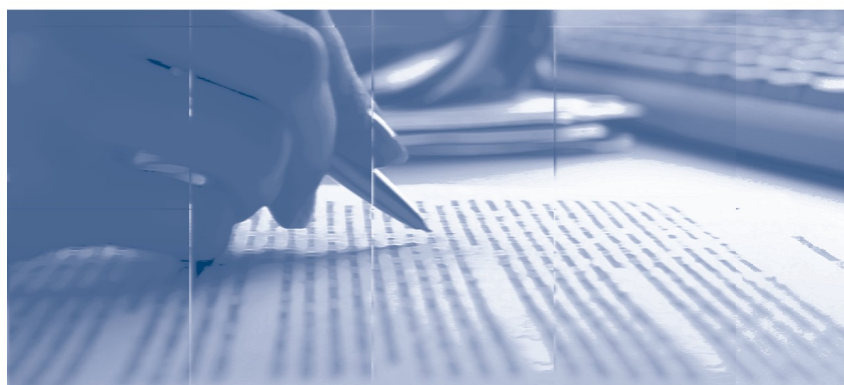


Sectoral Countercyclical Buffers in a DSGE Model with a Banking Sector

Marcos R. Castro

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Citizen Service Division

Banco Central do Brasil

Deati/Diate

SBS – Quadra 3 – Bloco B – Edifício-Sede – 2º subsolo

70074-900 Brasília – DF – Brazil

Toll Free: 0800 9792345

Fax: +55 (61) 3414-2553

Internet: <http://www.bcb.gov.br/?CONTACTUS>

Non-technical Summary

In response to the 2008 international financial crisis, the members of the Basel Committee on Banking Supervision agreed upon a new bank regulatory framework that, among other measures, introduced some new macroprudential instruments. One of them, the Counter-Cyclical Capital Buffer (CCyB), is intended to enhance bank resilience by requiring financial institutions to build additional bank capital chests during periods of high credit growth to provide more resilience during downturns, leaning against the wind of financial cycles. However, the CCyB is a blunt instrument that does not discriminate among distinct economic and credit sectors, responding mostly to aggregate credit evolution, and is not able to target specific credit sectors that might be originating the credit market imbalances, such as the mortgage sector in the USA prior to the global financial crisis. This lack of focus of the CCyB might be overcome by introducing a new set of more targeted capital requirement instruments able to address sectoral credit imbalances.

This article assesses the impact of a hypothetical introduction of a Sectoral Counter-Cyclical Capital Buffer (SCCyB) as a new macroprudential instrument in Brazil. While the former CCyB associates the counter-cyclical bank capital requirement to bank's total risk weighted assets, each new sectoral counter-cyclical buffer would apply only to risk-weighted assets of the respective credit sector, such as housing loans, commercial or consumer loans.

To evaluate the impact of introducing the SCCyB in Brazil, we develop and estimate a dynamic stochastic general equilibrium model featuring the three main bank credit categories in Brazil – housing loans, consumer loans to households and commercial loans to firms – as well as loans provided by the development bank. The model features bank capital requirement and both types of counter-cyclical capital buffers – the broad one and sectoral buffers for housing, consumer and commercial loans. We simulate alternative macroprudential frameworks involving different combinations of the broad CCyB and the sectoral buffers, and we compare the resulting performances. We conclude that introducing the sectoral buffers to the existing toolkit of macroprudential instruments may help enhancing macroeconomic and financial stabilization. However, introducing those additional instruments would require more frequent macroprudential intervention and careful coordination among policy instruments, adding more operational complexity to macroprudential policy.

Sumário Não Técnico

Em resposta à crise financeira internacional de 2008, o Comitê de Supervisão Bancária da Basileia propôs em 2010 um novo arcabouço de regulação bancária, introduzindo uma série de novos instrumentos macroprudenciais. Um dos mais importantes entre estes, o Adicional Contracíclico de Capital Principal (ACCP), procura aumentar a resiliência do sistema bancário ao exigir que as instituições financeiras acumulem maior volume de capital bancário nos períodos de expansão de crédito para fazer frente a períodos de crise econômica e retração do crédito, aumentando a resiliência do sistema financeiro. No entanto, o ACCP original é um requerimento de capital que não discrimina entre modalidades de crédito e setores econômicos, respondendo à evolução do crédito como um todo, e sem capacidade de se concentrar em modalidades específicas de crédito que eventualmente estejam originando desequilíbrios no mercado de crédito, tal como o mercado de crédito imobiliário nos Estados Unidos no período que antecedeu a crise de 2008. Essa falta de foco do ACCP nos leva a conjecturar sobre a possibilidade de introduzir instrumentos de requerimento de capital direcionados a segmentos específicos de crédito.

Este artigo procura avaliar os efeitos da introdução hipotética de um Adicional Setorial Contracíclico de Capital (ASCCP) como instrumento macroprudencial no Brasil. Enquanto o ACCP associa o requerimento contracíclico de capital ao volume total dos ativos ponderados pelo risco da instituição financeira, os adicionais setoriais teriam como base apenas os ativos ponderados por risco das respectivas modalidades de crédito, tais como empréstimos imobiliários, financiamentos à pessoa jurídica ou crédito ao consumidor.

Para avaliar os efeitos da introdução do Adicional Setorial no Brasil, foi desenvolvido e estimado um modelo dinâmico de equilíbrio geral capaz de representar o crédito bancário brasileiro e as principais modalidades de crédito – direcionado habitacional, crédito livre para pessoa física e jurídica, crédito direcionado do BNDES. O modelo também permite representar requerimentos de capital, e ambos os tipos de Adicionais Contracíclicos – o amplo e os setoriais para as modalidades de crédito direcionado habitacional e livres PF e PJ. Através de simulações comparando cenários alternativos de arcabouços macroprudenciais envolvendo o adicional amplo e/ou os adicionais setoriais, conclui-se que a introdução do novo instrumento permitiria melhor estabilização macroeconômica e do setor bancário. No entanto, a introdução de instrumentos adicionais exigiria maior esforço de coordenação dos instrumentos macroprudenciais, aumentando a complexidade operacional e de comunicação da política macroprudencial.

Sectoral Countercyclical Buffers in a DSGE Model with a Banking Sector

Marcos R. Castro *

Abstract

We develop and estimate a closed economy DSGE model with banking sector to assess the impact of introducing sectoral countercyclical capital buffers as a macroprudential tool. The model is developed to represent Brazilian bank credit markets. It features three types of bank credit — housing, consumer and commercial — as well as loans provided by a development bank. Loans are long-term, and government regulates housing loans, influencing both interest rates and loan supply. Banks are subject to bank capital requirement, and both broad (CCyB) and sectoral (SCCyB) countercyclical buffers can be introduced by macroprudential authorities. We simulate alternative policies using SCCyBs and CCyB with implementable nonlinear rules using broad and sectoral credit gaps as indicators, and compared the resulting performances. We conclude that, compared with CCyB alone, SCCyBs provide a more flexible set of instruments that allows achieving better macroeconomic stabilization in terms of variances of credit, total capital requirement and capital adequacy ratio. However, the marginal benefit of those SCCyB policies relative to the CCyB-only policy is lower than the improvements obtained by this latter policy compared with the reference scenario with no buffer. Also, SCCyB policies imply more frequent intervention, suggesting that in practice introducing these additional instruments may require more complex implementation procedures.

Keywords: DSGE models, Bayesian estimation, financial regulation, monetary policy, macroprudential policy

JEL classification: E4, E5, E6.

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*Research Department, Central Bank of Brazil,marcos.castro@bcbr.gov.br

1 Introduction

This paper introduces sectoral countercyclical buffers (SCCyBs) in a DSGE model with financial frictions and a banking sector to assess the extent to which these new instruments might help stabilizing the economy and enhancing resilience of the banking system. We start from a baseline scenario with no macroprudential policy and compare it to a benchmark scenario with only the broad countercyclical buffer (CCyB) and also to a few alternative macroprudential scenarios with rules involving both CCyBs and/or SCCyB. In order to produce more realistic macroprudential policy responses, we introduce nonlinear bounded macroprudential rules instead of simple linear rules. The model is developed to suit Brazilian credit market and banking sector, and it has been estimated with Bayesian methods using Brazilian data.

The model borrows from previous work by Carvalho et al. (2014) and Carvalho and Castro (2015a). It is a closed economy with a private banking sector that provides consumer and housing loans to households and commercial loans to firms. A separate development bank provides subsidized loans to entrepreneurs. These four credit categories represent the major bank loan types in Brazil, and are all simultaneously introduced to reproduce the effective leverage of agents in the economy. All loans are long term, which helps introducing nominal rigidities in aggregate loan interest rates. The representative bank is subject to fixed capital requirement and possibly countercyclical requirements in the form of broad (CCyB) and sectoral (SCCyB) buffers associated to consumer, housing and commercial loans. The model is a fully fledged DSGE model with patient and impatient households, entrepreneurs, retailers, intermediate goods producers, housing goods producers, banks and a government responsible for monetary, macroprudential, fiscal and subsidized loans policies.

In order to mimic the behavior of real countercyclical capital requirements, we resort to nonlinear discrete bounded macroprudential rules. For instance, the CCyB may respond with discrete 0.5pp increments for each 2pp increase in credit-to-GDP gap, up to a 2.5pp upper threshold. And, of course, it must always be non-negative. We introduce these nonlinear bounded rules to avoid some inconvenient results from simple linear rules, such as negative buffers (that might unrealistically reduce bank resilience in downturns) and extreme higher values (that would overestimate the strength of the instrument). Also, the discrete behavior allows evaluating the frequency of macroprudential intervention (a linear continuous rule implies changes every period) and assessing how additional instruments may add complexity to macroprudential policies. Of course, this approach has caveats. As the model is implemented as a first-order approximation, each nonlinear rule must be introduced in the model as an

exogenous autoregressive process, driving the macroprudential instruments with unexpected shocks from the point of view of the agents represented in the model. The size of the shocks that implement the rule are computed outside the model first-order approximation. As a result, agents can forecast macroprudential policy up to anticipated shocks informed by macroprudential authority. Hence, there are no rational expectations for macroprudential policy rule, that is, agents do not learn and anticipate macroprudential rules. Also, we are forced to resort to numerical simulations of alternative policies, which in practice makes searching for optimal rules very difficult.

We introduce four alternative macroprudential policy rules to be compared. The first (1) is a simple CCyB policy, bounded between 0 and 2.5pp, with discrete 0.5pp increments for each 2pp increase in total credit gap, starting from zero gap up to 10pp gap. As credit gap decreases from 10pp to zero, the CCyB decreases in a similar fashion down to zero. Each capital requirement change is announced immediately, but will take place 4 quarters ahead. We also allow for a sudden immediate release of the buffer in case of economic crisis, whenever GDP growth is 2 standard deviations below average. The second rule (2) features no CCyB and three independent SCCyB buffers (housing, consumer and commercial loans), each one operating in a similar way as the CCyB in policy rule 1, but targeting only the respective sectoral credit gap. The upper bound of those sectoral buffers is rescaled such that, when they are all simultaneously active, they result in the same bank capital requirement as the broad CCyB instrument. In the third policy (3), CCyB and SCCyB may be used, but only the SCCyB of the sector with the largest gap is activated and calibrated according to the respective sectoral credit gap. The CCyB is activated targeting the credit gap of aggregate remaining credit. Finally, the fourth rule (4) seeks to reproduce the same total capital requirement as rule 1, but uses SCCyBs instead of CCyB and seeks to distribute total capital requirement along sectors according to the respective contributions to total credit gap. The idea here is to evaluate how sectoral instruments may improve results while building exactly the same bank capital chest as policy 1 would with the CCyB alone.

We run numeric simulations with each alternative scenario and compare the results. Overall, the introduction of SCCyBs as additional macroprudential policy instruments allows achieving better macroeconomic stabilization in terms of variances of credit, total capital requirement and capital adequacy ratio. For instance, sectoral buffers achieved lower variances of sectoral credit gaps than CCyB alone (this might be helpful in practice if we need to target a specific sector). This result is hardly surprising, as in theory introducing more instruments allows better economic stabilization. But, in the simulated exercises, the marginal benefit of SCCyB policies relative to CCyB-only policy is smaller than the improvement obtained by this latter policy relative to the baseline scenario with no buffer. And, in some aspects, SCCyB policies may perform worse than CCyB-only, as in the case of stabilizing total credit gap.

Also, simulations show that the introduction of additional instruments require more frequent macroprudential intervention. Scenarios with SCCyB implied roughly twice as much periods of active buffer than the CCyB-only reference scenario. They also implied more frequent policy intervention, requiring as much as 70% more changes in total countercyclical capital requirement. And as there are more instruments, there are even more individual instrument changes in SCCyB policies than in the CCyB-only rule. Of course, we can expect that more granular sectoral policies would add even more complexity to the operational implementation of SCCyB.

We also compare the capital adequacy ratios banks present right before economic crises, signaling how resilient the banking system is to withstand the downturn. We find that all alternative rules simulated build roughly the same average bank capital chest before crises, with higher capital adequacy ratios than in the baseline scenario with no buffer. But as SCCyB policies tend to keep the buffer activated more frequently, they provide macroprudential relief in more crisis episodes.

In sum, our exercises suggest that introducing sectoral countercyclical capital buffers to central banks' macroprudential toolbox may help them stabilizing credit markets and enhance bank resilience, by targeting more directly sectoral imbalances. However, this additional instruments may add complexity to macroprudential policy and central bank communication, and as the marginal gains of introducing additional sectoral buffers gets smaller, it should be advisable making parsimonious use of sectoral instruments.

This paper is related to a growing literature that introduces credit, banks and macroprudential instruments in DSGE models. Financial frictions in our model are a variation of those in Bernanke et al. (1999). The banking system is similar to that in Gerali et al. (2010). The paper addresses the implementation of countercyclical macroprudential policies, like many other papers in the literature such as Agénor and Pereira da Silva (2017), Angelini et al. (2014), Alpanda et al. (2018), Benes and Kumhof (2015), Gertler and Karadi (2011), Mendicino et al. (2018), to name a few. In Brazil, Ferreira and Nakane (2015), Carvalho and Castro (2015a) and Areosa and Coelho (2013) introduce countercyclical macroprudential rules using capital requirement or reserve requirement. Few papers deal with sectoral macroprudential instruments. Carvalho et al. (2014) already presented sectoral instruments in the form of time-varying risk weights, and their macroprudential use was explored in Carvalho and Castro (2015b). Mendicino et al. (2018) found optimal constant sectoral risk weights but did not introduce countercyclical sectoral rules. Hodbod et al. (2018) introduce sectoral countercyclical risk weight rules in a DSGE model and suggest that it is a better alternative to the IRB approach to attenuate financial cycles, but they do not compare their suggested rule with Basel III countercyclical capital requirement. This paper, on the other hand, is mostly concerned

in comparing SCCyB to CCyB, and does not delve into other issues such as interaction with monetary policy.

The outline of the rest of the paper is the following. Section 2 presents a detailed description of the model. Section 3 discusses the procedure to calibrate and estimate the parameters. Section 4 describes a few important properties of the model. Section 5 discusses the policy exercises that compare alternative macroprudential rules involving sectoral and broad countercyclical buffers. Finally, section 6 sums up with concluding remarks.

2 Model

The model is a closed economy DSGE model similar to Carvalho et al. (2014) and it has been designed to allow for macroprudential policy exercises with capital requirement and both broad (CCyB) and sectoral (SCCyB) countercyclical capital buffers. As the model is intended to represent Brazilian banking and credit markets, it also features regulated subsidized loans with earmarked funding that accounts for a significant share of total bank credit. Credit is comprised of long-term loans, as opposed to the usual one-period loans of DSGE literature, in order to introduce nominal interest rates rigidities in a more realistic setup than usual Calvo rigidity.

The key agents in the model are households, firms, banks and the government. There are two types of households. Patient households receive dividends from firms and banks they own and spend on housing and consumption. They hold bank deposits and government bonds, which they use to smooth consumption over time. Impatient households supply labor to firms and use their wage income to consume and purchase houses. They also get consumer and housing loans from banks. Housing loans are used to purchase new housing stock that stands as collateral for those loans. Consumer loans are uncollateralized and can be used to smooth consumption over time. Borrowers refrain from defaulting loans for reputational reasons, but may be forced to default if they suffer adverse income shocks.

There are a few types of firms. Intermediate goods producers and retailers are as usual in the literature. The competitive intermediate goods producers rent capital and labor to produce goods to be sold to monopolistically competitive retailers. Entrepreneurs invest and accumulate productive capital and finance their holdings with commercial and subsidized collateralized bank loans. Housing stock producers invest to increase total housing stock.

Banks raise funds from deposits and equity to finance consumer, housing and commercial loans. Loans are long term with fixed interest rates, which introduce nominal rigidity in the decision process. Banks are subject to prudential regulation, namely total capital requirement and sectoral and broad countercyclical capital buffers. Housing loans are heavily regulated by

the government, with regulated interest rates and earmarked subsidized funding. There is also a development bank completely funded by the government that provides subsidized loans to entrepreneurs.

In the following, we present the main features and equations of the model. Complete detailed description of the model can be found in a separate technical appendix.

2.1 Households

There are two types of households. Patient households (or "savers") have higher subjective discount factors than impatient households (or "borrowers"). Hence, in equilibrium, patient households hoard most assets of the economy, whereas impatient households amass bank debt. Both types of households consume and accumulate housing stock. Savers own all firms and banks, as well as all bank deposits and government bonds, and derive their income from the respective dividends and interest payments. Impatient households supply labor services to firms, and use their wage income to consume, purchase houses and roll over their debt.

2.1.1 Patient households (Savers)

Savers choose their optimal allocation of consumption $C_{S,t}$, housing stock $H_{S,t}$ and financial investments in the form of investment fund shares $D_{S,t}^F$ in order to maximize the utility function

$$E_0 \left\{ \sum_{t \geq 0} \beta_S^t \left[\frac{1}{1 - \eta_{\chi S}} (\mathcal{X}_{S,t})^{1 - \eta_{\chi S}} \varepsilon_t^{\chi, S} \right] \right\}, \quad (1)$$

where

$$\mathcal{X}_{S,t} = \left[\begin{array}{l} \left(1 - \varepsilon_t^{H,S} \omega_{H,S} \right)^{\frac{1}{\eta_{\chi}}} \left(\frac{C_{S,t}}{\epsilon_{L,t} \epsilon_{A,t}} - \bar{h}_{S,C} \frac{C_{S,t-1}}{\epsilon_{L,t-1} \epsilon_{A,t-1}} \right)^{\frac{\eta_{\chi}-1}{\eta_{\chi}}} \\ + \left(\varepsilon_t^{H,S} \omega_{H,S} \right)^{\frac{1}{\eta_{\chi}}} \left(\frac{H_{S,t}}{\epsilon_{L,t} \epsilon_{A,t}} - \bar{h}_{S,H} \frac{H_{S,t-1}}{\epsilon_{L,t-1} \epsilon_{A,t-1}} \right)^{\frac{\eta_{\chi}-1}{\eta_{\chi}}} \end{array} \right]^{\frac{\eta_{\chi}}{\eta_{\chi}-1}}, \quad (2)$$

and $\varepsilon_t^{\chi, S}$ and $\varepsilon_t^{H,S}$ are preference shocks, $\omega_{H,S}$ is a scaling parameter, $\bar{h}_{S,C}$ and $\bar{h}_{S,H}$ represent habit persistence for consumption and housing, and $\eta_{\chi S}$ and η_{χ} are elasticity parameters. Variables $\epsilon_{L,t}$ and $\epsilon_{A,t}$ stand for population and labor productivity stochastic trends, respectively. They are introduced in the utility function to make the resulting first-order conditions compatible with a balanced growth path.

The investment fund holds a fixed-income portfolio which includes assets such as government bonds and bank deposits, and yields a one-period return rate R_t^F .

The patient household's budget constraint is given by

$$\begin{aligned} (1 + \tau_{C,t}) P_{C,t} C_{S,t} + P_{H,t} (H_{S,t} - (1 - \delta_H) H_{S,t-1}) + D_{S,t}^F \\ = R_t^F D_{S,t-1}^F + T_{S,t}^{Nom} + \Xi_{S,t}^{Nom} + T_{\Gamma,S,t}^{Nom}, \end{aligned} \quad (3)$$

where $P_{C,t}$ is the price of consumption goods, $P_{H,t}$ is the price of housing stock, and $\tau_{C,t}$ is the tax rate on consumption. House ownership implies depreciation proportional to the value of housing stock with parameter δ_H .

Savers also receive lump sum transfers $T_{S,t}^{Nom}$ from the government, in addition to net-of-tax profits $\Xi_{S,t}^{Nom}$ from firms, entrepreneurs, and banks. $T_{\Gamma,S,t}^{Nom}$ are adjustment costs from capital utilization, which we assume are distributed as lump-sum transfers to savers. One-period return on investment fund quotas $D_{S,t-1}^F$ in period t is R_t^F .

Investment fund We introduce a separate investment fund that takes financial investment decision on behalf of the patient households in order to separate the saving decisions by households from arbitrage conditions among distinct financial assets.

The investment fund portfolio D_t^F comprises one-period return assets such as government bonds B_t and bank time deposits D_t^T , and long-term bonds with fixed or floating interest rates. The one-period government bond B_t is remunerated at the short-term base interest rate R_t , such that the household receives $B_t R_t$ in period $t + 1$. Analogously, time deposits D_t^T present one-period return R_t^T , such that the household receives $D_t^T R_t^T$ in period $t + 1$. The other bonds are long-term with geometrically decaying amortization schedules analogous to Woodford (2001) and yield fixed and/or floating interest rates. The net supply of these bonds will be zero, and they are added to the model only to introduce new long-term interest rates that may be used to index some types of loans presented further in the model. As the workings of these long-term bonds are quite similar to long-term loans, we omit the details here as thorough description is presented in the borrowers' section. Also, the complete derivation of investment fund first-order conditions can be found in the technical appendix.

The total value of the investment fund and total return are given by

$$D_t^F = B_t + D_t^T, \quad (4)$$

and

$$R_t^F D_{t-1}^F = R_{t-1} B_{t-1} + R_{t-1}^T D_{t-1}^T. \quad (5)$$

2.1.2 Impatient households (Borrowers)

The borrowers' group consists of a continuum $[0, 1]$ of impatient households who can obtain loans by offering future wage income and houses as collateral. Household i chooses his optimal allocation $\{C_{B,i,t}, H_{B,i,t}, N_{B,i,t}\}$ of consumption, housing and labor supply to maximize the utility function

$$E_0 \left\{ \sum_{t \geq 0} \beta_B^t \left[\frac{1}{1-\eta_X B} (\mathcal{X}_{B,i,t})^{1-\eta_X S} \varepsilon_t^{X,B} \right] \right\},$$

where

$$\mathcal{X}_{B,i,t} = \left[\begin{aligned} & \left(1 - \varepsilon_t^{H,B} \omega_{H,B} \right)^{\frac{1}{\eta_X}} \left(\frac{C_{B,i,t}}{\varepsilon_{L,t} \varepsilon_{A,t}} - \bar{h}_{B,C} \frac{C_{B,i,t-1}}{\varepsilon_{L,t-1} \varepsilon_{A,t-1}} \right)^{\frac{\eta_X-1}{\eta_X}} \\ & + \left(\varepsilon_t^{H,B} \omega_{H,B} \right)^{\frac{1}{\eta_X}} \left(\frac{H_{B,i,t}}{\varepsilon_{L,t} \varepsilon_{A,t}} - \bar{h}_{B,H} \frac{H_{B,i,t-1}}{\varepsilon_{L,t-1} \varepsilon_{A,t-1}} \right)^{\frac{\eta_X-1}{\eta_X}} \end{aligned} \right]^{\frac{\eta_X}{\eta_X-1}}. \quad (6)$$

There is external habit formation in consumption and housing stock, represented in the utility function by parameters $\bar{h}_{B,C}$ and $\bar{h}_{B,H}$, respectively. Parameters $\psi_{B,N}$ and $\omega_{H,B}$ are scaling parameters. The preference shocks $\varepsilon_t^{X,B}$, $\varepsilon_t^{H,B}$ and ε_t^L follow AR(1) processes.

Loans with long-term amortization schedules and mixed interest rates There are two distinct types of credit — housing loans and consumer loans. In order to represent these loans and also loans provided to entrepreneurs, we introduce a generic long-term loan with mixed fixed and floating interest rates. The geometrically decaying amortization scheme is analogous to the exponentially decaying coupon bonds presented in Woodford (2001). Mixed fixed and floating interest rates are introduced to allow for a general representation of loan yields, encompassing fixed and floating interest rates, as well as other alternative payment schemes, such as fixed real long term interest rates.

Let's start with the representation of a generic type X loan. In period t , household i borrows an amount $NL_{B,i,t}^X$ of new loans to be redeemed in the future with geometrically decaying amortization such that principal decays at a constant rate $\rho_{L,X} < 1$. In period $t+k$, $k > 0$, a fraction $(1 - \rho_{L,X}) \rho_{L,X}^{k-1}$ of the original principal $NL_{B,i,t}^X$ will be redeemed, such that the sum of all amortization payments over time equals the total value originally borrowed:

$$\sum_{k=1}^{\infty} (1 - \rho_{L,X}) \rho_{L,X}^{k-1} NL_{B,i,t}^X = NL_{B,i,t}^X.$$

Each new loan adds to the previously existing credit stock, such that total loan stock or

principal $L_{B,i,t}^X$ is given by

$$\begin{aligned} L_{B,i,t}^X &= \sum_{k=0}^{\infty} \rho_{L,X}^k NL_{B,i,t-k}^X \\ &= NL_{B,i,t}^X + \rho_{L,X} L_{B,i,t-1}^X. \end{aligned} \quad (7)$$

Analogously, total amortization $A_{B,i,t}^{L,X}$ of principal to be paid in period t is given by

$$\begin{aligned} A_{B,i,t}^{L,X} &= (1 - \rho_{L,X}) NL_{B,i,t-1}^X + \rho_{L,X} A_{B,i,t-1}^{L,X} \\ &= (1 - \rho_{L,X}) L_{B,i,t-1}^X. \end{aligned} \quad (8)$$

Notice that it is implicit in the formulation above that there is no default on loan principal, although there can be default on debt service, defined below.

Loan interest rates are a composition of fixed and floating interest rates. They will accrue upon current accumulated loan stock (or principal) and will be completely paid to the lender every period, such that interest does not cumulate over time. The fixed multiplicative interest rate $R_{B,i,t}^{L,X, \text{fixed}}$ negotiated with the lending branch in period t for new loans $NL_{B,i,t}^X$ will accrue over remaining loan stock $\rho_{L,X}^{k-1} NL_{B,i,t}^X$ in period $t+k$, $k > 0$, in addition to multiplicative floating interest rate $R_{B,i,t+k}^{L,X, \text{float}}$. As a result, total interest payments $J_{B,i,t}^{L,X}$ due in period t sum to

$$J_{B,i,t}^{L,X} = \sum_{k=1}^{\infty} \left(R_{B,i,t}^{L,X, \text{float}} R_{B,i,t-k}^{L,X, \text{fixed}} - 1 \right) \rho_{L,X}^{k-1} NL_{B,i,t-k}^X.$$

For convenience, we introduce a couple of auxiliary variables. Let $B_{B,i,t}^{L,X} = J_{B,i,t}^{L,X} + L_{B,i,t-1}^{L,X}$ be the total outstanding debt owed at the beginning of period t . From the previous relations it is possible to write $B_{B,i,t}^{L,X}$ in a recursive way

$$B_{B,i,t}^{L,X} = R_{B,i,t}^{L,X, \text{float}} R_{B,i,t-1}^{L,X, \text{fixed}} NL_{B,i,t-1}^X + \frac{R_{B,i,t}^{L, \text{float}}}{R_{B,i,t-1}^{L, \text{float}}} \rho_{L,X} B_{B,i,t-1}^{L,X}. \quad (9)$$

Let $S_{B,i,t}^{L,X} = J_{B,i,t}^{L,X} + A_{B,i,t}^{L,X}$ be the debt service to be paid in period t . It is straightforward to write it as a function of $B_{B,i,t}^{L,X}$:

$$S_{B,i,t}^{L,X} = B_{B,i,t}^{L,X} - L_{B,i,t}^X + NL_{B,i,t}^X. \quad (10)$$

Since principal $L_{B,i,t}^X$ is not subject to default, but debt service $S_{B,i,t}^{L,X}$ is, the recursive representations above are not affected by default. If principal is also subject to default, the

equations get slightly different, as presented below:

$$\begin{aligned} L_{B,i,t}^X &= \sum_{k=0}^{\infty} \rho_{L,X}^k NL_{B,i,t-k}^X \prod_{n=0}^{k-1} \left(1 - F_B(\bar{\omega}_{B,i,t-n}^X)\right) \\ &= NL_{B,i,t}^X + \left(1 - F_B(\bar{\omega}_{B,i,t}^X)\right) \rho_{L,X} L_{B,i,t-1}^X, \end{aligned} \quad (11)$$

$$A_{B,i,t}^{L,X} = (1 - \rho_{L,X}) L_{B,i,t-1}^X, \quad (12)$$

$$\begin{aligned} B_{B,i,t}^{L,X} &= R_{B,i,t}^{L,X,float} R_{B,i,t-1}^{L,X,fixed} NL_{B,i,t-1}^X \\ &\quad + \frac{R_{B,i,t}^{L,X,float}}{R_{B,i,t-1}^{L,X,float}} \rho_{L,X} \left(1 - F_B(\bar{\omega}_{B,i,t-1}^X)\right) B_{B,i,t-1}^{L,X}, \end{aligned} \quad (13)$$

$$J_{B,i,t}^{L,X} = B_{B,i,t}^{L,X} - L_{B,i,t-1}^X, \quad (14)$$

$$\left(1 - F_B(\bar{\omega}_{B,i,t}^X)\right) S_{B,i,t}^{L,X} = \left(1 - F_B(\bar{\omega}_{B,i,t}^X)\right) B_{B,i,t}^{L,X} - L_{B,i,t}^X + NL_{B,i,t}^X, \quad (15)$$

where $F_B(\bar{\omega}_{B,i,t}^X)$ is the probability of default in period t , to be detailed in further sections.

In the pure fixed interest rate case ($R_{B,i,t}^{L,X,float} = 1$) the conditions above can also be represented as

$$\begin{aligned} B_{B,i,t}^{L,X} &= R_{B,i,t-1}^{L,X,fixed} NL_{B,i,t-1}^X + \rho_{L,X} \left(1 - F_B(\bar{\omega}_{B,i,t-1}^X)\right) B_{B,i,t-1}^{L,X}, \\ S_{B,i,t}^{L,X} &= \left(R_{B,i,t-1}^{L,X,fixed} - \rho_{L,X}\right) NL_{B,i,t-1}^X + \rho_{L,X} \left(1 - F_B(\bar{\omega}_{B,i,t-1}^X)\right) S_{B,i,t-1}^{L,X}. \end{aligned}$$

In the pure floating interest rate case ($R_{B,i,t-1}^{L,X,fixed} = 1$) the expressions above simplify further to

$$\begin{aligned} B_{B,i,t}^{L,X} &= R_{B,i,t}^{L,X,float} L_{B,i,t-1}^X, \\ S_{B,i,t}^{L,X} &= \left(R_{B,i,t}^{L,X,float} - \rho_{L,X}\right) L_{B,i,t-1}^X. \end{aligned}$$

Values of $L_{B,i,t}^X$, $A_{B,i,t}^{L,X}$, $B_{B,i,t}^{L,X}$, $J_{B,i,t}^{L,X}$ and $S_{B,i,t}^{L,X}$ must be interpreted as credit amounts before default. For instance, $L_{B,i,t}^X$ is loan principal to be carried over to the next period, $A_{B,i,t}^{L,X}$ is credit amortization due in period t before default, $B_{B,i,t}^{L,X}$ is outstanding debt (interest plus principal) due in period t before default, and so forth.

In the following, we introduce long-term consumer loans with amortization parameter $\rho_{L,C}$, no default of principal (only services) and fixed interest rate $R_t^{L,C}$, as most consumer loans in Brazil present fixed rates. Hence, the respective equations for consumer loans are

$$L_{B,i,t}^C = NL_{B,i,t}^C + \rho_{L,C} L_{B,i,t-1}^C, \quad (16)$$

$$B_{B,i,t}^{L,C} = R_{B,i,t-1}^{L,C} NL_{B,i,t-1}^{L,C} + \rho_{L,C} B_{B,i,t-1}^{L,C}, \quad (17)$$

$$S_{B,i,t}^{L,C} = \left(R_{B,i,t-1}^{L,C} - \rho_{L,C} \right) NL_{B,i,t-1}^{L,C} + \rho_{L,C} S_{B,i,t-1}^{L,C}. \quad (18)$$

Housing credit is represented by long-term loans with amortization rate $\rho_{L,H}$ and principal subject to default. In order to allow for policy exercises with alternative interest rate setups, we keep the general formulation for housing loans. Hence the equations that describe the behavior of housing loans are the following

$$L_{B,i,t}^H = NL_{B,i,t}^H + \left(1 - F\left(\bar{\omega}_{B,i,t}^H\right) \right) \rho_{L,H} L_{B,i,t-1}^H, \quad (19)$$

$$B_{B,i,t}^{L,H} = R_{B,i,t}^{L,H,float} R_{B,i,t-1}^{L,H,fixed} NL_{B,i,t-1}^H + \frac{R_{B,i,t}^{L,H,float}}{R_{B,i,t-1}^{L,H,float}} \rho_{L,H} \left(1 - F\left(\bar{\omega}_{B,i,t-1}^H\right) \right) B_{B,i,t-1}^{L,H}, \quad (20)$$

$$\left(1 - F\left(\bar{\omega}_{B,i,t}^H\right) \right) S_{B,i,t}^{L,H} - NL_{B,i,t}^H = \left(1 - F\left(\bar{\omega}_{B,i,t}^H\right) \right) B_{B,i,t}^{L,H} - L_{B,i,t}^H. \quad (21)$$

where $L_{B,i,t}^H$, $NL_{B,i,t}^H$, $B_{B,i,t}^{L,H}$ and $S_{B,i,t}^{L,H}$ are housing loans principal, new loans, outstanding debt and debt service, $R_{B,i,t}^{L,H,fixed}$ and $R_{B,i,t}^{L,H,float}$ are fixed and floating interest rates and $F_B\left(\bar{\omega}_{B,i,t}^H\right)$ is housing loans default probability, to be explained in the next subsection.

Credit Default In the model, borrowers' labor income is subject to idiosyncratic shocks $\omega_{B,i,t}$, a shortcut for idiosyncratic income shocks that do not affect households' aggregate income but affect the borrowers' ability to repay their debt. These shocks are independent and identically distributed with mean one, with differentiable cumulative distribution function $F_B(\omega_{B,i,t})$ on the domain $[0, \infty)$ such that the expected value of $\omega_{B,i,t}$ is 1.

After realization of shock $\omega_{B,i,t}$, borrower i 's net-of-tax nominal labor income is

$$\omega_{B,i,t} (1 - \tau_{W,t}) N_{B,i,t} W_t,$$

where W_t is the wage negotiated between firms and unions and $\tau_{W,t}$ is the labor income tax rate. At period t , impatient household i must honor debt service $S_{B,i,t}^{L,C} + S_{B,i,t}^{L,H}$ of consumer and housing loans cumulated in the previous period. Housing loans have housing stock as collateral, whereas consumer loans have no tangible collateral. However, there are reputational costs involved in defaulting any kind of loan. In more detailed models (for instance, Nikolov (2012)), reputational costs involve losing access to credit markets, permanently or temporarily. But implementing this in a model usually involves introducing heterogeneity among borrowers, and we'd rather avoid this by resorting to a simple but plausible shortcut. We assume that households will be willing

to pay their debts to avoid intangible reputational costs as long as their residual income net of debt service is higher than a given subsistence threshold \bar{C}_t^C . If the idiosyncratic shock $\omega_{B,i,t}$ is adverse enough to make this threshold trespassed, the borrower will default on credit service. As housing loans are collateralized and consumer loans are not, the borrower will default on consumer loans first, and will default on housing loans only after complete default on consumer loans.

Hence, at period t , the impatient household chooses to default on consumer loans if $\omega_{B,i,t} < \bar{\omega}_{B,i,t}^C$, where

$$\bar{\omega}_{B,i,t}^C (1 - \tau_{W,t}) N_{B,i,t} W_t = S_{B,i,t}^{L,H} + S_{B,i,t}^{L,C} + \bar{C}_t^C. \quad (22)$$

For simplicity, we impose that consumer loans default affects only credit service (interest plus amortization) and the remaining consumer debt stock is rolled over to the next period. If we allowed default on principal, an incentive to consumer loans default would show up in first-order conditions, as those loans are not collateralized and borrowers would not be penalized with collateral arrest. The main penalty delinquent borrowers face in real life is reputational – losing access to credit markets – and we want to avoid introducing the complex microfounded reputational costs in the model. By introducing default only in credit service and properly calibrating loss given default, it is possible to make loan interest rates be affected by default rates.

A similar behavior applies to housing loans default. But, in this case, the subsistence threshold is $\bar{C}_t^H \leq \bar{C}_t^C$, because borrowers incur the nuisance of foreclosure in addition to the reputational costs. Hence, housing loans default will happen if $\omega_{B,i,t} < \bar{\omega}_{B,i,t}^H$, where

$$\bar{\omega}_{B,i,t}^H (1 - \tau_{W,t}) N_{B,i,t} W_t = S_{B,i,t}^{L,H} + \bar{C}_t^H. \quad (23)$$

In this case, the borrower is forced to default on housing loans and has its real estate collateral arrested by the bank.

The probability of default on consumer loans is $F_B(\bar{\omega}_{B,i,t}^C)$. In case of default, banks incur a proportional loss $\mu_{B,C}$ of the recovered amount. The probability of housing loans default is $F_B(\bar{\omega}_{B,i,t}^H)$ and in case of default banks may arrest enough housing collateral to redeem total outstanding housing debt $B_{B,i,t}^{L,H}$, but incur proportional costs $\mu_{B,H}$ on the recovered amount.

For simplicity, we define the subsistence threshold \bar{C}_t^C and \bar{C}_t^H as fractions $\gamma_{t-1}^{B,C}$ and $\gamma_{t-1}^{B,H}$ of disposable income

$$\bar{C}_t^C = \gamma_{t-1}^{B,C} (1 - \tau_{W,t}) N_{B,i,t} W_t, \quad (24)$$

$$\bar{C}_t^H = \gamma_{t-1}^{B,H} (1 - \tau_{W,t}) N_{B,i,t} W_t, \quad (25)$$

where $\gamma_t^{B,C}$ and $\gamma_t^{B,H}$ are AR(1) processes.

Now we may express the probability of consumer and housing loans default as

$$F_B(\bar{\omega}_{B,i,t}^C) = F_B \left(\frac{S_{B,i,t}^{L,H} + S_{B,i,t}^{L,C}}{(1 - \tau_{W,t}) N_{B,i,t} W_t} + \gamma_{t-1}^{B,C} \right),$$

$$F_B(\bar{\omega}_{B,i,t}^H) = F_B \left(\frac{S_{B,i,t}^{L,H}}{(1 - \tau_{W,t}) N_{B,i,t} W_t} + \gamma_{t-1}^{B,H} \right),$$

which are increasing functions of debt service-to-income ratio. In this formulation, higher amount of housing credit increases the probability of default of consumer loans, but the opposite is not true, because in practice housing loans are senior to consumer loans. That means that the collateralized housing loans crowd out uncollateralized consumer credit.

There is a competitive bank lending branch that gets its funding from the bank conglomerate and provides loans to impatient households. These funds must be repaid at the same amortization schedule of the respective loans, with exponentially decaying rate $\rho_{L,C}$ but fixed interest rate $R_{B,t}^{F,C}$. Therefore, principal $L_{B,i,t}^{F,C}$, outstanding debt $B_{B,i,t}^{F,C}$ and debt service $S_{B,i,t}^{F,C}$ due to the bank conglomerate as a result of loans to households i are given by

$$L_{B,i,t}^{F,C} = NL_{B,i,t}^C + \rho_{L,C} L_{B,i,t-1}^{F,C} = L_{B,i,t}^C, \quad (26)$$

$$B_{B,i,t}^{F,C} = R_{B,t-1}^{F,C} NL_{B,i,t-1}^C + \rho_{L,C} B_{B,i,t-1}^{F,C}, \quad (27)$$

$$S_{B,i,t}^{F,C} = (R_{B,t-1}^{F,C} - \rho_{L,C}) NL_{B,i,t-1}^C + \rho_{L,C} S_{B,i,t-1}^{F,C}. \quad (28)$$

There is no default between the lending branch and the bank conglomerate (which owns the branch). Each period, the lending branch transfers the profits or losses of the lending activity to the bank conglomerate. The lending branch zero expected profit condition is given by

$$\beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} S_{B,i,t+1}^{F,C} \quad (29)$$

$$= \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \left[1 - \mu_{B,C} F_B(\bar{\omega}_{B,i,t+1}^C) \right] S_{B,i,t+1}^{L,C}.$$

An intuitive interpretation for this equation is that the lending branch chooses a lending rate $R_{B,i,t}^{L,C}$ for new loans $NL_{B,i,t}^C$ such that the expected return on total loans net of default losses equals total funding costs. Also, the higher the amount of new loans $NL_{B,i,t}^C$ provided to the borrower, the higher the interest rate $R_{B,i,t}^{L,C}$ charged on these loans to compensate for higher

default costs.

Housing loans and housing stock Impatient households can use housing loans only to purchase additional housing stock, up to a loan-to-value restriction. In opposition to most of the literature on household credit constraints, in this model households cannot pledge their homes as collateral for loans to smooth consumption over time. We intend to replicate the housing credit market in Brazil, where banks usually do not accept housing as collateral for general use credit. Housing collateral is usually accepted only for housing loans, i.e., loans to be used specifically to purchase homes. These loans are seldom refinanced, and borrowers usually keep them until they are completely redeemed.

In order to buy this additional housing stock $NH_{B,i,t}$ at market price P_t^H , household i can borrow up to a fraction $\gamma_t^{LTV,H}$ of total purchase value $P_t^H NH_{B,i,t}$. The household has no influence on this loan-to-value constraint, which banks impose uniformly on all households. Therefore, the LTV credit constraint is

$$0 \leq NL_{B,i,t}^H \leq \gamma_t^{LTV} P_t^H NH_{B,i,t}, \quad (30)$$

where $NL_{B,i,t}^H$ represent new housing loans to be added to existing credit stock.

In each period t , impatient households who default on housing loans have their depreciated housing stock arrested and auctioned at market prices by the bank to quit their outstanding housing debt. The bank uses part of the proceeds to quit the outstanding debt $B_{B,i,t}^{L,C}$ and the remaining receipts accrue back to the household. Non-defaulting households must sell a fixed fraction κ_H of its previously cumulated housing stock. This exogenous turnover accounts for people moving to other neighborhoods, upgrading to bigger homes or even dying and having their houses sold. It is introduced to disentangle housing credit growth from borrowers' housing stock growth in the steady-state calibration (otherwise, steady-state housing loans stock ought to be a fixed fraction of the steady-state housing stock of impatient households).

The idiosyncratic shock introduces heterogeneity of income, default, housing stock and housing loans among borrowers, leading to difficulties in aggregation. In order to circumvent this problem, we introduce an insurance contract among impatient households. This insurance contract comprises a contingent payment and an agreement to sell or buy housing stock, and is presented in detail in the technical appendix. As the contingent contract eliminates heterogeneity among households, they will all feature the same budget constraints and housing stocks:

$$\begin{aligned}
& (1 + \tau_{C,t}) P_{C,t} C_{B,i,t} + P_{H,t} N H_{B,i,t} \\
& = (1 - \tau_{W,t}) N_{B,i,t} W_t^N + T_{B,t}^{Nom} + \Xi_{B,t}^{Nom,LU} \\
& - S_{B,i,t}^{L,C} \left(1 - \mu_{B,C} F_B \left(\bar{\omega}_{B,i,t}^C \right) \right) + N L_{B,i,t}^C + L_{B,i,t}^H - B_{B,i,t}^{L,H} \\
& + \left[1 - \left(1 - F_B \left(\bar{\omega}_{B,i,t}^H \right) \right) (1 - \kappa_H) \right] P_{H,t} (1 - \delta_H) H_{B,i,t-1},
\end{aligned} \tag{31}$$

$$H_{B,i,t} = N H_{B,i,t}^H + \left(1 - F_B \left(\bar{\omega}_{B,i,t}^H \right) \right) (1 - \kappa_H) (1 - \delta_H) H_{B,i,t-1}, \tag{32}$$

where $T_{B,t}^{Nom}$ and $\Xi_{B,t}^{Nom,LU}$ are lump-sum transfers from the government and from labor unions. Equation (32) represent the dynamic evolution of impatient households' housing stock, given by new home purchases $N H_{B,i,t}^H$ in period t plus last period outstanding housing stock net of depreciation and sales originated from housing loan default and house turnover.

The Borrowers Program We may rewrite the budget constraint as the following

$$\begin{aligned}
& (1 + \tau_{C,t}) P_{C,t} C_{B,i,t} + P_{H,t} N H_{B,i,t} \\
& = (1 - \tau_{W,t}) N_{B,i,t} W_t^N + \kappa_H (1 - \delta_H) P_{H,t} H_{B,i,t-1} \\
& - B_{B,i,t}^{L,C} + L_{B,i,t}^C - B_{B,i,t}^{L,H} + L_{B,i,t}^H \\
& + T_{B,t}^{Nom} + \Xi_{B,i,t}^{Def} + \Xi_{B,t}^{Nom,LU},
\end{aligned}$$

where

$$\begin{aligned}
\Xi_{B,i,t}^{Def} & = S_{B,i,t}^{L,C} \mu_{B,C} F_B \left(\bar{\omega}_{B,i,t}^C \right) \\
& + F_B \left(\bar{\omega}_{B,i,t}^H \right) (1 - \kappa_H) (1 - \delta_H) P_{H,t} H_{B,i,t-1},
\end{aligned}$$

is the average nominal cash flow associated with default.

We assume that borrowers always consider beforehand that they will pay their loans in full next period, even though they might be eventually forced to default next period. Banks, on the other hand, take into account that possibility in their decision process. Hence, we introduced variable $\Xi_{B,t}^{Def}$ in the budget constraint to be treated as a lump-sum transfer to avoid introducing a non realistic incentive for risk taking, since, in the real economy, borrowers would also face adverse reputational and pecuniary costs that would discourage default. In the usual BGG financial accelerator setup (Bernanke et al. (1999)), the borrower is aware of the gains from defaulting, but he is also aware that his leverage will have an adverse impact on the interest rates charged by banks, which may surpass that initial gain. In this version, our borrower does

not take into account the possibility of default when deciding on consumption and loans, but banks are aware of the implications of default, and they will consider it in their decision process, to be detailed in the specific section.

Also, we introduce the possibility that the amount of housing loans is exogenously given, that is

$$L_{B,i,t}^H = L_{B,i,t}^{H,Ear}.$$

This is the case of earmarked housing loans, in which both the amount and the interest rate of housing credit is determined by the government. In this case, credit is rationed, and households obtain less housing loans than they would be willing to borrow at the prevailing interest rate. We introduce the possibility of credit rationing by resorting to an additional Lagrange multiplier $\varphi_{B,i,t}^{H,Ear}$ associated with the equation above. If $\varphi_{B,i,t}^{H,Ear} = 0$, the constraint is not binding, and vice versa if $\varphi_{B,i,t}^{H,Ear} \neq 0$.

The optimization problem for the representative borrower is therefore:

$$\max E_0 \left\{ \sum_{t \geq 0} \beta_B^t \left[\frac{1}{1-\eta_{\chi B}} (X_{B,t})^{1-\eta_{\chi S}} \mathcal{E}_t^{\chi, B} \right] \right\}$$

s.t.

$$\begin{aligned} & (1 + \tau_{C,t}) P_{C,t} C_{B,t} + P_{H,t} N H_{B,t} \\ & = (1 - \tau_{W,t}) N_{B,t} W_t^N + \kappa_H P_{H,t} (1 - \delta_H) H_{B,t-1} - B_{B,t}^{L,C} + L_{B,t}^C, \\ & \quad - B_{B,t}^{L,H} + L_{B,i,t}^H + T_{B,t}^{Nom} + \Xi_{B,t}^{Nom,LU} + \Xi_{B,t}^{Def} \end{aligned}$$

$$X_{B,t} = \left[\begin{array}{l} \left(1 - \mathcal{E}_t^{H,B} \omega_{H,B}\right)^{\frac{1}{\eta_{\chi}}} \left(\frac{C_{B,t}}{\epsilon_{L,t} \epsilon_{A,t}} - \bar{h}_{B,C} \frac{C_{B,t-1}}{\epsilon_{L,t-1} \epsilon_{A,t-1}}\right)^{\frac{\eta_{\chi}-1}{\eta_{\chi}}} \\ + \left(\mathcal{E}_t^{H,B} \omega_{H,B}\right)^{\frac{1}{\eta_{\chi}}} \left(\frac{H_{B,t}}{\epsilon_{L,t} \epsilon_{A,t}} - \bar{h}_{B,H} \frac{H_{B,t-1}}{\epsilon_{L,t-1} \epsilon_{A,t-1}}\right)^{\frac{\eta_{\chi}-1}{\eta_{\chi}}} \end{array} \right]^{\frac{\eta_{\chi}}{\eta_{\chi}-1}},$$

$$L_{B,t}^C = N L_{B,t}^C + \rho_{L,C} L_{B,t-1}^C,$$

$$L_{B,t}^H = N L_{B,t}^H + \rho_{L,H} L_{B,t-1}^H,$$

$$B_{B,t}^{L,H} = R_{B,t}^{L,H,float} R_{B,t-1}^{L,H,fixed} N L_{B,t-1}^H + \frac{R_{B,t}^{L,H,float}}{R_{B,t-1}^{L,H,float}} \rho_{L,H} B_{B,t-1}^{L,H},$$

$$B_{B,t}^{L,C} = R_{B,t-1}^{L,C} N L_{B,t-1}^C + \rho_{L,C} B_{B,t-1}^{L,C},$$

$$N L_{B,t}^H = \gamma_t^{LTV} P_{H,t} N H_{B,t},$$

$$H_{B,t} = N H_{B,t}^H + (1 - \kappa_H) (1 - \delta_H) H_{B,t-1},$$

$$\varphi_{B,t}^{H,Ear} (L_{B,t}^H - L_{B,t}^{H,Ear}) = 0.$$

The first-order conditions are the following

$$v_{B,t}^X = (\mathcal{X}_{B,t})^{-\eta_{X^S}} \varepsilon_t^{X,B}, \quad (33)$$

$$\Lambda_{B,t} (1 + \tau_{C,t}) = \left(\frac{\frac{C_{B,t}}{\epsilon_{L,t} \epsilon_{A,t}} - \bar{h}_{B,C} \frac{C_{B,t-1}}{\epsilon_{L,t-1} \epsilon_{A,t-1}}}{\left(1 - \varepsilon_t^{H,B} \omega_{H,B}\right) \mathcal{X}_{B,t}} \right)^{-\frac{1}{\eta_X}} v_{B,t}^X, \quad (34)$$

$$\begin{aligned} & (1 - \eta_{B,t}^{NL} \gamma_t^{LTV}) \frac{P_{H,t}}{P_{C,t}} \\ &= \beta_B E_t \frac{\Lambda_{B,t+1}}{\Lambda_{B,t} g_{L,t+1} g_{A,t+1}} \left[1 - \eta_{B,t+1}^{NL} \gamma_{t+1}^{LTV} (1 - \kappa_H) \right] \frac{P_{H,t+1}}{P_{C,t+1}} (1 - \delta_H) \\ &+ \left(\frac{\frac{H_{B,t}}{\epsilon_{L,t} \epsilon_{A,t}} - \bar{h}_{B,H} \frac{H_{B,t-1}}{\epsilon_{L,t-1} \epsilon_{A,t-1}}}{\varepsilon_t^{H,B} \omega_{H,B} \mathcal{X}_{B,t}} \right)^{-\frac{1}{\eta_X}} \frac{v_{B,t}^X}{\Lambda_{B,t}}, \end{aligned} \quad (35)$$

$$\Lambda_{B,t} \frac{W_t^N}{P \epsilon_{A,t}} (1 - \tau_{W,t}) = \psi_{B,N} \left(\frac{N_{B,t}}{\epsilon_{L,t}} \right)^{\eta_L} \varepsilon_t^L, \quad (36)$$

$$\zeta_{B,t}^{L,H} = \eta_{B,t}^{NL} + \beta_B E_t \frac{\Lambda_{B,t+1}}{\Lambda_{B,t} \Pi_{C,t} g_{L,t+1} g_{A,t+1}} \zeta_{B,t+1}^{B,H} R_{B,t+1}^{L,H,float} R_{B,t}^{L,H,fixed}, \quad (37)$$

$$1 + \rho_{L,H} \beta_B E_t \frac{\Lambda_{B,t+1}}{\Lambda_{B,t} \Pi_{C,t} g_{L,t+1} g_{A,t+1}} \zeta_{B,t+1}^{B,H} \frac{R_{B,t+1}^{L,H,float}}{R_{B,t}^{L,H,float}} = \zeta_{B,t}^{B,H}, \quad (38)$$

$$1 + \beta_B E_t \frac{\Lambda_{B,t+1}}{\Lambda_{B,t} \Pi_{C,t} g_{L,t+1} g_{A,t+1}} \zeta_{B,t+1}^{L,H} \rho_{L,H} + \varphi_{B,t}^{H,Ear} = \zeta_{B,t}^{L,H}, \quad (39)$$

$$\beta_B E_t \frac{\Lambda_{B,t+1}}{\Lambda_{B,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \left(1 + \frac{\rho_{L,C}}{R_{B,t+1}^{L,C} - \rho_{L,C}} \right) = \frac{1}{R_{B,t}^{L,C} - \rho_{L,C}}, \quad (40)$$

where $\Lambda_{B,t}$ is the Lagrange multiplier associated to real budget constraint and $v_{B,t}^X$, $\zeta_{B,t}^{L,H}$, $\zeta_{B,t}^{B,H}$ and $\eta_{B,t}^{NL}$ are Lagrange multipliers associated to $\mathcal{X}_{B,t}$, $L_{B,t}^H$, $B_{B,t}^H$ and LTV equations, respectively. In order to get some intuition from the expressions above, we solve recursively the last equation. The resulting expression is

$$E_t \sum_{k=1}^{\infty} (\beta_B)^k \frac{\Lambda_{B,t+k}}{\Lambda_{B,t} \Pi_{C,t+k} g_{L,t+k} g_{A,t+k}} (\rho_{L,C})^{k-1} (R_{B,t}^{L,C} - \rho_{L,C}) = 1.$$

It states that the present value of expected debt service $(\rho_{L,C})^{k-1} (R_{B,t}^{L,C} - \rho_{L,C})$ discounted with the stochastic discount factor is equal to 1. That is, $R_{B,t}^{L,C}$ is chosen such that the present value of

expected future cash flows generated by additional \$1 borrowed in t equals the current value in the same period.

If we solve equations (37), (38) and (39) by eliminating Lagrange multipliers $\zeta_{B,t}^{L,H}$ and $\zeta_{B,t}^{B,H}$ and setting $\varphi_{B,t}^{H,Ear} = 0$ (no rationing), we obtain an analogous expression for housing loans:

$$\eta_{B,t}^{NL} = 1 - E_t \sum_{k=1}^{\infty} \frac{(\beta_B)^k \Lambda_{B,t+k} (\rho_{L,H})^{k-1}}{\Lambda_{B,t} \Pi_{C,t,t+k} \mathcal{G}_{L,t,t+k} \mathcal{G}_{A,t,t+k}} \left(R_{B,t}^{L,H,fixed} R_{B,t+k}^{L,H,float} - \rho_{L,H} \right),$$

where $\eta_{B,t}^{NL}$ represents the net present value of additional \$1 housing loan borrowed in t . If housing loan interest rates are lower than the consumer loan rate, then $\eta_{B,t}^{NL} > 0$ and it is convenient to borrow more housing loans. However, housing loans are tied to house purchase. Hence, if $\eta_{B,t}^{NL} > 0$, there is an implicit subsidy to house purchases, as borrowers substitute cheaper housing loans for consumer loans while keeping the same total debt stock. This shows up when we solve recursively equation (35):

$$\begin{aligned} & (1 - \eta_{B,t}^{NL} \gamma_t^{LTV}) p_{H,t} \\ &= \kappa_H E_t \sum_{k=1}^{\infty} (\beta_B)^k \frac{\Lambda_{B,t+k} [(1 - \delta_H)(1 - \kappa_H)]^{k-1} P_{H,t+k}}{\Lambda_{B,t} \mathcal{G}_{L,t,t+k} \mathcal{G}_{A,t,t+k} P_{C,t+k}} (1 - \delta_H) \\ &+ E_t \sum_{k=0}^{\infty} (\beta_B)^k \frac{\Lambda_{B,t+k} [(1 - \delta_H)(1 - \kappa_H)]^k \left(\frac{h_{B,t+k} - \bar{h}_{B,H} h_{B,t+k-1}}{\varepsilon_{t+k}^{H,B} \omega_{H,B} \chi_{B,t+k}} \right)^{-\frac{1}{\eta_X}} v_{B,t+k}^X}{\Lambda_{B,t} \mathcal{G}_{L,t,t+k} \mathcal{G}_{A,t,t+k}} \frac{v_{B,t+k}^X}{\Lambda_{B,t+k}}, \end{aligned}$$

where $h_{B,t} = H_{B,t} / (\epsilon_{L,t} \epsilon_{A,t})$ and $p_{H,t} = P_{H,t} / P_{C,t}$. The left-hand side represents the marginal real cost of housing stock, where subsidy $\eta_{B,t}^{NL}$ is proportional to loan-to-value constraint γ_t^{LTV} . The right-hand side term represents the expected marginal benefit of housing stock, and it is the sum of the discounted expected value of turnover housing sales and the discounted expected marginal utility of housing stock.

2.2 Wages

Wages adjustment is sluggish with nominal Calvo rigidity. Complete derivation is presented in the technical appendix. The recursive representation of the resulting wage Phillips curve is given by the equations below:

$$\frac{W_t^O}{W_t} = \mu_w \frac{\mathcal{H}_{1,t}^W}{\mathcal{H}_{2,t}^W}, \quad (41)$$

$$\mathcal{H}_{1,t}^W = \frac{W_t^N}{W_t} \frac{L_t}{\epsilon_{L,t}} + E_t \frac{\beta_S \xi^W \Lambda_{S,t+1} \Pi_{t+1}^W}{\Lambda_{S,t} \mathcal{G}_{A,t+1} \Pi_{C,t+1}} \left(\frac{\tilde{\Pi}_{t+1}^W}{\Pi_{t+1}^W} \right)^{-\frac{\mu_w}{\mu_w - 1}} \mathcal{H}_{1,t+1}^W, \quad (42)$$

$$\mathcal{H}_{2,t}^W = \frac{L_t}{\epsilon_{L,t}} + E_t \frac{\beta_S \xi^W \Lambda_{S,t+1} \Pi_{t+1}^W}{\Lambda_{S,t} g_{A,t+1} \Pi_{C,t+1}} \left(\frac{\tilde{\Pi}_{t+1}^W}{\Pi_{t+1}^W} \right)^{-\frac{1}{\mu^W}} \mathcal{H}_{2,t+1}^W, \quad (43)$$

$$\tilde{\Pi}_t^W = [\Pi_{t-1}^C g_{A,t-1}]^{\gamma^W} [\bar{\Pi} g_A]^{1-\gamma^W}, \quad (44)$$

$$1 = (1 - \xi^W) \left(\frac{W_t^O}{W_t} \right)^{\frac{1}{1-\mu^W}} + \xi^W \left(\frac{\tilde{\Pi}_t^W}{\Pi_t^W} \right)^{\frac{1}{1-\mu^W}}, \quad (45)$$

where W_t is the final wage paid by firms, Π_t^W is the respective inflation rate, $\tilde{\Pi}_t^W$ is the wage indexation rule, W_t^N is the wage received by impatient households, and W_t^O is the wage set by the labor unions allowed to choose their wages in the Calvo setup. Total labor supply is L_t , and $\mathcal{H}_{1,t}^W$ and $\mathcal{H}_{2,t}^W$ are Lagrange multipliers that allow for the recursive representation of the wage Phillips curve.

2.3 Entrepreneurs

Entrepreneurs own the stock of productive capital in the economy and are responsible for investment decisions. In each period, entrepreneur i rents his stock of capital $K_{i,t-1}$ accumulated in the previous period to intermediate goods producers, and receives a proportional rental rate R_t^K . After being used in production, the stock of capital depreciates and is further augmented by the entrepreneur with new capital investment $I_{i,t}$. The entrepreneur borrows from banks to finance his capital holdings. Credit is comprised of long-term loans, with geometrically decaying amortization schedules and subject to default. There are two kinds of loans – commercial loans and subsidized loans – provided by commercial banks and a development bank, respectively.

In period t , entrepreneur i borrows an amount $NL_{E,i,t}$ of new commercial loans from banks. These long-term loans have geometrically decaying amortization rate $\rho_{L,E}$, fixed interest rate $R_{E,i,t}^{L, \text{fixed}}$, floating interest rate $R_{E,i,t}^{L, \text{float}}$ and default rate $F_E(\bar{\omega}_{E,i,t})$. Using the general representation for long term loans presented in the borrower's section, the dynamic evolution of commercial loans principal $L_{E,i,t}$, outstanding debt $B_{E,i,t}^L$ and debt service $S_{E,i,t}^L$ are given by

$$L_{E,i,t} = NL_{E,i,t} + (1 - F_E(\bar{\omega}_{E,i,t})) \rho_{L,E} L_{E,i,t-1}, \quad (46)$$

$$\begin{aligned} B_{E,i,t}^L &= R_{E,i,t}^{L, \text{float}} R_{E,i,t-1}^L NL_{E,i,t-1} \\ &+ \frac{R_{E,i,t}^{L, \text{float}}}{R_{E,i,t-1}^{L, \text{float}}} \rho_{L,E} (1 - F_E(\bar{\omega}_{E,i,t-1})) B_{E,i,t-1}^L, \end{aligned} \quad (47)$$

$$(1 - F_E(\bar{\omega}_{E,i,t})) S_{E,i,t}^L = (1 - F_E(\bar{\omega}_{E,i,t})) B_{E,i,t}^L - L_{E,i,t} + NL_{E,i,t}. \quad (48)$$

Subsidized credit is supplied by the Development Bank, owned by the government. Its interest rate is also determined by the government, it is always lower than the market interest rate $R_{E,t}^L$ and is invariant to the amount of subsidized loans $L_{E,i,t}^{DB}$. That means that the entrepreneur will always prefer subsidized credit over market rate loans, which will be crowded out unless there is some sort of rationing. The development bank may provide subsidized loans to finance part of every new investment. As the entrepreneur engages in investment $I_{i,t}$ in period t , the development bank may commit to provide new loans $NL_{E,i,t}^{DB}$ to finance a fraction $\gamma_t^{DB,E}$ of investment expenditures $P_{IK,t} I_{i,t}$.

$$NL_{E,i,t}^{DB} = \gamma_t^{DB,E} P_{IK,t} I_{i,t}.$$

This formulation implicitly states that firms must make investment in order to obtain subsidized loans. As a result, subsidized loans provide an effective incentive for additional investment, as any marginal increase in investment implies marginal increase of cheaper subsidized credit that can substitute for more expensive private bank loans.

In a possible alternative formulation, the Development Bank supplies an exogenous aggregate amount $NL_{E,t}^{DB,NB}$ of loans to be distributed to all firms according to the relative size of their capital stock in the previous period, with no binding restrictions associated with investment decisions. In this case, the direct impact of these loans on entrepreneurs' decisions is equivalent to providing lump-sum interest subsidies, as they substitute cheaper subsidized loans for more expensive private bank loans. If all firms have identical amount of capital, $NL_{E,i,t}^{DB}$ is given by:

$$NL_{E,i,t}^{DB} = NL_{E,t}^{DB,NB} \frac{K_{E,i,t}}{\int K_{E,i,t} di} = NL_{E,t}^{DB,NB}.$$

A general representation that encompasses both alternatives is

$$NL_{E,i,t}^{DB} = \gamma_t^{DB,E} P_{IK,t} I_{i,t} + NL_{E,t}^{DB,NB}. \quad (49)$$

and it allows the model to represent any intermediate case among both extremes (that is, subsidized loans providing only partial incentive in investment decision).

Subsidized loans follow a geometrically decaying payment schedule with decaying rate $\rho_{L,DB}$, and interest rates are a mixture of fixed $R_{B,i,t}^{L,DB,fixed}$ and floating $R_{B,i,t}^{L,DB,float}$ interest rates. As a result, outstanding principal $L_{E,i,t}^{DB}$ in period t , debt service $S_{E,i,t}^{L,DB}$ due in period t and the outstanding debt $B_{E,i,t}^{DB}$ are given by:

$$L_{E,i,t}^{DB} = NL_{E,i,t}^{DB} + \left(1 - F_E\left(\bar{\omega}_{E,i,t}^{DB}\right)\right)\rho_{L,DB}L_{E,i,t-1}^{DB}, \quad (50)$$

$$\left(1 - F_E\left(\bar{\omega}_{E,i,t}^{DB}\right)\right)S_{E,i,t}^{L,DB} = \left(1 - F_E\left(\bar{\omega}_{E,i,t}^{DB}\right)\right)B_{E,i,t}^{L,DB} - L_{E,i,t}^{DB} + NL_{E,i,t}^{DB}, \quad (51)$$

$$\begin{aligned} B_{B,i,t}^{L,DB} &= R_{B,i,t}^{L,DB,float}R_{B,i,t-1}^{L,DB,fixed}NL_{B,i,t-1}^{DB} \\ &+ \frac{R_{B,i,t}^{L,DB,float}}{R_{B,i,t-1}^{L,DB,float}}\rho_{L,DB}\left(1 - F_E\left(\bar{\omega}_{E,i,t-1}^{DB}\right)\right)B_{B,i,t-1}^{L,DB}, \end{aligned} \quad (52)$$

where $F_E\left(\bar{\omega}_{E,i,t}^{DB}\right)$ is the probability of default of subsidized loans.

As in Bernanke et al. (1999), at the beginning of period t , entrepreneur i 's capital is subject to an idiosyncratic shock $\omega_{E,i,t}$ with distribution $F_E(\cdot)$ with mean 1 and dispersion dependent on risk variable $\sigma_{E,t}$. The entrepreneur knows the actual value of $\sigma_{E,t}$ at the end of period $t-1$, immediately before making his decision on capital stock $K_{i,t-1}$. At the beginning of period t , shock $\omega_{E,i,t}$ realizes, and the real value of physical capital becomes $\omega_{E,i,t}K_{i,t-1}$. This is rented out to producers of intermediate goods at rate R_t^K , and, at the end of the period, it depreciates at rate δ_K . Therefore, the average nominal return of entrepreneurs' capital at period t is given by

$$\begin{aligned} R_t^{TK} &\equiv \int_0^\infty \omega \left[R_t^K + P_{K,t}(1 - \delta_K) \right] dF_E(\omega) \\ &= R_t^K + P_{K,t}(1 - \delta_K). \end{aligned} \quad (53)$$

The entrepreneur owes $B_{E,i,t}^L$ to the commercial lending branch and $B_{E,i,t}^{L,DB}$ to the development bank, due in period t . Only a fraction γ_{t-1}^E of assets can be pledged as collateral by banks in period t . This fraction is represented as an exogenous AR(1) process, and might be viewed as a measure of financial deepening. The higher the value of γ_t^E , the higher the availability of credit to firms.

If the value of this pledgeable collateral is lower than total debt, the entrepreneur is better off if he defaults and has the fraction γ_t^E of his assets arrested by the banks. Therefore, the minimum value $\bar{\omega}_{E,i,t}$ of $\omega_{E,i,t}$ at which it will still be optimal for the entrepreneur to repay its debt in full at t is given by:

$$\bar{\omega}_{E,i,t}\gamma_{t-1}^E R_t^{TK} K_{i,t-1} = B_{E,i,t}^L + B_{E,i,t}^{L,DB}. \quad (54)$$

If $\omega_{E,i,t} < \bar{\omega}_{E,i,t}$, the entrepreneur goes bankrupt and the pledgeable fraction γ_{t-1}^E of his capital is arrested by the banks. The entrepreneur keeps the remaining capital and can use it as collateral in the next period with no reputational costs. If $\omega_{E,i,t} \geq \bar{\omega}_{E,i,t}$, it is better for the

entrepreneur to roll over his debt. In case of default, the development bank loans have priority over the commercial loans. As a result, if $\bar{\omega}_{E,i,t}^{DB} \leq \omega_{E,i,t} < \bar{\omega}_{E,i,t}$, where

$$\bar{\omega}_{E,i,t}^{DB} \gamma_{t-1}^E R_t^{TK} K_{i,t-1} = B_{E,i,t}^{L,DB}, \quad (55)$$

the development bank loans suffer no losses and only commercial loans face partial default. Finally, if $\omega_{E,i,t} < \bar{\omega}_{E,i,t}^{DB}$, the commercial loans are not paid at all, whereas subsidized loans face partial default. Both the commercial lending branch and the development bank incur monitoring costs represented by fractions μ_E and $\mu_{E,DB}$ of the total value of recovered assets, respectively.

The lending branch gets its funding from the bank conglomerate. These funds must be repaid at the same amortization schedule of the respective loans, with exponentially decaying rate $\rho_{L,E}$. Interest rates are a mixture of fixed $R_{E,t}^{F,fixed}$ and floating $R_{E,t}^{F,float}$ interest rates. There is no default between the lending branch and the bank conglomerate (which owns the branch). Therefore, principal $L_{E,i,t}^F$, total outstanding debt $B_{E,i,t}^F$ and total service $S_{E,i,t}^F$ due to the bank conglomerate as a result of loans to entrepreneur i are given by

$$L_{E,i,t}^F = NL_{E,i,t} + \rho_{L,E} (1 - F_E(\bar{\omega}_{E,i,t})) L_{E,i,t-1}^F = L_{E,i,t}, \quad (56)$$

$$\begin{aligned} B_{E,i,t}^F &= R_{E,i,t}^{L,float} R_{E,i,t-1}^{F,fixed} NL_{E,i,t-1} \\ &\quad + \frac{R_{E,i,t}^{L,float}}{R_{E,i,t-1}^{L,float}} \rho_{L,E} (1 - F_E(\bar{\omega}_{E,i,t})) B_{E,i,t-1}^F, \end{aligned} \quad (57)$$

$$(1 - F_E(\bar{\omega}_{E,i,t})) S_{E,i,t}^F = (1 - F_E(\bar{\omega}_{E,i,t})) B_{E,i,t}^F - L_{E,i,t}^F + NL_{E,i,t}. \quad (58)$$

The zero expected profit condition for the lending branch implies the following equation:

$$\begin{aligned} &\beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} B_{E,i,t+1}^F \\ &= \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \gamma_t^E K_{i,t} R_{t+1}^{TK} G_E(\bar{\omega}_{E,i,t+1}, \bar{\omega}_{E,i,t+1}^{DB}), \end{aligned} \quad (59)$$

where

$$\begin{aligned} G_E(\bar{\omega}_{E,i,t}, \bar{\omega}_{E,i,t}^{DB}) &= (\bar{\omega}_{E,i,t} - \bar{\omega}_{E,i,t}^{DB}) (1 - F_E(\bar{\omega}_{E,i,t})) \\ &\quad + (1 - \mu_E) (Q_E(\bar{\omega}_{E,i,t}) - Q_E(\bar{\omega}_{E,i,t}^{DB})) \\ &\quad - (1 - \mu_E) \bar{\omega}_{E,i,t}^{DB} (F_E(\bar{\omega}_{E,i,t}) - F_E(\bar{\omega}_{E,i,t}^{DB})), \end{aligned} \quad (60)$$

and

$$Q_E(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega dF_E(\omega).$$

An intuitive interpretation for this equation is that the lending branch chooses a lending rate $R_{E,i,t}^{L,fixed}$ for new loans $NL_{E,i,t}$ such that the expected return on total loans net of default losses equals total funding costs. Also, the higher the amount of new loans $NL_{E,i,t}$ provided to the borrower, the higher the interest rate $R_{E,i,t}^{L,fixed}$ charged on these loans to compensate for higher default costs. This zero expected profit condition is similar to that in Bernanke et al. (1999), but slightly modified to allow for long-term loans and fixed instead of contingent interest rates.

The idiosyncratic shock introduces wealth heterogeneity among entrepreneurs, leading to difficulties in aggregation. In order to circumvent this problem, we introduce insurance contracts among entrepreneurs that eliminate the impact of idiosyncratic shocks on consumption and investment decisions. Details can be found in the technical appendix. We end up with the same budget constraint and capital accumulation equations for all entrepreneurs:

$$P_{C,t}C_{E,t} = R_t^K K_{t-1} - H_E(\bar{\omega}_{E,t}) \gamma_{t-1}^E R_t^{TK} K_{t-1} + L_{E,t} + L_{E,t}^{DB} - P_{IK,t} I_t, \quad (61)$$

$$K_t = (1 - \delta_K) K_{t-1} + \left(1 - \Gamma_K \left(\frac{I_t}{g_{L,t} g_{A,t} I_{t-1}} \varepsilon_t^{IK} \right) \right) I_t, \quad (62)$$

where

$$\ln(\varepsilon_t^{IK}) = \rho_{\varepsilon IK} \ln(\varepsilon_{t-1}^{IK}) + v_t^{\varepsilon IK}. \quad (63)$$

In the capital accumulation equation (62), capital in period t is given by last period outstanding capital stock net of depreciation plus new investment I_t , minus proportional investment adjustment costs given by quadratic function Γ_K .

2.3.1 Optimization program

The representative entrepreneur's problem is to maximize its utility function

$$E_0 \left\{ \sum_{t \geq 0} \beta_E^t \left[\frac{1}{1 - \eta_E} \left(\frac{C_{E,t}}{\epsilon_{L,t} \epsilon_{A,t}} \right)^{1 - \eta_E} \right] \varepsilon_t^{\beta, E} \right\},$$

where $\varepsilon_t^{\beta,E}$ is an AR(1) preference shock. He is subject to a budget constraint and investment and capital accumulation constraints:

$$P_{C,t}C_{E,t} = R_t^K K_{t-1} + P_{K,t}dK_t - P_{IK,t}I_t + L_{E,t} - B_{E,t}^L + L_{E,t}^{DB} - B_{E,t}^{L,DB} + \Xi_{E,t}^{Def},$$

$$\Xi_{E,t}^{Def} = B_{E,t}^L + B_{E,t}^{L,DB} - \gamma_{t-1}^E H_E(\bar{\omega}_{E,t}) R_t^{TK} K_{t-1},$$

$$K_t = (1 - \delta_K) K_{t-1} + \left(1 - \Gamma_K \left(\frac{I_t}{g_{L,t}g_{A,t}I_{t-1}} \varepsilon_t^{IK}\right)\right) I_t - dK_t,$$

where dK_t is an amount of capital that the entrepreneurs choose to sell at market price $P_{K,t}$. In symmetric equilibrium $dK_{i,t} = 0$, and this variable was introduced only to equal the market price of capital to the shadow price of capital in the entrepreneur's problem.

Again, we assume that entrepreneurs always consider beforehand that they will pay their loans in full next period, but banks respond to the likelihood that entrepreneurs might default on their debt. Hence, we introduce variable $\Xi_{E,t}^{Def}$ in the budget constraint to be treated as a lump sum transfer, where $\Xi_{E,t}^{Def}$ is the nominal gain from defaulting.

The resulting first-order conditions are the following

$$\left(\frac{C_{E,t}}{\epsilon_{L,t}\epsilon_{A,t}}\right)^{-\eta_E} \varepsilon_t^{\beta,E} = \Lambda_{E,t}, \quad (64)$$

$$P_{K,t} = \beta_E E_t \frac{\Lambda_{E,t+1}}{\Lambda_{E,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \left(P_{K,t+1} (1 - \delta_K) + R_{t+1}^K\right), \quad (65)$$

$$\begin{aligned} \frac{P_{IK,t}}{P_{C,t}} \left(1 - \eta_{E,t}^{DB} \gamma_t^{DB,E}\right) &= \frac{P_{K,t}}{P_{C,t}} \left(\begin{array}{c} 1 - \Gamma_K \left(\frac{i_t}{i_{t-1}} \varepsilon_t^{IK}\right) \\ -\Gamma'_K \left(\frac{i_t}{i_{t-1}} \varepsilon_t^{IK}\right) \frac{i_t}{i_{t-1}} \varepsilon_t^{IK} \end{array} \right) \\ &+ \beta_E E_t \frac{\Lambda_{E,t+1}}{\Lambda_{E,t}} \frac{P_{K,t+1}}{P_{C,t+1}} \Gamma'_K \left(\frac{i_{t+1}}{i_t} \varepsilon_{t+1}^{IK}\right) \left(\frac{i_{t+1}}{i_t}\right)^2 \varepsilon_{t+1}^{IK}, \end{aligned} \quad (66)$$

$$\begin{aligned} \frac{\zeta_{E,t}^L}{R_{E,t}^L} &= \rho_{L,E} \beta_E E_t \frac{\Lambda_{E,t+1}}{\Lambda_{E,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \frac{\zeta_{E,t+1}^L}{R_{E,t+1}^L} \\ &+ \beta_E E_t \frac{\Lambda_{E,t+1}}{\Lambda_{E,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} R_{E,t+1}^{L,float}, \end{aligned} \quad (67)$$

$$\zeta_{E,t}^L = 1 + \rho_{L,E} \beta_E E_t \frac{\Lambda_{E,t+1}}{\Lambda_{E,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \zeta_{E,t+1}^L, \quad (68)$$

$$\begin{aligned} \frac{\zeta_{E,t}^{L,DB} - \eta_{E,t}^{DB}}{R_{E,t}^{L,DB,fixed}} &= \rho_{L,DB} \beta_E E_t \frac{\Lambda_{E,t+1}}{\Lambda_{E,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \frac{\zeta_{E,t+1}^{L,DB} - \eta_{E,t+1}^{DB}}{R_{E,t+1}^{L,DB,fixed}} \\ &+ \beta_E E_t \frac{\Lambda_{E,t+1}}{\Lambda_{E,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} R_{E,t+1}^{L,DB,float}, \end{aligned} \quad (69)$$

$$1 + \rho_{L,DB} \beta_E E_t \frac{\Lambda_{E,t+1}}{\Lambda_{E,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \zeta_{E,t+1}^{L,DB} = \zeta_{E,t}^{L,DB}, \quad (70)$$

where

$$\Gamma_K \left(\frac{i_t}{i_{t-1}} \varepsilon_t^{IK} \right) = \frac{\phi_K}{2} \left(\frac{i_t}{i_{t-1}} \varepsilon_t^{IK} - 1 \right)^2, \quad (71)$$

$$\Gamma'_K \left(\frac{i_t}{i_{t-1}} \varepsilon_t^{IK} \right) = \phi_K \left(\frac{i_t}{i_{t-1}} \varepsilon_t^{IK} - 1 \right), \quad (72)$$

and $i_t = I_t / (\epsilon_{L,t} \epsilon_{A,t})$.

As in the borrowers' case, we can substitute and eliminate Lagrange multipliers $\zeta_{E,t}^L$ and $\zeta_{E,t}^{L,DB}$ to find the equations that define the endogenous fixed interest rate $R_{E,t}^L$ and subsidy $\eta_{E,t}^{DB}$:

$$1 = E_t \sum_{k=1}^{\infty} \frac{(\beta_E)^k \Lambda_{E,t+k} (\rho_{L,DB})^{k-1}}{\Lambda_{E,t} \Pi_{C,t+k} g_{L,t+k} g_{A,t+k}} \left(R_{E,t+k}^L R_{E,t+k}^{L,float} - \rho_{L,E} \right), \quad (73)$$

$$\eta_{E,t}^{DB} = 1 - E_t \sum_{k=1}^{\infty} \frac{(\beta_E)^k \Lambda_{E,t+k} (\rho_{L,DB})^{k-1}}{\Lambda_{E,t} \Pi_{C,t+k} g_{L,t+k} g_{A,t+k}} \left(R_{E,t+k}^{L,DB,fixed} R_{E,t+k}^{L,DB,float} - \rho_{L,DB} \right) \quad (74)$$

2.4 Intermediate goods producers and Retailers

The representative intermediate goods producer operates under perfect competition. It rents capital K_{t-1} at cost R_t^K and hire labor L_t with wages W_t in order to produce intermediate goods Z_t^D to be sold at market price MC_t . The production technology is given by:

$$Z_t^D = A \cdot \varepsilon_t^A [u_t K_{t-1}]^\alpha (\epsilon_{A,t} L_t)^{1-\alpha}, \quad (75)$$

where A is a scaling constant, $\epsilon_{A,t}$ is the labor productivity stochastic trend, and ε_t^A is a temporary shock to total factor productivity that follows an AR(1) process.

The intermediate goods producers sell their production at the competitive market price MC_t and seek to maximize their net cash flow each period. The first-order conditions from his optimization problem are the usual ones associated with Cobb-Douglas production functions:

$$R_t^K = P_{C,t} (\Gamma'_u(u_t) u_t - \Gamma_u(u_t)), \quad (76)$$

$$\frac{W_t L_t}{(R_t^K + \Gamma_u(u_t) P_{C,t}) K_{t-1}} = \frac{(1 - \alpha)}{\alpha}, \quad (77)$$

$$MC_t = \frac{1}{A_t \varepsilon_t^A} \left[\frac{(R_t^K + \Gamma_u(u_t) P_{C,t})}{\alpha} \right]^\alpha \left(\frac{W_t}{(1 - \alpha) \varepsilon_{A,t}} \right)^{1-\alpha}, \quad (78)$$

where $\Gamma_u(u_t)$ is an adjustment cost of capacity utilization.

Retailers introduce price rigidity in the model, as is usual in the literature. From their profit optimization program we can obtain the usual Phillips curve and aggregate price index:

$$\begin{aligned} & \frac{P_{D,t}^o}{P_{C,t}} E_t \sum_{k=0}^{\infty} (\beta_S \xi^D)^k \frac{\Lambda_{S,t+k}}{\Lambda_{S,t}} \frac{\tilde{\Pi}_{D,t,t+k}}{\tilde{\Pi}_{C,t,t+k}} \left(\frac{\tilde{\Pi}_{D,t,t+k}}{\tilde{\Pi}_{D,t,t+k}} \right)^{-\frac{\mu_D}{\mu_D-1}} \frac{Y_{t+k}^D}{\varepsilon_{L,t+k} \varepsilon_{A,t+k}} \\ & = \mu_D E_t \sum_{k=0}^{\infty} (\beta_S \xi^D)^k \frac{\Lambda_{S,t+k}}{\Lambda_{S,t}} \left(v_{t+k}^D \frac{MC_{t+k}}{P_{C,t+k}} \right) \left(\frac{\tilde{\Pi}_{D,t,t+k}}{\tilde{\Pi}_{D,t,t+k}} \right)^{-\frac{\mu_D}{\mu_D-1}} \frac{Y_{t+k}^D}{\varepsilon_{L,t+k} \varepsilon_{A,t+k}}, \end{aligned} \quad (79)$$

$$1 = (1 - \xi^D) \left(\frac{P_{D,t}^o}{P_{D,t}} \right)^{\frac{1}{1-\mu_D}} + \xi^D \left(\frac{\tilde{\Pi}_{D,t}}{\tilde{\Pi}_{D,t}} \right)^{\frac{1}{1-\mu_D}}, \quad (80)$$

where $\Pi_{D,t}$ is the inflation rate of final goods, Y_t^D is final goods total production, MC_{t+k} is the marginal cost of intermediate goods, and $\tilde{\Pi}_{D,t}$ is the indexation rule of the Calvo rigidity setup.

$$\tilde{\Pi}_{D,t} = \Pi_{D,t-1}^{\gamma_D} \bar{\Pi}^{1-\gamma_D} \tilde{\Pi}_{D,t}, \quad (81)$$

The final goods can be purchased as consumer, government, capital investment ou housing investment goods. Hence

$$P_{D,t} = P_{C,t} = P_{G,t} = P_{IK,t} = P_{IH,t},$$

$$\Pi_{D,t} = \Pi_{C,t} = \Pi_{G,t} = \Pi_{IK,t} = \Pi_{IH,t},$$

and

$$Y_t^D = C_t + G_t + I_t^K + I_t^H.$$

Complete derivation of both retailers and intermediate good can be found in the technical appendix.

2.5 Housing goods producers

Perfectly competitive firms produce new houses that add to the existing stock of housing. In period t , the representative house producing firm decides to produce new housing stock NH_t , which will be available to households at period t . As soon as the new houses are finished, they are sold at market price $P_{H,t}$. As the housing stock producer is owned by impatient households, it features the same preferences and stochastic discount factor. Housing investment is subject to quadratic adjustment costs

$$NH_t = \left[1 - \Gamma_H \left(\frac{I_{H,t}}{g_{L,t}g_{A,t}I_{H,t-1}} \varepsilon_t^{IH} \right) \right] I_{H,t},$$

where $\Gamma_H(r) \equiv \phi_H/2(r-1)^2$, and ε_t^{IH} is a shock on investment adjustment costs, given by a AR(1) process. The first-order conditions of his profit optimization program is as usual in the literature.

$$\begin{aligned} \frac{P_{IH,t}}{P_{C,t}} = \frac{P_{H,t}}{P_{C,t}} \left[\begin{array}{l} 1 - \Gamma_H \left(\frac{I_{H,t}}{g_{L,t}g_{A,t}I_{H,t-1}} \varepsilon_t^{IH} \right) \\ - \Gamma'_H \left(\frac{I_{H,t}}{g_{L,t}g_{A,t}I_{H,t-1}} \varepsilon_t^{IH} \right) \frac{I_{H,t}}{g_{L,t}g_{A,t}I_{H,t-1}} \end{array} \right] \\ + E_t \frac{(\beta_S) \Lambda_{S,t+1}}{\Lambda_{S,t}g_{L,t+1}g_{A,t+1}} \frac{P_{H,t+1}}{P_{C,t+1}} \Gamma'_H \left(\frac{I_{H,t+1}}{g_{L,t+1}g_{A,t+1}I_{H,t}} \varepsilon_{t+1}^{IH} \right) \left(\frac{I_{H,t+1}}{g_{L,t+1}g_{A,t+1}I_{H,t}} \right)^2 \varepsilon_{t+1}^{IH}. \end{aligned} \quad (82)$$

The newly produced housing stock NH_t is added to the existing stock of depreciated capital $(1 - \delta_H) H_{t-1}$ to yield the total amount of housing capital H_t available to households.

$$H_t = (1 - \delta_H) H_{t-1} + NH_t, \quad (83)$$

and it is distributed among patient and impatient households.

$$H_t = H_{S,t} + H_{B,t}. \quad (84)$$

2.6 Banking sector

The banking sector is composed of a representative competitive bank which obtains funding from deposits and provides credit to entrepreneurs and households. The bank is subject to regulatory capital requirements and can only accumulate capital through profit retention. It collects time deposits D_t^T and supplies commercial loans $L_{E,t}$ to entrepreneurs and housing and consumption loans $L_{B,t}^H$ and $L_{B,t}^C$ to households. Its balance sheet is given by:

$$L_{E,t} + L_{B,t}^H + L_{B,t}^C = D_t^T + D_t^{Ear.H} + K_t^{bank}, \quad (85)$$

where K_t^{bank} is net worth or bank capital and $D_t^{Ear,H}$ are earmarked funds provided by the government for housing loans. The bank complies with prudential regulation, making strategic decisions on capital accumulation, interest rates, portfolio allocation and taking into account the interaction with other banks in the credit market.

The bank accumulates capital from the net cash flow from its operations net of distributed dividends, $P_{C,t}C_{Bank,t}$. Capital accumulation is subject to an AR(1) shock $\varepsilon_t^{bankcap}$ that may capture operational losses or any other shocks that change banks' net worth. The capital accumulation rule is given by:

$$\begin{aligned}
K_t^{bank} = & -P_{C,t}C_{Bank,t} \\
& + \gamma_{t-1}^E R_t^{TK} K_{t-1} G_E(\bar{\omega}_{E,t}, \bar{\omega}_{E,t}^{DB}) - (\tau_{L,E,t-1} + s_{t-1}^{adm,E}) L_{E,t-1} \\
& + B_{B,t}^{L,C} - \mu_{B,C} F_B(\bar{\omega}_{B,t}^C) S_{B,t}^{L,C} - (\tau_{L,C,t-1} + s_{t-1}^{adm,C}) L_{B,t-1}^C \\
& + (1 - \mu_{B,H} F_B(\bar{\omega}_{B,t}^H)) B_{B,t}^{L,H} - (\tau_{L,H,t-1} + s_{t-1}^{adm,H}) L_{B,t-1}^H \\
& - R_{t-1}^T D_{t-1}^T - R_{t-1}^S D_{t-1}^{Ear,H} \\
& - \Gamma_{bankK} (K_{t-1}^{buff}) K_{t-1}^{bank} - \varepsilon_t^{bankcap} K_t^{bank} + T_{bank,t},
\end{aligned} \tag{86}$$

where $s_t^{adm,E}$, $s_t^{adm,H}$ and $s_t^{adm,C}$ represent administrative costs, which we assume to be proportional to the respective loan portfolio, $\tau_{L,E,t}$, $\tau_{L,H,t}$ and $\tau_{L,C,t}$ are tax rates on credit operations, and $T_{bank,t}$ are lump sum transfers. Loans are subject to default due to the idiosyncratic shocks on entrepreneurs' capital and households' income, and banks incur monitoring costs represented by fractions μ_E , $\mu_{B,H}$ and $\mu_{B,C}$ of the total value of recovered collateral.

The total values of outstanding consumer, housing and commercial loans net of default losses are

$$\begin{aligned}
& B_{B,t}^{L,C} - \mu_{B,C} F_B(\bar{\omega}_{B,t}^C) S_{B,t}^{L,C}, \\
& (1 - \mu_{B,H} F_B(\bar{\omega}_{B,t}^H)) B_{B,t}^{L,H}, \\
& \gamma_{t-1}^E R_t^{TK} K_{t-1} G_E(\bar{\omega}_{E,t}, \bar{\omega}_{E,t}^{DB}).
\end{aligned}$$

Housing loans are heavily regulated. Government supplies earmarked funds $D_t^{Ear,H}$ to finance housing for low-income families, with subsidized funding cost R_t^S represented as a function of the base interest rate R_t :

$$\ln\left(\frac{R_t^S}{R_t}\right) = \varphi_{R^S} \ln\left(\frac{R_t}{R}\right) + \ln(\varepsilon_t^{R,S}), \tag{87}$$

$$\varepsilon_t^{R,S} = \varphi_{\varepsilon,R,S} \varepsilon_{t-1}^{R,S} + \nu_t^{R,S}.$$

The total amount of earmarked funds for housing loans is given by:

$$L_{B,t}^{H,Ear} = D_t^{Ear,H}.$$

Interest rates of earmarked housing loans are also regulated by the government, and consist of the funding rate R_t^S plus a spread to compensate operational and default costs. Hence, the interest rate on new housing loans is composed of two components: a floating rate $R_{B,t}^{L,H,float}$, given by:

$$R_{B,t}^{L,H,float} = R_t^S, \quad (88)$$

and a fixed spread $R_{B,t}^{L,H,fixed}$ which is endogenously determined by the banks or exogenously set by the government. We will develop both cases below.

We introduce the possibility that housing loans are rationed to the amount of earmarked housing funds. The constraint below can represent both cases:

$$\varphi_{bank,t}^{H,Ear} [L_{B,t}^H - L_{B,t}^{H,Ear}] = 0 \quad (89)$$

If banks are not willing to provide more housing loans than the earmarked funds $L_{B,t}^{H,Ear}$, we impose that $\varphi_{bank,t}^{H,Ear} > 0$ and the constraint $L_{B,t}^H = L_{B,t}^{H,Ear}$ is binding. In the alternative case, $\varphi_{bank,t}^{H,Ear} = 0$ and $L_{B,t}^H > L_{B,t}^{H,Ear}$. Notice that we also introduced housing loans rationing in the borrowers' problem associated with Lagrange multiplier $\varphi_{B,t}^{H,Ear}$. Hence, to keep the model coherent, if $\varphi_{bank,t}^{H,Ear} = 0$ then $\varphi_{B,t}^{H,Ear} = 0$. As constraint $L_{B,t}^H = L_{B,t}^{H,Ear}$ is already presented in the borrowers' program, there is one missing equation to complete the system. In this case, we impose that the fixed housing loans interest rate spread $R_{B,t}^{L,H,fixed}$ is set exogenously by the government, according to the following simple AR(1) equation:

$$\log(R_{B,t}^{L,H,fixed} / R_B^{L,H,fixed}) = \rho_{R,L,H} \log(R_{B,t-1}^{L,H,fixed} / R_B^{L,H,fixed}) + \nu_t^{R,L,H,exog}. \quad (90)$$

As presented in the entrepreneur's section, the interest rate on commercial loans is composed of fixed and floating interest rates. The fixed rate is endogenously determined in the model. We impose that the floating rate $R_{E,t}^{L,float}$ is equal to the short-term policy interest rate R_t , as in most of working capital loans in Brazil, which are the most important category of commercial loans:

$$R_{E,t}^{L,float} = R_t. \quad (91)$$

The laws of motion of $\varepsilon_t^{bankcap}$ and the administrative costs are assumed to be simple AR(1) processes.

Banks have to comply with a simplified version of Basel capital requirement, which is based on the computation of capital adequacy ratios after weighting bank assets according to their risk factors. Although internal financing is usually costlier than external financing, a capital buffer above regulatory requirement has a potential signaling effect of banks' soundness, with positive effects on wholesale funding costs and on the probability of sudden stops in funding facilities. In addition, capital buffers can also prevent banks from falling short of the required minimum, an event that could result in undesired supervisory intervention.

Hence, we introduce a precautionary capital buffer by letting banks face an appropriate cost function when deviating from the total capital requirement, which is the sum of minimum capital requirement γ_t^{CapReq} , the countercyclical capital buffer γ_t^{CCyB} and sectoral countercyclical capital buffers $\gamma_t^{CCyB,E}$, $\gamma_t^{CCyB,H}$ and $\gamma_t^{CCyB,C}$. Hence, the capital buffer K_t^{buff} represented as a fraction of total capital requirement is given by

$$K_t^{buff} = \frac{K_t^{bank}}{\left[\begin{array}{l} \tau_{\chi E,t} \vartheta_t^E (\gamma_t^{CCyB} + \gamma_t^{CapReq} + \gamma_t^{CCyB,E}) L_{E,t} \\ + \tau_{\chi H,t} \vartheta_t^H (\gamma_t^{CCyB} + \gamma_t^{CapReq} + \gamma_t^{CCyB,H}) L_{B,t}^H \\ + \tau_{\chi C,t} \vartheta_t^C (\gamma_t^{CCyB} + \gamma_t^{CapReq} + \gamma_t^{CCyB,C}) L_{B,t}^C \\ + RWA_t^{other} (\gamma_t^{CCyB} + \gamma_t^{CapReq}) \end{array} \right]}, \quad (92)$$

where $\tau_{\chi E,t}$, $\tau_{\chi H,t}$ and $\tau_{\chi C,t}$ are the regulatory risk weights and ϑ_t^E , ϑ_t^H and ϑ_t^C are possibly time varying risk indicators of the respective credit sectors. It is easy to see in equation (92) that the effects sectoral countercyclical buffers and sectoral risk weights are equivalent in the model, up to a scale factor. The additional term RWA_t^{other} is introduced in the denominator to account for other assets that compose risk weighted assets in the observed series, and it is represented as a simple AR process:

$$RWA_t^{other} = \varepsilon_t^{RWA,other} \left[\tau_{\chi E,t} \vartheta_t^E L_{E,t} + \tau_{\chi H,t} \vartheta_t^H L_{B,t}^H + \tau_{\chi C,t} \vartheta_t^C L_{B,t}^C \right], \quad (93)$$

$$\log \left(\frac{\varepsilon_t^{RWA,other}}{\varepsilon^{RWA,other}} \right) = \rho_{\varepsilon RWA,other} \log \left(\frac{\varepsilon_{t-1}^{RWA,other}}{\varepsilon^{RWA,other}} \right) + \nu_t^{\varepsilon RWA,other}. \quad (94)$$

This additional term is necessary if we want to estimate the model using actual data.

Since the model solution is linearized around the balanced-growth path, it suffices to introduce a cost function $\Gamma_{bankK}(K_t^{buff})$ that fulfills conditions $\Gamma'_{bankK}(K_t^{buff}) < 0$, $\Gamma''_{bankK}(K_t^{buff}) > 0$, and, at the steady-state value K^{buff} , $\Gamma_{bankK}(K^{buff}) = 0$, where $K^{buff} > 1$. For convenience, and without loss of generality since the cost parameters that affect the model dynamics are estimated, we choose the following quadratic representation:

$$\Gamma_{bankK}(K_t^{buff}) = \frac{\chi_{bankK,2}}{2} (K_t^{buff})^2 + \chi_{bankK,1} (K_t^{buff}) + \chi_{bankK,0}. \quad (95)$$

Since this cost function is supposed to represent opportunity cost of equity, it will show up in the budget constraint of the bank multiplying total bank capital K_t^{bank} .

Banks are constrained by a minimum capital requirement, γ_t^{CapReq} , modeled as an AR(1) with very high persistence.

$$\ln\left(\frac{\gamma_t^{CapReq}}{\gamma^{CapReq}}\right) = \rho_{\gamma^{CapReq}} \ln\left(\frac{\gamma_{t-1}^{CapReq}}{\gamma^{CapReq}}\right) + \nu_t^{\gamma^{CapReq}} \quad (96)$$

This variable may be seen as the minimum capital requirement plus conservation buffer and, in principle, it might be regarded as a constant. However, we introduce it as a persistent AR(1) process to allow for policy exercises in which the regulation changes, such as when the new Basel III regulation was implemented.

The sectoral time varying risk indicators measure the credit risk associated with the respective credit stocks, and may depend on current and past default rates:

$$\log(\vartheta_t^X) = \rho_{\vartheta^X} \log(\vartheta_{t-1}^X) + (1 - \rho_{\vartheta^X}) \kappa^{\vartheta^X} \left(F(\bar{\omega}_{B,t}^X) - F(\bar{\omega}_B^X) \right) + \nu_t^{\vartheta^X}.$$

Notice that these risk indicators do not represent a precise measurement of actual risk, but rather a supervisory proxy for risk in the spirit of Basel II recommendations.

Banks choose the stream of real dividend distribution $\{C_{Bank,t}\}$ to maximize

$$E_0 \left\{ \sum_{t \geq 0} \beta_{Bank}^t \left[\frac{1}{1 - \eta_{Bank}} \left(\frac{C_{Bank,t}}{\epsilon_{L,t} \epsilon_{A,t}} \right)^{1 - \eta_{Bank}} \right] \epsilon_t^{\beta, Bank} \right\},$$

where $P_{C,t} C_{Bank,t}$ are banks' nominal dividends and $\epsilon_t^{\beta, Bank}$ is an AR(1) preference shock. We assume that banks' intertemporal discount factor, β_{Bank} , is lower than that of banks' stockholders. This is a shortcut to risk considerations in patient households' investment choices, since, in practice, bank shareholders demand a return on their risky investment in bank operations that is higher than the risk-free opportunity cost R_t .

First-order conditions are the following

$$\Lambda_{bank,t} = \left(\frac{C_{Bank,t}}{\epsilon_{L,t} \epsilon_{A,t}} \right)^{-\eta_{Bank}} \epsilon_t^{\beta, Bank}, \quad (97)$$

$$1 + \varepsilon_t^{bankcap} = \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \begin{bmatrix} R_t^T - \Gamma_{bankK} (K_t^{buff}) \\ -\Gamma'_{bankK} (K_t^{buff}) K_t^{buff} \end{bmatrix}, \quad (98)$$

$$\begin{aligned} \zeta_{\beta_{bank,t}}^{E,L} &= \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \begin{bmatrix} (\tau_{L,E,t} + s_t^{adm,E}) + R_t^T \\ -\tau_{\chi E,t} \vartheta_t^E R_t^{K,buff} \end{bmatrix} \\ &+ \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \zeta_{\beta_{bank,t+1}}^{E,L} (1 - F_E(\bar{\omega}_{E,t+1})) \rho_{L,E}, \end{aligned} \quad (99)$$

$$\begin{aligned} \frac{\zeta_{\beta_{bank,t}}^{E,L}}{R_{E,t}^L} &= \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \frac{G_E(\bar{\omega}_{E,t+1}, \bar{\omega}_{E,t+1}^{DB})}{\bar{\omega}_{E,t+1} - \bar{\omega}_{E,t+1}^{DB}} R_{E,t+1}^{L,float} \\ &+ \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \rho_{L,E} (1 - F_E(\bar{\omega}_{E,t+1})) \frac{\zeta_{\beta_{bank,t+1}}^{E,L}}{R_{E,t+1}^L}, \end{aligned} \quad (100)$$

$$\zeta_{\beta_{bank,t}}^{B,C,L} = \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \begin{bmatrix} (\tau_{L,C,t} + s_t^{adm,C}) + R_t^T \\ -\tau_{\chi C,t} \vartheta_t^C R_t^{K,buff} \\ +\rho_{L,C} \zeta_{\beta_{bank,t+1}}^{B,C,L} \end{bmatrix}, \quad (101)$$

$$\zeta_{\beta_{bank,t}}^{B,C,L} = \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \begin{bmatrix} \zeta_{\beta_{bank,t+1}}^{B,C,B} R_{B,t}^{L,C} \\ +\zeta_{\beta_{bank,t+1}}^{B,C,S} (R_{B,t}^{L,C} - \rho_{L,C}) \end{bmatrix}, \quad (102)$$

$$\zeta_{\beta_{bank,t}}^{B,C,S} = -\mu_{B,C} F_B(\bar{\omega}_{B,i,t}^C) + \rho_{L,C} \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \zeta_{\beta_{bank,t+1}}^{B,C,S}, \quad (103)$$

$$\zeta_{\beta_{bank,t}}^{B,C,B} = 1 + \rho_{L,C} \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \zeta_{\beta_{bank,t+1}}^{B,C,B}, \quad (104)$$

$$\begin{aligned} \zeta_{\beta_{bank,t}}^{B,H,L} &= \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \begin{bmatrix} (\tau_{L,H,t} + s_t^{adm,H}) + R_t^T \\ -\tau_{\chi H,t} \vartheta_t^H R_t^{K,buff} \end{bmatrix} \\ &+ \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \zeta_{\beta_{bank,t+1}}^{B,H,L} (1 - F_B(\bar{\omega}_{B,t+1}^H)) \rho_{L,H} \\ &- \varphi_{bank,t}^{H,Ear}, \end{aligned} \quad (105)$$

$$\begin{aligned} \frac{\zeta_{bank,t}^{B,H,L}}{R_{B,t}^{L,H,fixed}} &= \beta_{Bank} E_t \frac{\Lambda_{bank,t+1} \left[1 - \mu_{B,H} F_B \left(\bar{\omega}_{B,t+1}^H \right) \right]}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} R_{B,t+1}^{L,H,float} \\ &+ \beta_{Bank} E_t \frac{\Lambda_{bank,t+1} \rho_{L,H} \left(1 - F_B \left(\bar{\omega}_{B,t+1}^H \right) \right)}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \frac{\zeta_{bank,t+1}^{B,H,L}}{R_{B,t+1}^{L,H,fixed}}, \end{aligned} \quad (106)$$

where

$$R_t^{K,buf} = \left(\gamma_t^{CCyB} + \gamma_t^{CapReq} + \gamma_t^{CCyB,E} \right) \Gamma'_{bankK} \left(K_t^{buf} \right) \left(K_t^{buf} \right)^2. \quad (107)$$

In the particular case of $\rho_{L,C} = \rho_{L,H} = \rho_{L,E} = 0$, these conditions simplify to

$$\begin{aligned} &\beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \frac{G_E \left(\bar{\omega}_{E,t+1}, \bar{\omega}_{E,t+1}^{DB} \right)}{\bar{\omega}_{E,t+1} - \bar{\omega}_{E,t+1}^{DB}} R_{E,t+1}^{L,float} R_{E,t}^L \\ &= \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \left[\begin{array}{c} \left(\tau_{L,E,t} + s_t^{adm,E} \right) + R_t^T \\ -\tau_{\chi E,t} \vartheta_t^E R_t^{K,buf} \end{array} \right], \end{aligned}$$

$$\begin{aligned} &\beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \left[1 - \mu_{B,C} F_B \left(\bar{\omega}_{B,t+1}^C \right) \right] R_{B,t}^{L,C} \\ &= \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \left[\begin{array}{c} \left(\tau_{L,C,t} + s_t^{adm,C} \right) + R_t^T \\ -\tau_{\chi C,t} \vartheta_t^C R_t^{K,buf} \end{array} \right], \end{aligned}$$

$$\begin{aligned} &\beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \left[1 - \mu_{B,H} F_B \left(\bar{\omega}_{B,t+1}^H \right) \right] R_{B,t+1}^{L,H,float} R_{B,t}^{L,H,fixed} \\ &= \beta_{Bank} E_t \frac{\Lambda_{bank,t+1}}{\Lambda_{bank,t} \Pi_{C,t+1} g_{L,t+1} g_{A,t+1}} \left[\begin{array}{c} \left(\tau_{L,H,t} + s_t^{adm,H} \right) + R_t^T \\ -\tau_{\chi H,t} \vartheta_t^H R_t^{K,buf} \end{array} \right] - \varphi_{bank,t}^{H,Ear}. \end{aligned}$$

The conditions above state that the expected return of loans net of default losses must equal the funding costs. In the case of housing loans, this happens if there is no rationing, that is, $\varphi_{bank,t}^{H,Ear} = 0$.

If we impose housing loans rationing, we must also set

$$\log \left(\frac{R_{B,t-1}^{L,H,fixed}}{R_B^{L,H,fixed}} \right) = \rho_{R,L,H} \log \left(\frac{R_{B,t-1}^{L,H,fixed}}{R_B^{L,H,fixed}} \right) + \nu_t^{R,L,H,exog}. \quad (108)$$

Otherwise, we must impose

$$\varphi_{bank,t}^{H,Ear} = 0. \quad (109)$$

We also define $T_{bank,t}$ as the nominal sum of bank capital costs and administrative costs. If we assume administrative costs comprise mostly markups and risk premiums associated with each sort of loan, $T_{bank,t}$ represents the additional compensations for risk that banks must receive or pay for the risky components of their balance sheet. It should be a function of higher order moments of returns of each component, but this is not implementable in a first-order approximation of the model. Hence, we introduce them in the budget constraint as costs to impact the first-order conditions, but they are transferred back to banks as a lump sum.

$$T_{bank,t} = s_{t-1}^{adm,E} L_{E,t-1} + s_{t-1}^{adm,B} L_{B,t-1}^C + s_{t-1}^{adm,H} L_{B,t-1}^H + \Gamma_{bankK} \left(K_{t-1}^{buff} \right) K_{t-1}^{bank}. \quad (110)$$

2.7 Government

The government is composed of monetary, fiscal and macroprudential authorities. The monetary authority sets the base interest rate of the economy. The macroprudential authority regulates capital requirements. The fiscal authority purchases goods, issues public bonds, levies taxes, and makes lump-sum transfers to households.

The base interest rate is set according to:

$$\begin{aligned} \log\left(\frac{R_t}{R_{t-1}}\right) &= \rho_R \log\left(\frac{R_{t-1}}{R_{t-2}}\right) \\ &\quad - \alpha_R \left[\log\left(\frac{R_{t-1}}{R}\right) - \frac{\gamma_\pi}{4} \log\left(E_t \frac{P_{C,t+3}}{P_{C,t-1}} \frac{1}{\left(\bar{\Pi}_t^4\right)}\right) - \gamma_Y \log\left(\frac{gdp_t}{gdp}\right) \right] + \nu_t^R, \end{aligned} \quad (111)$$

where unsubscribed R is the equilibrium nominal interest rate of the economy given the steady-state inflation $\bar{\Pi}$, $\bar{\Pi}_t^4$ is a time-varying yearly inflation target for the current and the next three periods, and $gdp_t = GDP_t / (P_{C,t} \epsilon_{A,t} \epsilon_{L,t})$ is the stationary level of nominal output:

$$GDP_t = P_{C,t} C_t + P_{IK,t} I_{K,t} + P_{IH,t} I_{H,t} + P_{G,t} G_t. \quad (112)$$

The time-varying inflation target follows

$$\ln\left(\frac{\bar{\Pi}_t^4}{\bar{\Pi}^4}\right) = \rho_{\bar{\pi}} \ln\left(\frac{\bar{\Pi}_{t-1}^4}{\bar{\Pi}^4}\right) + \nu_t^{\bar{\pi}}. \quad (113)$$

The macroprudential authority is in charge of prudential regulation and controls the countercyclical buffer γ_t^{CCyB} and the sectoral countercyclical buffers $\gamma_t^{CCyB,E}$, $\gamma_t^{CCyB,H}$ and $\gamma_t^{CCyB,C}$ as well as sectoral risk weights $\tau_{\chi E,t}$, $\tau_{\chi H,t}$ and $\tau_{\chi C,t}$. We initially set all countercyclical buffers equal to zero, as none of these instruments have been used in Brazil so far, and no policy rule can be estimated. Countercyclical policy rules may be presented in alternative policy exercises. For simplicity, we also assume that risk weights $\tau_{\chi E,t}$, $\tau_{\chi H,t}$ and $\tau_{\chi C,t}$ are simple AR(1) processes.

The development bank provides subsidized funds to entrepreneurs to finance part of their new investment. As explained in the entrepreneurs' section, in period t , the development bank commits to finance a fraction $\gamma_t^{DB,E}$ of new investment project started in period t . These long-term loans have geometrically decaying amortization schedules with parameter $\rho_{L,DB}$ and subsidized interest rate $R_{E,t}^{L,DB} = R_{E,t}^{L,DB,float} R_{E,t}^{L,DB,fixed}$, where $R_{E,t}^{L,DB,float}$ and $R_{E,t}^{L,DB,fixed}$ are floating and fixed interest rates, respectively. For simplicity, we may assume that both follow persistent AR(1) processes which loosely track the base interest rate and the long term nominal interest rate $R_t^{LT,nom}$:

$$\frac{R_{E,t}^{L,DB,float}}{R_E^{L,DB,float}} = \left(\frac{R_{E,t-1}^{L,DB,float}}{R_E^{L,DB,float}} \right)^{\rho_{R,DB,float}} \left(\frac{R_t}{R} \right)^{(1-\rho_{R,DB,float})\kappa_{R,DB,float}} \exp(u_t^{R,DB,float}), \quad (114)$$

$$\frac{R_{E,t}^{L,DB,fixed}}{R_E^{L,DB,fixed}} = \left(\frac{R_{E,t-1}^{L,DB,fixed}}{R_E^{L,DB,fixed}} \right)^{\rho_{R,DB,fixed}} \left(\frac{R_t^{LT,nom}}{R^{LT,nom}} \right)^{(1-\rho_{R,DB,fixed})\kappa_{R,DB,fixed}} \exp(u_t^{R,DB,fixed}). \quad (115)$$

The bank also decides the amount of new loans provided every period. The formal instrument it uses is $\gamma_t^{DB,E}$, and we assume that the development bank might want to use subsidized loans to boost aggregate investment, according to the rule

$$\gamma_t^{DB,E} = (\gamma_{t-1}^{DB,E})^{\rho_{\gamma^{DB,E}}} \left(\gamma_{t-1}^{DB,E} \left(\frac{P_{IK,t} I_t}{GDP_t} \frac{gdp}{p_{IK} \cdot i} \right)^{\kappa_{\gamma^{DB,E}}} \right)^{(1-\rho_{\gamma^{DB,E}})} \exp(u_t^{\gamma^{DB,E}}). \quad (116)$$

As discussed in the entrepreneur's section, this instrument implies that the borrower must effectively perform the investment in order to receive the loans. An alternative policy just states that the development bank provides an aggregate amount of new loans $NL_{E,t}^{DB,NB}$ to be distributed to entrepreneurs with no strings attached, i.e., with no commitment to perform any investment:

$$\frac{NL_{E,t}^{DB,NB}}{\epsilon_{A,t} \epsilon_{L,t}} = \left(\frac{NL_{E,t-1}^{DB,NB}}{\epsilon_{A,t-1} \epsilon_{L,t-1}} \right)^{\rho_{DB,NB}} \left(n_E^{DB,NB} \left(\frac{P_{IK,t} I_t}{GDP_t} \frac{gdp}{p_{IK} \cdot i} \right)^{\kappa_{DB,NB}} \right)^{(1-\rho_{DB,NB})} \exp(u_t^{DB,NB}). \quad (117)$$

The first case above implies the higher impact of loans on investment decisions, whereas the second case implies the lower impact. A general implementation of this instrument encompassing both cases has already been presented in the entrepreneur's section, and

comprises a linear combination of both specifications.

The development bank net cash flow $\Xi_{G,t}^{Nom,DB}$ is transferred to the government as a lump sum.

The fiscal authority consumes final goods according to the rule:

$$g_t = (1 - \rho_g) [g - \mu_{B,G} (b_{t-1} - b)] + \rho_g g_{t-1} + v_t^G, \quad (118)$$

where $g_t = G_t / (\epsilon_{A,t} \epsilon_{L,t})$, $b_t = B_t / (P_{C,t} \epsilon_{A,t} \epsilon_{L,t})$ and g is the steady-state value of stationarized government consumption g_t . Government consumption has a role in stabilizing gross public sector debt.

For simplicity, we assume that tax rates $\tau_{C,t}$, $\tau_{W,t}$, $\tau_{\Pi,t}$, $\tau_{L,E,t}$, $\tau_{L,C,t}$ and $\tau_{L,H,t}$ are constant.

The joint public sector budget constraint is given by

$$\begin{aligned} P_{G,t} G_t + T_t^{Nom} + R_{t-1} B_{t-1} + D_t^{Ear,H} \\ = \tau_{W,t} \Xi_t^{Nom,LU} + \tau_{W,t} W_t^N N_t + \tau_{\Pi,t} \Xi_t^{Nom} + \tau_{C,t} P_{C,t} C_t \\ + \tau_{L,E,t-1} L_{E,t-1} + \tau_{L,C,t-1} L_{B,t-1}^C + \tau_{L,H,t-1} L_{B,t-1}^H \\ + B_t + R_{t-1}^S D_{t-1}^{Ear,H} + \Xi_{G,t}^{Nom,DB}, \end{aligned} \quad (119)$$

where G_t is government consumption, T_t^{Nom} are lump-sum transfers to households, and $D_t^{Ear,H}$ are subsidized funds to earmarked housing loans. The remaining terms account for tax revenues and the evolution of public debt. Lump-sum transfers, which behave as an AR(1) process, are distributed to savers and borrowers according to a fixed proportion

$$\frac{T_t^{Nom}}{P_{C,t} \epsilon_{A,t} \epsilon_{L,t} \bar{T}} \frac{1}{\bar{T}} = \left(\frac{T_{t-1}^{Nom}}{P_{C,t-1} \epsilon_{A,t-1} \epsilon_{L,t-1} \bar{T}} \frac{1}{\bar{T}} \right)^{\rho_T} \exp(v_t^T), \quad (120)$$

$$T_t^{Nom} = T_{S,t}^{Nom} + T_{B,t}^{Nom}, \quad (121)$$

$$T_{S,t}^{Nom} = \kappa_S^T T_t^{Nom} \quad (122)$$

For simplicity, we represent the evolution of earmarked housing loans funding $D_t^{Ear,H}$ as a simple AR(2) process, to account for the persistence observed in real data:

$$\log \left(\frac{d_t^{Ear,H}}{d_{t-1}^{Ear,H}} \right) = \rho_{D,Ear,H} \log \left(\frac{d_{t-1}^{Ear,H}}{d_{t-2}^{Ear,H}} \right) + \alpha_{D,Ear,H} \left(\frac{d_{t-1}^{Ear,H}}{d_{t-2}^{Ear,H}} \right) + \varepsilon_t^{D,Ear,H}, \quad (123)$$

where $d_t^{Ear,H} = D_t^{Ear,H} / (P_{C,t} \epsilon_{A,t} \epsilon_{L,t})$.

2.8 Market clearing, aggregation, and the resource constraint of the economy

Market clearing requires that the following equations hold:

$$Q_t^G = G_t, \quad (124)$$

$$Q_t^{IK} = I_{K,t}, \quad (125)$$

$$Q_t^{IH} = I_{H,t}, \quad (126)$$

$$Q_t^C = C_t. \quad (127)$$

Aggregation of consumption at the group level implies

$$C_t = C_{S,t} + C_{B,t}. \quad (128)$$

The resource constraint of the economy is a redundant equation once every budget constraint presented previously is included in the model.

$$Y_t^D = Y_t^{C,D} + Y_t^{IK,D} + Y_t^{IH,D} + Y_t^{G,D}. \quad (129)$$

3 Calibration and estimation

In order to facilitate estimation, we make variables stationary in the model by dividing real variables by technology and population trends $\epsilon_{A,t}$ and $\epsilon_{L,t}$, and nominal variables by both trends and consumer price $P_{C,t}$. To match these stationary variables to effective time series data, we also use stationary observable variables, most of them real or nominal variables represented as shares of GDP. Complete description of observable time series and observation equations can be found in the appendix. The sample period ranges from 2001Q4 to 2017Q2, which begins shortly after the introduction of inflation targeting in Brazil, when most of the credit time series are available. We used these sample time series both to calibrate steady-state values in the model with the respective sample averages and to estimate other parameters with Bayesian methods.

3.1 Steady-state values and calibration

The steady-state values of endogenous variables in the model are a function of parameter values. In our calibration procedure, we set the steady-state values of some important endogenous variables and find the respective implied parameter values. The detailed analytical procedure to map parameters from the steady-state values can be found in the technical appendix. Calibrated steady-state values and parameters are presented in Table 6. Most of the steady-state values were computed from sample means of the observable time series used to estimate the model.

The steady-state value of nominal GDP was set to 1, as well as the steady state value of relative prices p_D , p_C , p_{IK} , p_{IH} and p_G . Productivity growth rate g_A was obtained from a respective time series average computed as described in the appendix. Long-run labor force growth rate was set to zero (that is, the multiplicative factor $g_L = 1$). Steady-state inflation target was set to 4.5% yearly, as this has been the inflation target in Brazil for most of the last two decades up to 2018. The steady-state value of consumption, government spending, productive capital investment and housing investment were given by the average of the respective sample time series described in the appendix. Wage income as a share of GDP was set to 0.51, computed as the share of gross mixed income plus employees compensation in GDP. We also used observable time series averages to obtain the steady-state values of consumer (L_B^C), housing (L_B^H), commercial (L_E) and Development Bank (L_E^{DB}) loan stocks, as well as loan interest rates $R_B^{L,C}$, $R_B^{L,H}$, R_E^L , and $R_E^{L,DB}$. Nominal base rate was set as the average sample spread over CPI inflation plus the steady-state inflation target. The floating housing loans funding cost steady-state value $R_B^{L,H,float}$ was set equal to the sample average of savings deposits interest rate R^S from 2000 to 2017. Commercial loans floating rate steady-state $R_E^{L,float}$ was set equal to the base interest rate R . Development bank funding cost $R_E^{L,DB,float}$ was given by the respective sample mean of the Long Term Interest Rate (TJLP). Commercial and consumer loans loss-given-default ratios (LGD_E and LGD_C) were given by sample averages from 2007 to 2014 provided by the Bank Economics Report (Relatório de Economia Bancária) from the Central Bank of Brazil. Development Bank loans loss-given-default LGD_E^{DB} was set arbitrarily to 1pp, as it does not affect model dynamics because the respective loan interest rate is exogenous in the model. For the same reason, housing loans monitoring cost $\mu_{B,H}$ was set arbitrarily to 10pp. Default rates of consumer and commercial loans are calibrated with the respective sample means. Sample observable time series of default rates of housing and development bank loans are not available for the whole sample period, only from 2011 on. However, we used these short sample average values to calibrate the respective steady-state values.

Bank capital requirement was set to 11pp, as this was the regulatory requirement both

under Basel I and Basel II regulations. Bank steady-state capital adequacy ratio and total bank capital as a share of GDP were computed also from sample averages, as well as the ratio of RWAs other than credit RWA (RWA^{other}) to credit RWA. Consumer and commercial loans idiosyncratic risks σ_B^C and σ_E were arbitrarily set as zero, without loss of generality. Tax rates on consumption (τ_C), wage (τ_W) and profits (τ_{PIE}) were computed from the respective GDP shares of these tax categories in 2015. Government debt b is given by the average sovereign debt-to-GDP ratio from 2009 to 2012. Taxes on consumer ($\tau_{L,C}$) and commercial ($\tau_{L,E}$) loans were set to 3pp and 1.5pp yearly, according to the respective IOF (financial operations tax) rates. The tax rate on housing loans ($\tau_{L,C}$) was set arbitrarily to zero because it does not affect model dynamics, as housing loans lending rates are exogenous in the model.

The steady-state values of wage and price markups were calibrated arbitrarily to 1pp, and have little influence on model dynamics. Housing depreciation is set to 1pp yearly. Risk weights on commercial, housing and consumer loans were calibrated as 1.0, 0.5 and 1.0, close to average regulatory risk weights. Housing loans LTV ratio is 70%. The loan stock geometrically decaying rate of consumer, housing, commercial and development bank loans were chosen such that the respective weighted average remaining terms are 18.5, 120, 16 and 55 months, respectively. These figures represent the respective loan term averages from a 2011-2017 sample. Persistence of capital requirement shocks is calibrated to 0.999, in order to represent permanent policy shocks. Analogously, persistence of broad and sectoral countercyclical buffer is also set to 0.999. Bank's impatience parameter β^{bank} is arbitrarily set to 0.97, and this parameter has little impact on dynamics or loan interest rates steady-states (which are determined by respective sectoral administrative costs). Elasticities of default rates relative to idiosyncratic risks are calibrated without loss of generality to 1, as the standard deviation of the respective shock will be estimated.

3.2 Estimation

The model is estimated with Bayesian methods. We run two Markov chains with 1 million draws per chain. Sample time series used to estimate the model are described in the appendix. Table 7 presents the estimation results, as well as prior and posterior distributions. Definition of prior distributions was based mostly on previous results in Carvalho et al. (2014), as well as on preliminary impulse response exercises. Prior distribution of standard deviations of shocks were chosen such that the resulting theoretical second-order moments of observable variables in the model were similar to the respective sample moments. Both theoretical and sample moments of observable variables can be seen in Table 8. In general, unconditional model standard deviations are larger than the respective sample figures. This may be due to a certain extent to model misspecification, as even stationarized sample time series such as credit-to-gdp and bank

capital-to-GDP ratios exhibit evident short term trends the theoretical model is not designed to represent. Variance decomposition Table 9 presents some expected results. Overall, each endogenous variable is mostly affected by a few specific shocks closely related to the variable in the model. For instance, investment is strongly influenced by the investment adjustment shock, the most important shocks to explain private consumption variance are household preference shocks, and so on. Only the transitory productivity shock does show some important influence on many variables across the model.

4 Properties of the estimated model

4.1 Monetary policy shock

Figure 3 presents the impulse response functions to a 100bps shock to the policy interest rate. As the Taylor rule features two autoregressive terms, the policy interest rate response is hump-shaped. The pass-through to credit interest rates is higher in the case of commercial loans, as its floating component is equal to the base rate. Pass-through is lower for long term fixed consumer loans rate and for regulated housing loans rates. As a result of higher interest rates, consumption and investment fall, as well as GDP. Borrowers real wage income falls as a combination of lower real wages and lower hours worked. Real consumer credit increases slightly as borrowers try to smooth their consumption after facing an adverse temporary income shock, and also as a result of lower inflation rate (the impulse response becomes negative as expected if we introduce a more persistent interest rate dynamics). On the other hand, commercial loans decrease as entrepreneurs face higher interest rates and lower collateral values. New housing loans are controlled by the government in the mode and are not affected by the monetary policy shock. The real value of housing loans stock increases slightly as a result of lower inflation rates. Capital adequacy ratio increases as credit stock decreases more than bank capital. Total capital requirement is not affected, as there is no buffer rule in operation.

4.2 Capital requirement policy shocks

In the model, banks make lending decisions while trying to smooth expected dividend distribution over time subject to capital requirement regulation. Hence, if they have information about forthcoming capital requirement surcharges, they are able to make anticipated decisions about loan interest rates and dividend distribution. As a result, banks' responses to changes in capital requirement depend on whether these changes are anticipated or not. In order to

illustrate both cases, we present two alternative capital requirement shocks using the broad countercyclical buffer (CCyB) as instrument - an unanticipated 2.5pp positive shock, that can be viewed as an immediate buffer increase, and an anticipated gradual 2.5pp buffer increase announced four quarters before implementation, taking four quarters to gradually reach 2.5pp. Next, we repeat the same exercise with the sectoral countercyclical buffers (SCCyBs). The impulse responses presented below - anticipated buffers activation and surprise buffer releases - will provide the building blocks of the simulated policy exercises in the next section.

4.2.1 Broad countercyclical capital buffer shock

Figure 4 presents selected impulse response functions to an unanticipated and persistent 2.5pp CCyB increment. As excess bank capital over total requirements decreases, banks become willing to provide less loans and increase their interest rates of consumer and commercial loans, inducing households and entrepreneurs to consume and invest less. Subsequent reduction in GDP and inflation induces the monetary authority to lower the policy rate, with pass-through to loans rates. Notice that housing loans interest rate is not directly affected by the CCyB because it is regulated by the government and it is a function of the base interest rate. The stock of commercial and consumer loans decreases faster than bank capital, and capital adequacy ratio increases. New housing loans are determined by the government and are not affected, but the real value of housing loans stock increases as inflation is lower. The overall effect of the CCyB increase is contractionary. Notice that comparison with monetary policy impulse responses is not straightforward, since the monetary policy shock is short-lived whereas the CCyB shock is permanent. Dotted lines represent the alternative impulse responses when monetary policy response is muted by introducing monetary policy shocks that keep the rate constant. It is easy to see that active monetary policy response has an important role attenuating the impact of the macroprudential shock.

Impulse responses to an anticipated CCyB activation present similar behavior, with slightly delayed timing, as presented in Figure 5. Even before the buffer activation takes place, CAR starts to increase as banks accumulate dividends and hoard more bank capital to cope with future additional capital requirement. As all agents (banks, households and firms) anticipate the increase in lending spreads, demand falls, GDP and inflation decrease, and monetary authority starts reducing the policy interest rate. Although lending spreads increase, lending rates initially fall influenced by the policy rate. Commercial and consumer loans stock fall as a result of lower credit demand. Again, real housing loan stock fluctuates slightly as a result of lower inflation rates. As in the previous case, muted monetary policy response allows for a stronger impact of the macroprudential shock.

4.2.2 Sectoral countercyclical capital buffer shocks

Figure 6 shows the impulse responses to a sudden 2.5pp activation of commercial loans SCCyB. The impact on total capital requirement is 0.6pp, as commercial loans account for roughly one quarter of total risk weighted assets. As expected, there is an immediate effect on commercial lending rate, and a more muted direct impact on consumer lending rate. Higher commercial lending rates induce entrepreneurs to invest less. GDP and inflation decrease and the monetary authority lowers the policy rate. Consumer and housing loans rates fall as a consequence, and households demand more consumption and housing, as well as more commercial loans (real housing loans stock fluctuates only because of lower inflation). The overall effect of this SCCyB shock is contractionary, but sectoral effects are heterogeneous, as commercial loan rates increase and consumer rates fall. If we mute the monetary policy response, we obtain a more pronounced negative impact of the macroprudential shock. However, we also observe that consumer lending rates increase slightly. This represents the indirect impact of the sectoral commercial SCCyB shock on consumer loans. As the sectoral requirement increases, total bank capital buffer gets smaller, and as bank capital gets scarcer, this higher opportunity cost of bank capital also results in a minor increase of consumer lending rates. If monetary policy is active, the base interest rate response more than offsets this initial impact.

Responses to consumer sectoral countercyclical buffer shocks are analogous (Figure 7). A 2.5pp positive sudden shock to consumer SCCyB leads to a 0.47pp elevation of total capital requirement, as consumer loans account for less than 20% of total risk weighted assets. Now, consumer loans lending spreads increase, and households consume less. As GDP and inflation fall as a consequence, monetary policy rate decreases and contributes to lower commercial and housing lending rates. Consumer loans decrease and commercial loans increase. This heterogeneous reaction of credit sectors is much less intense when monetary policy is muted. In this case, both lending rates increase, with lower impact on commercial rates, and both sectors witness a reduction of loan volumes and respective sectoral demand.

The impact of housing loans SCCyB in the estimated model is considerably smaller than the previous cases, mostly because the share of housing loans in total risk weighted assets is small. The steady-state value of housing loans stock is roughly 1/3 of commercial loans stock and 40% of consumer loans stock. The calibration was based on the respective average sectoral shares in the Brazilian bank credit market from 2000Q4 to 2017Q2. As opposed to most developed countries, mortgages and housing loans are not the most important type of household loans. The transmission channel of the SCCyB is further weakened by a lower risk weight factor, such that the final impact of a 2.5pp SCCyB increase on total capital requirement is just 0.08pp. A further mitigating factor is the regulation of interest rates and funding. In

Brazil, housing loans are mostly financed by earmarked subsidized funding, with a regulated funding cost only partially influenced by monetary policy interest rate. Finally, one state-owned bank alone is responsible for 70% of all housing loans in the period. Hence, the best modeling alternative found for both lending volumes and rate are exogenous rules not influenced by other bank and credit variables. As a result, the only transmission channel of a housing SCCyB increase to bank lending spreads is by slightly decreasing excess bank capital, with secondary impact on commercial and consumer lending rates. As such, the sudden increase of the housing SCCyB produces only a mild elevation of lending spreads and negative impact on GDP, inflation and base interest rate, as presented in Figure 8. The responses when monetary policy is muted are slightly more pronounced.

5 Policy exercises with countercyclical capital buffers

5.1 Methodology

We intend to investigate the impact of introducing a countercyclical capital buffer (CCyB) and sectoral countercyclical capital buffers (SCCyB) as macroprudential instruments. But instead of relying on simple linear rules, we will resort to a more complex nonlinear policy setup.

As the model is implemented as a first-order approximation, the simplest way to introduce a countercyclical policy would be a linear rule like the Taylor rule for monetary policy. However, macroprudential rules such as countercyclical buffers cannot be realistically implemented this way. For instance, a usual recommendation for implementing the CCyB is activating the buffer gradually and releasing it swiftly. As policy makers detect some signal of overheating credit conditions such as excessive credit growth, they announce they will start increasing the buffer within one year, and may take several additional quarters to increase the buffer gradually from 0pp to 2.5pp. The whole process might take years starting from the announcement day. On the other hand, if the buffer is active and policy makers see serious distress in credit markets, they may decide to release the buffer immediately from 2.5pp to zero. Hence, usual CCyB policy recommendations hardly resemble a linear rule. The instrument is bounded (between 0 and 2.5pp, for instance), and both timing and indicators used to activate or release the buffer may be asymmetric.

In order to implement this sort of nonlinear policy rule in our first-order approximation model, we will resort to numerical simulations and exogenous implementation of policy rules. We introduce macroprudential instruments in the model as simple persistent autoregressive processes, and we implement policy rules as algorithms that set exogenous shocks to

macroprudential instruments as functions of the state of the economy.

In the case of the broad CCyB for instance, we start by introducing it in the model as a persistent AR(1) process with many anticipated shocks:

$$\gamma_t^{CCyB} = 0.9999\gamma_{t-1}^{CCyB} + \sum_{k=0}^{10} \varepsilon_{t-k}^{CCyB,k}, \quad (130)$$

where $\varepsilon_t^{CCyB,k}$ are exogenous shocks realized in t that will affect the buffer k periods ahead.¹ Hence, if the macroprudential authority announces in period t that the buffer will be activated four quarters later and will take additional four periods to reach 2.5pp, the corresponding shocks will be $\varepsilon_t^{CCyB,4} = \varepsilon_t^{CCyB,5} = \varepsilon_t^{CCyB,6} = \varepsilon_t^{CCyB,7} = 2.5pp/4$. On the other hand, if the buffer is announced to be completely released in the next period from a current level of 2.5pp, we need only $\varepsilon_t^{CCyB,1} = -2.5pp$. The equation implies that, in the rational expectation model, agents will expect the buffer to stay at the current level, except for anticipated shocks that happened up to ten periods ago.

We introduce policy equation (130) in the model, solve the first-order approximation rational expectations problem, and use the resulting state space representation to simulate the model. Starting at period t , the exogenous macroprudential policy algorithm checks the current state space values α_t for up or down trigger conditions and introduces appropriate macroprudential shocks $\{\varepsilon_t^{CCyB,k}\}$ if they are called for. A random sample ε_{t+1} of the remaining shocks is generated, and the whole set of shocks $\varepsilon_{t+1}^{All} = \{\varepsilon_{t+1}, \varepsilon_t^{CCyB,k}\}$ is introduced in the model to obtain next period state-space values α_{t+1} . The algorithm proceeds iteratively to produce a long simulation of $T = 300,000$ periods. To compare alternative macroprudential policies, we use the same random shocks sample $\{\varepsilon_t\}$ to simulate each policy, as well as a benchmark case with no macroprudential policy. We analyze the results by comparing summary sample statistics of endogenous variables such as means, variances and distributions.

We can use the model to study the introduction of sectoral countercyclical buffers (SCCyBs) as new alternative macroprudential instruments. As these instruments can be regarded as an extension of the broad CCyB - the CCyB can be reproduced perfectly with a particular combination of SCCyBs - we will compare alternative policy rules using SCCyB to a benchmark CCyB scenario as well as to a no macroprudential policy scenario.

The alternative macroprudential buffer rules to be compared are therefore:

0. No CCyB or SCCyB,
1. A reference CCyB only policy,

¹We set the autoregressive coefficient in the policy equation slightly lower than 1 because the software where the model is implemented is not prepared to handle unit root processes.

2. Independent SCCyBs policy, with each buffer targeting its own sectoral credit gap,
3. Coordinated SCCyB and CCyB policies, with the SCCyB targeting the sector with higher sectoral credit gap and the CCyB targeting all remaining sectors,
4. A SCCyB only policy that generates the same total capital requirement as the reference CCyB policy, but sectorally distributes the capital requirement according to each sector contribution to the RWA gap.

In the reference CCyB policy, the size of the buffer is determined by total credit-to-gdp gap. If credit gap is lower than 2pp, the CCyB is announced to be zero four quarters ahead. If the gap exceeds 2pp but is less than 4pp, the CCyB is announced to be set at 0.5pp four quarters ahead. If the credit gap is in the interval 4pp to 6pp, the CCyB will be set to 1.0pp and so forth, up to 2.5pp when total credit gap exceeds 10pp. Let's call "general proportional rule" this bounded discrete proportional rule to be applied to all other policies: a discrete 0.5pp buffer increase for each additional 2pp credit gap increase, with buffer bounded between 0pp and 2.5pp.

The policy of independent SCCyBs is similar to the reference CCyB policy, but with each SCCyB buffer targeting its own sectoral credit gap according to the general proportional rule, regardless of what is happening in other sectors.

In the coordinated SCCyB and CCyB policy, we single out in each period the sector with higher proportional credit expansion and compute the aggregated credit volume of all remaining sectors. Then we compute the CCyB level by applying the proportional rule to this aggregate credit gap and compute the SCCyB for the most exuberant sector applying the proportional rule to that sector. To avoid double incidence, we subtract the CCyB from this SCCyB value. For all other sectors, the respective sectoral buffers will be set to zero.

Finally, we introduce a SCCyB policy intended to replicate the same resilience provided by the reference CCyB policy, but with a more targeted sectoral focus. We apply the general rule to total credit gap and find the same total capital requirement as the reference CCyB rule. Next, we compute the sectoral credit gaps for each sector and single out those with positive values. Then we distribute that total capital requirement among these sectors, proportionally to sectoral credit gaps weighted by the respective RWA shares.

In all policies above, we also impose that all active buffers will be released completely in the next period if GDP growth in the current period is 2 standard deviations below average, and they will all be kept at zero level for the next eight quarters. Here we intend to reproduce the role of the macroprudential policies in mitigating the effects of an economic crisis.

This nonlinear methodology is convenient because it allows for nonlinear macroprudential policies which look more realistic than simple linear rules. On the other hand, as these policies

are implemented exogenously, they are not anticipated by agents in the model. This might be a considerable caveat for policies instruments that are frequently modified, such as the monetary policy interest rate. However, macroprudential instruments such as the countercyclical buffers are not intended to change frequently, so it is a good approximation supposing that agents' expectations about these policies are limited to the signals informed by the macroprudential authority.

Another caveat is that it is not possible to find optimal policy rules, as we resort to simulations. We must restrain the analysis to a few arbitrary alternative policies. This may not be a big issue. As models never incorporate all real world restrictions policy makers face, it is usual to obtain optimal linear policy rules that are not realistic or implementable. In the case of linear CCyB rules, unbounded buffer fluctuation and negative buffer values would show up. Our setup, on the other hand, is suitable to compare many alternative nonlinear implementable rules policy makers may come up with.

In order to make the results more easily comparable, we modified the model to increase the influence of the SCCyBs to the same extent as the broad CCyB. Our definition of excess bank capital in equation (92) is given by

$$K_t^{buff} = \frac{K_t^{bank}}{\left[\begin{array}{l} \tau_{\chi E,t} \vartheta_t^E (\gamma_t^{CCyB} + \gamma_t^{CapReq} + \gamma_t^{CCyB,E}) L_{E,t} \\ + \tau_{\chi H,t} \vartheta_t^H (\gamma_t^{CCyB} + \gamma_t^{CapReq} + \gamma_t^{CCyB,H}) L_{B,t}^H \\ + \tau_{\chi C,t} \vartheta_t^C (\gamma_t^{CCyB} + \gamma_t^{CapReq} + \gamma_t^{CCyB,C}) L_{B,t}^C \\ + RWA_t^{other} (\gamma_t^{CCyB} + \gamma_t^{CapReq}) \end{array} \right]},$$

where we introduced additional risk weighted assets RWA_t^{other} just to match the model with observed data, as housing, consumer and commercial loans comprise only roughly 50% of the RWA in the Brazilian banking system. Hence, variable RWA_t^{other} just shows up in the denominator to reduce the share of those loans categories in total risk weighted assets as observed in real data. It is modeled as an exogenous process and does not have any other role in the mode. But, in the formulation above, broad CCyB impacts both bank loans and additional RWAs, whereas sectoral loans affect only their respective RWA share. As a result, the CCyB instrument has a broader incidence base and is more powerful than the sum of all SCCyBs. To correct for this without delving into thorough modeling of RWA_t^{other} , we use for

the policy exercises in this section an alternative representation of K_t^{buff} :

$$K_t^{buff} = \frac{K_t^{bank}}{\left[\begin{array}{l} \tau_{\chi E,t} \vartheta_t^E (\gamma_t^{CCyB} + \gamma_t^{CapReq} + \gamma_t^{CCyB,E}) L_{E,t} \\ + \tau_{\chi H,t} \vartheta_t^H (\gamma_t^{CCyB} + \gamma_t^{CapReq} + \gamma_t^{CCyB,H}) L_{B,t}^H \\ + \tau_{\chi C,t} \vartheta_t^C (\gamma_t^{CCyB} + \gamma_t^{CapReq} + \gamma_t^{CCyB,C}) L_{B,t}^C \end{array} \right] \times (1 + \varepsilon_t^{RWA,other})}, \quad (131)$$

where other risk weighted assets are represented as a multiplicative term. With this formulation, the impact of all sectoral buffer together is the same as the CCyB, and it is possible to replicate the broad CCyB using the SCCyBs. This is equivalent to multiplying each sectoral requirement by $(1 + \varepsilon^{RWA,other})$, a number close to 2.2, such that SCCyBs' upper bound increases to 5.5pp.

5.2 Results

We simulate the model with each of those alternative policies. We perform simulations of 300,000 periods and compute the simulated moments of endogenous variables, presented in Table 1.

As compared to the benchmark scenario with no buffer, the average total capital requirement increases generated by the alternative policies are quite similar, ranging from 0.76pp to 0.81pp. As a result, the average impact on total loans and GDP is also roughly the same across policies. The introduction of buffers produces a small reduction of average credit volume as compared to the reference scenario with no buffer, roughly 2.6% of total credit volume. The average impact on GDP gap is negative but negligible, close to -0.026pp. Housing loans are little affected by buffer policies, as those loans are heavily regulated by the government in the model.

The impact on credit and GDP variances is not so homogeneous. The CCyB alone (policy rule 1) introduces a considerable reduction of credit variances, except for housing loans (not affected by macroprudential policies in the model). The impact on GDP variance is small. Policies 2 and 3, that involve sectoral buffers, are more efficient in reducing sectoral credit volatility, but do not perform as well as the CCyB to reduce total credit volatility. However, those SCCyB policies involve less total capital requirement volatility, which means they produce weaker aggregate countercyclical buffer reactions than the CCyB policy. Rule number 4 circumvents this by generating the same total capital requirement as the CCyB rule,

Table 1: Simulated sample mean and standard deviation of selected variables

Variable	Mean				
	No buffer	1 CCyB only	2 Independent SCCyBs	3 Coordinated SCCyB and CCyB	4 SCCyB replicating CCyB policy
log(Commercial loans/SS)	0	-0.0397	-0.0406	-0.0416	-0.0385
log(Consumer loans/SS)	0	-0.0184	-0.0196	-0.0203	-0.0152
log(Housing loans/SS)	0	0.0002	0.0003	0.0003	0.0002
log(Total loans/SS)	0	-0.0261	-0.0269	-0.0277	-0.0244
log(New commercial loans/SS)	0	-0.0413	-0.0422	-0.0433	-0.0401
log(New consumer loans/SS)	0	-0.0185	-0.0197	-0.0204	-0.0153
log(New housing loans/SS)	0	0.0000	0.0000	0.0000	0.0000
log(Total new loans/SS)	0	-0.0315	-0.0325	-0.0334	-0.0296
GDP gap	0	-0.00026	-0.00028	-0.00028	-0.00025
CAR	0.1684	0.1807	0.1814	0.1814	0.1805
Total capital requirement	0.1100	0.1177	0.1181	0.1181	0.1176

Variable	Standard Deviation				
	No buffer	1 CCyB only	2 Independent SCCyBs	3 Coordinated SCCyB and CCyB	4 SCCyB replicating CCyB policy
log(Commercial loans/SS)	0.487	0.461	0.458	0.465	0.454
log(Consumer loans/SS)	0.428	0.422	0.417	0.420	0.417
log(Housing loans/SS)	0.774	0.775	0.774	0.774	0.775
log(Total loans/SS)	0.342	0.322	0.327	0.331	0.323
log(New commercial loans/SS)	0.557	0.532	0.529	0.536	0.525
log(New consumer loans/SS)	0.481	0.477	0.472	0.474	0.472
log(New housing loans/SS)	1.187	1.187	1.187	1.187	1.187
log(Total new loans/SS)	0.388	0.367	0.370	0.376	0.367
GDP gap	0.0378	0.0374	0.0374	0.0374	0.0374
CAR	0.0345	0.0335	0.0332	0.0329	0.0334
Total capital requirement	0	0.01104	0.00848	0.00900	0.01104

but distributing this requirement to credit sectors according to their contribution to total RWA gap. As a result, policies 1 and 4 generate the same total capital requirement volatility, and almost the same total credit gap variance. However, as policy 4 features more targeted sectoral instruments, it can also achieve lower sectoral credit volatility.

Compared to the benchmark scenario with no buffer, we observe a marginal contribution of the CCyB policy in reducing total and sectoral credit volatility. The further introduction of sectoral instruments allows for additional reduction of sectoral credit volatility, but this marginal improvement is not as important as the first CCyB contribution. Nevertheless, the policy exercises suggest that the availability of additional sectoral instruments may help better stabilizing total and sectoral credit volumes. This result is hardly surprising, as more

instruments as a rule allow for better economic stabilization.

Notice that none of the alternative policies produces a relevant reduction in credit volatility. For instance, the best performing CCyB-only policy can reduce total credit volatility from 0.341 to 0.322, hardly a strong reduction. There are two main reasons for this result. First, as can be seen in variance decomposition (Table 9), most of the variance in credit volumes is explained by non financial shocks such as productivity and preference shocks, and macroprudential instruments are better suited to counterbalance financial shocks. Second, the low calibration of instruments' upper bounds, such as the CCyB 2.5pp upper bound in rule 1, may be limiting the impact of macroprudential policy. But instead of trying alternative rules with stronger responses, we keep a more conservative approach and restrict our simulations to those original rules which do not deviate too much from those effectively advocated in our current regulatory Basel III framework.

The availability of more instruments does require more frequent macroprudential intervention. Table 2 presents the relative frequency of total capital requirement change under each alternative rule. An increase means that the net result of all CCyB and SCCyB changes in that period implies higher total capital requirement than in previous period. A decrease is the same in the opposite direction, when a gradual movement is triggered by changes in total or sectoral credit gaps. A release is defined as a reduction triggered by a recession, when all active buffers will be released at once. Rule 1 implies that the CCyB is modified in only 12.3% of the periods. Rule 4 presents a similar number - 11.9% - as it seeks to replicate the aggregate capital requirement movements of rule 1. Scenario 2 is the busiest, with changes 22.6% of all periods, as each sectoral buffer moves independently. Finally, rule 3 implies aggregate total capital requirement change in 21.6%, as CCyB and some SCCyB may be working together. Hence, compared to rule 1, which uses only the CCyB, rules 2 and 3 require almost twice as much intervention, as they resort to two or more instruments to stabilize credit.

Table 2: Probability of increase, decrease or release of total countercyclical capital requirement, per quarter

Direction	Scenario			
	1 CCyB only	2 Independent SCCyBs	3 Coordinated SCCyB and CCyB	4 SCCyB replicating CCyB policy
Increase	0.062	0.107	0.102	0.056
Decrease	0.052	0.104	0.100	0.054
Release	0.009	0.015	0.014	0.009

A more detailed description of these movements is presented in Table 3. Only scenarios 1 to 3 are worth looking into, as they represent buffer changes through discrete 0.5pp spaced

values, from 0pp to 2.5pp. By construction, rule 4 tries to replicate discrete rule 1 by distributing its discrete total capital requirement changes continuously among sectoral buffers, which may assume infinitely many real values. Hence, changes in scenario 4 are continuous and much more frequent. By comparing scenarios 2 and 3 to the CCyB only scenario, it is easy to notice that every individual instrument exhibits a movement frequency similar to the CCyB in scenario 1. Hence, the operational complexity measured as the number of instrument changes increases roughly proportionally to the number of instrument in use.

Table 3: Probability of increase, decrease or release of each individual buffer, per quarter

Instrument	Direction	Scenario			
		1 CCyB only	2 Independent SCCyBs	3 Coordinated SCCyB and CCyB	4 SCCyB replicating CCyB policy
CCyB	Increase	0.062	0	0.048	0
	Decrease	0.052	0	0.041	0
	Release	0.009	0	0.011	0
SCCyB Commercial	Increase	0	0.056	0.047	0.394
	Decrease	0	0.046	0.042	0.345
	Release	0	0.010	0.010	0.008
SCCyB Consumer	Increase	0	0.052	0.045	0.373
	Decrease	0	0.043	0.042	0.326
	Release	0	0.009	0.010	0.008
SCCyB Housing	Increase	0	0.030	0.020	0.363
	Decrease	0	0.022	0.019	0.322
	Release	0	0.009	0.007	0.008

It is also possible to compare the alternative rules by looking at the average frequency each instrument is active. Table 4 shows the frequency of all possible combinations of instruments simultaneously active in any given period. Notice that rule 2 and 3 that resort to more instruments result in more frequent use of macroprudential instruments, respectively 70% and 64% of periods with some buffer active, compared to only 38% under policy rule 1 with CCyB only.

One of the alleged purposes of these macroprudential tools is building capital buffers to be available to banks in periods of distress, to reduce the probability that credit losses drive some institutions to bankruptcy. Our model does not feature bank failure, but a good proxy of bank resilience is the amount of bank capital held when a buffer release is called for. We sampled the periods in the simulation when a buffer release occurred triggered by an economic crisis and computed CAR mean and variance, as well as CAR relative frequency. We also sampled the respective values of CAR in counterfactual scenario with no buffer. The results are presented in Table 5.

All policies yield almost the same average CAR right before crises and the same variance,

Table 4: Relative frequency

Instrument	Scenario			
	1 CCyB only	2 Independent SCCyBs	3 Coordinated SCCyB and CCyB	4 SCCyB replicating CCyB policy
None	0.621	0.302	0.363	0.632
CCyB only	0.379	0	0.157	0
CCyB, Commercial SCCyB	0	0	0.024	0
CCyB, Consumer SCCyB	0	0	0.024	0
CCyB, Housing SCCyB	0	0	0.010	0
SCCyB, Commercial only	0	0.090	0.165	0.031
SCCyB, Consumer only	0	0.103	0.181	0.018
SCCyB, Housing only	0	0.127	0.076	0.040
SCCyB, Comm. and Cons.	0	0.085	0	0.062
SCCyB, Comm. and Hous.	0	0.096	0	0.063
SCCyB, Cons. and Hous.	0	0.096	0	0.047
SCCyB, Comm., Cons. and Hous.	0	0.100	0	0.108

Table 5: CAR distribution before buffer release

		Scenario			
		1 CCyB only	2 Independent SCCyBs	3 Coordinated SCCyB and CCyB	4 SCCyB replicating CCyB policy
Mean	Scenario (with buffer)	0.179	0.179	0.178	0.179
	Counterfactual (no buffer)	0.158	0.165	0.163	0.157
Standard deviation	Scenario (with buffer)	0.033	0.033	0.033	0.033
	Counterfactual (no buffer)	0.033	0.034	0.034	0.033

suggesting they all help building roughly the same average bank capital chest to endure adverse economic periods. Releases are more frequent in scenarios 2 and 3 (see Table 2) but the additional resilience they provide compared to the counterfactual scenario is lower than the CCyB-only rule. Figure 9 presents the CAR relative frequency histograms.

6 Concluding remarks

We develop and estimate a closed economy DSGE model with banking sector to assess the impact of introducing sectoral countercyclical capital buffers as a macroprudential tool. The model is developed to suit Brazilian bank credit markets. It features three types of bank credit - housing, consumer and commercial - as well as loans provided by a development bank. Loans are long term, and the government strongly regulates housing loans, influencing both

interest rates and loan supply. Banks are subject to bank capital requirement, and both broad (CCyB) and sectoral (SCCyB) countercyclical buffers can be introduced by macroprudential authorities. We simulate alternative policies using SCCyBs and CCyB with realistic nonlinear rules using broad and sectoral credit gaps as indicators, and compare the resulting performances. We conclude that, compared to CCyB, SCCyBs provide a more flexible set of instruments that allows achieving better macroeconomic stabilization in terms of variances of credit, total capital requirement and CAR. However, the marginal benefit of those SCCyB policies relative to the CCyB-only policy is lower than the improvements obtained by this latter policy compared to the reference scenario with no buffer. Also, SCCyB policies require more frequent intervention, suggesting that in practice introducing these additional instruments may require more complex implementation procedures.

Although the model was tailored for Brazil, some general conclusions could possibly be extended to other countries. First, the introduction of sectoral countercyclical buffers in the macroprudential toolbox may help enhance macroeconomic and credit stabilization, as well as make stabilization more efficient (lower credit variances) for the same amount of total capital requirement. Also, as a targeted instrument, the sectoral buffer is well suited to address imbalances concentrated in specific credit sectors. Finally, the availability of more instruments will demand more policy intervention and will require more attention to coordination among macroprudential instruments.

However, the model presents some country-specific characteristics that may not apply to many other countries. First and foremost, housing loans in Brazil are heavily regulated by the government. Significant amount of funding comes compulsorily from earmarked savings deposits with subsidized regulated interest rates. Housing loans interest rates are also regulated by the government. Most of the housing loans (roughly 70%) are provided by state owned banks. And housing loans respond for only 15% of total credit supply in the model, whereas real state loans share in advanced economies usually exceeds 50%. As a result, the impact of macroprudential instruments on housing loans is strongly subdued in the model and the simulations. Another country-specific feature is the low bank credit-to-GDP ratio in Brazil - roughly 50% - as compared to most advanced economies. This implies weaker impact of macroprudential instruments on the economy as a whole. In the opposite direction, the biggest chunk of credit in Brazil is supplied by the banking system, with limited room for a shadow banking sector which might weaken the impact of bank macroprudential instruments.

Finally, as the model resorts to one representative bank, it is not suited to address potential benefits of introducing sectoral buffers when banks are heterogeneous with respect to their loan portfolios. For instance, if a particular group of banks has larger exposure to housing loans and some of those banks are systemically important, the introduction of sectoral buffers may help increase resilience of the credit market, by focusing on localized sectoral vulnerabilities not

quite evident when one looks only at the aggregate credit statistics.

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A Data and observation equations

A.1 Real sector, inflation and policy interest rates

- Consumer inflation ($\Pi_{C,t}^{obs}$): inflation index used to assess compliance with the inflation target (IPCA - Índice de Preços ao Consumidor Amplo - IBGE). Quarterly seasonally adjusted log variation.

$$\Pi_{C,t}^{obs} = \log(\Pi_{C,t}) \quad (132)$$

- Inflation target ($\overline{\Pi}_t^{4,obs}$): Quarterly interpolation of yearly inflation target set by the Monetary Policy Committee (COPOM) of the Central Bank of Brazil's (BCB). Percentage points anual rate.

$$\overline{\Pi}_t^{4,obs} = 100 \cdot (\overline{\Pi}_t^4 - 1) \quad (133)$$

- Nominal interest rate ($R_t^{mean,obs}$): annualized mean quarterly effective nominal base rate (Selic). Percentage points.

$$R_t^{mean,obs} = (R_t^4 - 1) \cdot 100 \quad (134)$$

- GDP gap, GDP components and stochastic productivity trend.

As the model is a closed economy model, total GDP does not include exports or imports and is the sum of private consumption, government consumption, housing investment and productive capital investment. In order to match those variables with the real open economy GDP data, we compute an aggregate measure of GDP that excludes imports and exports and find the shares of each demand component of this GDP measure.

More specifically, we start with nominal time series of private consumption (C_t^{Nom}), government consumption (G_t^{Nom}), construction ($I_t^{H,Nom}$), and total gross formation of fixed capital (I_t^{Nom}) from the System of Quarterly National Accounts provided by IBGE. We compute the respective real values by dividing those series by cumulated IPCA index to obtain real time series C_t^{real} , G_t^{real} , $I_t^{H,real}$ and I_t^{real} . Next, we find the seasonally adjusted series $C_t^{real,sa}$, $G_t^{real,sa}$, $I_t^{H,real,sa}$ and $I_t^{real,sa}$ by applying the X12 filter to each real series independently. We define real seasonally adjusted GDP as

$$Y_t^{real,sa} = C_t^{real,sa} + G_t^{real,sa} + I_t^{real,sa}$$

and the shares of each GDP components are calculated as

$$C_t^{share,obs} = \frac{C_t^{real,sa}}{Y_t^{real,sa}}, G_t^{share,obs} = \frac{G_t^{real,sa}}{Y_t^{real,sa}}, I_t^{H,share,obs} = \frac{I_t^{H,real,sa}}{Y_t^{real,sa}}$$

The observation equation for private consumption GDP share is

$$C_t^{share,obs} = \frac{c_t}{c_t + g_t + i_t^H + i_t^K} \quad (135)$$

where c_t is defined as

$$c_t = \frac{C_t}{P_{C,t} \epsilon_{A,t} \epsilon_{L,t}}$$

and the same applies for g_t , i_t^H and i_t^K . Analogously, we have the observation equations of $G_t^{share,obs}$ and $I_t^{H,share,obs}$:

$$G_t^{share,obs} = \frac{g_t}{c_t + g_t + i_t^H + i_t^K} \quad (136)$$

$$I_t^{H,share,obs} = 2 \frac{i_t^H}{c_t + g_t + i_t^H + i_t^K} \quad (137)$$

The factor 2 multiplying the last equation stands for the fact that residence building accounts for roughly half of nominal GDP construction component $I_t^{H,share,obs}$ (the other half is comprised by infrastructure and corporate real state construction). The i_t^K share is implied by the previous equations.

To find GDP gap y_t , we start by computing an observable trend of economically active population $\epsilon_{L,t}^{obs}$ using data from IBGE. Next, we use it to find real GDP per capital series $Y_t^{real,sa,pc} = Y_t^{real,sa} / \epsilon_{L,t}^{obs}$, and finally find GDP gap $y_t^{gap,obs}$ by applying a HP filter with $\lambda = 6400$. The GDP gap observation equation is

$$y_t^{gap,obs} = \log\left(\frac{y_t}{\bar{y}}\right) \quad (138)$$

where $\bar{y} = 1$ is the steady-state value of y_t . We also use the HP trend $\epsilon_{A,t}^{trend,obs}$ to obtain the observation equation of productivity growth rate

$$\log\left(\frac{y_t^{trend,obs}}{y_{t-1}^{trend,obs}}\right) = \log(g_{A,t}) \quad (139)$$

- Nominal wage change ($\Pi_t^{W,sa,obs}$): quarterly log variation in an interpolated series built from IBGE's nominal wage series. Seasonally adjusted.

We start with the monthly nominal wage series, divide it by monthly CPI index) IPCA to obtain the monthly real wage series. Next, we compute the average quarterly real wage and

apply a X12 filter to remove seasonality and obtain a quarterly seasonally adjusted real wage series. Then, we multiply this series with the seasonally adjusted CPI index to find a seasonally adjusted quarterly nominal wage series. Finally, we log-differentiate this series to obtain the quarterly seasonally adjusted nominal wage log variation series $\Pi_t^{W,sa,obs}$.

In the model, wages feature a stochastic trend associated both to productivity and consumer prices. The stationarized real wage variable is defined as $w_t = w_t / (P_{C,t} \epsilon_{A,t})$, where $P_{C,t}$ is the CPI index and $\epsilon_{A,t}$ is the productivity stochastic trend. Hence, the observation equation is given by

$$\begin{aligned} \Pi_t^{W,sa,obs} &= \log\left(\frac{W_t}{W_{t-1}}\right) = \log\left(\frac{w_t P_{C,t} \epsilon_{A,t}}{w_{t-1} P_{C,t-1} \epsilon_{A,t-1}}\right) = \log\left(\frac{w_t}{w_{t-1}} \Pi_{C,t} g_{A,t}\right) \\ \text{or} \\ \Pi_t^{W,sa,obs} &= \log\left(\Pi_t^W g_{A,t}\right) \end{aligned} \quad (140)$$

where, by definition

$$\Pi_t^W = \frac{w_t}{w_{t-1}} \Pi_{C,t}$$

- Nominal housing price variation ($\Pi_t^{H,obs}$): quarterly log variation of IVG-R (Residential Real Estate Collateral Value Index) computed by the Central Bank of Brazil

$$\Pi_t^{H,obs} = \log\left(\frac{P_{H,t}}{P_{H,t-1}}\right) = \log\left(\frac{p_{H,t}}{p_{H,t-1}} \Pi_{C,t}\right) \quad (141)$$

where $p_{H,t} = P_{H,t} / P_{C,t}$ is the relative price of housing.

- Employment rate ($N_t^{obs,sa}$): seasonally adjusted quarterly employment rate. This series is an econometric interpolation of two (un)employment series produced by distinct IBGE surveys: PME (discontinued in 2016) and PNAD contínua (initiated in 2012).

The observation equation is

$$N_t^{obs,sa} = N_t \cdot \bar{N}^{obs}$$

where \bar{N}^{obs} is the calibrated steady-state value of employment rate.

A.2 Loans

The data on bank loans came from two distinct databases from the Central Bank of Brazil comprising different time spans and classifications of credit categories. The first database starts in 2003 and is discontinued in Dec/2012. The second one starts on Mar/2011 and was still operative in Dec/2017. The most recent dataset is richer, with more detailed

disaggregate information, but their aggregate time series could not be seamlessly concatenated with the former database as they span different credit categories and present different credit classifications. In order to obtain long aggregate time series of credit volumes, interest rates and default rates, we had to look into disaggregated time series and find the most appropriate way to concatenate those series.

For instance, in the case of consumer loans, we identified the most important credit categories present in both databases - namely personal credit, vehicles and other goods financing, and vehicles and other goods leasing - and concatenated the respective time series of loan interest rates, loan stock and default rate. We did not include overdraft and credit card loans, as these categories present very short loan terms and very high interest rates, and can be seen rather as means of payment than traditional credit instruments. To compute aggregate consumer interest rates and default rates, we calculated the respective averages weighted by loan stock. We considered this preferable to weighting with new loan volumes because there is considerable term differences among categories, and weighting with loan stocks give more weight to longer term loan categories. Similar approach was used to compute concatenated aggregate time series of commercial loans. It was not possible to obtain time series for interest rates and default rates of housing loans and development bank loans, as there is available data only from 2011 on. Only the respective subsidized funding costs were available.

Finally, in order to produce stationary series of credit stock, computed the credit-to-GDP for each credit category,

The resulting observable credit time series are

- Commercial loans-to-GDP ratio ($l_{E,t}^{Y,obs}$): stock outstanding of commercial loans granted by banks with freely allocated funds as a share of quarterly nominal GDP.

$$l_{E,t}^{Y,obs} = \frac{l_{E,t}}{c_t + g_t + i_t^H + i_t^K}$$

- Development bank loans-to-GDP ratio ($l_{E,t}^{DB,Y,obs}$): stock outstanding of subsidized loans granted by the development bank with subsidized rates as a share of quarterly nominal GDP.

$$l_{E,t}^{DB,Y,obs} = \frac{l_{E,t}^{DB}}{c_t + g_t + i_t^H + i_t^K}$$

- Consumer loans-to-GDP ratio ($l_{B,t}^{C,Y,obs}$): stock outstanding of consumer loans granted by banks with freely allocated funds as a share of quarterly nominal GDP.

$$l_{B,t}^{C,Y,obs} = \frac{l_{B,t}^C}{c_t + g_t + i_t^H + i_t^K}$$

- Housing loans ($I_{B,t}^{H,Y,obs}$): stock outstanding of housing loans to households as a share of quarterly nominal GDP.

$$I_{B,t}^{H,Y,obs} = \frac{I_{B,t}^H}{C_t + g_t + i_t^H + i_t^K}$$

- Commercial loans interest rate ($R_{E,t}^{L,obs}$): quarterly effective nominal interest rate on commercial loans granted with freely allocated funds and the base rate.

$$R_{E,t}^{L,obs} = \left[\left(R_{E,t}^L \right)^4 - 1 \right] \cdot 100$$

- Consumer loans interest spread ($R_{B,t}^{L,C,obs}$): quarterly effective nominal interest rate on consumer loans granted with freely allocated funds and the base rate.

$$R_{B,t}^{L,C,obs} = \left[\left(R_{B,t}^{L,C} \right)^4 - 1 \right] \cdot 100$$

- Housing loans regulated funding rate ($R_t^{S,obs}$): quarterly Referential Rate series, in percentage points of yearly rate. This rate is officially expressed as a spread over a fixed 0.5% monthly interest rate, in annual terms. The observation equation is

$$R_t^{S,obs} = \left[\left(\left(R_t^S \right)^{\frac{1}{3}} - 0.005 \right)^{12} - 1 \right] \cdot 100$$

- Development Bank subsidized floating lending rate ($R_{E,t}^{L,DB,float,obs}$): long-term interest rate (TJLP). Quarterly series in percentage points of yearly rate.

$$R_{E,t}^{L,DB,float,obs} = \left[\left(R_{E,t}^{L,DB,float} \right)^4 - 1 \right] \cdot 100$$

- Default rate on consumer loans ($F_{B,t}^{C,obs}$): retail loans in arrears for over 90 days as a share of total outstanding consumer loans, in percentage points. In the model, $F_B(\bar{\omega}_{B,t}^C)$ stands for the share of outstanding loans defaulted in the current period only. As observable series $F_{B,t}^{C,obs}$ stands for loans overdue from 3 months to one year, the observation equation is

$$F_{B,t}^{C,obs} = 100 \cdot \left[F_B(\bar{\omega}_{B,t}^C) + F_B(\bar{\omega}_{B,t-1}^C) + F_B(\bar{\omega}_{B,t-2}^C) \right]$$

- Default rate on commercial loans ($F_{E,t}^{obs}$): commercial loans in arrears for over 90 days as a share of total outstanding retail loans.

$$F_{E,t}^{obs} = 100 \cdot \left[F_B(\bar{\omega}_{E,t}) + F_B(\bar{\omega}_{E,t-1}) + F_B(\bar{\omega}_{E,t-2}) \right]$$

A.3 Banks

- Bank Capital ($K_t^{bank,Y,obs}$): Brazilian financial system's core capital as defined by the Central Bank of Brazil, as a share of quarterly nominal GDP. Source: Central Bank of Brazil.

$$K_t^{bank,Y,obs} = \frac{k_t^{bank}}{c_t + g_t + i_t^H + i_t^K}$$

- Capital Adequacy Ratio (CAR_t^{obs}): ratio of bank capital to aggregate risk weighted assets of Brazilian banking system. Source: Central Bank of Brazil.

$$CAR_t^{obs} = \frac{K_t^{bank}}{\tau_{\chi E,t} \vartheta_t^E L_{E,t} + \tau_{\chi H,t} \vartheta_t^H L_{B,t}^H + \tau_{\chi C,t} \vartheta_t^C L_{B,t}^C + RWA_t^{other}}$$

- Credit share in risk weighted assets ($RWA_t^{L,share}$): ratio of credit related risk weighted assets to total risk weighted assets. Source: Central Bank of Brazil.

$$RWA_t^{L,share} = \frac{\tau_{\chi E,t} \vartheta_t^E L_{E,t} + \tau_{\chi H,t} \vartheta_t^H L_{B,t}^H + \tau_{\chi C,t} \vartheta_t^C L_{B,t}^C}{\tau_{\chi E,t} \vartheta_t^E L_{E,t} + \tau_{\chi H,t} \vartheta_t^H L_{B,t}^H + \tau_{\chi C,t} \vartheta_t^C L_{B,t}^C + RWA_t^{other}}$$

- Housing loans risk weight factor ($\tau_{\chi H,t}$): this is the only observable effective risk weight factor explicitly associated to one of the credit categories in the model. It is not straightforward obtaining risk weight factors for aggregate commercial or consumer loans as these loan categories are not explicitly discriminated in the data, hence $\tau_{\chi H,t}$ is the only observable risk factor in the model. Source: Central Bank of Brazil.

B Tables

Table 6: Steady state and calibrated parameters

	Description	Value
Steady State Values		
g_A	Productivity growth rate (% per annum)	2.38
g_L	Labor Force growth rate (% per annum)	0.00
π_C	CPI Inflation (% per annum)	4.50
R	Nominal interest rate (% per annum)	8.80
R^S	Savings deposits interest rate (% per annum)	7.93
R^T	Time deposits interest rate (% per annum)	8.80
i^H	Investment in housing (% of GDP)	1.59
i^K	Investment in capital (% of GDP)	17.0
g	Government spending (% of GDP)	19.3
c	Private spending (% of GDP)	62.1
$w \times L$	Wage Income (% of GDP)	51.0
$L_B^{L,C}$	Consumption Loans (% of GDP)	7.9
$L_B^{L,C}$	Housing Loans (% of GDP)	3.5
L_E	Commercial Loans (% of GDP)	11.2
L_E^{DB}	Development Bank Loans (% of GDP)	7.3
$R_B^{L,C}$	Consumer Loans nominal interest rate (% per annum)	41.9
$R_B^{L,H}$	Housing Loans nominal interest rate (% per annum)	9.0
R_E^L	Commercial Loans nominal rate (% per annum)	25.1
$R_E^{L,DB}$	Development Bank Loans nominal rate (% per annum)	6.6
$R_B^{L,H,float}$	Housing Loans nominal interest rate (% per annum)	7.9
$R_E^{L,float}$	Commercial Loans floating nominal rate (% per annum)	8.8
$R_E^{L,DB,float}$	Development Bank Loans floating nominal rate (% per annum)	7.4
$F_B(\varpi_B^C)$	Consumer Loans default probability (% per quarter)	1.55
$F_B(\varpi_B^H)$	Housing Loans default probability (%)	0.64
$F_E(\varpi_E)$	Commercial Loans default probability (% per quarter)	1.30
LGD_C	Consumer Loans loss given default (% per quarter)	122.59
LGD_E	Commercial Loans loss given default (% per quarter)	73.40
LGD_E^{DB}	Development Bank Loans loss given default (% per quarter)	1.00
CAR	Capital adequacy ratio	16.9
γ^{CapReq}	Capital requirement ratio	11.0
K^{bank}	Bank capital (% of GDP)	9.3

Continued on next page

Table 6 – (cont.)

	Description	Value
RWA^{other}	Ratio of other RWA to credit RWA	1.183
σ_B^C	Consumer Loans idiosyncratic risk	0.000
σ_E	Commercial Loans idiosyncratic risk	0.000
τ^C	Tax rate on consumption (%)	20.9
τ^W	Tax rate on wages (%)	23.5
τ^π	Tax rate on profits (%)	21.5
$\tau_{L,C}$	Tax rate on consumer loans (% per annum)	3.0
$\tau_{L,E}$	Tax rate on commercial loans (% per annum)	1.5
$\tau_{L,H}$	Tax rate on housing loans (% per annum)	0.0
b	Government Debt (in % of GDP)	62.7
Calibrated Parameters		
μ_W	Wage markup (percentage point)	1.0
μ_D	Domestic goods price markup (percentage point)	1.0
δ_H	Housing depreciation (% per annum)	1.0
$\mu_{b,G}$	Gov. consumption coef. on total gov. debt	0.005
τ_{χ^E}	Risk weight on commercial loans	1.00
τ_{χ^C}	Risk weight on consumption loans	1.00
τ_{χ^H}	Risk weight on housing loans	0.50
$\mu_{E,DB}$	Devel. Bank Loans monitoring costs (percentage)	1.000
$\mu_{B,H}$	Housing Loans monitoring costs (percentage)	10.000
LTV_H	Housing Loans Loan-to-Value rate	0.70
$\rho_{L,C}$	Consumer Loans geom. decaying rate	0.84
$\rho_{L,H}$	Housing Loans geom. decaying rate	0.97
$\rho_{L,E}$	Commercial Loans geom. decaying rate	0.81
$\rho_{L,DB}$	Develop. Bank Loans geom. decaying rate	0.95
$\rho_{R,DB,fixed}$	Persist. of Dev. Bank interest rate	0.772
$\rho_{\gamma^{CapReq}}$	Persist. of capital requirement	0.999
$\rho_{\gamma^{CCyB}}$	Persist. of broad CCyB	1.000
$\rho_{\gamma^{CCyB,E}}$	Persist. of commercial CCyB	1.000
$\rho_{\gamma^{CCyB,C}}$	Persist. of consumer CCyB	1.000
$\rho_{\gamma^{CCyB,H}}$	Persist. of housing CCyB	1.000
β_{Bank}	Bank's utility time discount factor	0.970
$\phi_{\omega^E}^{\sigma^E}$	Commercial Loans default rate elasticity to risk	1.000
$\phi_{\omega^{DB}}^{\sigma^E}$	Devel. Bank default rate elasticity to risk	1.000

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Table 6 – (cont.)

Description	Value
$\phi_{\frac{\sigma_B^C}{\omega_B^C}}$ Consumer Loans default rate elasticity to risk	1.000
$\phi_{\frac{\sigma_B^H}{\omega_B^H}}$ Housing Loans default rate elasticity to risk	1.000

Table 7: Estimated Parameters and Shocks

Description	Prior Distribution		Posterior Distribution				
	Distribution	Mean	Std Dev	Credible set			
Preference and Technology							
$\bar{h}_{S,C}$	Savers' Consumption Habit persistence	Beta	0.75	0.05	0.629	0.557	0.705
$\bar{h}_{B,C}$	Borrowers' Consumption Habit persistence	Beta	0.75	0.05	0.771	0.696	0.849
$\bar{h}_{S,H}$	Housing Habit persistence	Beta	0.90	0.05	0.931	0.895	0.967
η_{χ^S}	Savers' Inverse Intertemporal Elast. of Subst.	Gamma	2.00	0.25	1.553	1.260	1.842
η_{χ^B}	Borrowers' Inverse Intertemporal Elast. of Subst.	Gamma	1.00	0.25	1.253	0.849	1.647
η_{χ}	Elast. of Subst. between Housing and Consumption	Gamma	2.00	0.50	1.114	0.820	1.410
η_L	Inverse Frisch elasticity of labor	Gamma	5.00	0.50	5.044	4.197	5.835
$\phi_{u,2}$	Capital utilization cost	Beta	0.01	0.01	0.005	0.001	0.008
η_E	Entrepreneurs' Inverse Intertemporal Elast. of Subst.	Gamma	1.00	0.50	0.816	0.561	1.069
μ_E	Commercial Loans Prop. Monit. Costs	Beta	0.50	0.25	0.628	0.350	0.902
ϕ_K	Adjustment cost of capital investment	Gamma	4.00	1.00	3.342	2.200	4.444
ϕ_H	Adjustment cost of housing investment	Gamma	8.00	2.00	15.677	11.308	19.847
Nominal Rigidities							
ξ_D	Calvo - domestic goods price	Beta	0.90	0.02	0.891	0.868	0.914
ξ_W	Calvo - wages	Beta	0.90	0.02	0.886	0.863	0.911
γ_D	Domestic Price indexation	Beta	0.50	0.25	0.267	0.098	0.431
γ_W	Wage indexation	Beta	0.50	0.25	0.116	0.002	0.234
Policy rules							
ρ_R	Interest rate smoothing	Beta	0.50	0.25	0.499	0.411	0.589

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Table 7 – (cont.)

Description	Prior Distribution		Posterior Distribution	
	Distribution	Mean	Std Dev	Credible set
α_R	Beta	0.50	0.25	0.141 0.116 0.167
γ_π	Gamma	2.00	0.25	1.826 1.552 2.097
ρ_g	Beta	0.50	0.25	0.714 0.582 0.844
ϕ_{RS}	Beta	0.50	0.10	0.385 0.335 0.435
$\alpha_{\bar{\pi}}$	Beta	0.10	0.05	0.111 0.058 0.163
$\kappa_{DB,NB}$	Beta	0.50	0.28	0.738 0.464 1.000
Financial Frictions				
η_{Bank}	Gamma	0.35	0.10	0.409 0.267 0.544
$\chi_{bankK,2}$	Gamma	0.05	0.03	0.049 0.032 0.066
$Elast_{FE,BE}^L$	Gamma	1.50	0.50	0.832 0.506 1.149
$Elast_{B,B}^{C,B,C}$	Gamma	1.50	0.50	1.142 0.602 1.658
Autoregressive Coefficients				
$\rho_{\mathcal{E}^K}$	Beta	0.50	0.20	0.457 0.328 0.588
$\rho_{\mathcal{E}^{H}}$	Beta	0.50	0.20	0.734 0.668 0.801
$\rho_{\mathcal{E}^{N,S}}$	Beta	0.50	0.20	0.232 0.077 0.378
$\rho_{\mathcal{E}^{N,B}}$	Beta	0.50	0.20	0.604 0.424 0.789
$\rho_{\mathcal{E}^{H,S}}$	Beta	0.50	0.20	0.941 0.933 0.949
$\rho_{\mathcal{E}^A}$	Beta	0.50	0.20	0.887 0.852 0.925
$\rho_{\mathcal{E}^{B,E}}$	Beta	0.50	0.20	0.307 0.130 0.482
ρ_{μ_D}	Beta	0.50	0.05	0.493 0.416 0.570
ρ_{μ_W}	Beta	0.50	0.20	0.541 0.403 0.683

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Table 7 – (cont.)

Description	Prior Distribution		Posterior Distribution	
	Distribution	Mean	Std Dev	Credible set
Autoregressive financial shocks				
ρ_{R_S}	Beta	0.90	0.05	0.969 0.949 0.990
$\rho_{adm,E}$	Beta	0.80	0.10	0.892 0.848 0.937
$\rho_{adm,C}$	Beta	0.80	0.10	0.897 0.858 0.938
ρ_{σ_B}	Beta	0.50	0.20	0.907 0.858 0.959
ρ_{σ_E}	Beta	0.50	0.20	0.811 0.727 0.896
$\rho_{DB,NL}$	Beta	0.50	0.20	0.540 0.395 0.682
$\rho_{NL,Ear,H}$	Beta	0.90	0.05	0.951 0.923 0.979
$\rho_{\tau_{yH}}$	Beta	0.95	0.03	0.949 0.914 0.986
$\rho_{sRWA,other}$	Beta	0.90	0.05	0.877 0.810 0.944
Non-financial shocks				
ν_R	Inv. Gamma	0.01	Inf	0.002 0.002 0.002
ν_G	Inv. Gamma	0.01	Inf	0.005 0.004 0.005
ν_{I_K}	Gamma	0.03	0.01	0.044 0.035 0.053
ν_{I_H}	Gamma	0.10	0.02	0.151 0.124 0.177
ν_{χ_S}	Gamma	0.10	0.02	0.143 0.113 0.173
ν_{χ_B}	Gamma	0.10	0.02	0.117 0.093 0.140
$\nu_{H,S}$	Gamma	0.50	0.10	0.823 0.644 0.997
ν_A	Gamma	0.02	0.01	0.026 0.020 0.031
$\nu_{\beta,E}$	Gamma	0.05	0.01	0.056 0.042 0.069
ν_{μ_D}	Gamma	0.05	0.01	0.063 0.050 0.076

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Table 7 – (cont.)

Description	Prior Distribution			Posterior Distribution	
	Distribution	Mean	Std Dev	Mean	Credible set
v_{μ_w} Wage markup	Gamma	0.05	0.01	0.089	0.076 0.102
$v_{\bar{\pi}}$ Inflation target	Inv. Gamma	0.01	Inf	0.001	0.001 0.002
Financial shocks					
$v_{adm,E}$ Markup on commercial loans	Inv. Gamma	0.01	Inf	0.009	0.007 0.010
$v_{adm,C}$ Markup on retail loans	Inv. Gamma	0.03	Inf	0.013	0.011 0.016
$v_{\beta,Bank}$ Banks' preference	Inv. Gamma	0.50	Inf	0.203	0.136 0.269
v_{σ_B} Risk shock to retail loans	Inv. Gamma	0.80	Inf	0.358	0.257 0.453
v_{σ_E} Risk shock to commercial loans	Inv. Gamma	1.00	Inf	0.379	0.291 0.462
$v_{eRWA,other}$ Non credit RWA share	Inv. Gamma	0.10	Inf	0.051	0.044 0.058
$v_{g^{All}}$ Exogenous Credit risk	Inv. Gamma	0.01	Inf	0.030	0.025 0.034
$v_{DB,NL}$ Dev, Bank New Loans	Inv. Gamma	0.01	Inf	0.010	0.009 0.012
$v_{NL,Ear,H}$ New Housing Loans	Inv. Gamma	0.01	Inf	0.002	0.002 0.003
$v_{R^{DB,float}}$ Dev. Bank funding cost	Inv. Gamma	0.00	Inf	0.001	0.001 0.001
v_{R^S} Housing Loans Risk Weight	Inv. Gamma	0.00	Inf	0.001	0.001 0.001
$v_{T_{y,H}}$ Housing Loans Risk Weight	Inv. Gamma	0.05	Inf	0.029	0.025 0.033

Table 8: Standard Deviations of Observable Series

Observable Variable	Sample Std. Dev.	Model Std. Dev.
Private Consumption Share in GDP	0.012	0.022
Government Consumption Share in GDP	0.007	0.017
Housing Investment Share in GDP	0.007	0.025
GDP gap (HP filtered)	0.037	0.038
Productivity Growth Rate	0.005	0.004
Employment rate	0.016	0.028
Nominal Wage Growth Rate	0.008	0.015
CPI Inflation	0.009	0.012
Nominal House price Growth Rate	0.020	0.045
Inflation Target	0.771	0.320
Base Interest Rate	5.335	6.239
Referencial Rate (Housing loans funding cost)	1.168	2.598
TJLP (Devel. Bank funding cost)	2.403	2.715
Retail Loans to GDP ratio	0.101	0.134
Housing Loans to GDP ratio	0.101	0.107
Commercial Loans to GDP ratio	0.076	0.213
Devel. Bank Loans to GDP ratio	0.086	0.061
Commercial Loans Interest Rate	3.835	5.399
Consumer Loans Interest Rate	11.082	5.724
Commercial Loans Default Rate	1.156	1.859
Consumer Loans Default Rate	0.766	2.409
Bank Capital to GDP	0.072	0.103
Capital Adequacy Ratio	0.012	0.035
Credit share in RWAs	0.024	0.027
Housing Loans Risk Weight	0.038	0.046

Table 9: Variance Decomposition of Selected Variables

Shock	Output Gap	Inflation	Interest Rate	Private Consumption	Government Consumption	Capital Investment
$u_{\chi,S}$	9.08	1.28	1.83	16.08	0.11	0.33
$u_{\chi,B}$	13.71	2.14	4.64	21.76	3.83	1.32
$u_{H,S}$	0.41	0.52	1.19	4.99	0.46	0.23
u_{I_K}	16.75	1.45	1.75	5.70	16.17	36.23
u_{I_H}	0.76	5.56	11.30	4.53	1.84	2.47
u_A	17.98	29.36	47.16	11.75	17.81	10.47
u_{μ_D}	4.96	49.57	9.41	5.11	1.11	2.64
u_{μ_W}	0.48	2.86	3.87	0.35	0.83	0.81
$u_{\beta,E}$	6.70	0.92	1.59	4.14	19.85	13.95
u_G	3.23	0.55	0.82	0.48	15.97	0.15
u_R	4.85	0.34	9.17	4.21	3.36	2.14
$u_{adm,C}$	4.28	1.05	1.39	9.29	1.03	1.38
$u_{adm,E}$	8.16	2.12	3.04	3.96	0.17	17.65
u_{σ_E}	0.71	0.08	0.10	0.38	0.23	1.43
u_{σ_B}	1.59	0.35	0.61	2.69	1.08	0.44
$u_{\beta,Bank}$	0.36	0.11	0.16	0.22	0.01	0.29
$u_{DB,NL}$	1.29	0.09	0.11	0.74	3.53	2.42
$u_{NL,Ear,H}$	0.62	0.72	1.41	0.86	3.35	0.19
$u_{\bar{\pi}}$	0.91	0.68	0.04	0.79	0.44	0.24
$u_{\tau_{\chi H}}$	0.00	0.00	0.00	0.00	0.00	0.00
$u_{\epsilon_{RWA,other}}$	0.06	0.01	0.01	0.04	0.00	0.05
$u_{R^{DB,float}}$	2.48	0.01	0.02	1.66	8.39	4.47
$u_{\beta^{All}}$	0.60	0.24	0.38	0.25	0.39	0.72
u_{RS}	0.00	0.00	0.00	0.01	0.03	0.00
ϵ_g^A	0.00	0.00	0.00	0.00	0.00	0.00

Table 9 - (cont.)

Shock	Commercial Loans	Dev. Bank Loans	Retail Loans	Housing Loans	Commercial Lending Rate	Retail Lending Rate	Housing Lending Rate
$u_{\chi,S}$	0.71	0.29	2.78	0.04	1.41	0.08	1.42
$u_{\chi,B}$	2.77	0.69	28.44	0.11	3.20	8.14	3.62
$u_{H,S}$	0.71	0.20	0.24	0.03	0.72	1.02	0.93
u_{I_K}	3.94	0.24	3.83	0.03	6.15	0.52	1.36
u_{I_H}	5.54	1.83	2.20	0.26	7.78	7.34	8.79
u_A	20.10	7.59	15.35	1.08	36.84	18.16	36.62
u_{μ_D}	2.05	2.57	0.34	0.31	13.61	10.18	7.00
u_{μ_W}	1.37	0.60	1.95	0.08	3.32	0.62	3.00
$u_{\beta,E}$	5.99	0.22	0.39	0.03	0.85	0.51	1.23
u_G	0.32	0.13	1.04	0.02	0.63	0.02	0.63
u_R	1.09	0.03	0.11	0.00	12.99	6.61	5.99
$u_{adm,C}$	1.45	0.14	22.94	0.02	1.79	34.32	1.08
$u_{adm,E}$	32.73	0.33	4.09	0.04	7.21	1.89	2.36
u_{σ_E}	2.41	0.01	0.19	0.00	1.20	0.16	0.07
u_{σ_B}	0.80	0.08	12.98	0.01	0.56	9.94	0.47
$u_{\beta,Bank}$	0.68	0.02	0.06	0.00	0.12	0.15	0.12
$u_{DB,NL}$	5.30	84.60	0.06	0.00	0.10	0.04	0.09
$u_{NL,Ear,H}$	0.47	0.23	0.30	97.92	0.96	0.10	1.10
$u_{\bar{\pi}}$	0.21	0.16	0.03	0.02	0.24	0.01	0.03
$u_{\tau_{xH}}$	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$u_{\epsilon RWA,other}$	0.05	0.00	0.00	0.00	0.04	0.10	0.01
$u_{R^{DB,float}}$	0.68	0.00	0.09	0.00	0.03	0.02	0.01
$u_{\beta^{All}}$	10.62	0.05	2.60	0.01	0.23	0.07	0.30
u_{R^S}	0.00	0.00	0.00	0.01	0.00	0.00	23.78
ϵ_{σ^A}	0.00	0.00	0.00	0.00	0.00	0.00	0.00

C Figures

Figure 1: Observable Variables

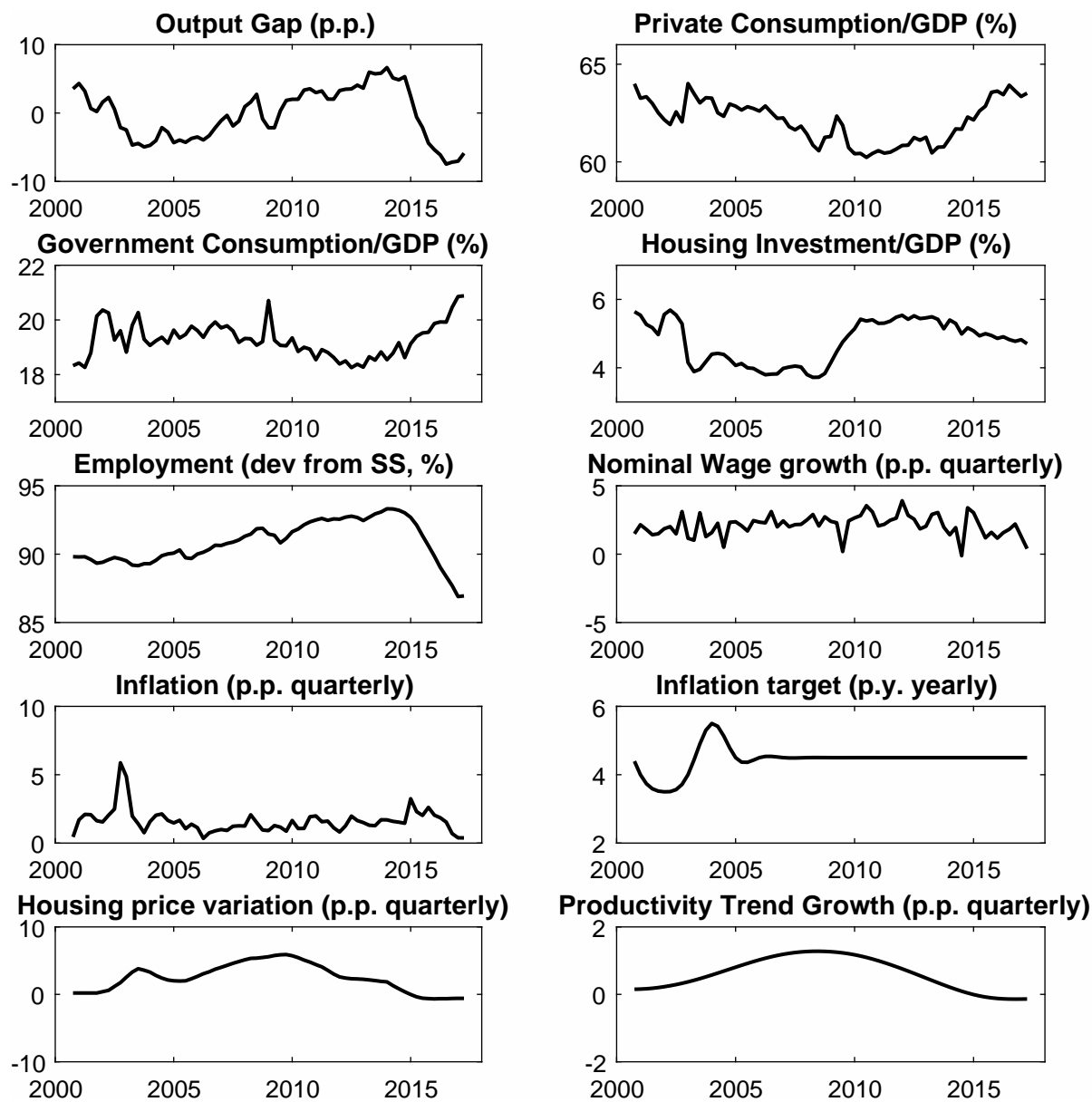


Figure 1: Observable Variables (cont.)

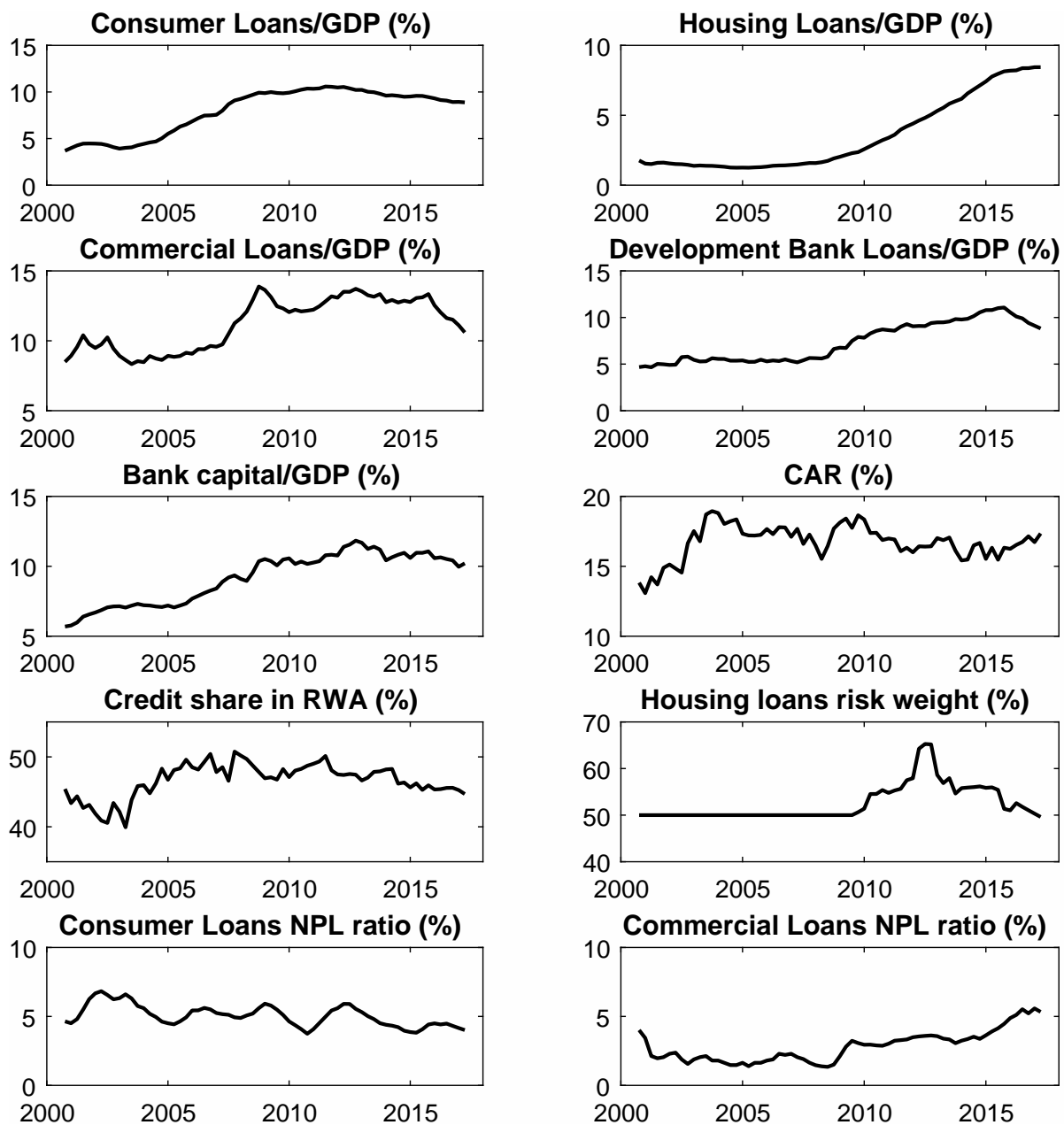


Figure 1: Observable Variables (cont.)

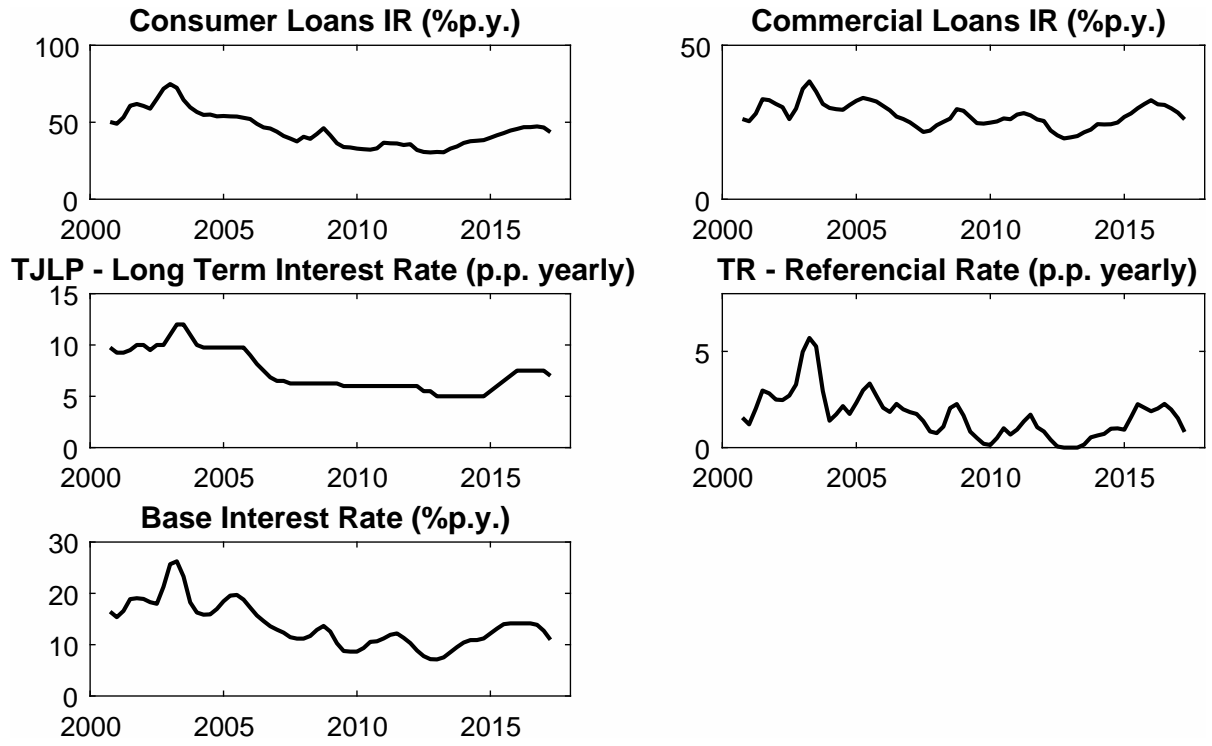


Figure 2: Priors and Posteriors

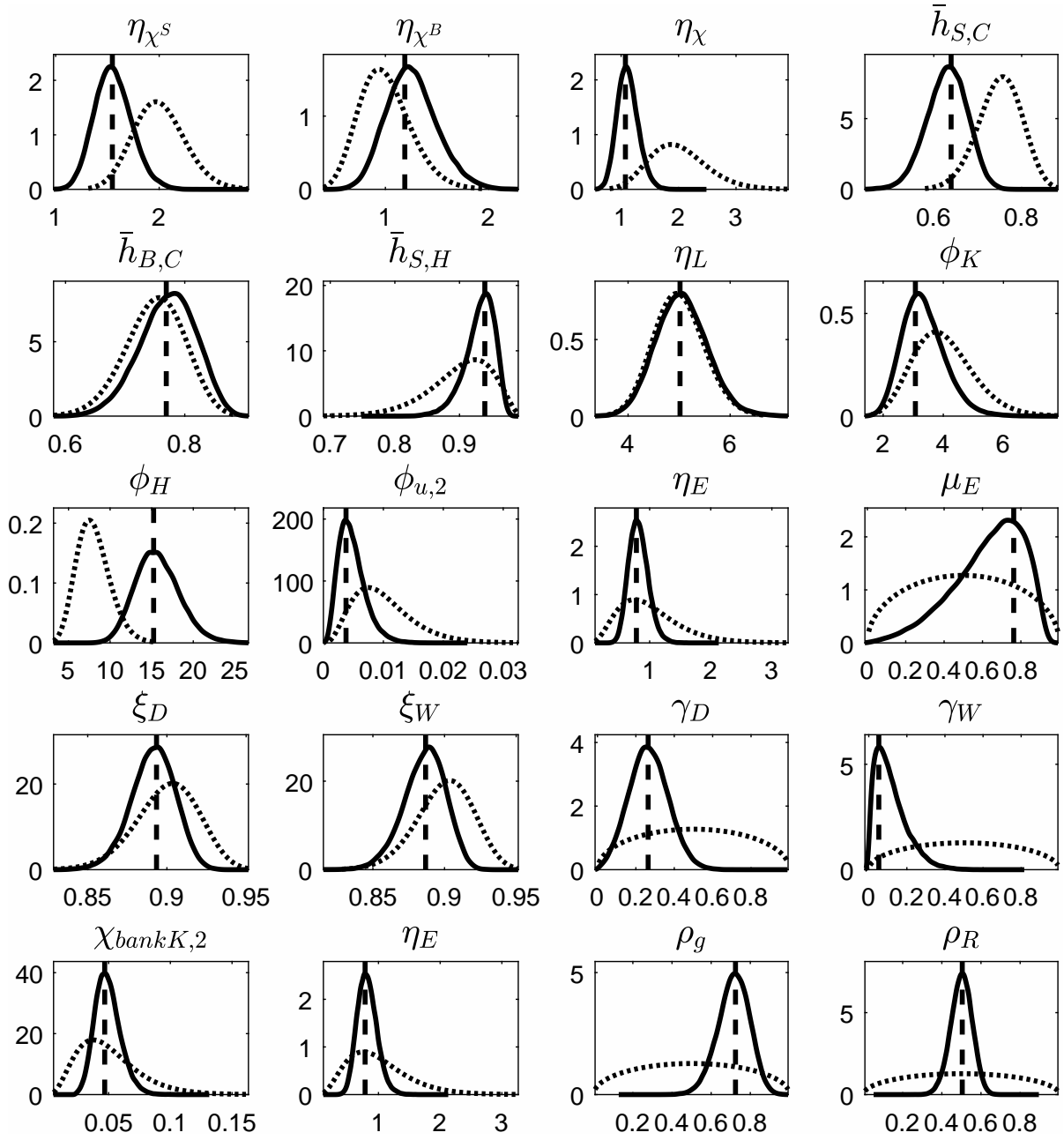


Figure 2: Priors and Posteriors (cont.)

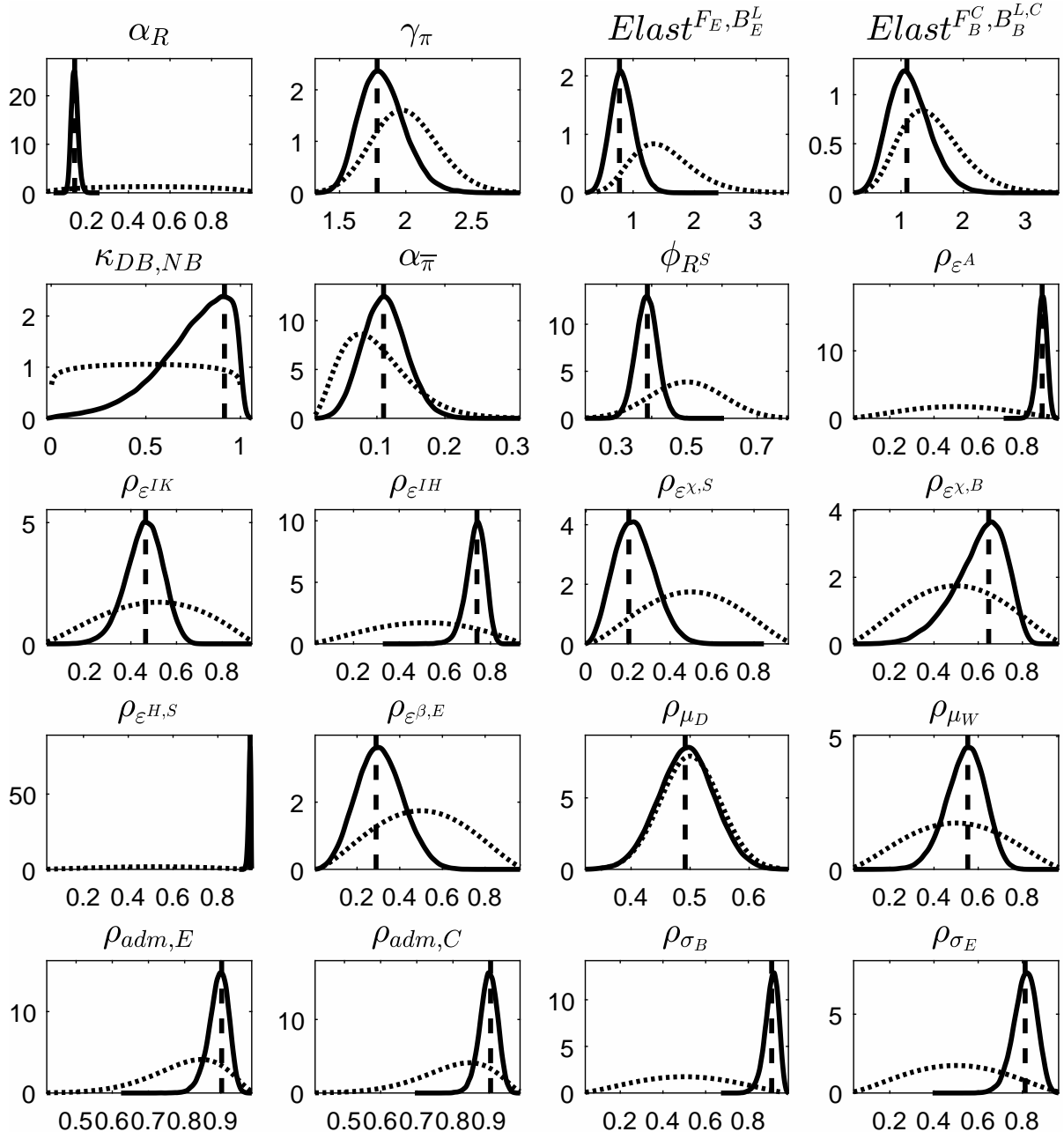


Figure 2: Priors and Posteriors (cont.)

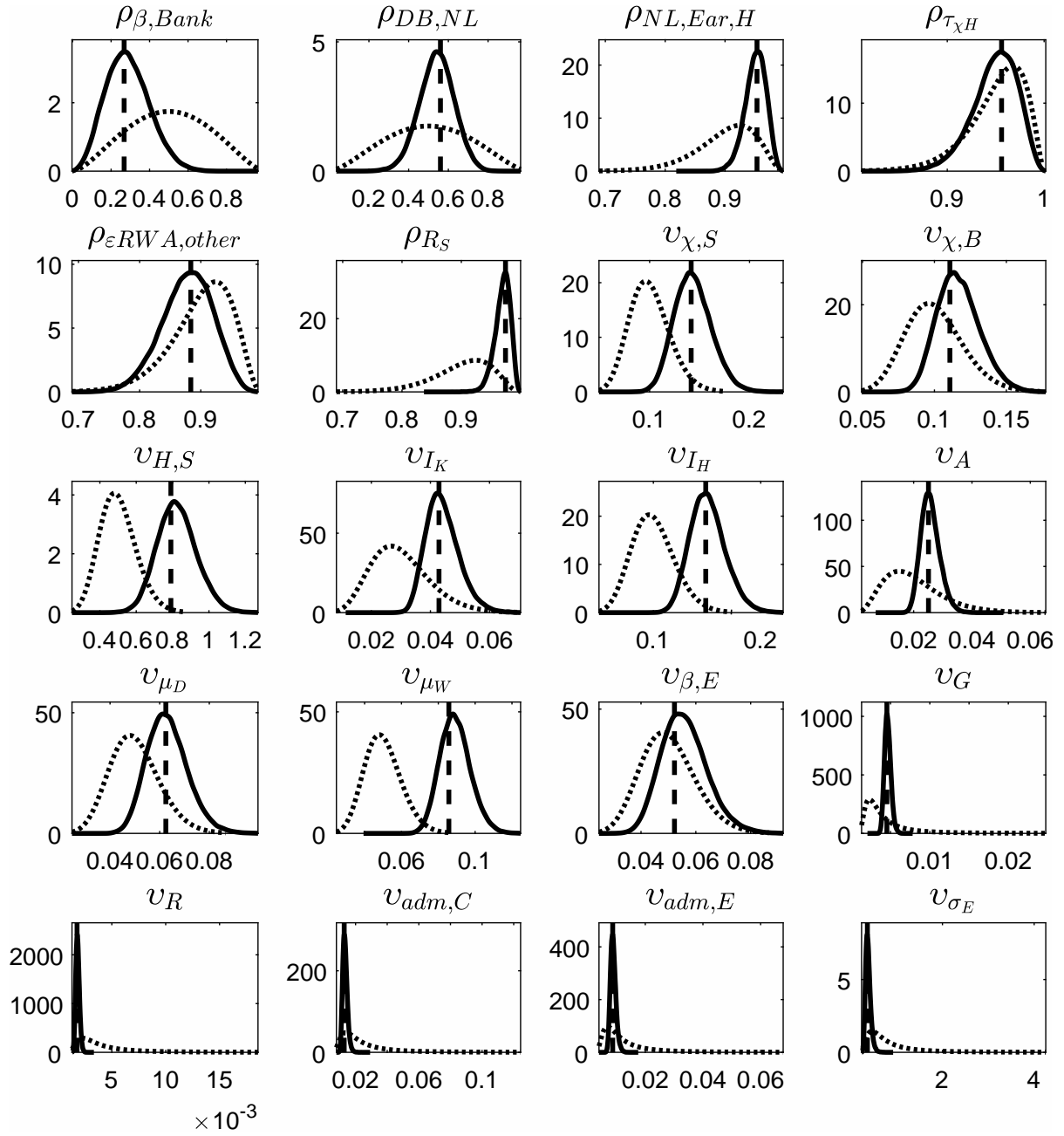


Figure 2: Priors and Posteriors (cont.)

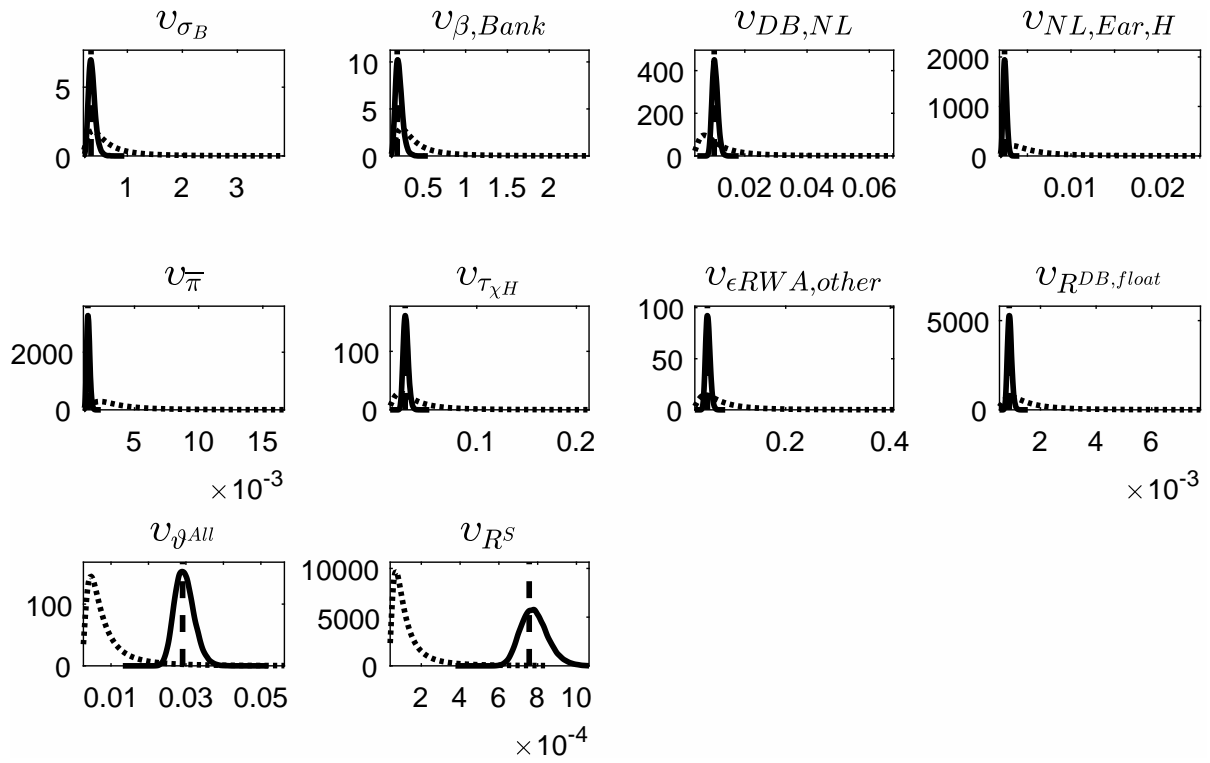
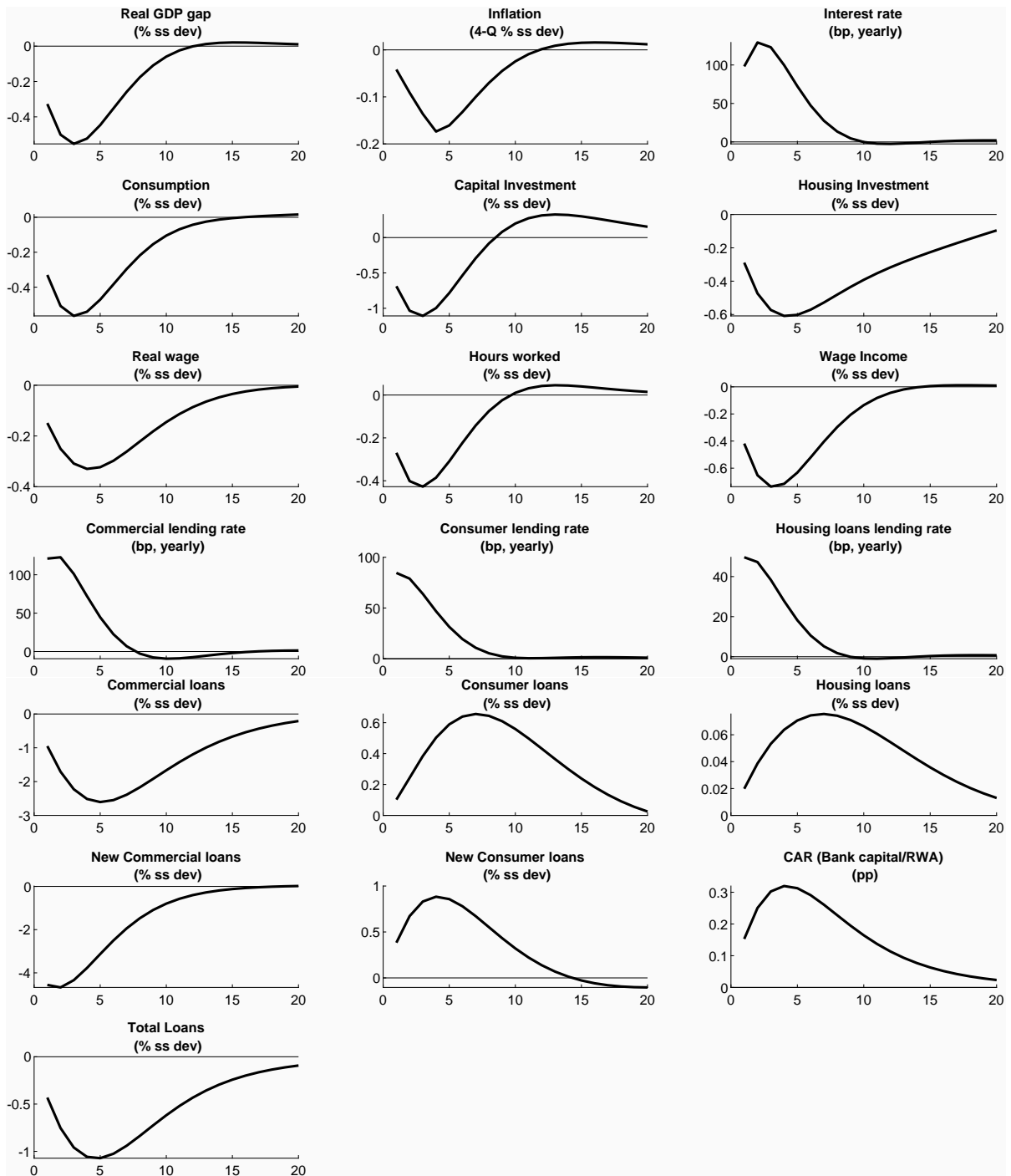
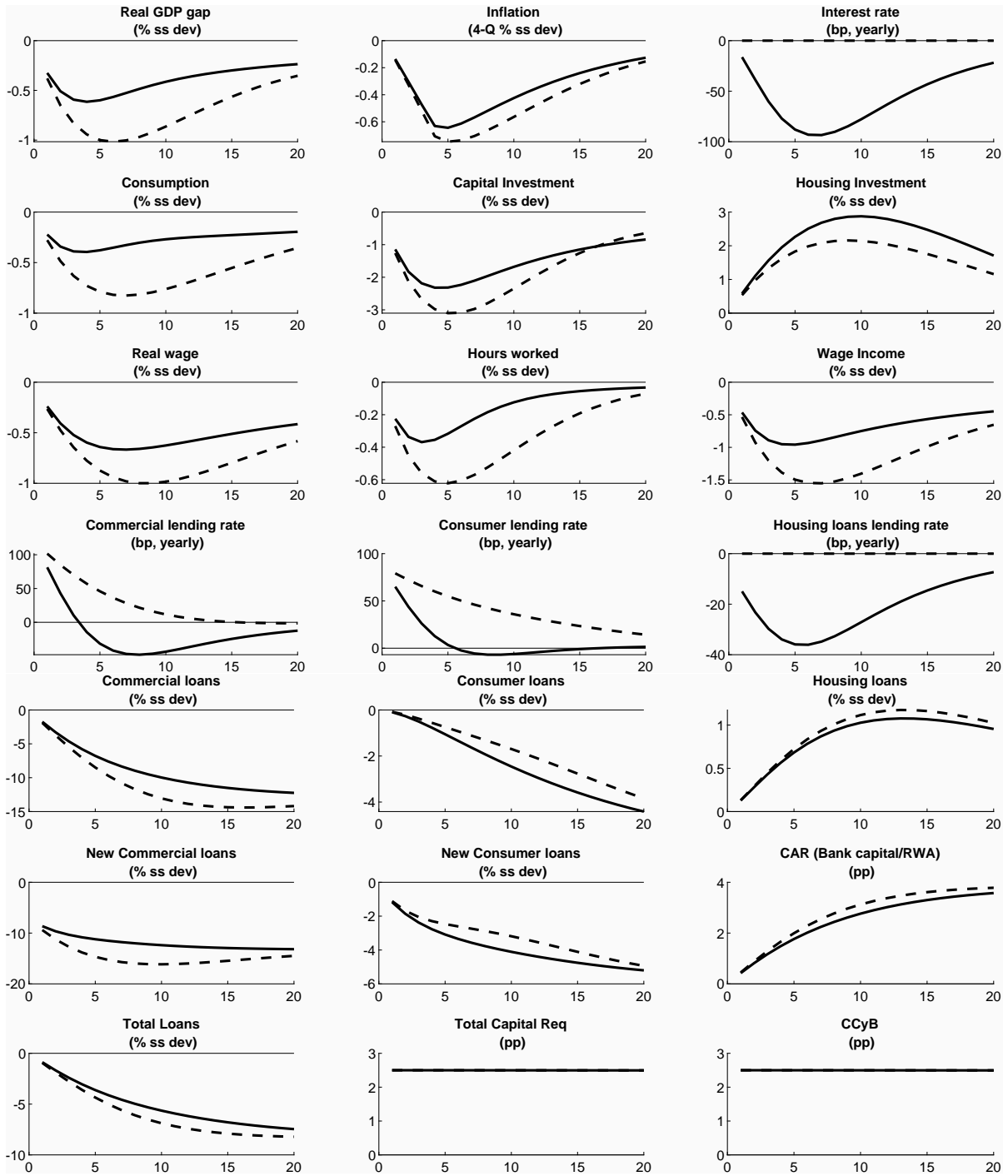


Figure 3: Monetary policy shock impulse responses



Note: loan stocks and new loans figures represent real values.

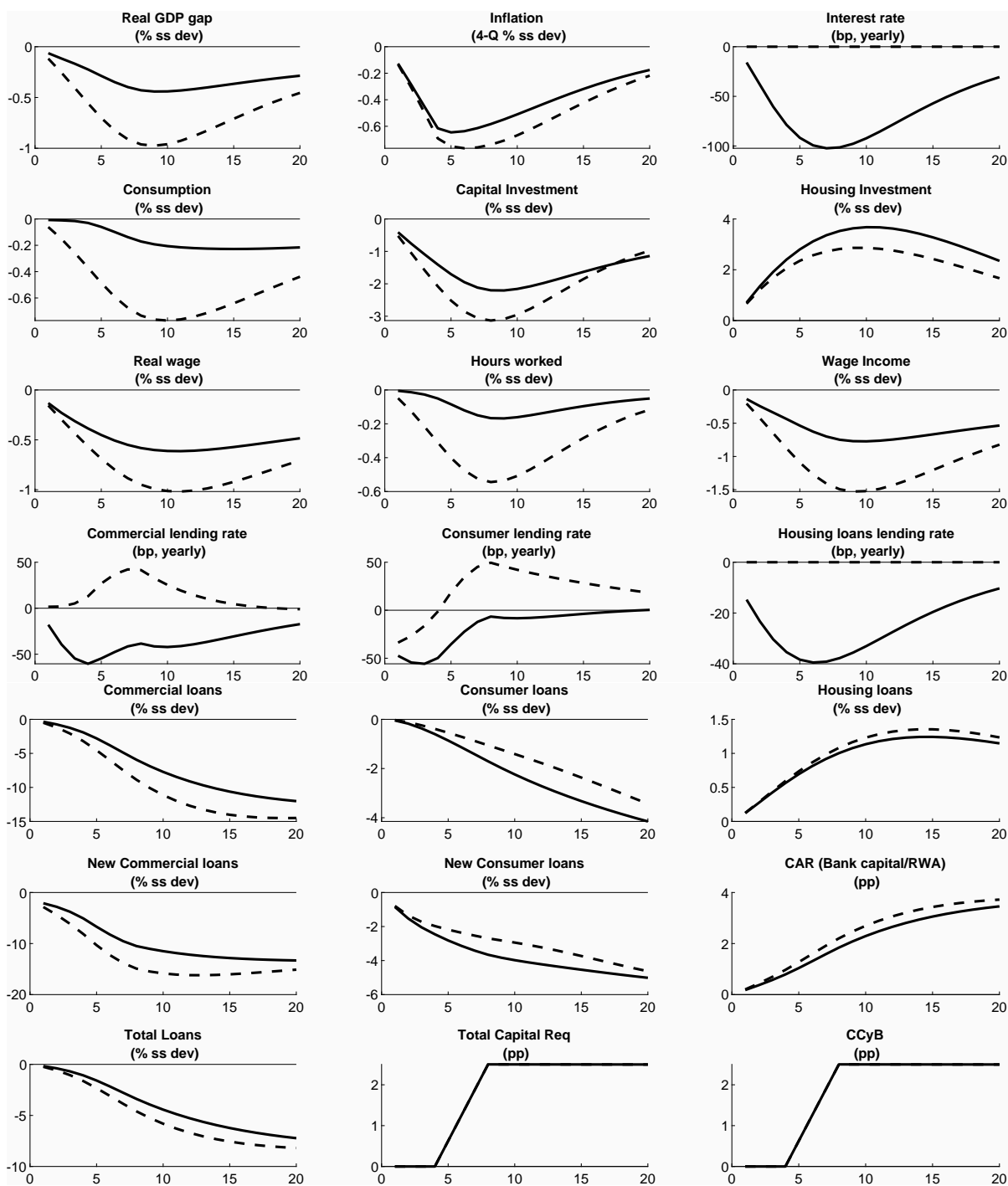
Figure 4: Unanticipated CCyB shock (2.5pp)



Continuous Lines: with monetary policy reaction (base interest rate follows Taylor rule).

Dashed Lines: no monetary policy reaction (base interest rate kept constant).

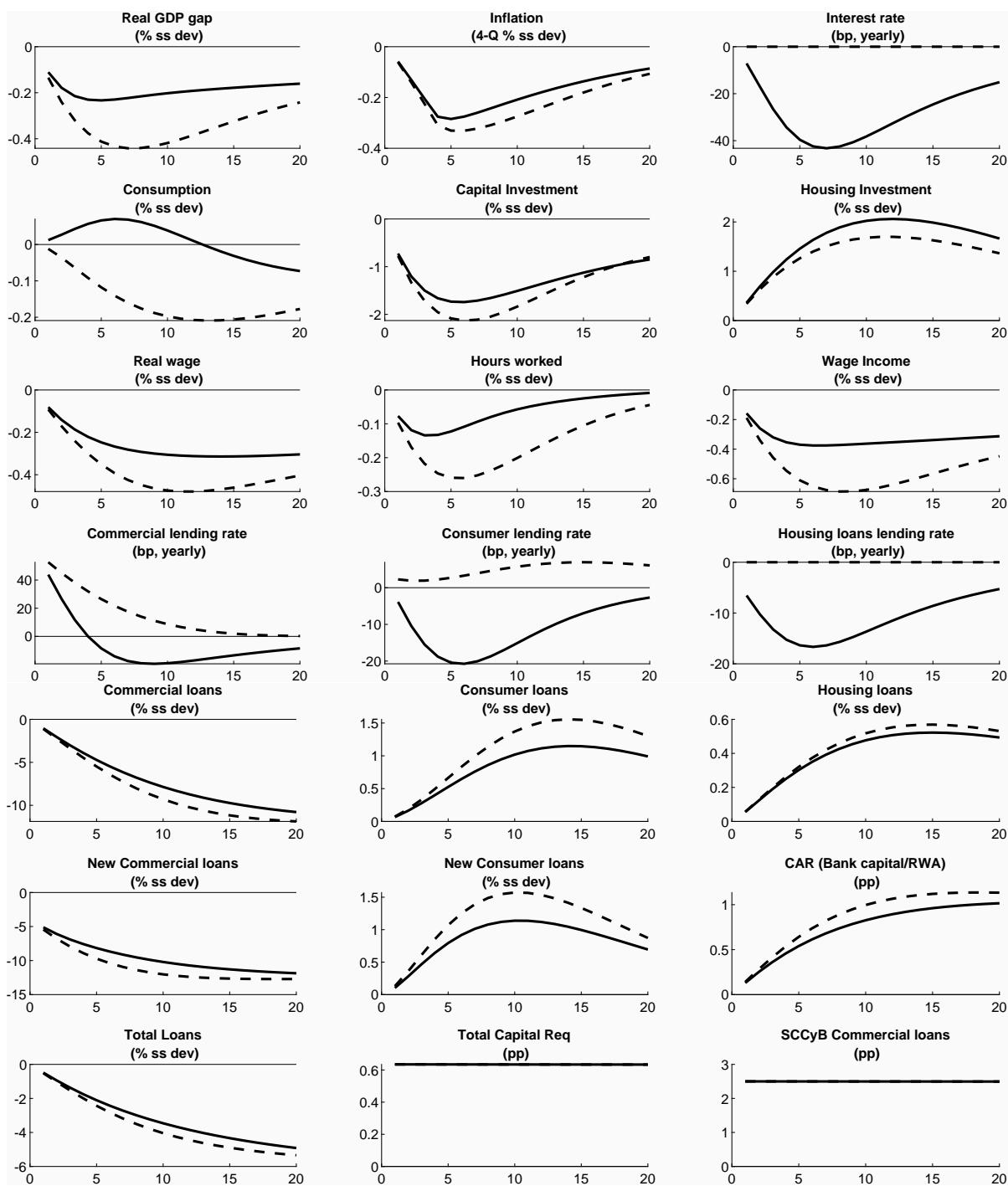
Figure 5: Anticipated CCyB shock (2.5pp)



Continuous Lines: with monetary policy reaction (base interest rate follows Taylor rule).

Dashed Lines: no monetary policy reaction (base interest rate kept constant).

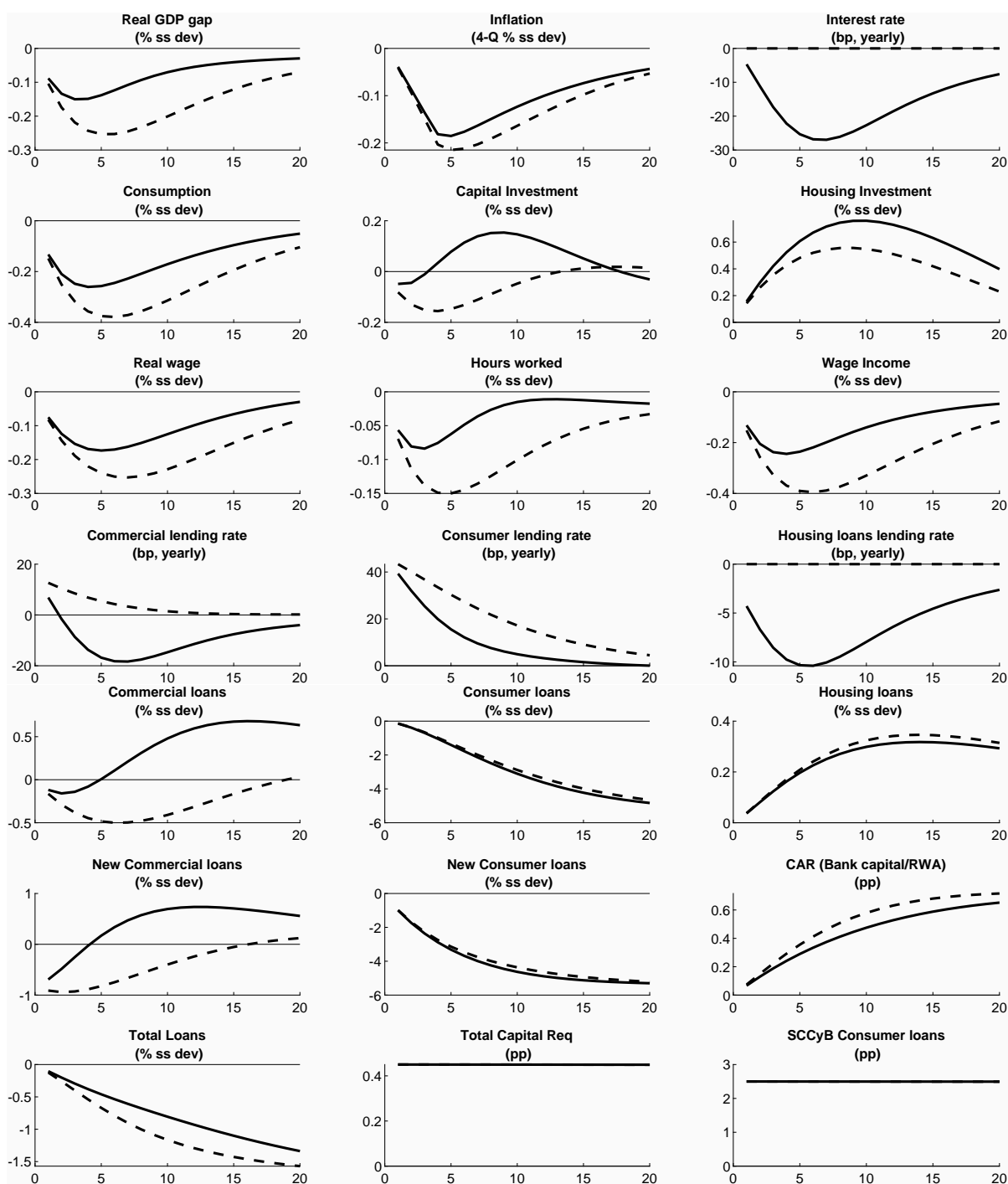
Figure 6: Unanticipated Commercial Loans SCCyB shock (2.5pp)



Continuous Lines: with monetary policy reaction (base interest rate follows Taylor rule).

Dashed Lines: no monetary policy reaction (base interest rate kept constant).

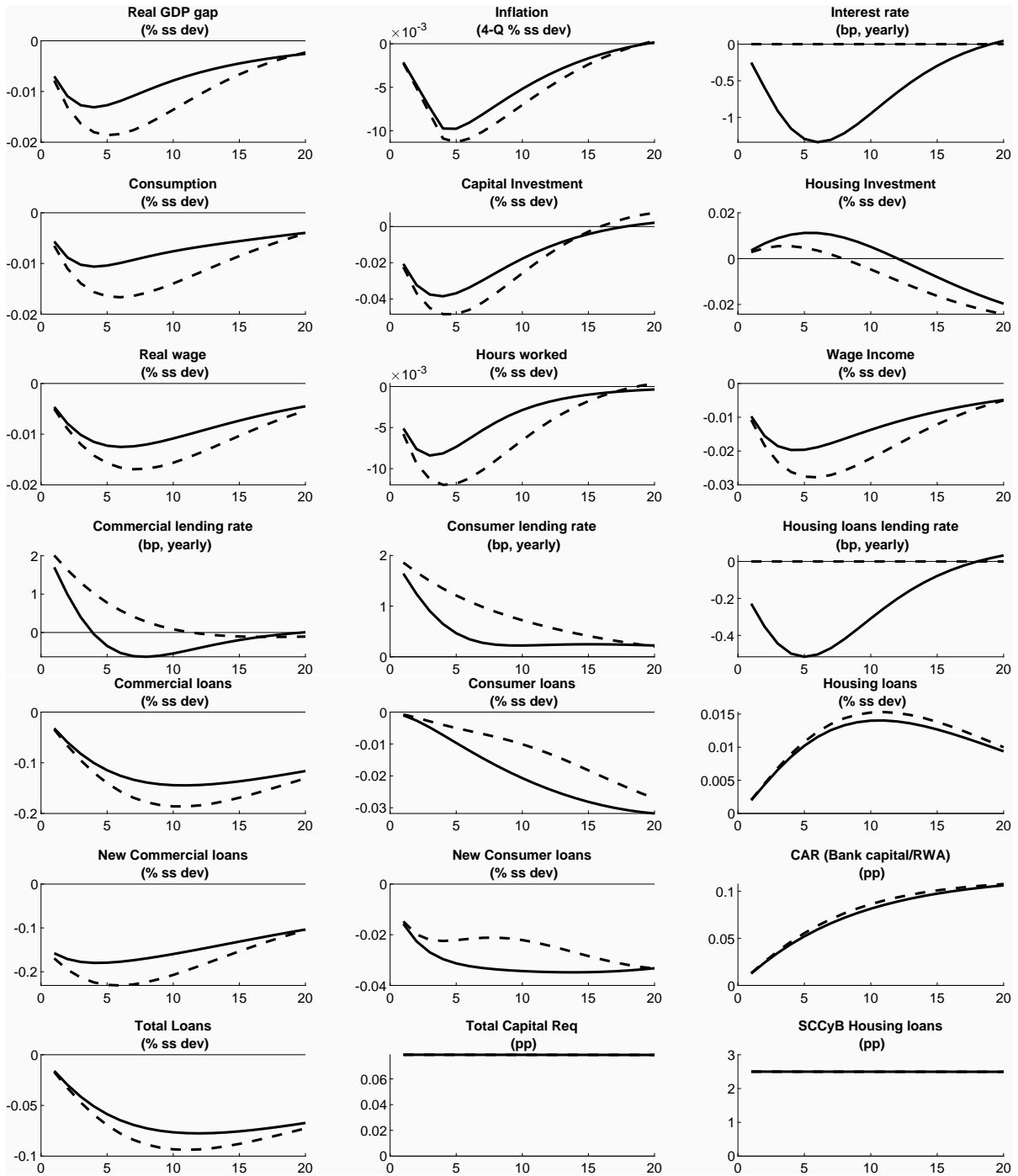
Figure 7: Unanticipated Consumer Loans SCCyB shock (2.5pp)



Continuous Lines: with monetary policy reaction (base interest rate follows Taylor rule).

Dashed Lines: no monetary policy reaction (base interest rate kept constant).

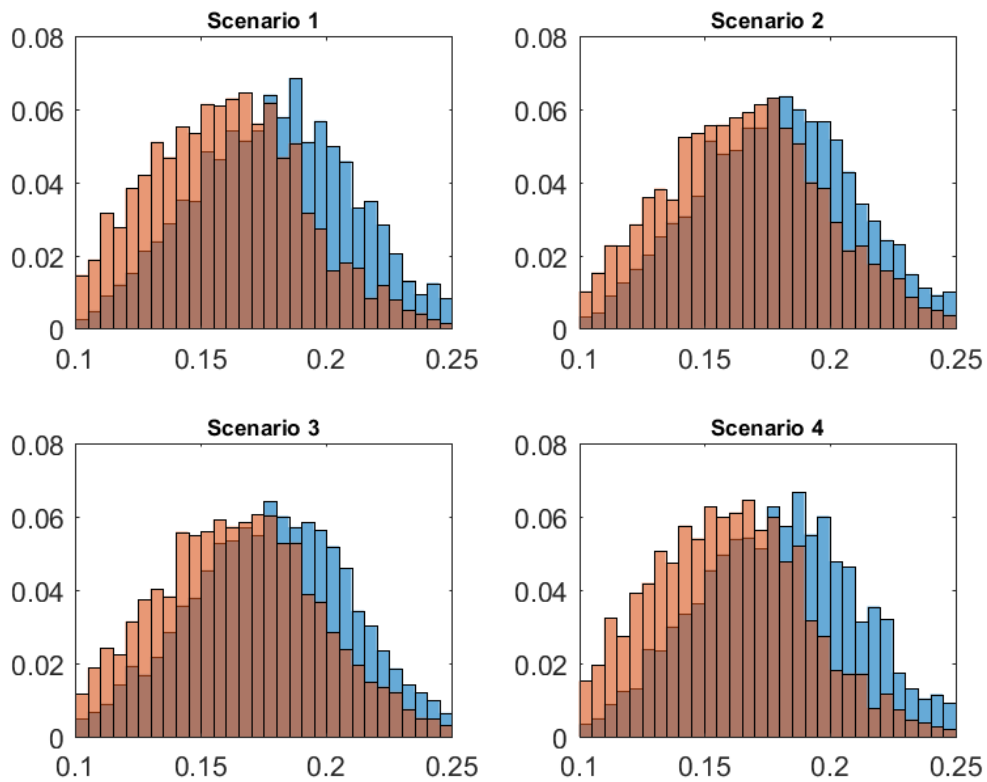
Figure 8: Unanticipated Housing Loans SCCyB shock (2.5pp)



Continuous Lines: with monetary policy reaction (base interest rate follows Taylor rule).

Dashed Lines: no monetary policy reaction (base interest rate kept constant).

Figure 9: Capital Adequacy Ratios (CAR) distribution before buffer releases



Blue bars represent CAR distribution under alternative countercyclical buffer policy rules.
Red bars represent distributions in the respective counterfactual scenario without macroprudential policies.
Scenarios 1 to 4 correspond to policy rules 1 to 4.