ISSN 1518-3548

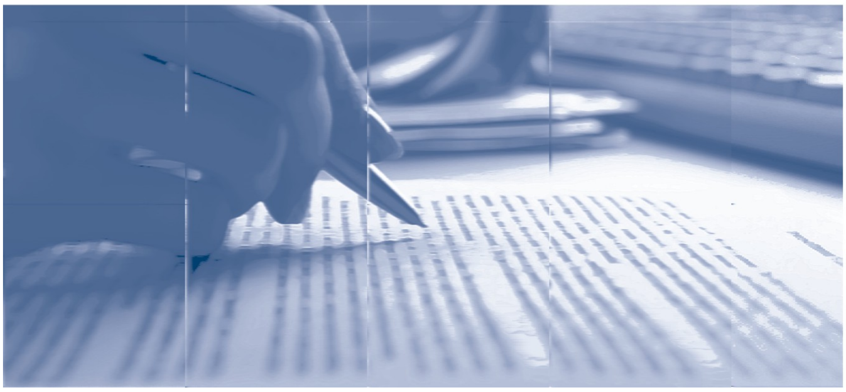
BANCO CENTRAL DO BRASIL

Earmarked Credit and Monetary Policy Power: micro and macro considerations

Pedro Henrique da Silva Castro

September 2019

Working Papers



505

ISSN 1518-3548
CGC 00.038.166/0001-05

Working Paper Series	Brasília	no. 505	September	2019	p. 1-45

Working Paper Series

Edited by the Research Department (Depep) – E-mail: workingpaper@bcb.gov.br

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Non-technical Summary

Is monetary policy power (i.e., the size of the effect that a given change in the policy rate has on the economic aggregates, such as GDP and inflation) reduced with the pervasiveness of earmarked credit in Brazil, which usually features interest rates that are insensitive to the policy rate defined by the Central Bank? A broad consensus seems to have been reached in the public debate, that "yes", this being one of the arguments for a recent reform of the benchmark interest rate for loans granted by the Brazilian development bank BNDES. This paper argues that the answer to this question is still unclear.

Few were, and still are, the academic works on this issue. Among the results of the literature, one that stands out is one that shows that firms with more access to earmarked credit show smaller variation of their employment level following changes in the monetary policy rate. But this result is not necessarily informative about the macroeconomic effect of interest.

First, this paper shows analytically how the macro effect of interest can be decompoed into the sum of the micro effect and an effect I call "external", which captures captures feedback mechanisms operating inside the economy, technically referred to as *general equilibrium effects*. This result is general and model independent.

Next, a simplified macroeocnomic model of the same class (dynamic stochastic general equilibrium) widely used in studies of monetary policy and calibrated with usual values, is used to exemplify the possibility of a macroeconomic effect if irrelevant size to coexist with a microeconomic effect of significant magnitude. The same model is also used to exemplify the possibility of the monetary policy power over inflation increasing with higher prevalence of earmarked credit, even with the possible reduction of its power over GDP.

Sumário Não Técnico

A prevalência do crédito direcionado no país, concedido a taxas de juros pouco sensível à taxa básica definida pelo Banco Central, reduz a potência da política monetária (i.e., o magnitude do efeito que dada mudança na taxa básica tem sobre agregados macroeconômicos, tais como o PIB ou a taxa de inflação)? No debate público o consenso que parece ter se formado é o de que "sim", tendo sido este um dos argumentos para recente reforma da taxa de referência para os empréstimos do BNDES. Este trabalho argumenta que a resposta a essa pergunta não está clara, ainda.

Poucos eram, e ainda são, os trabalhos acadêmicos dedicados ao tema. Entre os resultados da literatura, destaca-se o de que firmas com maior acesso a crédito direcionado apresentam menor variação em seu nível de emprego após mudanças na taxa básica de juros. Mas esse resultado não é necessariamente informativo sobre o efeito macroeconômico de interesse.

Primeiro, mostra-se analiticamente como o efeito macro pode ser decomposto na soma do efeito micro e um efeito que chamo de "externo", que captura a existência de mecanismos de retroalimentação na economia, tecnicamente chamados de *efeitos de equilíbrio geral.* Este resultado é geral e independe do modelo.

Em seguida, um modelo macroeconômico simplificado, da mesma classe (equilíbrio geral dinâmico e estocástico) que a amplamente utilizada no estudo de política monetária, calibrado com valores usuais, é utilizado para exemplificar a possibilidade de um efeito macroeconômico de tamanho irrelevante coexistir com um efeito significativo no nível micro. O mesmo modelo é ainda usado para exemplificar a possibilidade da política monetária ter sua potência sobre a inflação aumentada com a maior disponibilidade de crédito direcionado, mesmo com eventual redução de sua potência sobre o PIB.

Earmarked Credit and Monetary Policy Power: micro and macro considerations*

Pedro Henrique da Silva Castro†

Abstract

Is monetary policy power reduced in the presence of governmental credit with subsidized interest rates, insensitive to the monetary cycle? I argue this question has not yet been reasonably answered even though a virtual consensus seems to have been reached. Using a general analytical decomposition I show that the available microeconometric evidence is not necessarily informative about the macroeconomic effect of interest, due to the presence of general equilibrium effects. Moreover, evidence of decreased power over output does not imply that power over inflation is also decreased. A simple New Keynesian model where firms take credit, from both the market and the government, to finance working capital needs is presented to exemplify those possibilities.

Keywords: Monetary economics, Earmarked credit, Cost channel.

JEL Classification: E51, E52, H81

*The views expressed here are of my own and do not necessarily reflect those of the Central Bank of Brazil. This paper is part of the thesis written as a requirement to a doctorate degree in Economics at PUC-Rio. I thank Márcio Garcia, Tiago Berriel, Eduardo Zilberman, Carlos Viana de Carvalho, Marco Bonomo, Bernardo Guimarães, Ricardo Sabadini, Bruno Perdigão, Eduardo Amaral, Fernando Mattar, André Minella, as well as an anonymous reviewer, for valuable comments and suggestions on previous versions of this work.

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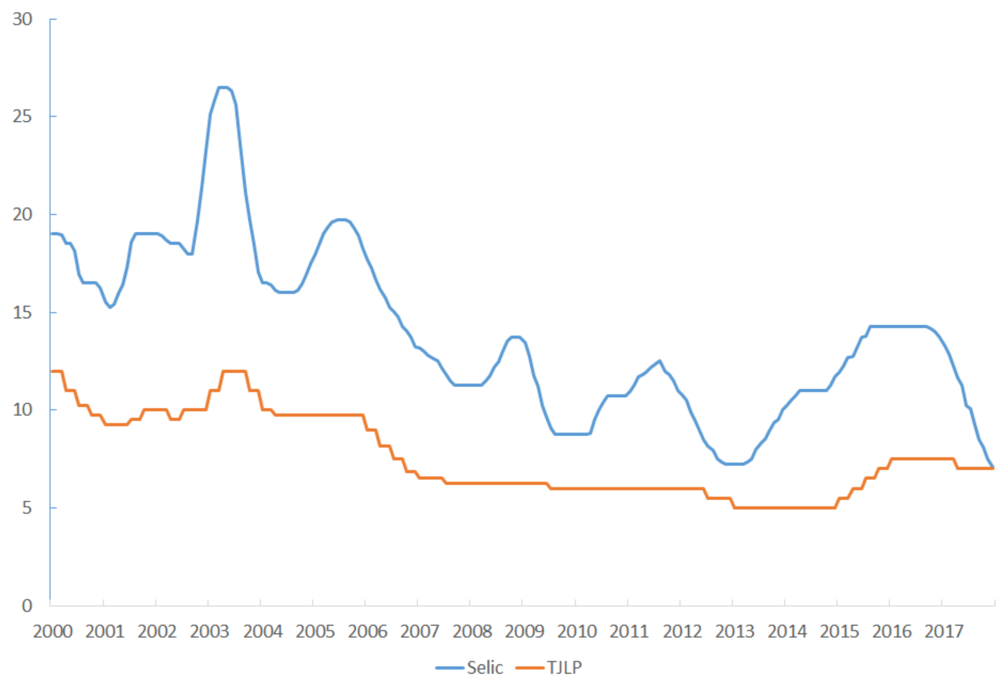
1 Introduction

Government is responsible for a large share of the credit supply in Brazil, by owning banks and by earmarking credit, channeling it to desired sectors and modalities. In December 2017 credit provided by government controlled banks amounted to 54.1% of total outstanding bank loans; earmarked loans corresponded to 48.7%[1]. A significant share of these earmarked loans has interest rates that are lower than the prevailing market rate, and nearly insensitive to the monetary policy rate (Selic). Both these features can be seen in Figure 1.1, which compares the trajectories of the Selic and TJLP[2] rates from 2000 to 2017.

Figure 1.1: Selic × TJLP (% p.a.)



It has been argued[3] that this pervasiveness of earmarked credit reduces the efficacy of monetary policy. This would occur because earmarked credit does not tighten in response to monetary policy tightening, or at least not as much as market credit does, and agents who can access it

[1] Note that there is significant overlap between earmarked loans and loans provided by government controlled banks. For example, BNDES is a government controlled bank and most of its loans are earmarked. Nonetheless, the concepts are different. Banco do Brasil is counted as a government controlled bank, but many of its loans are in the 'free' (i.e., not earmarked) segment. Bradesco is private controlled, but some of its loans are earmarked.

[2] The benchmark rate for BNDES credit operations, from December 1994 to December 2017. I focus on the TJLP because it certainly has drawn most of the attention. BNDES credit operations, for both households and non-financial companies, amounts to 36.5% of the total stock of earmarked credit in Brazil (as of October 2017). BNDES credit to companies amount to 69.2% of earmarked credit to companies. Other modalities of earmarked credit are real-estate (41.1%), rural (15.8%) and others (0.6%), which includes micro-credit.

[3] For instance, Arida (2005), Bacha (2010), Schwartsman (2011), Garcia (2011), Pinheiro (2014), among many others.

would not have to adjust their spending and investment as much as they would if they faced market interest rates. Such obstruction of monetary policy's transmission channel would make harder the job of the Central Bank in stabilizing the economy and might imply more volatile interest rates, as the Central Bank would have to increase its policy rate by more to achieve a given contraction in demand, if needed. In fact, such concern was one of the motivations for a recent policy change, as made clear by MP 777's exposition of motives[4]. This *Medida Provisória*, later converted into Law 13,438/2017, created a new benchmark rate for BNDES operations, the TLP, in substitution to the TJLP. Unlike its predecessor, the TLP is linked to the yield on 5-year inflation-indexed government bonds and, hence, affected by changes in policy rate. The effective TLP is phased in over 5 years, linearly increasing from the TJLP to the new benchmark.

A broad agreement was reached, thus, despite the fact that few are the academic works dedicated to study the relationship between earmarked credit and the monetary policy power. The most known work on this subject is the one by Bonomo and Martins (2016), who use firm-level employment and credit micro-data to assess how monetary policy transmission is affected by government-driven (both earmarked and by government controlled banks) loans, exploring variation in governmental credit access across firms. They find that access to government-driven credit does help insulate firms from the effects of interest rate changes: for instance, after a 1.p.p. hike in the policy rate, employment growth falls 1.2 p.p. in firms without access to governmental credit, but only 0.7 p.p. in firms totally financed by the government.

But how informative is this result about the question of interest, namely, the extent to which earmarked credit reduces monetary policy traction on the aggregate economy? Can we extrapolate the results from the cross-section domain (micro effects) to the aggregate domain (macro effects), thus corroborating the hypothesis that interest-insensitive earmarked credit renders monetary policy less effective?

Of course, the external validity of a result is not necessarily warranted and one must be cautious with extrapolations. I show there is a good reason why caution should be applied here as well. A firm's output response to monetary policy does not depend only on its own access to earmarked credit but also on all other firms'. Because of that, the macroeconomic effect depends not only on how firms' response to monetary shocks is affected by how much earmarked credit they receive (microeconomic effect), but also by how it is affected by other firms receiving it (external effect). In fact, I show that we can decompose the macro effect

[4] Which can be found here (in Portuguese): http://www.planalto.gov.br/ccivil_03/_ato2015-2018/2017/Exm/Exm-MP-777-17.pdf. Other motivations were: (i) improving the dynamics of the public debt, by eliminating subsidies that were invisible to the federal budget; (ii) fostering the securitization of loans by federal banks, by making their benchmark interest rate compatible to market rates; (iii) fostering further development of the Brazilian capital markets; and (iv) improving the efficiency of capital allocation.

into the sum of average micro and average external effects.

This general result is then explored in the context of a very simple New-Keynesian model which includes a *working-capital channel*, through which monetary policy shocks affect firms differently, depending on their reliance on earmarked credit. The model is able to reproduce the microeconometric evidence that employment is less responsive to monetary shocks in firms with more access to earmarked credit. In the model aggregate output is also less responsive to monetary shocks the more important government is in supplying credit. But the magnitude of micro and macro effects differ, the later usually being considerably larger than the former.

Another interesting result, in the model, is that inflation becomes more responsive the higher is the importance of earmarked credit — contrarily to the popular view. This happens both in the micro and macro level, but again with different magnitudes. The reason is the presence of a cost-channel induced by firms' working capital needs: interest rate hikes increase firms marginal costs, offsetting in part the deflationary pressure that comes from lower aggregate demand. But this cost-channel is weaker the more insulated firms are from variation in the market interest rate.

Related literature. As emphasized, the academic literature on the relationship between earmarked credit and monetary policy power is sparse. In Santin (2013), BNDES lending is countercyclical and reduces the response of the economy to monetary shocks. But in his model the credit policy follows Gertler and Karadi (2011)'s model of unconventional monetary policy, meaning that interest rate on government credit is no different from the one in the private market, which is at odds with the data and with our motivation. Rosa (2015) builds a DSGE model where earmarked credit finances firms' working capital needs. In his model earmarked credit is entirely financed with distortionary taxation on households' labor income. A balanced-budget is assumed and this, together with a fixed tax-rate, implies that earmarked credit interest rates must endogenously respond to monetary policy, which is at odds with the observations of insensitiveness[5]. Silva *et al.* (2016) is the closest to this paper. They extend the model of Hülsewig *et al.* (2009) by assuming that a share of the monopolistically competitive banks is government-owned and provide cheaper credit at a constant interest rate. Firms take credit in order to finance their working capital needs, opening space to a cost-channel. They find that both output and inflation responses to a monetary shock become more muted when the presence of earmarked credit is higher. But, importantly, because they find a significant *price-puzzle*[6], what happens is that inflation *rises less* following a monetary policy tightening. In a sense, this is similar to my result that inflation *falls more*. Finally, Castro (2017) is a

[5]Unfortunately, the paper does not show how the interest rate on earmarked credit respond to a monetary shock.

[6] *Price-puzzle* is the name given to the phenomenon of higher inflation following a monetary contraction.

parallel work where again I show, in the context of a model where earmarked credit finances firms' investment, that monetary policy power over inflation may not be much affected.

Besides the aforementioned work by Bonomo and Martins (2016), a more recent set of papers also empirically study this topic. Perdigão (2018) uses a factor-augmented VAR (FAVAR) approach to study the response of sector-level variables to a monetary shock, and shows that loan rates, output, employment, prices and real wages respond less the higher the share of earmarked credit on total sectoral bank debt. While acknowledging that sectoral-level evidence does not imply the aggregate-level evidence, he points to the possibility that, by capturing some "intra-sector" general equilibrium effects, his approach may get closer to the macroeconomic effect of interest than a more granular, firm-level, approach. Drawing only on aggregate, time series dimension of the data, Vieira (2019) uses Jordà (2005)'s local projection approach to compute impulse response functions of inflation and GDP to a monetary shock, allowing the effect to vary with the credit cycle of public and private banks. Among other things, he finds that following a monetary shock inflation *rises less* when the supply of credit by public banks is higher.

This paper also relates to the cost-channel literature — see Barth III and Ramey (2002), Christiano *et al.* (2005), Ravenna and Walsh (2006). This literature posits that interest rates changes work not only through demand channels (such as households' consumption-savings decisions) but also through supply channels, as higher interest rates may increase firms' operational costs. This, in turn, could be a possible explanation for the *price-puzzle*. The cost channel arises in our model because credit to firms is introduced through working capital needs — as in other DSGE models with this feature — but the cost-channel is more general than firms relying on working capital credit. It arises whenever there is a delay between paying production costs and receiving for sales.

Finally, this papers is also close to the literature showing that macro and micro elasticities can be very different. Classical papers are Houthakker (1955) — showing that the aggregation of fixed inputs-technology firms (hence, with zero elasticity of substitution) can give rise to an aggregate Cobb-Douglas production function (hence, with unitary elasticity), due to extensive margins — and Caballero (1991) — providing an example of asymmetric hiring and firing on the firm level that do not occur in the aggregate. For a sample of recent papers who take seriously these difference between micro and macro elasticities, see Nakamura and Steinsson (2014), Oberfield and Raval (2014), Beraja *et al.* (2016) and Baqaee and Farhi (2017).

Guideline. Section 2 provides a general analysis (i.e., not model-specific) of the relationship between the micro and macro elasticities of IRFs with respect to earmarked credit. Section 3 provides specific analysis, based on a New Keynesian model with a cost-channel. Section 4 concludes.

2 A general analysis

2.1 Distinguishing between macro and micro effects

When discussing whether, and to what extent, the presence of earmarked credit makes monetary policy less effective, our interest mostly lies in the response of *aggregate* output and inflation to monetary shocks, and how these responses change with the importance of earmarked credit in the economy. In this work I use the expression *macroeconomic effect* to describe this sort of consequences, distinguishing it from *microeconomic effects* that take place at firm level. In order to be precise I provide formal definitions of these objects:

Definition: The **macroeconomic effect** that earmarked credit has over variable Z's response to a monetary shock is given by:

$$\frac{\partial}{\partial \zeta}\left(\frac{\partial Z}{\partial R}\right),$$

where R is the policy rate and ζ is measure of the overall importance that earmarked credit has in the economy. Both changes in policy (earmarked credit and monetary) must be exogenous in order not to be confounded with other factors.

Definition: The **microeconomic effect** that earmarked credit has over firm i variable Z's response is given by:

$$\frac{\partial}{\partial \zeta_i}\left(\frac{\partial Z_i}{\partial R}\right).$$

Note that the effect is measured by exogenously changing firm i's access to earmarked credit (ζ_i) while holding fixed all other firms' access to earmarked credit. If firm i's size is negligible economy-wise, as I assume, then the overall importance of earmarked credit (ζ) is also fixed.

Before I present the main result of this paper it will prove useful to introduce one more definition. This is motivated by the fact that firm i's behavior is not only affected by its own access to earmarked credit, but also by all other firms access. For instance, its business is likely to be harmed if its competitors are able to find cheaper credit.

Definition: The **external effect** that earmarked credit has over firm i variable Z's response is given by:

$$\frac{\partial}{\partial \zeta_{-i}}\left(\frac{\partial Z_i}{\partial R}\right),$$

where ζ_{-i} is a measure of the overall importance that earmarked credit has to all other (than i) firms in the economy.

With these definitions in place we are ready to proceed to one of the main results of this paper. In order to focus on the essence of the argument, in the main text I only provide a proof of the proposition for a case with a finite number of firms, leaving the extension for infinitely countable and uncountable number of firms for the appendices A.1 and A.2. For concreteness I focus on output, but the analysis is similar for other variables.

Let N be the number of firms in the economy and denote by Y_i firm i's output. The equilibrium value for this variable potentially depends on many factors and, among them, the stance of monetary policy (R) and the earmarked credit access of each firm in the economy $(\zeta_1, \zeta_2, \ldots, \zeta_N)$. Because of that we write $Y_i = Y_i\big(\ R\ ;\ \zeta_1, \zeta_2, \ldots, \zeta_N\ ;\ \cdot\ \big)$. Let us define aggregate output as an average of firm's output, i.e., $Y = \frac{1}{N}\sum_{i=1}^{N} Y_i$. I use the average and not the sum for convenience. First, note that this can be considered just a choice of scale. Second, this is more consistent with the definition of aggregate output in a model with a unit measure continuum of firms, as is typical in DSGE models. In the same spirit, let us also define the aggregate importance of earmarked credit in the economy as the cross-section average of firms' access: $\zeta = \frac{1}{N}\sum_{i=1}^{N} \zeta_i$. Accordingly, $\zeta_{-i} = \frac{1}{N}\sum_{j \neq i1} \zeta_j$.

Proposition: The macroeconomic effect of interest is given by the sum of microeconomic and external effects averaged over the set of firms, i.e.,

$$\underbrace{\frac{\partial}{\partial \zeta}\left(\frac{\partial Y}{\partial R}\right)}_{\text{Macro effect}} = \underbrace{\mathbb{E}_i\left[\frac{\partial}{\partial \zeta_i}\left(\frac{\partial Y_i}{\partial R}\right)\right]}_{\text{Average of micro effects}} + \underbrace{\mathbb{E}_i\left[\frac{\partial}{\partial \zeta_{-i}}\left(\frac{\partial Y_i}{\partial R}\right)\right]}_{\text{Average of external effects}}.$$

Proof: Total differentiation of aggregate output with respect to earmarked credit variables $\{\zeta_j\}$ yields:

$$\mathrm{d}Y = \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{j=1}^{N}\frac{\partial Y_i}{\partial \zeta_j}\mathrm{d}\zeta_j\right).$$

Now, let us consider changes in earmarked access such that $d\zeta_i = d\zeta$, for all i. Hence,

$$\begin{aligned}\frac{\partial Y}{\partial \zeta} &= \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial Y_i}{\partial \zeta_j} \\ &= \frac{1}{N}\sum_{i=1}^{N}\frac{\partial Y_i}{\partial \zeta_i} \quad + \quad \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{j\neq i}\frac{\partial Y_i}{\partial \zeta_j}\right).\end{aligned}$$

Note that the term inside parenthesis in the last expression captures how firm i's output is affected by changes in all other firms ($j \neq j$) access to earmarked credit. We can write $\frac{\partial Y_i}{\partial \zeta_{-i}} = \sum_{j\neq i}\frac{\partial Y_i}{\partial \zeta_j}$. Derive the resulting expression with respect to R to complete the proof. □

As the derivation makes clear the result above is pretty much an identity. It just relies on the fact that a firm's output potentially depends not only on its own access to earmarked credit, but also on all other firms access. How to define the aggregate variable may have practical implications [7], but it does not change the essence of the argument. I have framed the proposition for our objects of interest (which are second-order mixed partial derivatives with respect to monetary and earmarked credit policies), but it is clear that a similar result is valid for many objects[8]. Thus, the result is very general and does not hinge on strong hypothesis. In particular, it is model-independent.

The generality of the result is a strength and, at the same time, a weakness. It carries no information about the sign and magnitude of each of the defined effects (macro, micro and external). In other words, the result tells us nothing whether and to what extent the presence of earmarked credit reduces monetary policy power. Nonetheless, it is still useful because it helps us to better understand the available microeconometric evidence, making clear how misleading extrapolating it can be.

2.2 External effect and general equilibrium

What is the nature of the *external effect*? Pragmatically, it depends only on the hypothesis that a firm's output depends not only on its own access to earmarked credit but also on other firms access. Why would it be the case?

[7] Defining the aggregate as a sum, instead of an average, would make the macro effect to be the sum of total micro and external effects. Also, if the aggregate is a weighted average then the weights would be carried over to the decomposition.

[8] For instance, we could be interested in whether the presence of earmarked credit increases steady-state aggregate output. And it can also be useful to study things unrelated to earmarked credit policy as well, for instance, elasticities of substitutions as in Houthakker (1955).

One does not need to rely on the existence of real (or technological) externalities in order to justify this assumption. In fact, what I have in mind is the existence of pecuniary externalities associated with general equilibrium forces. Consider the case with atomistic firms. The microeconomic effect captures a partial equilibrium effect in the sense that prices (including factor prices) and hence, the allocation of all other agents, are unchanged when a single atomistic firm is given more cheap credit. One would expect this firm to be able to hire more workers, capital, etc, and to produce more. When all firms in the economy are granted cheaper credit, however, prices are expected to change. For instance, if all firms want to hire more workers in response to the increased availability of credit, then wages should rise, and this in turn should mitigate the initial partial equilibrium effect (on the marginal cost). This general equilibrium force can be isolated by giving all other firms more credit, and then examining the unfavored atomistic.

2.3 A naive extrapolation

Consider this reduced form equation estimated by Bonomo and Martins (2016):

$$\Delta Y_{it} = \eta G_{i,t-1} + \pi \Delta R_t + \beta \left(G_{i,t-1} \cdot \Delta R_t \right) + \boldsymbol{\gamma}' \boldsymbol{X}_{it} + a_i + \varepsilon_{it},$$

where Y_{it} is an output measure[9] for firm i in year t, $G_{i,t-1}$ is firm's earmarked credit access in the previous year, R_t is the policy interest rate, and $\boldsymbol{X}_{it}$ is a vector of controls. The microeconomic effect is here captured by the parameter β.

Because this equation is assumed to be valid for all firms in the cross section, a naive analyst could be tempted to aggregate it in order to obtain an estimate of the macro effect.[10] For instance, defining $Y = \int Y_i \mathrm{d}i$ as the aggregate output, and doing the same for earmarked credit, controls and error terms, a simple integration of the equation (across firms) yields

$$\Delta Y_t = \eta G_{t-1} + \pi \Delta R_t + \beta \left(G_{t-1} \cdot \Delta R_t \right) + \boldsymbol{\gamma}' \boldsymbol{X}_t + a + \varepsilon_t,$$

and one would conclude that the macro effect would also be given by β. But from our decomposition we know that this is generally not the case. A limitation with this procedure is that the estimated cross-section equation omits the external effect. For instance, suppose

[9] They use employment, but this is the same for our purpose.

[10] To be clear, Bonomo and Martins (2016) do not extrapolate their findings. But such extrapolation has been made elsewhere and has influenced the debate. For instance, see the report of the parliamentary commission constituted to study the MP 777 (in Portuguese): https://legis.senado.leg.br/sdleg-getter/documento?dm=7139530&ts=1567534921484&disposition=inline

that we also include terms associated with the overall importance of earmarked credit:

$$\Delta Y_{it} = \eta G_{i,t-1} + \pi \Delta R_t + \beta \left(G_{i,t-1} \cdot \Delta R_t \right) + \boldsymbol{\gamma}' \boldsymbol{X}_{it} + \tilde{\eta} G_{t-1} + \tilde{\beta} \left(G_{t-1} \cdot \Delta R_t \right) + a_i + \varepsilon_{it}.$$

Now the aggregation yields:

$$\Delta Y_t = \left(\eta + \tilde{\eta} \right) G_{t-1} + \pi \Delta R_t + \left(\beta + \tilde{\beta} \right) \left(G_{t-1} \cdot \Delta R_t \right) + \boldsymbol{\gamma}' \boldsymbol{X}_t + a + \varepsilon_t,$$

and it becomes clear that the macro effect is now given by $\beta + \tilde{\beta}$, which is the sum of the micro effect and the external effect, as defined.

In principle this approach may be tried in order to disentangle micro and macro effects. But a problem that arises is that a good estimate of $\tilde{\beta}$ is much harder to obtain than a good estimate of β, since the identification of $\tilde{\beta}$ relies only on the time series dimension of the data, taking no advantage of the cross-section dimension. All aggregate time-varying effects that are correlated with $G_{t-1} \cdot \Delta R_t$ must be accounted for and, at the same time, one can not use time-effect dummies.

3 A model as example

I have argued that the available microeconometric evidence is not necessarily informative about the macroeconomic effect we are interested in. But in principle the external effect could be zero or very small, implying that micro and macro effects are quantitatively similar. As I have emphasized, one weakness of the general decomposition is that it is silent about the sign and magnitudes of each effect.

In this section I examine the sign and magnitude of each effect in the context of a very simple model: a textbook-like[11] New Keynesian model which includes working-capital needs by firms and, hence, a cost-channel, as in Christiano *et al.* (2005), Ravenna and Walsh (2006) and Christiano *et al.* (2010). A share of these loans is earmarked by the government, with interest rates that are subsidized and constant (hence, insensitive to the monetary policy rate). I allow for firm-level heterogeneity in access to earmarked credit in order to capture the microeconomic effect as well as the macro one.

Why would I work with a model of earmarked credit that emphasizes working capital credit instead of investment credit, as would be expected given the recent discussion on the TJLP rate, the importance of BNDES in the total supply of earmarked credit (around 40%) and

[11] Based on Galí (2008)'s chapter 3.

its focus on financing investments (around 95%)? First, by ignoring capital accumulation I can work with an analytically solvable model, giving formulas for the micro, macro and external effects. This is fine since one of this paper's main objectives is to give an example of the decomposition and of the fact that one cannot rely on microeconomic estimates to draw conclusion on the macroeconomic effect of interest. Second, related theoretical works on this subject — Santin (2013), Silva *et al.* (2016) — also embed earmarked credit in a model of working capital needs. But they do not explore the differences in macro and micro effects; and they do not thoughtfully examine the mechanism driving their results. Hence, in some sense this paper complements a previous literature. Third, we do see earmarked credit financing working capital needs. Although working capital credit to firms corresponds only to 2.5% of BNDES outstanding loans, working capital is very common in the rural credit (74% of its total[12]), which amounts to 15% of earmarked outstanding loans. The model should thus be useful when discussing such modality. Fourth, the cost-channel surpasses the existence and extent of working capital credit. All that is needed for it to be operative is for payments for input and factor use to occur before the production revenues. This time lag between payments and incomes introduces the opportunity cost of money in the marginal production cost, and is passed to prices. That is why Barth III and Ramey (2002) measure the importance of working capital and the value of inventories plus trade receivables (net of trade payables). Finally, there is nothing specific to investment in the claim that monetary policy becomes less effective when earmarked credit is present. The same obstruction-based argument could be applied every time a decision depends on the interest rate.

3.1 Model description

Because the model is very standard and in order to conserve space, in what follows I present the model without fully deriving it. Anyway, the full set of equilibrium conditions that characterize the model is presented in appendix A.5, and the log-linearized version of the model is presented in appendix A.6.

[12] Source: BCB — *Matriz de Dados do Crédito Rural*, for the year 2016. I consider as working capital the contracts financing current expenditures (*custeio*) and commercialization. The other major modality is investment.

3.1.1 Households

The representative household chooses consumption (C_t), labor supply (H_t) and security holdings, both real (D_t) and nominal (D_t^n), so as to maximize his expected lifetime utility

$$\max_{\{C,H,D,D^n\}} \qquad \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \frac{H_{t+s}^{1+\eta}}{1+\eta} \right] \right\} ,$$

subject to a set of flow budget constraints

$$C_t + D_t + \frac{D_t^n}{P_t} = W_t H_t + R_t D_{t-1} + R_t^n \frac{D_t^n}{P_t} + T_t \, ,$$

where T_t captures government net transfers and dividends from the ownership of firms.

3.1.2 Final Good Assemblers

The final good assembler operates in a perfectly competitive environment, producing the final consumption good from a continuum of varied retail goods, indexed by i. Its production technology is given by $Y_t = \left(\int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right)^{\frac{\varepsilon}{\varepsilon-1}}$. Conditional demand for each variety can be found by cost-minimization, and is given by $Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t$. Free entry in this market drives profit down to zero in each period and implies that the aggregate price level is given by $P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} \mathrm{d}i \right)^{\frac{1}{1-\varepsilon}}$.

3.1.3 Firms

There is a unit mass continuum of monopolistically competitive firms indexed by i. Each produces a differentiated good, but the technology used is the same, represented by the production function $Y_{it} = H_{it}^{1-\alpha}$.

Following Christiano *et al.* (2010) I introduce a 'working capital channel' by requiring that a fraction ψ of each firm's wage bill is to be externally financed. It should be acknowledged, however, that this is a simple modeling device used in the literature and that the existence of a cost-channel is much more general and can be derived from other micro-foundations. Let R_{it}^w be the gross real interest rate on working capital loans that firm i faces. Its real total cost is given by $\mathrm{Cost}_{it} = W_t H_{it} \Big(1 + \psi \left(R_{it}^w - 1 \right) \Big)$. The real marginal cost of production is given by $\mathrm{MC}_{it} = \left(\frac{1}{1-\alpha} \right) \Big(1 + \psi \left(R_{it}^w - 1 \right) \Big) W_t \frac{H_{it}}{Y_{it}}$.

A fraction ζ_i of firm i's financing needs is supplied by the government at the constant real rate R^s (s for subsidized). The other fraction must be financed at the market rate R_t. The average (and also the marginal) real interest rate firm i faces is then given by $R^w_{it} = R_t + \zeta_i\big(R^s - R_t\big)$.[13]

Firms are subject to nominal Calvo price rigidities, and with probability θ they are unable to reset prices. Retailer i price-setting problem is

$$\max_{P^*_{i,t}} \qquad \mathbb{E}_t\left\{\sum_{s=0}^{\infty}\theta^s\Lambda_{t,t+s}\left(\frac{P^*_{i,t}}{P_{t+s}} - \mathrm{MC}_{i,t+s|t}\right)\left(\frac{P^*_{i,t}}{P_{t+s}}\right)^{-\varepsilon}Y_{t+s}\right\},$$

where we have already substituted in the expression for demand. Let $p^*_{it} = P^*_{it}/P_t$ be the real optimal reset price. Taking into account the relationship between the marginal cost of firms setting prices in time t, $\mathrm{MC}_{i,t|t}$ and the average marginal cost of firm of the same type, $\mathrm{MC}_{i,t}$,[14] the first-order condition for this problem can be rewritten as

$$p^*_{it} = \left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\theta^s\Lambda_{t,t+s}\Pi_{t,t+s}^{\frac{\varepsilon}{1-\alpha}}Y_{t+s}\mathrm{MC}_{i,t+s}\right\}}{\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\theta^s\Lambda_{t,t+s}\Pi_{t,t+s}^{\varepsilon-1}Y_{t+s}\right\}}\right]^{\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}},$$

and states that the optimal reset price is a constant mark-up over a weighted average of expected marginal costs (which is just the price of wholesale goods). This optimality condition is the core of the New-Keynesian Phillips curve. Firms of the same type (i.e., with the same level of access to earmarked credit) chose the same $p^*_{i,t}$ when allowed to reset prices in the same period. But firms of different types choose different prices, and this gives rise to a multitude of Phillips curves — one for each type of firm.

3.1.4 Credit Policy

Credit policy is defined by the cost of the subsidized earmarked credit (R^s) and by the distribution of ζ_i, which represents accessibility. The interest rate is constant and, hence, is not

[13] A remark on this specification. One could alternatively have specified that the government provides to the firm a fixed *amount* in credit, instead of a fixed *fraction* of the firm's total credit need. This alternative specification might be seen as more plausible but its implications are, maybe, less appealing. This is because what should matter to a firm is the marginal credit cost, not the average credit cost. In a *fixed amount* setting firms output and hiring decisions would only be affected by earmarked credit if the amount of credit the firm needs is lower then the amount the government is willing to provide. But most firms are not totally financed by the government and, for those, private credit would be the marginal credit and hence the one affecting decision-making.

[14] Technically, I consider that for each firm i there is another continuum of identical firms, some able to readjust their prices and some not. This is necessary in order for the usual Calvo pricing algebra to follow.

influenced by monetary policy. I also assume that each firm's access to earmarked credit (ζ_i) is exogenous and fixed. The cost of this credit policy depends on the interest rate differential and on the amount of loans extended by the government. I assume it is entirely financed through lump-sum taxes.

3.1.5 Monetary Policy

The central bank is assumed to set nominal interest rates following a simple Taylor rule: $R_t^n = \left(R^n\right)\Pi_t^{\phi}U_t^m$, where the monetary policy shock is assumed to follow an AR(1) process with auto-regressive coefficient ρ. It is assumed that fiscal policy is passive: the government uses lump-sum taxes in order to satisfy its inter-temporal budget constraint for any sequence of price levels.

3.1.6 Market clearing

In this simple model there is no government spending, investment or foreign trade. Hence, final goods are all consumed: $Y_t = C_t$. Clearing in the labor market requires households' supply to equal wholesalers' demand: $H_t = \int_0^1 H_{jt}\mathrm{d}j$.

3.1.7 Equilibrium and solution

Equilibrium is defined as a sequence for endogenous variables that satisfies households optimality conditions, firms' optimality conditions, the government policy rule, and market clearing conditions, simultaneously, given the realized sequence of the exogenous stochastic process. In order to solve the model I log-linearize it around the deterministic steady state. In linearized models shocks enter additively and, because our goal is to compute impulse responses, there is no need to detail other stochastic process besides the monetary shock of interest.

3.2 A representative firm

Our model has a continuum of heterogeneous firms and this may be a nuisance for the solution of the model. For instance, if we approximate the model to have a hundred firms this would lead to $10 * 100 + 9 = 1009$ equations, according to the model's summary in appendix A.5. Of course one can simplify the equations before going for the solution, but it would still be the case that we would have at least one Phillips curve for each type of firm.

Fortunately our simple model admits a representative firm, up to a first-order approximation, as I show in appendix A.7. This is very useful as it allows us to ignore the distribution of ζ_i in the population of firms when computing the response of aggregate variables, like GDP and inflation. We only have to use the distribution of ζ_i to compute an appropriate average importance of earmarked credit in the economy, ζ, and work as if all firms in this economy have this same ζ access to earmarked credit. Given the solution for aggregate variables, we can go back and compute the solution for any given zero-measure firm with arbitrary access ζ_i. Hence, we are able to study both micro and macro effects with minimum computational difficulty, in fact, analytically.

3.3 Solving for macro variables

Because the model admits a representative firm we can solve for aggregate variables while ignoring what is happening to individual firms. As I show in appendix (A.8) the model can be reduced to a 3-equation system — comprised of an IS curve, a Phillips curve and a policy rule — for three variables — output, inflation and the real interest rate:

$$\begin{aligned} y_t &= \mathbb{E}_t\{y_{t+1}\} - \sigma^{-1} r_t \\ \pi_t &= \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa y_t + \gamma r_t \\ r_t &= \left(\phi \pi_t + u_t^m\right) - \mathbb{E}_t\left\{\pi_{t+1}\right\} \end{aligned}$$

where u_t^m follows an AR(1) process with root ρ. In addition to the structural parameters we have the following reduced-form parameters:

$$\lambda = \left(\frac{(1-\theta)(1-\beta\theta)}{\theta}\right)\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\,, \quad \kappa = \lambda\left(\sigma + \frac{\eta+\alpha}{1-\alpha}\right)\,, \quad \gamma = \lambda\left(\frac{\psi(1-\zeta)\beta^{-1}}{1+\psi(R^w-1)}\,,\right)$$

Note that the parameters λ and κ were defined exactly as in Galí (2008)[15]. Additionally this model also features a parameter γ, which captures the strength of the cost-channel. Note that γ/λ is the elasticity of the representative firm's real marginal cost to changes in the real interest rate, and that by setting $\gamma = 0$ (from either $\psi = 0$ or $\zeta = 1$) we recover the canonical textbook model. Also note that ζ, the overall importance of earmarked credit, only affects the equilibrium through this reduced form parameter γ and that:

$$\frac{\partial \gamma}{\partial \zeta} = \frac{-\lambda\psi\beta^{-1}\left(1+\psi\left(R^s-1\right)\right)}{\left[1+\psi(R^w-1)\right]^2} \quad < \quad 0\,.$$

[15] See chapter 3, which introduces the New-Keynesian model.

such that the higher the importance of earmarked credit the lower is the strength of the cost-channel. This is to be expected, since the less firms rely on private credit to finance their working capital needs then less they suffer when the market interest rate rises.

We can find the solution to this model by guess-and-verify. First, assume that for any variable z_t the solution is given by $z_t = b_z\, u_t^m$. This and the AR(1) nature of the driving force u_t^m imply that $\mathbb{E}_t\{z_{t+1}\} = \rho z_t$. Substituting the policy rule in the other two equations, and also the expectational terms:

$$\pi_t = \left[\frac{-\sigma(1-\rho)}{\phi-\rho}\right] y_t + \left[\frac{-1}{\phi-\rho}\right] u_t^m \qquad \text{IS, demand}\,,$$
$$\pi_t = \left[\frac{\kappa}{1-\beta\rho-\gamma(\phi-\rho)}\right] y_t + \left[\frac{\gamma}{1-\beta\rho-\gamma(\phi-\rho)}\right] u_t^m \qquad \text{PC, supply}\,.$$

This is a linear system of two equations for (y_t, π_t), where the exogenous term depends linearly on u_t^m. Hence the solution will be linear in u_t^m and the guess is verified. The solution for inflation and output has the following coefficients:

$$b_\pi = \frac{-\big[\kappa-\gamma(1-\rho)\big]}{(1-\beta\rho)(1-\rho)\sigma + (\phi-\rho)\big(\kappa-\gamma(1-\rho)\big)}\,,$$
$$b_y = \frac{-(1-\beta\rho)}{(1-\beta\rho)(1-\rho)\sigma + (\phi-\rho)\big(\kappa-\gamma(1-\rho)\big)}\,.$$

Models with a cost-channel may feature a "wrong" inflation response to monetary policy shocks, in principle, since interest rate changes trigger two effects with different signs. First, there is the usual aggregate demand effect, which decreases inflation for any given output level. Second, there is also an aggregate supply, cost-channel, effect, where inflation rises along with marginal costs. If the later dominates the former, then inflation may rise after a contractionist monetary shock. Fortunately this awkward response does not arise in this model[16].

Macro effects. With these solutions in hand we can then find the (macro) effects that

[16] To see this, note that the inflation's response will be well-behaved if $\kappa - \gamma(1-\rho) > 0$. Also, note that $\kappa - \gamma > 0$ is sufficient, since $\rho \in [0,1]$. This condition boils down to

$$\left(1 + \frac{\eta+\alpha}{1-\alpha}\right) > \frac{\psi(1-\zeta)\beta^{-1}}{(1-\psi)+\psi\zeta R^s + \psi(1-\zeta)\beta^{-1}}\,,$$

and it is clear that the left-hand side is bigger than one while the right-hand side is smaller than one ($\psi \in [0,1]$). Hence, $b_\pi < 0$ and $b_y < 0$.

earmarked credit has on output and inflation. Remember that

$$\text{Macro effect: output} \equiv \frac{\partial}{\partial \zeta}\left(\frac{\partial y_t}{\partial u_t^m}\right) = \frac{\partial b_y}{\partial \zeta} = \frac{\partial b_y}{\partial \gamma}\frac{\partial \gamma}{\partial \zeta}$$
$$\text{Macro effect: inflation} \equiv \frac{\partial}{\partial \zeta}\left(\frac{\partial \pi_t}{\partial u_t^m}\right) = \frac{\partial b_\pi}{\partial \zeta} = \frac{\partial b_\pi}{\partial \gamma}\frac{\partial \gamma}{\partial \zeta}$$

where the last equalities come from the fact that the overall importance of earmarked credit in the economy (ζ) only affects the economy through γ. We already have the value of $\frac{\partial \gamma}{\partial \zeta}$, and it has negative sign. Now:

$$\frac{\partial b_y}{\partial \gamma} = \left[\frac{(\phi-\rho)(1-\rho)}{(1-\beta\rho)(1-\rho)\sigma + (\phi-\rho)\big(\kappa-\gamma(1-\rho)\big)}\right] b_y \quad < \quad 0\,,$$
$$\frac{\partial b_\pi}{\partial \gamma} = \left[\frac{-\sigma(1-\rho)^2}{(1-\beta\rho)(1-\rho)\sigma + (\phi-\rho)\big(\kappa-\gamma(1-\rho)\big)}\right] b_y \quad > \quad 0\,.$$

Hence, $\frac{\partial b_y}{\partial \zeta} > 0$ and $\frac{\partial b_\pi}{\partial \zeta} < 0$. Because both b_y and b_π are negative we conclude that earmarked credit reduces the power of monetary policy shocks over aggregate output — as the common sense predicts — but increases the power over inflation — contrary to the common sense.

Figure 3.1: Shifts in supply and demand due to monetary policy tightening

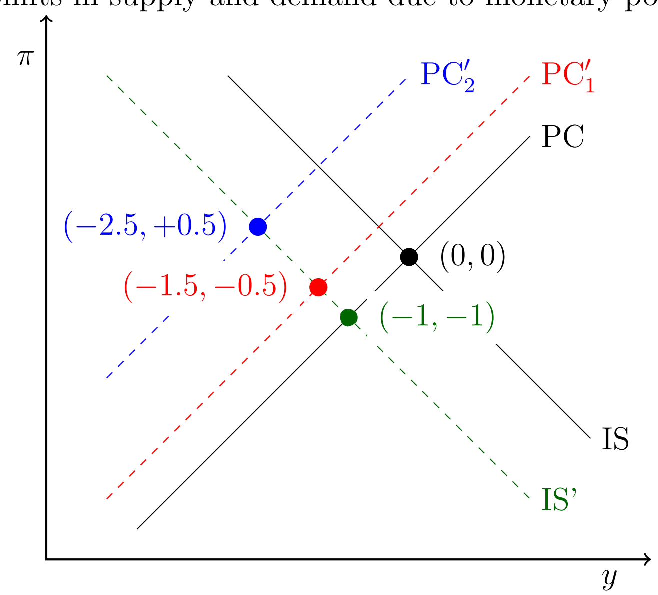


Figure 3.1 illustrates what is happening[17]. Suppose the economy is initially at steady state, represented by $(0, 0)$. An interest rate hike shifts aggregate demand (IS) curve inwards from

[17] The graph used in this example is not a precise description of the demand and supply curves we have found — for instance, it ignores the fact that changes in γ changes not only the shift size of the supply curve but the slope of this curve. I do this to simplify the exposition.

IS to IS', as consumption spending is cut down in favor of savings. If the cost-channel is not operative then aggregate supply PC does not shift, and the new equilibrium is represented by $(-1, -1)$: both inflation and output fall. If the cost-channel is operative the supply curve shifts up, however, as the rise in marginal cost caused by the higher interest rates is passed to prices, generating inflation. The curve PC'_1 represents the case where the cost-channel is operative but not sufficiently strong to dominate the aggregate demand effect — the case that always happens in our model. The equilibrium is now $(-1.5, -0.5)$: inflation and output still fall, output more than before while inflation less[18]. Now, remember that the presence of earmarked credit reduces the strength of the cost-channel. Hence, it mitigates the effect of the monetary policy on aggregate output but reinforces the effect over inflation.

3.4 Solving for micro variables

The Phillips curve for a given individual firm can be written as:

$$\pi_{i,t} = \beta \mathbb{E}_t \big\{ \pi_{i,t+1} \big\} + \kappa y_t + \gamma_i r_t - \delta p_{it} ,$$

where γ_i is firm i's analogue of the aggregate γ, and δ how pricing decisions depend on firms' relative price, given aggregate conditions (the higher the relative price, the less it needs to be increased):

$$\gamma_i = \lambda \left(\frac{\psi(1-\zeta_i)\beta^{-1}}{1+\psi(R_i^w - 1)} \right) \qquad\qquad \delta = \lambda \left(\frac{1-\alpha+2\alpha\varepsilon}{1-\alpha} \right) .$$

Using the equation that determines the evolution of this firm's relative price

$$p_{it} = p_{i,t-1} + \pi_{it} - \pi_t ,$$

to substitute for π_{it} in the Phillips curve, and noting from the aggregate Phillips curve that $\pi_t - \beta \mathbb{E}_t \{\pi_{t+1}\} = \kappa y_t - \gamma_i r_t$, we can then write the following equation for the relative price:

$$p_{it} = \left(\frac{1}{1+\beta+\delta} \right) p_{i,t-1} + \left(\frac{\beta}{1+\beta+\delta} \right) \mathbb{E}_t \{p_{i,t+1}\} - \left(\frac{\gamma_i - \gamma}{1+\beta+\delta} \right) r_t .$$

Note that in this equation the real interest r_t is the exogenous driving force, whose dynamics were already computed using the set of equations for the aggregate economy. As I show in

[18] PC'_2 represents cases where the cost-channel dominates the aggregate demand channel, and it gives rise to a price-puzzle. Again, this does not arise in this model, but may arise in others.

appendix A.9 the solution for this firm's relative price is given by:

$$p_{i,t} = Ap_{i,t-1} + B\big(\gamma_i - \gamma\big) r_t \,.$$

where:

$$A = \frac{\big(1+\beta+\delta\big) - \sqrt{\big(1+\beta+\delta\big)^2 - 4\beta}}{2\beta} \quad \in [0,1] \quad \text{and} \quad B = \frac{1}{(1-\beta A) + \beta(1-\rho) + \delta} \quad > 0\,.$$

Note that neither A nor B depend on earmarked credit. Because $B > 0$ it is clear that the relative price of firms with below-average access to earmarked credit ($\gamma_i > \gamma$) increases after an interest rate hike, while the relative price of firms with above-average access decreases.

With the solution for p_{it} it is then possible to back-out the solution for the firm's output using the conditional demand for firms' products:

$$\begin{aligned} y_{i,t} &= \underbrace{y_t}_{\text{Aggregate demand}} + \underbrace{\big(-\varepsilon\big)p_{i,t}}_{\substack{\text{Relative price} \\ \Longrightarrow \text{Market share}}} \\ &= -\varepsilon A p_{i,t-1} + \Big[1 + \varepsilon\sigma(1-\rho)B\big(\gamma_i - \gamma\big)\Big]\, b_y u^m_t \,. \end{aligned}$$

I have highlighted that variation in a firm's output must be related to two causes: (i) variation in aggregate demand; (ii) variation in the firm's relative price, which determines its market share. Now note that the aggregate demand effect is equal across firms, which means that heterogeneity in the response of firms to a monetary policy shock must come only from market share variations. This is another reason why it makes no sense to extrapolate the microeconomic effect — which relies on market share changes — to the macroeconomic level — for which there is no sense talking about market shares.

Note that it is possible for the output of some firms to rise after a contractionist monetary shock. This would occur for firms with $\big(\gamma_i - \gamma\big) < -1\big/\varepsilon\sigma(1-\rho)B$ — i.e., for firms with a particularly *high* access to earmarked credit. For this to be possible it is necessary that the market-share effect is sufficiently strong, more than compensating than the aggregate demand effect which has the "correct" sign. Weak restrictions over the parametric space cannot rule out this possibility[19], but I have checked that this does not happen within conventional bounds for parameter values — at least on impact, which is our focus[20].

[19] For instance, we can generate such response pattern for $\gamma_i = 1$ firms using a basic calibration for all parameters except for the inverse elasticity of substitution, for which we set $\gamma = 1000$.

[20] It happens, though, for firms' response to have the "wrong sign" over longer horizons. In fact, figure 3.2 exemplifies this.

The solution for a firm's "inflation" (the rate it changes its own price) can be backed-out using its definition:

$$\begin{aligned}\pi_{i,t} &= \pi_t + p_{i,t} - p_{i,t-1} \\ &= (A-1)\, p_{i,t-1} + \left[b_\pi - \sigma(1-\rho)B\left(\gamma_i - \gamma\right)b_y\right] u_t^m \,,\end{aligned}$$

and, again, it is theoretically possible that this solution has the wrong sign — firm's inflation rising after a contractionist shock. This would happen for firms with $\left(\gamma_i - \gamma\right) > \left[\kappa - \gamma(1-\rho)\right] / \varepsilon\sigma(1-\beta\rho)(1-\rho)B$ — i.e., for firms with a particularly *low* access to earmarked credit. Although there is this possibility, I again have checked that this does not happen with conventional values for the parameters.

Micro effects — output:

$$\begin{aligned}\text{Micro effect : output} \quad &\equiv \frac{\partial}{\partial \zeta_i}\left(\frac{\partial y_{i,t}}{\partial u_t^m}\right) \\ &= \left[\varepsilon\sigma(1-\rho)B\left(\frac{\partial \gamma_i}{\partial \zeta_i} - \frac{\partial \gamma}{\partial \zeta_i}\right)\right] b_y + \left[1 + \varepsilon\sigma(1-\rho)B\left(\gamma_i - \gamma\right)\right] \frac{\partial b_y}{\partial \gamma}\frac{\partial \gamma}{\partial \zeta_i} \\ &= \varepsilon\sigma(1-\rho)Bb_y\frac{\partial \gamma_i}{\partial \zeta_i} \quad > \quad 0\,,\end{aligned}$$

where the last equality uses the fact that $\gamma = \int \left(p_i\right)^{1-\varepsilon}\gamma_i \mathrm{d}i$ (see appendix A.7) and, hence, that $\frac{\partial \gamma}{\partial \zeta_i} = \left(p_i\right)^{1-\varepsilon}\mathrm{d}i \approx 0$. Intuitively, giving more subsidized credit to a zero-measure firm has negligible effect on the overall importance of earmarked credit in the economy. The positive sign means that the output of a firm *falls less* when it has more access to earmarked credit, following a contractionary monetary shock[21]. Averaging across firms:

$$\begin{aligned}\text{Avg. micro effect : output} \quad &\equiv \int_0^1 \left(p_i\right)^{1-\varepsilon}\left[\frac{\partial}{\partial \zeta_i}\left(\frac{\partial y_{i,t}}{\partial u_t^m}\right)\right]\mathrm{d}i \\ &= \varepsilon\sigma(1-\rho)Bb_y\frac{\partial \gamma}{\partial \zeta} \quad > \quad 0\,,\end{aligned}$$

where the weights take into account the fact that firms have different steady-state output levels. Also, I have used the fact that $\gamma = \int \left(p_i\right)^{1-\varepsilon}\gamma_i \mathrm{d}i$.

[21] Or *rise more*, if the individual firm response has the wrong sign. This is unusual, however.

Micro effects — inflation:

$$\begin{aligned}
\text{Micro effect : inflation} \quad &\equiv \frac{\partial}{\partial \zeta_i}\left(\frac{\partial \pi_{i,t}}{\partial u_t^m}\right) \\
&= \frac{\partial b_\pi}{\partial \gamma}\frac{\partial \gamma}{\partial \zeta_i} - \sigma(1-\rho)B\left[\big(\gamma_i - \gamma\big)\frac{\partial b_y}{\partial \gamma}\frac{\partial \gamma}{\partial \zeta_i} + b_y\left(\frac{\partial \gamma_i}{\partial \zeta_i} - \frac{\partial \gamma}{\partial \zeta_i}\right)\right] \\
&= -\sigma(1-\rho)Bb_y\frac{\partial \gamma_i}{\partial \zeta_i} \quad < \quad 0\,,
\end{aligned}$$

and, averaging:

$$\begin{aligned}
\text{Avg. micro effect : inflation} \quad &\equiv \int_0^1 \big(p_i\big)^{1-\varepsilon}\left[\frac{\partial}{\partial \zeta_i}\left(\frac{\partial \pi_{i,t}}{\partial u_t^m}\right)\right]\mathrm{d}i \\
&= -\sigma(1-\rho)Bb_y\frac{\partial \gamma}{\partial \zeta} \quad < \quad 0\,,
\end{aligned}$$

where again I have used the fact that $\frac{\partial \gamma}{\partial \zeta_i} \approx 0$.

Relation between output and inflation average micro effects. Note that:

$$\text{Avg. micro effect : output} = \big(-\varepsilon\big)\ \text{Avg. micro effect : inflation}\,,$$

meaning that (i) these effects have opposite signs; and that (ii) the higher the elasticity of substitution across goods varieties the higher is the micro effect over output, given the micro effect over inflation. This is to be expected, since the micro effect comes from market-share variations induced by variation in relative prices.

External effects — output: Remember that the external effect is defined as the change in the impulse response function of a firm when the access to earmarked credit *of all other firms* varies. Hence:

$$\begin{aligned}
\text{External effect : output} \quad &\equiv \frac{\partial}{\partial \zeta_{-i}}\left(\frac{\partial y_{i,t}}{\partial u_t^m}\right) \\
&= \left[\varepsilon\sigma(1-\rho)B\left(\frac{\partial \gamma_i}{\partial \zeta_{-i}} - \frac{\partial \gamma}{\partial \zeta_{-i}}\right)\right]b_y + \big[1 + \varepsilon\sigma(1-\rho)B\big(\gamma_i - \gamma\big)\big]\frac{\partial b_y}{\partial \gamma}\frac{\partial \gamma}{\partial \zeta_{-i}} \\
&= \left[-\varepsilon\sigma(1-\rho)Bb_y + \Big(1 + \varepsilon\sigma(1-\rho)B\big(\gamma_i - \gamma\big)\Big)\frac{\partial b_y}{\partial \gamma}\right]\frac{\partial \gamma}{\partial \zeta}\,,
\end{aligned}$$

where we used the fact that $\frac{\partial \gamma_i}{\partial \zeta_{-i}} = 0$ and that $\zeta_{-i} = \zeta - \big(p_i\big)^{1-\varepsilon}\mathrm{d}i \approx \zeta$, and, hence, $\frac{\partial \gamma}{\partial \zeta_{-i}} \approx \frac{\partial \gamma}{\partial \zeta}$.

The average external effect is less complicated:

$$\begin{aligned}\text{Avg. external effect : output} \quad &\equiv \int_0^1 \left(p_i\right)^{1-\varepsilon} \left[\frac{\partial}{\partial \zeta_{-i}} \left(\frac{\partial y_{i,t}}{\partial u_t^m}\right)\right] \mathrm{d}i \\ &= \left[\frac{\partial b_y}{\partial \gamma} - \varepsilon\sigma(1-\rho)Bb_y\right]\frac{\partial \gamma}{\partial \zeta} \quad \gtreqless 0\,,\end{aligned}$$

but its sign is still ambiguous, related to the fact the a firm's output is affected by both aggregate demand and market-share considerations. Aggregate demand follows the macro effect: when all other firms (but i) have more access to earmarked credit aggregate demand falls less when there is a contractionary shock, so demand for firm i's goods also falls less, *given* relative prices. But relative price of a firm also changes: when all other firms have more access to earmarked credit their prices fall by more following a contractionary shock, meaning that firm i's relative price rises, reducing the demand for its goods, *given* aggregate demand.

External effects — inflation:

$$\begin{aligned}\text{External effect : inflation} \quad &\equiv \frac{\partial}{\partial \zeta_{-i}} \left(\frac{\partial \pi_{i,t}}{\partial u_t^m}\right) \\ &= \frac{\partial b_\pi}{\partial \gamma}\frac{\partial \gamma}{\partial \zeta_{-i}} - \sigma(1-\rho)B\big(\gamma_i - \gamma\big)\frac{\partial b_y}{\partial \gamma} - \sigma(1-\rho)Bb_y\left(\frac{\partial \gamma_i}{\partial \zeta_{-i}} - \frac{\partial \gamma}{\partial \zeta_{-i}}\right) \\ &= \left[\frac{\partial b_\pi}{\partial \gamma} - \sigma(1-\rho)B\big(\gamma_i - \gamma\big)\frac{\partial b_y}{\partial \gamma} + \sigma(1-\rho)Bb_y\right]\frac{\partial \gamma}{\partial \zeta}\,,\end{aligned}$$

and, averaging:

$$\begin{aligned}\text{Avg. external effect : inflation} \quad &\equiv \int_0^1 \left(p_i\right)^{1-\varepsilon} \left[\frac{\partial}{\partial \zeta_{-i}} \left(\frac{\partial \pi_{i,t}}{\partial u_t^m}\right)\right] \mathrm{d}i \\ &= \left[\frac{\partial b_\pi}{\partial \gamma} + \sigma(1-\rho)Bb_y\right]\frac{\partial \gamma}{\partial \zeta} \quad \gtreqless 0\,,\end{aligned}$$

The sign of this average external effect is also ambiguous, and again there are two forces. On one hand, a firm needs to raise its prices when there is inflation if it wants to keep its relative price fixed: when all other firms (but i) have more access to earmarked credit, inflation falls by more following a contractionary monetary shock and, by this channel, firm i also wants to cut the price it charges. On the other hand, the firm may want to change its relative price: when all other firms have more access to earmarked credit, the marginal cost of firm i increases by more than the marginal cost of its competitors, and this is an incentive for firm i to increase the price it charges.

The decomposition works. With the formulas for the average micro and external effects it is easy to check that the decomposition

$$\text{Macro effect} \quad = \quad \text{Avg. micro effect} \quad + \quad \text{Avg. external effect} \,.$$

works for both output and inflation.

3.5 A quantitative assessment

The analysis so far has been all analytical, and this approach was very useful to find some answers that are not conditional on the parameterization and also to better understand the forces at play. For instance, it allowed us to show that in the model the presence of earmarked credit reduces the power of monetary policy shocks over output, but increases it over inflation, and allowed us to understand how it is linked to the cost-channel. Also, we could check that the micro and macro effects are indeed different objects, with different formulas for their computation.

However, some answers could not be obtained by relying only on analytical derivation. For example, the signs of the external effects are ambiguous, and its not clear how big they are. In order to proceed we need to to put some values on the parameters. To this end I consider two approaches: (i) looking at a particular parameter vector of interest; and (ii) considering a prior distribution for the parameters and computing the resulting distribution of macro, micro and external effects.

Table 3.1: Prior distributions

Parameter	Meaning	Distribution	Mean	Std. Dev
β	Discount factor	Beta	0.99	0.01
α	Capital share	Beta	0.33	0.05
σ	Inverse elasticity of intertemporal substitution	Gamma	1	0.5
η	Inverse Frisch Elasticity	Gamma	1	0.5
θ	Calvo nominal rigidity	Beta	0.66	0.2
$\varepsilon - 1$	Elasticity of substitution among goods	Gamma	5	5
$\phi_m - 1$	Taylor rule coefficient	Gamma	0.5	0.25
ρ_m	Monetary shock persistence	Beta	0.5	0.25
R_s	Governmental credit interest rate	Fixed	1	-
ψ	Working capital need	Fixed	1	-
ζ	Overall importance of governmental credit	Fixed	$\{ 0 \ , \ 1 \}$	-

Table 3.1 presents the considered priors I use in the analysis that follows. For the distribution shapes, I consider beta or gamma distributions depending on the parametric space. For simplicity, I choose as prior means values from Galí (2008)[22]. For standard deviations I set somewhat *ad hoc* values reflecting our own uncertainty about parameter values. Of course, the textbook can not guide the choice of values for ψ and R^s, as they are specific to this model. For those I just fix a value instead of specifying a distribution, because it is trivial how they affect our effects of interest. I set $\psi = 1$, implying that firms must finance the entirety of its wage bill. I do so not for realism but to maximize the potential effect of earmarked credit on the economy. I set $R_s = 1$, so that the annualized real interest rate on earmarked loans is 4 p.p. lower than the policy rate in steady state. For ζ I just consider the values of 0 and 1 in order to compute macro and external effects[23].

For the approach using a specific parameter vector I employ prior mean shown in table 3.1, with one twist: $\alpha = 0$. I do so for a pedagogical purpose, in order to obtain more pronounced micro, macro and external effects. Also, $\alpha = 0$ is itself a benchmark case (constant returns to scale).

3.5.1 Assessment using a particular parameter vector

Figure 3.2 is a graphical representation of the macro, micro and external effects, for both output (upper panel) and annualized inflation (lower panel), following a monetary policy shock of 1 p.p. (annualized). Tables 3.2 and 3.3 present the numerical values shown on Figure 3.2 for the horizon $t = 1$, i.e., on impact.

Table 3.2: IRF for output, on impact

	$\zeta = 0$	$\zeta = 1$	$\Delta(\zeta)$
Agg.	-0.249	-0.213	+0.036
Firm($\zeta_j = 0$)	-0.249	-0.321	-0.072
Firm($\zeta_j = 1$)	-0.123	-0.213	-0.091
Δ(Firm)	+0.126	+0.108	-

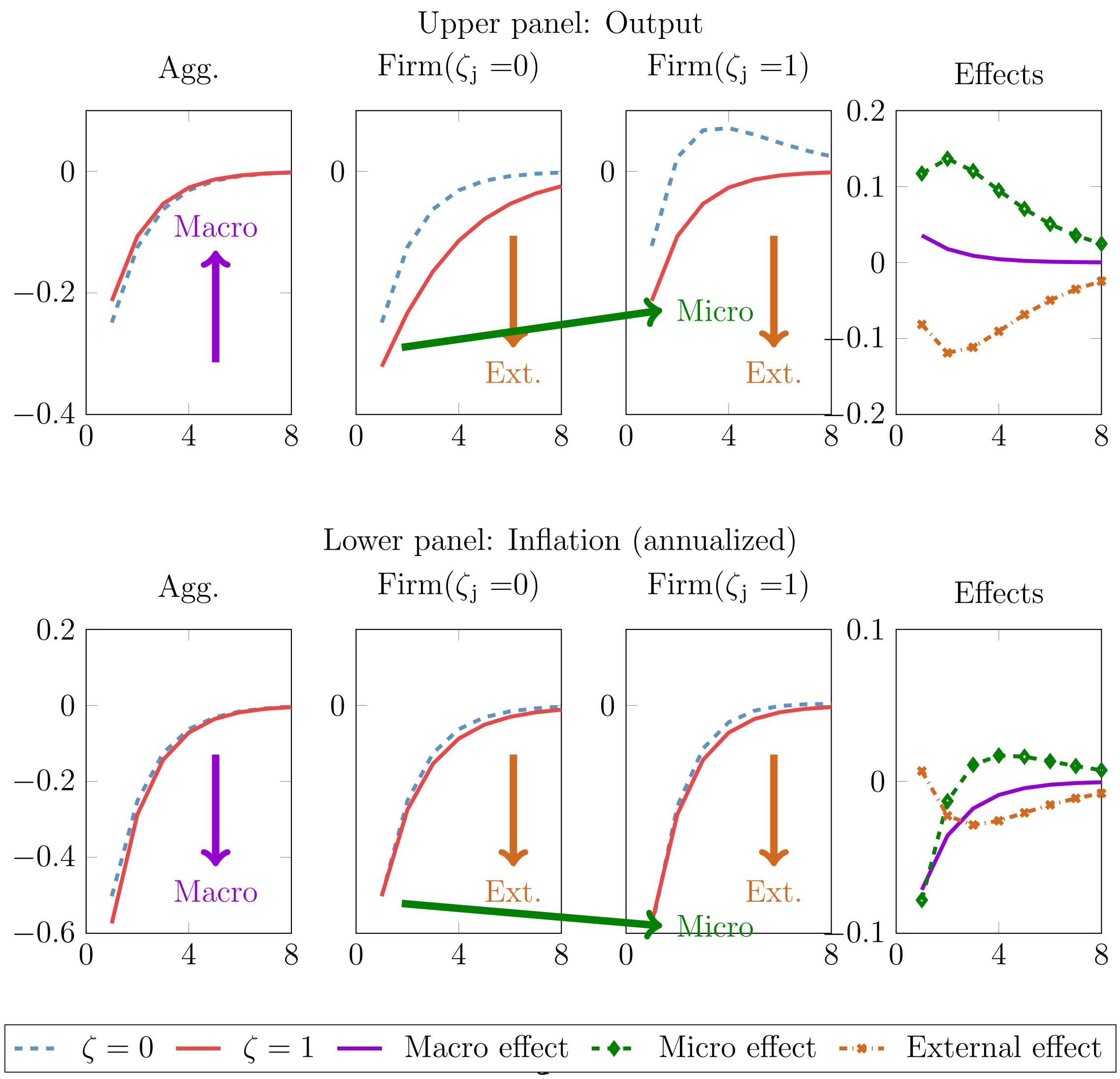
Table 3.3: IRF for inflation, on impact

	$\zeta = 0$	$\zeta = 1$	$\Delta(\zeta)$
Agg.	-0.503	-0.574	-0.071
Firm($\zeta_j = 0$)	-0.503	-0.502	+0.001
Firm($\zeta_j = 1$)	-0.587	-0.574	+0.013
Δ(Firm)	-0.084	-0.072	-

[22] Chapter 3, page 52.

[23] Appendix A.4 shows how to perform the decomposition when the considered changes in ζ and ζ_j are discrete (i.e. not infinitesimal). Anyway, it turns out that the function solution coefficients are almost linear in ζ or ζ_j on the domain $[0, 1]$, such that it does not matter whether one computes the marginal difference or a discrete difference.

Figure 3.2: IRFs to a 1 p.p. contractionary M.P. shock; and macro, micro and external effects



Note: In each panel, the leftmost box plots the response of the aggregate variable following a 1 p.p. annualized contractionary monetary policy shock, both when $\zeta = 0$ (blue, dashed line) and $\zeta = 1$ (red, continuous line), and comparison between these two lines captures the "macro effect". The two central boxes plot firm-level impulse responses: one box plots the response of a firm without any access to earmarked credit ($\zeta_j = 0$), the other the response of a firm completely financed by the government ($\zeta_j = 1$). Comparison between these central boxes capture the differences in responses across firms, i.e., the "micro effect". Again the lines correspond to different scenarios of the overall importance of earmarked credit, and the difference between them captures the "external effect". Finally, the rightmost box simultaneously plots the macro, micro and external effects.

As expected, both aggregate inflation and output fall on impact. In accordance with our previous discussion, when earmarked credit is present output falls less — from -0.249% in the economy with $\zeta = 0$ to -0.213% in the economy with $\zeta = 1$, for a macro effect of +0.036 p.p. — and inflation falls more — from -0.503% in the economy with $\zeta = 0$ to -0.574% in the economy with $\zeta = 1$, in annualized terms, for a macro effect of -0.071 p.p. These macro effects are relatively small, in comparison to the respective IRF, barely noticeable. For output, the average micro effect (of +0.117) is three times larger than the macro effect. Hence, by observing the cross-section a large and significant effect of earmarked credit on firms' response does not necessarily means that the same large and significant effect is present at the aggregate level.

3.6 Sensitivity to other parameterizations

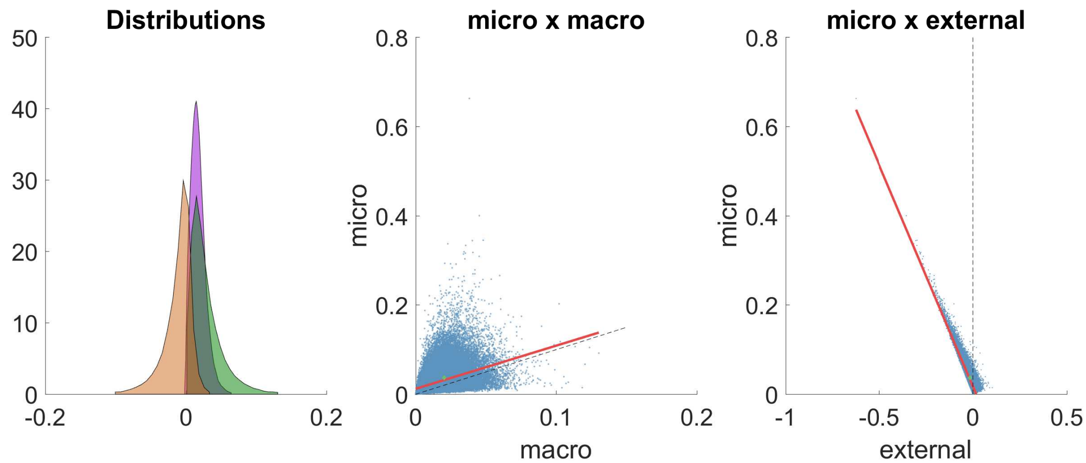
I consider a random sample of 100,000 draws from the prior distribution and for each I compute the associated micro, macro and external effects, for both output and inflation. Figure 3.3 plots the results.

For output (upper panel) we see that the distribution of macro and micro effects has support over positive numbers, as expected, which means that monetary policy power is always reduced when earmarked credit is present, both at the firm and at the aggregate level. The distribution of external effects is mostly concentrated on negative numbers, implying that in general the external effect mitigates the micro effect, implying a macro effect which is lower than the micro effect. But there are cases where the external effect is positive and, hence, the macro effect is higher. The size of the macro effect is positively correlated with the size of the micro effect, in this prior, but for some parameter variation — for instance, for ϵ — the correlation is negative (not shown).

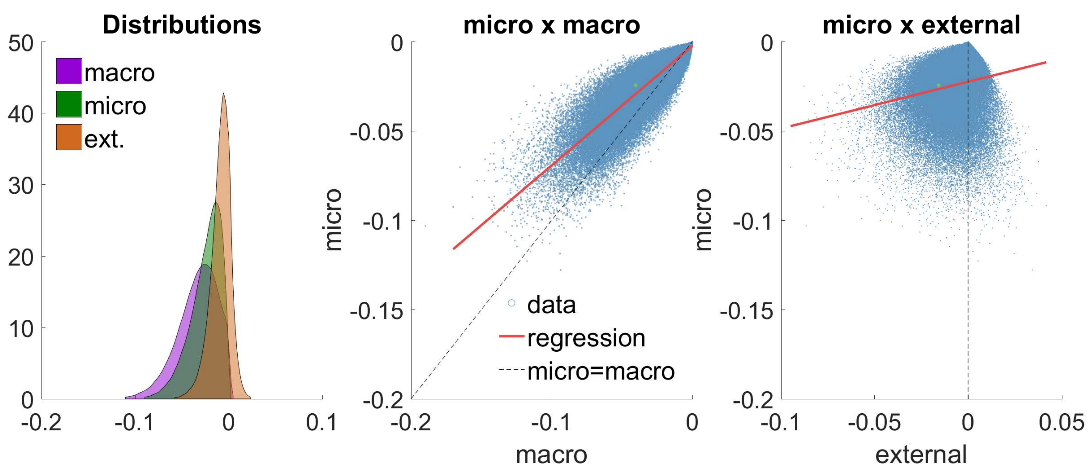
For inflation (lower panel) we see that the distribution of macro and micro effects has support over negative numbers, as expected — meaning that monetary policy power is increasing in the importance of earmarked credit. The distribution of external effects is mostly concentrated on negative numbers — which means that micro and external effects generally reinforce each other and result in a larger macro effect.

Figure 3.3: Macro, micro and external effects associated with the prior parameter distribution

Upper panel: Output



Lower panel: Inflation (annualized)



4 Conclusion

It seems that a broad agreement has been reached among Brazilian economists that monetary policy becomes significantly less effective in the presence of earmarked credit featuring subsidized and monetary cycle-insensitive interest rates. In this paper I argue that such a question should be reexamined.

First, the available microeconometric evidence that firms with more access to earmarked credit respond less to monetary policy shocks is not necessarily informative about the macroeconometric effect that economists are mostly interested in. I show this theoretically and also in a toy, exemplifying model. In particular I show the possibility of a large effect on the cross-section of firms to coexist with a small effect on the aggregate level.

Second, monetary policy affects many variables, and the presence of earmarked credit may affect differently each variable's responsiveness. In the exemplifying model aggregate output's response does indeed become milder, as expected, but inflation's responsiveness becomes stronger. In the model this is the case because, by financing firms' working capital needs, earmarked credit reduces the strength of the cost channel.

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A Appendix

A.1 Decomposition with an infinite but countable number of firms

Take the derivation with a finite number N of firms:

$$\frac{\partial Y}{\partial \zeta} \;=\; \frac{1}{N}\sum_{i=1}^{N}\frac{\partial Y_i}{\partial \zeta_i} \;+\; \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{j\neq i}\frac{\partial Y_i}{\partial \zeta_j}\right)$$

and now let $N \to \infty$. Assume that

$$\lim_{N\to\infty}\frac{\partial Y_i}{\partial \zeta_j} = 0 \qquad \text{and} \qquad \lim_{N\to\infty}(N-1)\frac{\partial Y_i}{\partial \zeta_j} \text{ is bounded}$$

for all i and $j \neq i$. I am just asking for a case where firm j's ability to influence firm i's decision to vanish, in an appropriate velocity, when the number of firms in the economy grows large. This assumption is reasonable — when the number of firms in the economy are very big and the distribution of size in the economy is well-behaved (meaning that no firm in the economy is too big), no firm is supposed to affect the aggregate and, through that, other firms. This assumption gives us sufficient regularity to apply the law of the large numbers and define:

$$\frac{\partial Y_i}{\partial \zeta_{-i}} = \lim_{N\to\infty}\frac{1}{N-1}\sum_{j\neq i}\left[(N-1)\frac{\partial Y_i}{\partial \zeta_j}\right]$$

With this, we can write:

$$\frac{\partial Y}{\partial \zeta} = \mathbb{E}_i\left[\frac{\partial Y_i}{\partial \zeta_i}\right] + \mathbb{E}_i\left[\frac{\partial Y_i}{\partial \zeta_{-i}}\right]$$

A.2 Decomposition of aggregate effect with an uncountable number of firms

I assume that there is a unitary measure of firms. Firm i's access to earmarked credit is denoted by $\zeta_i = f(i)$, where $f : [0,1] \to [0,1]$ is a function that describes how much access each firm has. In equilibrium firm i's output is a function of R and a functional of $f(\cdot)$:

$$Y_i = Y_i\big(\,R\,;f\big)$$

Total differentiation of firm i's output with respect to earmarked credit access variables yields:

$$\begin{aligned} \mathrm{d}Y_i &= \int_0^1 \frac{\partial Y_i}{\partial \zeta_j} \mathrm{d}f(j) \\ &= \frac{\partial Y_i}{\partial \zeta_i} \mathrm{d}f(i) + \int_{\mathcal{C}_i} \frac{\partial Y_i}{\partial \zeta_j} \mathrm{d}f(j) \end{aligned}$$

where $\mathcal{C}_i = [0,1] \setminus [i]$. We have separated ζ_i from the other ζ_j, to make clear that the "own effect" of ζ_i over Y_i has a very different nature than the "external effect" of ζ_i, $j \neq i$. In fact, as discussed on the main text, it is likely that, by itself, the access of any given firm $j \neq i$ has negligible effect over the product of firm i. But when aggregated with all other firms the total external effect might be relevant.

Aggregate earmarked credit is defined as $\zeta = \int_0^1 \zeta_i \mathrm{d}i$, and we again consider the policy change $\mathrm{d}\zeta_i = \mathrm{d}\zeta$ for all i. Hence:

$$\frac{\partial Y_i}{\partial \zeta} = \frac{\partial Y_i}{\partial \zeta_i} + \int_{\mathcal{C}_i} \left(\frac{\partial Y_i}{\partial \zeta_j} \frac{1}{\mathrm{d}j} \right) \mathrm{d}j$$

Note that in the second right-hand side term we are integrating $\frac{\partial Y_i}{\partial \zeta_j} \frac{1}{\mathrm{d}j}$, not $\frac{\partial Y_i}{\partial \zeta_j}$. This is consistent with the case with an infinite but countable number of firms (appendix A.1), where the "external effect" was the average of $(N-1)\frac{\partial Y_i}{\partial \zeta_j}$, not of $\frac{\partial Y_i}{\partial \zeta_j}$[24]. Again, we assume that:

$$\frac{\partial Y_i}{\partial \zeta_j} \text{ is infinitesimal and } \mathcal{O}\big(\mathrm{d}i\big)$$

so that $\frac{1}{\mathrm{d}i} \frac{\partial Y_i}{\partial \zeta_j}$ is bounded. Then, define

$$\frac{\delta Y_i}{\delta \zeta_{-i}} = \int_{\mathcal{C}_i} \left[\frac{1}{\mathrm{d}j} \frac{\partial Y_i}{\partial \zeta_j} \right] \mathrm{d}j$$

Note that we have used the operator δ of functional derivative, instead of ∂ of the partial derivative, respecting the fact that ζ_{-i} is in fact a functional of a function $\mathcal{C}_i \to \mathbb{R}$.

The decomposition of the effects over firm i can then be written as:

$$\frac{\partial Y_i}{\partial \zeta} = \frac{\partial Y_i}{\partial \zeta_i} + \frac{\delta Y_i}{\delta \zeta_{-i}}$$

[24] Remember that $\mathrm{d}i$ is the limit of $\Delta i = N^{-1}$.

Now we define aggregate output as $Y = \int_0^1 Y_i \mathrm{d}i$. We then have:

$$\begin{aligned}
\frac{\partial Y}{\partial \zeta} &= \int_0^1 \frac{\partial Y_i}{\partial \zeta_i}\mathrm{d}i + \int_0^1 \frac{\delta Y_i}{\delta \zeta_{-i}}\mathrm{d}i \\
&= \mathbb{E}_i\left[\frac{\partial Y_i}{\partial \zeta_i}\right] + \mathbb{E}_i\left[\frac{\delta Y_i}{\delta \zeta_{-i}}\right]
\end{aligned}$$

which is the same expression we derived for the case with a countable number of firms.

A.3 Decomposition with sectors and firms

Consider an economy with S sectors and N firms in each sector $s \in \{1, \ldots, S\}$. Assume that the output firm i, who operates in sector s, is, in equilibrium $Y_{si} = Y_{si}\big(\ R\ ;\ \{\zeta_{s'i'}\}\ \big)$, where $\{\zeta_{s'i'}\}$ lists the access of all firms in all sectors: $\zeta_{11}, \zeta_{12}, \cdots, \zeta_{1N}, \zeta_{21}, \cdots, \zeta_{SN}$.

Sectoral output is given by:

$$Y_s = \frac{1}{N}\sum_{i=1}^{N} Y_{si}$$

And aggregate output is given by:

$$\begin{aligned}
Y &= \frac{1}{S}\sum_{s=1}^{S} Y_s \\
&= \frac{1}{NS}\sum_{s=1}^{S}\sum_{i=1}^{N} Y_{si}
\end{aligned}$$

Aggregate and sectoral levels of access to earmarked credit — ζ and ζ_s — are defined accordingly. Total differentiation of the aggregate output with respect to all sector-firms levels of access imply:

$$\mathrm{d}Y = \frac{1}{NS}\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{\tilde{s}=1}^{S}\sum_{\tilde{i}=1}^{N} \frac{\partial Y_{si}}{\partial \zeta_{\tilde{s}\tilde{i}}}\mathrm{d}\zeta_{\tilde{s}\tilde{i}}$$

Again, we consider a change in credit policy such that $\mathrm{d}\zeta_{si} - \mathrm{d}\zeta$, for all sectors and firms. We then separate the sums:

$$\frac{dY}{d\zeta} = \frac{1}{NS}\left[\sum_{s=1}^{S}\sum_{i=1}^{N}\frac{\partial Y_{si}}{\partial \zeta_{si}}\right] + \frac{1}{NS}\left[\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{\tilde{i}\neq i}\frac{\partial Y_{si}}{\partial \zeta_{s\tilde{i}}}\right] + \frac{1}{NS}\left[\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{\tilde{s}\neq s}\sum_{\tilde{i}}\frac{\partial Y_{si}}{\partial \zeta_{\tilde{s}\tilde{i}}}\right]$$

The first term on the right-hand side accounts for "own-effects", i.e., the effect over one firm's output when we change their own access to earmarked credit. The second and third terms account for "external-effects", with the difference that the former considers same-sector externalities while the latter considers cross-sector externalities.

In order to make make the notation easier, define:

$$\frac{\partial Y_{si}}{\partial \zeta_{s,-i}} = \sum_{\tilde{i}\neq i}\frac{\partial Y_{si}}{\partial \zeta_{s\tilde{i}}}$$

to denote the effect on sector s-firm i's output when all other firms in the same sector have their levels of access changed. Also, define:

$$\frac{\partial Y_{si}}{\partial \zeta_{-s}} = \sum_{\tilde{s}\neq s}\sum_{\tilde{i}}\frac{\partial Y_{si}}{\partial \zeta_{\tilde{s}\tilde{i}}}$$

to denote the effect on sector s-firm i's output when the levels of firms in all other sectors change. With this investment in notation, we can finally write:

$$\underbrace{\frac{\partial Y}{\partial \zeta}}_{\text{Macro effect}} = \underbrace{\underbrace{\mathbb{E}_i\left[\frac{\partial Y_{si}}{\partial \zeta_{si}}\right]}_{\text{Average of micro effects}} + \underbrace{\mathbb{E}_i\left[\frac{\partial Y_{si}}{\partial \zeta_{s,-i}}\right]}_{\substack{\text{Average of external effects}\\ \text{same sector}}}}_{\text{Sectoral micro effect}} + \underbrace{\mathbb{E}_i\left[\frac{\partial Y_{si}}{\partial \zeta_{-s}}\right]}_{\substack{\text{Average of external effects}\\ \text{other sectors}}}$$

Now, note that at the sectoral level we can do the following decomposition:

$$\underbrace{\frac{\partial Y_s}{\partial \zeta_s}}_{\text{Sectoral micro effect}} = \underbrace{\mathbb{E}_i\left[\frac{\partial Y_{si}}{\partial \zeta_{si}}\right]}_{\text{Average of micro effects}} + \underbrace{\mathbb{E}_i\left[\frac{\partial Y_{si}}{\partial \zeta_{s,-i}}\right]}_{\substack{\text{Average of external effects}\\ \text{same sector}}}$$

A.4 Decomposition for discrete changes in accessibility

The decomposition still holds if one consider discrete variations in access to earmarked credit. For simplicity, here we show that this is the case for a simple case with two firms.

Again, in equilibrium each firm's variable of interest is a function of the monetary policy instance and of all firms' access to earmarked credit:

$$Y_j = Y_j\big(\, u^m \,; \zeta_1, \zeta_2\big) \qquad j = \{1, 2\}$$

and we define the aggregate variable of interest to be $Y = \frac{1}{2}\,(Y_1 + Y_2)$. Now we consider the effect from moving the economy from a state of no government financing ($\zeta_1 = \zeta_2 = 0$) to a state of total government financing ($\zeta_1 = \zeta_2 = 1$). By definition, the macroeconomic effect of such a move is given by:

$$\text{macro effect} = Y(1,1) - Y(0,0)$$

where we have omitted the monetary policy argument in order to save on notation. Using the definition of the aggregate:

$$\text{macro effect} \quad = \quad \frac{1}{2}\Big[Y_1(1,1) - Y_1(0,0)\Big] + \frac{1}{2}\Big[Y_2(1,1) - Y_2(0,0)\Big]$$

Now, add and subtract both $\frac{1}{2}Y_0(0,1)$ and $\frac{1}{2}Y_1(1,0)$. Rearranging the terms in a convenient way, we get:

$$\begin{aligned} \text{macro effect} \quad &= \quad \underbrace{\frac{1}{2}\Bigg\{\Big[Y_1(1,1) - Y_1(0,1)\Big] + \Big[Y_2(1,1) - Y_2(1,0)\Big]\Bigg\}}_{\text{Avg. micro effect}} \\ &\quad + \quad \underbrace{\frac{1}{2}\Bigg\{\Big[Y_1(0,1) - Y_1(0,0)\Big] + \Big[Y_2(1,0) - Y_2(0,0)\Big]\Bigg\}}_{\text{Avg. external effect}} \end{aligned}$$

Note that the first line indeed captures the micro effect: we consider that happens to firm 1 when we change its access from $\zeta_1 = 0$ to $\zeta_1 = 1$ while keeping $\zeta_2 = 1$. And the second line captures the external effect: for firm 1 we change *firm 2's access* from $\zeta_2 = 0$ to $\zeta_2 = 1$, while keeping $\zeta_1 = 0$.

Because variation is discrete the decomposition is sensitive to the baseline chosen for comparison. In the decomposition above we have calculated the micro effect holding the other firms' access equal to 1, but we could have proceeded by holding other firms' access equal to 0, if we had added and subtracted $\frac{1}{2}Y_0(1,0)$ and $\frac{1}{2}Y_1(0,1)$ instead. Such problems are very common with decompositions. A reasonable compromise in this case is to take the average.

A.5 Summary of model's equations

Households

$$C_t^{-\sigma} = \beta R_t \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \right\} \tag{A.1}$$

$$C_t^{\sigma} H_t^{\eta} = W_t \tag{A.2}$$

$$R_t^n = R_t \mathbb{E}_t \big\{ \Pi_{t+1} \big\} \tag{A.3}$$

$$\Lambda_t = \beta \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \tag{A.4}$$

Firms: $\forall \boldsymbol{i} \in [\mathbf{0}, \mathbf{1}]$

$$Y_{it} = \big(p_{it}\big)^{-\varepsilon} Y_t \tag{A.5}$$

$$Y_{it} = H_{it}^{1-\alpha} \tag{A.6}$$

$$R_{it}^w = R_t + \zeta_i \big(R^s - R_t \big) \tag{A.7}$$

$$\mathrm{MC}_{it} = \left(\frac{1}{1-\alpha} \right) W_t \bigg(1 + \psi \left(R_{it}^w - 1 \right) \bigg) \big(Y_{it} \big)^{\frac{\alpha}{1-\alpha}} \tag{A.8}$$

$$p_{it}^* = \left[\left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{u_{it}}{z_{it}} \right]^{\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}} \tag{A.9}$$

$$u_{it} = Y_t \mathrm{MC}_{it} + \theta \mathbb{E}_t \bigg\{ \Pi_{t+1}^{\frac{\varepsilon}{1-\alpha}} \Lambda_{t+1} u_{i,t+1} \bigg\} \tag{A.10}$$

$$z_{it} = Y_t + \theta \mathbb{E}_t \bigg\{ \Pi_{t+1}^{\varepsilon-1} \Lambda_{t+1} z_{i,t+1} \bigg\} \tag{A.11}$$

$$\Pi_{it} = \Pi_t \frac{p_{it}}{p_{i,t-1}} \tag{A.12}$$

$$1 = \theta \Pi_{i,t}^{\varepsilon-1} + (1-\theta) \left(\frac{p_{it}^*}{p_{it}} \right)^{1-\varepsilon} \tag{A.13}$$

Monetary policy

$$R_t^n = \big(R^n \big) \Pi_t^{\phi} U_t^m \tag{A.14}$$

$$U_t^m = \big(U_{t-1}^m \big)^{\rho} \exp \left(\varepsilon_t^m \right) \tag{A.15}$$

Market clearing and price-level

$$Y_t = C_t \tag{A.16}$$

$$H_t = \int_0^1 H_{jt} \mathrm{d}j \tag{A.17}$$

$$1 = \int_0^1 p_{it}^{1-\varepsilon} \mathrm{d}i \tag{A.18}$$

Note that we have 9 "aggregate equations" (A.1-A.4, A.14-A.18) and 9 aggregate variables (C, H, Λ, Π, R, R^n, Y, W, U^m). Also, we have 9 "firm equations" (A.5 - A.13) for 9 firm variables (Y_i, H_i, R_i^w, MC_i, p_i^*, p_i, Π_i, u_i, z_i).

A.6 The log-linearized model

The set of equations below represent the log-linearized version of the set of equations presented in appendix A.5. The only adaptation is that I have eliminated the auxiliary variables u_i and z_i, together with the respective equations, in order to write down a Phillips curve for each type of firm.

Households

$$c_t = \mathbb{E}_t\{c_{t+1}\} - \sigma^{-1} r_t \tag{A.19}$$

$$w_t = \sigma c_t + \eta h_t \tag{A.20}$$

$$r_t^n = r_t + \mathbb{E}_t\{\pi_{t+1}\} \tag{A.21}$$

Firms: $\forall \boldsymbol{i} \in [\mathbf{0}, \mathbf{1}]$

$$y_{it} = y_t - \varepsilon p_{it} \tag{A.22}$$

$$y_{it} = (1-\alpha) h_{it} \tag{A.23}$$

$$r_{it}^w = \left[\frac{(1-\zeta_i)\beta^{-1}}{R_i^w}\right] r_t \tag{A.24}$$

$$\mathrm{mc}_{it} = w_t + \left[\frac{\alpha}{1-\alpha}\right] y_{it} + \left[\frac{\psi R_i^w}{1+\psi(R_i^w - 1)}\right] r_{it}^w \tag{A.25}$$

$$\pi_{it} = \beta \mathbb{E}_t\{\pi_{i,t+1}\} + \left[\frac{(1-\theta)(1-\beta\theta)}{\theta}\right] \left[\left(\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\right) \mathrm{mc}_{it} - p_{it}\right] \tag{A.26}$$

$$\pi_{it} = \pi_t + p_{it} - p_{i,t-1} \tag{A.27}$$

$$\pi_{it} = \left[\frac{1-\theta}{\theta}\right] \left(p_{it}^* - p_{it}\right) \tag{A.28}$$

Monetary policy

$$r_t^n = \phi \pi_t + u_t^m \tag{A.29}$$

$$u_t^m = \rho u_{t-1}^m + \varepsilon_t^m \tag{A.30}$$

Market clearing

$$y_t = c_t \tag{A.31}$$

$$h_t = \int \left(\frac{H_i}{H}\right) h_{it} \, \mathrm{d}i \tag{A.32}$$

$$0 = \int \left(p_i\right)^{1-\varepsilon} p_{it} \, \mathrm{d}i \tag{A.33}$$

A.7 A representative firm

Further simplifying the set of equations in each firm's block, we find:

$$\pi_{it} = \beta \mathbb{E}_t \{\pi_{i,t+1}\} + \lambda \left[w_t + \left(\frac{\alpha}{1-\alpha} \right) y_t \right] + \lambda \left[\left(\frac{\psi(1-\zeta_i)\beta^{-1}}{1+\psi(R_i^w - 1)} \right) \right] r_t - \delta p_{it}$$

where, for simplicity, I have defined the following coefficient:

$$\lambda = \left(\frac{(1-\theta)(1-\beta\theta)}{\theta} \right) \left(\frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \right) \qquad \delta = \lambda \left(\frac{1-\alpha+2\alpha\varepsilon}{1-\alpha} \right)$$

Note that we have two firm specific variables in this Phillips curve: p_i, π_i and y_i. And also note that:

$$0 = \int \big(p_i\big)^{1-\varepsilon} \hat{p}_{it} \mathrm{d}i \qquad \hat{\pi}_t = \int \big(p_i\big)^{1-\varepsilon} \hat{\pi}_{it} \mathrm{d}i \qquad y_t = \int \big(p_i\big)^{1-\varepsilon} y_{it} \mathrm{d}i$$

where the weights in the equation for aggregate output come from the conditional demand relation $\frac{Y_i}{Y} = \big(p_i\big)^{-\varepsilon}$. This means that if we aggregate firms' Phillips curves using weights $\big(p_i\big)^{1-\varepsilon}$ we can write:

$$\pi_t = \beta \mathbb{E}_t \{\pi_{t+1}\} + \lambda \left[w_t + \left(\frac{\alpha}{1-\alpha} \right) y_t \right] + \lambda \left[\int \big(p_i\big)^{1-\varepsilon} \left(\frac{\psi(1-\zeta_i)\beta^{-1}}{1+\psi(R_i^w - 1)} \right) \mathrm{d}i \right] r_t$$

Now, if we define ζ — and, hence, $R^w = \beta^{-1} + \zeta\big(R^s - \beta^{-1}\big)$ — such that:

$$\left(\frac{\psi(1-\zeta)\beta^{-1}}{1+\psi(R^w - 1)} \right) = \int \big(p_i\big)^{1-\varepsilon} \left(\frac{\psi(1-\zeta_i)\beta^{-1}}{1+\psi(R_i^w - 1)} \right) \mathrm{d}i$$

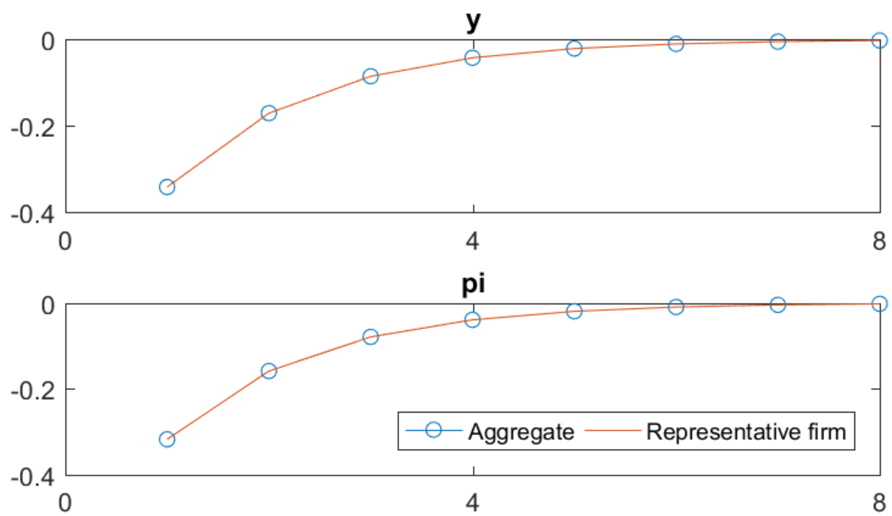
we recover an aggregate Phillips curve that is very similar to a firm-level Phillips curve — the difference is the absense of the term p_{it}:

$$\pi_t = \beta \mathbb{E}_t \{\pi_{t+1}\} + \lambda \left[w_t + \left(\frac{\alpha}{1-\alpha} \right) y_t \right] + \lambda \left[\left(\frac{\psi(1-\zeta)\beta^{-1}}{1+\psi(R^w - 1)} \right) \right] r_t$$

Now, in a model without heterogeneity where the firm has a level ζ of access to earmarked credit — ζ defined as above — its relative real price level will be equal to 1, by definition, and its log-deviation will be equal to zero. Hence, the aggregate Phillips curve we have just derived also represents the Phillips curve in this economy with a representative firm.

To illustrate the validity of our result I numerically solve a model with many firms without using this representative firm shortcut, and then compare the results with the ones I get by using a representative firm. Using 100 types of firms with ζ_i uniformly distributed over $[0,1]$ implies that the representative firm has $\zeta = 0.501$, even though the mean of the distribution is exactly 0.5. Figure A.1 plots the impulse response functions we obtain by using both approaches, and shows that they are the same.

Figure A.1: Aggregate × representative firm's IRFs



A.8 Log-linearized model in canonical form

Starting from the aggregate Phillips curve from the last appendix section, and substituting into it the labor supply schedule together with the market-clearing condition for goods and labor, we find:

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \underbrace{\lambda\left[\sigma + \frac{\eta+\alpha}{1-\alpha}\right]}_{\equiv\kappa} y_t + \lambda \underbrace{\left[\left(\frac{\psi(1-\zeta)\beta^{-1}}{1+\psi(R^w-1)}\right)\right]}_{\equiv\gamma} r_t$$

which is the exactly the Phillips curve found in Galí (2008) textbook exposition of the New-Keynesian model (see page 49) except for the new term $\lambda\gamma r_t$, which captures the cost-channel.

Together with the usual IS curve (which combines households' Euler equation with the market-clearing condition for good):

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\sigma} r_t$$

and the monetary policy rule (which, here, we write in terms of the real interest rate with the help of the Fisher equation):

$$r_t = \phi_m \pi_t - \mathbb{E}_t \{\pi_{t+1}\} + u_t^m$$

this aggregate Phillips curve defines a simple three equation New-Keynesian model.

A.9 Solving for firm's relative price

Remember that the firm's relative price is determined by the following equilibrium condition:

$$p_{it} = \left(\frac{1}{1+\beta+\delta}\right) p_{i,t-1} + \left(\frac{\beta}{1+\beta+\delta}\right) \mathbb{E}_t \{p_{i,t+1}\} - \left(\frac{\gamma_i - \gamma}{1+\beta+\delta}\right) r_t$$

We can solve this by the method of undetermined coefficients. Suppose a solution given by:

$$p_{i,t} = Ap_{i,t-1} + Br_t$$

Substituting into the equilibrium condition, and remembering that $\mathbb{E}_t \{r_{t+1}\} = \rho r_t$, we get:

$$\left[1 - \left(\frac{\beta}{1+\beta+\delta}\right) A\right] p_t = \left(\frac{1}{1+\beta+\delta}\right) p_{i,t-1} + \left[\frac{\beta\rho B - (\gamma_i - \gamma)}{1+\beta+\delta}\right] r_t$$

Hence,

$$A = \left[1 - \left(\frac{\beta}{1+\beta+\delta}\right) A\right]^{-1} \left(\frac{1}{1+\beta+\delta}\right) \quad \text{and} \quad B = \left[1 - \left(\frac{\beta}{1+\beta+\delta}\right) A\right]^{-1} \left[\frac{\beta\rho B - (\gamma_i - \gamma)}{1+\beta+\delta}\right]$$

The first condition is a second degree equation in A

$$\beta A - (1+\beta+\delta) A + 1 = 0$$

with roots

$$A = \frac{(1+\beta+\delta) \pm \sqrt{(1+\beta+\delta)^2 - 4\beta}}{2\beta}$$

Let A_- and A_+ denote roots associated with the minus and plus signs, respectively. With some algebra we can place some bounds on this roots and find that $A_- \in [0, 1]$ while $A_+ > 1$. Because our solution concept rules out explosive solution we then have that $A = A_-$. Substituting this into the condition for B and rearranging we get:

$$B = \frac{(\gamma_i - \gamma)}{(1 - \beta A_-) + \beta(1 - \rho) + \delta}$$