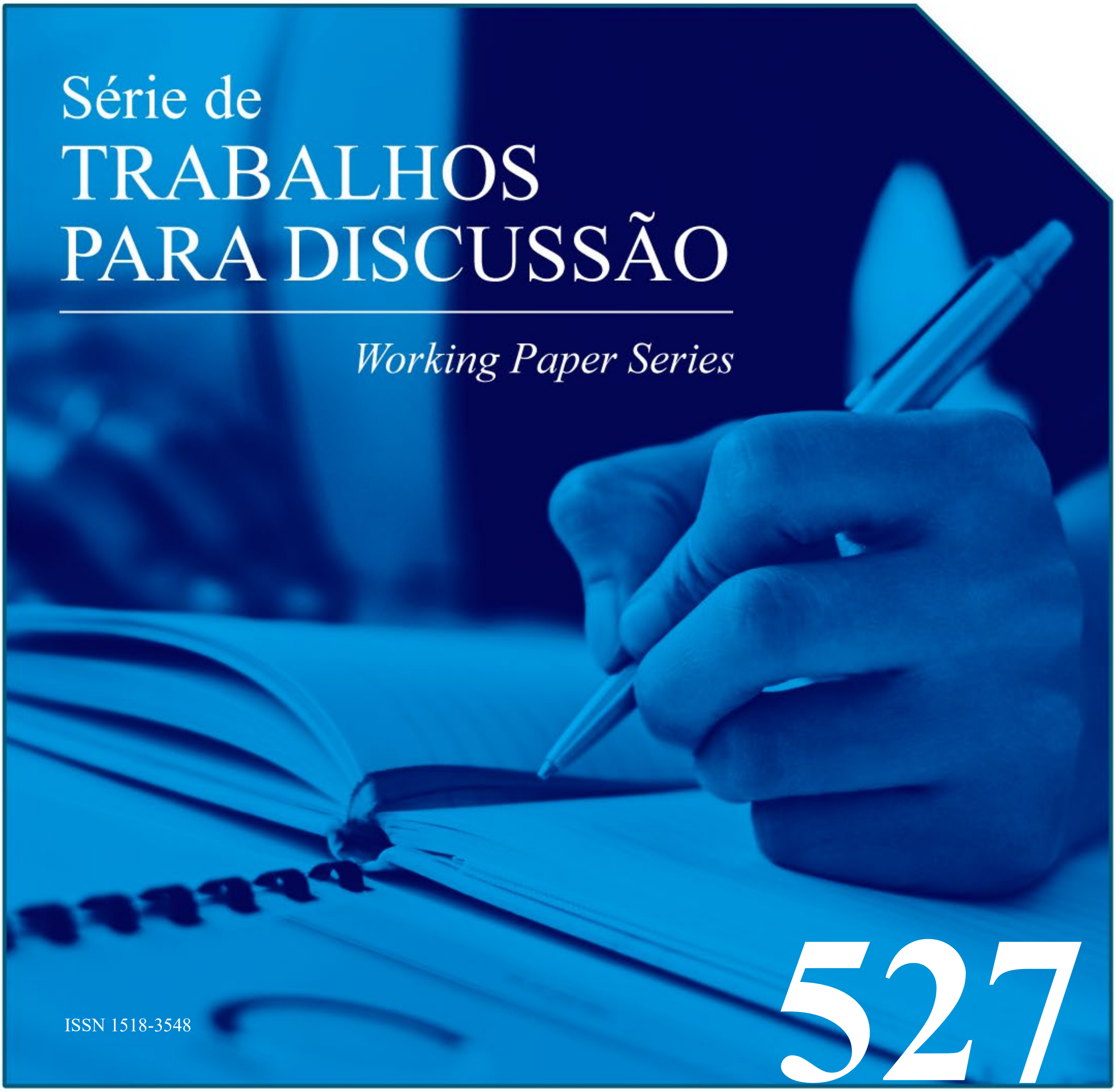

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Is the Equity Risk Premium Compressed in Brazil?

Alexandre de Carvalho, Thiago Trafane Oliveira Santos

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Deati/Diate

SBS – Quadra 3 – Bloco B – Edifício-Sede – 2º subsolo

70074-900 Brasília – DF – Brazil

Toll Free: 0800 9792345

Fax: +55 (61) 3414-2553

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Sumário Não Técnico

Uma premissa usual dos modelos econômicos é que os agentes são avessos ao risco. Logo, um investimento mais arriscado só será feito se o retorno que se espera for mais elevado. Analisando os dados brasileiros entre 1996 e 2017, verifica-se que o retorno do mercado de ações foi, de fato, superior ao retorno das aplicações de baixo risco. Contudo, o valor observado desse excesso de retorno, conhecido como *Equity Risk Premium* (ERP), é baixo frente ao observado em outros países. Existem apenas duas explicações possíveis para esse resultado:

i. o excesso de retorno esperado pelos investidores é baixo; e/ou
ii. o excesso de retorno observado foi menor do que o esperado pelos investidores. Como tais erros tendem a não ser sistemáticos, já que desvios persistentes levariam os investidores a corrigir suas expectativas, isso significa que a amostra de retornos não é longa o bastante para eliminar os erros de expectativas dos investidores.

Esse artigo procura avaliar essa primeira explicação, de excesso de retorno esperado baixo, utilizando duas abordagens distintas.

Na primeira abordagem avalia-se se a alta participação do governo no mercado acionário do Brasil estaria causando um baixo excesso de retorno esperado. Afinal, ao perceber benefícios não-monetários advindos desse investimento, o governo poderia requerer menores retornos monetários. Contudo, os resultados empíricos não corroboram essa tese, já que não se encontram evidências de que o governo exija um menor retorno monetário ao investir em ações. O que se encontra é que o fato de o governo ser o controlador de uma empresa tendeu a aumentar o risco percebido das ações dessa companhia, especialmente entre 2009 e 2016.

Na segunda abordagem inicialmente estima-se o excesso de retorno esperado das ações nos países emergentes e desenvolvidos. Essas estimativas foram então utilizadas para avaliar o excesso de retorno esperado no Brasil encontrado na primeira abordagem. Os resultados indicam que o excesso de retorno esperado das ações no Brasil é consistente com o excesso de retorno internacional.

Assim, os resultados de ambas as abordagens sugerem que o retorno requerido das ações no Brasil não é baixo e que, portanto, o baixo excesso de retorno observado é resultado da curta série de tempo disponível de retornos históricos do mercado acionário.

Non-technical Summary

A usual assumption in economic models is that agents are risk averse. Hence, a riskier investment will only be made if its expected return is higher. Analyzing the Brazilian data between 1996 and 2017, we find the return on the stock market was, in fact, higher than the return on low-risk investments. However, the observed excess return, known as the Equity Risk Premium (ERP), is low compared to those observed in other countries. There are only two possible explanations for this outcome:

i. the investors' expected excess return is low; and / or
ii. the observed excess return was less than the expected one. As such errors tend to be unsystematic, since persistent deviations would lead investors to correct their expectations, this means the sample of returns is not long enough to eliminate investors' expectations errors.

In this paper, we seek to evaluate this first explanation, of low expected excess return, using two different approaches.

In the first approach, we assess whether the high participation of the government in the Brazilian stock market is causing a low expected excess return. After all, as this investment may also provide non-monetary rewards to the government, it could require lower monetary returns to buy stocks. However, the empirical results do not corroborate this thesis, since we find no evidence the government requires a lower monetary return to invest in this market. What we find is that the perceived risk of firm's stocks tended to increase when the government is the controller, especially between 2009 and 2016.

In the second approach, we initially estimate the expected excess return in emerging and developed countries' stock markets. These estimates were then used to assess the expected excess return in Brazil found in the first approach. The results indicate the expected excess return of the Brazilian stock market is consistent with international excess returns.

Thus, the results from both approaches suggest the required stock return in Brazil is not low and, therefore, the low observed excess return is the result of the short time series of historical stock market returns.

Is the Equity Risk Premium Compressed in Brazil?[*]

Alexandre de Carvalho[**]

Thiago Trafane Oliveira Santos[***]

Abstract

The *ex-post* or historical equity risk premium in Brazil is low compared to other countries. In this paper we seek to evaluate whether this is a result of a compressed *ex-ante* equity risk premium, using two different approaches. First, we investigate the effects of government-controlled shareholders, which could lower the risk premium if the government is also interested in non-pecuniary benefits. To verify this, we estimate the Brazilian equity risk premium from 2002 to 2017 using cross-section regressions based on the CAPM and the Gordon model, but supposing stocks are priced differently by government and private investors. An important feature of this approach is that we control for the possible impact of the government as firm's manager on the perceived risk of the firm. Our results suggest the government does not compress the equity risk premium, although the government as a manager seems to influence the firms' risk. Second, we decompose the Brazilian equity risk premium using a global CAPM estimated with quarterly data from 47 countries and find it is consistent with international risk premia. Therefore, the findings from the two approaches indicate the low *ex-post* risk premium in Brazil seems to be a consequence of a relatively short time series rather than a Brazilian idiosyncrasy.

Keywords: Equity Risk Premium, CAPM, stock market, Brazil
JEL Classification: C33, G12, G14, G18

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[**] Central Bank of Brazil. E-mail: alexandre.carvalho@bcb.gov.br.
[***] Central Bank of Brazil. E-mail: thiago.trafane@bcb.gov.br.

1 Introduction

In Brazil, the annual geometric mean of the *ex-post* or historical equity risk premium (ERP) during the period of 1996-2017 is 0.9%[1], which is very low compared to the evidence from other countries. Table 1 shows historical ERP computed for 21 countries[2]. For the period 1900-2016, the average annualized ERP computed for this sample is 4.9%, well above our findings for the *ex-post* Brazilian ERP.

Table 1: Countries' historical ERP

Country	ERP 1900-2016 (%)	ERP 2000-2016 (%)
Australia	6.1	3.4
Austria	8.7	4.8
Belgium	3.0	3.1
Canada	4.2	3.9
Denmark	3.3	7.7
Finland	5.9	-1.3
France	6.0	1.4
Germany	5.7	1.9
Ireland	3.7	1.6
Italy	5.5	-2.2
Japan	6.1	0.7
Netherlands	4.4	0.1
New Zealand	4.5	4.0
Norway	3.2	5.6
Portugal	4.6	-2.3
South Africa	6.2	6.0
Spain	3.3	2.5
Sweden	4.1	4.3
Switzerland	3.6	2.6
United Kingdon	4.5	1.7
United States	5.6	3.2
Average	4.9	2.5

Assuming investors' expectations are unbiased, there are only two possible explana-

[1]Computed as the return of the IBrX 100 index less the effective Selic rate. Using the Ibovespa index, the annual geometric mean is -1.75%. Long-term rates as a proxy for the risk-free interest rate yields even lower risk premia.

[2]Countries' ERP are computed using data from Dimson et al. (2017), using the return on bills to measure the risk-free interest rate. Because of the lack of longer time series for emerging economies, the sample includes mostly advanced economies.

tions for this result. First, given that the *ex-post* ERP is very unstable, as shown in Figure 1[3], the time series may be not long enough to eliminate expectations' errors[4]. Second, the Brazilian *ex-ante* or expected ERP could be low. In this paper, we investigate the second possibility and find no evidence of a compressed *ex-ante* ERP, supporting the first explanation, of a short time series[5]. Indeed, Table 1 shows an average *ex-post* ERP of 2.5% for the short sample (2000-2016), a result that perhaps may also be explained by the limited period of computation. The absence of negative countries' ERP in the long sample (1900-2016) and the higher dispersion of countries' ERP in the short sample strengthen this hypotesis.

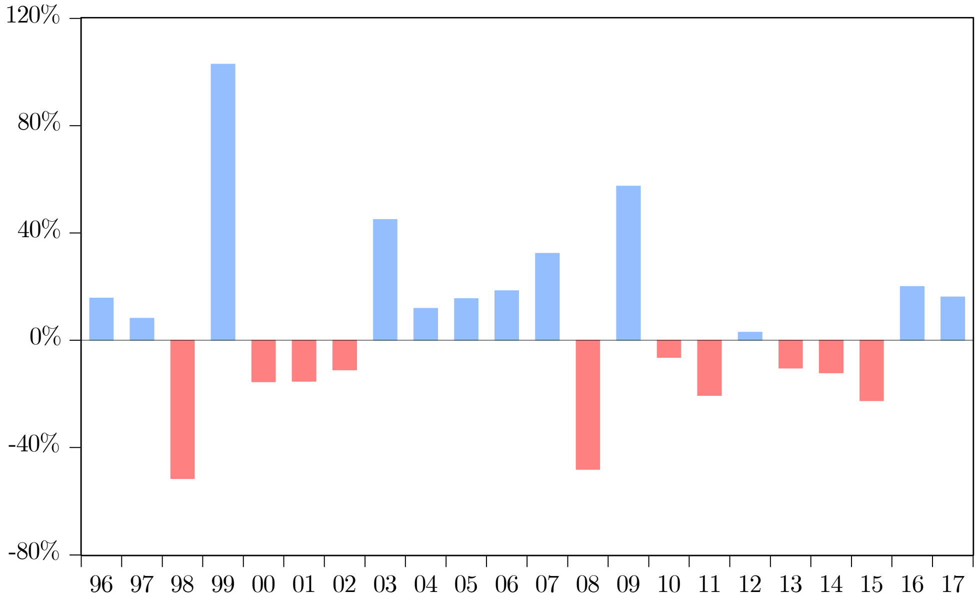

Figure 1: Brazilian annual historical ERP (1996-2017)

To evaluate the Brazilian *ex-ante* ERP, we use two different approaches. In the first one, we attempt to identify distinct characteristics of the Brazilian stock market that could justify a distorted *ex-ante* ERP. The more evident candidate seems to be the high share of the Brazilian government in the stock market. Buying stocks may provide monetary returns to the government and private investors, but for the government it may provide also non-monetary rewards. The government may use firms it controls to deliver social benefits or to pursue political objectives. Alternatively, by rising stock prices, the government

[3]This *ex-post* ERP is again computed as the return of the IBrX 100 index less the effective Selic rate. The volatility of this annual excess return can also be verified from its very high standard deviation, equal to 34.3%.

[4]This is related to the empirical evidence indicating that expected ERP estimates derived from short time series of *ex-post* ERP are highly volatile and sensitive to the time period (Damodaran (2012), Welch and Goyal (2007) and Dimson et al. (2017)).

[5]Risk premia in Brazil are usually estimated using short time series because monetary stability is a relatively recent phenomenon, starting in July 1994. Before that, the country experienced almost two decades of persistently high inflation, hyperinflation cycles and several monetary reforms.

may improve the public perception of the government performance. Since those non-pecuniary benefits are normally extracted by the government but not by the private sector, the former could accept lower expected monetary returns from stocks in comparison to private investors. Thus, this would imply a lower required or *ex-ante* ERP.

To verify this, we estimate the Brazilian ERP from 2002 to 2017 using quarterly cross-section regressions based on the CAPM and the Gordon model, as in Polk et al. (2006), but supposing stocks are priced differently by government and private investors. To correctly evaluate this effect of the government as an investor, we also control for the possible impact of the government as firm's manager on the perceived risk of the firm by assuming the true betas of the CAPM model could be different from the historical betas. Hence, our methodology innovates as it allows the estimation of the impact of the government on the stock market, both as investor and firm's manager. In order to do that, we identify which of the 2,684 shareholders of the 264 selected stocks comprising our sample are government-controlled.

The results from the first approach indicate the government does not affect the *ex-ante* ERP, although the government as a manager seems to influence the firms' risk. In fact, our estimates indicate the true betas for government-controlled firms were higher than the respective historical betas between 2009 and 2016, meaning investors required an extra-remuneration to finance these firms vis-à-vis what the historical risk measure would suggest.

In the second approach, we decompose the Brazilian ERP estimated in the first approach. To do that, we initially estimate the ERP of developed markets and emerging markets over the period 1995-2017, also through quarterly cross-section regressions, but using a sample of 47 countries' stock indexes rather than firms data, following Norges Bank (2016). Then, using a global CAPM, we decompose the Brazilian ERP into (i) developed markets effect, (ii) emerging markets effect, and (iii) Brazilian effect. Our findings point out the Brazilian ERP is consistent with international risk premia, as the Brazilian effect is not systematically different from zero.

Therefore, our findings from the two approaches indicate the *ex-ante* risk premium is not compressed in Brazil. As a result, the low *ex-post* risk premium seems to be a consequence of a relatively short time series, as argued by Giovannetti et al. (2016), rather than a Brazilian idiosyncrasy.

Our paper proceeds as follows. Section 2 discusses the estimation of the Brazilian ERP, including the methodology we use to identify government-controlled shareholders, and analyzes the impacts of the government in the stock market both as manager and investor. Section 3 presents the decomposition of the Brazilian *ex-ante* ERP estimated in the previous section using a global CAPM. Section 4 concludes.

2 Estimating the Brazilian ERP and the government impacts

2.1 Model

Following Norges Bank (2016), the literature in finance uses four main methods to estimate the *ex-ante* ERP: (i) long-term averages of the *ex-post* ERP; (ii) implied estimates from discounted dividend models; (iii) regression-based estimates; and (iv) surveys. We choose to estimate the ERP using cross-section regressions based on the CAPM and the Gordon's valuation model, as in Polk et al. (2006). By relying on the cross-section information, this method can be applied to short time series, and, as it is based on regressions, it is suitable for testing possible anomalies on the *ex-ante* ERP.

To obtain our model, let us start with a local standard CAPM,

$$E_t(ERP_i^{BR}) = \beta_{it}^{BR} E_t(ERP_M^{BR}) \tag{1}$$

where $E_t(\cdot)$ is the expectation operator conditional on the information set available at time t, ERP_i^{BR} represents the excess return of Brazilian stock i, ERP_M^{BR} is the excess return of the Brazilian market portfolio, and β_{it}^{BR} is a measure of the systematic risk of the stock, which assesses the sensibility of Brazilian stock i returns (R_{it}^{BR}) in relation to the returns of the Brazilian market portfolio (R_{Mt}^{BR}). Formally,

$$\beta_{it}^{BR} = \frac{cov(R_{it}^{BR}, R_{Mt}^{BR})}{\sigma^2(R_{Mt}^{BR})} \tag{2}$$

Thereafter our first approach innovates by adding a role for the government to affect the *ex-ante* ERP. As argued in the introduction, buying stocks may also provide non-monetary rewards for government investors. Thus, the government could accept lower expected monetary returns from stocks in comparison to private investors. A shortcut to incorporate this behavior into the CAPM framework is to assume government and private investors have different required or expected return on the stock market. In formal terms, instead of using the standard CAPM (equation (1)), we use

$$E_t^g(ERP_i^{BR}) = \beta_{it}^{BR} E_t^g(ERP_M^{BR}) \tag{3}$$

$$E_t^p(ERP_i^{BR}) = \beta_{it}^{BR} E_t^p(ERP_M^{BR}) \tag{4}$$

where $E_t^g(\cdot)$ and $E_t^p(\cdot)$ are the expectation operators conditional on the information set available at period t of government and private investors, respectively.

In this context of heterogeneous investors, $E_t(ERP_i^{BR})$ could be considered the expected excess return implied on the market price of the stock. If stock i is held only by private investors, it is natural to assume $E_t(ERP_i^{BR})$ equals $E_t^p(ERP_i^{BR})$. Analogous reasoning applies if the stock is owned only by the government. More generally, we could assume

$$E_t(ERP_i^{BR}) = \varphi_{it} E_t^g(ERP_i^{BR}) + (1-\varphi_{it}) E_t^p(ERP_i^{BR})$$
$$E_t(ERP_i^{BR}) = \beta_{it}^{BR}[E_t^p(ERP_M^{BR}) + \varphi_{it}\Omega_t] \quad (5)$$

where equations (3) and (4) are used in the last step, $\Omega_t = E_t^g(ERP_M^{BR}) - E_t^p(ERP_M^{BR})$, and $0 \leq \varphi_{it} \leq 1$ is a function f of the share of the Brazilian stock i owned by the government-controlled shareholders (GS_{it}^{BR}), being $f(0) = 0$ and $f(1) = 1$. Since f is unknown and could in principle be non-linear, we use ten different specifications. The first one is the linear specification, where $f(GS_{it}^{BR}) = GS_{it}^{BR}$. The other nine are dummy specifications: $f(GS_{it}^{BR}) = 1$ if $GS_{it}^{BR} > GS^c$ and 0 otherwise, where $GS^c = \{0.1, 0.2, ..., 0.9\}$.

In order to get unbiased estimates of the impact of the government as an investor on the *ex-ante* ERP, we should control for the possible impacts of the government as a manager on the firms' risk. After all, government-controlled firms may be seen as riskier than privately-controlled ones, given that the government may pursue goals not aligned with the ideal of maximizing its value.

Our conjecture is that this greater riskiness of government-controlled firms is reflected in higher individual betas[6]. We incorporate this by assuming that the effective betas used to price stocks are equal to the historical betas multiplied by a factor that depends on the controlling shareholder. Formally,

$$\beta_{it}^{BR} = \widehat{\beta}_{it}^{BR}(\gamma_t^p + d_{it}^g \omega_t) \quad (6)$$

where β_{it}^{BR} is the true or effective beta of Brazilian stock i, $\widehat{\beta}_{it}^{BR}$ is the historical beta of Brazilian stock i in relation to the Brazilian market portfolio, d_{it}^g is a dummy variable that equals one for government-controlled firms and zero for privately-controlled ones, γ_t^g and γ_t^p are beta adjustment factors for firms controlled by the government and private investors, respectively, and $\omega_t = \gamma_t^g - \gamma_t^p$.

According to equation (6), each investor, in computing the stocks' betas, makes a backward-looking analysis, based on the stocks' historical performance, and a prospective analysis, where the risks implied by the government management are accounted for. This approach to estimate the betas is another innovation of our methodology.

A property of the beta is that, for the market portfolio, it should equal one. Therefore, we need to impose some restriction on ω_t to guarantee that this holds for the true betas

[6]In a CAPM model the risk is reflected on the betas, justifying our assumption that the government risk alters the betas. In any case, we also tested a version where this risk is reflected on $E_t^g(ERP_M)$ and $E_t^p(ERP_M)$. When we estimate the model under this hypothesis, we find that the larger gaps between these two ERPs were systematically associated with increases in $E_t^g(ERP_M)$ and decreases in $E_t^p(ERP_M)$, a result more consistent with the assumption that the effects are reflected on the betas.

(β_{it}^{BR}). In Appendix A, we show this is ensured if

$$\omega_t = \frac{1-\gamma_t^p}{\theta_t^{BRg}\widehat{\beta}_t^{BRg}} \tag{7}$$

where θ_t^{BRg} is the share of government-controlled firms in the Brazilian market portfolio and $\widehat{\beta}_t^{BRg}$ is the historical beta computed for the portfolio of government-controlled Brazilian firms in relation to the Brazilian market portfolio.

Thus, substituting equation (7) into (6) and, then, into (5), we obtain

$$E_t(ERP_i^{BR}) = \widehat{\beta}_{it}^{BR}\left[\gamma_t^p\left(1-\frac{d_{it}^g}{\theta_t^{BRg}\widehat{\beta}_t^{BRg}}\right)+\left(\frac{d_{it}^g}{\theta_t^{BRg}\widehat{\beta}_t^{BRg}}\right)\right]\left[E_t^p(ERP_M^{BR})+\varphi_{it}\Omega_t\right] \tag{8}$$

In order to get a proxy for the left-hand-side of equation (8), we rely on Polk et al. (2006) and use the Gordon's valuation model (Gordon (1962)). It is a simpler dividend discount model (DDM)[7] as it assumes dividends grow at a constant rate. Under the Gordon's model,

$$E_t(ERP_i^{BR}) = \frac{D_{it}^{BR}[1+E_t(g_i^{BR})]}{P_{it}^{BR}} - rf_t^{BR} + E_t(g_i^{BR}) \tag{9}$$

where P_{it}^{BR} is the price of Brazilian stock i, D_{it}^{BR} is the dividend of Brazilian stock i, g_i^{BR} is the (constant) dividends growth rate of Brazilian stock i, and rf_t^{BR} is the Brazilian risk-free interest rate at time t.

Finally, substituting equation (9) into (8), we obtain our model:

$$\frac{D_{it}^{BR}}{P_{it}^{BR}} = \widehat{\beta}_{it}^{BR}\left[\gamma_t^p\left(1-\frac{d_{it}^g}{\theta_t^{BRg}\widehat{\beta}_t^{BRg}}\right)+\left(\frac{d_{it}^g}{\theta_t^{BRg}\widehat{\beta}_t^{BRg}}\right)\right]\left[E_t^p(ERP_M^{BR})+\varphi_{it}\Omega_t\right]-\psi_{it}+\varepsilon_{it} \tag{10}$$

where $\psi_{it} = E_t(g_i^{BR})\left(1+\frac{D_{it}^{BR}}{P_{it}^{BR}}\right) - rf_t^{BR}$[8]. In (10) we also add an error term (ε_{it}), with $\varepsilon_{it} \perp \varepsilon_{jt}\ \forall\ i \neq j$.

2.2 Estimation methodology and data

Equation (10) has three parameters that need to be estimated: γ_t^p, $E_t^p(ERP_M^{BR})$ and Ω_t. In principle, we could use Nonlinear Least Squares (NLS). However, it is reasonable to argue the errors are heteroskedastic across the stocks given, for instance, their different liquidity

[7]The DDM states the stock price equals the sum of the present value of the expected dividends.

[8]Polk et al. (2006) also uses others proxies than the dividend yield to measure the left-hand side of the equation. However, in these cases, they loose information about the magnitude of the ERP. Since estimating the level of the ERP is the main goal of this paper, we choose to rely only on the dividend yield, as the theoretical model (10) suggests. This empirical approach is also used in Norges Bank (2016).

levels. For this reason, we estimate equation (10) using Feasible Generalized Nonlinear Least Squares (FGNLS). We use robust standard errors to account for possible between period correlation. Since all parameters of equation (10) are time-variant, this model is essentially a cross-section model. The panel information is used only to estimate the cross-section weights of the FGNLS[9]. We use the software Eviews 10 for all estimations in this paper.

Our sample includes 264 stocks, listed in Appendix B, which were in the IBrX 100 index[10] for at least some period between 2002 and 2017. For this stock sample, we calculate quarterly proxies for the variables of equation (10) other than γ_t^p, $E_t^p(ERP_M^{BR})$, and Ω_t.

From the Economatica database, we extract , D_{it}^{BR}, P_{it}^{BR} and a proxy for $E_t(g_i^{BR})$. D_{it}^{BR} is the dividend per share of the last four quarters. P_{it}^{BR} is the average quarterly price of the stock. $E_t(g_i^{BR})$ is proxied by the return on equity multiplied by the share of dividends retained for reinvestments (the retention ratio)[11], both for the last four quarters, and both smoothed using the Hodrick-Prescott filter with the standard quarterly parameter of 1,600.

The risk-free interest rate rf_t^{BR} comes from the Gordon's model (9), which is a DDM. Thus, a good proxy for rf_t^{BR} would be the risk-free interest rate at the duration of the stocks. As this duration is expected to be large, we use a long term risk-free interest rate to measure rf_t^{BR}: the difference between the five-year interest rate and the four-year interest rate, both from the Pre-DI swap contract[12], extracted from Bloomberg.

To obtain $\widehat{\beta}_{it}^{BR}$, we initially calculate $\widetilde{\beta}_{it}^{BR} = \frac{\widehat{cov}\left(R_{it}^{BR}, R_{\widetilde{M}t}^{BR}\right)}{\widehat{\sigma^2}\left(R_{\widetilde{M}t}^{BR}\right)}$, where $\widehat{cov}\left(R_{it}^{BR}, R_{\widetilde{M}t}^{BR}\right)$ is the sample covariance between the return of Brazilian stock i (R_{it}^{BR}), extracted from Economatica, and the return of the IBrX 100 index ($R_{\widetilde{M}t}^{BR}$), extracted from Bloomberg, and $\widehat{\sigma^2}\left(R_{\widetilde{M}t}^{BR}\right)$ is the sample variance of $R_{\widetilde{M}t}^{BR}$. In this calculation, we use five-year rolling windows of quarterly data, discarding any stock with less than two years of return information in every window.

Because of these criteria and data constraints, our 64 cross-sections, one for each quarter between 2002 and 2017, present unequal sizes in terms of stocks, comprising an unbalanced panel. The cross-sections' size varies from 86 stocks (second quarter of 2003) to a maximum of 153 stocks (first quarter of 2012), totalizing 121 stocks per quarter on

[9]We also estimate the FGNLS on a cross-section approach, but this yields essentialy the same results of using NLS, as these cross-section weights are estimated using only one observation of each stock.

[10]The IBrX 100 is an index composed of the 100 stocks with the highest tradability in the Brazilian stock market. Our sample size is higher than 100 because the index portfolio is reviewed every four months, based on the assets' tradability along the prior twelve months.

[11]To see why this is a measure of the dividend growth rate in the long run, see Appendix C.

[12]The Pre-DI swap contract traded at the BM&FBovespa, the Brazilian Stock Exchange, is an interest rate swap where one of the parties agree to make pre-fixed interest payments in exchange for receiving floating interest payments based on the DI rate, whereas the other assumes a reverse position.

average.

As discussed in the previous section, a property of the beta is that, for the market portfolio, it should equal one, which implies that the weighted average of the stocks' betas should also equal one (for more details, see Appendix A). However, using $\widetilde{\beta}_{it}^{BR}$, this does not hold, for two reasons. First, our sample does not include all the stocks of the IBrX 100 index, although the weight of the stocks in our sample in the market index is higher than 95% in all quarters. Second, this property requires that the historical beta for a given period is computed using the market returns from the index composition of that period, not using the IBrX 100 index returns and its changing index composition, as we do.

In this context, $\widehat{\beta}_{it}^{BR}$ is obtained by correcting the results from $\widetilde{\beta}_{it}^{BR}$ in order to ensure that the weighted average of the betas equals one in each period, where the IBrX weights are adjusted to add to one across the stocks included in the sample[13]. This beta correction is done by multiplying each $\widetilde{\beta}_{it}^{BR}$ by a period-specific correction factor, including for the stocks that are not in the IBrX 100 index in that period. Intuitively, by using a new market portfolio reference, we alter the betas' scale, but this should not change the relative risk between any pair of stocks. For a more formal treatment of this issue, see Appendix D.

Based on the ownership structure of each stock and also on the classification of shareholders as government or privately-controlled, we can compute d_{it}^{g}, φ_{it}, θ_t^{BRg}, and $\widehat{\beta}_t^{BRg}$. The dummy variable d_{it}^{g} is equal to 1 for firms where the shareholders controlled by the government have at least 50% of the common shares, and 0 otherwise. φ_{it} is a function of GS_{it}^{BR}, where GS_{it}^{BR} is the share of government-controlled shareholders in each stock [14]. θ_t^{BRg} is the share of government-controlled firms ($d_{it}^{g} = 1$) on the IBrX 100 index. $\widehat{\beta}_t^{BRg}$ is the weighted average of $\widehat{\beta}_{it}^{BR}$ from government-controlled firms, being the weights adjusted to add to one across the stocks of our sample that belong to government-controlled firms.

While the ownership structure can be easily obtained from Economatica, the shareholder classification is not a straightforward process. This is discussed in the next section.

2.2.1 Identification of government-controlled shareholders

Initially, we extracted from Economatica database all the shareholders of the 264 selected stocks between 2002 and 2017. Following this procedure, we were able to list 2,684 shareholders, which could be individual investors, privately-held firms or publicly-held firms. We adopt a broad-based government definition, including central and regional

[13] In other calculations using IBrX weights, we always use these adjusted weights.

[14] For common and preferred stocks, we checked the government-controlled firms' ownership on the voting and non-voting stocks, respectively. As regards the units, we consider the government share in total stocks, regardless the mix of common and preferred stocks in each unit. In practice, this simplification is of little concern, since the market value of the units represents less than 5% of the IBrX 100 index in Brazil.

governments and public entities, as the Brazilian sovereign fund, state-owned firms and their pension funds (e.g., Caixa and Funcef), and the Brazilian Bank for Economic and Social Development (BNDES). Based on this database, we follow three steps to identify the government-controlled shareholders.

In the first step, we manually identify the shareholders controlled by selected firms, that is, firms whose stocks are included in our sample. This step required an in-depth research on the firms' equity structure using various sources, including Economatica, Bloomberg, newspapers and financial statements. In order to verify indirect controls, we try to reach, as far as possible, the layer of individual investors.

In the second step, we manually classified the shareholders that are not controlled by selected firms as government or privately-controlled, using the same manual procedure of the first step[15].

In the third step, we classify the shareholders that are controlled by selected firms as government or privately-controlled. However, this requires the classification of selected firms, which in turn demands the classification of their shareholders. Therefore, the identification is endogenous. As a result, it cannot be performed manually and must be solved simultaneously, using the information about the common shareholders' structure and the classifications from the two preceding steps[16].

We initiate this step constructing the following matrix of equity ownership of common stocks for each period:

$$\underset{(k+n)xn}{\Theta_t} = \overset{\text{Selected firms}}{\begin{bmatrix} \underset{kxn}{\Theta_t^{Others}} \\ ------ \\ \underset{nxn}{\Theta_t^{SF}} \end{bmatrix}} \text{Shareholders} \tag{11}$$

The first partition of matrix Θ_t, named Θ_t^{Others}, is a (kxn) matrix that shows the participation of each shareholder not controlled by selected firms on each selected firm's common stock. The second partition, named Θ_t^{SF}, is a (nxn) matrix that shows the participation of each selected firms on each selected firm's common stock, through the shareholders that they control. Accordingly, each column of matrix Θ_t add to one.

Our goal in the third step is to calculate a $(1xn)$ vector G_t^{SF}, where the j-th element

[15]The identification of the government-controlled shareholders was performed using various sources and not through a complete and structured database of shareholders, and for this reason it is subject to imprecisions. Additionally, in some cases the classifications "government-controlled" or "privately-controlled" were confirmed for some given period, but assumed for the entire period of our analysis, because of data constraints. This does not seem to be problematic, since we did not find evidence, along the period of our analysis, of shareholders that went from public to private control, or vice-versa.

[16]In Brazil, publicly traded firms are legally required to disclose the shareholders owning more than 5% of the common stocks, and their equity shares. These minority are treated here as private shareholders not controlled by selected firms.

is equal to one if the j-th selected firm is government-controlled, and zero otherwise. We assume government-controlled firms are those where the shareholders controlled by the government have at least 50% of the common shares. From the second step, we could obtain the $(1xk)$ vector G_t^{Others}, which similarly identifies, among the shareholders not controlled by selected firms, the government-controlled ones. Therefore, G_t^{SF} is defined by the following expression:

$$G_t^{SF} = I(G_t^{Others} * \Theta_t^{Others} + G_t^{SF} * \Theta_t^{SF}) \tag{12}$$

where $I(e_i) = 1$ if $e_i \geq 50\%$ and 0 otherwise.

In equation (12) G_t^{SF} appears in both sides. Given this endogeneity, we must simultaneously solve the problem of control identification for the selected firms and for the shareholders controlled by these firms. That is, we must find the vector G_t^{SF} that satisfies equation (12).

Since the indicator function $I(\cdot)$ is non-linear, equation (12) does not have an analytical solution, and should be solved numerically. The initial guess for G_t^{SF} is found using a second technique, the proportional method. This method distributes the shares of the partition Θ_t^{SF} for the shareholders of partition Θ_t^{Others} proportionally, creating a new (kxn) matrix for the participation of the shareholders not controlled by selected firms, named $\widetilde{\Theta}_t^{Others}$, whose columns add to 1. Then, calculating $G_t^{Others}\tilde{\Theta}_t^{Others}$, we obtain the government share in each selected firm and, consequently, a initial guess for G_t^{SF} using $I(\cdot)$.

This proportional distribution is the main advantage of this second approach to identify government-controlled firms, since it enables an analytical solution, as shown in Appendix E, but it is also the main weakness as the controls are not properly accounted for. For example, the proportional method could indicate that the government controls a selected firm because of minority participations in some shareholders.

As a final result, thirty selected companies were classified as government-controlled. Some of them remained in this condition during all the sample period (e.g., Petrobras, Banco do Brasil), others for only some quarters (e.g., Valefert)[17]. Panel A of Figure 2 presents the participation of government-controlled firms in the IBrX 100 index over time, which is declining from 2008 onwards (blue line). However, this does not seem to reflect government disinvestments in the stock market, but a decrease in the relative market value

[17]The thirty government-controlled firms are the following: Banco do Brasil, BB Seguridade, Banco Banestes, Nossa Caixa, BRF, Banco Banrisul, Companhia Energética de São Paulo (Cesp), Centrais Elétricas de Santa Catarina (Celesc), Caemi, Companhia Energética de Minas Gerais (Cemig), Companhia Paranaense de Energia (Copel), Companhia de Saneamento de Minas Gerais (Copasa), Liq, Eletrobras, Empresa Metropolitana de Águas e Energia (Emae), Valefert, Kepler-Weber, Light, Lupatech, Oi, Petrobras, Paranapanema, Companhia de Saneamento do Paraná (Sanepar), Sabesp, Telemar, Tele Norte, Companhia de Transmissão de Energia Elétrica Paulista, Tupy, Vale do Rio Doce, and Wiz.

of government-controlled firms. After all, the share of the government-controlled firms in the index would have remained practically constant, around 60%, had the firms' weight in the index been kept constant during the entire period at 2002Q1 levels (red line).

Panel B of Figure 2, in turn, shows the share of government-controlled shareholders in the IBrX 100 index. This is around 15%, much lower than that of the government-controlled firms (Panel A), suggesting that, rather than merely investing in the stocks of publicly-held companies, the main goal of the government in Brazil is to control them. Moreover, Panel B clearly shows the government as a non-prominent stock market investor, a preliminary result that weakens our initial credence that the government could compress the equity risk premium by investing in the stock market.

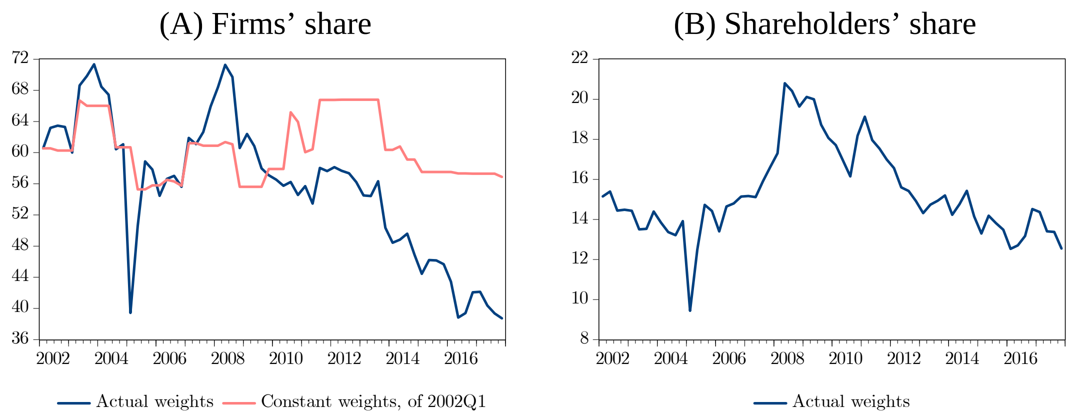

Figure 2: Share of government-controlled firms and shareholders in the IBrX 100 index

2.3 Results

Initially, we try to use equation (10) to estimate the parameters γ_t^p, $E_t^p(ERP_M^{BR})$, and Ω_t. However, although this model is consistent with the CAPM and the Gordon's model, we were not able to estimate it, as the paramaters did not converged. Investigating the reasons behind this, we find that the parameter that is multiplying ψ_{it} is not close to -1 as in equation (10). In fact, if we estimate a time-variant parameter ζ_{it} related to ψ_{it}, we would find estimates very close to zero (-0.03 on average) and not statistically significant at 10% in nearly 40% of the sample. Estimates were also statistically different from -1 in all periods.

This could indicate investors valuate assets using a rule of thumb. The extensive use of the dividend yield as a predictor variable for the equity risk premium, as documented in the literature (see, for instance, Damodaran (2012) and Ilmanen (2011)), assures some support for this understanding. Another possibility is that the expected dividend growth rate may be equal, or close to, the risk-free rate, as assumed in some papers (Norges Bank (2016), for example). As far as we know, there is no obvious reason for this result[18]. In

[18]Rozeff (1984) rationalizes the equality between dividend growth and risk-free interest rate using the

any case, we leave this issue for future extensions.

Whatever the reason for this fact, it seems to be empirically valid. As a result, we run a simpler version of equation (10), without ψ_{it}:

$$\frac{D_{it}^{BR}}{P_{it}^{BR}} = \widehat{\beta}_{it}^{BR} \left[\gamma_t^p \left(1 - \frac{d_{it}^g}{\theta_t^{BRg} \widehat{\beta}_t^{BRg}} \right) + \left(\frac{d_{it}^g}{\theta_t^{BRg} \widehat{\beta}_t^{BRg}} \right) \right] \left[E_t^p(ERP_M^{BR}) + \varphi_{it} \Omega_t \right] + \varepsilon_{it} \tag{13}$$

The estimates of $E_t^p(ERP_M^{BR})$ and $E_t^g(ERP_M^{BR})$ from 2002 to 2017 using equation (13), for the ten specifications of $f(GS_{it}^{BR})$ considered, are shown in Figure 3[19]. Panels A to I show the results for the dummy variable specification: $f(GS_{it}^{BR}) = 1$ if $GS_{it}^{BR} > GS^c$ and 0 otherwise, where $GS^c = \{0.1, 0.2, ..., 0.9\}$. Panel J presents the results for the linear specification: $f(GS_{it}^{BR}) = GS_{it}^{BR}$.

Our results do not support the view that the government compresses the *ex-ante* ERP. First, Ω_t is not statistically significant in most of the quarters, suggesting that government and private sector have similiar required ERP. Second, $E_t^g(ERP_M^{BR})$ remains above $E_t^p(ERP_M^{BR})$ in most periods, which is the opposite of what we would expect if the government were squeezing the *ex-ante* ERP in Brazil[20].

Given the previous results, we regress the model stated in equation (13) without the assumption that stocks are priced differently by the government and private investors. We still suppose the government as a manager may affect the stocks' beta. Formally,

$$\frac{D_{it}^{BR}}{P_{it}^{BR}} = \widehat{\beta}_{it}^{BR} \left[\gamma_t^p \left(1 - \frac{d_{it}^g}{\theta_t^{BRg} \widehat{\beta}_t^{BRg}} \right) + \left(\frac{d_{it}^g}{\theta_t^{BRg} \widehat{\beta}_t^{BRg}} \right) \right] E_t(ERP_M^{BR}) + \varepsilon_{it} \tag{14}$$

theory of economic growth, but under very restrictive conditions. It is necessary to assume the framework of the Solow growth model, a savings rate consistent with the golden rule for consumption, and the absence of risk premia of any nature.

[19]The model that includes $\zeta_{it}\psi_{it}$ yields quite similar results for all ten specifications. The more relevant differences occur in 2002 and 2003, when the ERP estimates from equation (13) are always more precise, justifying our choice for the simpler model, without ψ_{it}. See Appendix F for a comparison between these two models for the specification where $f(GS_{it}^{BR}) = GS_{it}^{BR}$. In this appendix we also show the results of these two models using NLS, whose estimates are more noisy and less precise.

[20]Considering the nine dummy specifications, we reject the null $\Omega_t = 0$ at 10% only in 23% of the quarters, at most ($GS^c = 0.5$ case). Furthermore, in only 13% of the quarters the rejection of the null suggests the government is squeezing the *ex-ante* ERP, again at most ($GS^c = 0.1$ case). Regarding the linear specification, the null is not rejected in 86% of the quarters, and in none of the quarters where the null is rejected the government is squeezing the *ex-ante* ERP.

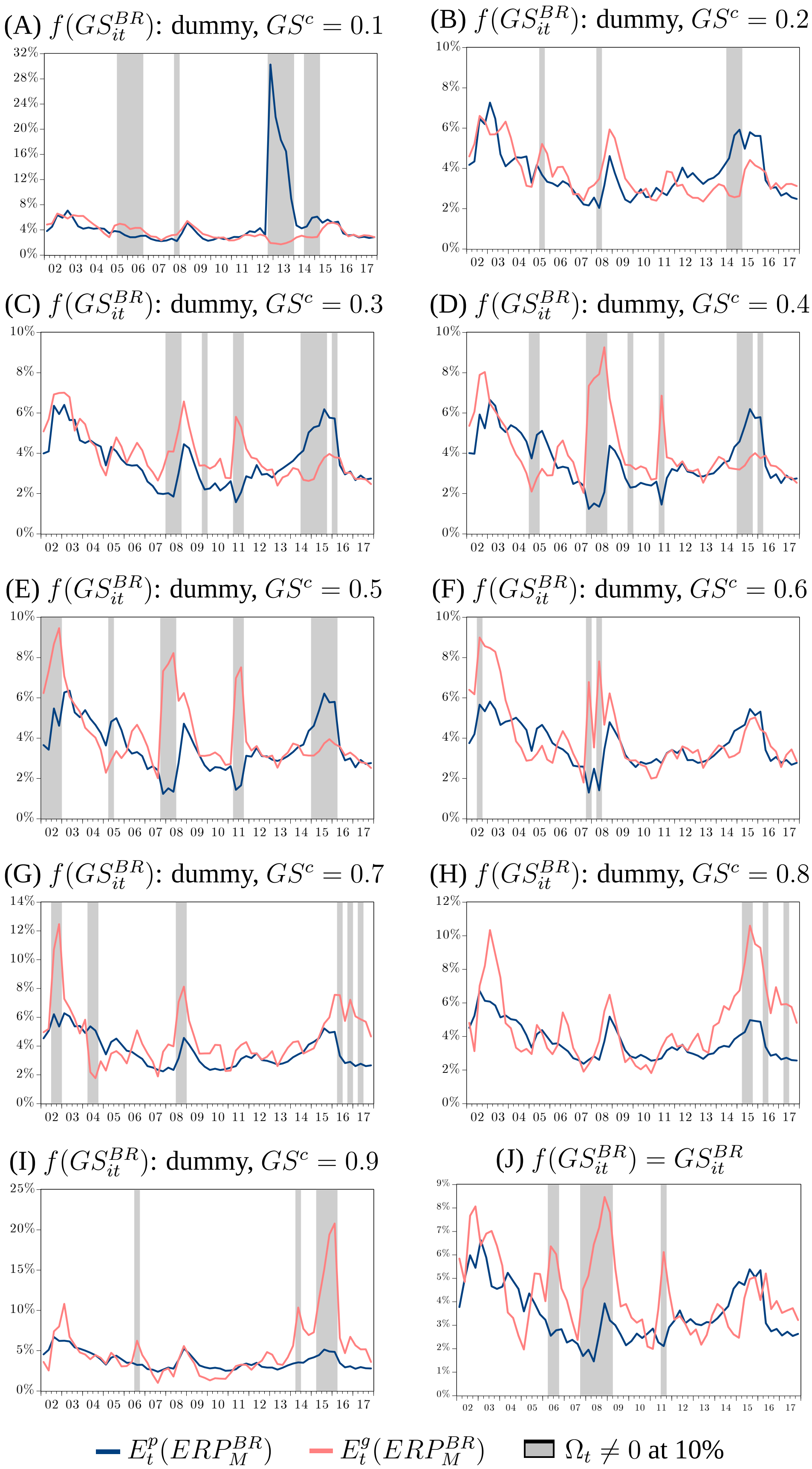

Figure 3: Brazilian ERP estimates: model with government control both as manager and investor

Figure 4 shows the $E_t(ERP_M^{BR})$ estimates using (14) along 2002-2017[21]. The geometric mean of $E_t(ERP_M^{BR})$ is 3.74%, much above the geometric mean of the *ex-post* ERP. All the estimates are statistically significant at 10%. As already documented[22], the ERP in Brazil peaked in three moments: during the confidence crisis prior to the national election of 2002, during the Global Financial Crisis (GFC) and along the political crisis that took place in Brazil between 2013 and 2016.

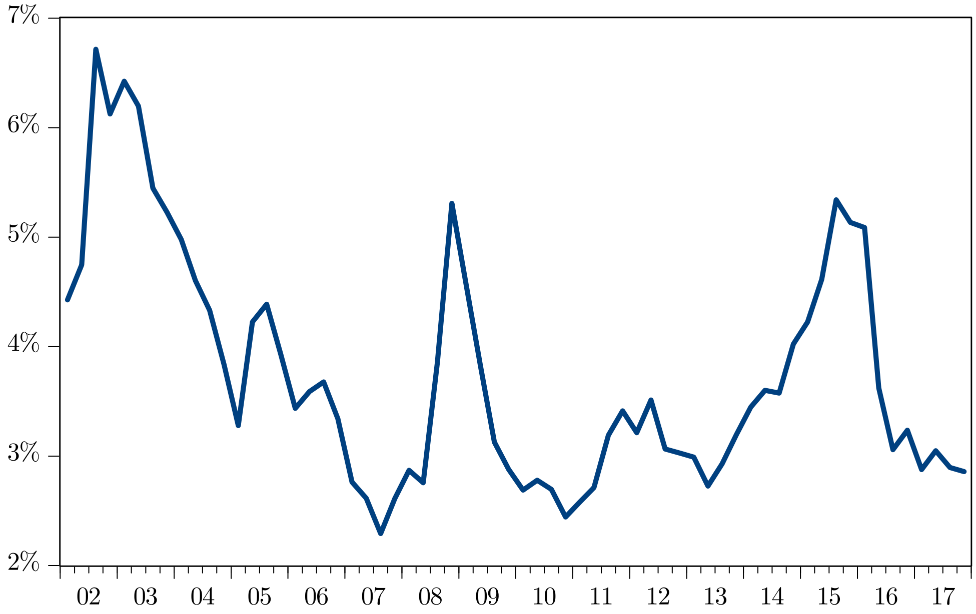

Figure 4: Brazilian ERP estimates: model with government control only as manager

Yet, our results point out that the government as manager do influence the firms' risk. Figure 5 shows the estimates of γ_t^g, from equation (14). For government-controlled firms, the true betas are systematically and statistically higher than the historical betas ($\gamma_t^g > 1$) between 2009 and 2016, meaning that investors required an extra-remuneration to finance these firms vis-à-vis what the historical risk measure would suggest. This result probably reflects the political crisis in Brazil. Similar results can be obtained also using the ten specifications of the more complete model (13), as shown in Appendix H.

[21]This model is linear in the parameters and, thus, it is estimated using Feasible Generalized Least Squares (FGLS). In Appendix G, we show some robustness tests. Initially, we include in equation (14) ψ_{it} multiplied by -1, as in the theoretical model (10). In this case, we were able to estimate it, but the results are not economically reasonable. This fact can be understood when we estimate (14) with $\zeta_{it}\psi_{it}$. After all, as in the case of the more complete model (13), the estimates of ζ_{it} are very close to zero (-0.03 on average) and not statistically significant at 10% in nearly 40% of the sample. Once more, the estimates of ζ_{it} are statistically different from -1 in all periods. Also similarly, the estimates of the ERP are very close to those of equation (14), except between 2002 and 2003, when the model without ψ_{it} is also more precise. These three specifications of (14) are also estimated using Ordinary Least Squares (OLS), whose estimates are again less precise and more noisy.

[22]See Sanvicente and Carvalho (2016).

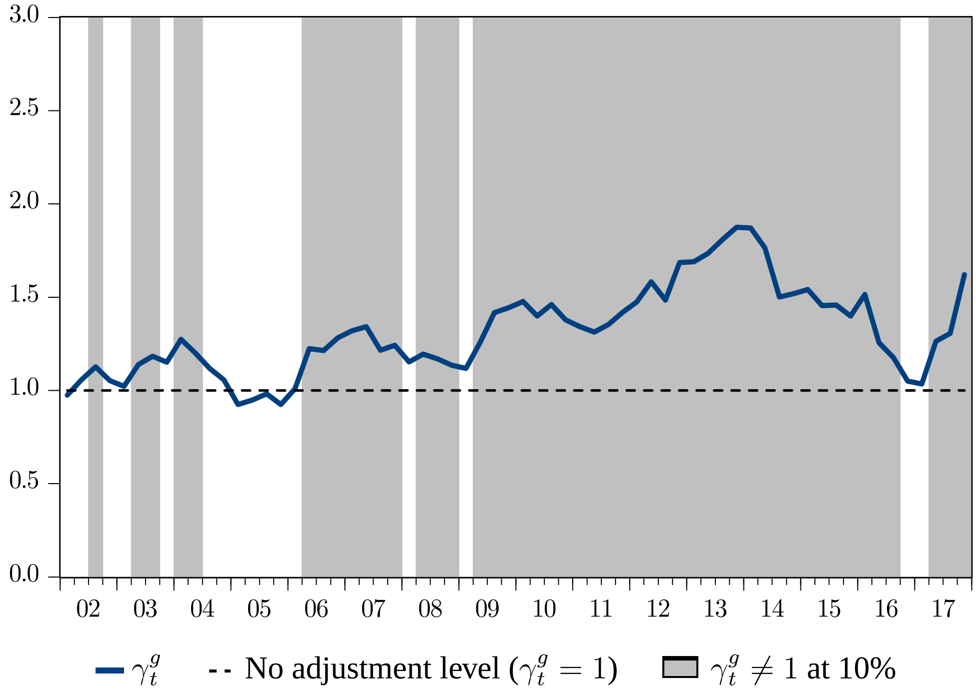

Figure 5: Betas' adjustment factor: model with government control only as manager

Since γ_t^g is statistically different from 1 in most of the sample (grey area of Figure 5)[23], the model where the government as manager affects the stocks' beta seems to be the most adequate to estimate $E_t(ERP_M)$. To access the importance of this government effect on the estimation of the ERP, we also estimate a model without any government control:

$$\frac{D_{it}^{BR}}{P_{it}^{BR}} = \widehat{\beta}_{it}^{BR} E_t(ERP_M^{BR}) + \varepsilon_{it} \tag{15}$$

Figure 6 shows the results for this model without government controls (equation (15)) and for the model with government control only as a manager (equation (14)). The correlation between these two estimates is elevated, of 0.967, but the ERP estimated with this government control is systematically higher. Hence, if we ignore this government effect, the ERP estimates will be biased. One advantage of (15) is it can be estimated since 1998 as it does not require government control data. Given the high correlation between the two estimates, one can evaluate the dynamic of the ERP between 1998 and 2001 from this simpler model estimates. As can be seen in Figure 6, these results indicate a sharp rise in the ERP in the second semester of 1998, which coincides with the period of exchange rate crisis in Brazil.

[23]We found γ_t^g after the estimation of γ_t^p using equations (6) and (7) and the parameters θ_t^{BRg} and $\widehat{\beta}_t^{BRg}$. Given these two equations, it is easy to show the test statistic for the null hypothesis $\gamma_t^g = 1$ is the additive inverse of those of testing $\gamma_t^p = 1$. Hence, since the test distribution is symmetric, we use the p-value of the test $\gamma_t^p = 1$ to obtain the significance of $\gamma_t^g - 1$.

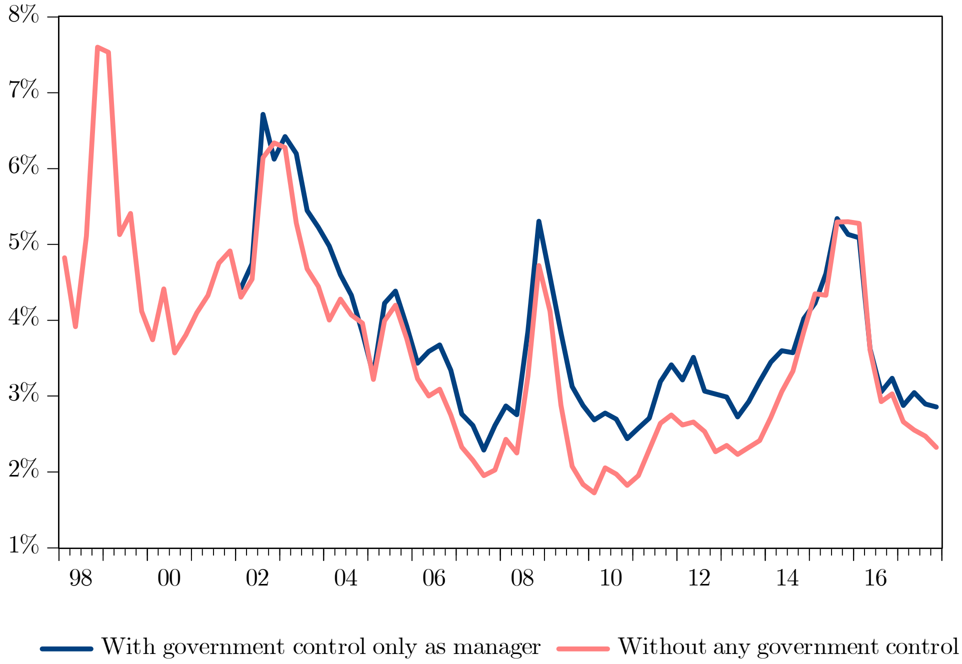

Figure 6: Brazilian ERP estimates: model with government control only as manager vesus model without any government control

3 Decomposition of the Brazilian ERP

In this section, we decompose the *ex-ante* ERP shown in Figure 4 and estimated using the model (14) that considers the effect of the government as a manager on stocks' betas. This decomposition is a second approach to identify possible idiosyncrasies in the Brazilian ERP. We initially estimate the ERP for developed economies and for emerging markets, also through quarterly cross-section regressions, but using a sample of countries' stock indexes rather than firms data, as in Norges Bank (2016)[24]. We follow two steps.

Firstly, we estimate quarterly betas for every country of our sample relatively to the MSCI All Country World Index (MSCI ACWI), using MSCI data of stocks returns in US dollar, extracted from Bloomberg[25]. Since we do not have data on the weights of the index, we do not perform any correction on these estimates.

Secondly, we estimate the *ex-ante* ERP using a global CAPM. We assume the histori-

[24]Our set of emerging markets is comprised of the 24 countries used to compute the MSCI Emerging Markets Index: Brazil, Chile, Colombia, Mexico, Peru, Czech Republic, Egypt, Greece, Hungary, Poland, Qatar, Russia, South Africa, Turkey, United Arab Emirates, China, India, Indonesia, Korea, Malaysia, Pakistan, Philippines, Taiwan, and Thailand. For developed economies, we used the sample of the MSCI World Index, which includes 23 countries: Canada, United States, Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Israel, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, Australia, Hong Kong, Japan, New Zealand, and Singapore.

[25]More formally, we regress the return of each country MSCI index on the return of the MSCI ACWI using a rolling five-years window, discarding any stock with less than two years of return information in every window. We also tested rolling ten-years windows, but the differences were not relevant for the decomposition of the Brazilian ERP.

cal betas are the true betas, but we do not impose the *ex-ante* ERP of emerging and developed markets are the same. After all, the investors' risk-aversion of each market may be different (e.g., several institutional investors are obligated to allocate their resources only in safer assets, usually located in advanced economies). Formally,

$$E_t(ERP_M^c) = \widehat{\beta}_t^c \left[E_t^{DM}(ERP_M) + d_{ct}^{EM}\delta_t\right] \tag{16}$$

where ERP_M^c is the excess return of the market portfolio of country c, $\widehat{\beta}_t^c$ is the historical beta of country c in relation to the global market portfolio, ERP_M is the excess return of the global market portfolio, $E_t^{DM}(\cdot)$ and $E_t^{EM}(\cdot)$ are the expectation operators conditional on the information set available at period t of developed and emerging markets investors, respectively, d_{ct}^{EM} is a dummy variable that equals one if the country is an emerging market and 0 otherwise, and $\delta_t = E_t^{EM}(ERP_M) - E_t^{DM}(ERP_M)$.

As done in Section 2, we use the dividend yield as proxy for the *ex-ante* ERP:

$$\frac{D_t^c}{P_t^c} = \widehat{\beta}_t^c \left[E_t^{DM}(ERP_M) + d_{ct}^{EM}\delta_t\right] + \epsilon_{ct} \tag{17}$$

where ϵ_{ct} is an error term, with $\epsilon_{it} \perp \epsilon_{jt} \ \forall \ i \neq j$, and $\frac{D_t^c}{P_t^c}$ is the dividend yield of country c, which is computed by the MSCI and extracted from Bloomberg[26].

Figure 7 shows the estimates for the *ex-ante* ERP of developed and emerging economies using equation (17) between 1995 and 2017[27]. We also plot the Brazilian ERP estimated previously using (14). As can be seen, all three ERPs rise during the GFC. Moreover, the ERP for developed economies lies systematically above the ERP for emerging markets, which is an unexpected result.

Given these estimates, we can decompose the Brazilian *ex-ante* ERP based on (16):

$$E_t(ERP_M^{BR}) = \widehat{\beta}_t^{BR} E_t^{DM}(ERP_M) + \widehat{\beta}_t^{BR}\delta_t + PBR_t \tag{18}$$

where $E_t(ERP_M^{BR})$ is the Brazilian *ex-ante* ERP estimated in Section 2 using equation (14), $E_t^{DM}(ERP_M)$ is the *ex-ante* ERP of developed markets estimated in this section, $\delta_t = E_t^{EM}(ERP_M) - E_t^{DM}(ERP_M)$ is also estimated in this section, $\widehat{\beta}_t^{BR}$ is the beta

[26] Norges Bank (2016) also uses equation (17), but without the term $d_{ct}^{EM}\delta_t$, which is not a problem there as its sample only includes developed countries.

[27] This linear model is estimated using FGLS. Again, we use robust standard errors to account for between period correlation. In Appendix I we present some robustness tests with this global model. As in section 2, we add $\zeta_t\psi_t$ to the model, finding again this parameter is very close to zero, being statistically different, at 10%, from -1 in all periods and from 0 in less than 25% of the quarters. As a consequence, the model without ψ_t yields much more similar results to the model with $\zeta_t\psi_t$ than the model that includes $-\psi_{it}$, as the theory would suggest. Furthermore, the estimates of the model that ignores ψ_{it} are more precise. We also present estimates of $E_t^{DM}(ERP_M)$ and $E_t^{EM}(ERP_M)$ of these specifications using OLS, which yields relatively similar estimates of the FGLS case, but with less precision.

coefficient of the Brazilian market relative to the MSCI ACWI[28], and PBR_t measures the differences in the *ex-ante* Brazilian ERP estimated using the local Brazilian model (14) and the global model (17), being calculated by residual.

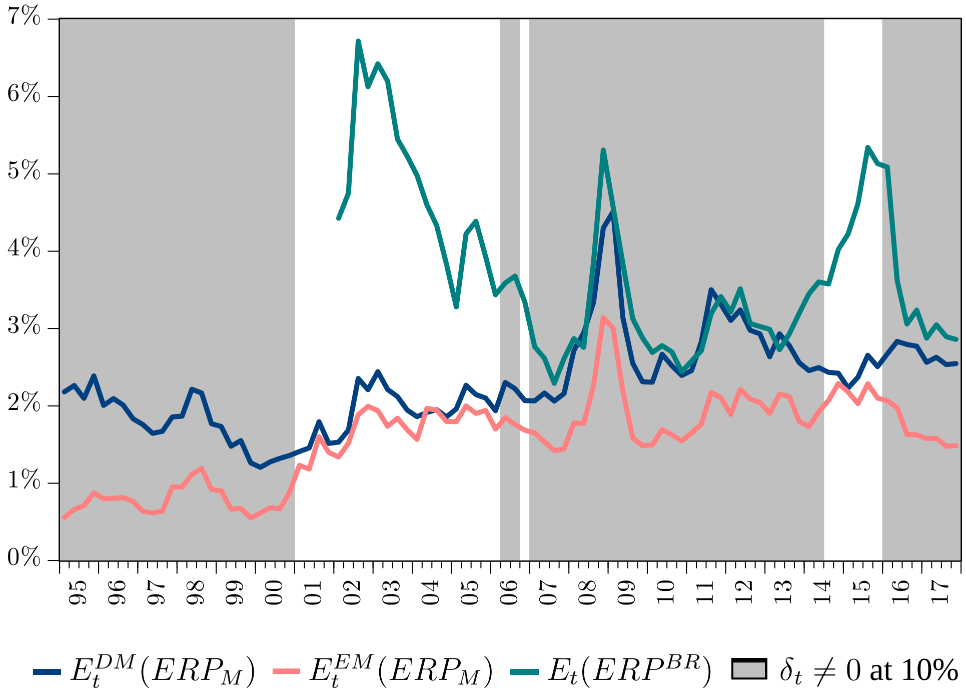

Figure 7: ERP estimates - Brazil, Developed Economies and Emerging Markets

From (18), we decompose $E_t(ERP_M^{BR})$ into (i) $\widehat{\beta}_t^{BR} E_t^{DM}(ERP_M)$, a developed markets effect, (ii) $\widehat{\beta}_t^{BR}\delta_t$, an emerging markets effect, and (iii) PBR_t, a country-specific effect. PBR_t should equal zero if Brazilian *ex-ante* ERP is consistent with international risk premia under a global CAPM. Thus, evaluating PBR_t is the main goal of this section.

In this regard, note, from equation (18), in order to calculate PBR_t for a given estimate of $E_t(ERP_M^{BR})$, all we need is a reliable estimate of the term $(\widehat{\beta}_t^{BR} E_t^{DM}(ERP_M) + \widehat{\beta}_t^{BR}\delta_t)$. This has an important implication. We argued in this section that the differences between emerging and developed markets are reflected in different expectations about the excess return of the global market portfolio (ERP_M), which is a shortcut to incorporate different investors' risk-aversion in the CAPM framework. However, the differences of these two groups could be reflected on the betas, as countries' perceived risk could depart from the historical risk, as discussed in Section 2 for the case of government-controlled firms. This being the case, our estimates of $E_t^{DM}(ERP_M)$ and δ_t based on equation (18) are biased, but our estimate of $(\widehat{\beta}_t^{BR} E_t^{DM}(ERP_M) + \widehat{\beta}_t^{BR}\delta_t)$ is not. Intuitively, the use of a bad proxy for the true beta is compensated in the estimate of the *ex-ante* ERP of each

[28] Since $E_t(ERP_M^{BR})$ is estimated in Section 2 based on the IBrX 100 index, $\widehat{\beta}_t^{BR}$ used in equation (18) is calculated using the return of this index, in dollar. In any case, this beta and the beta using the Brazilian MSCI index are very close.

group. As a result, our estimate of PBR_t is robust to both specifications, although the distinction between developed and emerging markets effects is not. For a more formal treatment to this issue, see Appendix J.

Based on equation (18), Figure 8 shows the decomposition of the *ex-ante* Brazilian ERP. Initially, note the increase in $E_t(ERP^{BR})$ around the GFC is mainly due to the developed economies effect (blue bar), which is an expected result since this crisis hit more severely the advanced economies. In contrast, an unexpected result is that the emerging markets effect (red bar) is negative between 2002 and 2017, which is consistent with the results shown in Figure 7. It becomes even more negative after 2008, which could be due to the nature of the GFC. Regarding the Brazilian effect (green bar), it is not systematically different from zero, being positive in some periods (2002-2003 or 2013-2016) and negative in others (2004-2007 or 2017). Thus, this empirical evidence does not support the view of a compressed *ex-ante* ERP in Brazil, as it is essentially consistent with international risk premia.

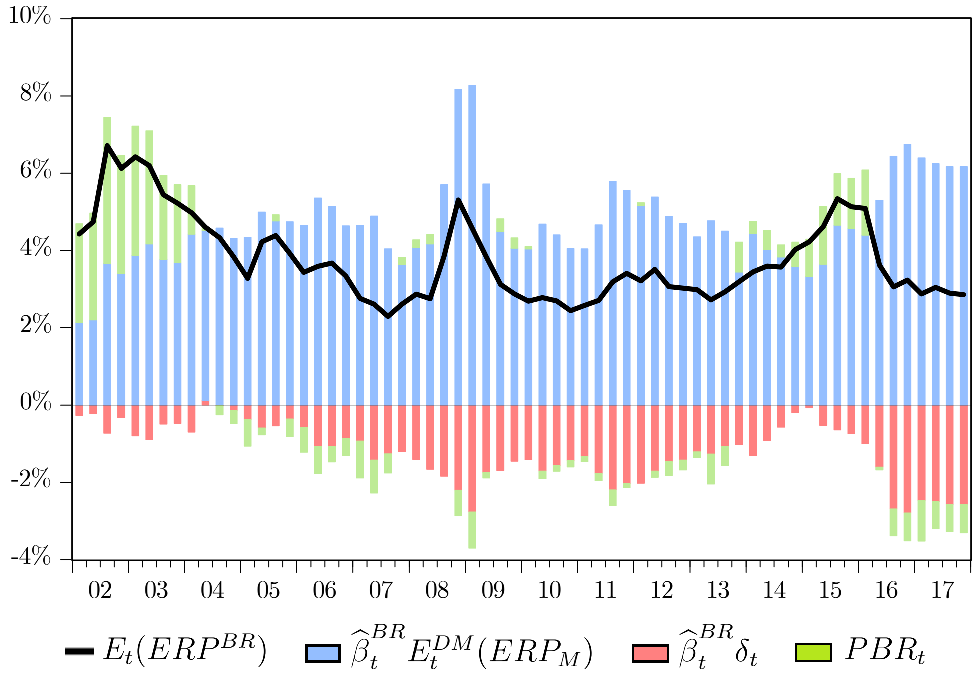

Figure 8: Brazilian ERP decomposition

4 Conclusion

In this paper we seek to evaluate whether the low *ex-post* equity risk premium in Brazil is a result of a compressed *ex-ante* equity risk premium, using two different approaches.

In the first one, we investigate possible distortions due to government-controlled shareholders. The results from the first approach suggest the government does not affect the *ex-ante* ERP, although the government as a manager seems to influence the firms' risk. In fact, for government-controlled firms, our estimates indicate the true betas were higher than the historical betas between 2009 and 2016, meaning that investors required an extra-remuneration to finance government-controlled firms vis-à-vis what the historical risk measure would suggest.

In the second approach, we decompose the Brazilian equity risk premium estimated in the first approach. Our findings point out the Brazilian equity risk premium is consistent with international risk premia.

Therefore, the findings from the two approaches indicate the *ex-ante* risk premium is not compressed in Brazil. As a result, the low *ex-post* risk premium seems to be consequence of a relatively short time series, as argued by Giovannetti et al. (2016), rather than a Brazilian idiosyncrasy.

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Appendix

A Historical beta correction term ω_t

From betas definition (equation (2)),

$$\beta_{Mt}^{BR} = \frac{cov(R_{Mt}^{BR}, R_{Mt}^{BR})}{\sigma^2(R_{Mt}^{BR})} = \frac{cov\left(\sum_{i=1}^{n} \theta_{it}^{BR} R_{it}^{BR}, R_{Mt}^{BR}\right)}{\sigma^2(R_{Mt}^{BR})} = 1$$

$$\sum_{i=1}^{n} \theta_{it}^{BR} \beta_{it}^{BR} = 1 \tag{A.1}$$

where θ_{it}^{BR} is the share of Brazilian stock i in the Brazilian market portfolio.

Hence, from equation (A.1), the weighted average of true betas should add to one. It is easy to show this is valid also for stocks' portfolios. In particular,

$$\theta_t^{BRp} \beta_t^{BRp} + \theta_t^{BRg} \beta_t^{BRg} = 1 \tag{A.2}$$

where θ_t^{BRp} and θ_t^{BRg} are the shares of privately and government-controlled firms on Brazilian market portfolio, respectively, and β_t^{BRp} and β_t^{BRg} are the true betas for the portfolio of privately and government-controlled firms, respectively. It is easy to show β_t^{BRp} and β_t^{BRg} equal the weighted average of the β_{it}^{BR} of each portfolio, where the weights are the shares of the stocks on the respective portfolio.

Substituting the expression for the true betas (equation (6)) into (A.2),

$$\theta_t^{BRp} \left(\widehat{\beta}_{it}^{BRp} \gamma_t^p\right) + \theta_t^{BRg} \left[\widehat{\beta}_{it}^{BRg} (\gamma_t^p + \omega_t)\right] = 1 \tag{A.3}$$

where $\widehat{\beta}_{it}^{BRp}$ and $\widehat{\beta}_{it}^{BRg}$ are the historical betas for the portfolio of privately and government-controlled firms in relation to the Brazilian stock market, respectively.

Finally, we need just to isolate ω_t in equation (A.3):

$$\omega_t = \frac{1 - \left(\theta_t^{BRp} \widehat{\beta}_{it}^{BRp} + \theta_t^{BRg} \widehat{\beta}_{it}^{BRg}\right) \gamma_t^p}{\theta_t^{BRg} \widehat{\beta}_{it}^{BRg}}$$

$$\omega_t = \frac{1 - \gamma_t^p}{\theta_t^{BRg} \widehat{\beta}_{it}^{BRg}} \tag{A.4}$$

where we use in the last step $\theta_t^{BRp} \widehat{\beta}_{it}^{BRp} + \theta_t^{BRg} \widehat{\beta}_{it}^{BRg} = 1$, since equation (A.2) should also be valid when sample variances and covariances are used, as in the historical betas.

B Selected stocks

Table 2: Selected stocks (1)

Ticker	Name	Ticker	Name	Ticker	Name	Ticker	Name
ABEV3	Ambev S/A ON	BPAC11	Btgp Banco UN	CRTP5	Crt Celular PN	EQTL3	Equatorial ON
ABYA3	Abyara ON	BPAN4	Banco Pan PN	CRUZ3	Souza Cruz ON	ESTC3	Estacio Part ON
ACES3	Am Inox BR ON	BRAP3	Bradespar ON	CSAN3	Cosan ON	ETER3	Eternit ON
ACES4	Am Inox BR PN	BRAP4	Bradespar PN	CSMG3	Copasa ON	EVEN3	Even ON
ACGU3	Guarani ON	BRFS3	BRF SA ON	CSNA3	Sid Nacional ON	EZTC3	Eztec ON
AEDU3	Anhanguera ON	BRKM5	Brasken PN	CSPC4	Cosipa PN	FESA4	Ferbasa PN
AELP3	AES Elpa ON	BRML3	BR Malls Par ON	CSTB4	Sid Tubarão PN	FFTL4	Valefert PN
AGEI3	Agre Emp Imo ON	BRPR3	BR Propert ON	CTAX4	Liq PN	FHER3	Fer Heringer ON
AGIN3	Agra Incorp ON	BRSR6	Banrisul PN	CTIP3	Cetip ON	FIBR3	Fibria ON
ALLL11	Rumo SA UN	BRTP3	Brasil T Par ON	CTNM4	Coteminas ON	FJTA4	Forjas Taurus PN
ALLL3	Rumo SA ON	BRTP4	Brasil T Par PN	CVCB3	CVC Brasil	FLRY3	Fleury ON
ALLL4	Rumo SA PN	BTOW3	B2W Digital ON	CYRE3	Cyrela Realt ON	GETI3	AES Tiete ON
ALPA4	Alpargatas PN	CARD3	Csu Cardsyst ON	DASA3	Dasa ON	GETI4	AES Tiete PN
ALSC3	Aliansce ON	CCIM3	CC Des Imob	DAYC4	Daycoval PN	GFSA3	Gafisa ON
ALUP11	Alupar UN	CCPR3	Cyre Com-Ccp	DIRR3	Direcional ON	GGBR3	Gerdau ON
AMIL3	Amil ON	CCRO3	CCR SA ON	DTEX3	Duratex ON	GGBR4	Gerdau PN
ANIM3	Anima ON	CESP3	Cesp ON	DURA4	Duratex-Old PN	GOAU4	Gerdau Met PN
ARCE3	Arcelor BR ON	CESP5	Cesp PN	EBTP3	Embratel Part ON	GOLL4	Gol PN
ARCZ6	Aracruz PN	CESP6	Cesp PN	EBTP4	Embratel Part PN	GRND3	Grendene ON
ARTR3	Arteris ON	CGAS5	Congas ON	ECOR3	Ecorodovias ON	GUAR3	Guararapes ON
BBAS3	Brasil ON	CIEL3	Cielo ON	EGIE3	Engie Brasil ON	GVTT3	GVT Holding ON
BBAS4	BrasilPN	CLSC4	Celesc PN	ELET3	Eletrobras ON	HGTX3	Cia Hering ON
BBDC3	Bradesco ON	CMET4	Caemi PN	ELET6	Eletrobras PN	HYPE3	Hypera ON
BBDC4	Bradesco PN	CMIG3	Cemig ON	ELEV3	Eleva ON	IDNT3	Ideiasnet ON
BBRK3	BR Brokers ON	CMIG4	Cemig PN	ELPL3	Eletropaulo ON	IGTA3	Iguatemi ON
BBSE3	BBSeguridade ON	CNFB4	Confab PN	ELPL4	Eletropaulo PN	INEP4	Inepar PN
BDLL4	Bardella PN	COCE5	Coelce PN	ELPL5	Eletropaulo PN	IRON3	Anglo Brazil ON
BEEF3	Minerva ON	CPCA4	Trikem PN	EMAE4	Emae PN	ITSA4	Itausa PN
BEES3	Banestes ON	CPFE3	CPFL Energia ON	EMBR3	Embraer ON	ITUB3	ItauUnibanco ON
BICB4	BicBanco PN	CPLE3	Copel ON	ENBR3	Energias BR ON	ITUB4	ItauUnibanco PN
BISA3	Brookfield ON	CPLE6	Copel PN	ENEV3	Eneva ON	JBSS3	JBS ON
BNCA3	Nossa Caixa ON	CPNY3	Company ON	ENGI11	Energisa UN	JHSF3	JHSF Part ON
BOBR4	Bombril PN	CPSL3	Copesul ON	EQTL11	Equatorial UN	KEPL3	Kepler Weber ON

Table 3: Selected stocks (2)

Ticker	Name	Ticker	Name	Ticker	Name	Ticker	Name
KLBN11	Klabin S/A UN	OIBR4	Oi PN	SAPR4	Sanepar PN	TNCP3	Tele Nort Cl ON
KLBN4	Klabin S/A PN	OSXB3	OSX Brasil ON	SBSP3	Sapesp ON	TNCP4	Tele Nort Cl PN
KROT3	Kroton ON	PCAR4	Pão de Açúcar PN	SDIA4	Sadia S/A PN	TNEP3	Tele Nor Cel ON
KSSA3	Klabin Segall ON	PDGR3	PDG Realt ON	SEER3	Ser Educa ON	TNEP4	Tele Nor Cel PN
LAME3	Lojas Americ ON	PETR3	Petrobrás ON	SLCE3	SLC Agrícola ON	TNLP3	Telemar ON
LAME4	Lojas Americ PN	PETR4	Petrobrás PN	SLED4	Saraiva Livr PN	TNLP4	Telemar PN
LEVE3	Metal Leve ON	PINE4	Pine PN	SMTO3	Sao Martinho ON	TOTS3	Totvus ON
LIGT3	Light S/A ON	PLAS3	Plascar Part ON	STBP11	Santos Brp UN	TRPL3	Tran Paulist ON
LINX3	Linx ON	PLDN4	Polialden PN	STBR11	Santos Bras UN	TRPL4	Tran Paulist PN
LOGN3	Log-In ON	PMAM3	Paranapanema ON	SULA11	Sul America UN	TSEP4	Tele Sud Cel PN
LREN3	Lojas Renner ON	PMAN4	Paranapanema PN	SUZB3	Suzano Papel ON	TUPY3	Tupy ON
LUPA3	Lupatech ON	POMO4	Marcopolo PN	SUZB5	Suzano Papel PN	UBBR11	Unibanco ON
MAGG3	Magnesita SA ON	POSI3	Positivo Tec ON	SZPQ4	Quattor Petr PN	UBBR4	Unibanco PN
MAGS5	Magnesita PN	PRIO3	Petrorio ON	TAEE11	Taesa UN	UGPA3	Ultrapar ON
MDIA3	M. Dias Branco ON	PRML3	Prumo ON	TAMM4	Tam S/A PN	UGPA4	Ultrapar PN
MEDI3	Medial Saude ON	PRTX3	Portx ON	TBLE3	Engie Brasil PN	UNIP6	Unipar PN
MGLU3	Magaz Luiza ON	PSSA3	Porto Seguro ON	TCOC4	Tele C Oeste PN	UOLL4	Uol PN
MILS3	Mills On	PTIP4	Ipiranga Petr PN	TCSA3	Tecnisa ON	USIM3	Usiminas ON
MMXM3	MMX Miner ON	QGEP3	Qgep Part ON	TCSL4	Tim Part S/A PN	USIM5	Usiminas PN
MNDL4	Mundial PN	QUAL3	Qualicorp ON	TDBH3	Telefonica Hld ON	VALE3	Vale ON
MPLU3	Multiplus ON	RADL3	Raia Drogasil ON	TDBH4	Telefonica Hld PN	VALE5	Vale PN
MRFG3	Marfrig ON	RAIL3	Rumo S.A. ON	TEND3	Tenda ON	VCPA4	Fibria PN
MRV3	MRV ON	RAPT4	Randon Part. PN	TERI3	Tereos ON	VIVO3	Vivo ON
MSAN3	Bunge Brasil ON	RDCD3	Redecard ON	TESA3	Terra Santa ON	VIVO4	Vivo PN
MSAN4	Bunge Brasil PN	RENT3	Localiza ON	TIET11	AES Tietê E UN	VIVR3	Viver ON
MULT3	Multiplan ON	RIPI4	Ipiranga Ref PN	TIMP3	Tim Part S/A ON	VIVT3	Telef Brasil ON
MYPK3	Iochp-Maxion ON	RLOG3	Cosan Log ON	TLCP3	Tele Leste Cel ON	VIVT4	Telef Brasil PN
MYPK4	Iochp-Maxion PN	RPMG3	Pet Manguinhos ON	TLCP4	Tele Leste Cel PN	VLID3	Valid ON
NATU3	Natura ON	RPSA4	Ripasa PN	TMAR3	Telemar N L ON	VVAR11	Viavarejo UN
NETC4	Net PN	RSID3	Rossi Residencial ON	TMAR5	Telemar N L PNA	VVAX11	Vivax UN
ODPV3	Odontoprev ON	RUMO3	Rumo Log ON	TMAR6	Telemar N L PNB	WEGE3	Weg ON
OGXP3	OGX Petroleo ON	SALM4	Seara Alim PN	TMCP3	Telemig Part ON	WEGE4	Weg PN
OIBR3	Oi ON	SANB11	Santander BR UN	TMCP4	Telemig Part PN	WIZS3	Wiz ON

C Proxy for the dividend growth rate

By definition,

$$g_{it} = \frac{D_{it}}{D_{it-1}} - 1 = \frac{EQPS_{it-1}ROE_{it}(1-b_{it})}{EQPS_{it-2}ROE_{it-1}(1-b_{it-1})} - 1 \tag{C.1}$$

where $EQPS_{it}$ is the equity per share, ROE_{it} is the return on equity, and b_{it} is the retention rate, the share of dividends retained for reinvestments, all three of firm i at time t.

Also by definition,

$$EQPS_{it} = EQPS_{it-1}(1 + ROE_{it}b_{it}) \tag{C.2}$$

Substituting equation (C.2) into (C.1),

$$g_{it} = \frac{EQPS_{it-2}(1 + ROE_{it-1}b_{it-1})ROE_{it}(1-b_{it})}{EQPS_{it-2}ROE_{it-1}(1-b_{it-1})} - 1$$

$$g_{it} = (1 + ROE_{it-1}b_{it-1})\left[\frac{ROE_{it}(1-b_{it})}{ROE_{it-1}(1-b_{it-1})}\right] - 1 \tag{C.3}$$

Since in the steady state $ROE_{it+1} = ROE_{it} = ROE^*_{it}$ and $b_{it+1} = b_{it} = b^*_{it}$,

$$g^*_{it} = ROE^*_{it}b^*_{it} \tag{C.4}$$

Thus, from (C.4), $ROE^*_{it}b^*_{it}$ is a steady-state measure for the dividend growth rate g^*_{it}.

D Betas adjustment to a new market portfolio reference

Given the definitions $\widehat{\beta}^{BR}_{it} = \frac{\widehat{cov}(R^{BR}_{it}, R^{BR}_{Mt})}{\widehat{\sigma^2}(R^{BR}_{Mt})}$ and $\widetilde{\beta}^{BR}_{it} = \frac{\widehat{cov}\left(R^{BR}_{it}, R^{BR}_{\widetilde{M}t}\right)}{\widehat{\sigma^2}\left(R^{BR}_{\widetilde{M}t}\right)}$ and the Ordinary Least Squares (OLS) estimators, we can write

$$R^{BR}_{it} = \widehat{\alpha}_{i\underline{t}} + \widehat{\beta}^{BR}_{i\underline{t}} R^{BR}_{M_{\underline{t}}t} + \widehat{u}^{BR}_{it}, \text{ for } \underline{t} - w \leq t \leq \underline{t} \tag{D.1}$$

$$R^{BR}_{it} = \widetilde{\alpha}_{i\underline{t}} + \widetilde{\beta}^{BR}_{i\underline{t}} R^{BR}_{\widetilde{M}t} + \widetilde{u}^{BR}_{it}, \text{ for } \underline{t} - w \leq t \leq \underline{t} \tag{D.2}$$

where $R^{BR}_{M_{\underline{t}}t}$ is the return of the Brazilian market portfolio at time t considering the index's weights at time $\underline{t}$, $\widehat{u}^{BR}_{it}$ and $\widetilde{u}^{BR}_{it}$ are OLS error estimates, and $w > 0$.

Hence, from equations (D.1) and (D.2),

$$R^{BR}_{\widetilde{M}t} = \left(\frac{\widehat{\alpha}_{i\underline{t}} - \widetilde{\alpha}_{i\underline{t}}}{\widetilde{\beta}^{BR}_{i\underline{t}}}\right) + \left(\frac{\widehat{\beta}^{BR}_{i\underline{t}}}{\widetilde{\beta}^{BR}_{i\underline{t}}}\right) R^{BR}_{M_{\underline{t}}t} + \left(\frac{\widehat{u}^{BR}_{it} - \widetilde{u}^{BR}_{it}}{\widetilde{\beta}^{BR}_{i\underline{t}}}\right) \tag{D.3}$$

By construction, $\widehat{u}_{it}^{BR} \perp R_{M_{\underline{t}}t}^{BR}$ and $\widetilde{u}_{it}^{BR} \perp R_{\widetilde{M}t}^{BR}$. Assuming $\widetilde{u}_{it}^{BR} \perp R_{M_{\underline{t}}t}^{BR}$, we can estimate (D.3) using OLS. Hence,

$$\widehat{\beta}_{i\underline{t}}^{BR} = \widetilde{\beta}_{i\underline{t}}^{BR} \eta_{\underline{t}} \tag{D.4}$$

where $\eta_{\underline{t}} = \frac{\widehat{cov}\left(R_{\widetilde{M}t}^{BR}, R_{M_{\underline{t}}t}^{BR}\right)}{\widehat{\sigma^2}\left(R_{M_{\underline{t}}t}^{BR}\right)}$.

Therefore, changing the market portfolio reference does not alter stocks' relative historical risk, since $\eta_{\underline{t}}$ does not depend on i. This results only in a change in betas' scale. As a consequence, from the property shown in Appendix A that the weighted average of betas add to one,

$$\eta_{\underline{t}} = \frac{1}{\sum_{i=1}^{n} \theta_{it}^{BR} \widetilde{\beta}_{it}^{BR}} \tag{D.5}$$

where the weights θ_{it}^{BR} that we use in the calculation of $\eta_{\underline{t}}$ are the IBrX weights adjusted to add to one across the stocks included in the sample.

E Proportional method to identify firms' control

The proportional method is an alternative approach to verify whether a selected firm is controlled by the government or not. This method distributes the shares of partition Θ_t^{SF} for the shareholders of partition Θ_t^{Others} proportionally, creating a new (kxn) matrix for the participation of the shareholders not controlled by selected firms, named $\widetilde{\Theta}_t^{Others}$, whose columns add to 1. Thus, $\widetilde{\Theta}_t^{Others}$ measures the direct and indirect participation of the shareholders not controlled by selected firms on the selected firms.

The direct share of each shareholder not controlled by selected firms on the selected firms is given by Θ_t^{Others}. Adopting a proportional approach, the indirect share is the product between the participation of these shareholders, directly and indirectly, on the selected firms, given by $\widetilde{\Theta}_t^{Others}$, and the participation of the shareholders controlled by selected firms on the selected firms, given by Θ_t^{SF}. In formal terms,

$$\widetilde{\Theta}_t^{Others} = \Theta_t^{Others} + \widetilde{\Theta}_t^{Others} \Theta_t^{SF}$$

$$\widetilde{\Theta}_t^{Others} = \Theta_t^{Others} \left(I - \Theta_t^{SF}\right)^{-1} \tag{E.1}$$

Finally,

$$\widetilde{G}_t^{SF} = I(G_t^{Others} * \widetilde{\Theta}_t^{Others}) \tag{E.2}$$

where $\widetilde{G}_t^{SF}$ is the proportional method estimate of G_t^{SF}.

The proportional method is intuitive and provides an analytical solution for the identification of firms' control. Yet, it has obvious shortcomings, since the controls are not properly accounted for. The government could have 60% of the common shares of firm A, that owns 60% of the common shares of a given selected firm B. The method based on the controls would correctly indicate that B is government-controlled, while the proportional method would indicate private control. Moreover, the proportional method could indicate the government control a given selected firm because of participations in shareholders that it does not itself control. In any case, the initial guess provided by the proportional method converged fast to the final result.

F Robustness tests with the complete local model

Figure 9 shows the robustness tests with equation (13) under $f(GS_{it}^{BR}) = GS_{it}^{BR}$. Panel A is exacly the same of panel J of Figure 3, while Panel C is estimated using the same model of panel A, but using NLS instead of FGNLS. Panel B and C are estimated adding $\zeta_{it}\psi_{it}$ in (13) using FGNLS and NLS, respectively.

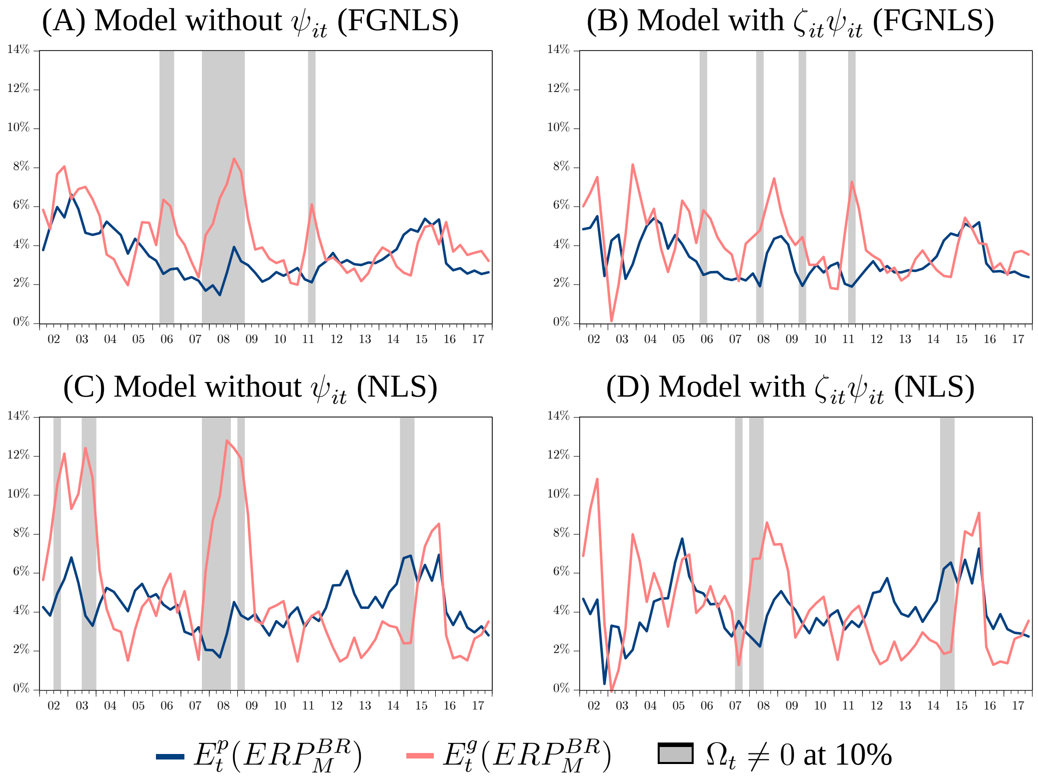

Figure 9: Robustness tests with the complete local model (linear specification)

G Robustness tests with the local model without impact of government as investor

Figure 10 shows the robustness tests with equation (14). Panels A and B show the results using FGLS[29], while in panels C and D we use OLS. In panels A and C we show the results

[29]Thus, the blue line of panels A and B are the same ERP of Figure 4.

for the three specifications (without ψ_{it}, with $\zeta_{it}\psi_{it}$, and with $-\psi_{it}$), while in panels B and D are shown only the results for the model without ψ_{it} and with $\zeta_{it}\psi_{it}$. This is done to facilitate the analysis of these two last specifications, given the very different behavior of the model with $-\psi_{it}$.

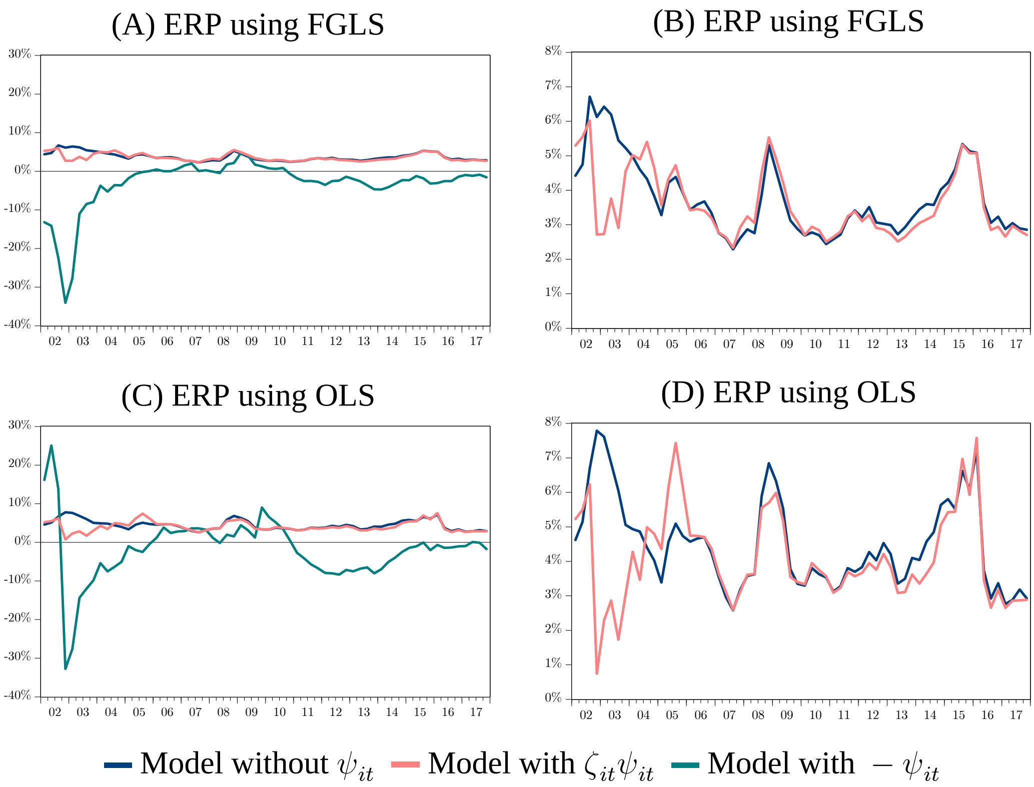

Figure 10: Robustness tests with the local model without impact of government as investor

H Estimates of γ_t^g with the complete local model

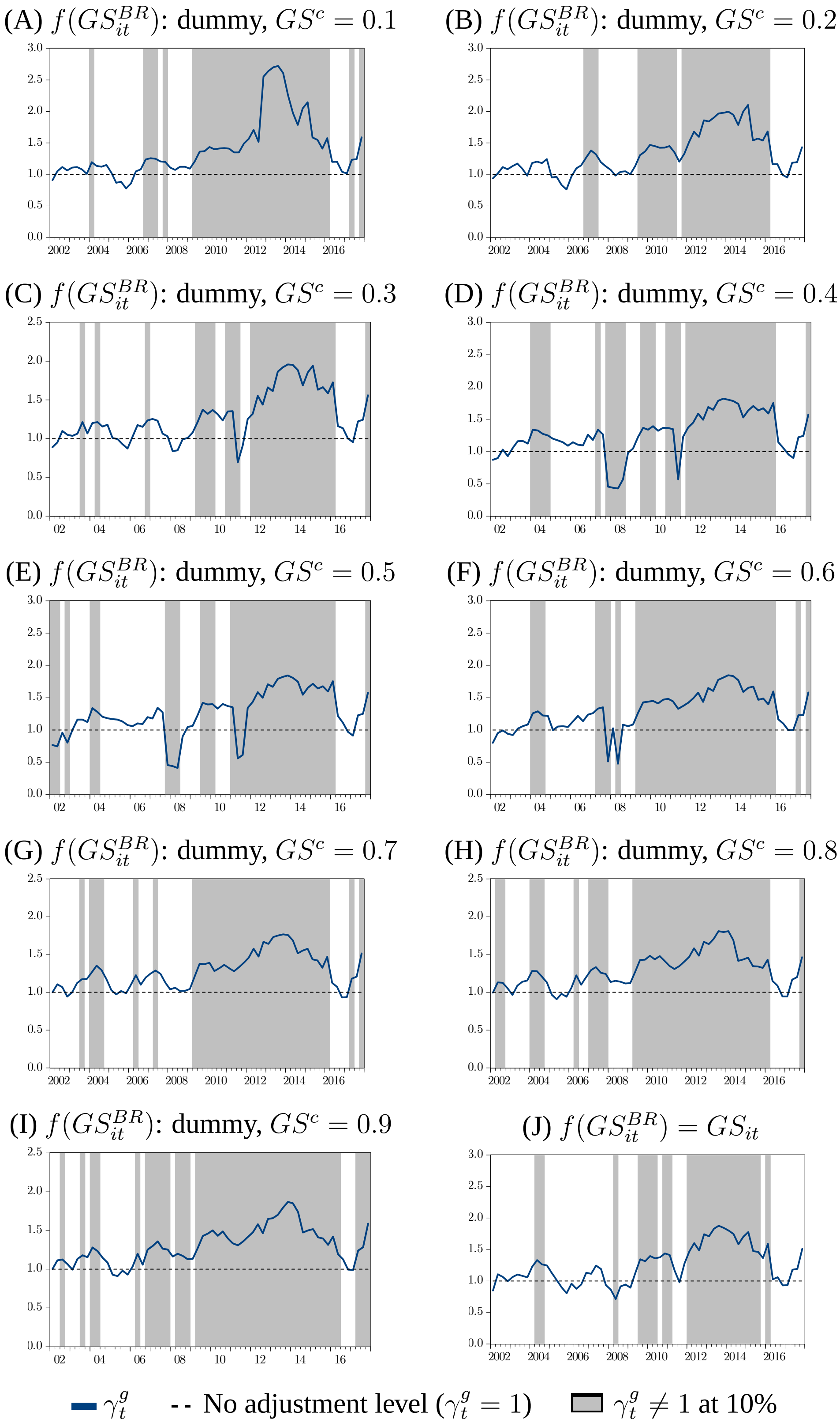

Figure 11: Betas' adjustment factor: model with government control both as manager and investor

I Robustness tests with the global model

Figure 12 shows the robustness tests with equation (17). Panels A and C show, respectively, FGLS and OLS estimates for $E_t^{DM}(ERP_M)$, while panels B and D present the FGLS and OLS estimates for $E_t^{EM}(ERP_M)$, respectively,. All panels show the results for the three specifications (without ψ_{it}, with $\zeta_{it}\psi_{it}$, and with $-\psi_{it}$).

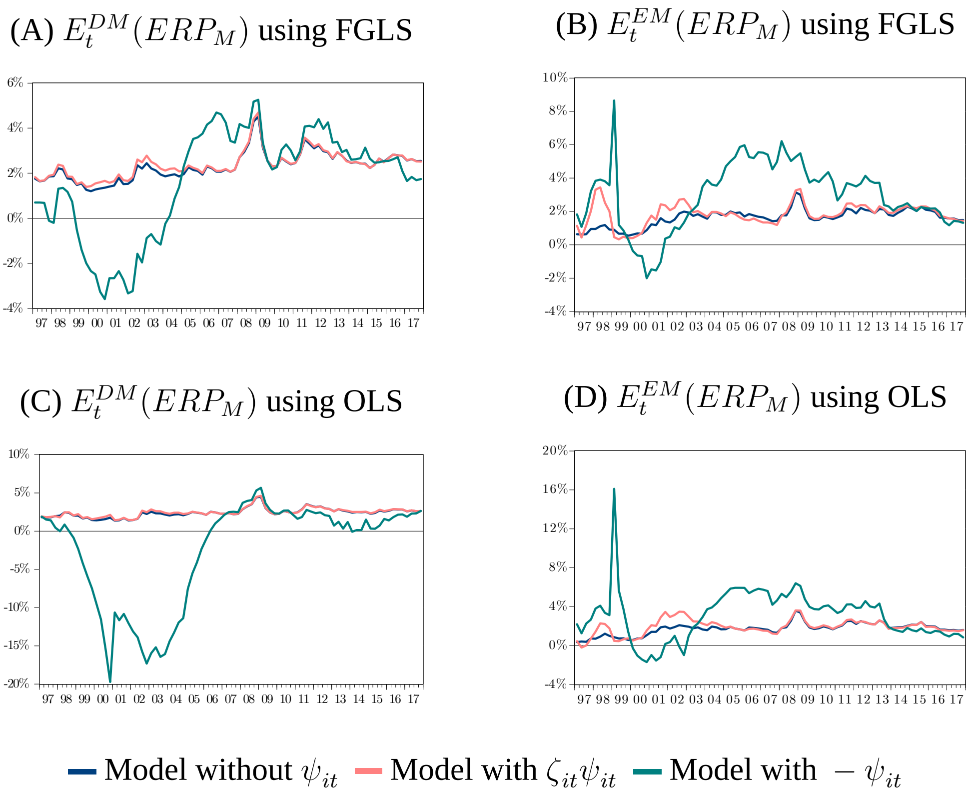

Figure 12: Robustness tests with the global model

J Robustness of the estimated country-specific effect

Instead of assuming the differences between emerging and developed markets are reflected in different expectations about ERP_M, as in equation (17), let us assume the differences of these two groups are reflected on the betas. Formally, we assume the correct model is

$$\frac{D_t^c}{P_t^c} = \beta_t^c E_t(ERP_M) + \epsilon_{ct} \tag{J.1}$$

where β_t^c is the true beta of country c in relation to the global market portfolio.

In addition, we assume the true beta, β_t^c, is a function of the historical beta, $\widehat{\beta}_t^c$. Following the discussion of Section 2, we have

$$\frac{D_t^c}{P_t^c} = \widehat{\beta}_t^c \left[\gamma_t^{DM} \left(1 - \frac{d_{it}^{EM}}{\theta_t^{EM} \widehat{\beta}_t^{EM}} \right) + \left(\frac{d_{it}^{EM}}{\theta_t^{EM} \widehat{\beta}_t^{EM}} \right) \right] E_t(ERP_M) + \epsilon_{ct}$$

$$\frac{D_t^c}{P_t^c} = \widehat{\beta}_t^c \left[\gamma_t^{DM} E_t(ERP_M) + d_{ct}^{EM} \left(\frac{1-\gamma_t^{DM}}{\theta_t^{EM} \widehat{\beta}_t^{EM}} \right) E_t(ERP_M) \right] + \epsilon_{ct} \tag{J.2}$$

where γ_t^{DM} is the beta adjustment factor for developed countries, θ_t^{EM} is the share of emerging countries in the world portfolio, and $\widehat{\beta}_t^{EM}$ represents the historical beta computed for the portfolio of emerging markets in relation to the global portfolio. The other variables are defined as before.

If we define $\gamma_t^{DM} E_t(ERP_M) \equiv E_t^{DM}(ERP_M)$ and $\left(\frac{1-\gamma_t^{DM}}{\theta_t^{EM} \widehat{\beta}_t^{EM}} \right) E_t(ERP_M) \equiv \delta_t$, we obtain

$$\frac{D_t^c}{P_t^c} = \widehat{\beta}_t^c \left[E_t^{DM}(ERP_M) + d_{ct}^{EM} \delta_t \right] + \epsilon_{ct} \tag{J.3}$$

which is the equation (17) used in Section 3.

Hence, the estimated PBR_t of Section 3 is robust to both approaches regarding the differences in the two group of countries. However, the distinction between developed and emerging markets effects is not. After all, in the alternative approach discussed here, the emerging markets effect is null by construction as there is only one *ex-ante* ERP.