (or, in technical terms, lower recovery rate), and (v) more concentrated financial system. This result is mainly driven by the cost components, as the changes in IOF tax, Selic rate, delinquency and recovery rate account for 89% of the lending rate reduction in the simulated scenario. In terms of bank interest spread, which is essentially the difference between the lending rate and the Selic rate and is a common measure of banks' margin, costs explain 78% of the decrease. Therefore, concentration is only of second-order importance: the higher lending rates in Brazil are largely a story of higher banks' cost.

High Lending Interest Rates in Brazil: cost or concentration?*

Thiago Trafane Oliveira Santos**

Abstract

Why are lending interest rates so high in Brazil? This paper seeks to answer this question through counterfactual exercises, which are simulated using a Cournot model. The proposed model is estimated for the nonearmarked market between March 2011 and May 2019. The estimates of the price elasticities of demand are always negative and statistically significant, implying the markup increases with concentration in my model. Using the estimated model, I performed counterfactual exercises, finding the higher lending rates in Brazil vis-à-vis other South American countries between 2012 and 2016 can be explained by (i) IOF tax, (ii) high level of the risk-free interest rate, (iii) higher probability of default, (iv) lower recovery rate, and (v) more concentrated financial system. This result is mainly driven by the cost components, as the changes in IOF tax, risk-free interest rate, recovery rate, and probability of default account for 89% of the optimal lending rate reduction in the counterfactual scenario. Or, in terms of optimal bank interest spread, costs explain 78% of the decrease. Therefore, concentration is only of second-order importance: the higher lending rates in Brazil are largely a story of higher banks' costs.

Keywords: lending interest rates, Brazil, taxes, reserve requirements, probability of default, recovery rate, concentration, risk-free interest rate.
JEL Classification: C22, E43, G21, L13.

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1 Introduction

Why are lending interest rates so high in Brazil? This paper seeks to answer this question through counterfactual exercises, which are simulated using a Cournot model applied to the nonearmarked credit market. The results indicate the higher lending rates in Brazil *vis-à-vis* other South American countries between 2012 and 2016 can be explained by (i) IOF tax, (ii) high level of the risk-free interest rate, (iii) higher probability of default, (iv) lower recovery rate, and (v) more concentrated financial system. This result is mainly driven by the cost components, as the changes in IOF tax, risk-free interest rate, recovery rate, and probability of default account for 89% of the optimal lending rate reduction in the counterfactual scenario. Or, in terms of optimal bank interest spread, the costs components explain 78% of the decrease. Therefore, concentration is only of second-order importance: the higher lending rates in Brazil are largely a story of higher banks' costs.

The Brazilian lending interest rates have been object of several studies over the last decades[1], since the country has been leading international rankings of credit cost (Nakane (2006)). This literature can be classified into five different groups. First, papers that use general equilibrium models to evaluate the high credit cost in Brazil. To the best of my knowledge, Souza-Sobrinho (2010) is the only paper of this kind[2], whose main conclusion is reserve requirements and earmarked loans account for about half of the spread in the model and a third of the actual spread. According to the author, this result is mainly driven by the impact of earmarked loans.

Second, papers that deal with accounting decompositions of the lending rate or the bank interest spread. This approach was originally proposed in Central Bank of Brazil (1999) and became a very important instrument to understand the high credit cost in Brazil, being continually updated over the years[3]. Central Bank of Brazil (2020) presents the latest version of this methodology, which presents a decomposition of the average interest rate of all active operations in the credit portfolio of the financial system. Regarding the nonearmarked market, the average results for 2017-2019 show the main component is the funding cost (28.3% of the lending rate), followed by delinquency (22.7%), administrative expenses (19.4%), taxes and Credit Guarantor Fund (15.4%), and financial margin (14.2%). In the earmarked market, in which most operations are subject to interest rate

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[1]For a survey of this literature, see Zeidan (2020).

[2]Alencar and Nakane (2004), D'erasmo (2016), and Capeleti et al. (2018) also develop general equilibrium models to study the credit market in Brazil, but they do not focus in evaluating the high credit cost issue.

[3]These accounting decompositions proposed by the Central Bank of Brazil are not the only ones. Reis Júnior et al. (2013) present an excelent review of this literature, including regarding the evolution of Central Bank's methodology, when they also propose an alternative accounting decomposition for the bank interest spread.

caps[4], the main component is still the funding cost (62.9% of the lending rate), followed by delinquency (26.7%), administrative expenses (10.9%), financial margin (0.3%), and taxes and Credit Guarantor Fund (-0.7%). Interesting to note financial margin is the smallest component in the nonearmarked market, which is supportive to the view that lack of competition among banks is not the main reason behind the high credit cost in Brazil. Also interesting is the fact that financial margin share in the earmarked lending rate is essentially zero (0.3%), against a share of 14.2% in the nonearmarked market, which is consistent with cross subsidization.

The third group consists of empirical works that use panel data to identify the determinants of lending rates or bank interest spreads, mostly using the methodology of Ho and Saunders (1981) (Afanasieff et al. (2002), Bignotto and Rodrigues (2005), Dantas et al. (2012), Manhiça and Jorge (2012), Jorgensen and Apostolou (2013), Da Silva and Pirtouscheg (2015), Almeida and Divino (2015), da Silva et al. (2016), Valente et al. (2018)). Fourth, empirical papers that evaluate specifically the impact of a more concentrated banking market on lending rates, bank spreads or competition[5] (Tonooka and Koyama (2003), de Araújo et al. (2006), Martins (2012), da Silva (2014), Joaquim and Van Doornik (2019)). Fifth, papers that assess specifically whether bank interest spreads are proportional to the funding rate (Nakane (2006), World Bank (2006)).

The empirical evidence provided by this third group of papers are very mixed, which makes it difficult to obtain a clear diagnosis of the high credit cost issue. In any case, when these last three groups are taken together, two important empirical facts arise. First, most papers that evaluate the market concentration effect find a more concentrated market increases lending rates, increases bank spreads or lowers competition (de Araújo et al. (2006), Martins (2012), Dantas et al. (2012), Manhiça and Jorge (2012), Almeida and Divino (2015), Joaquim and Van Doornik (2019)), although there is some contradictory evidence (Tonooka and Koyama (2003), da Silva (2014)). Second, most papers that evaluate the impact of the funding rate[6] on the bank interest spread find a positive effect (Afanasieff et al. (2002), Bignotto and Rodrigues (2005), Nakane (2006), World Bank (2006), Manhiça and Jorge (2012), da Silva et al. (2016), Castor and Ribeiro (2018))[7], even though some papers do not find a significant impact (Dantas et al. (2012), Almeida and Divino (2015)).

In this paper, I investigate the high lending rate using a different approach from those of the literature, based on a partial equilibrium economic model. More specifically, I choose to use a Cournot model, for three reasons. First, market concentration and firms'

[4]For a detailed description of the earmarked credit market in Brazil, see Pazarbasioglu et al. (2017).

[5]Measured, for instance, by the H-statistic of Panzar and Rosse (1987) or the Boone (2008) indicator.

[6]As proxied, for instance, by the Selic rate or the DI rate.

[7]This result is also found in other countries (Turgutlu (2010) for the Turkish banking industry; Almarzoqi and Naceur (2015) for Caucasus and Central Asia; Gelos (2006) for a world database with 85 countries).

markup are positive related in a Cournot model, being consistent with the first empirical fact just highlighted. Second, firms' markup is multiplicative with respect to the marginal cost in a Cournot model, which is consistent with a positive relation between funding rate and bank interest spread (second empirical fact)[8]. Third, it seems natural to assume firms compete on quantity when their products are essentially homogeneous.

The structure of the Cournot model is very simple, as the model is static, banks are quite similar, being all risk neutral, and features of the credit operation other than quantity and interest rate (e.g. maturity, duration and probability of default) are assumed to be exogenous. To reconcile a static model with the eminently dynamic feature of credit operations, where payments occur only in the future, I consider risk-neutral banks maximize the present value of the expected profit. By doing that, I can evaluate more accurately the economic costs of lending money. For instance, as default occurs in principal and interest payments, not directly in interest rates, this approach allows a more precise estimate of nonperforming loans (NPLs) cost. Furthermore, given this approach, I can consider specific details of taxes calculation in Brazil. Therefore, although the structure of the Cournot model is quite simple, the model is very rich in terms of the treatment given to banks' cost. As a consequence, the optimal lending rate expression I obtain in this model is quite rich as it links the optimal rate with several traditional determinants of the lending rate (taxes, risk-free interest rate, probability of default, recovery rate, reserve requirements, market concentration, among others). To the best of my knowledge, this approach is new in the literature.

I use two different specifications of the model. In the first one, all banks are profit maximizers, meaning their behavior are consistent with the optimal lending interest rate expression. In the second one, public banks are allowed to deviate persistently from the optimal behavior as the price elasticity of demand considered by them could be different from true or actual elasticity.

Using monthly data from March 2011 to May 2019, I estimate these two specifications of the model applying maximum likelihood and the Kalman filter. To do that, I assume the observable lending rate is equal to the optimal rate plus a random shock. The inverse of the price elasticities of demand are treated as state variables that follow random walk processes, which required the linearization of the aggregated markup of each specification. This linearization occurs around the sample average of the estimated markup. Hence, the estimation is done iteratively. The actual lending rate considered is the marginal lending rate for nonearmarked credit operations, since it is more suitable for a model

[8]World Bank (2006) presents three possible explanations for a bank interest spread that is proportional to the funding rate and one is exactly firms with market power. They show this result using a Bertrand model with differentiated products, but the same result can be obtained in other competition models, as is the case of a Cournot model. The other two possible explanations presented by World Bank (2006) are: (i) regulatory or operational costs that are proportional to the deposits (reserve requirements, for example) combined with a zero profit condition, and (ii) adverse selection considerations.

that evaluates the economic decision of lending money.

The estimated aggregated price elasticity of demand is essentially the same in both specifications. Furthermore, these estimated elasticities are always negative and statistically significant, meaning the markup increases with concentration. Another interesting issue is that between 2012 and 2014 the demand became more elastic, lowering the markup, which is consistent with the use of public banks to reduce the lending rates in the same period.

Using the optimal rate expressions, I investigate the high lending interest rate in Brazil through counterfactual simulations. These simulations are essentially conditional expectation exercises: I evaluate what is expected to happen with the lending rate when I alter one or more of its determinants, maintaining the other determinants unchanged. In this sense, I consider only the direct effect of each determinant, ignoring possible indirect effects through other determinants. This recommends caution in analyzing the results since indirect effects can be very important.

Therefore, my goal here is not to identify the specific reforms or measures that could lower the Brazilian lending rate. I am at an earlier stage of investigation: I want to identify whether the lending rate in Brazil is consistent with the international standard when we control for some of its determinants. By doing that, I try to draw a general picture about the high credit cost issue in Brazil, which could be useful for future investigations regarding the specific reforms or measures.

I initially use the estimated model to simulate three scenarios: (i) instantly recognition of principal losses due to NPLs for profit tax purpose, (ii) null taxes over profit and revenue, and (iii) no reserve requirements. The results show these variables are of little importance in explaining the high credit cost in Brazil, at least directly.

To evaluate what can explain it, I use as a benchmark a group of six South American countries (Argentina, Chile, Colombia, Paraguay, Peru, and Uruguay) during the 2012-2016 period, which all have lower credit cost than Brazil. In fact, the Brazilian interest rate is almost two times the median rate of this benchmark group. In this context, I simulate a new scenario, eliminating the IOF tax, which seems to be a Brazilian idiosyncrasy, and setting the risk-free interest rate, the probability of default, the recovery rate, and the market concentration to the median values of these six countries. I find this counterfactual rate is close to half of the observable Brazilian lending rate, being essentially equal to the median rate of the benchmark group. Thus, these five variables can explain all the higher lending rate in Brazil over the 2012-2016 period.

Using a natural logarithm approximation of the interest rate, I decompose the overall decrease in the optimal lending rate. I find this reduction is essentially due to costs, as they explain 89% of the difference. The costs components are the most important drivers even in terms of bank interest spread, accounting for 78% of the reduction. Hence, concentration explains only 11% and 22% of the decrease in the optimal lending rate and in the optimal spread, respectively, indicating higher lending rates in Brazil are essentially a result of higher banks' cost. This is consistent with the accounting decomposition, as it shows costs account for most of the nonearmarked lending interest rate, with financial margin share in the lending rate being equal to only 14.2% for the 2017-2019 period (Central Bank of Brazil (2020)). It is also consistent with Nakane and Costa (2005), as they find the Brazilian Lerner index is much closer to the international standard than the absolute bank interest spread, although it is still higher.

The remainder of the paper proceeds as follows. Section 2 uses a Cournot model to obtain the optimal lending interest rate and the aggregated markup in the two specifications of the model. Section 3 discusses the data and the methodology used to estimate both specifications. Section 4 shows the estimated price elasticities of demand. Section 5 estimates counterfactual lending rates in order to identify the main determinants of the high credit cost in Brazil. Finally, Section 6 concludes.

2 Model

2.1 Profit at present value

The optimal lending interest rate is obtained using a static Cournot model. Risk-neutral banks produce a homogeneous product and, in each period, choose their quantities simultaneously in order to maximize the present value of the expected profit, taking the quantities set by its competitors as given. Features of the credit operation other than quantity and interest rate are assumed to be exogenous.

In formal terms, the objective function is

$$\begin{aligned} \Pi_{it} &= (EPV_{it} - V_{it}) - VC_{it} - FC \\ \Pi_{it} &= (EPV_{it}/V_{it} - VC_{it}/V_{it})\, V_{it} - FC_{it} \end{aligned} \tag{1}$$

where Π_{it} is the net profit, at present value, of bank i at time t, EPV_{it} is the expected present value of the stream of cash flows from the credit operations of bank i granted at time t, VC_{it} is the present value of the variable cost net of taxes of the credit operations of bank i granted at time t, FC_{it} is the fixed cost net of taxes of bank i at time t, and V_{it} is the value of credit operations of bank i granted at time t. The expected present value (EPV_{it}) is net of the costs with taxes and nonperforming loans (NPLs). I assume the revenue from the credit operations granted in previous periods are not affected by banks' decisions in this period. Thus, they can be treated as fixed costs. Or, alternatively, one can think the banks securitize their credit operations and sell them in each period, receiving EPV_{it}. Furthermore, the variable cost at present value (VC_{it}) includes only costs related to the funding of the credit operation. Other costs like administrative expenses are treated as fixed costs in this model. The essentially null accounting profit in earmarked operations (Central Bank of Brazil (2020)), which may suggest negative economic profit in this market, could be seen as another potential source of fixed costs.

2.2 Expected present value per monetary unit of credit

The expected present value (EPV_{it}) in Brazil could be calculated using

$$EPV_{it} = \sum_{k=1}^{m_t} \left[\frac{P_{it}^k (1 - \gamma_t \phi_t) + I_{it}^k (1 - \gamma_t \phi_t - \tau_t^r)(1 - \tau_t^p)}{(1 + f_t)^k} + \frac{P_{it}^k \gamma_t \phi_t \tau_t^p}{(1 + f_t)^{k + \widetilde{m}_t}} \right] \qquad (2)$$

where P_{it}^k is the principal payment at $t + k$ of the credit operations of bank i granted at time t, I_{it}^k is the interest payment at $t+k$ of the credit operations of bank i granted at time t, γ_t is the probability of default at time t, ϕ_t is the loss given default (LGD) at time t, τ_t^r is the PIS/Cofins tax rate banks pay over revenues at time t, τ_t^p is the IRPJ/CSLL tax rate over accounting gross profit at time t, f_t is the funding rate or discount rate net of PIS/Cofins and IRPJ/CSLL taxes at time t, m_t is the maturity of the credit operations at time t, and $\widetilde{m}_t$ is the time to recover profit tax paid over defaulted principal installments at time t. Note the probability of default γ_t, the LGD ϕ_t, and the maturity m_t are assumed to be the same for all banks.

From equation (2), expected present value EPV_{it} is the present value of the principal and interest payments net of taxes and NPLs costs. The first term in the right-hand side of the equation shows the losses due to NPLs occur both in principal and interest payments. Regarding the taxes, since principal payments are not considered as banks' revenue, the tax base of the PIS/Cofins tax is the interest payments, including the share banks do not receive due to NPLs. Similarly, the profit tax (IRPJ/CSLL tax) is applied only over interest payments.

Although the principal payments are not revenue, the principal losses due to NPLs affect banks' results. However, in Brazil these losses are not recognized instantly for profit tax purposes. This is done only after $\widetilde{m}_t$ periods (second term in the right-hand side of the equation). I assume, by simplification, this value has no monetary correction.

Assuming banks use the same amortization methods and given the maturity and the interest rate are the same for all banks in this model, principal payment to credit value ratio (P_{it}^j / V_{it}) and interest payment to credit value ratio (I_{it}^k / V_{it}) are invariant across banks. In this context, to simplify the presentation, I define $PPV_t = \sum_{j=1}^{m} \frac{P_{it}^j / V_{it}}{(1+f_t)^j}$ and $IPV_t =$

$\sum_{k=1}^{m} \frac{I_{it}^k / V_{it}}{(1+f_t)^k}$. Thus, equation (2) can be rewritten as

$$\frac{EPV_{it}}{V_{it}} = \frac{PPV_t \gamma_t \phi_t \tau_t^p}{(1+f_t)^{\widetilde{m}}} + (1 - \gamma_t \phi_t) PPV_t + (1 - \gamma_t \phi_t - \tau_t^r)(1 - \tau_t^p) IPV_t \quad (3)$$

In Appendix A I show $IPV_t = \left(\widetilde{i}_t / f_t\right)(1 - PPV_t)$, where $\widetilde{i}_t$ is the lending interest rate net of IOF tax of credit operations granted at time t. Thus, substituting in (3),

$$\frac{EPV_{it}}{V_{it}} = PPV_t (1 - \gamma_t \phi_t A_t) + (1 - \gamma_t \phi_t - \tau_t^r)(1 - \tau_t^p)(1 - PPV_t) \left(\frac{\widetilde{i}_t}{f_t}\right) \quad (4)$$

where $A_t = 1 - \frac{\tau_t^p}{(1+f_t)^{\widetilde{m}_t}}$.

I would like to have in equation (4) the lending rate that includes all costs (i_t) instead of $\widetilde{i}_t$, since from the borrower's point of view this is the relevant interest rate. By definition,

$$V_{it} = \sum_{k=1}^{m} \frac{P_{it}^k + I_{it}^k}{\left(1 + \widetilde{i}_t\right)^k} \quad (5)$$

$$V_{it} = V_{it}\tau_t^c + \sum_{k=1}^{m} \frac{P_{it}^k + I_{it}^k}{(1 + i_t)^k} \leftrightarrow V_{it}(1 - \tau_t^c) = \sum_{k=1}^{m} \frac{P_{it}^k + I_{it}^k}{(1 + i_t)^k} \quad (6)$$

where τ_t^c is the IOF tax rate at time t, which is applied over the value of the credit operation and is paid when the credit is granted.

Given equations (5) and (6), I use a first-order Macaulay approximation (Alps (2017)):

$$V_{it}(1 - \tau_t^c) \approx V_{it} \left(\frac{1 + \widetilde{i}_t}{1 + i_t}\right)^{d_t}$$

$$\widetilde{i}_t \approx (1 + i_t)(1 - \tau_t^c)^{1/d_t} - 1 \quad (7)$$

where $d_t = \sum_{k=1}^{m} \left[\frac{\left(P_{it}^k + I_{it}^k\right) / \left(1 + \widetilde{i}_t\right)^k}{V_{it}}\right] k$ is the Macaulay duration calculated using the stream of cash flows net of IOF tax from the credit operations granted at time t[9].

Thus, substituting equation (7) in (4),

$$\frac{EPV_{it}}{V_{it}} = PPV_t (1 - \gamma_t \phi_t A_t) + \left(\frac{B_t}{f_t}\right)(1 - \tau_t^c)^{1/d_t} \left[(1 + i_t) - (1 - \tau_t^c)^{-1/d_t}\right] \quad (8)$$

where $B_t = (1 - \gamma_t \phi_t - \tau_t^r)(1 - \tau_t^p)(1 - PPV_t)$.

[9]This approximation can also be done using the modified duration. However, Alps (2017) shows the Macaulay approximation is more accurate whenever the cash flow amounts are positive, as with credit operations.

2.3 Variable cost per monetary unit of credit

When a bank lends money, it increases its assets (credit) and its liabilities (deposits) by the same amount. As a result of the increase in deposits, the bank should rise its reserves in order to maintain the reserve to deposit ratio at the required level. Since the reserve to deposit ratio is typically lower than one, the bank would need to increase its reserves by just a share of the new credit.

This very usual description of the credit creation, although accurate, refers only to the beginning of the process. In fact, these new deposits could then be transformed into other liabilities within the bank or transferred to other banks.

The resources that are not transferred to other banks cause an increase in bank's liability and reserves, as one can see from the previous description of the credit creation. As a result, the cost of this share of the new resources is the sum of the liability cost and the cost of reserves, both being affected by the final composition of the bank's new liabilities. The liability cost depends on the interest rate the bank is expecting to pay on the new liabilities. The cost of reserves is the product between the quantity of new reserves, determined by the required reserve to liability ratio, and the cost of the reserves net of their remuneration. I use the discount rate f_t as a proxy for the cost of the reserves per currency unit.

The resources that are transferred to other banks cause an equivalent reduction in bank's reserves, resulting in a cost to the bank as it needs to restore the reserves. Thus, the cost of this share of the new resources is the cost of reserves f_t.

These costs per currency unit apply over the outstanding balance. After all, as the credit is amortized, the increase in bank's assets, which is the factor that generated the costs, starts to revert. Furthermore, as the amortization represents an use of principal payments (P_{it}^j), they should also be included.

Based on the discussion above, one can obtain the variable cost of lending money at present value VC_{it}, where I use the market share of the bank (s_{it}) as a proxy for the share of the new deposits that is expected to remain within the bank:

$$
\begin{aligned}
VC_{it} &= \sum_{j=1}^{m} \frac{\left(V_{it} - \sum_{k=1}^{j-1} P_{it}^k\right)\left\{s_{it}\left[i_t^{lb} + \rho_t\left(f_t - i_t^r\right)\right] + (1-s_{it})f_t\right\} + P_{it}^j}{(1+f_t)^j} \\
\frac{VC_{it}}{V_{it}} &= PPV_t + \left\{s_{it}\left[i_t^{lb} + \rho_t\left(f_t - i_t^r\right)\right] + (1-s_{it})f_t\right\}\left[\sum_{j=1}^{m} \frac{\sum_{k=j}^{m} P_{it}^k / V_{it}}{(1+f_t)^j}\right] \qquad (9)
\end{aligned}
$$

where i_t^{lb} is the liability cost, ρ_t is the required reserve to liability ratio, i_t^r is the reserves remuneration rate, $\left(V_{it}-\sum_{k=1}^{j-1}P_{it}^{k}\right)$ is the outstanding balance, $\left[i_t^{lb}+\rho_t\left(f_t-i_t^r\right)\right]$ is the cost of the new resources that remain within the bank, and f_t is the cost of the resources that are transferred to other banks. Note that I use, in the last part, $\sum_{k=1}^{m}P_{it}^{k}=V_{it}$.

From equation (9),

$$\begin{aligned}
\frac{VC_{it}}{V_{it}} &= PPV_t+\left\{s_{it}\left[i_t^{lb}+\rho_t\left(f_t-i_t^r\right)\right]+(1-s_{it})f_t\right\}\left[\sum_{k=1}^{m}\frac{P_{it}^k}{V_{it}}\sum_{j=1}^{k}\frac{1}{(1+f_t)^j}\right]\\
\frac{VC_{it}}{V_{it}} &= PPV_t+\left\{\frac{s_{it}\left[i_t^{lb}+\rho_t\left(f_t-i_t^r\right)\right]+(1-s_{it})f_t}{f_t}\right\}\left\{\sum_{k=1}^{m}\frac{P_{it}^k}{V_{it}}\left[1-\frac{1}{(1+f_t)^k}\right]\right\}\\
\frac{VC_{it}}{V_{it}} &= 1-C_t s_{it} \qquad (10)
\end{aligned}$$

where $C_t=(1-PPV_t)\left[1-\frac{i_t^{lb}+\rho_t(f_t-i_t^r)}{f_t}\right]$.

2.4 Optimal lending interest rate

Using equation (1), one can calculate the derivative of the profit (Π_{it}) with respect to the value of credit (V_{it}):

$$\frac{\partial\Pi_{it}}{\partial V_{it}}=\left(\frac{EPV_{it}}{V_{it}}-\frac{VC_{it}}{V_{it}}-1\right)+V_{it}\left(\frac{\partial EPV_{it}/V_{it}}{\partial V_{it}}-\frac{\partial VC_{it}/V_{it}}{\partial V_{it}}\right) \qquad (11)$$

From equation (8),

$$\begin{aligned}
V_{it}\left[\frac{\partial EPV_{it}/V_{it}}{\partial V_{it}}\right] &= V_{it}\left(\frac{B_t}{f_t}\right)(1-\tau_t^c)^{1/d_t}\left[\frac{\partial\left(1+i_t\right)}{\partial V_t}\frac{\partial V_t}{\partial V_{it}}\right]\\
V_{it}\left[\frac{\partial EPV_{it}/V_{it}}{\partial V_{it}}\right] &= \left(\frac{B_t}{f_t}\right)(1-\tau_t^c)^{1/d_t}\left(1+i_t\right)\left(-\frac{s_{it}}{|\varepsilon_t^d|}\right) \qquad (12)
\end{aligned}$$

where V_t is the value of credit operations of all banks granted at time t and $\frac{1}{|\varepsilon_t^d|}=-\frac{\partial(1+i_t)}{\partial V_t}\left(\frac{V_t}{1+i_t}\right)$ is the inverse of the absolute value of the price elasticity of demand at time t.

From equation (10),

$$\begin{aligned}
V_{it}\left[\frac{\partial VC_{it}/V_{it}}{\partial V_{it}}\right] &= -V_{it}C_t\left(\frac{V_t-V_{it}}{V_t^2}\right)\\
V_{it}\left[\frac{\partial VC_{it}/V_{it}}{\partial V_{it}}\right] &= -C_t\left(s_{it}-s_{it}^2\right) \qquad (13)
\end{aligned}$$

Thus, substituting equations (8), (10), (12) and (13) in (11), one can obtain the optimal lending rate (i_t^*) by making $\frac{\partial \Pi_{it}}{\partial V_{it}} = 0$:

$$\begin{aligned}
(1+i_t^*) &= \frac{1 - PPV_t\left(1-\gamma_t\phi_t A_t\right) - C_t\left(2s_{it} - s_{it}^2\right) + \left(\frac{B_t}{f_t}\right)}{\left(\frac{B_t}{f_t}\right)\left(1-\tau_t^c\right)^{1/d_t}\left(1-\frac{s_{it}}{|\varepsilon_t^d|}\right)} \\
(1+i_t^*) &= MC_{it}\left(\frac{1}{1-\frac{s_{it}}{|\varepsilon_t^d|}}\right) \qquad (14)
\end{aligned}$$

where $MC_{it} = \left[\frac{1}{(1-\tau_t^c)^{1/d_t}}\right]\left\{1 + f_t\left[\frac{1-PPV_t(1-\gamma_t\phi_t A_t)-C_t\left(2s_{it}-s_{it}^2\right)}{(1-\gamma_t\phi_t-\tau_t^r)\left(1-\tau_t^p\right)(1-PPV_t)}\right]\right\}$ is the total marginal cost of the bank i at time t and $\left(1-\frac{s_{it}}{|\varepsilon_t^d|}\right)^{-1}$ is the bank's markup. Note that in the definition of MC_{it} I use $B_t = \left(1-\gamma_t\phi_t-\tau_t^r\right)\left(1-\tau_t^p\right)\left(1-PPV_t\right)$.

I use two different specifications of the model. In the standard specification, I assume all banks are maximizers, being consistent with equation (14). In this case, the aggregated markup, which is the markup over the weighted average total marginal cost, is

$$\begin{aligned}
\sum_{i=1}^{n} s_{it}\left(1+i_t^*\right)\left(1-\frac{s_{it}}{|\varepsilon_t^d|}\right) &= \sum_{i=1}^{n} s_{it} MC_{it} \\
(1+i_t^*) &= MC_t\left(\frac{1}{1-\frac{HHI_t}{|\varepsilon_t^d|}}\right) \qquad (15)
\end{aligned}$$

where $MC_t = \left[\frac{1}{(1-\tau_t^c)^{1/d_t}}\right]\left\{1 + f_t\left[\frac{1-PPV_t(1-\gamma_t\phi_t A_t)-C_t(2HHI_t-HHI3_t)}{(1-\gamma_t\phi_t-\tau_t^r)\left(1-\tau_t^p\right)(1-PPV_t)}\right]\right\}$ is the aggregated total marginal cost[10], n is the number of banks, $HHI_t = \sum_{i=1}^{n} s_{it}^2$ is the Herfindahl-Hirschman concentration index (HHI), and $HHI3_t = \sum_{i=1}^{n} s_{it}^3$.

In the alternative specification, public banks can deviate from the optimal behavior. This is incorporated by assuming these banks consider a different price elasticity of demand. In formal terms, their markup is $\left(1-\frac{s_{it}}{\theta_t|\varepsilon_t^d|}\right)^{-1}$, implying public banks' markup are lower than the optimal level when $\theta_t > 1$. Hence, the aggregated markup is

[10] I assume, except from the market share s_{it}, the other determinants of the marginal cost MC_{it} do not vary across banks. Naturally, this is a simplification. More generally, MC_t as defined here is only a first-order approximation of the weighted average marginal costs ($\overline{MC_t}$). A simple approach to evaluate the approximation error is estimating the model under the assumption $\overline{MC_t} \approx \alpha + \beta MC_t$ or $\ln \overline{MC_t} \approx \alpha + \beta \ln MC_t$. In empirical exercises, I do not find strong statistical evidence to reject $\alpha = 0$ and $\beta = 1$. So, I choose to maintain $\overline{MC_t} \approx MC_t$.

$$
\begin{aligned}
\left[\sum_{i\in P} s_{it}\left(1-\frac{s_{it}}{|\varepsilon_t^d|}\right)+\sum_{j\in G} s_{jt}\left(1-\frac{s_{jt}}{\theta_t|\varepsilon_t^d|}\right)\right](1+i_t^*) &= \sum_{i=1}^{n} s_{it} MC_{it} \\
\left[1-(s_t^p)^2\left(\frac{HHI_t^p}{|\varepsilon_t^d|}\right)-(s_t^g)^2\left(\frac{HHI_t^g}{\theta_t|\varepsilon_t^d|}\right)\right](1+i_t^*) &= MC_t \\
(1+i_t^*) &= MC_t\left(\frac{1}{1-\frac{HHI_t}{\left|\overline{\varepsilon_t^d}\right|}}\right) \quad (16)
\end{aligned}
$$

where P and G are the sets of private and public banks, respectively, s_t^p and s_t^g are the market share of private and public banks at time t, respectively, $s_{it}^p = \frac{s_{it}}{s_t^p}$, $s_{jt}^g = \frac{s_{jt}}{s_t^g}$, $HHI_t^p = \sum_{i\in P}\left(s_{it}^p\right)^2$ and $HHI_t^g = \sum_{j\in G}\left(s_{jt}^g\right)^2$ are the HHI's for private banks and public banks markets at time t, respectively, and $\left|\overline{\varepsilon_t^d}\right|^{-1} = \left[\frac{\left(s_t^p\right)^2 HHI_t^p}{HHI_t}\right]|\varepsilon_t^d|^{-1} + \left[\frac{\left(s_t^g\right)^2 HHI_t^g}{HHI_t}\right]\left(\theta_t|\varepsilon_t^d|\right)^{-1}$ is the inverse of the absolute value of the aggregated price elasticity of demand adjusted for non-optimal behavior of public banks[11].

Naturally, in both specifications (equations (15) and (16)), the optimal lending rate i_t^* increases with marginal cost MC_t and with the markup. This markup is multiplicative, which is consistent with the idea the bank interest spread is affected by the funding rate, the second empirical fact presented in the introduction of the paper. As expected, a higher market concentration (HHI_t) implies a higher aggregated markup. But the effect depends on the aggregated elasticity of demand ($|\varepsilon_t^d|$ or $\left|\overline{\varepsilon_t^d}\right|$ for the standard and alternative specifications, respectively), since a higher elasticity implies a smaller effect of concentration on markup. In this context, the degree of competition, as measured by companies' markup as in the Lerner index, is a function not only of market concentration, but also of the elasticity, which depends, for example, on the availability of close substitutes to the domestic credit market (essentially, domestic capital markets and external funding sources)[12]. Thus, in this Cournot model, competition and concentration are not the same, as a country with a high concentration can have a low markup if its elasticity of demand is high, but changes in concentration tend to affect the degree of competition in a given country.

Regarding the marginal cost MC_t, a more concentrated market (HHI_t), which increases $2HHI_t - HHI3_t$[13], implies a lower MC_t if $C_t > 0$ and $(1-\gamma_t\phi_t-\tau_t^r) > 0$, as happens in Brazil. A sufficient but not necessary condition for $C_t > 0$ is the liability

[11] If $\theta_t = 1$, indicating all banks are profit maximizing, $\left|\overline{\varepsilon_t^d}\right| = |\varepsilon_t^d|$ as expected, since $HHI_t = \left(s_t^p\right)^2 HHI_t^p + \left(s_t^g\right)^2\left(HHI_t^g\right)$.

[12] As a result, measures aimed at developing the domestic capital markets can increase the degree of competition in the domestic credit market.

[13] See Appendix B for more details.

cost to be less than the discount rate ($i_t^{lb} < f_t$) and the reserves remuneration rate be close to the liability cost ($i_t^r \approx i_t^{lb}$), which is valid for the Brazilian case. To understand this result, assume $i_t^r = i_t^{lb}$ in equation (9). The cost per monetary unit of credit of the resources that remain within the bank is $(1-\rho_t)\, i_t^{lb} + \rho_t f_t$, while the cost of the resources that are transferred is f_t. Consequently, if $i_t^{lb} < f_t$, the resources that remain within the banks are cheaper, implying the MC_t should decrease with concentration. After all, in a more concentrated market the share of the resources expected to remain within each bank increases on average.

Thus, a higher market concentration increases the markup, but lowers the marginal cost. The net effect on the lending rate depends on the relative intensity of each effect. As shown in Section 5, at least for the Brazilian case, where C_t is close to zero, the effect on the markup is much stronger, implying a traditional positive impact of concentration on the lending rate.

It is easy to show the effects of the other variables on the marginal cost are the expected ones if, as in Brazil, $C_t\,(2HHI_t - HHI3_t) \approx 0$ and $(1 - \gamma_t\phi_t - \tau_t^r) > 0$. First, the marginal cost MC_t increases with the discount rate f_t, as the impact of changes in f_t on the present value of principal payments PPV_t and C_t is small. Second, the marginal cost MC_t also increases with the probability of default γ_t and the LGD ϕ_t, reflecting the increased NPLs cost. Third, a higher time to recover profit tax paid over defaulted principal installments $\widetilde{m}_t$ also implies a higher MC_t, as it increases the cost, at present value, of profit taxes.

Fourth, the marginal cost MC_t increases with a lower duration d_t. The effect of the duration occurs directly, on the term $(1-\tau_t^c)^{1/d_t}$, related to the IOF tax, but also indirectly through the present value of principal payments PPV_t, since a higher duration lowers this present value. The intuition is similar to the effect of NPLs on the lending rate analyzed in Box 8 of Central Bank of Brazil (2018): in a credit of higher duration the costs can be distributed over more periods, representing a lower share of the interest rate charged in each period. Fifth, MC_t increases with the required reserve to liability ratio ρ_t. This impact is larger the greater the rate differential between the discount rate and the reserves remuneration rate ($f_t -$ i_t^r), since it rises the opportunity cost of the holding reserves. As a consequence, a lower reserve remuneration rises the lending rate, as expected. This intensity also increases with $2HHI_t - HHI3_t$, as it rises the expected value of the reserve requirements over the credit operation. After all, a higher market concentration implies a higher probability of the resources remaining within the bank. In fact, if the market is competitive, meaning $2HHI_t - HHI3_t = 0$, the effect of ρ_t on the lending rate is null.

Finally, the marginal cost MC_t increases with the tax rates (τ_t^r, τ_t^p, and τ_t^c). In the case of the PIS/Cofins tax rate (revenue tax rate) τ_t^r and the IRPJ/CSLL tax rate (profit tax rate) τ_t^p, this is valid for a given discount rate f_t. After all, since f_t is also net of PIS/Cofins and IRPJ/CSLL taxes, higher τ_t^r and τ_t^p decrease f_t, lowering the lending rate. The net effects are not clear in principle, but one could expect small impacts. Intuitively, as these taxes are levied on revenue and also on variable cost, since the discount rate f_t is net of taxes, by considering these taxes we are basically multiplying the objective function by a constant, which should have no effect on firms' optimal decision. Similarly, the fixed costs do not affect the lending rate directly, as they do not alter the first-order condition of the optimization problem.

However, indirect effects can be very important. Taxes or fixed costs alter the net profit of banks, which could induce banks to enter or to leave the market, affecting market concentration and, then, the optimal lending rate. In this sense, the level of concentration does not reflect only variables more closely linked to the competition environment, notably the size of the barriers to entry or to leave the market, but also factors that affect the profit level. As a result, even a market without barriers to entry or to leave the market can have some steady-state concentration, since a markup higher than one can be necessary to ensure a null economic profit.

3 Data and estimation methodology

Given the regulated nature of the earmarked market of banking loans, where most of the operations are subject to interest rate caps, I estimate the model using data only from the nonearmarked market. The monthly sample starts in March 2011 and ends in May 2019, totalizing 99 observations. All variables were seasonally adjusted using Census X-13.

Usually, the aggregated or average lending rate is calculated as a weighted average of the lending rate of each credit product, where the weights are given by credit outstanding values. In fact, this method is the one used in the official data released by the Central Bank of Brazil. However, as discussed in Box 4 of Central Bank of Brazil (2018), the internal rate of return "[...] is, conceptually, the best methodology to express the average rate of a credit portfolio of transactions with distinct rates and maturities" and the use of weighted averages tend to overestimate the internal rate of return[14], especially if one aggregates using annual interest rates[15].

Thereby, I calculate the lending rate i_t as the internal rate of return of nonearmarked new credit operations, that is, it is the interest rate that makes the present value of the stream of cash flows from these credit operations equal the value of the credit granted.

[14]This bias of weighted averages is also shown in Box 9 of Central Bank of Brazil (2019).

[15]The weighted average lending rate would not be the same if one aggregates using monthly interest rates, annualizing only after aggregation, or aggregates using directly annual rates, given Jensen's inequality and the fact these changes in the compounding period are not linear transformations. Since in principle there are no ideal compounding period, this choice is essentially arbitrary and, as a consequence, the average rate is, to some extent, arbitrary. This fact supports, from a conceptual point of view, calculating a weighted average is not the best method for aggregating interest rates.

The streams of principal and interest payments, which are not observed, are obtained by assuming an amortization method for each nonearmarked credit product[16] and using credit outstanding value, maturity and lending interest rate data from the Central Bank of Brazil[17], similar to what is done in Box 9 of Central Bank of Brazil (2019)[18]. Based on the streams of cash flows, I also calculate the lending interest rate net of IOF tax $\widetilde{i}_t$ and, then, the Macaulay duration from the stream of cash flows net of IOF tax d_t.

The gross discount rate (f_t^{gr}) is the nonearmarked average funding rate calculated by the Central Bank of Brazil, which can be obtained by the difference between the average interest rate and average spread, both for nonearmarked new credit operations. Essentially, this rate is the risk-free interest rate at the average maturity of the credit operations.

Regarding the net discount rate f_t,

$$f_t = f_t^{gr} \left(1 - \tau_t^r\right) \left(1 - \tau_t^p\right) \tag{17}$$

Using f_t and the stream of principal payments obtained in the calculation of i_t, I calculate PPV_t. Regarding $\widetilde{m}_t$, based on law 9.430/96, I assume the time to recover profit tax over principal equals two years for collateralized loans and one year for uncollateralized loans[19]. As a consequence, by classifying each credit product loans as collateralized or uncollateralized (Appendix C), I obtain the time to recover of each product. Finally, I calculate $\widetilde{m}_t$ as the weighted average of these credit products' times, where the weights are given by credit outstanding values.

The liability cost i_t^{lb} is calculated as a weighted average:

$$\begin{aligned} i_t^{lb} &= \alpha_t^{sd} i_t^{sd} + \alpha_t^{dd} 0 + \left(1 - \alpha_t^{sd} - \alpha_t^{dd}\right) f_t \\ i_t^{lb} &= \alpha_t^{sd} i_t^{sd} + \left(1 - \alpha_t^{sd} - \alpha_t^{dd}\right) f_t \end{aligned} \tag{18}$$

where α_t^{sd} is the share of savings deposits in total liability, i_t^{sd} is the interest rate net of taxes in savings deposits, and α_t^{dd} is the share of demand deposits in total liability. The liability data and the gross interest rate in savings deposits ($i_t^{sd\_gr}$) are from the Central

[16] See Appendix C.

[17] The expanses with IOF tax and operating charges are treated separately, by assuming these expenses occur in the same month the credit is granted. Therefore, the interest rates used to calculate the cash flows under the assumed amortization methods are net of these two expenses.

[18] The main difference is that I seek to measure the aggregated lending interest rate for nonearmarked new credit operations (nonearmarked marginal lending rate), while the internal rate of return estimated in Box 9 of Central Bank of Brazil (2019) is a proxy for the aggregated lending rate of all active operations in the credit portfolio of the financial system, including earmarked operations and credit granted in previous periods.

[19] According to the law 9.430/96, given certain legal procedures, the following credits can be recorded as losses for profit tax purposes: 1) against a debtor declared insolvent, bankrupt or in bankruptcy; 2) without collateral, of value: a) up to R$ 5,000.00 (R$ 15,000.00 after October 2014), overdue for more than six months; b) above R$ 5,000.00 (R$ 15,000.00 after October 2014), overdue for more than one year; 3) with collateral, overdue for more than two years.

Bank of Brazil[20]. Following the calculation of f_t,

$$i_t^{sd} = i_t^{sd\_gr} \left(1 - \tau_t^r\right) \left(1 - \tau_t^p\right) \tag{19}$$

Similarly to the liability cost, the reserves remuneration rate is

$$\begin{aligned} i_t^r &= \beta_t^{sd} i_t^{sd} + \beta_t^{dd} 0 + \left(1 - \beta_t^{sd} - \beta_t^{dd}\right) f_t \\ i_t^r &= \beta_t^{sd} i_t^{sd} + \left(1 - \beta_t^{sd} - \beta_t^{dd}\right) f_t \end{aligned} \tag{20}$$

where β_t^{sd} and β_t^{dd} are the shares of the reserve requirements on savings deposits and demand deposits on total reserve requirement, respectively. The reserve requirement data are also from the Central Bank of Brazil.

From the data used to calculate the liability cost i_t^{lb} and the interest rate net of taxes in savings deposits i_t^{sd}, I obtain ρ_t as the total reserve requirement to total liability ratio. The concentration index HHI_t, $HHI3_t$ and the market share of private and public banks (s_t^p, and s_t^g, respectively) are calculated considering credit market data of commercial banks[21]. Using IF.data database of the Central Bank of Brazil, I calculate these variables on a quarterly basis[22]. The monthly series are obtained using quadratic interpolation.

Regarding the taxes, τ_t^c is the weighted average IOF tax rate on nonearmarked new credit operations. The IOF tax data come from the Central Bank of Brazil. The IRPJ/CSLL tax rate τ_t^p equals 45% between September 2015 and December 2019 and 40% otherwise. The PIS/Cofins tax rate τ_t^r equals 4.65% for all periods. The probability of default γ_t is the share of problematic assets among nonearmarked loans, as measured by the Central Bank of Brazil. The definition of problematic assets takes into account 90 days past due loans, restructured debt and "E to H" rated loans[23]. The LGD ϕ_t is calculated using the recovery rate of São Paulo from the Doing Business Report[24]. As this rate is reported annually, the monthly series is obtained using Catmul-Rom Spline interpolation.

[20] Total liability is the sum of savings deposits, demand deposits, time deposits, financial bonds, exchange bills, and others private securities.

[21] I also tested using data from two other segments of the Brazilian credit market: (i) banks or (ii) banks and non-banks financial institution. Besides level shifts, the elasticities estimated using the three credit segments are essentially the same. One can understand this result by comparing the concentration indexes of these segments, which are highly correlated (pairwise correlation is always higher than 0.995) but have different levels.

[22] I use data aggregated in terms of banking financial conglomerates, since individual banks data underestimate the banking concentration and overestimate the intensity of competition (Cardoso et al. (2016)).

[23] This variable is available since December 2012. For the March 2011 to November 2012 period, I estimated it using a model that relates the change in share of problematic assets with the change in the share of 90 days past due nonearmarked loans, which is calculated by the Central Bank of Brazil and is available for the entire sample. This extrapolation model is estimated from January 2013 to June 2015.

[24] Since the Doing Business Report of 2014, there is also a recovery rate for Rio de Janeiro, but the two rates are always the same. Therefore, the recovery rate of São Paulo can be seen as the Brazilian recovery rate.

Based on this database, I need to estimate only the aggregated elasticity of demand ($|\varepsilon_t^d|$ or $\left|\overline{\varepsilon_t^d}\right|$ for the standard and alternative specifications, respectively), since I have the marginal cost MC_t and the concentration index HHI_t. I apply the Kalman filter to estimate the state variables $|\varepsilon_t^d|^{-1}$ (in both specifications) and $\left(\theta_t|\varepsilon_t^d|\right)^{-1}$ (only in the alternative specification), which are assumed to follow random walk processes. The variance of the state variables are estimated by maximum likelihood. In order to do that, I eliminated the nonlinearities in the state variables in the equation of each specification (equations (15) and (16)). First, I use the natural logarithm in these equations:

$$\ln(1+i_t^*) = \ln MC_t - \ln\left(1 - \frac{HHI_t}{|\varepsilon_t^d|}\right) \quad (21)$$

$$\ln(1+i_t^*) = \ln MC_t - \ln\left\{1 - \left[(s_t^p)^2\left(\frac{HHI_t^p}{|\varepsilon_t^d|}\right) + (s_t^g)^2\left(\frac{HHI_t^g}{\theta_t|\varepsilon_t^d|}\right)\right]\right\} \quad (22)$$

Second, I linearized the second term in equations (21) and (22) by applying a first-order Taylor series approximation:

$$\begin{aligned}\ln(1-x) &\approx \ln(1-x_0) + \left[\left.\frac{\partial \ln(1-x)}{\partial(1-x)}\right|_{x_0}\right]\left[(1-x)-(1-x_0)\right] \\ \ln(1-x) &\approx \alpha_0 - \alpha_1 x \end{aligned} \quad (23)$$

where $\alpha_0 = \ln(1-x_0) + x_0(1-x_0)^{-1}$, $\alpha_1 = (1-x_0)^{-1}$, and $x = \frac{HHI_t}{|\varepsilon_t^d|}$ (standard specification) or $x = \left[(s_t^p)^2\left(\frac{HHI_t^p}{|\varepsilon_t^d|}\right) + (s_t^g)^2\left(\frac{HHI_t^g}{\theta_t|\varepsilon_t^d|}\right)\right]$ (alternative specification).

Finally, using equation (23) in (21) and (22),

$$\ln(1+i_t) \approx \ln MC_t - \alpha_0 - \alpha_1\left(\frac{HHI_t}{|\varepsilon_t^d|}\right) + \epsilon_t \quad (24)$$

$$\ln(1+i_t) \approx \ln MC_t - \alpha_0 - \alpha_1\left[(s_t^p)^2\left(\frac{HHI_t^p}{|\varepsilon_t^d|}\right) + (s_t^g)^2\left(\frac{HHI_t^g}{\theta_t|\varepsilon_t^d|}\right)\right] + \epsilon_t \quad (25)$$

where I also use $\ln(1+i_t^*) = \ln(1+i_t) + \epsilon_t$, $\epsilon_t \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$[25].

I choose to approximate x around the sample average of x, which seems to be a natural choice. This is done using an iterative approach: in each step, I estimate the state variables using x_0 equal to the sample average of the estimated x in the previous step. The procedure ends when x_0 converges. To initialize the method, I use $x_0 = 0$, implying

[25] Initially, I adopted a more flexible approach by modelling the markup using an error correction model, given the empirical evidence indicating a persistent characteristic of banks' margins or lending rates in Brazil (Dantas et al. (2012), Manhiça and Jorge (2012), Da Silva and Pirtouscheg (2015), Almeida and Divino (2015), da Silva et al. (2016), Valente et al. (2018), Castor and Ribeiro (2018)) and in other countries (Maudos and Solís (2009) for Mexico; Turgutlu (2010) for Turkey; Almarzoqi and Naceur (2015) for Caucasus and Central Asia). However, I did not find empirical evidence of such a smooth behavior in the markup, as the estimates for the parameter that measures the spead of adjustment were close to 1.

$\alpha_0 = 0$ and $\alpha_1 = 1$.

4 Model estimates

Figure 1 shows the estimated inverse of the aggregated price elasticity of demand: $|\varepsilon_t^d|$ for the standard specification (S1, solid blue line), in which all banks are profit maximizers, and $\left|\overline{\varepsilon_t^d}\right|$ for the alternative specification (S2, solid red line), in which public banks are allowed to deviate persistently from the optimal behavior. I also present the 95% confidence interval for the standard specification (dashed blue line). As can be seen, the results from both specifications are essentially the same. The estimated values are always positive, as expected, and quite precise given the narrow confidence interval. This result means a higher concentration increases the markup (or decreases the competition) and, consequently, the lending interest rate in the nonearmarked market[26]. This result is consistent with most of the empirical literature on this topic, as discussed in the introduction.

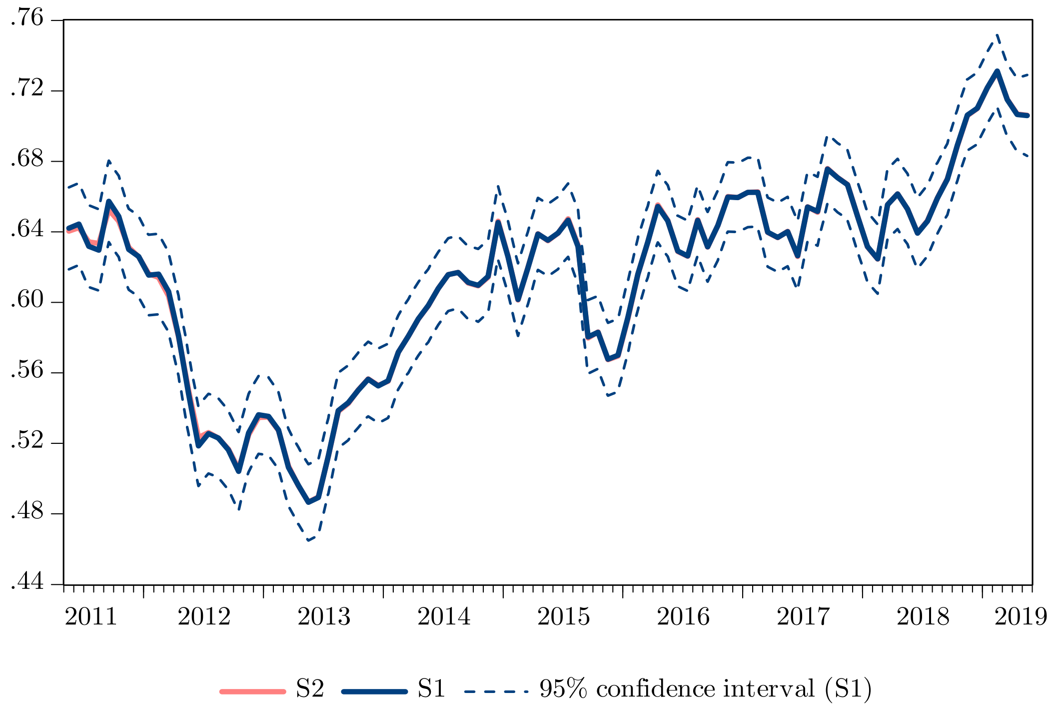


Figure 1: Inverse of the absolute value of the 'aggregated' price elasticity of demand: standard and alternative specifications (S1 and S2, respectively)

Another interesting issue regarding Figure 1 is that between 2012 and 2014 the demand became more elastic, lowering the markup, which is consistent with the use of public banks to reduce the lending rates in the same period. As a result, in the alternative specification, I initially used a dummy variable for this period of intervention as an

[26]As discussed in Section 2.4, a higher concentration also lowers the marginal cost, but the net effect is positive in Brazil, as shown in Section 5.

additional source of shock in $\left(\theta_t|\varepsilon_t^d|\right)^{-1}$. This resulted in a null estimated variance of $|\varepsilon_t^d|^{-1}$, regardless of the initial guesses. Thus, in the final alternative specification, whose results are shown in Figure 2, I choose to assume the 'elasticity of private banks' ε_t^d is time-invariant and do not use any dummy on the estimation of the 'elasticity of public banks' $\theta_t|\varepsilon_t^d|$, given the difficulty of identifying precisely the period of intervention.

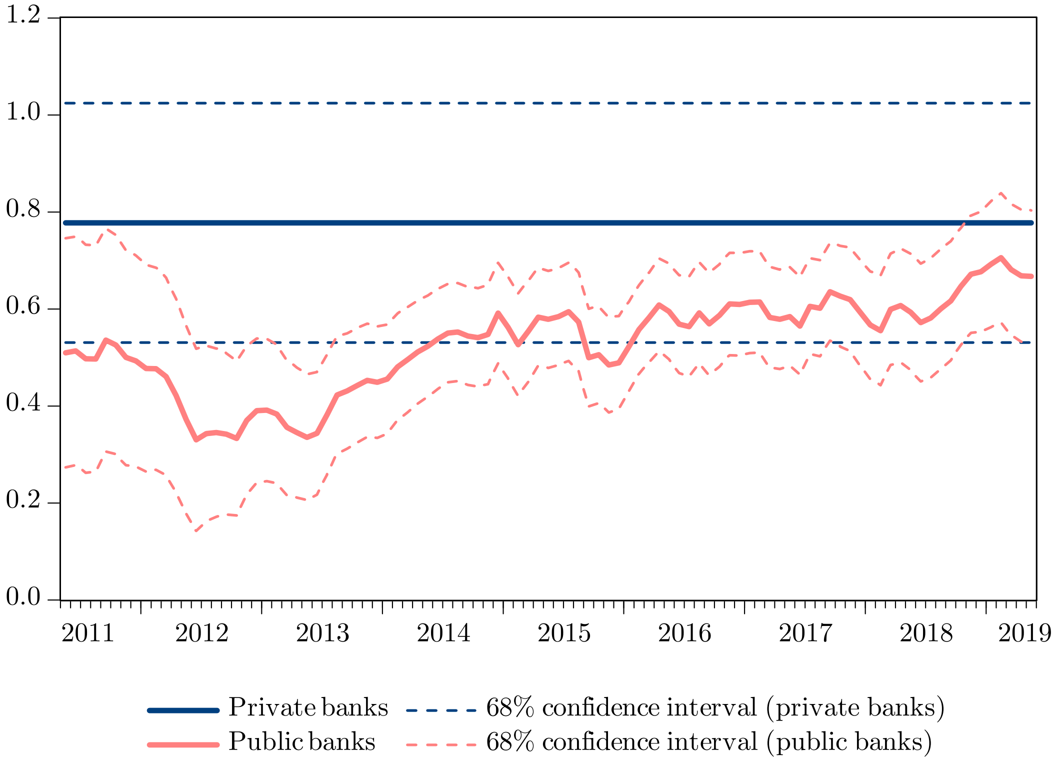


Figure 2: Inverse of the absolute value of the price elasticity of demand: alternative specification

As can be seen, the 68% confidence intervals are very wide. Thus, although the estimate of $\left|\overline{\varepsilon_t^d}\right|^{-1}$ is quite precise, the estimates of its components are not[27]. This recommends caution when analyzing point estimates of the elasticities of private and public banks. With this caveat in mind, it should be noted $\left(\theta_t|\varepsilon_t^d|\right)^{-1}$ is always lower than $|\varepsilon_t^d|^{-1}$, indicating public banks put a downward pressure on the lending rate. This was especially true between 2012 and 2014, indicating a period of more intense use of public banks to reduce the lending rate. After mid 2018, the elasticity of public banks has decreased in absolute terms, indicating public banks are behaving more similarly to private banks.

Lastly, it is worth mentioning that, given the estimated aggregated elasticities, the difference between actual markups, from equation (15) or (16), and their linearized versions, from equation (24) or (25), is negligible for sample index concentration HHI_t data. This

[27]In fact, the use of a dummy variable for the period of intervention does not alter the estimates of $\left|\overline{\varepsilon_t^d}\right|^{-1}$, but has a great impact on the estimates of the elasticities of private and public banks.

seems to support my approach of approximating the markup around its sample average, as discussed in Section 3.

5 Investigating the high lending interest rate in Brazil

I investigate the high lending interest rate in Brazil through counterfactual exercises. I show the effect on the lending rate of changes in tax rates (τ_t^c, τ_t^r and τ_t^p), time to recover profit tax paid over defaulted principal installments $\widetilde{m}$, required reserve to liability ratio ρ_t, gross discount rate f_t^{gr}, probability of default γ_t, LGD ϕ_t, and concentration index HHI_t. In these exercises, I consider only the direct effects of each determinant on the lending rate, ignoring possible indirect effects through other determinants. This recommends caution in analyzing the results since indirect effects can be very important, as discussed in Section 2.4. In particular, the degree of concentration may be affected, depending on the barriers to entry or to leave the market, by changes in the other determinants of the lending rate, as they alter the net profit of banks.

5.1 Methodology of the counterfactual exercises

Given the similarity of the estimated aggregated inverse of the elasticity in both specifications (Figure 1), counterfactual results would not be affected by the choice of the specification. In any case, the simulations of this section use the alternative specification of the model. More specifically, I use the original equation of the alternative specification (16)[28], not its linearized version (25). This avoids any approximations errors in the markup when new levels of concentration are simulated.

In all counterfactual simulations, I adjust the duration d_t using a second-order Macaulay approximation (Alps (2017)):

$$d_t^{CF} \approx d_t \left(\frac{1+\widetilde{i}_t}{1+\widetilde{i}_t^{CF}} \right)^{c_t} \left\{ 1 + \left(\frac{\widetilde{i}_t^{CF} - \widetilde{i}_t}{1+\widetilde{i}_t} \right)^2 \left[\frac{c3_t - c_t^2}{2} \right] \right\} \tag{26}$$

where $\widetilde{i}_t^{CF}$ is the counterfactual $\widetilde{i}_t$, $c_t = \sum_{k=1}^{m} \left[\frac{\left(P_{it}^k + I_{it}^k\right)/\left(1+\widetilde{i}_t\right)^k}{V_{it}} \right] k^2$ is the Macaulay convexity calculated using the stream of cash flows net of IOF tax from the credit operations granted at time t, and $c3_t = \sum_{k=1}^{m} \left[\frac{\left(P_{it}^k + I_{it}^k\right)/\left(1+\widetilde{i}_t\right)^k}{V_{it}} \right] k^3$ [29] [30].

[28]Again, I use $\ln(1+i_t^*) = \ln(1+i_t) + \epsilon_t$, where ϵ_t in the counterfactual scenario is the estimated ϵ_t.

[29]After all, looking at the duration as a price of a bond, one can see the duration of this bond is the convexity of the original stream of cash flows at present value.

[30]In any case, I find very similar counterfactual lending rates when I ignore this effect of the lending rate on the duration ($d_t^{CF} = d_t$). This suggests the assumption of an exogenous duration adopted in Section 2 does little harm.

In exercises with the concentration index HHI_t, I made equal proportional adjustment in the concentration indexes for private banks and public banks (HHI_t^g and HHI_t^p, respectively), meaning the aggregated elasticity of the alternative specification $\left|\overline{\varepsilon_t^d}\right|$ is not altered in the counterfactual scenarios. Furthermore, when HHI_t is altered, one should adjust $HHI3_t$ accordingly. Assuming each bank has the same market share, it is easy to show

$$\widehat{HHI3}_t = HHI_t^2 \tag{27}$$

where $\widehat{HHI3}_t$ is a estimate of $HHI3_t$ consistent with a given value of HHI_t under this market share assumption[31].

The correlation between $HHI3_t$ and $\widehat{HHI3}_t$ is very high, of 0.995, but $\widehat{HHI3}_t$ is always lower than $HHI3_t$ (21% on average). In this context, I obtain the counterfactual $HHI3_t$ ($HHI3_t^{CF}$) using

$$HHI3_t^{CF} = HHI3_t \left(\frac{\widehat{HHI3}_t^{CF}}{\widehat{HHI3}_t} \right) \tag{28}$$

where $\widehat{HHI3}_t^{CF} = \left(HHI_t^{CF}\right)^2$ and HHI_t^{CF} is the counterfactual concentration index.

In the exercises with the gross discount rate f_t^{gr}, I consider its effect on the lending rate also through the present value of principal payments PPV_t and C_t. Regarding PPV_t, I obtain f_t from equation (17) and then, as in the duration case, use a second-order Macaulay approximation (Alps (2017)):

$$PPV_t^{CF} \approx PPV_t \left(\frac{1+f_t}{1+f_t^{CF}} \right)^{d_t^{PPV}} \left\{ 1 + \left(\frac{f_t^{CF} - f_t}{1+f_t} \right)^2 \left[\frac{c_t^{PPV} - \left(d_t^{PPV}\right)^2}{2} \right] \right\} \tag{29}$$

where PPV_t^{CF} and f_t^{CF} are the counterfactual PPV_t and f_t, respectively, $d_t^{PPV} = \sum_{k=1}^{m} \left[\frac{P_{it}^k/(1+f_t)^k}{PPV_t} \right] k$ is the Macaulay duration of PPV_t, and $c_t^{PPV} = \sum_{k=1}^{m} \left[\frac{P_{it}^k/(1+f_t)^k}{PPV_t} \right] k^2$ is the Macaulay convexity of PPV_t. I use f_t and the stream of principal payments of PPV_t to calculate d_t^{PPV} and c_t^{PPV}.

In the case of C_t, I just need to calculate the counterfactual gross interest rate in savings deposits $i_t^{sd\_gr}$. The legislation since 2012 establishes $i_t^{sd\_gr}$ equals 70% of the Selic rate if the Selic rate is lower than or equal to 8.5% per annum and $i_t^{sd\_gr}$ equals 0.5% per month plus the TR rate if the Selic rate is higher than 8.5% per annum[32]. In this

[31] After all, under this assumption, HHI_t would be n^{-1} and $HHI3_t$ would be n^{-2}. An alternative approach is to assume the market shares follow a geometric progression. As shown in Appendix D, under this assumption $HHI3_t$ would be $4HHI_t^2/\left(3+HHI_t^2\right)$. Empirically, I find, besides level differences, both approaches yield essentially the same results. So, I choose to use the simpler approach of an equally distributed market among banks.

[32] In reality, according to the letter of the law, $i_t^{sd\_gr}$ equals 70% of the Selic rate plus the TR rate if the Selic rate is lower than or equal to 8.5%. However, since at this level of the Selic rate the TR rate is

context, I initially calculate a counterfactual Selic rate by applying the same proportional change of the discount rate f_t^{gr}. Since in the scenario I consider here the Selic rate is always lower than 8.5%, the counterfactual $i_t^{sd\_gr}$ equals 70% of the new Selic rate.

5.2 What does not explain the high lending rate

I start showing some determinants of the lending rates have little direct impact on the credit cost in Brazil, even under extreme scenarios as the ones considered here. I perform three simulations. First, I estimated the lending rate if the principal losses due to NPLs were recognized instantly for profit tax purpose ($\widetilde{m} = 0$). Second, I simulated a scenario with null taxes over profit and revenue ($\tau_t^r = \tau_t^p = 0$). Third, I eliminated the reserve requirements ($\rho_t = 0$). The results are shown in Table 1. As expected, the three average counterfactual lending rates are lower than the actual rate[33]. However, the effects are very low: 0.3pp for $\widetilde{m} = 0$[34], 0.6pp for $\tau_t^r = \tau_t^p = 0$, and 0.2pp for $\rho_t = 0$.

Table 1: Lending interest rates (2011M3-2019M5 average)

Actual	Scenario 1 ($\widetilde{m} = 0$)	Scenario 2 ($\tau_t^r = \tau_t^p = 0$)	Scenario 3 ($\rho_t = 0$)
31.0%	30.7%	30.4%	30.8%

This low impact could be surprising, especially regarding the last two scenarios. In the case of the taxes scenario ($\tau_t^r = \tau_t^p = 0$), it is worth mentioning that, as discussed in Section 2.4, since these taxes apply over revenue and also over the variable cost, one should not expect a meaningful impact on firms' optimal decision. In the case of the reserve requirement scenario ($\rho_t = 0$), the effect increases with the difference between the discount rate and the reserves remuneration rate ($f_t - i_t^r$) and with $2HHI_t - HHI3_t$, as discussed in Section 2.4. However, the difference between f_t and i_t^r is lower than 2pp and $2HHI_t - HHI3_t$ is around 0.3, yielding a relatively low direct effect of the required reserve to liability ratio ρ_t.

Therefore, the direct effects of $\widetilde{m}$, τ_t^r, τ_t^p, and ρ_t are essentially null, although indirect effects can be very important. In any case, the results of these three simulations suggest one should not expect meaningful short-run impact of these variables on the lending rate.

essentially null, in practice one can ignore the TR rate in this case.

[33] I calculate the counterfactual lending rates for each month of the sample, obtaining, then, the average values shown here.

[34] As discussed in Section 2.2, I assume no monetary correction is applied over the principal losses not recognized instantly for profit tax purposes. However, this is just a simplification. Hence, the impact on the lending rate of $\widetilde{m} = 0$ is probably even lower than 0.3%.

5.3 What does explain the high lending rate

In this section I show the results for the simulations with IOF tax rate τ_t^c, gross discount or funding rate f_t^{gr}, which is essentially a measure of the risk-free interest rate, probability of default γ_t, LGD ϕ_t, and concentration index HHI_t. Since a tax like IOF seems to be a Brazilian idiosyncrasy, I assume $\tau_t^c = 0$ in the counterfactual scenario. The other four variables are adjusted to fit the observed in other South American countries. The choice of this benchmark group is due to its cultural similarities with Brazil, which may be supportive to the implicit assumption in these exercises of similar price elasticities of demand among the countries.

In order to make fair comparisons, I just consider those countries that have all variables available for the entire sample period considered in these exercises, between 2012 and 2016. The countries left in the sample are Argentina, Chile, Colombia, Paraguay, Peru, and Uruguay. Since the methodologies used to calculate the average lending interest are not homogeneous across countries (Nakane and Costa (2005)), I do not perform simulations using data of specific countries. Instead, I only use median values for the group of countries in the sample, which should minimize measurement problems.

It is worth mentioning that all countries in the sample seem to calculate their aggregated lending rate using weighted averages. This would suggest my comparison is not fair, as the lending interest rates released by the countries in the sample would be overestimated. However, most of these countries seem to consider only credit operations of specific segments of the credit market to calculate their averages (for instance, using only credit to non-financial firms or considering solely short-term loans). As a result, it is not clear the median lending rates of these countries are overestimated. In any case, this lack of standardization is a major challenge for international comparisons like this one.

With this caveat in mind, Table 2 presents the observed values of the four counterfactual variables in Brazil and in the benchmark group. Given the lack of international data for problematic assets, I use the share of 90 days past due loans as a proxy for the probability of default γ_t. For similar reasons, regarding the concentration index HHI_t, I use an estimate for the HHI for commercial banks assets, which is calculated using data of assets of three and five largest commercial banks as a share of total commercial banking assets[35].

As can be seen, the four variables analyzed contribute to a higher lending rate in Brazil. First, the gross funding rate f_t^{gr} is higher in Brazil, more than two times the rate in South America excluding Brazil. Second, the share of 90 days past due loans in Brazil is also more than two times higher. Third, the Brazilian recovery rate is lower, around 20%, while in the group of countries analyzed this number is close to 30%. Finally, the banking

[35] See Appendix E for details.

market seems to be more concentrated in Brazil than in the group of countries analyzed.

Table 2: Variables of the counterfactual exercise: 2012-2016 average

Variable	Brazil	Benchmark group[a]
Gross funding rate (f_t^{gr})	11.1%[b]	4.7%[c]
90 days past due loans (proxy for γ_t)	5.0%[d]	2.2%[e]
Recovery rate ($1-\phi_t$)	19.9%[b]	29.9%[f]
Concentration measure (proxy for HHI_t)	0.165[g]	0.143[g]

[a] Argentina, Chile, Colombia, Paraguay, Peru, and Uruguay.

[b] Sources for these Brazilian data are those presented in Section 3.

[c] Deposit rate, countries' median. Source: International Financial Statistics.

[d] Share of 90 days past due nonearmarked loans. Source: Central Bank of Brazil.

[e] Share of 90 days past due loans, countries' median. Source: Global Financial Development Database.

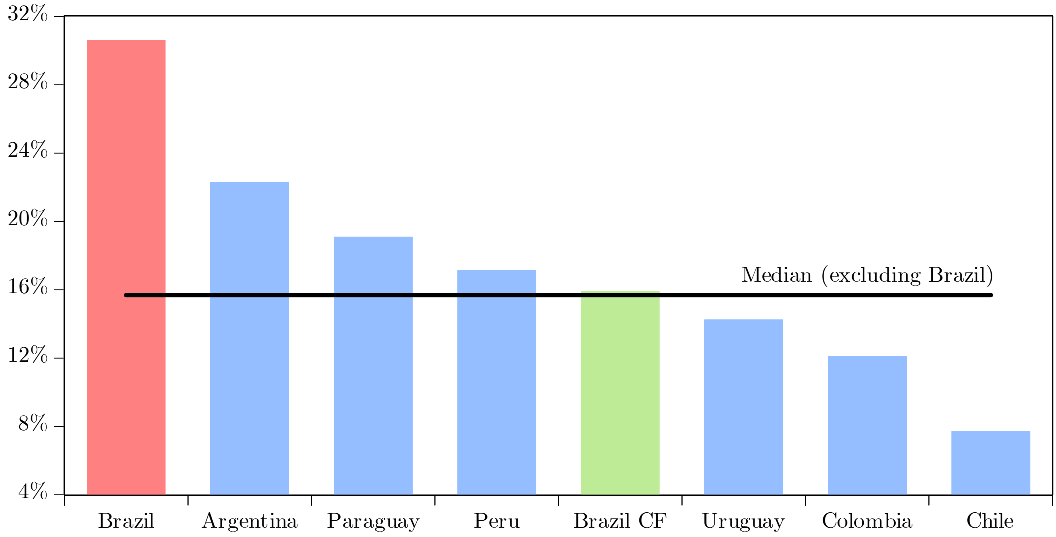
[f] Countries' median. Source: Doing Business Report.

[g] The proxy for each country is calculated using data from the Global Financial Development Database, as shown in Appendix E. Then, for the group of countries, I obtain the median value.

Therefore, in the counterfactual exercise, I adjust f_t^{gr}, γ_t, ϕ_t, and HHI_t by multiplying each of them by the ratio between its 2012-2016 counterfactual average (third column of Table 2) and its 2012-2016 Brazilian average (second column of Table 2)[36], for each month between 2012 and 2016. The results of this simulation, which also uses $\tau_t^c = 0$, can be seen in Figure 3 that shows the 2012-2016 average lending interest rate for South American countries, including the counterfactual Brazilian rate (Brazil CF, green bar).

These data confirm the Brazilian lending interest rate is very high by international standards (red bar *vis-à-vis* blue bars). It is the highest in this sample of countries, almost two times the median rate (solid black line). Furthermore, the counterfactual rate is close to half of the observable Brazilian lending rate, a reduction of 14.7pp. As a result, the counterfactual rate is essentially equal to the median rate (15.9% and 15.7%, respectively). Hence, essentially all the higher credit cost in Brazil compared to other South American countries between 2012 and 2016 can be explained by these five variables.

[36] Therefore, HHI_t and γ_t are adjusted based on the their proxies.



Source for other countries than Brazil: International Financial Statistics.

Figure 3: Lending interest rates (2012-2016 average)

To identify which of these five variables contributes most for the higher credit cost in Brazil in the period analyzed, let us use the natural logarithm approximation of the optimal lending interest rate i_t^* on equation (16), which is more accurate the lower the interest rate[37]:

$$\ln\left(1+i_t^*\right) = \ln\left[\frac{1}{\left(1-\tau_t^c\right)^{1/d_t}}\right] + \ln \widetilde{MC}_t - \ln\left(1 - \frac{HHI_t}{\left|\overline{\varepsilon_t^d}\right|}\right) \tag{30}$$

where $\widetilde{MC}_t = 1 + f_t\left[\frac{1-PPV_t(1-\gamma_t\phi_t A_t)-C_t(2HHI_t-HHI3_t)}{(1-\gamma_t\phi_t-\tau_t^r)\left(1-\tau_t^p\right)(1-PPV_t)}\right]$ is the marginal cost disregarding the IOF tax (that is, MC_t when $\tau_t^c = 0$).

Using equation (30), I decompose the reduction of the approximated optimal lending rate $\ln\left(1+i_t^*\right)$ in the counterfactual simulation for each month of the sample into (i) IOF tax effect, which is the change in the first element in the right-hand side of the equation, (ii) other costs effect, which captures the effect of the discount rate f_t^{gr}, the probability of default γ_t, and the LGD ϕ_t, from the change in the second element, and (iii) market concentration effect, which equals the change in the third element. Subtracting the change in $\ln(1+f_t^{gr})$ between the two scenarios from both sides of equation (30), I can also obtain the importance of each determinant on the variation of the approximated optimal spread $\ln\left(1+i_t^*\right) - \ln(1+f_t^{gr})$. In this case, the other costs effect is net of the change in $\ln(1+f_t^{gr})$.

The 2012-2016 average results from these decompositions are shown in Figure 4[38].

[37] To see that, let $f(x) = \ln(1+x)$ and apply a first-order Taylor series approximation around 0: $f(x) \approx f(0) + f'(0)(x-0) \leftrightarrow \ln(1+x) \approx x$.

[38] Note the sum of the effects of the three components is lower than the 14.7pp difference between the

From Panel A, one can see that, in terms of the change in the approximated optimal lending rate, the most important effect is related to the other costs, of 9.3pp or 78% of the total reduction. The effect related to the IOF tax equals 1.3pp, which represents 11% of the reduction. Therefore, 89% of the reduction in $\ln(1+i_t^*)$ is due to costs. Only 11%, or 1.3pp, is determined by concentration. In terms of the approximated bank interest spread, shown in Panel B, the most important effect is still that related to f_t^{gr}, γ_t, and ϕ_t, accounting for 56% of the reduction, followed by IOF tax and concentration (22% of the reduction for each). So, even in terms of spread, the reduction is essentially due to costs, as they explain 78% of the difference.

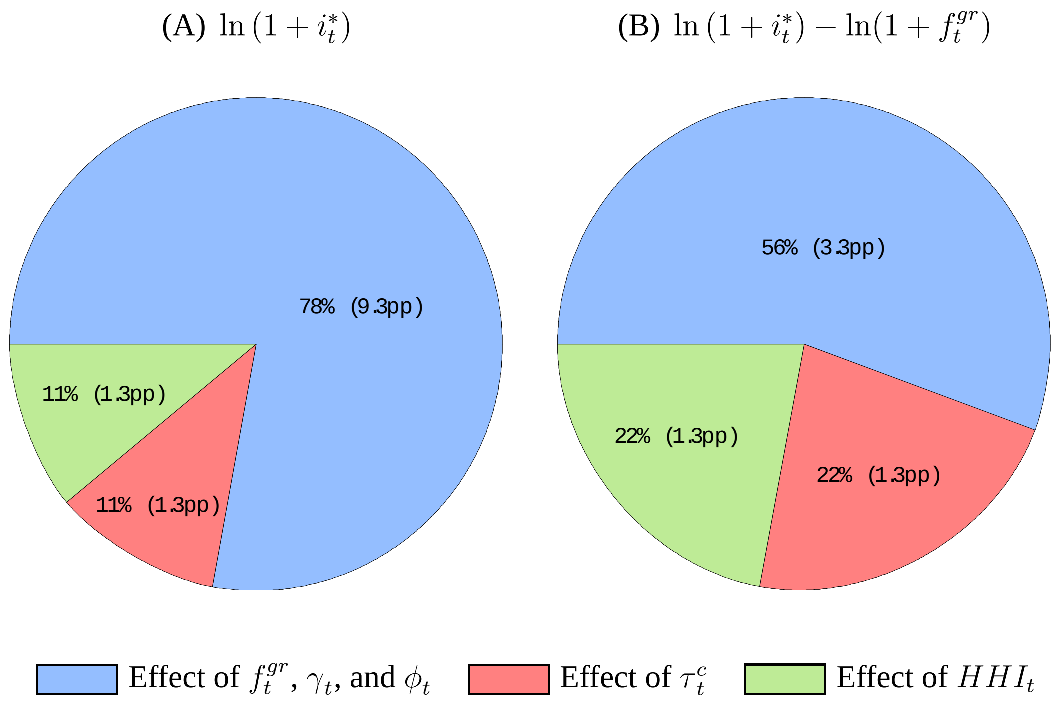


Figure 4: Determinants of the lower optimal counterfactual credit cost (2012-2016 average)

I find the results shown in Panel B are essentially invariant to changes in the gross discount rate f_t^{gr}. This suggests the effect of the other costs on the approximated optimal spread is mainly given by the probability of default γ_t and the LGD ϕ_t. The same conclusion arises regarding the exact spread $i_t - f_t^{gr}$, which decreases 8.3pp in this counterfactual exercise, but only 1.6pp when just f_t^{gr} is altered in the simulated scenario and 7.7pp when all variables except f_t^{gr} are set to their counterfactual levels. This evidence does not support the view that high bank interest spread in Brazil is largely a story of high opportunity cost of funds (World Bank (2006)). This may reflect, at least partially, the

counterfactual rate and the observable credit cost shown in Figure 3. This is essentially due to the approximation error, since here I am using natural logarithm approximation of the interest rates (equation (30).

strong reduction in the monetary policy rate (Selic rate) in the last fifteen years, since "the direct impact on spreads of the traditional microeconomic culprits [...] is likely to be of second-order importance at high levels of the Selic rate but will 'bite' increasingly as this rate declines" (World Bank (2006)).

But what explains the high levels of the gross discount rate f_t^{gr}, the probability of default γ_t, the LGD ϕ_t, and concentration index HHI_t in Brazil? A full assessment of this issue is beyond the scope of this paper, but I can make some comments about it.

First, on the one hand, the high level of f_t^{gr}, which is essentially a measure of the nominal risk-free interest rate, is due to higher inflation in Brazil. The 2012-2016 average inflation rate in Brazil is 7%, while the median value for the sample of countries analyzed is 4%. However, between 2017 and 2019, the Brazilian inflation rate was close to the median rate of the group of countries analyzed. And this lower inflation environment is expected to be persistent, given the recent reductions in the Brazilian inflation target, from 4.5% in 2018 to 4.25% in 2019, 4% in 2020, 3.75% in 2021, 3.5% in 2022, and 3.25% in 2023.

On the other hand, the higher level of f_t^{gr} is due to the historically high real risk-free interest rate in Brazil. Santos (2020), based on a general characterization of the capital cost, identifies expressions for the natural real interest rate. The results from the application of this approach to Brazil indicate the historically high natural real interest rate in the country is justified by (i) the impact of the subsidized lending by the Brazilian Development Bank (BNDES) and, most importantly, by both (ii) the low level of firms' markup in product markets and (iii) the low total savings rate in the country.

As discussed in Santos (2020), the first of these problems is essentially solved given the new BNDES funding rate (TLP). Regarding the other two problems, the low level of markup could be a reflection of the high share of prices regulated by the government (25% to 30% of the consumer price index), while the low savings rate seems to reflect the low households' incentives to save in the country, given, for example, the very benevolent pension system (see Brito and Minari (2015)). In this context, it is worth mentioning the recent approval of the pension system reform.

Second, the higher level of the probability of default γ_t could reflect, besides cyclical considerations, adverse selection in light of informational asymmetries in the Brazilian credit market. This highlights the importance of the recent improvement in the positive credit bureau (Cadastro Positivo)[39]. Furthermore, the high level of the LGD ϕ_t in Brazil could also increase the incentives to default, as the borrower expect to lose just a small share of the collateral in the event of a default.

Earmarked credit at subsidized levels could also induce an increase in γ_t in the non-

[39]The main improvement is the implementation of an opt-out mechanism in substitution to the current opt-in, which tends to strongly increase the number of participants in the credit bureau.

earmarked credit market. On the one hand, given the lower lending rates of the earmarked market, it can get the less risky borrowers (selection effect). On the other hand, borrowers' incentive is to default preferably in nonearmarked credit operations, since the earmarked credit are cheaper and with higher maturity (incentive effect). At a first glance, this hypothesis seems to be consistent with data: while 2012-2016 average of the share of 90 days past due loans is 5.0% in the nonearmarked credit market, it is only 1.2% for earmarked credit operations. In this context, a simple proxy for this measure of delinquency rate without possible distortions from the earmarked credit is the average value considering data from both markets, which is 3.3%, much closer to the level of 2.2% observed in other South American countries (Table 2).

Third, the higher level of the LGD ϕ_t is related to the concept of jurisdictional uncertainty proposed by Arida et al. (2005), which "is an uncertainty of a diffuse character that permeates the decisions of the executive, legislative, and judiciary and manifests itself predominantly as an anti-saver and anti-creditor bias"[40]. This diffuse character implies this anomaly is not a result of a single inefficiency. In fact, Oliver Wyman (2018) presents a very long list of measures to improve the Brazilian recovery rate, such as the modernization of the bankruptcy law, the review of the seniority rank, and the review of the legislation of foreclosures. The high level of the risk-free interest rate could also contribute to a high ϕ_t by lowering the present value of the recovered resources[41]. This effect could be especially important in Brazil given its longer time to resolve an insolvency[42].

Finally, the more concentrated financial system, as measured by the concentration index HHI_t, could reflect higher barriers to entry or to leave the market, but could also reflect factors that reduce the level of banks' economic profit as administrative costs, profit taxes, reserve requirements, and economic costs due to earmarked credit operations. After all, even without barriers to entry or leave the market, the steady-state markup is higher if, for example, the fixed costs are also higher, in order to guarantee a null economic profit, implying in a higher steady-state concentration. In this sense, on the one hand, if the high concentration is mainly due to high barriers to entry or to leave the market, the economic profit is expected to be high in the average interest rate. On the other hand, if the high concentration is mainly due to these factors that reduce the profit level, the economic profit is expected to be low.

According to the accounting decomposition of Central Bank of Brazil (2020), the

[40] Arida et al. (2005) develop this concept of uncertainty to explain the high level of the real risk-free interest rate in Brazil. However, empirical evidence does not seem to support this hypothesis (Gonçalves et al. (2007); Bacha et al. (2009)). In any case, this concept could help us understand the low level of the recovery rate given the anti-creditor bias, which is a component of the high bank interest spread.

[41] Although da Silva et al. (2009) find empirical evidence supporting a non-expected negative relation between the Brazilian policy rate (Selic rate) and their estimates for the LGD.

[42] According to data from Doing Business reports 2013-2017, the average time to resolve a insolvency in Brazil is 4 years, while in the group of countries analyzed here it is around 3 years.

financial margin accounts for only 11.8% of the 2017-2019 average lending rate (earmarked and nonearmarked markets). This may suggest the higher concentration in Brazil is mainly due to these factors that reduce the profit level rather than higher barriers to entry and to leave the market.

In any case, Ribeiro and Castor (2017) present evidence of low market contestability, as the number of banks entering and leaving the Brazilian financial market is low and as the entering banks have difficulties in gaining market share[43]. Informational asymmetries in the credit market could contribute to this outcome, since established banks may have competitive advantage against the incoming ones as they have more information about the borrowers based on previous relationships. Therefore, the improvement made in the positive credit bureau (Cadastro Positivo) can lower the credit cost in Brazil through two channels: (i) decreasing the probability of default γ_t by diminishing adverse selection and (ii) strengthening the market competition by increasing market contestability.

6 Conclusion

This paper seeks to identify the main reasons for the high level of nonearmarked Brazilian lending interest rate through counterfactual exercises, which are simulated using a Cournot model. The results indicate the higher lending rates in Brazil *vis-à-vis* other South American countries between 2012 and 2016 can be explained by (i) IOF tax, (ii) high level of the risk-free interest rate, (iii) higher probability of default, (iv) lower recovery rate, and (v) more concentrated financial system. This result is mainly driven by the cost components. They account for 89% of the optimal lending rate reduction in the counterfactual scenario. The market concentration explains only 11%. In terms of the optimal bank interest spread, the concentration becomes more relevant, explaining 22% of the difference, but the cost components are still the main determinants (78% of the decrease).

These results draw a general picture about the high credit cost issue in Brazil. Investigating these determinants of the high lending interest rates in Brazil is an interesting topic for future research. It can enrich the diagnosis presented here, especially regarding the specific reforms and measures required to reduce the Brazilian lending rate.

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[43]The low market contestability could also prevent more efficient banks from entering the market. As a result, the low level of economic profit on the average lending rate can be, at least partially, due to established banks inefficiency. This investigation is an interesting area for future research.

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Appendix

A Present value of interest payments

Let us start with the calculation of the interest payment I_{it}^{k}:

$$I_{it}^{k} = \left(V_{it} - \sum_{j=1}^{k-1} P_{it}^{j} \right) \widetilde{i}_t \tag{A.1}$$

where $\widetilde{i}_t$ is the lending interest rate net of IOF tax of credit operations granted at time t.

Thus, the present value of interest payments (IPV_{it}) is

$$\begin{aligned}
IPV_t &\equiv \sum_{k=1}^{m} \frac{\left(I_{it}^{k}/V_{it}\right)}{(1+f_t)^k} = \sum_{k=1}^{m} \frac{\left(V_{it} - \sum_{j=1}^{k-1} P_{it}^{j} \right) \widetilde{i}_t / V_{it}}{(1+f_t)^k} \\
IPV_t &= \widetilde{i}_t \left\{ \sum_{k=1}^{m+1} \left[\frac{1}{(1+f_t)^k} \right] \left(1 - \sum_{j=1}^{k-1} \left(P_{it}^{j}/V_{it} \right) \right) \right\} \\
IPV_t &= \widetilde{i}_t \left\{ \sum_{h=1}^{m+1} \frac{1}{(1+f_t)^h} - \sum_{k=2}^{m+1} \left[\frac{1}{(1+f_t)^k} \right] \sum_{j=1}^{k-1} \left(P_{it}^{j}/V_{it} \right) \right\} \\
IPV_t &= \widetilde{i}_t \left\{ \sum_{h=1}^{m+1} \frac{1}{(1+f_t)^h} - \sum_{j=1}^{m} \left(P_{it}^{j}/V_{it} \right) \sum_{k=j+1}^{m+1} \left[\frac{1}{(1+f_t)^k} \right] \right\}
\end{aligned} \tag{A.2}$$

where I use $\sum_{k=1}^{m+1} \left[\frac{1}{(1+f_t)^k} \right] \left[1 - \sum_{j=1}^{k-1} \left(P_{it}^{j}/V_{it} \right) \right] = \sum_{k=1}^{m} \left[\frac{1}{(1+f_t)^k} \right] \left[1 - \sum_{j=1}^{k-1} \left(P_{it}^{j}/V_{it} \right) \right]$ since $\sum_{j=1}^{m} P_{it}^{j} = V_{it}$.

Using the expression for the sum of the elements of a geometric progression,

$$\begin{aligned}
\sum_{k=j+1}^{m+1} \frac{1}{(1+f_t)^k} &= (1+f_t)^{-(j+1)} \left[\frac{1-(1+f_t)^{-(m-j+1)}}{1-(1+f_t)^{-1}} \right] \\
\sum_{k=j+1}^{m+1} \frac{1}{(1+f_t)^k} &= (1+f_t)^{-j} \left[\frac{1-(1+f_t)^{-(m-j+1)}}{f_t} \right] \\
\sum_{k=j+1}^{m+1} \frac{1}{(1+f_t)^k} &= \frac{(1+f_t)^{-j} - (1+f_t)^{-(m+1)}}{f_t}
\end{aligned} \tag{A.3}$$

$$\sum_{h=1}^{m+1} \frac{1}{(1+f_t)^h} = \frac{1-(1+f_t)^{-(m+1)}}{f_t} \tag{A.4}$$

Finally, substituting (A.3) and (A.4) in (A.2), I obtain the desired expression:

$$IPV_t = \left(\frac{\widetilde{i}_t}{f_t}\right)\left\{1-(1+f_t)^{-(m+1)}-\sum_{j=1}^{m}\left(P_{it}^{j}/V_{it}\right)\left[(1+f_t)^{-j}-(1+f_t)^{-(m+1)}\right]\right\}$$

$$IPV_t = \left(\frac{\widetilde{i}_t}{f_t}\right)(1-PPV_t) \tag{A.5}$$

where I use $\sum_{j=1}^{m}\left(P_{it}^{j}/V_{it}\right)(1+f_t)^{-(m+1)} = (1+f_t)^{-(m+1)}$ since $\sum_{j=1}^{m}P_{it}^{j} = V_{it}$.

B The impact of changes in $\mathbf{HHI_t}$ on $\mathbf{2HHI_t\text{-}HHI3_t}$

By definition,

$$\frac{\Delta\left(2HHI_t - HHI3_t\right)}{\Delta HHI_t} = 2 - \frac{\Delta HHI3_t}{\Delta HHI_t} = 2 - \frac{\sum_{i=1}^{n}\Delta s_{it}^{3}}{\sum_{j=1}^{n}\Delta s_{jt}^{2}} \tag{B.1}$$

Applying a first-order Taylor series approximation on $\Delta HHI3_t$,

$$\frac{\Delta HHI3_t}{\Delta HHI_t} = \frac{\sum_{i=1}^{n}\Delta\left(s_{it}^{2}\right)^{3/2}}{\sum_{j=1}^{n}\Delta s_{it}^{2}} \approx \frac{\left(\frac{3}{2}\right)\sum_{i=1}^{n}s_{it}\Delta s_{it}^{2}}{\sum_{j=1}^{n}\Delta s_{it}^{2}}$$

$$\frac{\Delta HHI3_t}{\Delta HHI_t} \approx \frac{\left(\frac{3}{2}\right)\left[\sum_{i\in H}s_{it}\Delta s_{it}^{2} - \sum_{j\in L}s_{jt}\left(-\Delta s_{jt}^{2}\right)\right]}{\sum_{k\in H}\Delta s_{kt}^{2} - \sum_{o\in L}\left(-\Delta s_{ot}^{2}\right)} \tag{B.2}$$

where H and L are the sets of banks whose market shares have not decreased and have decreased, respectively. Thus, $\Delta s_{it}^{2} \geq 0$ if $i \in H$ and $-\Delta s_{jt}^{2} > 0$ if $j \in L$.

From equation (B.1), the lowest value of $\Delta\left(2HHI_t - HHI3_t\right)/\Delta HHI_t$ is achieved at the highest level of $\Delta HHI3_t/\Delta HHI_t$. In this context, let us consider a upper bound of $\Delta HHI3_t/\Delta HHI_t$, identifying the distribution of the changes in the market shares that maximize $\Delta HHI3_t$, for a given change in HHI_t. This occurs by maximizing $\sum_{i\in H}s_{it}\Delta s_{it}^{2}$ and minimizing $\sum_{j\in L}s_{jt}\left(-\Delta s_{jt}^{2}\right)$, which happens when the biggest bank in the initial period, whose initial share is s_{1t}, obtains the totality of the market in the final period. From equation (B.2),

$$
\begin{aligned}
\frac{\Delta HHI3_t}{\Delta HHI_t} &\leq \frac{\left(\frac{3}{2}\right)\left[s_{1t}\left(1-s_{1t}^2\right)-\sum_{j\in L} s_{jt}\left(s_{jt}^2-0\right)\right]}{\left(1-s_{1t}^2\right)-\sum_{j\in L}\left(s_{jt}^2-0\right)} \\
\frac{\Delta HHI3_t}{\Delta HHI_t} &\leq \frac{\left(\frac{3}{2}\right)\left[s_{1t}\left(1-s_{1t}^2\right)-\sum_{j\in L} s_{jt}^3\right]}{\left(1-s_{1t}^2\right)-\sum_{j\in L} s_{jt}^2} \leq \frac{\left(\frac{3}{2}\right) s_{1t}\left(1-s_{1t}^2\right)}{\left(1-s_{1t}^2\right)-\sum_{j\in L} s_{jt}^2} \qquad \text{(B.3)}
\end{aligned}
$$

Since the changes in the market shares should add to zero,

$$
\begin{aligned}
1-s_{1t} &= \sum_{j\in L} s_{jt} \\
\left(1-s_{1t}\right)^2 &= \left(\sum_{j\in L} s_{jt}\right)^2 \geq \sum_{j\in L} s_{jt}^2 \qquad \text{(B.4)}
\end{aligned}
$$

Thus, using equation (B.4) in (B.3),

$$
\begin{aligned}
\frac{\Delta HHI3_t}{\Delta HHI_t} &\leq \frac{\left(\frac{3}{2}\right) s_{1t}\left(1-s_{1t}^2\right)}{\left(1-s_{1t}^2\right)-\left(\sum_{j\in L} s_{jt}\right)^2} \\
\frac{\Delta HHI3_t}{\Delta HHI_t} &\leq \frac{\left(\frac{3}{2}\right) s_{1t}\left(1+s_{1t}\right)}{\left(1+s_{1t}\right)-\left(1-s_{1t}\right)} \\
\frac{\Delta HHI3_t}{\Delta HHI_t} &\leq \left(\frac{3}{4}\right)\left(1+s_{1t}\right) \leq \frac{3}{2} \qquad \text{(B.5)}
\end{aligned}
$$

where in the last part I use the maximum value of s_{1t} is one.

Finally, using equation (B.5) in (B.1),

$$
\frac{\Delta\left(2HHI_t-HHI3_t\right)}{\Delta HHI_t} \geq 2-\frac{3}{2}=\frac{1}{2}>0 \qquad \text{(B.6)}
$$

Hence, from (B.6), $2HHI_t - HHI3_t$ increases with HHI_t.

C Credit products assumptions

Table 3 shows the assumptions regarding the amortization method and the presence of collateral for each credit product. In relation to the amortization methods, I follow Box 9 of Central Bank of Brazil (2019).

Table 3: Credit products assumptions

Borrower	Nonearmarked credit product	Amortization	Collateral?
NFC[a] and HH[b]	Overdraft	S[c]	No
	Goods financing	F[d]	Yes
	Goods leasing	F[d]	Yes
	Credit card revolving credit	S[c]	No
	Credit card financing	F[d]	No
	Credit card purchases	S[c]	No
NFC[a]	Discount of trade and credit card bills	S[c]	Yes
	Working capital	F[d]	No
	Guaranteed over draft accounts	S[c]	No
	Vendor	S[c]	Yes
	Compror	S[c]	Yes
	Advances on exchange contracts	S[c]	Yes
	Imports and exports financing	S[c]	Yes
	Foreign on lendings	F[d]	Yes
HH[b]	Personal credit	F[d]	No
	Payroll-deducted personal loans	F[d]	No

[a]Non-financial corporations.
[b]Households.
[c]Single payment.
[d]French method.

D HHI3$_t$ when the market shares follow a geometric progression

Consider the market shares follow a geometric progression:

$$s_{it} = s_{1t}\kappa_t^{i-1} \tag{D.1}$$

where s_{it} is the market share of the i-st largest bank and $0 < \kappa_t < 1$.

From equation (D.1), I need to calculate two parameters to determine the market shares: s_{1t} and κ_t. However, since the market shares should add to one, I can write s_{1t} as a function of κ_t. In formal terms,

$$\begin{aligned} \sum_{i=1}^{n} s_{1t}\kappa_t^{i-1} &= 1 \\ s_{1t} &= (1 - \kappa_t) \end{aligned} \tag{D.2}$$

Substituting equation (D.2) in (D.1),

$$s_{it} = (1 - \kappa_t)\, \kappa_t^{i-1} \tag{D.3}$$

Using equation (D.3) and the concentration index HHI_t, one can calculate κ_t:

$$\begin{aligned} HHI_t &= \sum_{i=1}^{n} \left[(1-\kappa_t)\,\kappa_t^{i-1}\right]^2 = \frac{(1-\kappa_t)^2}{1-\kappa_t^2} \\ 0 &= (1+HHI_t)\kappa_t^2 - 2\kappa_t + (1-HHI_t) \\ \kappa_t &= \frac{2 \pm \sqrt{4 - 4\,(1+HHI_t)\,(1-HHI_t)}}{2\,(1+HHI_t)} = \frac{1 \pm HHI_t}{1+HHI_t} \end{aligned} \tag{D.4}$$

where I can discard $\kappa_t = \frac{1+HHI_t}{1+HHI_t} = 1$, since $0 < \kappa_t < 1$.

Finally, using equations (D.3) and (D.4), one can calculate $\widetilde{HHI3}_t$, which is a estimate of $HHI3_t$ consistent with a given value of HHI_t under the market share assumption of equation (D.1):

$$\begin{aligned} \widetilde{HHI3}_t &= \sum_{i=1}^{n} \left[(1-\kappa_t)\,\kappa_t^{i-1}\right]^3 = \frac{(1-\kappa_t)^3}{1-\kappa_t^3} \\ \widetilde{HHI3}_t &= \frac{(2HHI_t)^3}{(1+HHI_t)^3 - (1-HHI_t)^3} = \frac{4HHI_t^2}{3+HHI_t^2} \end{aligned} \tag{D.5}$$

E Calculating the HHI using concentration ratios data

Given the lack of detailed bank data to calculate the HHI for other countries, I estimate this concentration index using data of assets of three and five largest commercial banks as a share of total commercial banking assets, from the Global Financial Development Database of the World Bank. To do that, I use three assumptions: (i) the three largest banks have the same size, (ii) the fourth and the fifth largest banks have the same size, and (iii) the market share of the sixth largest onwards decline following a geometric progression, similar to discussion of Appendix D. In formal terms,

$$s_{c1t} = s_{c2t} = s_{c3t} = \frac{C3_{ct}}{3} \tag{E.1}$$

$$s_{c4t} = \frac{C5_{ct} - C3_{ct}}{2} \tag{E.2}$$

$$s_{cit} = \left(\frac{C5_{ct} - C3_{ct}}{2}\right) \kappa_{ct}^{(i-5)}, i = 5, 6, 7, ... \tag{E.3}$$

where s_{cit} is the market share of the i-st largest bank in country c at time t, $C3_{ct}$ is the three-commercial bank concentration ratio in country c at time t, $C5_{ct}$ is the five-commercial bank concentration ratio in country c at time t, and $0 < \kappa_{ct} < 1$.

The HHI for assets of commercial banks under these assumptions for country c at time t ($\widehat{HHI}^{a}_{ct}$) is:

$$\begin{aligned}
\widehat{HHI}^{a}_{ct} &= \sum_{i=1}^{\infty} s^2_{cit} = 3\left(\frac{C3_{ct}}{3}\right)^2 + \left(\frac{C5_{ct}-C3_{ct}}{2}\right)^2 + \sum_{i=5}^{\infty}\left[\left(\frac{C5_{ct}-C3_{ct}}{2}\right)\kappa_{ct}^{(i-5)}\right]^2 \\
\widehat{HHI}^{a}_{ct} &= 3\left(\frac{C3_{ct}}{3}\right)^2 + \left(\frac{C5_{ct}-C3_{ct}}{2}\right)^2 + \frac{\left(\frac{C5_{ct}-C3_{ct}}{2}\right)^2}{1-\kappa^2_{ct}} \\
\widehat{HHI}^{a}_{ct} &= 3\left(\frac{C3_{ct}}{3}\right)^2 + \left(\frac{C5_{ct}-C3_{ct}}{2}\right)^2\left(1+\frac{1}{1-\kappa^2_{ct}}\right) \qquad \text{(E.4)}
\end{aligned}$$

Since the market shares should add to one, I can obtain κ_{ct}:

$$\begin{aligned}
1 &= \sum_{i=1}^{\infty} s_{cit} = 3\left(\frac{C3_{ct}}{3}\right) + \left(\frac{C5_{ct}-C3_{ct}}{2}\right) + \sum_{i=5}^{\infty}\left(\frac{C5_{ct}-C3_{ct}}{2}\right)\kappa_{ct}^{(i-5)} \\
1 &= C3_{ct} + \left(\frac{C5_{ct}-C3_{ct}}{2}\right) + \frac{\left(\frac{C5_{ct}-C3_{ct}}{2}\right)}{1-\kappa_{ct}} \\
\kappa^c_t &= 1 - \frac{\left(\frac{C5_{ct}-C3_{ct}}{2}\right)}{1-C3_{ct}-\left(\frac{C5_{ct}-C3_{ct}}{2}\right)} \qquad \text{(E.5)}
\end{aligned}$$

To evaluate this estimate, I calculate the HHI in a quarterly basis for six different segments of the Brazilian banking market, as I considered three types of financial institutions (commercial banks, banks or banks and non-banks financial institutions) and two types of assets (credit or total assets). For each of these six segments, I also calculate the estimate of the concentration index in a quarterly basis based on equations (E.4) and (E.5), using solely the corresponding concentration ratios. The average results for the 2010Q4-2019Q2 period are shown in Table 4, which indicate the HHI estimates are quite precise, at least for the Brazilian case.

Table 4: HHI for Brazil: actual versus estimate (2010Q4-2019Q2 average)

Segment		Actual	Estimate
Credit	Commercial banks	0.158	0.158
	Banks[b]	0.149	0.150
	Banks and non-banks financial institutions[c]	0.121	0.123
Assets	Commercial banks[a]	0.137	0.142
	Banks[b]	0.131	0.137
	Banks and non-banks financial institutions[c]	0.109	0.114