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Fiscal Risk and Financial Fragility

Thiago Christiano Silva, Solange Maria Guerra and Benjamin Miranda Tabak

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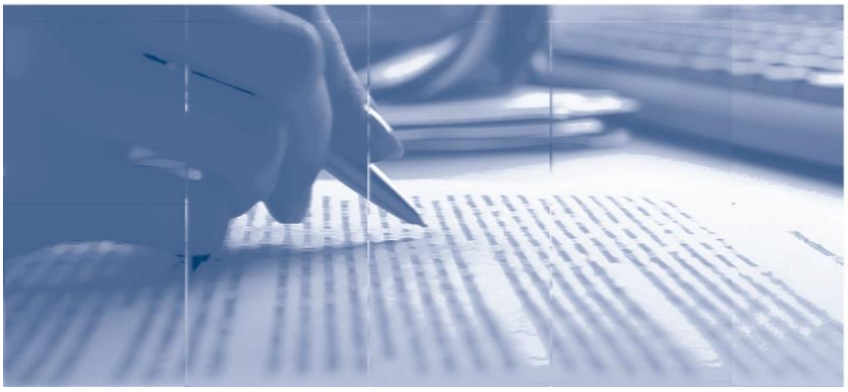

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Non-Technical Summary

Brazilian states have increased their bank indebtedness since 2014. Bank indebtedness coupled with the effects of the recessionary period of the Brazilian economy on revenues aggravated the fiscal situation of the states, leading many of them to exceed the maximum limits of indebtedness imposed by the Fiscal Responsibility Law. Given this momentum of rising bank debt by federated states and the negative macroeconomic conditions in Brazil, it becomes important to understand to what extent financial stability could be compromised.

This paper contributes with the literature on public finance and financial economics in three important points. We first develop a new methodology to estimate the likelihood of noncompliance of states under the legal restrictions imposed by the Fiscal Responsibility Law. We innovate in the sense that the methodology not only uses historical data but also embodies expectations or prospects for real GDP in the future. Second, we use a comprehensive systemic risk model that encompasses the real and financial sector to quantify and understand the resilience of the financial system to credit defaults from Brazilian states. Last, we combine the states' noncompliance probability and their corresponding impact on the financial system to compose a state-specific statistical indicator of expected loss that they would impose to the financial sector.

We find banks are resilient to credit defaults from Brazilian states, notably those under severe fiscal stress. Financial contagion remains small due to the high levels of capitalization that Brazilian banks, on average, maintain. State-owned banks are the most sensitive to credit defaults from Brazilian states, partly because they are the largest creditor banks and because they take central positions in the network structure, thus making them more susceptible to financial contagion.

Sumário Não Técnico

Os estados brasileiros vêm aumentando seu endividamento bancário desde 2014. O endividamento bancário associado aos efeitos do período recessivo da economia brasileira sobre as receitas agrava a situação fiscal dos estados, levando muitos deles a exceder os limites máximos de endividamento impostos pela Lei de Responsabilidade Fiscal. Dados esse crescente endividamento bancário por estados federados e as condições macroeconômicas adversas no Brasil, torna-se importante entender em que medida a estabilidade financeira poderia ser comprometida.

Este artigo contribui com a literatura sobre finanças públicas e economia bancária em três pontos importantes. Primeiramente, desenvolvemos uma nova metodologia para estimar a probabilidade de descumprimento, por parte dos estados, das restrições legais impostas pela Lei de Responsabilidade Fiscal. A metodologia não usa apenas dados históricos, mas também incorpora expectativas ou perspectivas para o PIB real. Em segundo lugar, usamos um modelo abrangente de risco sistêmico que engloba o setor real e financeiro para quantificar a resiliência do sistema financeiro ao *default* de crédito bancário dos estados. Por fim, combinamos a probabilidade de não conformidade dos estados e seu impacto correspondente no sistema financeiro para compor um indicador estatístico de perda esperada que os estados imporiam ao setor financeiro.

Encontramos que os bancos são resilientes a *defaults* de crédito dos estados, inclusive daqueles que estão sob estresse fiscal severo. O contágio financeiro continua baixo devido aos altos níveis de capitalização que os bancos brasileiros, em média, mantêm. Os bancos públicos são os mais sensíveis a *defaults* de crédito dos estados, em parte porque são os maiores bancos credores dos estados e também por ocuparem posições centrais na estrutura da rede, tornando-se mais suscetíveis ao contágio financeiro.

Fiscal Risk and Financial Fragility

Thiago Christiano Silva*

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Benjamin Miranda Tabak***

Abstract

This paper proposes a new methodology to evaluate the importance of fiscal risk to financial stability. We first develop a novel method to estimate the probability of default of public entities, which takes into account a strict legal framework is mandatory for governments. Using options theory, we model the volatile public revenues using country macroeconomic expectations while allowing expenses, which cannot be easily reduced, to grow with inflation. Next, we compute the expected losses due to fiscal risk using a combination of the probability of default with potential losses that the public sector would impose on the economy using a complex network model. Motivated by the crisis on Brazilian states after 2015, we use Brazil to illustrate the usefulness of our model. We estimate the probability of default of states using legal restrictions on consolidated debt and personnel expenses. While most states are struggling to comply with limits on personnel expenses, the richest states have trouble to comply with limits on the consolidated debt. Using a network model that embeds counterparty and funding risks to estimate losses, we find state-owned banks are most likely to be affected if states default on bank credit. Financial contagion is small mostly because the banks that are more exposed to the public sector are highly capitalized.

Keywords: fiscal risk; financial stability; interconnectivity; networks; probability of default; contagion
JEL Classification: G21, G28, C63

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1 Introduction

The crisis in the Euro area highlighted the importance of monitoring the sustainability of the public sector when dealing with financial stability.[1] Magkonis and Tsopanakis (2016) document that there is a strong interconnection between financial and fiscal stress. In this way, unsustainable fiscal conditions can affect banks and, as a result, be further transmitted not just to the entire economy, but also abroad through financial linkages.

Even though fiscal risk can lead to detrimental effects on financial stability, there are no models explaining such relationship while explicitly accounting for the role of financial interconnectedness and the legal framework by which public entities must abide. In order to fill this gap, we develop a novel method to estimate the public entities probability of default and test the resilience of the banking system to credit default by these public entities. Our analysis of probability of default observes the legal rigidities with which governments must continuously comply. We use options theory to model volatile public revenues that depend on country macroeconomic expectations while allowing expenses, which are hard to reduce, to grow with inflation. Then, we evaluate the probability of default in terms of the fiscal sustainability of the public entity by looking at the balance of revenues and expenses. We implement our model to the Brazilian case at the state level (27 states). Brazil provides an ideal case study because fiscal sustainability in Brazil is heterogeneous among different states, some of which are struggling to meet minimum requirements while others have well-managed public finances.

We use a network approach to estimate the resilience of the banking system to credit defaults of public entities. Our network encompasses the interbank market (bank-bank network) and bank lending to the real sector (bank-firm network). Each link connecting banks and firms represents some kind of financial fragility, either counterparty risk (credit default) or funding risk (risk of not rolling over short-term debt). The default of a public entity causes losses to creditor banks and, as a result, to the rest of the interbank market through successive credit deterioration and to the real sector through credit crunches. We evaluate the resilience of the banking system by analyzing to what extent banks are able to absorb losses and not go bankrupt.

Our work extends a variety of papers that assess financial distress for municipalities in different countries, such as Greece (Cohen et al. (2012)) and France (Galariotis et al. (2016)). Our results can also be used in the construction of performance measures for different states and to improve discussions on transfers from the federal government to states in financial distress (Lin et al. (2011)). We propose a methodological contribution to assess financial distress in different states, that takes

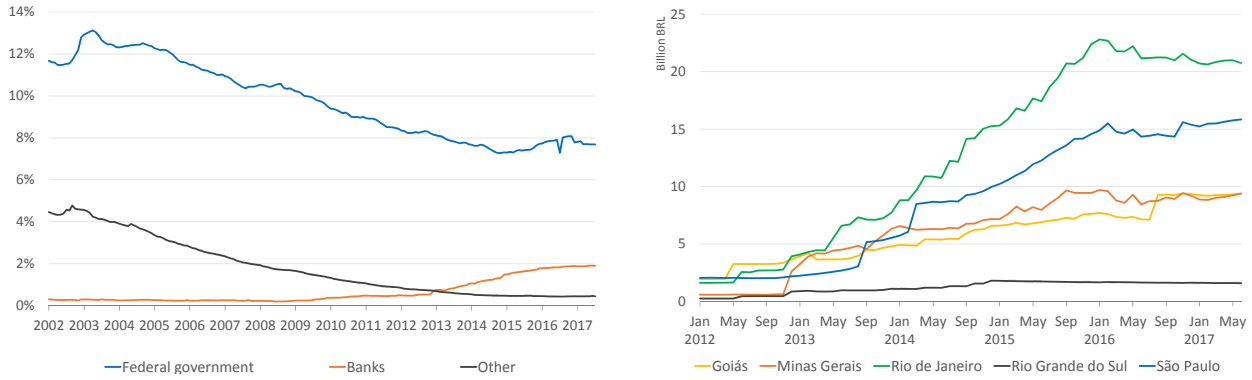
[1] Problems due to the fiscal unsustainability have become more severe across countries after the global financial crisis. The ratio of government debt to GDP in the developed world has risen above 100% of GDP in the 2010s, from less than 75% prior to the global financial crisis. Increased public indebtedness can also be explained by country social characteristics. For instance, long-term sustainability of public finances of some countries is being threatened in view of increased pension and health care liabilities.

into account the legislative framework that binds local government's decisions[2].

The Brazilian case is of uttermost importance to illustrate the usefulness of our model. The crisis in the state of Rio de Janeiro after 2015, also experienced by other neighboring and economically dependent states and even by the federal government, highlights the weakness of public finances at both state and federal levels in Brazil. Such fragility becomes evident when macroeconomic conditions deteriorate, such as during the 2014 Brazilian recession, when states struggled to repay their debt and to provide basic services to society.

High levels of public debt is a recurrent issue faced by Brazilian states. After 2000, the Fiscal Responsibility Law (FRL) came into force and imposed limits on public debt that would be achieved through responsible fiscal management and transparency to the society. As evidenced by Figure 1a, the FRL was effective as consolidated debt of Brazilian states fell until 2013, when Brazil again experienced adverse macroeconomic conditions triggered by a new recession.

Such negative event started in 2014 and took its toll on Brazilian states, which had to resort to bank credit as a way to maintain necessary public investments. Figure 1a makes clear this change of debt dynamics: the consolidated debt started to rise again, mainly as a consequence of bank credit. Figure 1b depicts the evolution of bank debt broken down by Brazilian states, which confirms that such phenomenon of rising bank debt was generalized across the wealthiest Brazilian states.



(a) *Consolidated debt*

(b) *Outstanding bank debt of states*

Figure 1: *(a) Consolidated debt of states as a percentage of the Brazilian GDP from 2002 to 2017 broken down by credit from banks and from the federal government, and other debts. (b) Outstanding bank debt of five Brazilian states that are facing fiscal difficulties. The curve in (b) is constructed using 5 states out of the 27 that compose the total credit from banks shown in (a). We only report data from 2012 onwards because such cutoff point represents the beginning of rising bank debt of Brazilian states.*

The rising exposure of the financial system to the public sector in Brazil is an important risk channel for financial stability due to numerous reasons. First, banks play a central role in the economy and thus are strongly connected to different market segments. As such, localized credit shocks arising from the public sector can develop into system-wide events due to spillover effects mediated by the banking sector. For instance, these credit shocks can cause losses to banks, which could reflect in

[2]See also Liu et al. (2015) for a different model to assess future defaulters.

credit crunches for the real sector (bank lending channel).[3] Second, capital markets in Brazil are underdeveloped, in a way that firms would be significantly affected by a sudden reduction of bank credit. Therefore, it becomes important to analyze how resilient the financial system is to credit shocks from the public sector.

The rising bank debt coupled with the effects of the recessionary period of the Brazilian economy on revenues aggravated the fiscal situation of the states, leading many of them to exceed the maximum limits of public debt imposed by the FRL. In addition, most of the public expenses are inertial and cannot be easily reduced by the government due to legal restrictions, such as public staff expenditure, including pension and social security.

Given this momentum of rising bank debt by states, their inability to easily reduce expenses, and their poor revenues given by the weak macroeconomic performance in Brazil, it becomes important to understand to what extent financial stability could be compromised. Despite the importance for public provisioning and government decision-making, there are no theoretical models that allow the calculation of these fiscal risks under a legal framework that binds state debt growth to sustainable levels.

This paper seeks to fill this theoretical–empirical gap and contributes with the literature on public finance and financial economics in three important points. The first contribution is the development of a new methodology to estimate the likelihood of non-compliance of states with the legal restrictions imposed by the FRL. The great innovation of this methodology is that it uses not only historical data but also incorporates expectations of macroeconomic conditions. We estimate expected revenues under specific macroeconomic conditions and inspect how they cope with government planned expenses. The second contribution is the quantification of the impact that the states would bring on the Brazilian financial system if budget availability ceases and bank credit is not honored. We use a systemic risk model that encompasses banks and firms. Third, we combine the states non-compliance probability and their corresponding impact on the financial system to compose a state-specific statistical indicator of expected loss that they would impose on the financial sector.[4]

Our statistical indicator evaluates the extent of damage that each state would cause to the financial system using two components: the non-compliance probability and the losses banks would incur in the case of a default. It may be the case that a specific state can cause large losses to the financial system. Even so, our indicator would yield a small number if its non-compliance probability is negligible. Our indicator not only captures the damage level to the financial system from a default,

[3] Another classic example comes from the subprime crisis in 2008. The shocks originally started inside the banking system, leading to a massive destruction of bank assets. However, later, they also heavily affected external markets, with relevant real effects.

[4] See Battiston and Martinez-Jaramillo (2018) for a discussion on the importance of the use of networks to unveil systemic risk. We contribute using a new approach to show the relationship between fiscal risk and financial stability. See also Roukny et al. (2018), Berndsen et al. (2018), Corsi et al. (2018), Cajueiro and Tabak (2008), Guerra et al. (2016), Kanno (2015), Silva et al. (2017c) and Souza et al. (2015).

but it also brings the likelihood of such event.

The Brazilian National Treasury estimates the fiscal health of Brazilian states through a rating system that looks at financial and economic indicators.[5] The method uses data from current year and also the previous two years and combine them using a decay weighting scheme. In this way, it performs a backward-looking analysis to assign the fiscal health rating, which we can conceptualize as proportional to the probability of insolvency of that state in the last three years.

Our methodology estimates the fiscal health of states in a more comprehensive way. We estimate how likely a state is to violate at least one of the following FRL financial limits: consolidated debt or personnel expenses. The violation has legal consequences both to states and to responsible managers. In this way, these limits are carefully watched by states and are worth being modeled. We model the probability of non-compliance using the contingent assets model and option theory (Merton (1974)). Our probability of non-compliance has a prospective component in that it is conditional on changes in the real GDP in the near future.[6] As a practical example, our method is able to evaluate how a hypothetical deepening of the Brazilian recession would affect fiscal risk of states.

The dependence of the probability of non-compliance on the future real GDP allows us to construct stress scenarios to evaluate the resilience of states fiscal conditions. If a given scenario is likely to occur, it is possible to take measures to reduce the potential impact—even before it occurs—from a prudential perspective. In this manner, the model allows us to think about fiscal risk in terms of legal compliance and balance-sheet deterioration of states and thus provides prospective insights for conducting public policies.

We also estimate the resilience of the financial system to credit shocks from states. Due to the very rich data sets maintained by the Central Bank of Brazil, we are able to map out every credit operation of the entire financial and real sectors. The model takes into account the network of loans between financial institutions and federated entities, the interbank market, and also credit operations that banks provide to firms. With this loan-level data, we are able to evaluate the systemic importance of each federated entity to the financial system by verifying the amount of losses that it would impose on the economy if the federated entity defaults on its outstanding bank credit.

Our model uses microdata to construct the network of exposures among banks and firms. In this way, we are able to analyze possible paths through which shocks can propagate and amplify, either inside or outside of the financial system. The model also allows us to think about the phenomenon of financial stability in a broader way by also bringing in the role of the government besides banks and firms. Shocks in state finances can negatively reflect on the financial system, which in turn can

[5] The methodology looks at eight financial and economic indicators: public debt, debt service in net current income, primary debt service result, personnel expenses and social charges in net current income, generation of own savings, participation of investments in total expenses, participation of Welfare Policy contributions and remunerations in social security expenditures, tax revenues in costing expenses.

[6] We model the change in states expected revenue as a function of real GDP, after controlling for other macro variables, such as the exchange rate, interest rate (Selic) and inflation.

amplify these shocks through credit crunches to the real sector and repricing mechanisms. Less credit to the real sector induces lower investment, which can impact firm revenues and thus affect the amount of taxes that the government collects. It is thus important to also analyze the fiscal side of federated entities when dealing with financial stability.

Despite the relevance of federated entities to the financial system, it is unclear whether the Brazilian financial system would be resilient in the scenario of rising fiscal risk of these entities. Our model sheds light on this issue by not only estimating the level of resilience of the financial system to credit defaults from the public sector, but also highlighting which groups of banks would be most affected. We find that state-owned banks would be most affected if states default on bank credit. Financial contagion is small mostly due to the fact that the most exposed banks to the public sector are highly capitalized.

Our methodology is not restricted to the particularities of the Brazilian legislation. In addition, it is not limited to modeling states fiscal fragility, as it can also trivially support modelling at country or municipal levels. The extension point of our methodology lies in modeling the legal limits inside the contingent assets approach. For instance, we could use our methodology to the Euro area with the following modifications: First, we model the balance sheets of groups of countries instead of states in a same country. Second, we model legal limits on maximum budget deficit and public debt in relation to GDP, which are two legal restrictions with which Euro countries must continuously comply.

The paper proceeds as follows. Section 2 discusses the state-of-the-art literature on systemic risk and financial stability models. Section 3 compiles relevant background regarding Brazilian states and how their debt largely increased in the last decades. Section 4 details our systemic risk methodology to gauge fiscal risk. Sections 5 and 6 show our applied work using Brazilian data, in which we describe the data and results, respectively. Section 7 draws our conclusions.

2 Related literature

Economies and financial systems are highly interconnected. The global financial crisis of 2008 is a clear example, in which we saw how localized shocks unfolded into system-wide events that put at stake the financial stability of several economies (Gai et al. (2011)). The shock propagation not only depends on the interconnection patterns among economic agents but also on the legal environment. The lack of proper regulation had a critical role in the extent of loss amplification worldwide. Regulatory framework in place before the global financial crisis was preponderantly microprudential, which mainly aimed at preventing the costly failure of financial institutions and thus, by construction, focused on the partial equilibrium of the system. After the crisis, macroprudential approaches have gained importance as they look at general equilibrium effects and seek to safeguard the economy in its entirety (Arnold et al. (2012); Hanson et al. (2011)).

The financial system can be conveniently expressed in a network (directed graph), which partly explains the rapid increase of network-based methods in the systemic risk and financial stability literature after the global financial crisis. Systemic risk estimation using network models broadly falls into two categories: loss- (Eisenberg and Noe (2001)) and stress-based (Battiston et al. (2012)) methods.[7] While loss-based models are useful to identify the real consequences of large shocks or bank defaults in a financial system, stress-based models are important quantitative tools to understand systemic risk buildup. The second methodology is more sensitive to small shocks and are more suitable when banks become financially distressed but do not default. Our paper proposes a stress-based method to understand fiscal risk in the Brazilian economy.

Network-based models are useful as stress-testing tools for regulators (Anand et al. (2018); Glasserman and Young (2015)). Given the current state of the economy, we can easily design counterfactual scenarios to assess how the current financial network can amplify exogenous shocks. To date, there are no models that analyze the risks to the financial system posed by fiscal problems of federated entities using complex networks theory. Our paper adds to this literature in that we analyze how interconnections of the financial system to public entities take place and affect the economy in a hypothetical stressed scenario of rising fiscal risk.

Our paper is also closely related to the literature of fiscal sustainability and financial stress. Magkonis and Tsopanakis (2016) highlight the need for a broad array of fiscal conditions indicators to measure fiscal sustainability. In this sense, most indicators essentially look at historical data.[8] In this paper, we construct a forward-looking probability of default that not only considers historical data but also has a prospective component. We use a contingent assets approach that is conditioned on GDP growth prospects to quantify the fragility of public finances.

3 Background: the rising public debt of Brazilian states

In this section, we provide an overview of how states public debt evolved in the last decades and the measures that the federal government took in an attempt to ensure public debt sustainability.

Until the mid-1990s, Brazil experienced a persistent inflationary process that culminated in hyperinflation. During this period, public debt control was not stringent and states and municipalities made use of inflationary revenue to obtain primary surpluses.[9] After the implementation of the *Plano Real* in 1994, inflation stabilized and fiscal problems of states became more evident.

[7] Several extensions have been proposed in the network modeling for systemic risk after these seminal papers. We highlight the models in Anand et al. (2015); Aoyama (2014); Bardoscia et al. (2015); Battiston et al. (2016); Blume et al. (2013); Hojman and Szeidl (2008); Lux (2015); Poledna et al. (2015); Silva et al. (2017a, 2016b, 2017b); Souza et al. (2016).

[8] Classic indicators include the overall fiscal balance to GDP ratio and net financial debt to GDP.

[9] In this case, revenues kept increasing due to the high inflation while expenses, mainly administrative costs, such as personnel expenses, would not follow the same trajectory.

The states debt rose sharply due to the reduction of inflationary revenue and to the high interest rates. Consequently, states with high public debt had problems to roll over their debt in the financial market. To avoid states defaults, the federal government stepped in to restore their fiscal health. On the one hand, the federal government would assume the states debt towards the financial market and therefore would became their new creditor. In this debt agreement, the federal government extended the debt maturity up to 30 years and established lower interest rates. On the other hand, the states would commit to restructuring their finances to obtain primary surpluses. There were also agreements on privatization of state-owned firms, including banks.

The high public debt of states has roots in the management inefficiencies regarding debt sustainability. After the debt restructuring of Brazilian states, the federal government took measures to enforce structural changes to ensure public debt control. To this end, the FRL came into force in 2000 and imposed on public entities new protocols, responsible fiscal management, and prudential rules to debt growth.

The FRL established guidelines for budget planning, required the fulfillment of fiscal targets, and enhanced transparency of the entire budget execution process to society. Among the forbidden actions that would cause state indebtedness, the FRL prohibited new credit operations of public entities with controlled companies. Not only public entities that are non-compliant with the FRL are subject to penalties but also the responsible managers, who can face fines, impeachment and even imprisonment. This accountability of public finances had profound impact on public budget execution.

In addition to the guidelines and actions aimed at improving new public management practices, the FRL imposed limits on personnel expenses[10] and on the levels of public debt of the federal government, states and municipalities. In the case of states, the most important limits are:

- *level of public debt*: the states consolidated debt cannot exceed 200% of their net current revenue.
- *personnel expenses*: states cannot spend more than 60% of the net current revenue with personnel expenses: 49% for the Executive branch, 3% for the Legislative branch, 6% for the Judiciary and 2% for the public prosecution services.

If states violate one of these legal limits, they must take steps to recomply or otherwise will suffer sanctions, such as the impossibility of receiving further voluntary transfers from the federal government and of engaging in new bank credit operations.

One difficulty for complying with these legal limits is that states are not allowed to freely maneuver their budget. For instance, social and political costs allied with the rigidity of the legislation make it difficult for states to reduce their expenses. In addition, the FRL prohibited the further in-

[10]The reduction of personnel costs by public entities faces a number of obstacles, since they cannot reduce salaries and civil servants can be fired or dismissed only in exceptional cases that are listed in the law.

crease of their public debt.

When Brazil entered a recession in 2014,[11] which led to a strong reduction in the state revenues, the fiscal situation of the most highly indebted states became critical. Some states even delayed the payment of salaries to their employees, reduced the quality of public services, and substantially reduced their investments. In view of this negative macroeconomic scenario, the federal government established a fiscal agreement with states to rebalance the growing states public debt in 2016.

4 Methodology

In this section, we discuss the intuition and the underpinnings of our methodology.

4.1 Probability of non-compliance of states

We model the potential non-compliance of states with the FRL legal limits using an adaptation of the Merton (1974)'s contingent assets approach. In our approach, we consider the balance sheet of states instead of firms with some methodological modifications that reflect the specific nature of public entities.

In the original model, Merton (1974) models firm's total assets as an European purchase option that matures T periods ahead, with an exercise price equal to its obligations. If the company goes bankrupt, shareholders get nothing. Otherwise, shareholders receive the difference between the values of assets and obligations of the firm. We instead model the evolution of revenues, expenses and debt of states while also account for the legal restrictions of the FRL.

We model two FRL legal restrictions with which states must comply: (i) the consolidated debt against net current revenue (CD/NCR), which cannot exceed 200% and (ii) personnel expenses against net current revenue (PE/NCR), which cannot go above 49%.[12] In the following, we describe the way we estimate the evolution of states revenues, expenses, and debt, which will then be used to monitor the potential non-compliance of states with the two FRL legal restrictions.

Evolution of revenues: Following the recurrent challenges faced by states, we consider that there is uncertainty about the evolution of the net current revenue (NCR) over time. We attribute changes of the net current revenue to a certainty component (drift) and to an uncertainty component (random or stochastic term). The drift represents the average expected value of the revenue growth. The stochastic term is a random walk in which the variance captures unexpected events that impact states

[11] According to the Brazilian Business Cycle Dating Committee (CODACE) from the *Fundação Getúlio Vargas*, which establishes reference chronologies for the Brazilian economic cycles, the recession started in the second quarter of 2014 and ended in the forth quarter of 2016 (11 quarters), reaching a cumulative growth from peak to trough of -8.6%.

[12] The FRL establishes as 60% the limit of personnel expenses of the entire state (Executive, Legislative, Judiciary). Due to data unavailability, we only model the limit imposed on the Executive, which is 49%.

revenues, such as recessions and booms. Thus, the evolution dynamics of the net current revenue follows a diffusion process:

$$\frac{dNCR}{NCR} = \mu_{NCR}dt + \sigma_{NCR}dZ \tag{1}$$

in which μ_{NCR} is the NCR growth rate (drift), σ_{NCR} is the NCR volatility, and dZ is a Wiener process normally distributed with zero mean and unit variance. We assume the drift term is the current inflation. For this process, we evaluate the NCR value t periods ahead as follows:

$$NCR_t = NCR_0 \exp\left[\left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t + \sigma_{NCR}\varepsilon\sqrt{t}\right] \tag{2}$$

in which NCR_0 is the current net current revenue and ε is the realization of a normal random variable with zero mean and unit variance. While NCR_0 is deterministic, NCR_t is stochastic in view of the Wiener process dZ.

Evolution of debt and expenses: In contrast, we model the evolution of the consolidated debt (CD) and personnel expenses (PE) in a deterministic way. We do so because states are not allowed to freely maneuver their budget due to stringencies in the legislation, in such a way that the evolution of debt and expenses tends to be much less volatile and more predictable than that of revenues. Therefore, we consider these terms have only a drift term μ_{CD}, μ_{PE}, which we model as the current inflation. Mathematically,

$$CD_t = CD_0 \exp\left[\mu_{CD}t\right], \tag{3}$$

$$PE_t = PE_0 \exp\left[\mu_{PE}t\right], \tag{4}$$

i.e., the t-step ahead consolidated debt CD_t and personnel expenses PE_t are simply given by the current consolidated debt CD_0 and personnel expenses PE_0, respectively, corrected by the current inflation.

Probability of non-compliance with CD/NCR: We estimate the probability that the legal limit CD/NCR is violated t periods ahead, i.e., $\mathbb{P}\left(\frac{CD_t}{NCR_t} > 200\%\right)$, using (2) and (3) as follows:

$$
\begin{aligned}
\mathbb{P}\left(\frac{CD_t}{NCR_t} > 200\%\right) &= \mathbb{P}(CD_t > 2NCR_t) \\
&= \mathbb{P}\left(CD_t > 2NCR_0 \exp\left[\left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t + \sigma_{NCR}\varepsilon\sqrt{t}\right]\right) \\
&= \mathbb{P}\left(\log\left(\frac{CD_t}{2NCR_0}\right) > \left[\left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t + \sigma_{NCR}\varepsilon\sqrt{t}\right]\right) \\
&= \mathbb{P}\left(\log\left(\frac{CD_t}{2NCR_0}\right) - \left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t > \sigma_{NCR}\varepsilon\sqrt{t}\right) \\
&= \mathbb{P}\left(-\frac{\log\left(\frac{2NCR_0}{CD_t}\right) + \left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t}{\sigma_{NCR}\sqrt{t}} > \varepsilon\right) \qquad (5)
\end{aligned}
$$

If $d_{\mathrm{CD}}(t) = \frac{\log\left(\frac{2NCR_0}{CD_t}\right) + \left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t}{\sigma_{NCR}\sqrt{t}}$, and recalling that $\varepsilon \sim \mathcal{N}(0,1)$, the probability of not complying with the legal limit $\frac{CD_t}{NCR_t} > 200\%$ t periods after—denoted here by $PN_{\mathrm{CD}}(t)$—is:

$$
\begin{aligned}
PN_{\mathrm{CD}}(t) &= \mathbb{P}\left(-d_{\mathrm{CD}}(t) > \varepsilon\right) \\
&= \mathcal{N}\left(-d_{\mathrm{CD}}(t)\right). \qquad (6)
\end{aligned}
$$

The value $-d_{\mathrm{CD}}(t)$ measures, in standard deviations of the net current revenue, whether the state will be outside the limits established by the FRL t periods ahead.

When applying this methodology to private firms, Merton (1974) uses firm's assets volatility as a proxy for the stochastic component of the total assets evolution. Consistent with the fact that revenues are dependent on macroeconomic performance, we use as stochastic component (volatility) the sensitiveness of states net current revenue to changes in real GDP. With this modification, we can give a prospective view to the model rather than looking only at historical data.

We estimate the following panel to capture the sensitivity or elasticity of the net current revenue of each state to variations of real GDP:[13]

$$
\Delta NCR_{it} = \alpha_i + \eta_t + \sum_{i \in \mathscr{E}} \beta_i \cdot \Delta GDP_t + \gamma^T \cdot \mathrm{varControl}_t + \mu_{it}, \qquad (7)
$$

in which i indexes the Brazilian states, t represents time, and $\mathscr{E}$ is the set of states in Brazil. Since the

[13] Ideally, it would be better to use state-level real GDP. However, we use national-level GDP due to data unavailability.

net current value of states also depends on other macroeconomic conditions of the country, we control for the exchange rate, inflation, and the interest rate in the vector varControl. We add state fixed effect α_i and time fixed effects η_t, to absorb non-observed and time-independent regional particularities of states and time-dependent homogeneous (across states) macroeconomic tendencies, respectively. We estimate the specification (7) using clustered errors at the state level.

Our coefficient of interest is β_i, which gives the individual sensitivity or elasticity of each state i, $i \in \mathscr{E}$, to changes in the real GDP, ΔGDP_t.

We then estimate the volatility of state i, σ_{NCR}, for an expected variation of the real GDP t periods ahead as follows:

$$\sigma_{NCR}(\theta) = \beta_i \times \theta, \tag{8}$$

in which $\theta \in \mathbb{R}$ is the expected variation of real GDP t periods ahead. In this way, the probability of non-compliance of the states is conditional on the expected variation of real GDP in the future. Such conditioning can be useful to understand how the fiscal health of the states would behave in the case of a hypothetical recession or expansion t periods ahead.

Probability of non-compliance with PE/NCR: Analogously, we estimate the probability that the legal limit PE/NCR is exceeded t periods ahead $\mathbb{P}\left(\frac{PE_t}{NCR_t} > 49\%\right)$ using (2) and (4) as follows:

$$\begin{aligned} PN_{\mathrm{PE}}(t) &= \mathbb{P}\left(\frac{\mathrm{PE}_t}{NCR_t} > 49\%\right) \\ &= \mathbb{P}\left(\mathrm{PE}_t > 0.49 \cdot NCR_t\right) \\ &= \mathbb{P}\left(-d_{\mathrm{PE}}(t) > \varepsilon\right) \\ &= \mathscr{N}\left(-d_{\mathrm{PE}}(t)\right), \end{aligned} \tag{9}$$

in which $d_{\mathrm{PE}}(t) = \frac{\log\left(\frac{0.49NCR_0}{\mathrm{PE}_t}\right) + \left(\mu_{NCR} - \frac{\sigma_{NCR}^2}{2}\right)t}{\sigma_{NCR}\sqrt{t}}$ and $PN_{\mathrm{PE}}(t)$ is the probability of non-compliance of the personnel expenses limit $\frac{\mathrm{PE}_t}{NCR_t} > 49\%$ t periods ahead of time. We assume that personnel expenses increase deterministically, according to the current inflation.

Probability of non-compliance of states: FRL sanctions are executed when states violate a single legal limit. Consistent with this rule, we express the probability of non-compliance of states t periods ahead—$PN(t)$—as the probability of not complying with the rule $\frac{CD_t}{NCR_t} > 200\%$ *or* the rule $\frac{\mathrm{PE}_t}{NCR_t} > 49\%$, i.e.:

$$PN(t \mid \Delta GDP = \theta) = \mathbb{P}\left(\frac{CD_t}{NCR_t} > 200\% \bigcup \frac{\text{PE}_t}{NCR_t} > 49\%\right)$$
$$= PN_{\text{CD}}(t) + PN_{\text{PE}}(t) - \mathbb{P}\left(\frac{CD_t}{NCR_t} > 200\% \bigcap \frac{\text{PE}_t}{NCR_t} > 49\%\right), \quad (10)$$

in which we make explicit that the probability of non-compliance depends on GDP growth prospects that directly affect states net current revenues.

Due to data limitation in the empirical part of the paper, we will not be able to evaluate the joint probability of both legal limits in (10). Instead, we identify the minimum and maximum feasible $PN(t)$ values, given the marginal probabilities of each legal limit. The identification of such region is important for systemic risk analysis, because we are able to map down the minimum and maximum losses to which the financial system would be subject for a given external shock.

The minimum value of (10) occurs when $\mathbb{P}\left(\frac{CD_t}{NCR_t} > 200\% \bigcap \frac{\text{PE}_t}{NCR_t} > 49\%\right)$ is maximal, i.e., when $\mathbb{P}\left(\frac{CD_t}{NCR_t} > 200\% \bigcap \frac{\text{PE}_t}{NCR_t} > 49\%\right) = \min\left[PN_{\text{CD}}(t), PN_{\text{PE}}(t)\right]$. Substituting this expression in (10), the minimum value for PN(t)—denoted here as $\text{PN}_{\min}(t)$—is:

$$\text{PN}_{\min}(t) = PN_{\text{CD}}(t) + PN_{\text{PE}}(t) - \min\left[PN_{\text{CD}}(t), PN_{\text{PE}}(t)\right]$$
$$= \max\left[PN_{\text{CD}}(t), PN_{\text{PE}}(t)\right] \quad (11)$$

The maximum value of $PN(t)$ occurs when $PN_{\text{CD}}(t)$ and $PN_{\text{PE}}(t)$ are independent events, such that $\mathbb{P}\left(\frac{CD_t}{NCR_t} > 200\% \bigcap \frac{\text{PE}_t}{NCR_t} > 49\%\right) = 0$. In this case, the maximum value for PN(t)—denoted here as $\text{PN}_{\max}(t)$—is:

$$\text{PN}_{\max}(t) = PN_{\text{CD}}(t) + PN_{\text{PE}}(t) \quad (12)$$

Since $\text{PN}_{\min}(t) \leq PN(t) \leq \text{PN}_{\max}(t)$ by construction, we will always report both limits in the empirical part of the paper. Our estimated probability of non-compliance takes into account the potential non-compliance of states with *any* of the two legal limits. We condition such probability on the change of real GDP t periods ahead, which directly affects the evolution of revenues. As a useful application, our method can estimate the probability of non-compliance in the case of a recession or boom in the economy.

4.2 Contagion and amplification in the financial system

We take a close look at how defaults of bank loans of these public entities impact the banking system, since the financial system can potentially exacerbate the initial shock through spillovers in the interbank market and also to the real sector. We use an adaptation of Silva et al. (2017a)'s, which considers the balance-sheet composition of each and every firm and bank in the economy and also the different network relationships.[14]

The model takes as input the losses that each bank would suffer from the credit default of states and evaluates to what extent the network amplifies the shock through financial exposures among banks (interbank market), among banks and firms (bank lending to the real sector), and among firms (corporate trade network). To do so, we represent the financial and real sectors as a two-layer complex network. The financial sector layer includes banks that can expose themselves to each other through unsecured credit operations. The real sector layer contains the network of firms, in which supplier companies grant credit purchases to client companies and thus are exposed to the default of the latter. Exposures between economic agents—banks and/or firms—pave the way for a potential channel of contagion, and hence of amplification of losses. The model considers two types of financial vulnerability: credit and funding risk.

Credit risk is a financial vulnerability in the asset-side of the balance sheet of banks. If borrowers—which can be both firms and banks—do not pay back, banks register losses and their net worth will reduce accordingly. Credit losses do not only occur through defaults. Banks constantly monitor the financial soundness of their debtors and always register their exposures to these agents as a function of their creditworthiness, measured by their net worth. In this way, as the firms' net worth deteriorate, lenders see that and reprice down their investment toward them.

Funding risk relates to a negative event in the liability-side of the balance sheet of banks and firms. As lenders become financially distressed, the funding risk of borrowers rises in the sense that lenders are not willing to rollover their short-term debt anymore. Depending on their liquidity conditions, borrowers may have to firesell their illiquid assets and thus incur losses. Such effect can be attenuated or exacerbated depending on the ability of borrowers to replace one lender with another one whenever bank deny rolling over the borrowers' debt.

The agents' behavior to funding and credit risk gives rise to a financial accelerator between pairs of economic agents. For example, if a creditor bank of a specific Brazilian state suffers a credit default, its net worth drops. Consequently, it registers losses and the funding risk of the state increases in view of the financial distress of its creditor bank. Thus, economic agents amplify the initial received negative shock.

This dynamic process continues until the entire system reaches equilibrium to a unique fixed

[14] See also Silva et al. (2018) and Silva et al. (2016a).

point. Silva et al. (2017a) demonstrate that the contagion mechanism between banks and firms with feedback between the financial and real sectors can be modeled with the following dynamic nonlinear system:

$$\mathbf{b}_i(t+1) = \min\left[1, \mathbf{b}_i(t) + \sum_{j\in\mathscr{B}} \mathbf{V}_{ij}^{(\text{bank-bank})}\Delta\mathbf{b}_j(t) + \sum_{u\in\mathscr{F}} \mathbf{V}_{iu}^{(\text{bank-firm})}\Delta\mathbf{f}_u(t)\right], \tag{13}$$

$$\mathbf{f}_k(t+1) = \min\left[1, \mathbf{f}_k(t) + \sum_{u\in\mathscr{F}} \mathbf{V}_{ku}^{(\text{firm-firm})}\Delta\mathbf{f}_u(t) + \sum_{j\in\mathscr{B}} \mathbf{V}_{kj}^{(\text{firm-bank})}\Delta\mathbf{b}_j(t)\right], \tag{14}$$

in which:

- $\mathbf{b}_i(t) \in [0,1]$ e $\mathbf{f}_k(t) \in [0,1]$ is the financial stress of bank i and firm k in iteration t. If $\mathbf{b}_i(t) = \mathbf{f}_k(t) = 0$, then bank i and firm k are not in financial distress (no net worth was lost). If $\mathbf{b}_i(t) = \mathbf{f}_k(t) = 1$, then bank i and firm k are in default, since they have lost all their net worth. These vectors are defined in terms of the net worth loss relatively to the initial net worth.
- $\mathscr{B}$ e $\mathscr{F}$ are the sets of banks and firms, respectively.
- The matrices $\mathbf{V}^{(\text{bank-bank})}$, $\mathbf{V}^{(\text{bank-firm})}$, $\mathbf{V}^{(\text{firm-firm})}$ e $\mathbf{V}^{(\text{firm-bank})}$ are financial vulnerability matrices that model the funding and credit risk between every pair of economic agents in the economy.

In our setup, we assume that the financial stress in all banks $i \in \mathscr{B}$ that have granted loans to federated entities is $\mathbf{b}_i(0) > 0$ and 0 otherwise. The more bank i lends to the public sector, the more it is subject to losses, which will reflect in larger coefficients $\mathbf{b}_i(0)$ in the model.

We evaluate the losses L of a given initial scenario (state default) as follows:

$$L = \frac{1}{C}\sum_{i\in\mathscr{B}} \mathbf{b}_i(T)e_i(0), \tag{15}$$

in which $\mathbf{e}_i(0)$ is the initial net worth of bank i, $\mathbf{b}_i(T)$ is the fraction of net worth loss relatively to the initial net worth $\mathbf{e}_i(0)$ at the convergence iteration T,[15] and $C = \sum_{i\in\mathscr{B}} \mathbf{e}_i(0)$ is the total capitalization of the financial system.

[15] Silva et al. (2017a) demonstrate that the dynamical nonlinear system described by (13) and (14) always converges to a unique fixed point that depends on the network relationships and the initial shock. The iteration T denotes the state of system once the convergence takes place.

4.3 Probability of default

According to the FRL, once a single legal expenditure limit is violated, the state already becomes subject to sanctions. Equation (10) reflects this feature, in that it considers the state's probability of non-compliance as a probabilistic union of not complying with each of the legal restrictions.

We assume that states default once they become non-compliant to the FRL. This is a plausible assumption, given that states can no more receive voluntary transfers from other public entities and engage in new credit operations. Following this reasoning, the probability of default is given by:

$$PD(t) = PN(t \mid \Delta GDP = \theta). \tag{16}$$

in which the probability of default is conditioned on the GDP growth prospects. Following the same reasoning of the probability of non-compliance, we also report lower and upper bounds for the probability of default.

4.4 Expected losses in the financial system due to state defaults

In this section, we evaluate the expected losses in the financial system due to state defaults in the upcoming year. Our expected losses are conditioned on the GDP growth expectations. The strategy is to weigh the potential loss that the financial system would suffer if states default on their bank loans with the probability of occurrence of such event. To compose the state-specific expected loss, we combine the probability of default evaluated according to Section 4.3 and the losses that the financial system would suffer when a state defaults using the systemic risk model in Section 4.2.

Therefore, we evaluate the expected losses as follows:

$$EL(t) = PD(t \mid \Delta GDP = \theta)L \tag{17}$$

in which $PD(t \mid \Delta GDP = \theta)$ is the probability of default of some state conditioned on a exogenous prospect of GDP growth (Equation (16)), and L represents the losses relatively to the entire financial system given the occurrence of the state default (Equation (15)). This weighting rules out events that would impose large losses on the system but are otherwise negligible in terms of actual feasibility.

5 Data

We first delineate entities that we consider as the public sector. To get a comprehensive view on the exposures of the financial sector, we consider bank credit exposures to public entities linked to the Executive, Legislative and Judicial branches of each Brazilian state. In addition, we consider as public sector state-owned companies whose regulation is subject to public law.

We map every outstanding credit operation that the public sector takes from the financial sector using proprietary data that comes from the Credit Bureau (SCR) of the Central Bank of Brazil.[16] The SCR is a large-scale and loan-level database that contains information on credit operations and securities with credit characteristics and respective guarantees contracted by individuals and legal entities from financial institutions in Brazil. Every financial institution in Brazil must report their credit operations with an outstanding amount greater than R$ 200 (roughly US$ 60) on a monthly basis, which reinforces the representativeness of our data set.

If a federated state does not pay its bank debts, the corresponding creditor banks suffer losses and thus have their net worth reduced. The net worth decrease has implications for both the real and the financial sectors. In the financial sector, connected banks reprice their investments toward those that suffered losses from the public sector; therefore, incurring losses as well. We identify connected banks using proprietary bilateral exposures data compiled by the Central Bank of Brazil, which collects and standardize financial transactions from several custodian agents in Brazil.[17] The set of all bilateral exposures makes up the Brazilian interbank market. We only consider non-collateralized transactions between commercial, investment and development banks. Interbank exposures encompass the classical interfinancial deposits, credit operations, swaps, among many others.

The contagion model also takes into account effects in the real sector, which is simplified, in this empirical exercise, to consider only publicly traded companies in the Brazilian stock market. This subset of firms is economically relevant and is responsible for a large part of the payment flows in the trade network in Brazil.[18] We consider credit risk arising from outstanding bank loans to the real sector. We segregate from these loans the parcels that are due in the short term, which are responsible for the funding risk of firms. Balance-sheet information from firms comes from Economatica.

Regarding the information on the fiscal situation of states, we collect public data from the National Treasury Secretariat, which receives and maintains accounting data received by every public entity, through the Brazilian Public Sector Accounting and Tax Information System (Siconfi) and the Accounting Data Collection System (SISTN).

[16] The tasks of collecting, matching and processing all supervisory data were conducted in secured sites inside the Central Bank of Brazil exclusively by its staff.

[17] For example, SCR is maintained by the Central Bank of Brazil and has credit operations among banks, while B3 is a private company that has information on private securities, swaps and options and must report these operations to the regulator.

[18] The aggregate total assets of this subset of firms corresponds to 54% of Brazil's nominal GDP in 2017.

In the following sections, we provide an exploratory analysis of our collected data.

5.1 Fiscal health of the federated entities

We first analyze the evolution of important accounting variables that the FRL employs to construct the prudential limits to states: the consolidated debt, the net current revenue and the personnel expenses of states. Then, we present the evolution of the legal restrictions on public expenses imposed by the Brazilian FRL, which are: the ratio of consolidated debt to net current revenue and the ratio of personnel expenses to net current revenue. These data are reported every four months.

5.1.1 Accounting variables

The left panel of Figure 2 portrays the nominal consolidated debt, net current revenue, and personnel expenses over the period April 2008 to 2017 of Goiás, Minas Gerais, Rio de Janeiro, Rio Grande do Sul and São Paulo. We will take a close look at these five states because they either have large bank debt or are the most fragile in terms of public finances and are nearing non-compliance with the FRL. In addition, the right panel of Figure 2b shows average values of the same quantities of the other Brazilian states, which we have further divided into two subgroups, depending on their levels of public debt, net current revenues, and personnel expenses.[19]

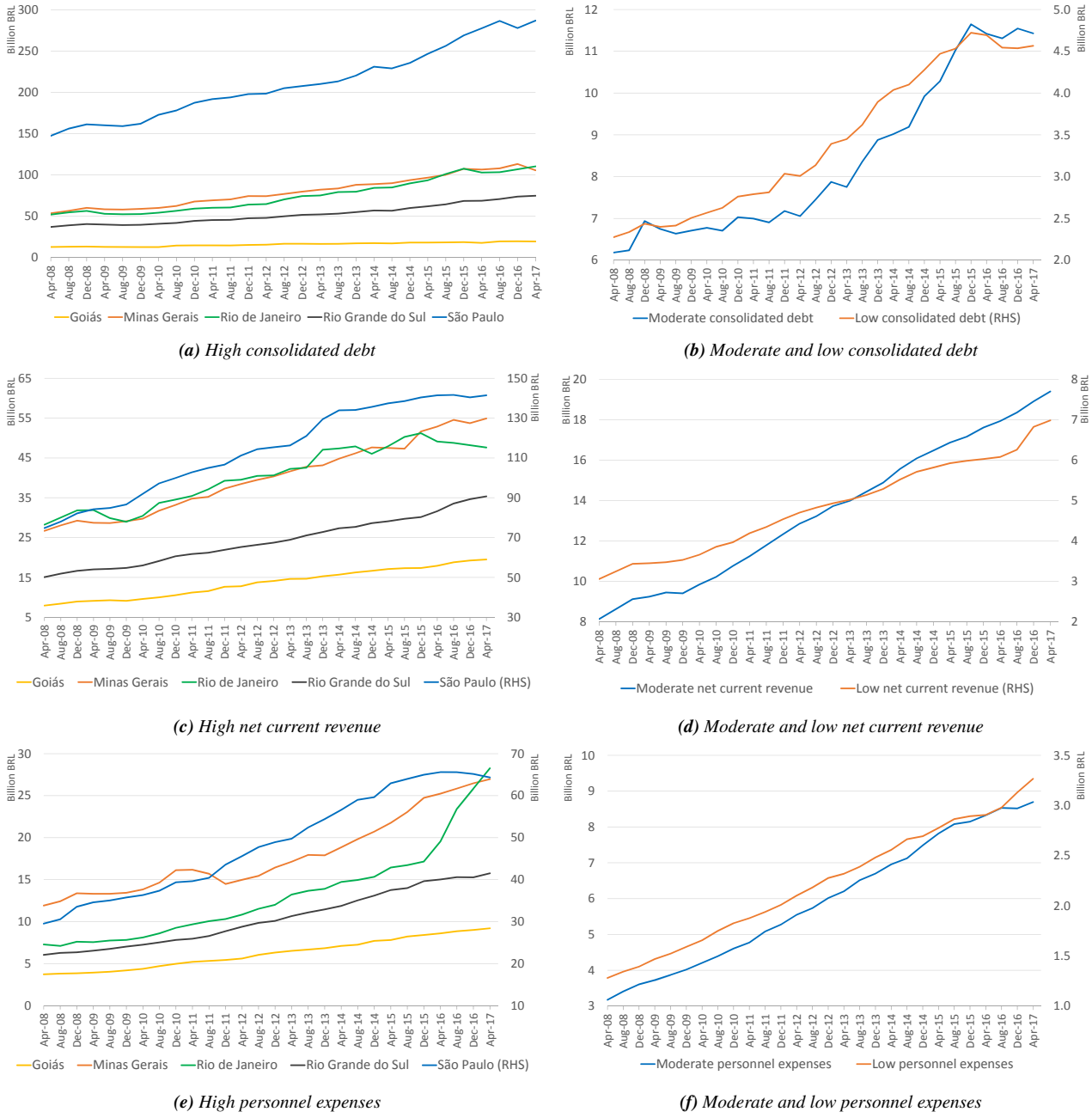
Overall, there is a growing public debt in nominal terms of public entities,[20] especially for the state of São Paulo. Minas Gerais and Rio de Janeiro have the second largest consolidated debt. States with moderate and low consolidated debt also become increasingly more indebted, especially in the period from 2011 to 2015, after which the debt growth ceases.

The net current revenue, in general, increases due to inflation and GDP growth. There is also a considerable decrease in the growth rate of São Paulo's net current revenue as from the beginning of 2014 and a decrease in Rio de Janeiro's net current revenue as from 2016. States with low and moderate net current revenue also have a consistent growth of the net current revenue.

There is a consistent increase in states personnel expenses for most states, especially for Rio de Janeiro, which rapidly increased after 2016. Contrary to that tendency, São Paulo's personnel expenses declined after 2016.

[19] States classified as "moderate": Amazonas, Bahia, Ceará, Distrito Federal, Espírito Santo, Maranhão, Mato Grosso, Pará, Paraná, Pernambuco, and Santa Catarina. States classified as "low": Acre, Alagoas, Amapá, Mato Grosso do Sul, Paraíba, Piauí, Rio Grande do Norte, Rondônia, Roraima, Sergipe, and Tocantins.

[20] Even though the consolidated debt over GDP tends to decrease until the end of 2014 (Figure 1a), the nominal consolidated debt of states constantly increases throughout the entire sample period (first row of Figure 2), which is mostly due to inflation.



(a) *High consolidated debt*

(b) *Moderate and low consolidated debt*

(c) *High net current revenue*

(d) *Moderate and low net current revenue*

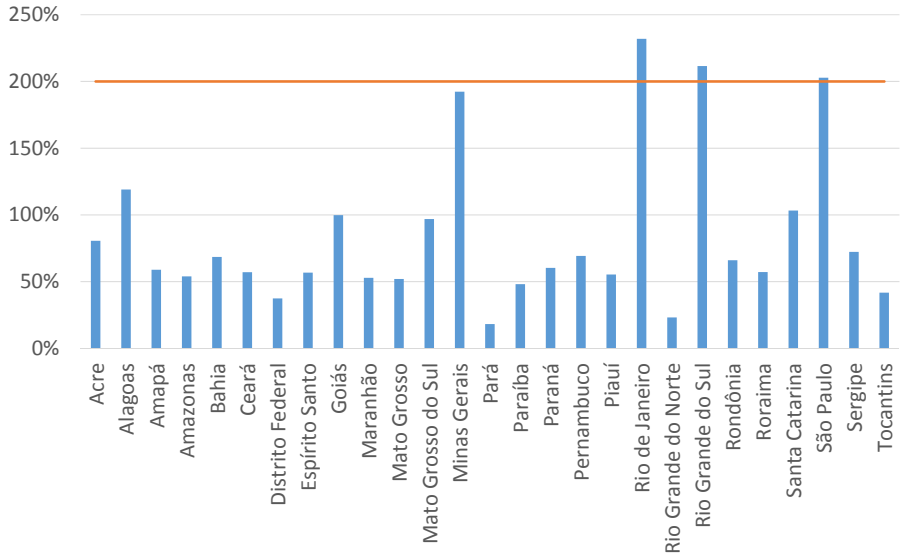
(e) *High personnel expenses*

(f) *Moderate and low personnel expenses*

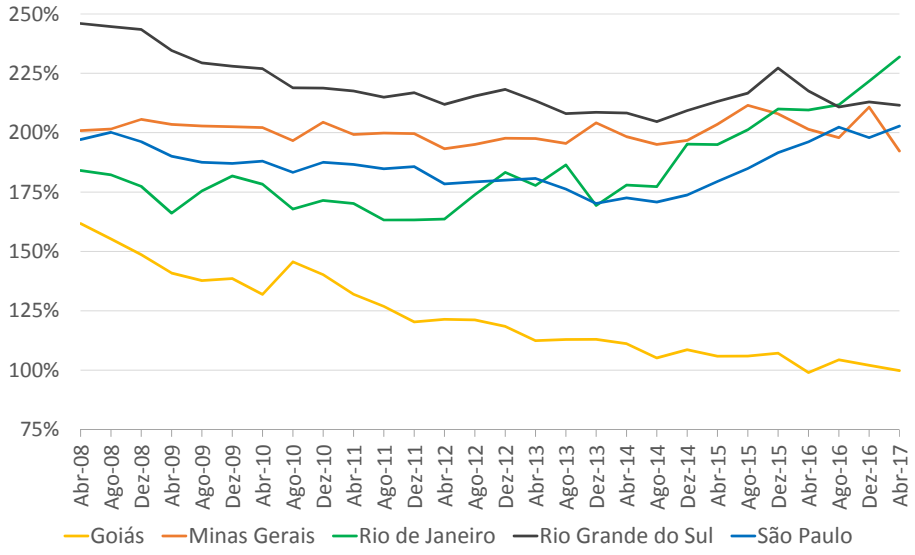
Figure 2: *Evolution of the nominal consolidated debt of Brazilian states (first row), net current revenue (second row), and personnel expenses (third row) from April 2008 to 2017. We report the variables for (a) Goiás, Minas Gerais, Rio de Janeiro, Rio Grande do Sul and São Paulo (left panel) and (b) the other states (right panel). In the latter, we show average values for each division of states. Our classification of the level of states public debt, net current revenue, and personnel expenses concerns only the absolute value and does not take into account the state size. The list of states classified as moderate or low is given in footnote 19.*

5.1.2 FRL legal limits

Consolidated debt to net current revenue. Figure 3a illustrates the CD/NCR legal constraint for all Brazilian states in January 2017. Rio de Janeiro, Rio Grande do Sul, and São Paulo were non-compliant with the 200% legal limit from the FRL in 2017. Therefore, they were subject to sanctions, including the impossibility of celebrating new credit operations.



(a) *Legal constraint CD/NCR in January 2017*

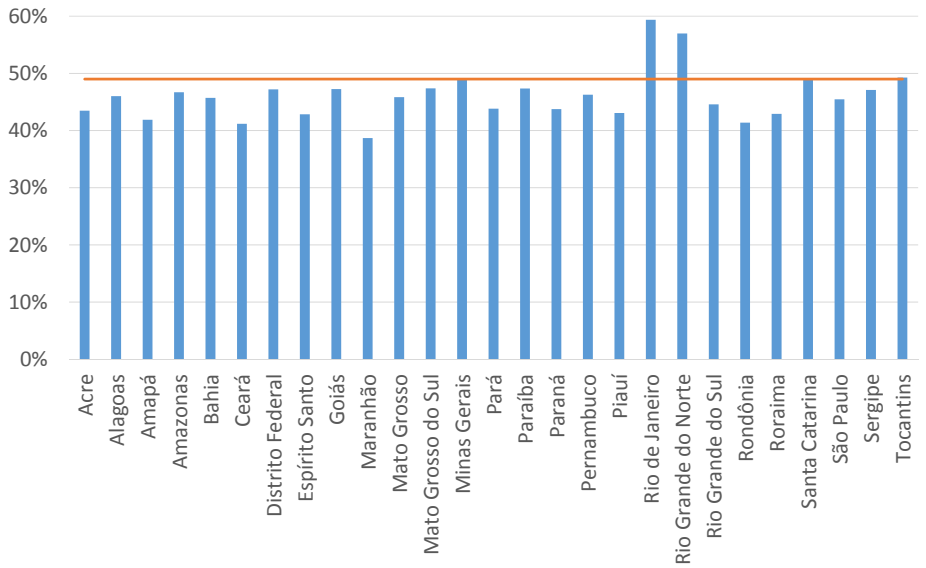


(b) *Trajectory of the legal constraint CD/NCR for some states*

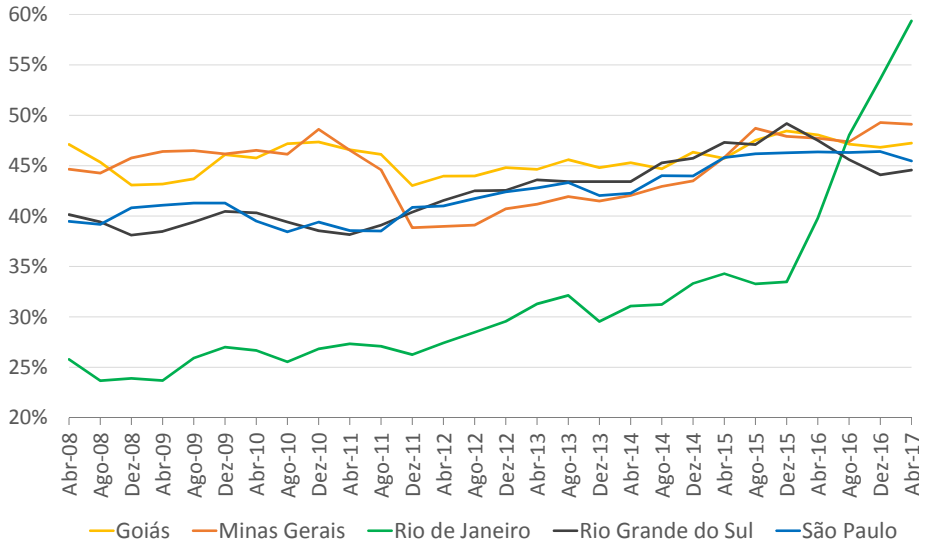
Figure 3: *Fiscal situation of Brazilian states regarding the legal limit on the consolidated debt to net current revenue. According to the FRL, states must maintain this index under 200%. If they do not comply with this legal restriction, they are subject to sanctions, including the impossibility of celebrating new bank credit operations.*

Figure 3b displays the trajectory of the legal constraint CD/NCR of the states Goiás, Minas Gerais, Rio de Janeiro, Rio Grande do Sul and São Paulo from 2008 to 2017. Rio Grande do Sul remains above the FRL legal limit of 200% throughout the period, even though with a perceivable decline. São Paulo shows a U-shaped behavior, with the lowest index reached in 2014. Goiás consistently reduces its index over the period, while Minas Gerais' index does not significantly change. Rio de Janeiro has a consistent and rapid growth rate of the index, going from 180% in April 2008 (below the legal threshold) to 230% in April 2017 (well above the legal threshold). Figure 3b makes clear that the most developed Brazilian states are having problems to comply with the CD/NCR legal index.

Personnel expenses to net current revenue. Figure 4a illustrates the PE/NCR legal expenses limit for all Brazilian states in January 2017. Due to data unavailability, we report prudential limits only for the Executive branch of states. The FRL disciplines that the Executive branch of states must maintain the index PE/NCR below 49%. We observe that almost all states are at the verge of not complying with this prudential limit on personnel expenses. Some states—such as Rio de Janeiro and Rio Grande do Norte —have exceeded this legal threshold in January 2017 and therefore have become non-compliant. Minas Gerais, Santa Catarina, and Tocantins are at the verge of becoming non-compliant.



(a) *Legal constraint PE/NCR (Executive) in January 2017*



(b) *Trajectory of the legal constraint PE/NCR (Executive) for some states*

Figure 4: *Fiscal situation of Brazilian states regarding the legal restriction of personnel expenses to net current revenue. According to the FRL, the Executive branch of states must maintain this personnel expenses index lower than 49%.*

Figure 4b shows the evolution of the PE/NCR index for our five selected states from 2008 to 2017. While Rio de Janeiro had a compliant and rather low index of 25% for the personnel expenses in

2008, it grew at an explosive rate by the end of 2015, quickly surpassing the legal limit. Since August 2016, Rio de Janeiro is non-compliant with the personnel expenses limit. The other states show a similar pattern of positive but small growth rates, all of which being at the verge of not complying with the personnel expenses limit.

Most critical (or binding) legal restrictions to states. Table 1 reports which of the two FRL legal limits is more critical (or binding) to states, i.e., more difficult to meet. According to the FRL, once a single legal expenditure limit is violated, the state already becomes subject to sanctions, which include the impossibility of celebrating credit operations or of receiving voluntary transfers from other public entities. Our measure of probability of default takes this feature into account and combines both legal expenditure limits.

Of the 27 federated entities, 5 (18.5%) are more critical in complying with the CD/NCR index, while 22 (81.5%) are more critical with the PE/NCR index. The richest states in Brazil are struggling to meet the legal expenditure limit related to their consolidated debt, while medium and small states, in general, tend to have more difficulty in complying with the limit linked to personnel expenses.

Table 1: *Most critical (or binding) legal expenditure limit that the FRL imposes on states: (i) ratio consolidated debt to net current revenue or (ii) ratio personnel expenses on the Executive Branch to net current revenue. Of the 27 federated entities, 5 (18.5%) are more critical in complying with the CD/NCR index, while 22 (81.5%) are more critical in the personnel expenses / NCR index. We evaluate these legal expenditure limits using averages of the entire sample period.*

State	Worst FRL criterion	State	Worst FRL criterion
Acre	PE/NCR	Paraíba	PE/NCR
Alagoas	CD/NCR	Paraná	PE/NCR
Amapá	PE/NCR	Pernambuco	PE/NCR
Amazonas	PE/NCR	Piauí	PE/NCR
Bahia	PE/NCR	Rio de Janeiro	CD/NCR
Ceará	PE/NCR	Rio Grande do Norte	PE/NCR
Distrito Federal	PE/NCR	Rio Grande do Sul	CD/NCR
Espírito Santo	PE/NCR	Rondônia	PE/NCR
Goiás	PE/NCR	Roraima	PE/NCR
Maranhão	PE/NCR	Santa Catarina	PE/NCR
Mato Grosso	PE/NCR	São Paulo	CD/NCR
Mato Grosso do Sul	PE/NCR	Sergipe	PE/NCR
Minas Gerais	CD/NCR	Tocantins	PE/NCR
Pará	PE/NCR		

5.2 Bank credit to Brazilian states

We first aggregate loan-level outstanding bank credit data of states on a monthly basis. The funding portfolio of Brazilian states comes in great part from refinancing operations with the federal government and from banks. Table 2 breaks down the representativeness of these two sources of funding in December 2015 and June 2016.

The most representative creditor of these five states is the federal government, with exception for the state of Goiás, whose debt is roughly equally divided between the federal government and banks. Thus, the states public debt is largely to the federal government and not the banking system.

Table 2: *Funding portfolio of Brazilian states broken down by creditor type: federal government and banks. The amounts are reported for two dates: December 2015 and June 2016. We show the four states with the largest bank debt to the financial system, as well as the state of Rio Grande do Sul, which is experiencing fiscal problems.*

In billions R$

	December 2015			June 2016		
State	**Federal Government**	**Bank**	**Total**	**Federal Government**	**Bank**	**Total**
Goiás	9.50	7.63	17.12	8.98	7.13	16.11
Minas Gerais	78.88	9.45	88.33	83.04	8.44	91.48
Rio de Janeiro	56.34	22.39	78.73	59.48	21.18	80.66
Rio Grande do Sul	51.91	1.67	53.58	54.72	1.65	56.37
São Paulo	220.14	14.57	234.71	232.77	14.35	247.12

Nonetheless, it stands as a relevant issue to study the effects of a potential default of the bank debt to the financial system, because banks are highly connected to other markets and therefore have the potential to amplify shocks.

There are other indirect channels of shock transmission to the financial system that can be triggered by states, which we abstract away in this paper. For example, the nonpayment to civil servants from a specific state would lead to an increase in the delinquency rate of the bank loans granted to these employees. In addition, government indebtedness can force a reduction of investments and therefore affect private firms in the form of reduced demand and incentives, which in turn could again hit the financial sector through further loan defaults.

In order to understand which banks are more susceptible to defaults from the public entities, Table 3 discriminates the outstanding bank credit to Brazilian states in terms of bank control, which can be state-owned,[21] and private (domestic and foreign) banks. The great majority of bank credit that Brazilian states take comes from state-owned banks, which highlights their exposure to the public sector.

Table 3: *Proportion of outstanding bank credit that Brazilian states took from the Brazilian financial system in December 2015 and June 2016. We discriminate these amounts by state-owned and private (domestic and foreign) banks. We list the four states with the largest bank debts in the Brazilian banking system, as well as the state of Rio Grande do Sul, which is experiencing fiscal problems. For each state and date, we depict in bold the prevailing portion of bank credit to the corresponding state.*

	December 2015		June 2016	
State	**State-owned banks**	**Private banks**	**State-owned banks**	**Private banks**
Goiás	**100.00**%	0.00%	**100.00**%%	0.00%
Minas Gerais	**100.00**%	0.00%	**99.60**%	0.40%
Rio de Janeiro	**94.60**%	5.40%	**93.70**%	6.30%
Rio Grande do Sul	**100.00**%	0.00%	**100.00**%	0.00%
São Paulo	**100.00**%	0.00%	**100.00**%	0.00%

State-owned banks are therefore the main targets of a potential rise of fiscal risk in Brazil. However, it is important to understand how capitalized these banks are in face of their exposure amounts

[21] While the FRL forbids states to take credit operations with controlled financial institutions, that same state can celebrate credit contracts with financial institutions controlled by the federal government or even by other states.

to the public sector. Figure 5 portrays the total outstanding bank credit to states in relation to the total capitalization of the corresponding creditor banks. We use the bank net worth as capitalization. Capitalization serves as a way to absorb potential defaults from unexpected defaults. This ratio then gives us a hint on how banks would withstand losses coming from the public entities. When losses surpass bank capital, then defaults take place.

Overall, we observe banks would be able to absorb losses from Brazilian states. However, we should note that an underlying assumption of this analysis is defaults take place in an idiosyncratic manner. That is, we look at default of single states and not groups of states.

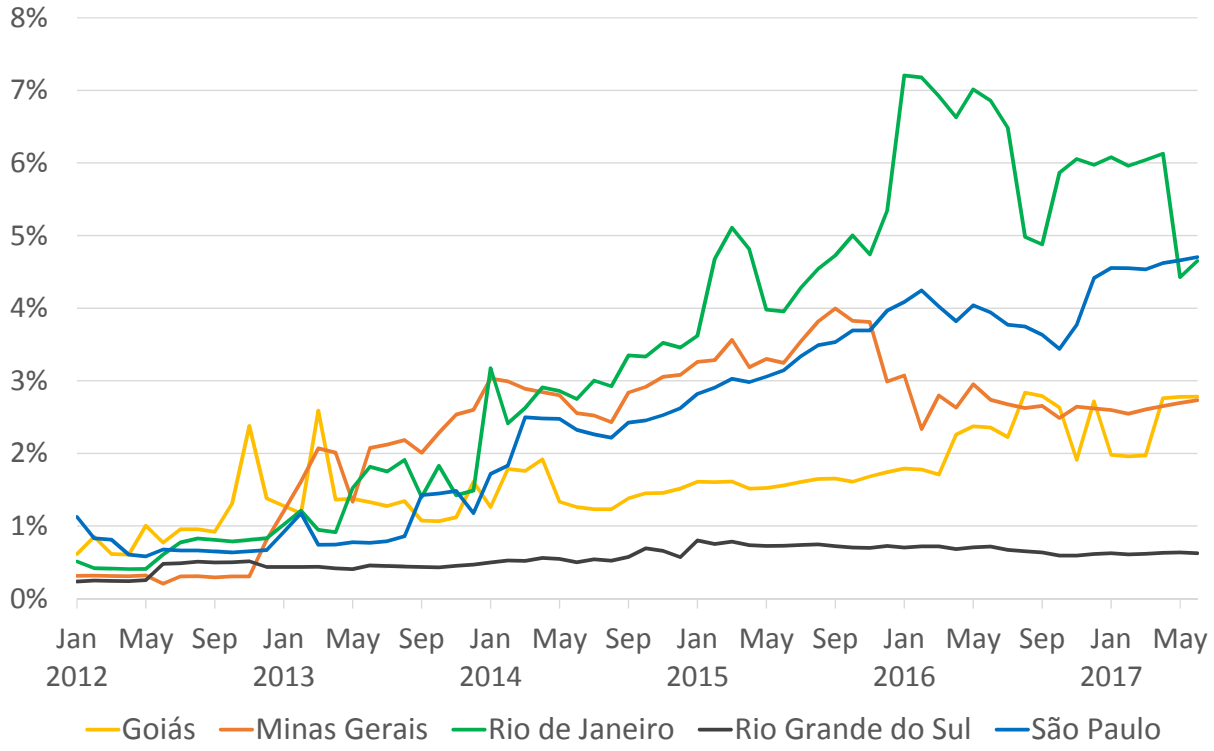


Figure 5: *Ratio of the states outstanding bank credit to the total capitalization of corresponding creditor banks. Given that banks incur losses when states default, which in turn directly affects their net worth, this ratio gives us a hint on how banks would withstand losses coming from the public entities. When losses surpass bank capital, then defaults take place.*

6 Results

In this section, we present the main results of the paper. We start with the estimation of the probability of non-compliance of states in a hypothetical scenario that simulates a deepening of the recession that took place in Brazil in 2014–2016. We then analyze the impact that the financial system would suffer in the event that states do not honor their bank debt. Finally, we combine both the probability of default and the impact in the financial system to compose a statistical indicator of expected loss in the financial system.

6.1 Probability of non-compliance of Brazilian states under a stressed scenario

In this section, we estimate the probability of non-compliance of Brazilian states using a hypothetical scenario in which the Brazilian real GDP falls by 5% in the following year. This exercise simulates negative prospects of growth in the short term. Estimating the probability of non-compliance of states is a useful tool to classify, in a forward-looking way, the fiscal health of these entities and hence our method is useful to evaluate fiscal risk.

We evaluate the probability of non-compliance of states according to the FRL legal expenditure limits $CD/NCR \leq 200\%$ and $PE/NCR \leq 49\%$ for a one-year horizon with negative prospects of real GDP growth of -5%. Such analysis can be useful in times of recession, in which we observe rising expenses and falling revenues. We simulate this hypothetical recession in the upcoming year for each point in our sample that ranges from 2008 to 2017. For instance, if we are at the point referring to April 2014, then our estimated probability of non-compliance of states refers to a potential non-compliance of these states in the next relative year, i.e., from the period from May 2014 to April 2015.

The expected negative prospect in real GDP for the next year generates a decrease in net current revenues of states, which we estimate using the empirical specification in Section 4.1. The reduction of net current revenues conjugated with non-decreasing expenses required by law raises the probability of non-compliance with the FRL.

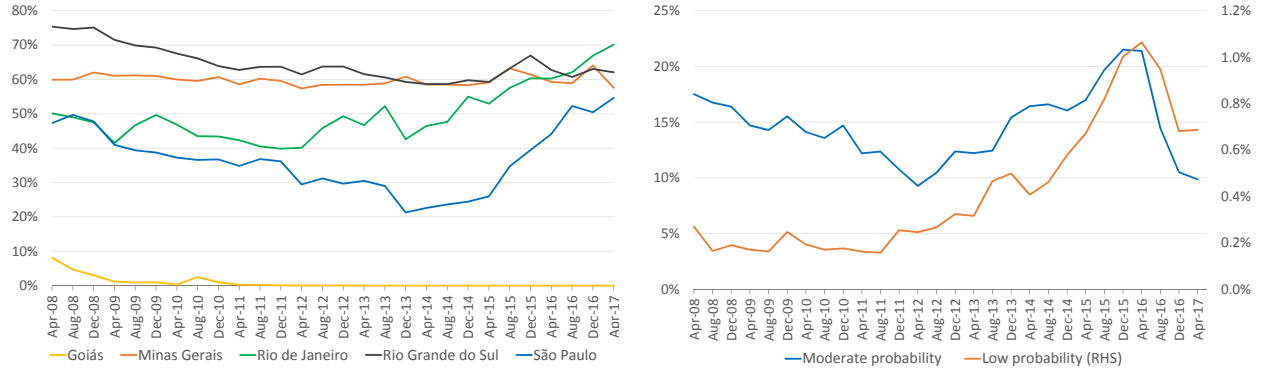
Elasticity of net current revenue of states to changes in the real GDP. Table 4 reports our estimates for the elasticities of the net current revenue of each Brazilian state to real GDP. We use these elasticities to estimate how the net current revenue of states would decrease in the negative scenario of a negative real GDP growth of 5%. We use the stressed net current revenue values together with an inflationary increase of the consolidated debt and personnel expenses to evaluate the probability of non-compliance in the upcoming year. In Table 4, for example, São Paulo would have a projected reduction of $0.04 * 5 = 0.20\%$ of its net current revenue relatively to the current year for the upcoming year as a result of a hypothetical decrease of 5% in real GDP.

There is heterogeneity in the sensitivity or elasticity of net current revenue of states in relation to real GDP. The most sensitive state is Amapá, followed by Amazonas, Acre, Minas Gerais, Sergipe and Rio de Janeiro. We consider that net current revenue is inelastic for cases in which the coefficient is statistically insignificant, such as for the states of Pará, Rondônia and Santa Catarina.

Probability of non-compliance with the legal limit CD/NCR $\leq$ 200%. Figure 6a displays our estimates for the states probability of non-compliance with the legal expenses limit $CD/NCR \leq 200\%$ in the following relative year for Goiás, Minas Gerais, Rio de Janeiro, Rio Grande do Sul, and São Paulo. Figure 6b shows the average value of the same information for the remainder of states, which we further divided into groups with moderate and low probability of non-compliance.

Table 4: *Annual elasticity of the net current revenue (NCR) relative to real GDP changes. For example, a 1% increase in real GDP over the previous period would cause NCR of the state of São Paulo to increase by 0.04% in the following year. We indicate statistical significance by means of the p-value: *** corresponds to a p-value of less than 1%, **, at a p-value between 1% and 5% and *, at a p-value between 5% and 10%.*

State	Elasticity NCR-GDP	Estado	Elasticity NCR-GDP
Acre	0.12***	Paraíba	0.08***
Alagoas	0.06***	Paraná	0.02*
Amapá	0.20***	Pernambuco	0.04***
Amazonas	0.13***	Piauí	0.06***
Bahia	0.06***	Rio de Janeiro	0.09***
Ceará	0.11***	Rio Grande do Norte	0.08***
Distrito Federal	0.10***	Rio Grande do Sul	0.07***
Espírito Santo	0.03**	Rondônia	0.00
Goiás	0.03***	Roraima	0.08***
Maranhão	0.06***	Santa Catarina	-0.01
Mato Grosso	0.05***	São Paulo	0.04***
Mato Grosso do Sul	0.05***	Sergipe	0.11***
Minas Gerais	0.11***	Tocantins	0.09***
Pará	-0.02		



(a) *Goiás, Minas Gerais, Rio de Janeiro, Rio Grande do Sul, and São Paulo*

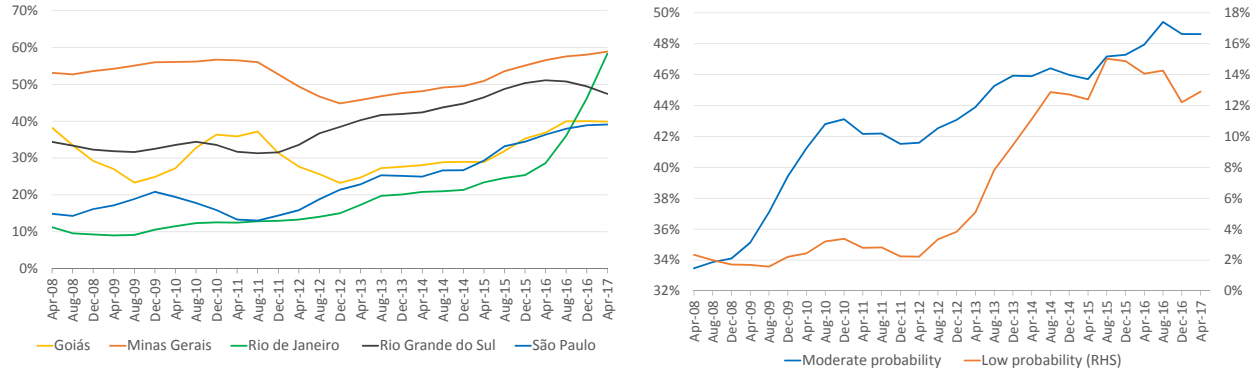
(b) *Groups of the remainder states*

Figure 6: *Trajectory of the probability of non-compliance for the legal expenses limit $CD/NCR \leq 200\%$ in the upcoming year for Brazilian states. For each point in the curves, we evaluate the probability of non-compliance in the upcoming year in an independent manner. For instance, with regard to the point referring to April 2014 for the state of São Paulo, we evaluate its probability of non-compliance until April 2015. The list of states classified as moderate or low is given in footnote 19.*

The probability of non-compliance with the limit CD/NCR of Rio de Janeiro rapidly increases after the end of 2013, surpassing that of the state of Rio Grande do Sul by April 2016, which was the state with the highest probability of non-compliance until then. Minas Gerais also has a high probability of non-compliance, but without positive or negative trends. There is also a significant increase in São Paulo's probability of non-compliance with the limit CD/NCR after the first semester of 2015. Goiás maintains the probability of non-compliance virtually at zero throughout the period.

Probability of non-compliance with the legal limit $PE/NCR \leq 49\%$. Figures 7a and 7b show our estimates for the probability of non-compliance with the legal expenses limit $PE/NCR \leq 49\%$ (Executive branch limit) of Brazilian states in the next year for Goiás, Minas Gerais, Rio de Janeiro, Rio Grande do Sul, São Paulo, and subgroups of the remainder states.

In general, we observe a trend of increasing values for the probability of non-compliance after 2013. Rio de Janeiro stands out due to the very rapid increase of its probability of non-compliance after 2016. By the end of 2015, Rio de Janeiro had a probability of non-compliance nearing 25%, which increased up to 60% by April 2017. Minas Gerais has indices that are consistently above the FRL legal limit on personnel expenses.



(a) *Goiás, Minas Gerais, Rio de Janeiro, Rio Grande do Sul, and São Paulo*

(b) *Groups of the remainder state*

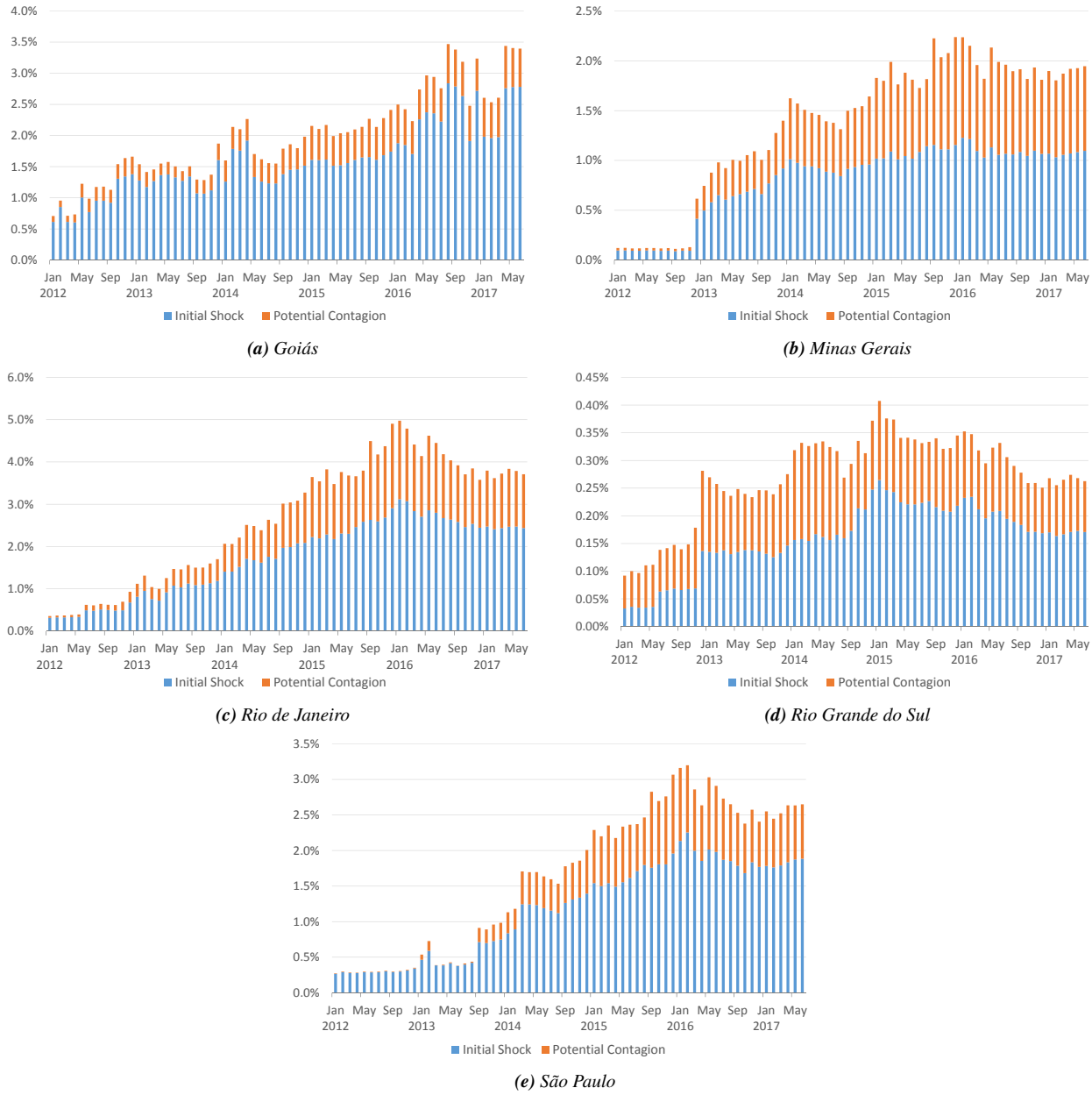
Figure 7: *Trajectory of the probability of non-compliance with the legal expenses limit $PE/NCR \leq 49\%$ (Executive branch) in the following relative year for Brazilian states. For each point in the curves, we evaluate the probability of non-compliance in the upcoming year in an independent manner. For instance, with regard to the point referring to April 2014 for the state of São Paulo, we evaluate its probability of non-compliance until April 2015. The list of states classified as moderate or low is given in footnote 19.*

6.2 Resilience of the financial system to defaults of Brazilian states

In this section, we quantify the resilience of the Brazilian financial system in the event that states default on their bank debt. Although bank debt is not the most representative funding source of Brazilian states, we should carefully look at these exposures because the financial system is strongly interconnected and mostly serves as a backbone through which several markets communicate. The crisis of 2008 is an example that clarifies how localized shocks could spread worldwide due to financial linkages and therefore develop micro events into systemic events.

We first estimate the rate at which the financial system amplifies losses in the event that states do not honor their bank debt, disregarding the probability of occurrence of such event. This empirical exercise is important for financial stability supervisors in the sense that it provides estimates of losses in view of specific state defaults.

Figure 8 shows the short-term consequences of a hypothetical credit default to the financial system of the states of Goiás, Minas Gerais, Rio de Janeiro, Rio Grande do Sul, and São Paulo. We measure the overall impact in terms of the banking system's total capital losses. In addition to this direct hit, we evaluate the negative spillover effects due to successive credit deterioration and credit crunches, which can be seen as sources of financial contagion.



(a) Goiás

(b) Minas Gerais

(c) Rio de Janeiro

(d) Rio Grande do Sul

(e) São Paulo

Figure 8: *Initial shock and the corresponding amplification of the financial system. The results are reported in terms of the banking system's total capital. The initial shock component represents the losses that banks would suffer if states default on their bank debt. The amplification arises due to successive credit deterioration and credit crunches, which can be seem as a form of financial contagion. We evaluate the initial shock and the corresponding amplification for each point in time in an independent manner.*

The default of Rio de Janeiro on its bank debt has increasing negative consequences for the financial system until the end of 2015. After this date, the outstanding bank credit starts to slowly decrease while creditor banks of the Rio de Janeiro state increase capitalization. Both factors lead to a decrease of the credit default impact of Rio de Janeiro on the financial system. São Paulo has a similar pattern. However, unlike Rio de Janeiro, potential contagion starts to take significant amounts only after September 2013, when São Paulo took new loans in the financial system. The parcel due to potential contagion is the largest for credit defaults from the state of Minas Gerais. This fact is

partly because their creditor banks are relatively less capitalized, which increases the vulnerability and likelihood that shocks will get amplified inside the financial network. This fact highlights the importance of the network topology in shaping how shocks can be amplified.

Overall, in the short term, the Brazilian financial system is resilient to shocks from Brazilian states in the sense that no banks would go bankrupt. We attribute this factor to the high levels of bank capitalization in the financial system. Bank capital serves as a shock barrier to external negative shocks as it is the primary source of loss absorption. Nonetheless, we see a significant downfall of the financial system capitalization, which could have further negative consequences for the real sector in the medium and long run that our systemic risk model cannot capture because it assumes the network structure as exogenous.

Table 5 segregates the total impact on the financial system—comprising the initial shock and the potential contagion—in terms of bank control: state-owned and domestic/foreign private banks. Credit defaults from the five Brazilian states that are struggling the most with fiscal problems would mostly hit state-owned banks. These banks not only are the major creditor banks to these states but also take central positions in the network, which make them susceptible to receiving shocks in the form of financial contagion. Both factors contribute to increasing financial stress over these state-owned banks. Domestic and foreign private banks would not be significantly affected, neither by the initial shock nor by financial contagion.

Table 5: *Proportion of losses that banks are subject to, in view of credit default from states. Losses are evaluated in December 2015 and 2016 and are broken down by bank control: state-owned and domestic/foreign private banks. We only report results for the five states that are struggling the most with fiscal problems by the end of the sample period: Goiás , Minas Gerais, Rio de Janeiro, Rio Grande do Sul, and São Paulo. We depict in bold the type of bank that suffers the most at each time point and state.*

	December 2015		June 2016	
State	**State-owned banks**	**Private banks**	**State-owned banks**	**Private banks**
Goiás	**95.31**%	4.69%	**97.08**%	2.92%
Minas Gerais	**97.00**%	3.00%	**98.62**%	1.38%
Rio de Janeiro	**96.53**%	3.47%	**98.19**%	1.81%
Rio Grande do Sul	**96.15**%	3.85%	**98.10**%	1.90%
São Paulo	**96.02**%	3.98%	**97.65**%	2.35%

6.3 Expected impact of states on the financial system under negative prospects of GDP growth

In this section, we weigh the potential loss that the financial system would suffer if states default on their bank loans with the probability of occurrence of such event. This weighting rules out events that would impose large losses on the system but are otherwise negligible in terms of actual feasibility.

As discussed before, we assume that states that do not comply with the legal expenses limits imposed by the FRL are likely to undergo serious fiscal problems, preventing them from fully paying

back their bank loans. We take the conservative approach here in terms of systemic risk analysis and assume that these states do not pay back their remaining outstanding bank debt. We estimate states probability of default as the highest of the probabilities of non-compliance with the legal expenses limits CD/NCR and PE/NCR. This is because the non-compliance of a single legal constraint already makes the state subject to legal sanctions.

Figure 9a shows the potential losses that the financial system would suffer from credit defaults from the states of Goiás, Minas Gerais, Rio de Janeiro, Rio Grande do Sul, and São Paulo. We report potential monetary losses, which are the sum of losses due to the credit default and the potential financial contagion that could happen inside the network. Figures 9b and 9c indicate the lower and upper bounds, respectively, of our estimates for the states probability of default, which takes into account the likelihood of not complying with the FRL legal restrictions. Figures 9d and 9e show lower and upper bounds, respectively, for the expected losses to the financial system due to the Brazilian states.

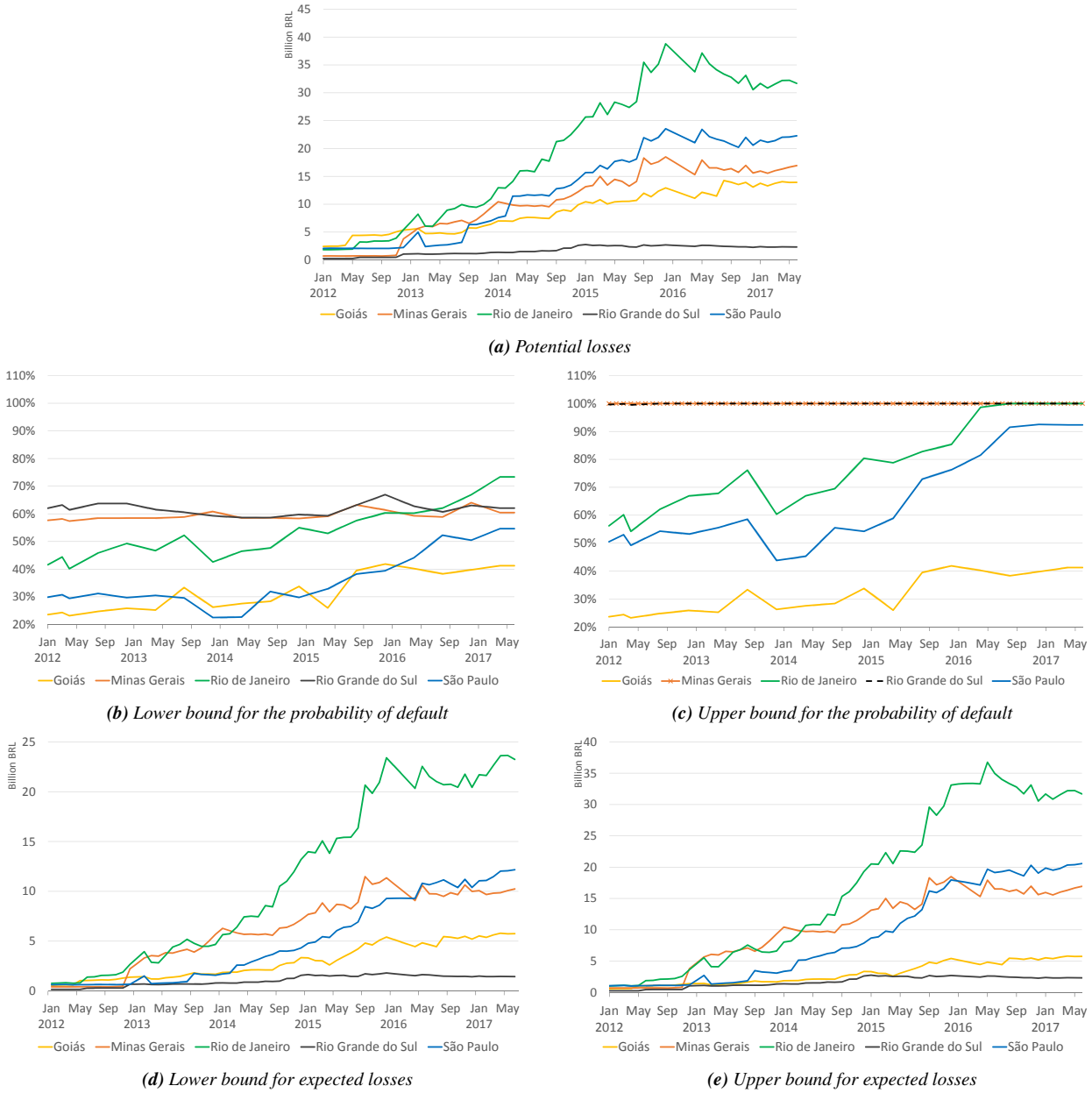
The expected losses from the most fragile states, on average, decrease from 2016 onwards, which may be related to the federal government's agreements with Brazilian states. Among these agreements, states can temporarily suspend payments to the Treasury, which, in theory, would give then surplus cash to pay back their bank debt and other obligations.

In 2012, Goiás was the state that would inflict the largest losses to the Brazilian financial system. However, the occurrence of this event of credit default was more unlikely comparatively to the state of Rio de Janeiro in the same year. In this way, the combination of the impact and probability explains the smaller expected loss to the financial system for the state of Goiás in relation to the state of Rio de Janeiro in the same year. From 2013 onwards, Rio de Janeiro rapidly becomes the most risky Brazilian state to the financial system, both unconditionally and conditionally.

If we do not consider the probability of occurrence of defaults, São Paulo would be the second most risky state to the Brazilian financial system from March 2014 onwards, followed by Minas Gerais. However, the expected losses to the financial system due to São Paulo are smaller than those caused by Minas Gerais until March 2016, due to its comparatively stronger fiscal health (Figures 9b and 9c).

7 Conclusions

This work contributes with the literature of public finance and banking in three important points. First, we develop a new methodology to estimate the probability of non-compliance of Brazilian states to a legal framework. We innovate in the sense that the methodology not only uses historical data but also embodies expectations or prospects for real GDP in the future. Second, we use a comprehensive systemic risk model that encompasses the real and financial sector to quantify and understand the



(a) *Potential losses*

(b) *Lower bound for the probability of default*

(c) *Upper bound for the probability of default*

(d) *Lower bound for expected losses*

(e) *Upper bound for expected losses*

Figure 9: *(a) Potential losses to the financial system if we assume idiosyncratic state defaults. (b) Lower and (c) upper bound for the probability of default when we assume a negative growth rate for the real GDP of 5% in the upcoming year. (d) Lower and (e) upper bounds for the expected losses to which the financial system is subject. Figures (d) and (e) are a Hadamart product between graphs (a) and (b) / (a) and (c), respectively.*

resilience of the financial system to credit defaults from Brazilian states. Third, we combine the probability of non-compliance and the corresponding impact on the financial sector to compose a state-specific statistical indicator of expected loss in the financial sector.

We find that banks are resilient to credit defaults from Brazilian states, notably those under severe fiscal stress. Financial contagion remains small due to the high levels of capitalization that Brazilian banks, on average, maintain. State-owned banks are the most sensitive to credit defaults from Brazilian states, partly due to the fact that they are the largest creditor banks and because they

take central positions in the network structure, thus making them more susceptible to financial contagion.

The major creditor of Brazilian states is the federal government, and not the banking sector, except for the state of Goiás. This notwithstanding, it is still important to understand the resilience of the financial system to these external shocks, because financial interconnections can largely amplify shocks, such as occurred with the last global financial crisis. Bank capitalization plays a major role as a financial shock barrier to hold back and control the level of amplification of financial shocks.

The model has some limitations. Other indirect channels of shock transmission to the financial system through public entities are relevant. For example, nonpayment to state employees would lead to an increase in their loan defaults to the financial sector. In addition, government indebtedness could force a reduction of investments, leading to increased unemployment and idleness of firms, and these, in turn, would have problems to honor their debts with the financial system. Finally, some of the bank credit that Brazilian states take are guaranteed by the federal government. Therefore, they are virtually risk-free if we assume that the federal government will not default. In this way, if we remove these guaranteed bank credit, then the exposure of the financial sector to fiscal risk would decrease.

References

Anand, K., Craig, B., and von Peter, G. (2015). Filling in the blanks: network structure and interbank contagion. *Quantitative Finance*, 15:625–636.

Anand, K., van Lelyveld, I., Ádám Banai, Friedrich, S., Garratt, R., Hałaj, G., Fique, J., Hansen, I., Jaramillo, S. M., Lee, H., Molina-Borboa, J. L., Nobili, S., Rajan, S., Salakhova, D., Silva, T. C., Silvestri, L., and de Souza, S. R. S. (2018). The missing links: A global study on uncovering financial network structures from partial data. *Journal of Financial Stability*, 35:107–119.

Aoyama, H. (2014). Systemic risk in Japanese credit network. In Abergel, F., Aoyama, H., Chakrabarti, B. K., Chakraborti, A., and Ghosh, A., editors, *Econophysics of Agent-Based Models*, New Economic Windows, pages 219–228. Springer International Publishing.

Arnold, B., Borio, C., Ellis, L., and Moshirian, F. (2012). Systemic risk, macroprudential policy frameworks, monitoring financial systems and the evolution of capital adequacy. *Journal of Banking and Finance*, 36(12):3125–3132.

Bardoscia, M., Battiston, S., Caccioli, F., and Caldarelli, G. (2015). DebtRank: a microscopic foundation for shock propagation. *PLoS ONE*, 10(6):e0130406.

Battiston, S., Farmer, J. D., Flache, A., Garlaschelli, D., Haldane, A. G., Heesterbeek, H., Hommes, C., Jaeger, C., May, R., and Scheffer, M. (2016). Complexity theory and financial regulation. *Science*, 351(6275):818–819.

Battiston, S., Gatti, D. D., Gallegati, M., Greenwald, B., and Stiglitz, J. E. (2012). Liaisons dangereuses: increasing connectivity, risk sharing, and systemic risk. *Journal of Economic Dynamics and Control*, 36(8):1121 – 1141.

Battiston, S. and Martinez-Jaramillo, S. (2018). Financial networks and stress testing: Challenges and new research avenues for systemic risk analysis and financial stability implications. *Journal of Financial Stability*, 35:6 – 16.

Berndsen, R. J., León, C., and Renneboog, L. (2018). Financial stability in networks of financial institutions and market infrastructures. *Journal of Financial Stability*, 35:120 – 135.

Blume, L., Easley, D., Kleinberg, J., Kleinberg, R., and Tardos, E. (2013). Network formation in the presence of contagious risk. *ACM Transactions on Economics and Computation*, 1(2):6:1–6:20.

Cajueiro, D. O. and Tabak, B. M. (2008). The role of banks in the Brazilian interbank market: Does bank type matter? *Physica A: Statistical Mechanics and its Applications*, 387(27):6825 – 6836.

Cohen, S., Doumpos, M., Neofytou, E., and Zopounidis, C. (2012). Assessing financial distress where bankruptcy is not an option: An alternative approach for local municipalities. *European Journal of Operational Research*, 218(1):270 – 279.

Corsi, F., Lillo, F., Pirino, D., and Trapin, L. (2018). Measuring the propagation of financial distress with Granger-causality tail risk networks. *Journal of Financial Stability*, 38:18 – 36.

Eisenberg, L. and Noe, T. H. (2001). Systemic risk in financial systems. *Management Science*, 47(2):236–249.

Gai, P., Haldane, A., and Kapadia, S. (2011). Complexity, concentration and contagion. *Journal of Monetary Economics*, 58(5):453–470.

Galariotis, E., Guyot, A., Doumpos, M., and Zopounidis, C. (2016). A novel multi-attribute benchmarking approach for assessing the financial performance of local governments: Empirical evidence from France. *European Journal of Operational Research*, 248(1):301 – 317.

Glasserman, P. and Young, H. P. (2015). How likely is contagion in financial networks? *Journal of Banking and Finance*, 50:383–399.

Guerra, S. M., Silva, T. C., Tabak, B. M., de Souza Penaloza, R. A., and de Castro Miranda, R. C. (2016). Systemic risk measures. *Physica A: Statistical Mechanics and its Applications*, 442:329 – 342.

Hanson, S. G., Kashyap, A. K., and Stein, J. C. (2011). A macroprudential approach to financial regulation. *Journal of Economic Perspectives*, 25(1):3–28.

Hojman, D. A. and Szeidl, A. (2008). Core and periphery in networks. *Journal of Economic Theory*, 139:295–309.

Kanno, M. (2015). Assessing systemic risk using interbank exposures in the global banking system. *Journal of Financial Stability*, 20:105 – 130.

Lin, M.-I., Lee, Y.-D., and Ho, T.-N. (2011). Applying integrated DEA/AHP to evaluate the economic performance of local governments in China. *European Journal of Operational Research*, 209(2):129 – 140.

Liu, F., Hua, Z., and Lim, A. (2015). Identifying future defaulters: A hierarchical bayesian method. *European Journal of Operational Research*, 241(1):202 – 211.

Lux, T. (2015). Emergence of a core-periphery structure in a simple dynamic model of the interbank market. *Journal of Economic Dynamics and Control*, 52(0):A11–A23.

Magkonis, G. and Tsopanakis, A. (2016). The financial and fiscal stress interconnectedness: The case of G5 economies. *International Review of Financial Analysis*, 46:62 – 69.

Merton, R. (1974). On the pricing of corporate debt: The risk structure of interest rates. *The Journal of Finance*, 29(2):449–470.

Poledna, S., Molina-Borboa, J. L., Martínez-Jaramillo, S., van der Leij, M., and Thurner, S. (2015). The multi-layer network nature of systemic risk and its implications for the costs of financial crises. *Journal of Financial Stability*, 20:70–81.

Roukny, T., Battiston, S., and Stiglitz, J. E. (2018). Interconnectedness as a source of uncertainty in systemic risk. *Journal of Financial Stability*, 35:93 – 106.

Silva, T. C., da Silva Alexandre, M., and Tabak, B. M. (2018). Bank lending and systemic risk: A financial-real sector network approach with feedback. *Journal of Financial Stability*, 38:98–118.

Silva, T. C., Guerra, S. M., Tabak, B. M., and de Castro Miranda, R. C. (2016a). Financial networks, bank efficiency and risk-taking. *Journal of Financial Stability*, 25:247 – 257.

Silva, T. C., Silva, M. A., and Tabak, B. M. (2017a). Systemic risk in financial systems: a feedback approach. *Journal of Economic Behavior and Organization*, 144:97–120.

Silva, T. C., Souza, S. R. S., and Tabak, B. M. (2016b). Network structure analysis of the Brazilian interbank market. *Emerging Markets Review*, 26:130–152.

Silva, T. C., Souza, S. R. S., and Tabak, B. M. (2017b). Monitoring vulnerability and impact diffusion in financial networks. *Journal of Economic Dynamics and Control*, 76:109–135.

Silva, T. C., Tabak, B. M., and Guerra, S. M. (2017c). Why do vulnerability cycles matter in financial networks? *Physica A: Statistical Mechanics and its Applications*, 471:592 – 606.

Souza, S. R., Tabak, B. M., Silva, T. C., and Guerra, S. M. (2015). Insolvency and contagion in the Brazilian interbank market. *Physica A: Statistical Mechanics and its Applications*, 431:140 – 151.

Souza, S. R. S., Silva, T. C., Tabak, B. M., and Guerra, S. M. (2016). Evaluating systemic risk using bank default probabilities in financial networks. *Journal of Economic Dynamics and Control*, 66:54–75.