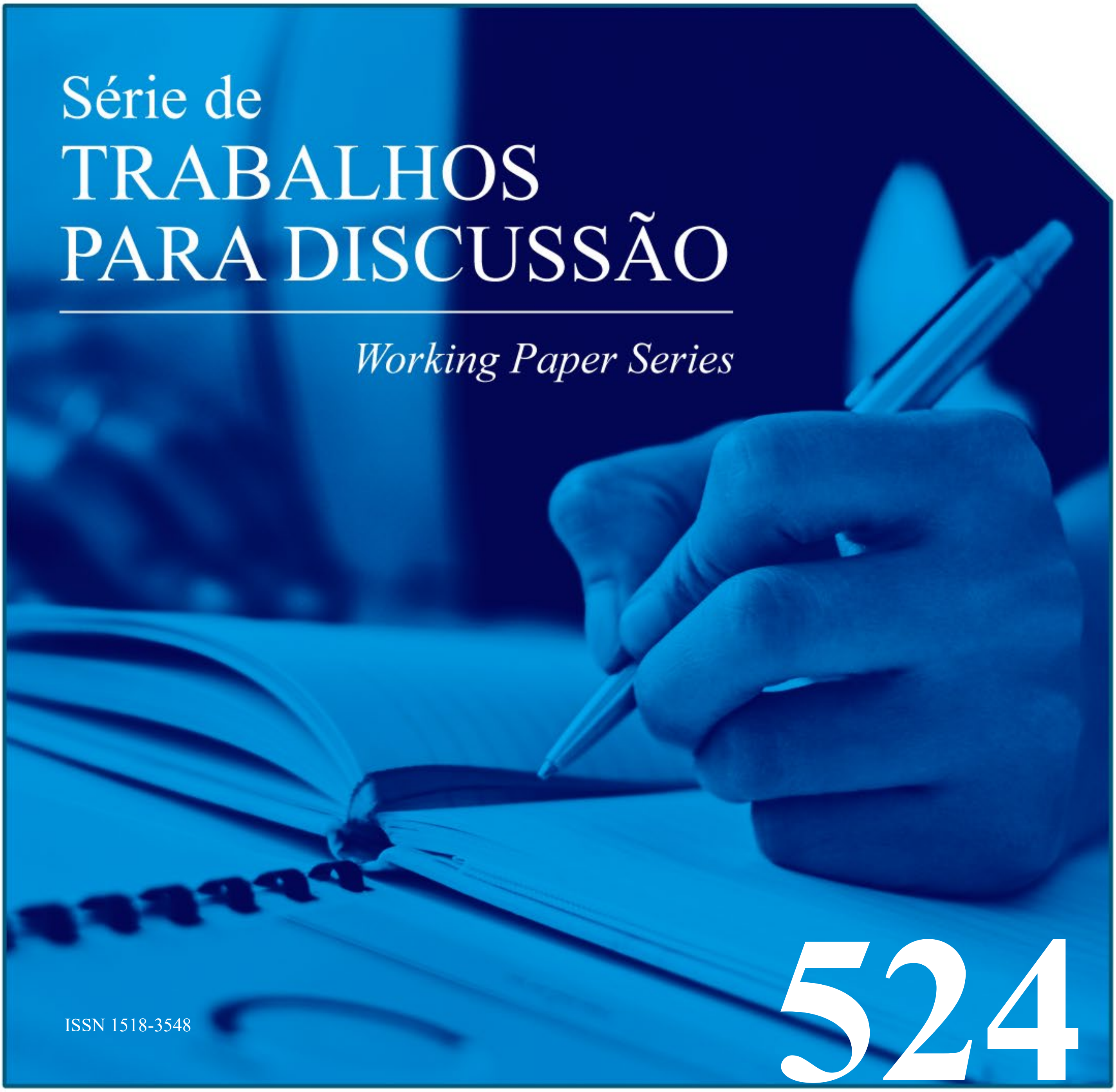

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A General Characterization of the Capital Cost and the Natural Interest Rate: an application for Brazil

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Sumário Não Técnico

Uma das principais funções de um banco central é a condução da política monetária. Modernamente, o principal instrumento de política monetária é a taxa básica de juros (taxa Selic, no caso brasileiro), que serve de referência para todas as demais taxas de juros da economia, como as taxas de juros dos empréstimos bancários.

A teoria econômica prediz que o banco central terá liberdade para escolher o nível da taxa básica apenas temporariamente. No longo prazo, a taxa básica de juros deve convergir para um nível de equilíbrio, que não depende diretamente das ações do banco central, sendo determinado por características estruturais da economia. O grande desafio é que essa taxa de equilíbrio, conhecida como taxa de juros natural, não é observada e sua expressão matemática pode variar a depender do referencial teórico considerado.

Esse artigo mostra que é possível obter, sob hipóteses razoavelmente gerais, expressões matemáticas para a taxa de juros natural. Dessa forma, essas expressões são consistentes com diversos referenciais teóricos, sendo um bom ponto de partida para a estimação e avaliação dos determinantes dessa taxa de juros de equilíbrio.

Essa metodologia foi aplicada para o caso brasileiro. Os resultados sugerem que a taxa natural recuou rapidamente entre 2009 e 2014, passando de 12% a.a. para cerca de 4% a.a., consequência da queda do crescimento potencial ou sustentável da economia. Afinal, menor crescimento deprime a rentabilidade dos investimentos das empresas, fazendo com que elas só invistam a uma taxa de juros mais baixa. Portanto, não fosse esse recuo do crescimento potencial, a taxa básica continuaria sendo alta.

Frente a Chile, Colômbia, México e Peru entre 2005 e 2012, a taxa de juros mais alta no Brasil é explicada por três fatores. Em primeiro lugar, os empréstimos do BNDES com taxas de juros abaixo das taxas de mercado. Afinal, se parte dos empréstimos tem juros mais baixos, as taxas de mercado devem ser mais elevadas a fim de que a taxa de juros média seja condizente com o nível de equilíbrio. Em segundo lugar, o baixo nível de poupança no país, que reduz os recursos disponíveis para investimento, tornando-os mais caros. Por fim, o menor poder de mercado das firmas brasileiras, que faz com que elas tenham menor capacidade de obter taxas de juros mais baixas. Dentre esses três fatores, baixa poupança e baixo poder de mercado das firmas são os mais relevantes para explicar o historicamente elevado nível da taxa básica brasileira.

Non-technical Summary

One of the main roles of a central bank is the conduct of the monetary policy. Modernly, the key monetary policy instrument is the policy rate (Selic rate, in the case of Brazil), which is a reference for all the other interest rates in the economy, as the banking lending interest rates.

The economic theory predicts the central bank is free to choose the level of the policy rate only temporarily. In the long run, the policy rate should converge to its equilibrium level, which is not directly affected by central bank measures, being determined by structural features of the economy. The main challenge is that this equilibrium rate, known as the natural interest rate, is not observed and its mathematical expression can be different according to the theoretical model analyzed.

In this paper I show it is possible to obtain, under relatively weak assumptions, mathematical expressions for the natural interest rate. As a result, these expressions are consistent with different theoretical models, being a good starting point to estimate this equilibrium interest rate and evaluate its determinants.

This methodology is applied for the Brazilian case. The results show the natural interest rate rapidly decreased between 2009 and 2014, from 12% p.a. to 4% p.a., due to the lowering of the potential or sustainable growth rate of the economy. After all, a lower growth reduces the profitability of firms' investments, causing them to invest only at a lower interest rate. Therefore, if the potential growth rate had not decreased, the policy rate in Brazil would still be high by international standards.

Comparing to Chile, Colombia, Mexico e Peru between 2005 and 2012, three factors explain the higher Brazilian policy rate. First, the Brazilian Development Bank (BNDES) loans at interest rates below market rates. After all, if a share of the loans has below market interest rates, the market interest rates should be higher in order to achieve an average interest rate that is consistent with the equilibrium level. Second, the low savings in the country, which reduces the resources available for investment, making them more expensive. Finally, the lower firms' market power, since it implies they are less capable of obtaining lower interest rates. Among these three factors, low savings and low firms' market power are the most relevant in explaining the historically high policy rate in Brazil.

A General Characterization of the Capital Cost and the Natural Interest Rate: an application for Brazil*

Thiago Trafane Oliveira Santos**

Abstract

Is there a general expression for the real natural interest rate? This paper shows, based on a general characterization of the steady state capital cost, it is possible to obtain natural interest rate equations for (i) the standard or single funding rate case and (ii) the dual funding rate case. These equations become economically interpretable when more theoretical structure is added, assuming, for instance, a CES production function and a price-setting rule. This allows the use of the equations not only to estimate the natural interest rate, but also to evaluate its results. As an application, I estimate the Brazilian natural rate between 2002Q2 and 2017Q4 using the dual funding rate equation for a Cobb-Douglas production function and a semi-structural macroeconomic model. The results indicate the real natural interest rate in Brazil has rapidly decreased between 2009 and 2014, from 12% p.a. to 4% p.a., mainly reflecting the lowering of the steady-state output growth rate. Given a higher potential growth, Brazil's natural rate would be still high by international standards. Comparing to Chile, Colombia, Mexico and Peru between 2005 and 2012, the higher Brazilian rate is justified by the impact of the subsidized lending by the BNDES and, most importantly, by both the low total savings rate and the low level of firms' markup.

Keywords: natural interest rate, Brazil, markup, savings rate, subsidized lending.
JEL Classification ou Classificação JEL: C32, C54, E43, E52.

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1 Introduction

Is there a general expression for the real natural interest rate? This paper shows, based on a general characterization of the steady-state capital cost, it is possible to obtain natural interest rate equations for (i) the standard or single funding rate case and (ii) the dual funding rate case.

For each of these cases, I show two versions of the natural rate equation. The first version is very general as it is mainly derived from definitions. However, this generality has a cost, which is the lack of economic interpretation of the equations. This issue is corrected in the second version when I consider additional theoretical structure, assuming a constant elasticity of substitution (CES) production function and a price-setting rule. The second version, although less general than the first one, is still quite general and has the advantage of being economically interpretable, allowing the use of the equations not only to estimate the natural rate, but also to evaluate its results.

In any case, even in their second versions, the natural rate equations are more closely associated with a semi-structural model, as they show equilibrium relations between the natural interest rate and some variables such as steady-state output growth rate, total savings rate, and risk premia. They are not structural equations, which identify the deep parameters. However, to identify the deep parameters it is necessary to define a specific structural model, with different models yielding different equations. Thus, there is a trade-off between generality and theoretical accuracy and, consequently, the proposed semi-structural approach should be viewed as complementary to structural approaches, rather than a substitute. As the proposed equations are quite general, they are natural starting points to understand the natural interest rate estimates both overtime and across countries. A structural approach can then enrich the diagnoses, exploring the deep parameters that could justify the results found previously, which makes the understanding about the causalities clearer.

In order to show the usefulness of the proposed approach, I initially estimate the Brazilian natural rate using the second version of the dual funding rate equation for a Cobb-Douglas production function, as I find empirical evidence supporting the assumption of an unitary elasticity of substitution, and a semi-structural macroeconomic model for the 2002Q2-2017Q4 period. This natural rate equation is consistent with the Brazilian economy, where two funding rates coexist, one linked to the monetary policy rate (Selic rate) and another to the Brazilian Development Bank (BNDES) funding rate. Thus, this equation explicitly accounts for the effect of the subsidized lending by the BNDES. The point estimates, which are subject to a high degree of uncertainty, indicate the real natural one-year interest rate was around 12% from 2003 to 2008, rapidly decreased between 2009 and 2014, and stabilized around 4% between 2015 and 2017.

Then, using the estimated natural interest rate equation, I evaluate the results obtained both overtime and across countries. These analyses should be interpreted essentially as conditional expectations exercises, since some caution is needed regarding the causalities, which could be precisely identified only in structural approaches. The results indicate the natural interest rate in Brazil has decreased between 2009 and 2014 mainly reflecting the lowering of the steady-state output growth rate. Hence, given a higher potential growth, Brazil's natural rate would be still high by international standards. Comparing to Chile, Colombia, Mexico and Peru between 2005 and 2012, the higher Brazilian rate is justified by the impact of the subsidized lending by the BNDES and, most importantly, by both the low total savings rate in the country and the low level of firms' markup. While the low markup is, to the best of my knowledge, a new explanation for the high level of risk-free interest rate in Brazil, the other two factors have already been identified in the empirical literature.

For example, Segura-Ubiergo (2012), using a panel error correction model with all emerging markets inflation targeting regimes, finds evidence that the low level of domestic savings is the main reason behind the high level of the real interest rate in the country by the end of the 2000s. However, according to the author, after controlling for everything else in the model, "Brazil's real interest rates are still about 2 percentage points higher than those of its inflation targeting peers", which could be due to, in the author's opinion, the non-tested effect of public lending at subsidized rates. In fact, De Bolle (2015) finds evidence that the expansion of credit by the BNDES tends to increase the real interest rate. According to this paper, "assuming linearity, if the flow of BNDES lending as a share of GDP were to be reduced by half – reverting to levels last seen in early 2004 – real interest rates could fall by as much as 1.3 percentage points".

The remainder of the paper proceeds as follows. Section 2 presents the general characterization of the steady-state capital cost. Section 3 shows the natural interest rate equations for (i) the standard or single funding rate case and (ii) the dual funding rate case. Section 4 estimates the Brazilian natural interest rate equation. Section 5 seeks to understand the decrease in the Brazilian natural interest rate between 2009 and 2014 and the country's historically high level of interest rate. Lastly, Section 6 concludes.

2 General characterization of the capital cost

My main theoretical goal in this paper is to obtain general expressions for the natural interest rate, which would be natural starting points to evaluate this equilibrium rate. To obtain these expressions, I first get a general characterization of the steady-state capital cost. Let K_t denote the capital stock, s_t the total nominal savings rate, that is, the nominal investment to nominal GDP ratio from National Accounts, p_t the price of the output, p_t^k

the price of capital, ζ_t the share of the real investments that alters the capital stock[1], Y_t the real product, δ the depreciation rate, g_t the growth rate of output, and * identifies the expected steady-state equilibrium of the variable.

Assumption A. $K_t = K_{t-1} + \left[\zeta_{t-1} s_{t-1} \left(p_{t-1}/p^k_{t-1}\right)\right] Y_{t-1} - \delta K_{t-1}$
Assumption B. $g^*_t + \delta > 0$

Assumption A is just a general law of motion for capital, while assumption B implicates the steady-state growth could not be negative and greater, in absolute terms, than the depreciation rate, which is around 4%-5% per annum (Feenstra et al. (2015)).

Proposition 1 *Suppose assumptions A and B hold. Then,*

$$r^*_t = KS^*_t \left(\frac{g^*_t + \delta}{\zeta^*_t s^*_t}\right) \tag{1}$$

where r_t is the real gross capital cost and KS_t is the ratio between capital owners' income and the GDP.

Proof. By definition,

$$KS_t = \frac{K_t p^k_t r_t}{Y_t p_t}$$

$$r^*_t = KS^*_t \left[\frac{1}{(K_t/Y_t)^*}\right] \left(\frac{p_t}{p^k_t}\right)^* \tag{2}$$

As presented in Appendix A, if assumptions A and B hold, it is easy to show

$$\left(\frac{K_t}{Y_t}\right)^* = \frac{\zeta^*_t s^*_t \left(p_t/p^k_t\right)^*}{g^*_t + \delta} \tag{3}$$

Substituting equation (3) into (2), one can obtain equation (1). ■

The generality of equation (1) is a strength, being consistent with a broad range of theories, but it is also a weakness given its lack of economic interpretation as it is mainly derived from definitions. In this regard, some additional theoretical structure is needed. Being MPK_t the marginal product of capital, μ_t a measure of firms' markup, A_{it} the productivity factors, and FP_{it} the other factors of production, with $i = 1, 2, ..., n$, I consider two additional assumptions:

Assumption C. $p_t = \left(\frac{r_t p^k_t}{MPK_t}\right) \mu_t$

[1]This share could be lower than 1 for at least two reasons: (i) National Accounts data include household capital formation and (ii) part of the current investment could leak and not turn into capital due to a number of factors, such as corruption (Gomes et al. (2005)).

Assumption D. $Y_t = \left[\alpha K_t^{\frac{\rho-1}{\rho}} + \sum_{i=1}^{n}\beta_i \left(A_{it}FP_{it}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$, where $\alpha + \sum_{i=1}^{n}\beta_i = 1$

From assumption C, p_t is determined by applying the markup μ_t over $r_t p_t^k / MPK_t$. This condition can be obtained from a model where firms are profit-maximizing, price takers in the capital market and price makers in the product market, when $r_t p_t^k / MPK_t$ is the marginal cost. Assumption D is imposing a CES production function with constant return to scale, where ρ is the elasticity of substitution[2].

Proposition 2 *Suppose assumptions A, B, C and D hold. Then,*

$$r_t^* = \left[\frac{\alpha}{\mu_t^*\left(p_t^k/p_t\right)^*}\right]\left[\frac{g_t^*+\delta}{\zeta_t^* s_t^*\left(p_t/p_t^k\right)^*}\right]^{1/\rho} \tag{4}$$

Proof. From assumptions C and D,

$$r_t = \frac{MPK_t}{\mu_t\left(p_t^k/p_t\right)} = \frac{\alpha\left(\frac{1}{K_t/Y_t}\right)^{1/\rho}}{\mu_t\left(p_t^k/p_t\right)}$$

$$r_t^* = \left[\frac{\alpha}{\mu_t^*\left(p_t^k/p_t\right)^*}\right]\left[\frac{1}{\left(K_t/Y_t\right)^*}\right]^{1/\rho} \tag{5}$$

Finally, given assumptions A and B, I can use equation (3). Substituting this equation into (5), one can obtain equation (4)[3]. ■

Naturally, equation (4) is consistent with (1) as $KS_t^* = \left(\frac{\alpha}{\mu_t^*}\right)\left(\frac{g_t^*+\delta}{\zeta_t^* s_t^*\left(p_t/p_t^k\right)^*}\right)^{\frac{1-\rho}{\rho}}$ under assumptions C and D. However, equation (4), although less general than (1), has the advantage of presenting greater economic intuition[4], as will be discussed in the next section.

3 Natural interest rate equations

Based on equations (1) and (4) of Section 2, I obtain the natural interest rate equations. To this end, I define the real gross capital cost as a weighted average of the real gross cost

[2]For the Cobb-Douglas case ($\rho \to 1$), one can drop the assumption of constant return to scale.

[3]This is similar to the gross marginal product of capital method of Muinhos and Nakane (2006). However, they use actual data for the capital-output ratio instead of Equation (3), not exploring the determinants of $(K_t/Y_t)^*$. Furthermore, they use this equation as a proxy for the natural interest rate, not for r_t^*, which probably explains the high level of their natural rate estimate from this method.

[4]If assumption B does not hold ($g_t^* + \delta \leq 0$), it is easy to show, using assumption A, $(K_t/Y_t)^* \to \infty$ and, consequently, $r_t^* \to 0$ (equation (5)). Thus, a more general expression for r_t^*, based only on assumptions A, C, and D, is $r_t^* = \max\left\{0, \left[\frac{\alpha}{\mu_t^*\left(p_t^k/p_t\right)^*}\right]\left[\frac{g_t^*+\delta}{\zeta_t^* s_t^*\left(p_t/p_t^k\right)^*}\right]^{1/\rho}\right\}$. However, since $g_t^* + \delta \leq 0$ is unusual in practice, I choose to use assumption B, simplifying the presentation.

of equity and debt:

$$r_t^* = (1-\theta_t^*)(i_t^* + TP_t^* + ERP_t^* + \delta) + \theta_t^*(f_t^* + DRP_t^* + \delta)$$

$$r_t^* = [(1-\theta_t^*)(i_t^* + TP_t^*) + \theta_t^* f_t^*] + ARP_t^* + \delta \tag{6}$$

where TP_t^* is the steady-state term premium at the steady-state average maturity of the investments in the economy ($\overline{m}_t^*$), i_t^* is the real natural spot interest rate, f_t is the real funding rate of credit operations for capital goods, ERP_t is the equity risk premium, DRP_t is the debt risk premium, θ_t is the debt weight on firms' capital structure, and $ARP_t = [(1-\theta_t)ERP_t + \theta_t DRP_t]$ is the average risk premium.

Since the steady-state debt cost is a function of the real funding rate f_t^*, the natural interest rate equation changes if f_t^* is linked or not to $i_t^* + TP_t^*$. In this context, I choose to determine the natural interest rate equation for two extreme cases. First, in the standard or single funding rate case, I assume that $f_t^* = i_t^* + TP_t^*$. Second, in the dual funding rate case, f_t^* and $i_t^* + TP_t^*$ are treated as independent.

3.1 Standard or single funding rate case

In the standard or single funding rate case, the funding rate of debt is linked to the funding rate of equity. In principle, they could be different given possible differences in the average maturity of the investments that are financed with equity and debt. However, in order to simplify the presentation, I consider an extreme case where they are exactly the same.

Assumption E. $r_t^* = i_t^* + TP_t^* + ARP_t^* + \delta$

Proposition 3 *Suppose assumptions A, B and E hold. Then,*

$$i_t^* = KS_t^* \left(\frac{g_t^* + \delta}{\zeta_t^* s_t^*} \right) - (TP_t^* + ARP_t^* + \delta) \tag{7}$$

Proof. Based on assumptions A and B, I can use, from proposition 1, equation (1). Combining this equation and assumption E, one can obtain equation (7). ■

In principle, if the only goal is to estimate the natural interest rate, equation (7) could be used. In this regard, note g_t^* is being multiplied by $KS_t^* / (\zeta_t^* s_t^*)$, which is not necessarily equal to 1. This suggests the assumption adopted by Holston et al. (2016) of one-for-one relationship between the trend growth and the natural rate is not necessarily adequate, at least for some countries.

However, in order to have an economic understanding about the natural interest rate, I should use equation (4) instead of (1) to obtain the natural rate equation.

Proposition 4 *Suppose assumptions A, B, C, D and E hold. Then,*

$$i_t^* = \left[\frac{\alpha}{\mu_t^*\left(p_t^k/p_t\right)^*}\right]\left[\frac{g_t^*+\delta}{\zeta_t^* s_t^*\left(p_t/p_t^k\right)^*}\right]^{1/\rho} - \left(TP_t^* + ARP_t^* + \delta\right) \tag{8}$$

Proof. Based on assumptions A, B, C and D, I can use, from proposition 2, equation (4). Combining this equation and the assumption E, one can obtain equation (8). ■

Let us evaluate the economic mechanisms behind the equilibrium relations expressed in the natural spot interest rate equation (8). A lower g_t^* or a higher effective real total savings rate $\zeta_t^* s_t^*\left(p_t/p_t^k\right)^*$ increases the capital intensity of the economy in the steady state (equation (3)), lowering MPK_t^* and, consequently, i_t^* (equation (5)). Similarly, a higher α increases i_t^* by its effect on MPK_t^*. As expected, the intensity of these effects increases with a lower elasticity of substitution ρ. After all, a given change in the capital intensity of the economy in the steady state should be followed by a higher change in the real price i_t^* under a low ρ. In fact, if $\rho \to \infty$, the capital intensity has no effect on i_t^*, as the CES converges to a perfect substitutes production function, whose isoquants are straight lines.

From assumption C, a higher real markup $\mu_t\left(p_t^k/p_t\right)$ decreases the real marginal cost r_t/MPK_t. As a result, a higher $\mu_t^*\left(p_t^k/p_t\right)^*$ implies a lower r_t^* and, consequently, i_t^*, since $MPK_t^* = \alpha\left[\frac{1}{(K_t/Y_t)^*}\right]^{1/\rho}$ is not a function of μ_t^*.

Furthermore, higher ARP_t^* and TP_t^* lower i_t^*. After all, as r_t^* is not affected by these variables (equation (4)), any change on the price of risk must be compensated by i_t^* in order to maintain r_t^* stable. An economic explanation for this equilibrium effect can be obtained by considering changes in investors' risk aversion. For example, if risk aversion increases, the relative demand for risky assets decreases, reducing their prices and increasing their yields; the contrary happens with the safe asset, which experiences an increase in the price and a fall in the yield. Thus, the risk premia will be higher and the risk-free interest rate lower.

A last comment regarding the term premium. Since the term structure of the natural interest rates is expected to be steeper under more uncertain scenarios, the natural rate for other maturities would increase with uncertainty for a given i_t^*. After all, the natural interest rate of any maturity can be obtained by adding the corresponding steady-state term premium on i_t^*. However, i_t^* is also affected by interest rate uncertainty, although the effect is not clear.

On the one hand, a higher uncertainty implicates higher term premium for each maturity. On the other hand, the steady-state average maturity of the investments in the economy ($\overline{m}_t^*$) tends to decrease with higher uncertainty, lowering TP_t^*. Therefore, given these two opposite effects, TP_t^* can increase or decrease with higher uncertainty, decreasing or increasing i_t^*, respectively. As a result, the effect on the natural rates of other maturities is not clear either and could be distinct for different maturities[5]. In fact, there are only two things that we can say about it. First, as higher uncertainty increases the slope of the yield curve, the natural rate for a sufficiently long maturity will rise. Second, the natural rate of maturity $\overline{m}_t^*$, $i_t^* + TP_t^*$, is not affected by changes in the term premium (equation (8)).

Regarding δ, the effect is not clear. On the one hand, the increase in δ makes i_t^* higher by lowering the capital intensity of the economy (equation (3)). On the other hand, it has a similar effect to the risk premia: a higher δ decreases i_t^*. The prevalent effect depends on the parametrization[6].

The current debate about the low levels of natural interest rates in advanced economies, especially in the US, seems to confirm the generality of equation (8), as the main explanations about this phenomenon are included in this equation. For example, based on Del Negro et al. (2017), one can identify three main explanations[7]. First of all, the lowering of the growth of trend output in these economies (e.g., Laubach and Williams (2003), Holston et al. (2016)), which is captured in equation (8) by g_t^*. Second, the increase in aggregate savings due to, for example, demographic transition or higher income inequality, which is captured in equation (8) by $\zeta_t^* s_t^* \left(p_t/p_t^k\right)^*$. Third, the increase in risk premia, which is the main contribution of Del Negro et al. (2017), captured in the equation by ARP_t^*.

3.2 Dual funding rate case

If there are two funding rates in the economy, as in Brazil, one cannot use assumption E. After all, in this case, the average funding rate f_t^* is at most just partially linked to i_t^*. In order to simplify the presentation, I assume f_t^* and $i_t^* + TP_t^*$ are independent:

Assumption F. $r_t^* = [(1-\theta_t^*)(i_t^* + TP_t^*) + \theta_t^* f_t^*] + ARP_t^* + \delta$

[5]This result indicates that the use of long-term market interest rates as proxies for natural short-term interest rates could be problematic, even if only to extract information about their dynamics. One example of this kind of approach to estimate the natural rate is the forward curve method of Fuentes and Gredig (2007), applied for the Chilean economy.

[6]In the case of a Cobb-Douglas production function ($\rho = 1$), if $KS_t^* / (\zeta_t^* s_t^*) > 1$, the first effect dominates and a higher δ increases i_t^*; if $KS_t^* / (\zeta_t^* s_t^*) < 1$, the second effect dominates and a higher δ decreases i_t^*; and if $KS_t^* / (\zeta_t^* s_t^*) = 1$, δ does not have any effect on i_t^*.

[7]They present four explanations for the current low level of observable real interest rates. However, the secular stagnation hypothesis (Summers (2014)) is mainly an explanation concerning the low level of observable interest rates, not the natural interest rate. In fact, this hypothesis can be viewed more as a consequence of an environment of very low natural rate rather than as an explanation of such environment.

Proposition 5 *Suppose assumptions A, B and F hold. Then,*

$$i_t^* = \frac{KS_t^* \left(\frac{g_t^*+\delta}{\zeta_t^* s_t^*}\right) - \left[TP_t^* \left(1-\theta_t^*\right) + \delta + ARP_t^* + \theta_t^* f_t^*\right]}{1-\theta_t^*} \tag{9}$$

Proof. Based on assumptions A and B, I can use, from proposition 1, equation (1). Combining this equation and assumption F, one can obtain equation (9). ■

Proposition 6 *Suppose assumptions A, B, C, D and F hold. Then,*

$$i_t^* = \frac{\left[\frac{\alpha}{\mu_t^* \left(p_t^k/p_t\right)^*}\right] \left[\frac{g_t^*+\delta}{\zeta_t^* s_t^* \left(p_t/p_t^k\right)^*}\right]^{1/\rho} - \left[TP_t^* \left(1-\theta_t^*\right) + \delta + ARP_t^* + \theta_t^* f_t^*\right]}{1-\theta_t^*} \tag{10}$$

Proof. Based on assumptions A, B, C and D, I can use, from proposition 2, equation (4). Combining this equation and assumption F, one can obtain equation (10). ■

There is a more interesting way of presenting equation (10). Initially, define i_t^{**} as the natural interest rate of the dual funding rate case if $f_t^* = i_t^* + TP_t^*$. Thus, i_t^{**} is equivalent to i_t^* of the standard case (equation (8)):

$$i_t^{**} = \left[\frac{\alpha}{\mu_t^* \left(p_t^k/p_t\right)^*}\right] \left[\frac{g_t^*+\delta}{\zeta_t^* s_t^* \left(p_t/p_t^k\right)^*}\right]^{1/\rho} - \left(TP_t^* + \delta + ARP_t^*\right) \tag{11}$$

Consequently, I could define the difference between i_t^* and i_t^{**} as the macroeconomic steady-state cross subsidy (MCS_t^*):

$$MCS_t^* \equiv i_t^* - i_t^{**} \tag{12}$$

Proposition 7 *Suppose assumptions A, B, C, D and F hold. Then,*

$$MCS_t^* = \left(\frac{\theta_t^*}{1-\theta_t^*}\right) \left[\left(i_t^{**} + TP_t^*\right) - f_t^*\right] \tag{13}$$

Proof. Based on assumptions A, B, C, D and F, I have, from proposition 6, (10). Given this equation and (11) and (12), one can obtain (13), as shown in Appendix B. ■

One can evaluate i_t^* in an economy with two funding rates using equations (11), (12) and (13). Since equation i_t^{**} is equivalent to i_t^* of the standard case, I can just analyze here MCS_t^*, which shows a cross subsidy effect: if $f_t^* < \left(i_t^{**} + TP_t^*\right)$, $MCS_t^* > 0$ and $i_t^* > i_t^{**}$. This cross subsidy increases with the intensity of the subsidy $\left[\left(i_t^{**} + TP_t^*\right) - f_t^*\right]$ (price effect) and with the weight of the subsidized lending on the economy θ_t^* (volume effect).

MCS_t^* is not a typical microeconomic cross subsidy, in which the effect happens within firms: it is a macroeconomic cross subsidy. The intuition behind this effect is similar to that of the risk premia presented in Section 3.1 for the standard case. After all, lowering f_t^* results in a compression in the DRP_t^* when it is calculated using the opportunity cost $i_t^* + TP_t^*$, as in the standard case. Thus, the relative demand for risky assets rises, increasing the safe asset yield.

4 Estimating the Brazilian natural interest rate

4.1 Model

I estimate the Brazilian natural interest rate using a quarterly semi-structural macroeconomic model with four blocks: (i) a Phillips curve, (ii) an IS curve, (iii) potential output equations, and (iv) a natural interest rate equation.

The Phillips curve adopted is

$$\pi_t^{free} = \psi_0 E_t\left(\pi_{t+1}\right) + \psi_1 \pi_{t-1} + \left(1 - \psi_0 - \psi_1\right) \pi_{t-1}^f + \psi_2 \widetilde{y}_t + \varepsilon_t^{ph} \tag{14}$$

where π_t^{free} is the free-price inflation rate, π_t is the inflation rate, $E_t\left(\cdot\right)$ is the expectation operator conditional on the information set available at period t, π_t^f is the inflation rate of imported goods, $\widetilde{y}_t$ is the output gap, and ε_t^{ph} is a normally distributed serially uncorrelated error. The parameters of $E_t\left(\pi_{t+1}\right)$, π_{t-1} and π_{t-1}^f add to 1, ensuring the long-run verticality of the Phillips curve.

The IS curve is

$$\widetilde{y}_t = \gamma_0 \widetilde{y}_{t-1} - \gamma_1 (r_{t-1}^1 - r_{t-1}^{1*}) - \gamma_2 GFC_t + \varepsilon_t^{IS} \tag{15}$$

where r_t^1 is the one-year cost of capital, GFC_t is a dummy variable to control for the Global Financial Crisis[8] and ε_t^{IS} is a normally distributed serially uncorrelated error. Equation (15) uses the short-term capital cost gap instead of the usual interest rate gap, as the investment decision should depend on the capital cost, no matter the source of this cost.

The Brazilian economy has two funding rates, one linked to the monetary policy rate (Selic rate) and another to the BNDES funding rate. Since my sample only includes the periods when the nominal funding rate for BNDES credit operations was the arbitrarily defined TJLP, I can use the results of Section 3.2. In this regard, I assume $\theta_t = \theta_t^{BNDES}$, $f_t = f_t^{BNDES}$ and $DRP_t = DRP_t^{BNDES}$, where θ_t^{BNDES} is the weight of BNDES lending in capital goods financing, f_t^{BNDES} is the real funding rate for BNDES credit operations, and DRP_t^{BNDES} is the spread between the lending interest rate and the funding

[8]The dummy variable assumes 1 for 2008Q4 and 2009Q1 and 0 otherwise.

rate in BNDES credit operations for capital goods. Thus, I assume the remaining share of the investments, equal to $1 - \theta_t^{BNDES}$, is financed through equity or debt with costs similar to equity[9].

Hence, substituting the definitions of r_{t-1}^1 and r_{t-1}^{1*} for Brazil based on assumption F into (15) and assuming θ_t^{BNDES}, f_t^{BNDES} and DRP_t^{BNDES} are martingales[10] and, consequently, $\theta_t^{BNDES*} = \theta_t^{BNDES}$, $f_t^{BNDES*} = f_t^{BNDES}$, and $DRP_t^{BNDES*} = DRP_t^{BNDES}$,

$$\begin{aligned} \widetilde{y}_t = \gamma_0 \widetilde{y}_{t-1} - \gamma_1 \left(1 - \theta_{t-1}^{BNDES}\right) \left[\left(i_{t-1}^1 + TP_{t-1}^1\right) - \left(i_{t-1}^* + TP_{t-1}^{1*}\right)\right] \\ -\gamma_1 \widetilde{ARP}_{t-1} - \gamma_2 GFC_t + \varepsilon_t^{IS} \end{aligned} \tag{16}$$

where i_t^1 is the expected average spot risk-free real interest rate from t to $(t+1)$, TP_t^1 is the one-year term premium, and $\widetilde{ARP}_t = (ARP_t - ARP_t^*)$ is the average risk premium gap.

Equation (16) presents two interesting facts. First, the presence of $\widetilde{ARP}_t$, capturing the impact of changes on the risk premia[11]. Second, the parameter of the interest rate gap, $\gamma_1 \left(1 - \theta_{t-1}^{BNDES}\right)$, implies BNDES credit decreases the power of monetary policy. This loss of power is expected during the period considered, as policy rate changes do not affect all investment decisions since, along the sample, f_t^{BNDES} is based on an arbitrarily defined nominal rate (TJLP). Thus, the presence of two funding rates not just alters i_t^*, but could also change the interest rate gap, since a higher level of interest rate gap volatility is required to maintain a given volatility of $\widetilde{y}_t$.

The potential output equations are those of Laubach and Williams (2003):

$$y_t^* = y_{t-1}^* + g_t^* + \varepsilon_t^{y^*} \tag{17}$$

$$g_t^* = g_{t-1}^* + \varepsilon_t^{g^*} \tag{18}$$

where $\varepsilon_t^{y^*}$ and $\varepsilon_t^{g^*}$ are normally distributed serially uncorrelated errors. Thus, in the short run, the growth of y_t^* can be different from g_t^* because of the shocks in the level of the potential output ($\varepsilon_t^{y^*}$).

Finally, the natural interest rate is estimated using two different specifications. In the benchmark specification, I assume the natural one-year interest rate follows a random walk, which is a very flexible assumption to estimate a latent variable[12]:

$$\left(i_t^* + TP_t^{1*}\right) = \left(i_{t-1}^* + TP_{t-1}^{1*}\right) + \varepsilon_t^{i_{RW}^*} \tag{19}$$

[9] According to Anbima data, total funding from domestic capital market has increased by 59% in 2019, after growing 74% in 2017 and 11% in 2018. Given this growing importance of capital markets, future research may need to handle this type of financing more carefully.

[10] After all, changes in these variables could be seen as unpredictable since they are essentially arbitrarily defined by the government.

[11] Given Equation (15), this term would also appear in the IS of the standard case.

[12] This kind of approach to estimate the natural rate is used, for example, in Perrelli and Roache (2014).

where $\varepsilon_t^{i^*_{RW}}$ is a normally distributed serially uncorrelated error.

In the proposed specification, I use equation (10) of Section 3.2 for $\rho = 1$ (Cobb-Douglas case[13]) as I find empirical evidence suggesting an unitary elasticity of substitution is a good approximation for Brazil[14]. The estimation of this natural rate equation elicits three comments. First, using assumption D for $\rho \to 1$, with labor being one of the other factors of production FP_{it}, and an assumption similar to C but defined in terms of labor instead of capital[15], one can obtain $LS_t = \beta/\mu_t$, where LS_t is the ratio between labor's income and the GDP and β is the share parameter of labor. Thus, under these assumptions,

$$\mu_t^* = \frac{\beta}{LS_t^*} \tag{20}$$

where LS_t can be estimated using National Accounts data[16,17].

Second, I assume the steady-state share of the real investments that alters the capital stock is time-invariant:

$$\zeta_t^* = \zeta \tag{21}$$

Third, regarding the term premium estimation, I use a methodology that is different from the literature as it adopts a very flexible approach regarding the interest rate expectations, assuming a more rigid term premium structure over the maturities[18]. In formal terms,

$$TP_t^m = \kappa_t \left(1 - \varphi^m\right) m \tag{22}$$

where $\kappa_t \geq 0$ is a time-variant parameter that measures the risk in the price, which depends on risk and risk aversion, φ is a parameter that captures the evolution of the term premium along the maturities, and m is the maturity in months.

[13] In fact, using a Cobb-Douglas production function instead of assumption D, one would get exactly equation (10) for $\rho = 1$.

[14] Estimating a model using (10) without imposing $\rho = 1$, I find ρ between 0.92 and 1.05 and not stastically different from 1 as the respective p-values are always higher than 0.25. In this model, I use the g_t^* estimated in the other two specifications, as I could not estimate it using (18), given the nonlinearity of g_t^* in (10). Furthermore, since I was not able to estimate ζ_t^*, I considered six values for it (0.5, 0.6, ..., 1). Hence, given the two g_t^* and the six ζ_t^*, I estimate twelve versions of this model.

[15] Formally, $p_t = \left(w_t p_t^w / MPL_t\right) \mu_t$, where MPL_t is the marginal product of labor, w_t is the real wage, and p_t^w is the price of consumption goods. Comparing this condition with assumption C, one can see $r_t p_t^k / MPK_t = w_t p_t^w / MPL_t$, which is a standard optimal condition when firms are price takers in the markets for factors of production.

[16] Conversely, if $\mu_t > 1$, estimating KS_t from National Accounts is not straightforward since $KS_t \neq 1 - LS_t$. In fact, considering labor and capital as the only factors of production, it is easy to show $KS_t = 1/\mu_t - LS_t$ if one uses assumptions C and D, along with the labor version of C used in equation (20).

[17] If $\rho = 1$ is not assumed, one can use $\mu_t^* = \left(\frac{1}{LS_t^*}\right)\left\{1 - \alpha\left[\frac{\zeta_t^* s_t^* \left(p_t/p_t^k\right)^*}{g_t^* + \delta}\right]^{(\rho-1)/\rho}\right\}$, which can be obtained from assumptions A, B, the labor version of C used in (20), and D with capital and labor as the only factors of production.

[18] See Appendix C for details.

Thus,

$$TP_t^* = TP_t^{\overline{m}_t^* *} = \kappa_t^* \left(1 - \varphi^{\overline{m}_t^*}\right) \overline{m}_t^* \tag{23}$$

where $\overline{m}_t^*$ is modeled as a function of κ_t^*:

$$\overline{m}_t^* = \eta_0 \exp\left(-\eta_1 \kappa_t^*\right) \tag{24}$$

Equation (24) has three interesting characteristics. First, the greater the interest rate uncertainty as measured by κ_t^*, the shorter is $\overline{m}_t^*$, with $\overline{m}_t^* \to 0$ when $\kappa_t^* \to \infty$. Second, $\overline{m}_t^*$ has an upper bound, which seems to be a reasonable assumption. This upper bound is given by η_0 as $\overline{m}_t^* = \eta_0$ when uncertainty is null ($\kappa_t^* = 0$). Third, this function is flexible in the sense that it can show, depending on η_0 and η_1, a positive or a negative effect of κ_t^* on TP_t^*, the two possibilities discussed in Section 3.1. In fact, as this function is nonlinear, the sign of this effect may depend on the level of κ_t^*. I will come back to this discussion in Section 4.3.

Substituting equations (20) and (21) into (10) and using once more $\rho = 1$, $\theta_t = \theta_t^* = \theta_t^{BNDES}$, $f_t = f_t^* = f_t^{BNDES}$ and $DRP_t = DRP_t^* = DRP_t^{BNDES}$, I get the natural rate equation of the proposed specification:

$$i_t^* = \frac{\left(\frac{\alpha/\beta}{\zeta}\right) LS_t^* \left(\frac{4g_t^* + \delta}{s_t^*}\right) - \left[TP_t^*\left(1 - \theta_t^{BNDES}\right) + \delta + ARP_t^* + \theta_t^{BNDES} f_t^{BNDES}\right]}{1 - \theta_t^{BNDES}} \tag{25}$$

where g_t^* is multiplied by 4 just to annualize the quarterly growth, since the other variables in the equation are in an annual basis, and TP_t^*, from equations (23) and (24), is given by the following expression:

$$TP_t^* = \kappa_t^* \left[1 - \varphi^{\eta_0 \exp(-\eta_1 \kappa_t^*)}\right] \eta_0 \exp\left(-\eta_1 \kappa_t^*\right) \tag{26}$$

4.2 Estimation methodology

From Section 4.1, the equations of the model in each specification are those shown in Table 1. I use quarterly exogenous estimates from 2002 to 2017 for π_t^{free}, $E_t\left(\pi_{t+1}\right)$, π_t, π_t^f, y_t, $\left(i_t^1 + TP_t^1\right)$, LS_t, δ, s_t, κ_t, φ, θ_t^{BNDES}, ERP_t, DRP_t^{BNDES}, and f_t^{BNDES}. A complete description of these exogenous variables can be found in Appendix D. Here, two comments are due. First, regarding the term premium, the parameters κ_t and φ are estimated previously, as discussed in Appendix C, while η_0 and η_1 are estimated in the model. Second, I use the average inflation expectation collected just in the first month of the quarter to measure $E_t\left(\pi_{t+1}\right)$. Since the first monthly inflation of the quarter is released only in the second month, this measure of $E_t\left(\pi_{t+1}\right)$ is not affected by π_t^{free}, avoiding endogeneity problems.

In the proposed specification, the number of steady-state variables is elevated and nonlinear relations exist between some of these latent variables in equation (25). In this context, I choose to estimate s_t^*, LS_t^*, ERP_t^* and κ_t^* first, by using the r-filter of Araujo et al. (2003), which is a Hodrick-Prescott filter (HP filter) generalization. I apply the r-filter of order 1, since it is more suitable than the HP filter (r-filter of order 2) for series that are stationary around a piecewise constant process, as is the case of these four variables[19]. In the benchmark specification, of these four variables, only ERP_t^* is included, which is also estimated first using r-filter of order 1, ensuring comparability.

Therefore, I need to estimate the parameters ψ_0, ψ_1, ψ_2, γ_0, γ_1, γ_2, σ^2_{ph}, σ^2_{IS}, $\sigma^2_{y^*}$, and $\sigma^2_{g^*}$ in both specifications, while $\sigma^2_{i^*_{RW}}$ is estimated only in the benchmark specification and $\left(\frac{\alpha/\beta}{\zeta}\right)$, η_0 and η_1 only in the proposed specification. I also need to estimate the latent variables y_t^* and g_t^*, besides $(i_t^* + TP_t^{1*})$ in the benchmark specification.

Table 1: Equations of the model: benchmark and proposed specifications

Block	Equations
Phillips	$\pi_t^{free} = \psi_0 E_t\left(\pi_{t+1}\right) + \psi_1 \pi_{t-1} + \left(1 - \psi_0 - \psi_1\right)\pi^f_{t-1}$ $+\psi_2 \widetilde{y}_t + \varepsilon_t^{ph}$ (14)
IS	$\widetilde{y}_t = \gamma_0 \widetilde{y}_{t-1} - \gamma_1\left(1 - \theta_{t-1}^{BNDES}\right)\left[\left(i^1_{t-1} + TP^1_{t-1}\right) - \left(i^*_{t-1} + TP^{1*}_{t-1}\right)\right]$ $-\gamma_1 \widetilde{ARP}_{t-1} - \gamma_2 GFC_t + \varepsilon_t^{IS}$ (16)
Potential Output	$y_t^* = y_{t-1}^* + g_t^* + \varepsilon_t^{y^*}$ (17) $g_t^* = g_{t-1}^* + \varepsilon_t^{g^*}$ (18)
Natural interest rate	Benchmark specification: $\left(i_t^* + TP_t^{1*}\right) = \left(i^*_{t-1} + TP^{1*}_{t-1}\right) + \varepsilon_t^{i^*_{RW}}$ (19) Proposed specification: $i_t^* = \frac{\left(\frac{\alpha/\beta}{\zeta}\right) LS_t^*\left(\frac{4g_t^*+\delta}{s_t^*}\right) - \left[TP_t^*\left(1-\theta_t^{BNDES}\right)+\delta+ARP_t^*+\theta_t^{BNDES} f_t^{BNDES}\right]}{1-\theta_t^{BNDES}}$ (25) $TP_t^* = \kappa_t^*\left[1 - \varphi^{\eta_0 \exp(-\eta_1 \kappa_t^*)}\right]\eta_0 \exp\left(-\eta_1 \kappa_t^*\right)$ (26)

Note: $\left(\varepsilon_t^{ph}, \varepsilon_t^{IS}, \varepsilon_t^{y^*}, \varepsilon_t^{g^*}, \varepsilon_t^{i^*_{RW}}\right) \overset{iid}{\sim} N(0, \sigma^2_\varepsilon I)$
$\sigma^2_\varepsilon = \left(\sigma^2_{ph}, \sigma^2_{IS}, \sigma^2_{y^*}, \sigma^2_{g^*}, \sigma^2_{i^*_{RW}}\right)$

As discussed in Laubach and Williams (2003), maximum likelihood estimates of $\sigma^2_{g^*}$ is likely to be biased towards 0 given the so-called pile-up problem. I therefore assume

$$\sigma^2_{g^*} = \sigma^2_{y^*}/\lambda \tag{27}$$

where λ is estimated using the median unbiased estimator of Stock and Watson (1998).

[19] The parameter is 40, which is equivalent to the 1600 standard parameter of the HP filter for quarterly data.

The method proceeds in an iterative manner. In the first step, I estimate the complete model using maximum likelihood and the Kalman filter[20], imposing a value for λ. In the second step, I calculate the exponential Wald statistic for an intercept shift at an unknown break date in the first difference of the potential output estimated in the previous step. The results of Stock and Watson (1998) are used to convert the Wald statistic into an estimate of λ. These two steps are repeated until λ converges[21].

The use of an iterative procedure here is the main difference from the approach of Laubach and Williams (2003)[22]. Thus, if the solution is unique, the initial guess of λ is not relevant in my iterative method.

4.3 Results

In order to verify if the solution to the iterative problem is unique, I use a numerical exercise. For each 100 initial λ obtained from 1 to 1,600 on a logarithmic scale[23], I estimate λ in both specifications using the methodology of Section 4.2, but without iterating. The results are shown in Figure 1.

Panels A and B show the estimated λ is always lower than 10, no matter the initial guess. Thus, in Panels C and D I plot the 45^o line along with the results from the numerical exercise for initial λ below 10. The solutions of the iterative process should be those points that cross the 45^o line, which seem to be unique, between 4 and 6. In fact, using the iterative methodology, I obtain 4.3 for the benchmark and 5.3 for the proposed specifications.

Based on these variance restrictions, I estimate the other parameters by maximum likelihood, which are shown in Table 2[24]. The estimates of the two specifications are very similar, having all the expected signs and being, except from ψ_1, all significant at 5%.

[20] The initial guesses of the parameters are: $\psi_0 = 0.6$, $\psi_1 = 0.3$, $\psi_2 = 0.1$, $\gamma_0 = 0.9$, $\gamma_1 = 0.3$, $\gamma_2 = 0.1$, $[\alpha/\beta]/\zeta = 1$, $\eta_0 = 60$ and $\eta_1 = 600$. For all the variances of the model, I use $\exp(-10)$.

[21] The convergence criteria is 0.1.

[22] There are two other differences. First, I do not use the Stock and Watson (1998) estimator to restrict the natural rate variance, since I find no problem in estimating it by maximum likelihood. Second, when calculating the Wald statistic, I follow the suggestion of Stock and Watson (1998) and correct the data for serial correlation, assuming the error follows an autoregressive process of order 3.

[23] Equations (17), (18) and (27) are those used in the estimation of the r-filter of order 1 of Araujo et al. (2003) through Kalman Filter. Thus, the usual parameter λ would be 40, equivalent to the 1600 parameter of the HP filter. In order to obtain a sufficient large interval of initial λ and given the logarithmic sensitivity of the filter to λ, I multiply and divide this standard parameter by 40 to calculate the interval of the numerical exercise (1 to 1600).

[24] In the case of the benchmark specification, the estimates for $\ln(\sigma^2_{ph})$, $\ln(\sigma^2_{IS})$, $\ln(\sigma^2_{y^*})$, and $\ln(\sigma^2_{i^*_{RW}})$ are -10.021,-10.456,-12.123, and -8.961, respectively. Regarding the proposed specification, estimates for $\ln(\sigma^2_{ph})$, $\ln(\sigma^2_{IS})$, and $\ln(\sigma^2_{y^*})$ are -10.125, -10.368, and -12.422. All these estimates are significant at 1%.

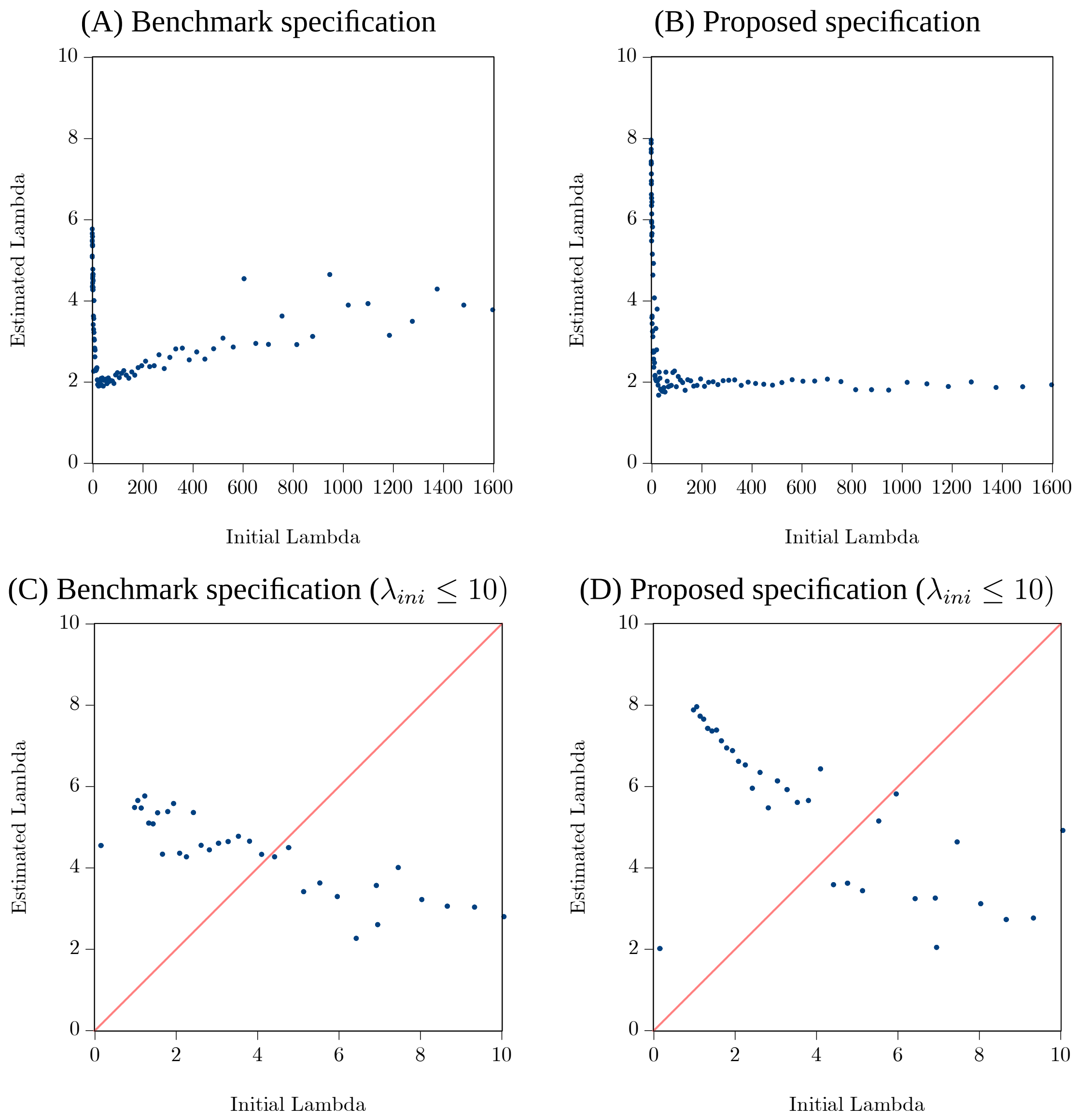


Figure 1: Robustness analysis of the estimated Lambda

As discussed in Section 3, $4g_t^*$ is multiplied in the natural rate equation by $KS_t^*/\left(\zeta_t^* s_t^*\right)$, which is equivalent here to $\left[\frac{\alpha/\beta}{\zeta}\right]\left(\frac{LS_t^*}{s_t^*}\right)$. Thus, in Brazil, $KS_t^*/\left(\zeta_t^* s_t^*\right)$ is around 2.5, much higher than 1, the assumption of Holston et al. (2016). The high level of $KS_t^*/\left(\zeta_t^* s_t^*\right)$ not only makes i_t^* more sensitive to changes in g_t^* but also increases i_t^*. I will come back to this discussion in Section 5.

As seen in Section 4.1, the loss of monetary policy power due to the dual funding rate elevates the interest rate gap volatility required to maintain a given volatility of the output gap. From equation (16), if subsidized lending did not exist in Brazil, the interest rate gap would need to be multiplied by $\left(1-\theta_{t-1}^{BNDES}\right)$ in order to maintain the same level of output gap, all else equal. As a result, the standard deviation of the interest rate gap would be reduced by 13%.

Table 2: Estimated Parameters

Block	Parameter	Regressor	Estimate Benchmark	Estimate Proposed
Phillips	ψ_0	$+E_t\left(\pi_{t+1}\right)$	0.645***	0.671***
			(0.239)	(0.175)
	ψ_1	$+\pi_{t-1}$	0.322	0.298*
			(0.241)	(0.174)
	ψ_2	$+\widetilde{y}_t$	0.104***	0.101***
			(0.000)	(0.000)
IS	γ_0	$+\widetilde{y}_{t-1}$	0.793***	0.894***
			(0.001)	(0.000)
	γ_1	$-(r^1_{t-1} - r^{1*}_{t-1})$	0.228***	0.302***
			(0.000)	(0.000)
	γ_2	$-GFC_t$	0.041***	0.040***
			(0.007)	(0.008)
Natural interest rate	$\frac{\alpha/\beta}{\zeta}$	$+LS^*_t\left(\frac{4g^*_t+\delta}{s^*_t}\right)$		0.994***
				(0.000)
	η_0	Nonlinear on TP^*_t		124.974**
				(58.034)
	η_1	Nonlinear on TP^*_t		1537.421**
				(725.721)

Note: Parenthesis: standard error; Significance: * (10%), ** (5%), *** (1%)
Method: Kalman filter / Maximum likelihood (EViews 10)
Sample: 2002Q2 – 2017Q4 (63 observations)

Panel A of Figure 2 presents the estimated natural one-year interest rate for both specifications and the 95% confidence interval for the benchmark specification[25]. The confidence interval is wide, close to 2pp, except in the last quarter of 2017 when it reaches 3pp. This recommends caution when analyzing point estimates, especially for real time

[25]Calculating the confidence interval for the proposed specification is not straightforward, given that it uses some exogenous estimates. Thus, I choose to focus on the confidence interval of the benchmark specification.

estimation. With this caveat in mind, note the two estimates are similar, being close to 12% until 2008, rapidly decreasing between 2009 and 2014, and stabilizing around 4% after 2015. This decrease in the natural interest rate of Brazil after the Global Financial Crisis has also been reported in other papers (e.g., Gottlieb (2013), Perrelli and Roache (2014)).

One can also estimate the natural interest rates of other maturities[26], as shown in Panel B of Figure 2 based on the proposed specification[27]. As discussed in Section 3, in periods of high uncertainty, the term structure of the natural interest rate becomes steeper. In fact, from 2003 to 2005, when the term premium is very high, as shown in Appendix C, the dispersion of the natural rates is also high. As the term premium decreased, the dispersion became smaller, with the natural interest rate of longer maturities falling rapidly between 2003 and 2006.

(A) Natural one-year interest rate of each specification

(B) Natural rates of different maturities (proposed specification)

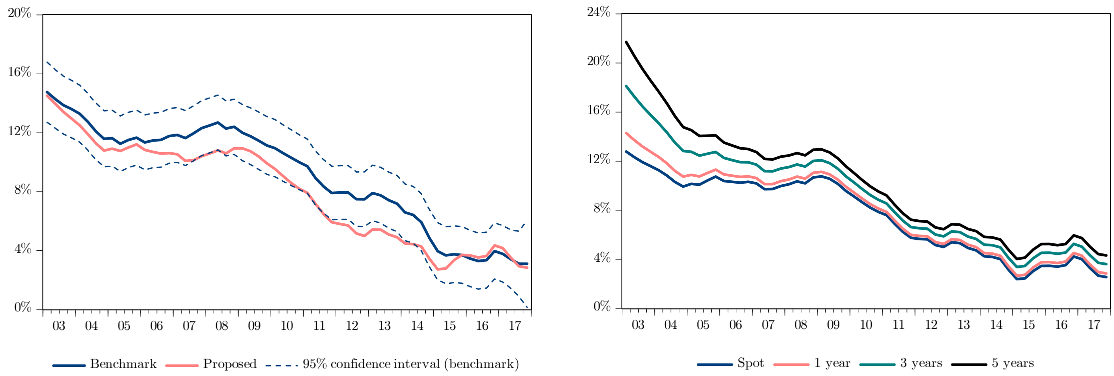


Figure 2: Natural interest rates estimates

Figure 3 shows the effect of changes in the steady-state interest rate uncertainty, as measured by κ_t^*, on the natural interest rate of different monthly maturities. I evaluate this effect in a period of high uncertainty (2003Q1, Panel A) and also in a period of low uncertainty (2017Q4, Panel B). The blue line shows the estimated curve, while the red and green lines are two simulated curves calculated using $\widetilde{\kappa}_t^*$ instead of κ_t^*, where $\widetilde{\kappa}_t^* = 2\kappa_t^*$ for the red line scenario and $\widetilde{\kappa}_t^* = \kappa_t^*/2$ for the green line scenario. As discussed in Section 3.1, the natural rate of the steady-state average maturity of the investments in the economy ($\overline{m}_t^*$), $i_t^* + TP_t^*$, is not affected by κ_t^*. This level is shown by the dashed black line. Therefore, the points where the blue, red, and green lines cross the dashed black line identify the $\overline{m}_t^*$ of each of these three cases.

This exercise shows some interesting results. First, as expected, when the uncertainty

[26]Using equation (22), I can obtain the term premium of any maturity and, consequently, estimate any natural interest rate, as discussed in Section 3.1.

[27]The results using the benchmark specification would be close, given the similarity of the estimated natural one-year interest rate in these two specifications (Panel A of Figure 2).

increases the curve becomes steeper and $\overline{m}_t^*$ decreases. Second, as discussed in Section 3.1, the natural rate of sufficiently long maturities always increases with uncertainty. Third, for shorter maturities, the effect depends on the initial level of κ_t^*. In periods of high uncertainty (Panel A), a higher κ_t^* decreases the TP_t^*, rising i_t^* and, consequently, the natural rate of all maturities. This happens because the direct positive effect of κ_t^* on TP_t^* (equation (23)) is more than compensated by the indirect negative impact of κ_t^* on TP_t^* through $\overline{m}_t^*$ (equation (24)). In periods of low uncertainty (Panel B), the direct positive effect of κ_t^* on TP_t^* is stronger: an increase in κ_t^* rises the TP_t^* and decreases i_t^*. As a consequence, the natural rate of shorter maturities are reduced by the higher uncertainty[28].

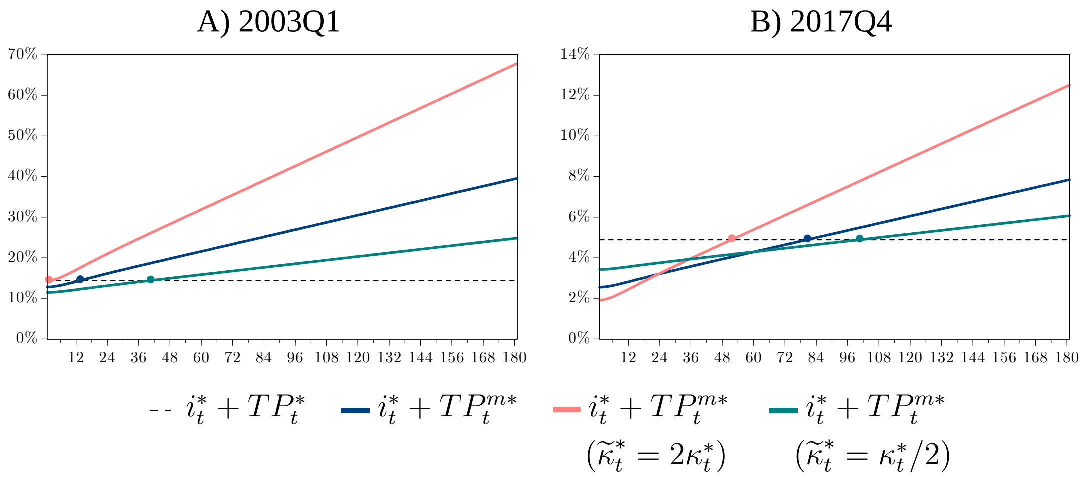


Figure 3: Natural interest rates of different monthly maturities (proposed specification)

5 Understanding the Brazilian natural interest rate

The goal of this section is to understand the results of Section 4.3. Since under the benchmark specification the natural rate follows a random walk, it is not able to address this kind of issue. Thus, one must use the proposed specification and its interest rate equation (25). This is the main strength of the proposed specification: it can be used not only to estimate the natural interest rate, but also to evaluate its results. This evaluation should be interpreted essentially as conditional expectations exercises, since some caution is needed regarding the causalities, which could be precisely identified only in structural approaches.

Initially, I want to understand the decrease in the Brazilian natural rate between 2009 and 2014. Figure 4 presents a decomposition of the natural spot interest rate using the proposed specification. I choose to decompose the natural rate in three steps, especially given the nonlinearities of (25). Firstly, as shown in Panel A of Figure 4, I decompose i_t^*

[28] The distinct effect of κ_t^* on TP_t^* on periods of low and high uncertainty is not a consequence of the initial guesses for η_0 and η_1. After all, under the initial values, the effect of κ_t^* on TP_t^* is always positive.

into i_t^{**} and MCS_t^* (equation (12)), where MCS_t^* comes from (13) and i_t^{**} from (11) for $\rho = 1$. Secondly, in Panel B, I decompose i_t^{**} into r_t^* and $-(TP_t^* + \delta + ARP_t^*)$ based on (4) and (11), both under $\rho = 1$. Finally, in Panel C, I decompose $\ln(r_t^*)$ using (4) under $\rho = 1$, (20) and (21), into three components: (i) $\ln\left[\left(\frac{\alpha/\beta}{\varsigma}\right) LS_t^*\right]$, which, from equation (20), has its dynamic behavior given by μ_t^*; (ii) $\ln(4g_t^* + \delta)$, linked to the dynamics of g_t^*; and (iii) $-\ln(s_t^*)$, which is only a function of s_t^*.

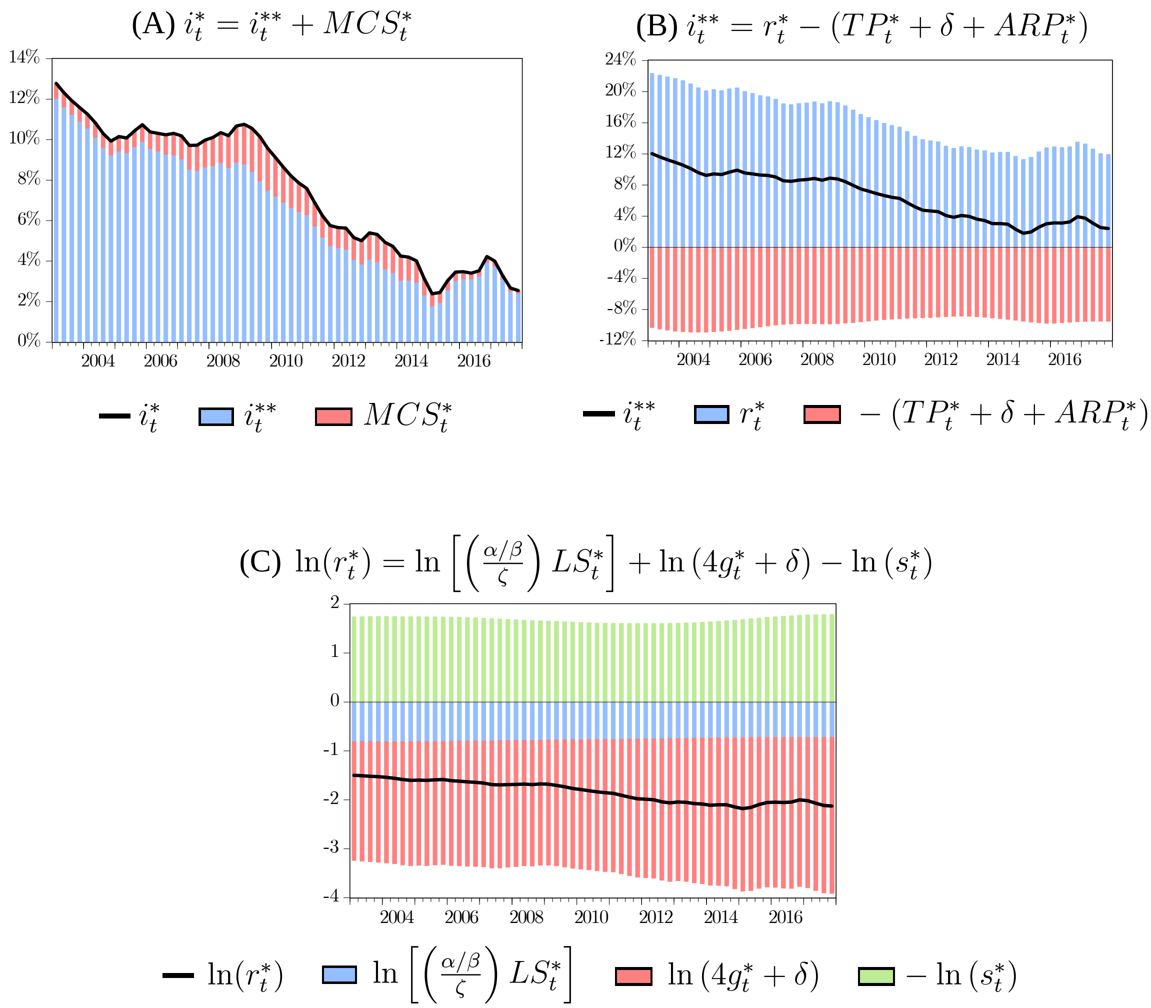
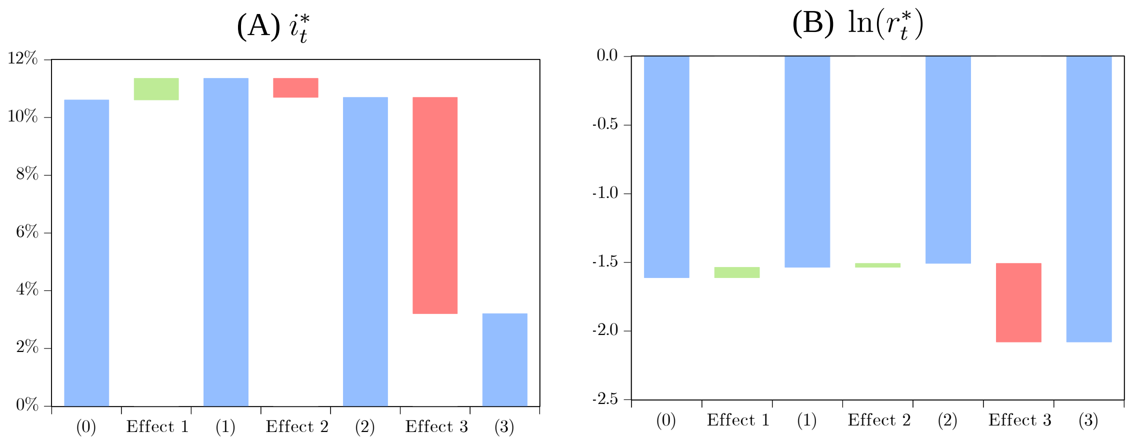


Figure 4: Natural spot interest rate decomposition (proposed specification)

Analyzing the results, in Panel A, one can see the decrease of i_t^* between 2009 and 2014 is predominantly due to i_t^{**}, although MCS_t^* has also contributed, especially after 2014. In fact, after oscillating around 1pp until 2014, achieving a maximum value close to 2pp by 2009, MCS_t^* is essentially null at the present. This new behavior of MCS_t^* between 2015 and 2017 seems to reflect a change in the economic policy orientation. On the volume effect side, the weight of BNDES lending in capital goods financing (θ_t^{BNDES}) fell sharply, from 18% in 2014 to 7% in 2017. On the price effect side, the real funding rate for BNDES credit operations (f_t^{BNDES}) increased rapidly, from 0% in 2014 to 3% in 2017, mainly reflecting the increase in its nominal funding rate (TJLP). It is worth mentioning that this estimation of MCS_t^*, by considering both price and volume effects, contributes to the empirical literature that has estimated this subsidy in Brazil, since that literature considered only volume data (e.g., De Bolle (2015), Goldfajn and Bicalho (2011)).

Panel B shows the decrease of i_t^{**} after the Global Financial Crisis is related to the fall in r_t^*. After all, $(TP_t^* + \delta + ARP_t^*)$ decreased slightly between 2003 and 2006 as the term premium and the equity risk premium fell (Appendix C, Carvalho and Santos (2020)), indicating an environment of lower risk in Brazil, but has not changed much since then. Finally, as shown in Panel C, the dynamics of $\ln(r_t^*)$ is mainly justified by the fall of g_t^* as the terms linked to μ_t^* and s_t^* has been essentially stable over the sample.

Therefore, the main explanation for the decrease in i_t^* seems to be the fall in g_t^*. This can be seen more easily evaluating the change of i_t^* between 2003-2008 and 2015-2017, using the average results of Figure 4 for these two periods. This is shown in Figure 5.



(0): i_t^* (2003-2008 average)
Effect 1: $-\Delta\left(TP_t^* + \delta + ARP_t^*\right)$
(1): i_t^* after effect 1
Effect 2: ΔMCS_t^*
(2): i_t^* after effects 1 and 2
Effect 3: Δr_t^*
(3): i_t^* (2015-2017 average)

(0): $\ln(r_t^*)$ (2003-2008 average)
Effect 1: $\Delta \ln\left[\left(\frac{\alpha/\beta}{\varsigma}\right) LS_t^*\right]$
(1): $\ln(r_t^*)$ after effect 1
Effect 2: $-\Delta\ln\left(s_t^*\right)$
(2): $\ln(r_t^*)$ after effects 1 and 2
Effect 3: $\Delta\ln\left(4g_t^* + \delta\right)$
(3): $\ln(r_t^*)$ (2015-2017 average)

Figure 5: Natural spot interest rate decomposition (proposed specification): 2003-08 versus 2015-17

From Panel A, 2003-2008 average of i_t^* is 10.6% (first bar), while 2015-2017 average of i_t^* is 3.2% (last bar). The decrease equal 7.4pp, which is explained by the positive effect of the decrease in the risk premia (effect 1, of 0.8pp) and negative effects of a lower MCS_t^* (effect 2, of -0.7pp) and of a lower r_t^* (effect 3, of -7.5pp). Thus, the main explanation for the fall in i_t^* is r_t^*.

The decomposition of the change in $\ln(r_t^*)$ is presented in Panel B of Figure 5. The main reason for the decrease in $\ln(r_t^*)$ is the fall in g_t^*, with the effects related to changes in μ_t^* and s_t^* being positive and very close to zero. Thus, the results of Figure 5 confirm that a lower g_t^* is the main explanation for the fall in i_t^* after 2009.

Hence, given a higher g_t^*, i_t^* would still be high by international standards. How can one explain this? From equations (11) and (12), i_t^* is a function of seven variables: μ_t^*, g_t^*, δ, TP_t^*, ARP_t^*, s_t^* and MCS_t^*. Table 3 presents a comparison of these variables in Brazil and in a group of four Latin American countries (Chile, Colombia, Mexico and Peru), for the 2015-2012 period. For ERP_t and DRP_t, I considered data for emerging market economies (EMEs). For s_t and LS_t, the Brazilian data sources used in this table are different from those used in the estimation, to ensure international comparability.

Table 3: The variables of the natural interest rate equation (2005-2012 average)

Variable	Brazil	Chile, Colombia, Mexico, and Peru	Analysis of the variable	Could explain a high i_t^*?
i_t^*	9%[a]	2%[b]	High	-
MCS_t^*	1.3%[a]	-	High	Yes
s_t	20.1%[c]	22.9%[c]	Low	Yes
LS_t	54.7%[d]	41.6%[d]	High	Yes
TP_t^{10}	3%[e]	3%[f]	Similar	No
ERP_t	3.3%[g]	2.6%[g] (EMEs)	Similar	No
DRP_t	3.8%[h]	3.4%[i] (EMEs)	Similar	No
δ	3.7%[j]	3.9%[d]	Similar	No
$4g_t$	3.9%[k]	4.6%[k]	Similar	No

[a]Source: own estimation, proposed specification.

[b]Countries' average. Source: Magud and Tsounta (2012).

[c]Total investment to GDP ratio, countries' median. Source: WEO/IMF database.

[d]Countries' median. Source: Feenstra et al. (2015).

[e]Estimated using the methodology presented in Appendix C.

[f]Countries' median. Source: Blake et al. (2015).

[g]Source: Carvalho and Santos (2020).

[h]BNDES lending rate for capital goods minus the TJLP rate. Source: BCB.

[i]J.P. Morgan Corporate EMBI composite blended spread. Source: Bloomberg.

[j]Source: Morandi and Reis (2004).

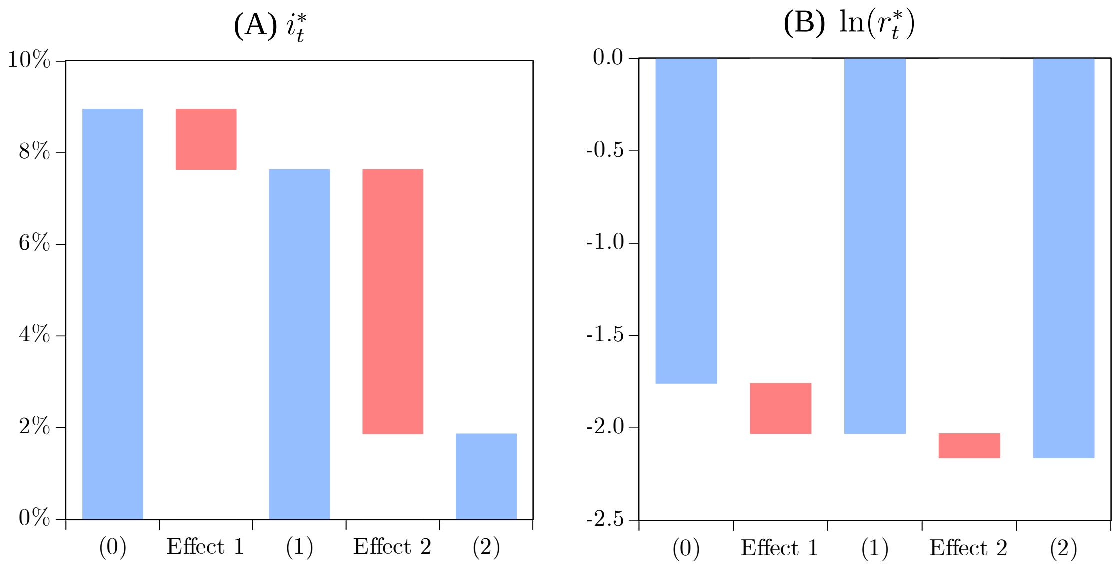
[k]Countries' median. Source: WEO/IMF database.

The first row of Table 3 shows the natural interest rate is much higher in Brazil (9% versus 2%). This higher level of the Brazilian natural rate seems to be justified, according to the other rows of the table, by MCS_t^*, s_t^* and $\mu_t^* = \beta/LS_t^*$ if it is assumed ζ, α, β, and $\overline{m}_t^*$ are similar internationally. While a low s_t^* and a positive MCS_t^* are usually cited in the Brazilian economic debate (e.g., Segura-Ubiergo (2012), De Bolle (2015)), a low μ_t^* is, to the best of my knowledge, a new explanation.

Figure 6 presents some counterfactual exercises in order to quantify the importance of each of these three variables in explaining the high level of interest rate in Brazil. In Panel A, I analyze i_t^*. The first bar shows the 2005-2012 average of i_t^*, equal to 8.9%. The second bar shows the 2015-2012 average effect of the macroeconomic cross subsidy, of 1.3pp. Thus, without this subsidy, the natural rate would be 7.6% (third bar). The fourth bar shows the average effect of a higher mark-up and savings rate in Brazil. More specifically, based on Table 3, LS_t^* and s_t^* of each quarter are multiplied by $\frac{41.6\%}{54.7\%}$ and $\frac{22.9\%}{20.1\%}$, respectively. As can be seen, the average effect is very large: 5.8pp.

Consequently, given that MCS_t^*, s_t^* and μ_t^* are at international levels, i_t^* would be 1.9% (fifth bar), which is essentially equal to the natural interest rate observed in the group of Latin American countries analyzed, close to 2% (first row of Table 3). Thus, controlled by these three variables, the Brazilian natural interest rate is not high.

Given the nonlinearities of the natural interest rate equation, one can separate the effects of μ_t^* and s_t^* only regarding $\ln(r_t^*)$. As shown in Panel B of Figure 6, both effects are relevant, but the impact of μ_t^* seems greater.



(0): i_t^* (2005-2012 average)
Effect 1: $MCS_t^* = 0$
(1): i_t^* after effect 1
Effect 2: $\uparrow\mu_t^*$, $\uparrow s_t^*$
(2): i_t^* after effects 1 and 2

(0): $\ln(r_t^*)$ (2005-2012 average)
Effect 1: $\uparrow\mu_t^*$
(1): $\ln(r_t^*)$ after effect 1
Effect 2: $\uparrow s_t^*$
(2): $\ln(r_t^*)$ after effects 1 and 2

Figure 6: Understanding the historically high Brazilian natural interest rate (proposed specification)

But what explains the levels of MCS_t^*, s_t^* and μ_t^* observed in Brazil? A full assessment of this issue is beyond the scope of this paper, but I can make a few comments about it. First, regarding MCS_t^*, in September of 2017 the National Monetary Council

(CMN) announced the creation of a new BNDES funding rate. Unlike the old rate (TJLP), which was arbitrarily defined, the new rate (TLP) is a market rate, as it is based on the 5-year inflation-indexed treasury bonds. The transition between the two funding rates will be smooth, with the real TLP being equal to the real TJLP at the start of 2018 and converging to the yield of the 5-year inflation-indexed treasury bond only in 2023. With this market-based funding rate, MCS_t^* tends to be closer to zero, contributing to a lower natural rate in Brazil. For instance, if the BNDES real funding rate between 2014 and 2017 had followed the yield of the 5-year inflation-indexed treasury, the average value of MCS_t^* in this period would be just -0.1pp, instead of the estimated 0.5pp.

Second, the low level of μ_t^* in Brazil could reflect the high share of prices that are regulated by the government, something between 25% and 30% of the consumer price index (IPCA). This investigation is an interesting avenue for future research.

Third, the low s_t^* in Brazil in the last two decades may reflect weak incentives to save, which would reduce private savings rate[29]. For example, according to Brito and Minari (2015), over 95% of Brazilians do not need to save during their working lives to maintain their consumption levels in retirement. Furthermore, the low level of public savings in Brazil may also contributed to this outcome, including indirectly, by increasing the sovereign risk and, consequently, reducing the external savings rate[30]. This highlights the importance of fiscal consolidation in reducing the natural interest rate, especially if fiscal reforms strengthen the incentives to save, as seen regarding the pension system's reform.

6 Conclusion

This paper shows, based on a general characterization of the steady-state capital cost, it is possible to obtain natural interest rate equations for (i) the standard or single funding rate case and (ii) the dual funding rate case. These equations become economically

[29] Another possible determinant for a low level of private savings is the elevated risk pertaining to the postponement of consumption in Brazil given the jurisdictional uncertainty, as argued in Arida et al. (2005). However, empirical evidence does not seem to support this hypothesis, since the connection between high jurisdictional uncertainty and high interest rates is not empirically clear (Gonçalves et al. (2007); Bacha et al. (2009)).

[30] The external savings rate is a function of the external equilibrium of the economy. Hence, generally, the natural interest rate can be seen as determined by the interaction between this external condition and the natural interest equation (8) or (10), which describes the internal equilibrium. Essentially, this is the approach adopted in this paper. However, it is worth mentioning under particular circumstances the external equilibrium can alone determine the natural rate (see de Holanda Barbosa et al. (2016) for an example of this type of external approach). For instance, in a small open economy where the uncovered interest rate parity (UIP) and the purchasing power parity (PPP) are valid, the natural interest rate equal the external natural rate plus a sovereign risk premium. Thus, if the sovereign risk premium is not affected by the current account balance, the natural rate would be determined solely by the external equation. Any difference between external and internal equilibrium conditions would be corrected in the last one, through changes in the external savings rate as deviations from the external equilibrium would cause exchange rate adjustments.

interpretable when more theoretical structure is added, assuming, for instance, a CES production function and a price-setting rule. This allows the use of the equations not only to estimate the natural interest rate, but also to evaluate its results. As an application, I estimate the Brazilian natural rate using the dual funding rate equation for a Cobb-Douglas production function and a semi-structural macroeconomic model.

The point estimates, which are subject to a high degree of uncertainty, indicate the natural one-year interest rate was around 12% from 2003 to 2008, rapidly decreased between 2009 and 2014, and stabilized around 4% between 2015 and 2017. This fall in the natural rate mainly reflects the lowering of the steady-state output growth rate. Given a higher potential growth, Brazil's natural rate would be still high by international standards. Comparing to Chile, Colombia, Mexico and Peru between 2005 and 2012, the higher Brazilian rate is justified by the impact of the subsidized lending by the BNDES and, most importantly, by both the low total savings rate in the country and the low level of firms' markup.

But what are the ultimate reasons behind these results? Which deep parameters justify them? A full assessment of such issues would enrich the diagnoses presented in this paper, being an interesting topic for future research, particularly using structural models.

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Appendix

A The capital-output ratio in the steady state

Initially, let us define the law of motion for capital:

Assumption A. $K_t = K_{t-1}(1-\delta) + \left[\zeta_{t-1t} s_{t-1}\left(p_{t-1}/p_{t-1}^k\right)\right] Y_{t-1}$

From, assumption A,

$$\left(\frac{K_t}{Y_t}\right) = \left[\zeta_{t-1t} s_{t-1}\left(p_{t-1}/p_{t-1}^k\right)\right]\left(\frac{1}{1+g_t}\right) + \left(\frac{1-\delta}{1+g_t}\right)\left(\frac{K_{t-1}}{Y_{t-1}}\right) \tag{A.1}$$

where $g_t \neq -1$.

Assuming $g_t^* + \delta \neq 0$, one can obtain the steady-state equilibrium from equation (A.1):

$$\Delta\left(\frac{K_t}{Y_t}\right) = 0 \rightarrow \left(\frac{K_t}{Y_t}\right)^* = \frac{\zeta_t^* s_t^* \left(p_t/p_t^k\right)^*}{g_t^* + \delta} \tag{A.2}$$

where I use $\zeta_t^* s_t^* \left(p_t/p_t^k\right)^* \equiv E_t\left[\lim_{\tilde{t}\longrightarrow\infty} \zeta_{\tilde{t}} s_{\tilde{t}}\left(p_{\tilde{t}}/p_{\tilde{t}}^k\right)\right] = E_t\left[\lim_{\tilde{t}\longrightarrow\infty} \zeta_{\tilde{t}-1} s_{\tilde{t}-1}\left(p_{\tilde{t}-1}/p_{\tilde{t}-1}^k\right)\right]$, with $E_t(\cdot)$ being the expectation operator conditional on the information set available at period t.

From equation (A.1), the capital-output ratio equilibrium of (A.2) is stable if, and only if, $\left|\frac{1-\delta}{1+g_t^*}\right| < 1$. Since $0 \leq \delta \leq 1$ and $g_t^* \geq -1$, $\frac{1-\delta}{1+g_t^*} \geq 0$, with $g_t^* \neq -1$. Thus, $\left|\frac{1-\delta}{1+g_t^*}\right| < 1$ is equivalent to the following assumption:

Assumption B. $g_t^* + \delta > 0$

Assumption B implicates $g_t^* + \delta \neq 0$ and $g_t^* > -1$, since $\delta \leq 1$. Thus, the conditions $g_t^* + \delta \neq 0$ and $g_t^* \neq -1$ are fulfilled under assumption B. Hence, under assumptions A and B, the capital-output ratio has a stable steady-state equilibrium, which is given by equation (A.2).

B The Brazilian macroeconomic steady-state cross subsidy

To obtain MCS_t^*, I should multiply i_t^* of the dual funding rate case (equation (10)) by $(1-\theta_t^*)$ and subtract i_t^{**} (equation (11)) from this expression:

$$(1-\theta_t^*)\, i_t^* - i_t^{**} = -\theta_t^*\left(f_t^* - TP_t^*\right)$$

$$(1-\theta_t^*)\, i_t^* - (1-\theta_t^* + \theta_t^*)\, i_t^{**} = -\theta_t^*\left(f_t^* - TP_t^*\right)$$

$$MCS_t^* \equiv i_t^* - i_t^{**} = \left(\frac{\theta_t^*}{1-\theta_t^*}\right)\left[\left(i_t^{**} + TP_t^*\right) - f_t^*\right] \tag{B.1}$$

where I use the definition of MCS_t^* (equation (12)).

C The estimation of the term premium[31]

As pointed out by Kim and Orphanides (2007) and Rudebusch, Sack and Swanson (2006), there are three categories of models to estimate the term premium: (i) regression-based models of rational expectations (e.g., Guillen and Tabak (2008)), (ii) arbitrage-free affine models (e.g., Duffie and Kan (1996)), and (iii) macro-finance models (e.g., Rudebusch, Swanson and Wu (2006)). These methodologies are usually based on the identification of the data generating process of the interest rate expectations, adopting a flexible term premium structure over the maturities. The approach proposed here does exactly the opposite: leave the interest rate expectations estimation very flexible, assuming a more rigid term premium structure over the maturities. This means that I can easily recover the term premium of any maturity, even for those that I do not have data.

This feature is of great value given the purpose of estimating the natural interest rate. After all, the natural rate is a function of the steady-state term premium at the steady-state average maturity of the investments in the economy. Since this average maturity is not known, I need to estimate it. Hence, I need to have a function that, for each period, gives me the term premium for any maturity, as in the methodology proposed here.

The term premium is usually defined as a reward for investing in a bond of long maturity instead of rolling over bonds of shorter maturities. In formal terms,

$$TP_t^m \equiv \ln\left(1 + j_t^m\right) - \sum_{i=1}^{m} \frac{E_t\left[\ln\left(1 + j_{t+i}^1\right)\right]}{m} \tag{C.1}$$

where TP_t^m is the annual term premium of maturity m at time t, j_t^m is the annual interest rate of a bond with maturity m at time t, and $E_t\left(\cdot\right)$ is the expectation operator conditional on the information set available at period t.

The methodology proposed here assumes this reward is only due to interest rate risk. More specifically, it considers investors seek to ensure that they will loose by investing in the long maturity bond only in the occurrence of a rare event. Investors that are more risk-averse require a higher term premium, as they want to be hedged against worse scenarios.

Since the losses due to interest rate variations occurs because of price reductions in the long maturity bond, I can approximate that using its duration. Thus,

$$TP_t^m = \sigma_t^m Z_t d_t^m \tag{C.2}$$

where σ_t^m is the standard deviation of $\sum\limits_{i=1}^{m} \frac{E_t\left[\ln\left(1+j_{t+i}^1\right)\right]}{m}$, Z_t is a variable that captures the

[31]This methodology was developed with Alexandre de Carvalho (alexandre.carvalho@bcb.gov.br).

degree of risk aversion as it defines how rare are the events that the investors want to hedge against, and d_t^m is the duration of the bond of long maturity.

Consider the long maturity bond has no coupon ($d_t^m = m$). Assume also σ_t^m follows an autoregressive process of order 1 over the maturities. Therefore, from equation (C.2),

$$TP_t^m = \left\{ \left[1 - (\varphi_t)^m \right] \sigma_t^\infty \right\} Z_t m \tag{C.3}$$

where $0 < \varphi_t < 1$ is the autoregressive parameter at time t and σ_t^∞ is the standard deviation of $\sum_{i=1}^{m} \frac{E_t\left[\ln\left(1+j_{t+i}^1\right)\right]}{m}$ for $m \to \infty$.

From equation (C.3), the term premium increases with the risk (σ_t^∞) and with the risk aversion (Z_t), as expected. This increase occurs in all maturities, but it is more intense for longer maturities. After all, σ_t^∞ and Z_t are multiplicative and the term premium should increase with the maturity, a property that can be verified in equation (C.3). Hence, when the risk or the risk aversion rises, the slope of the yield curve tends to be higher.

Substituting equation (C.3) into (C.1),

$$\ln\left(1 + j_t^m\right) = \sum_{i=1}^{m} \frac{E_t\left[\ln\left(1 + j_{t+i}^1\right)\right]}{m} + \left[1 - (\varphi_t)^m\right] \kappa_t m \tag{C.4}$$

where $\kappa_t = \sigma_t^\infty Z_t$ captures the risk in the price, which depends on the risk (σ_t^∞) and the risk aversion (Z_t).

I estimate the term premium using Kalman filter over the maturities, where equation (C.4) is the observation equation, and κ_t and φ_t are parameters of the model[32]. The state equations are

$$\sum_{i=1}^{m} \frac{E_t\left[\ln\left(1 + j_{t+i}^1\right)\right]}{m} = \sum_{i=1}^{m} \frac{E_t^s\left[\ln\left(1 + j_{t+i}^1\right)\right]}{m} + u_t \tag{C.5}$$

$$u_t = u_{t-1} + \varepsilon_t^u \tag{C.6}$$

where $E_t^s(\cdot)$ is the expectations at time t as captured by a survey and ε_t^u is a normally distributed serially uncorrelated error. Note the estimation of the expectations is very flexible given the presence of u_t.

Besides this standard specification, I use an alternative specification. In the phi-invariant specification, the state equations are still (C.5) and (C.6), but the observation equation is altered by assuming φ_t is time-invariant. The time-invariant estimate is the sample median φ_t from the standard model.

Regarding the data, $E_t^s\left(j_{t+i}^1\right)$ comes from the Focus survey of the Central Bank of

[32] After all, the Kalman filter is applied over the maturities and these two parameters do not vary in this dimension, only over time.

Brazil. It surveys the expected target policy rate (Selic rate) at the last working day of the period. The survey is made on a daily basis and the forecasts are of monthly or annual periodicity. Using this database, I follow five steps. First, I combine monthly and annual information of average forecasts[33], considering only the information surveyed on the last working day of each week. Second, the information for missing months is estimated by interpolation[34]. Third, I use the Monetary Policy Committee (Copom) meeting dates to obtain the expected policy rate target for each day. Fourth, assuming a log-normal distribution, I transform these daily interest rate expectations on expectations about the natural logarithm approximation of the interest rate. To do that, I use standard deviation data from the Focus survey[35]. Fifth, I calculate $\sum_{i=1}^{m} \frac{E_t^s\left[\ln\left(1+j_{t+i}^1\right)\right]}{m}$ for each monthly maturity.

With respect to j_t^m, I use Pre-DI swap data[36], from Bloomberg. For missing months, I also use interpolation[37]. Finally, I adjust these data by adding the difference between the actual target Selic rate and the spot DI rate for each surveyed day included in the sample[38].

The sample includes 865 weeks, from 11/9/2001 to 6/1/2018. Since the Kalman filter is applied over the maturities, the estimation of each week is independent from the estimation of other weeks. The sample of each week has 72 observations as I consider monthly maturities up to 72.

The estimated results for the two specifications are shown in Figure 7. In Panel A, I show the estimated one-year term premium, while the three-years term premium is shown in Panel B. As expected, the term premium of three years is always higher then the term premium of one year for each specification. Comparing the two specifications, one can see the results are essentially the same for the term premium of longer maturity (Panel B). The main differences occur in shorter maturities (Panel A), when the phi-invariant specification seems to yield less noisy estimates. For this reason, I choose to use the term premium from this alternative specification in the estimation of the natural interest rate.

[33]This is done by using annual information when monthly data are unavailable.

[34]I apply the cubic spline method. In any case, I find very similar results using the log-linear or the Catmull-Rom spline methods.

[35]To obtain the daily information, I follow the same first three steps just described regarding average expectations data.

[36]The Pre-DI swap contract traded at the BM&FBovespa, the Brazilian Stock Exchange, is an interest rate swap where one of the parties agree to make pre-fixed interest payments in exchange for receiving floating interest payments based on the DI rate, whereas the other assumes a reverse position.

[37]Again, using the cubic spline method, but the results are very similar if I consider the log-linear or the Catmull-Rom spline methods.

[38]This difference is smoothed by applying the Hodrick-Prescott filter, setting the lambda by Ravn Uhlig frequency rule, with power equal to 2.

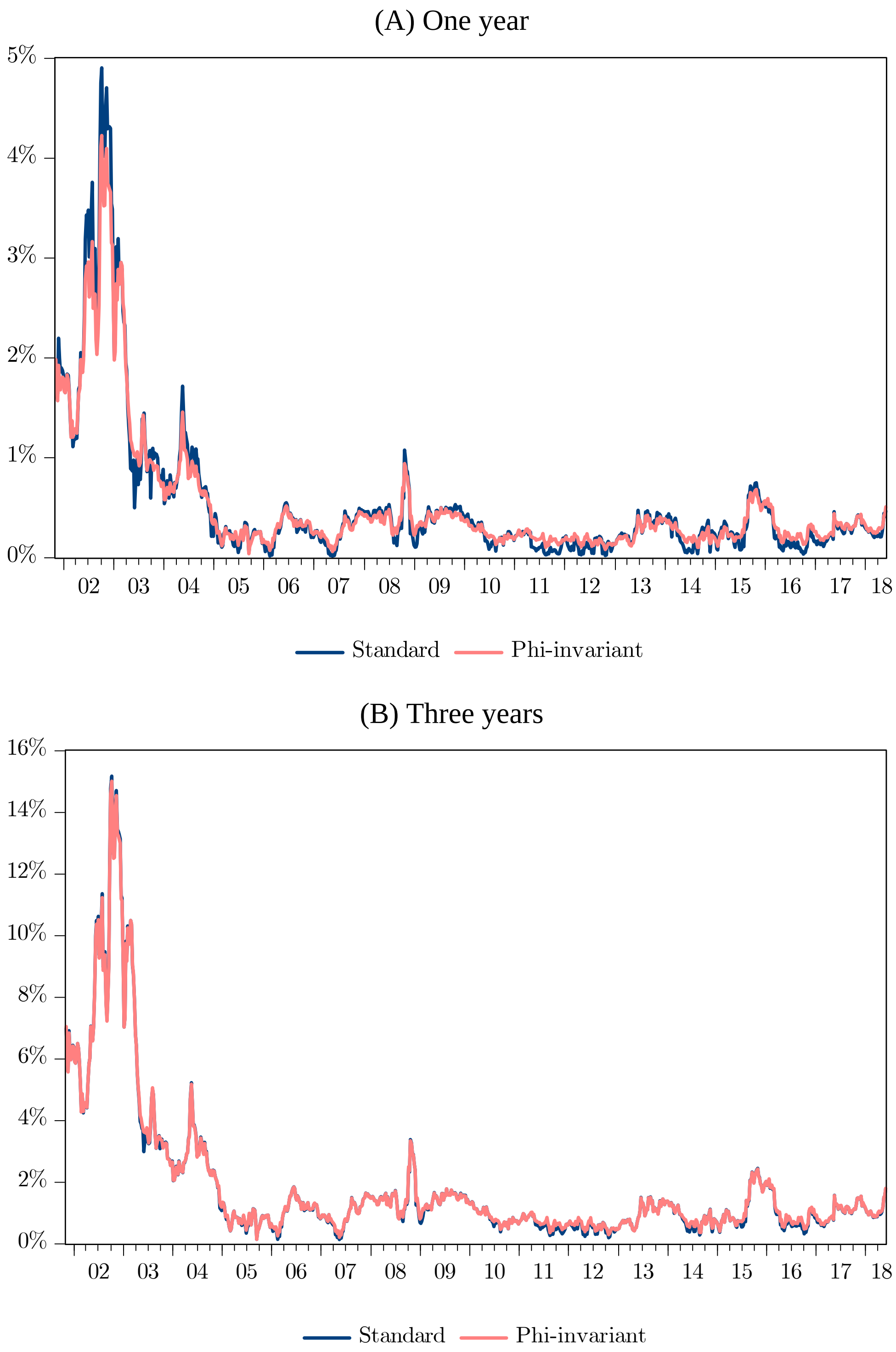

Figure 7: Estimated term premium

D Detailed description of the exogenous variables

Table 4: Detailed description of the exogenous variables

Variable	Definition	Source
π_t^{free}	Free-price IPCA inflation rate, SA[a]	BCB
$E_t\left(\pi_{t+1}\right)$	Next quarter IPCA inflation expectation in the first month of the quarter (Focus survey), SA[a]	BCB
π_t	IPCA inflation rate, SA[a]	IBGE
π_t^f	Variation of the imported goods prices in R$, SA[a]	BCB
y_t	Natural logarithm of the GDP seasonally adjusted	IBGE
$i_t^1+TP_t^1$	Pre-DI swap 360 days minus the expected 12-month IPCA inflation from the Focus survey (smoothed)	Bloomberg / BCB
LS_t	$\left(\frac{\text{compensation of employees}}{\text{Value added - mixed income}}\right)\left(\frac{\text{Value added}}{\text{GDP}}\right)$	IBGE[b]
δ	3.7% per annum	Morandi and Reis (2004)
s_t	Gross capital formation as a share of GDP	IBGE
κ_t	-	Appendix C
φ	-	Appendix C
θ_t^{BNDES}	$\frac{\text{BNDES lending for capital goods}}{\text{private sector gross capital formation}}$, filtered[c]	BNDES / IBGE[d]
ERP_t	-	Carvalho and Santos (2020)
DRP_t^{BNDES}	The BNDES lending rate for capital goods minus the TJLP, filtered[c]	BCB[e]
f_t^{BNDES}	TJLP minus the longest annual IPCA inflation expectation from Focus Survey, filtered[c]	BCB

[a]Seasonally adjusted (Census X-13, additive).
[b]Until 2015, I use the Annual National Accounts. For 2016-17, annual data are updated using the Quarterly National Accounts and the continuous PNAD. Quarterly data are obtained through interpolation.
[c]Hodrick-Prescott filter, with lambda of 10, in order to eliminate only high frequency elements.
[d]Until 2015, government investment comes from Annual National Accounts. For 2016-17, annual data are updated using National Treasury (STN) database. Quarterly data are obtained through interpolation.
[e]Prior to 2011, I use the series' historical average, disregarding the period 2012Q2-2015Q4.