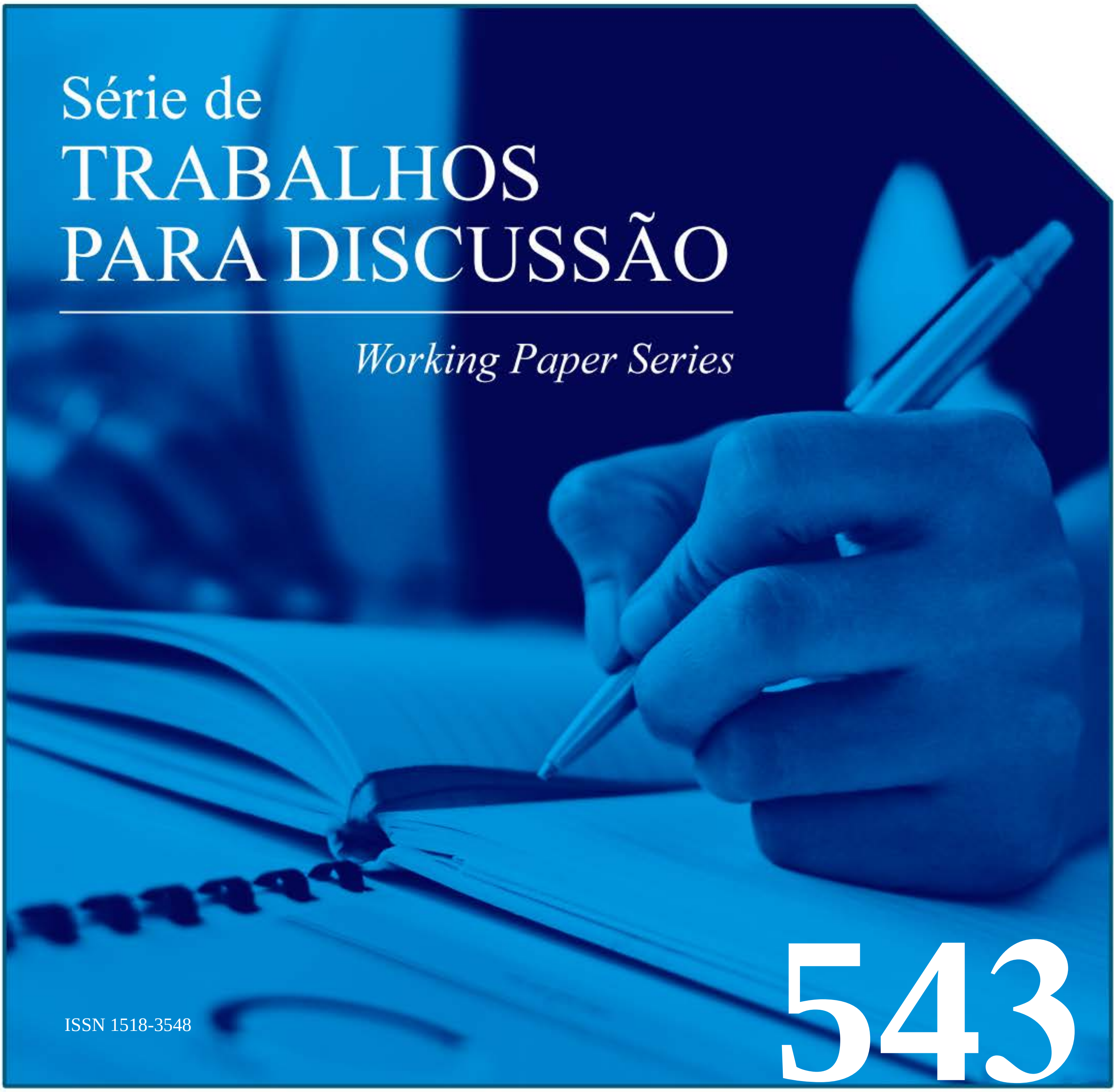

JANUARY 2021

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BANCO CENTRAL DO BRASIL

ISSN 1518-3548
CGC 00.038.166/0001-05

Working Paper Series	Brasília	no. 543	January	2021	p. 1-15

Working Paper Series

Edited by the Research Department (Depep) – E-mail: workingpaper@bcb.gov.br

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Non-Technical Summary

In the recent years, a broad class of arbitrage-free dynamic models has been proposed to fit nominal and real interest rates and then make forecasts of future inflation. These models rely on assumptions about the inflation risk premium, an unobserved quantity. In general, the estimation of arbitrage-free yield curve models is quite complex, suffering from the curse of dimensionality. In a model with only two factors, the number of parameters can reach 20. The likelihood surface will contain several plateaus and irregular behavior, making the optimization problem difficult.

This work proposes a new idea to circumvent the problem of estimating the inflation risk premium using arbitrage-free models. Instead of jointly modeling the nominal and real yield curves, as is usual in the literature, we study only the break-even inflation rate. As a result, there is a reduction in the number of factors and consequently an improvement in the quality of the estimation. However, this gain is not obtained for free. It was necessary to assume that the expected value of the exponential of the instantaneous inflation integral is equal to the break-even inflation rate. In other words, convexity terms have been neglected. Nonetheless, as several studies in the literature show, Jensen's correction is usually of small magnitude.

The results show the risk premium extracted by the proposed model is similar to that obtained by complex models (that is, model with a high number of factors). Furthermore, the model's inflation forecast errors reveal that it is a good competitor of traditional tools such as market and survey-based measures.

Sumário Não Técnico

Nos últimos anos modelos dinâmicos livre de arbitragem têm sido propostos para ajustar as taxas de juros nominal e real e em seguida realizar previsões da inflação futura. Esses modelos confiam em hipóteses sobre o prêmio de risco da inflação, uma grandeza não observada. Em geral, a estimação de modelos de curvas de juros livres de arbitragem é bastante complexa, sofrendo da maldição da dimensionalidade. Em um modelo com apenas dois fatores, o número de parâmetros pode chegar a 20. A superfície de verossimilhança conterá diversos platôs e comportamento irregular, dificultando o problema de otimização.

Este trabalho propõe uma nova ideia para contornar o problema de estimação do prêmio de risco de inflação usando modelos dinâmicos livre de arbitragem. Em vez de modelar conjuntamente a curva nominal e a curva real, como é de praxe na literatura, estudamos apenas a inflação implícita. Com isso, há uma redução do número de fatores e, consequentemente, uma melhora na qualidade da estimação. Mas esse ganho não é obtido de graça. Foi necessário supor que o valor esperado da exponencial da integral da inflação instantânea é igual à inflação implícita. Em outras palavras, termos de convexidade foram desprezados. Porém, como demonstram diversos trabalhos na literatura, a correção de Jensen geralmente é de magnitude pequena.

Os resultados mostram que o prêmio de risco extraído pelo modelo proposto é similar ao obtido por modelos complexos (isto é, modelos com um número maior de fatores). Além do mais, os erros de previsão de inflação do modelo revelam que ele é um bom competidor de ferramentas tradicionais, como a pesquisa Focus e a inflação implícita.

A Non-Knotty Inflation Risk Premium Model

José Valentim Machado Vicente*

Abstract

In this paper I estimate the inflation risk premium (IRP) using a low-dimensional arbitrage-free dynamic model through a novel strategy. Instead of modeling the nominal and real yields jointly, I make assumptions about the short-term inflation rate. More specifically, I assume it follows a Gaussian process. This framework has a closed-form expression for IRP. Since inflation yields are not observed, to estimate the model parameters I approximate them by the break-even inflation rate. This approximation works well because the convexity correction is very small. I find the estimated IRP is strongly correlated with those obtained using surveys or more complex models. Therefore, I provide an easier procedure to obtain IRP, avoiding the cumbersome estimation process of high-order models.

Keywords: Inflation risk premium, break-even inflation rate, affine models.
JEL Classification: E31, E43, E44.

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1 Introduction

The break-even inflation rate (the difference between nominal and real interest rates, hereafter BEIR) is a traditional estimator of future inflation. According to Söderlind (2011), BEIR has some advantages compared to other procedures usually employed to predict inflation. First, it is easily computed. Second, BEIR is based on prices, that is, on decisions involving gains and losses. Third, it is available on a daily basis, thus providing high frequency updating of inflation expectations. However, besides inflation expectation, BEIR contains the inflation risk premium (IRP). IRP is a component of nominal bond yields demanded by investors to compensate them for losses due to unexpected rise of prices.

Although IRP is a simple economic concept, its estimation is a challenging task. In this paper I propose a non-complex arbitrage-free dynamic term structure model to estimate IRP. More specifically, I adopt the affine model (see Duffie and Kan (1996) and Dai and Singleton (2002)) as the framework to capture the risk-neutral measure and the dynamics of the inflation rate. Of course, I am not the first to use dynamic term structure models to estimate IRP. A number of papers have addressed this issue through the lens of affine models (see, for example, Hördahl and Tristani (2007), Haubrich et al. (2012) and Vicente and Kubudi (2018)). However, the estimation of such models is cumbersome. In order to fit the nominal and real yields well, one needs a high-order interest rate process. Estimates of such models suffer from serious problems (see, for instance, Kim and Orphanides (2012) and Bauer et al. (2012)). Moreover, the risk premium parameters are the hardest to pin down, which complicates the IRP estimation.

My model circumvents these drawbacks. I address the problem in a novel way. IRP depends only on the dynamics of the instantaneous inflation rate process. Therefore, instead of modeling nominal and real yields, I just consider the instantaneous inflation rate as the variable of interest. Thus, I reduce the number of factors and parameters. As a result, I am able to make a robust estimation of the model. To the best of my knowledge, I am the first to attack the problem with this method. More specifically, I assume the instantaneous inflation rate follows a two-dimensional Vasicek (1977) model with an essentially affine market price of risk (Duffee, 2002). Vasicek (1977) model is in the Gaussian framework. Hence, the distribution of the integral of the instantaneous inflation rate is available in closed-form. Consequently, it is easy to compute the risk-neutral expected value of the discount factor. The negative of the logarithm of this expected value is denominated the inflation yield. However, inflation yields are not observed. Therefore, to estimate the model, I approximate the inflation yield by the BEIR, based on their similarity. The difference between them arises from the Jensen inequality, which implies a convexity correction in the computation of the expectation. Since the convexity correction is around 1 bps (see for instance, Ang et al. (2008) and Vicente and Kubudi (2018)), I assume inflation yields are equal to the BEIR. Using this approximation, I can use traditional methods such as maximum likelihood to estimate model parameters.

In the empirical exercise, I use data from the Brazilian economy. Some interesting features of the Brazilian inflation-linked bond market prompted this choice. First, it is the fifth largest fixed income market in the world. Second, the indexation lag of Brazilian real bonds is very small. While the US TIPS and UK Gilts have a lag of three months, Brazilian real bonds are lagged by only 15 days. Third, unlike TIPS, Brazilian real bonds are not protected against deflation. Deflation protection complicates the extraction of real yields from TIPS.

My main findings can be summarized as follows. First, on average, the IRP is less than 0.3% p.y. and its term structure is increasing. Second, it rises around turbulent periods, such as the subprime crisis and the political crisis that resulted in the impeachment of President Dilma Rousseff in 2016. Third, the IRP estimated in this work has high positive correlation with the one estimated by Vicente and Kubudi (2018), who studied the Brazilian nominal and real curves using a four-factor Gaussian model. Finally, the inflation forecasting performed using the estimated IRP has predictive power similar to that of the Focus survey and BEIR.[1] This last result is promising since it is well known that market and survey-based inflation forecasting outperform economic models (see Ang et al. 2007).

2 Model

In recent years, dynamic term structure models (DTSMs) have become the workhorse of interest rate modeling. DTSMs rely on assumptions about the short-term rate coupled with the absence of arbitrage hypothesis. A number of papers have used DTSMs to estimate the IRP (see, for instance, Joyce, Lildholdt and Sorensen (2010) and D'Amico, Kim and Wei (2010)). Basically, the idea of these papers is to specify the joint dynamics of nominal and real short-term interest rates in the risk-neutral and physical worlds. Next, the model is estimated using nominal and real bond data. Finally, the IRP is obtained. I face this problem by a simpler procedure. Instead of modeling nominal and real rates, I consider only the instantaneous inflation rate. Therefore, I decrease the dimensionality of the model, which in turn makes its estimation less complicated.

First I review some traditional measures of the bond market profitability. Let r_t^n be the instantaneous nominal rate. The time t nominal spot yield of time to maturity $\tau = T - t$ is defined as:

[1] Focus is a macroeconomic survey among professionals conducted by the Central Bank of Brazil.

$$y_t^n(\tau) = -\frac{\ln E_t^Q\left[e^{-\int_t^T r_u^n \mathrm{d}u}\right]}{\tau}, \tag{1}$$

where $E_t^Q[\cdot]$ is the risk-neutral expectation. Let m_t be the instantaneous inflation rate and I_t the corresponding price index. Then,

$$I_T = I_t e^{\int_t^T m_u \mathrm{d}u}. \tag{2}$$

I also define the real spot yield as:

$$y_t^r(\tau) = -\frac{\ln E_t^Q\left[e^{-\int_t^T r_u^r du}\right]}{\tau}, \tag{3}$$

where $r_t^r = r_t^n - m_t$ is called the instantaneous real rate.

Next, I assume m_t follows a two-dimensional Gaussian process:

$$m_t = \phi_0 + X_{1,t} + X_{2,t},$$

where the dynamics of the process $X_t = [X_{1,t} \;\; X_{2,t}]'$ is given by

$$\mathrm{d}X_t = -\kappa X_t \mathrm{d}t + \rho \mathrm{d}W_t^Q, \tag{4}$$

with $\kappa \in \mathbb{R}^{2\times 2}$ being a diagonal matrix, $\rho \in \mathbb{R}^{2\times 2}$ a lower triangular matrix and W_t^Q a standard two-dimensional Brownian motion under the risk-neutral measure Q.[2] The connection between the probability measure Q and objective probability measure P is given by Girsanov's Theorem, with an essentially affine market price of risk (Duffee, 2002):

$$\mathrm{d}W^P = \mathrm{d}W^Q - \left(\lambda^0 + \lambda^1 X_t\right)\mathrm{d}t, \tag{5}$$

where $\lambda^0 \in \mathbb{R}^{2\times 1}$ and $\lambda^1 \in \mathbb{R}^{2\times 2}$ is a diagonal matrix. Therefore,

$$\mathrm{d}X_t = \left(\kappa - \rho\lambda^1\right)\left[\frac{\rho\lambda^0}{\kappa - \rho\lambda^1} - X_t\right]\mathrm{d}t + \rho \mathrm{d}W_t^P = \tilde{\kappa}\left(\tilde{\theta} - X_t\right)\mathrm{d}t\rho \mathrm{d}W_t^P. \tag{6}$$

This model is in the affine class of DTSMs studied by Duffie and Kan (1996). The constant diffusion term of the Gaussian specification results in log-normality of factor dynamics, which facilitates the estimation of model parameters. Although constant volatility can be viewed as a caveat, this is not the case. Some papers have shown that Gaussian DTSMs have nice fitting properties (see, for instance, Dai and Singleton (2002) and Duf-

[2]The model identification is based on Dai and Singleton (2000).

fee (2002)) and outperform many stochastic volatility models.

The break-even inflation rate (BEIR) is the difference between nominal and real yields:

$$\eta_t(\tau) = y_t^n(\tau) - y_t^r(\tau). \tag{7}$$

BEIR is a market-based measure of expected inflation. It is a key variable for policymakers, who monitor its evolution daily.

The inflation rate from t to $T = t + \tau$ is defined as:

$$\pi_t(\tau) = \frac{1}{\tau}\ln\left(\frac{I_T}{I_t}\right) = \frac{1}{\tau}\left(\int_t^T m_u \mathrm{d}u\right). \tag{8}$$

I denote the expectation of $\pi_t(\tau)$ under Q and P conditioned on the information available up to time t by $\mu_t^Q(\tau)$ and $\mu_t^P(\tau)$, respectively. Therefore, IRP is given by:

$$IRP_t(\tau) = \mu_t^Q(\tau) - \mu_t^P(\tau).$$

Finally, I define the "inflation yield" as:

$$y^m(\tau) = -\frac{\ln E_t^Q\left[\mathrm{e}^{-\int_t^T m_u du}\right]}{\tau}. \tag{9}$$

Although $\eta_t(\tau)$, $\mu_t^Q(\tau)$ and $y^m(\tau)$ are closely related, they do not represent the same thing. Due to the convexity terms, there are differences between them. The normality property allows me to obtain simple expressions relating $\eta_t(\tau)$, $\mu_t^Q(\tau)$ and $y^m(\tau)$. Define the nominal and the inflation integrated factors $N_{t,T} = \int_t^T r_u^n \mathrm{d}u$ and $M_{t,T} = \int_t^T m_u \mathrm{d}u$, respectively. Note that under the assumptions of the Gaussian affine model, N and M follow normal distributions. Then $\mathrm{e}^{N_{t,T}}$ and $\mathrm{e}^{M_{t,T}}$ are log-normals. Therefore,

$$y_t^m(\tau) = \mu_t^Q(\tau) - \frac{1}{2\tau}\sigma_M^2, \tag{10}$$

$$\eta_t(\tau) = \mu_t^Q(\tau) - \frac{1}{2\tau}\left(\sigma_M^2 - 2\sigma_{N,M}\right), \tag{11}$$

and

$$\eta_t(\tau) = y_t^m(\tau) + \frac{1}{\tau}\left(\sigma_M^2 - \sigma_{N,M}\right), \tag{12}$$

where σ_M^2 and $\sigma_{N,M}$ are the variance of M and the covariance of M and N under the measure Q, respectively. From (12) note that BEIR is not equal to the inflation yield. Moreover, BEIR is not the risk-neutral expected value of the inflation rate. This point causes some confusion among practitioners. These three variables are related by convexity terms (variances and covariances on the right-hand sides).

Solving the stochastic differential Equation (6) leads to

$$\mu_t^P(\tau) = \phi^0 + \left[\tilde{\theta} + \frac{\tilde{\kappa}^{-1}}{\tau}\left(\mathrm{e}^{-\tilde{\kappa}\tau} - I_N\right)\left(\tilde{\theta} - X_t\right)\right]. \tag{13}$$

A similar expression applies to $\mu_t^Q(\tau)$, just by replacing $\tilde{\kappa}$ with κ and making $\tilde{\theta} = 0$.

Note that μ^Q and μ^P depend only on the m_t dynamics. Therefore, the calculation of the IRP also depends only on m_t parameters. Thus, to obtain IRP one does not need to know the data generation processes of nominal and real yields. This is a crucial point of this paper.

However, I do not observe any variable solely related to m. But I have market data of nominal and real yields. Thus, a common solution to estimate m_t parameters is to enlarge the model by including the dynamics of r_t^n. Of course, this solution increases the dimension, which leads to serious problems. Estimation of DTSMs of high dimension is a challenging task due to non-linearity and bad behavior of likelihood surfaces (see Hamilton and Wu, 2012). In order to keep the number of dimensions low, I propose a novel way to estimate m_t parameters. Observe that the difference between $\eta_t(\tau)$ and $y^m(\tau)$ just arises from the convexity correction. Many authors have tried to evaluate convexity terms. All in all, these papers show the convexity terms of the nominal and real yields are very small (around 1 bps).[3] Therefore, I approximate y^m (which depends only on m_t parameters) by BEIR (which is available from market data).[4]

To complete the set of equations, I need a formula to compute y^m. This can be done by solving a pair of ordinary differential equations (see, Duffie and Kan, 1996). In my simple model I have a close-form expression for y^m. According to Almeida and Vicente (2012):

$$\begin{aligned}\sigma_M^2 = {} & \textstyle\sum_{i=1}^2 \frac{1}{\kappa_i^2}\left(\tau + \frac{2}{\kappa_i}\mathrm{e}^{-\kappa_i\tau} - \frac{1}{2\kappa_i}\mathrm{e}^{-2\kappa_i\tau} - \frac{3}{2\kappa_i}\right)(\rho_{i1}^2 + \rho_{i2}^2) + \\ & +2\frac{1}{\kappa_1\kappa_2}\left(\tau + \frac{\mathrm{e}^{-\kappa_1\tau}-1}{\kappa_1} + \frac{\mathrm{e}^{-\kappa_2\tau}-1}{\kappa_2} - \frac{\mathrm{e}^{-(\kappa_1+\kappa_2)\tau}-1}{\kappa_1+\kappa_2}\right)(\rho_{11}\rho_{21} + \rho_{12}\rho_{22}).\end{aligned} \tag{14}$$

Substituting (14) and the equivalent expression of (13) for μ^Q in (10) results in a close-form expression for y^m.

[3] See for instance, Ang et al. (2008) and Vicente and Kubudi (2018).

[4] I also assume that BEIR is not influenced by the liquidity risk premium. Although this a controversy hypothesis, some papers support it for the Brazilian market (see, for example, Vicente and Graminho, 2015).

3 Data e estimation

I use data from the Brazilian market, which is one of the largest fixed income markets in the world. Specifically, I use monthly time series of Brazilian BEIR with maturities of 1, 2, 3, 4 and 5 years. The database covers the period from January 2007 to August 2019. BEIR is the difference between nominal and real yields. The nominal and real term structures are interpolated by the Svensson (1994) model based on the market prices of *Letras do Tesouro Nacional* and *Notas do Tesouro Nacional F* (Brazilian Treasury securities that allow investors to negotiate the nominal rate) and *Notas do Tesouro Nacional B* (inflation-linked bonds issued by the Brazilian Treasury).[5]

I estimate the model parameters using the maximum likelihood procedure proposed by Chen and Scott (1993). Since my model is two-dimensional, I need to choose two BEIRs to be priced without errors in order to obtain the latent factors. This choice is arbitrary. Thus, I test the ten possible two-combinations of the five BEIRs. I also consider many different identification strategies for the risk premium parameters. Finally, to avoid local maximums, I start the optimization from several different parameter vectors, and for each one, I search for the optimal point using the Nelder-Mead Simplex algorithm for nonlinear optimization and gradient-based optimization methods. Among all the possible combinations, the one that has the greatest likelihood was reached by inverting the 1- and 3-year BEIRs with $\lambda_1^0 = \lambda_{11}^1 = 0$. The 2-, 4- and 5-year BEIRs are assumed to be observed with Gaussian errors uncorrelated in the time dimension. Table 1 presents the estimated parameters (second line) and standard errors (third line). The standard errors were obtained by the BHHH method (see Davidson and MacKinnon, 1993). All parameters are significant at a 95% confidence level.

Table 1: Parameters and standard errors

ϕ_0	κ_1	κ_2	ρ_{11}	ρ_{21}	ρ_{22}	λ_0	λ_{22}
0.0725	0.0344	-0.0140	0.0142	1.0029	0.0901	-1.4329	-55.0667
0.0082	0.0033	0.0039	0.0011	0.1709	0.0421	0.7958	24.0264

Notes: This table presents parameter values (second line) and standard errors (third line) for the Gaussian model estimated by maximum likelihood. The 1- and 3-year BEIRs were priced exactly, and the other BEIRs were priced with i.i.d. Gaussian errors. Standard errors were obtained by the BHHH method.

4 Results

First, I discuss the model fit. The mean errors of estimated BEIRs with maturities of 2, 4, and 5 years are, respectively, 1.23 bps, -1.08 bps and -1.10 bps, while the absolute means

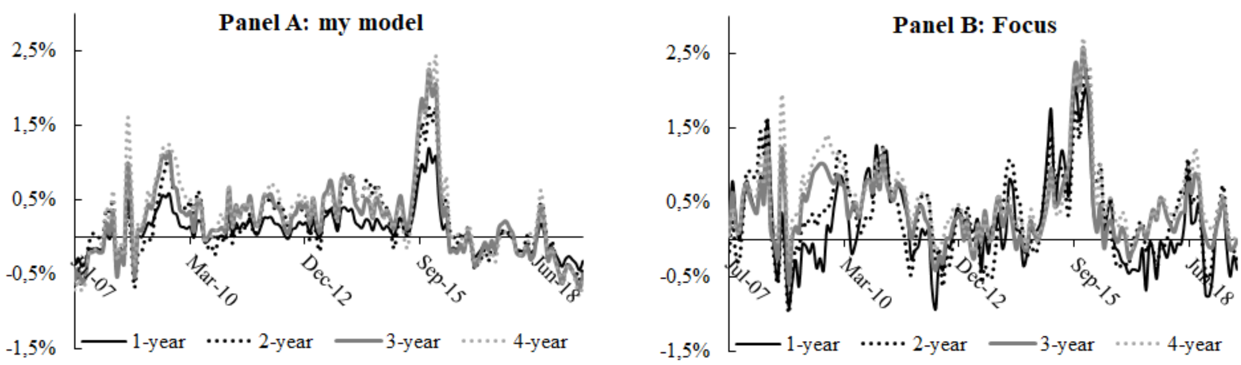
[5]For more details about Brazilian bonds, see Vicente and Kubudi (2018).

of these errors are 13.10 bps, 11.15 bps and 12.87 bps. These figures are around the bid-ask spread of Brazilian fixed income bonds (on average equal to five bps). Therefore, the model does a good job of fitting the BEIR term structure.

Next, I analyze the inflation risk premium. Figure 1 shows the time evolution of 1-, 2-, 3- and 4-year inflation risk premiums. Panel A presents the IRP implied by my model. The average IRP values are 0.07%, 0.15%, 0.21% and 0.26% for 1-, 2-, 3- and 4-year horizons, respectively. Therefore, the IRP term structure is upward-sloping. Note that IRP is not high.[6] However, at some moments, it reaches values greater than 1%. The IRP increase is associated with turbulent periods such as the subprime crisis (2008) and the Brazilian political crisis that resulted in the impeachment of President Dilma Rousseff (2016). Moreover, I observe negative IRP values at the beginning and end of the sample period. Although this finding is not expected, it was already reported by other studies (see, for instance, Haubrich et al. (2012) and Grishchenko and Huang (2013)).

Panel B presents the difference between BEIR (which is an approximation of μ^Q) and the Focus inflation forecast (which is an approximation of μ^P). The IRP implied by my model (Panel A) and the approximation of IRP obtained from Focus (Panel B) display similar patterns. The correlation coefficients support this observation, since they are equal to 42%, 44%, 71% and 76% for 1-, 2-, 3- and 4-year horizons, respectively. I also compare IRP implied by my model to the one obtained by Vicente and Kubudi (2018). Vicente and Kubudi (2018) propose a full model in which the nominal and real short-term rates follow a multidimensional Gaussian process. In this case, the correlations are even greater: 46%, 72%, 81% and 80%. All in all, these findings show evidence that my simple model capture the IRP dynamics.

Figure 1: Inflation risk premium



Notes: This figure shows the 1-, 2-, 3- and 4-year inflation risk premiums implied by my model and extracted from the Focus survey.

[6] In the sample period, the average monthly inflation rate is around 5.6% p.y. Therefore, the average IRP represents less than 5% of the inflation rate.

As a final exercise, I evaluate the inflation forecasting of my model. I compare the inflation expectation extracted from my model with the BEIR (market-based inflation forecast) and the Focus (survey-based inflation forecast). Table 2 presents the root mean square errors (RMSE) of these three competitors. Note that the RMSE are very similar across the competitors. Although at first glance this result may be disappointing, this not the case. Market and survey-based inflation are perfect benchmarks to test the predictive power of a model, since they outperform many forecasting methods (see, for instance, Ang et al., 2007). Therefore, a model not outperformed by them is a promising competitor.

Table 2: RMSE of inflation forecast

Horizon	My model	BEIR	Focus
1-year	1.55	1.53	1.60
2-year	1.50	1.49	1.43
3-year	1.39	1.42	1.38
4-year	1.25	1.13	1.40

Notes: This table reports the RMSE of the inflation forecasts of my model, BEIR and Focus survey for the horizons of 1, 2, 3 and 4 years.

5 Conclusion

In this paper, I propose a simple arbitrage-free model to estimate inflation risk premiums. Instead of modeling the nominal and real rates, I specify only the dynamics of the instantaneous inflation rate. In this way, I can work with a low-dimensional process which allows easier estimation of the parameters. In the estimation procedure, I ignore the convexity terms, since I do not observe inflation yields. However, I show this approximation has insignificant impact on the risk premium estimates. I find the inflation risk premium obtained by the proposed model is very close to the one extracted by the difference between the break-even inflation rate and inflation survey rate and the one estimated by a more complex model. Therefore, I show inflation risk premiums can be reliably estimated by a simple arbitrage-free model. Moreover, the proposed model is a promising inflation forecasting competitor with predictive ability similar to market and survey-based measures.

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