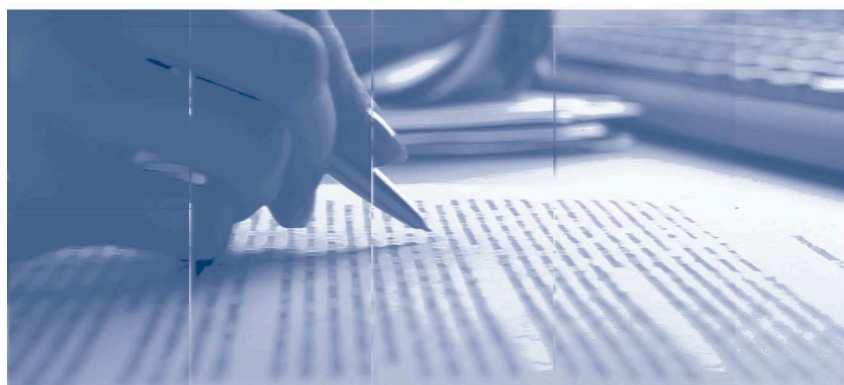


Breakeven Inflation Rate Estimation: an alternative approach considering indexation lag and seasonality

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Non-Technical Summary

The monitoring of economic agents' inflation expectations is crucial in an inflation-targeting regime. Through this monitoring, central banks can analyze the degree of anchoring of inflation expectations and the credibility of monetary policy. A widely used way to estimate this expectation is through the breakeven inflation embedded in financial instruments, calculated by the difference between nominal and real interest rates.

However, the seasonal behavior of inflation and the lag of inflation-linked instruments in relation to their indexation index generate distortions in the breakeven inflation calculation, especially for short-term estimates.

The objective of this article is to propose an alternative method of estimating breakeven inflation using only government bonds, as well as to determine whether this measure contains relevant information on future inflation in the short term, between 4 and 57 calendar days before the inflation index was released.

The empirical results show the proposed breakeven inflation measure: (i) reacts more quickly to unexpected price shocks; (ii) is superior to the breakeven inflation, which does not consider the indexation lag and seasonality; (iii) was statistically equal to or better than survey-based inflation expectations (in the case of Brazil, the Focus and Focus Top 5 surveys) and the breakeven inflation embedded in futures market (extracted through DAP contracts).

These findings are relevant for at least two reasons: (i) outdoing survey-based inflation expectations is not a trivial task, especially in the short run; and (ii) the breakeven inflation rate is constantly updated, allowing real-time monitoring of market agents' expectations by policymakers.

Sumário Não Técnico

O monitoramento das expectativas inflacionárias dos diversos agentes econômicos é crucial em um regime de metas de inflação. Por meio desse monitoramento, os bancos centrais podem analisar o grau de ancoragem das expectativas de inflação e a credibilidade da política monetária. Uma forma muito utilizada para se estimar esta expectativa é por meio da inflação implícita embutida em instrumentos financeiros, calculada pela diferença entre taxas de juros nominais e reais.

Entretanto, o comportamento sazonal da inflação e a defasagem dos instrumentos financeiros indexados à inflação em relação ao seu índice de indexação geram distorções no cálculo da inflação implícita, principalmente para estimações de curto prazo.

O objetivo deste artigo é propor um método alternativo de se estimar a inflação implícita que utiliza somente títulos públicos, assim como determinar se essa medida contém informação relevante sobre a inflação futura no curto prazo, entre 4 e 57 dias corridos antes da divulgação do índice de inflação.

Os resultados empíricos mostram que a medida de inflação implícita proposta: (i) reage mais rapidamente a choques inesperados de preços; (ii) é superior em relação à inflação implícita que não considera a defasagem de indexação e a sazonalidade; (iii) é estatisticamente igual ou superior às medidas baseadas em pesquisas de mercado (no caso brasileiro, as pesquisas Focus e Focus Top 5) e à inflação implícita extraída do mercado futuro (por meio de contratos de DAP).

Esses resultados são relevantes por pelo menos duas razões: (i) superar pesquisas de mercado sobre expectativa de inflação não é uma tarefa trivial, especialmente no curto prazo; e (ii) a inflação implícita é atualizada permanentemente, permitindo um acompanhamento em tempo real, por parte dos *policy makers*, das expectativas dos agentes de mercado.

Breakeven Inflation Rate Estimation: an alternative approach considering indexation lag and seasonality[†]

Flávio de Freitas Val [‡]

Gustavo Silva Araujo[§]

Abstract

The breakeven inflation rate (BEIR) is the difference between nominal and real interest rates. This measure can be obtained from fixed-rate and inflation-indexed government bonds. However, this task has two difficulties: a) inflation-indexed securities have an indexation lag in respect to inflation; and b) the seasonality of inflation implies seasonality of the real rate. The objective of this paper is to propose an alternative method for BEIR estimation using only government bonds, as well as to determine if this measure contains information on future short-term inflation. A good measure of BEIR estimated directly from bonds is relevant since these are the inflation-linked instruments with the highest liquidity. The empirical results show that the proposed BEIR is superior to the BEIR that do not take into account the problems of lag-indexation and seasonality and is statistically equal to or better than survey-based inflation expectations and the BEIR extracted from the future market when forecasting short-term inflation. Furthermore, the results show the lag and seasonal adjustments are more important for the estimation of the BEIR in the short term. An advantage of the proposed BEIR in relation to survey-based inflation expectations is that it allows for real-time monitoring, since it is continually updated.

Keywords: Implied Inflation, Breakeven Inflation, BEIR, Inflation Forecast, Inflation Expectations.

JEL Code: G12, E43, G17

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1. Introduction

Monitoring the inflation expectations of economic agents is crucial in the inflation-targeting regime. It allows central banks to analyze the degree of anchoring of inflation expectations and the credibility of monetary policy. One way to make this monitoring is through financial assets.

The prices of financial assets reflect the expectations and uncertainties of the market participants in relation to future prices. Therefore, it is possible to extract expectations directly from assets spot prices. Implied price expectations have some advantages over survey expectations (see, for example, Sörderlind, 2011): the updating takes place in real time if there is enough liquidity of financial instruments, and no incentives are required to update the prices at high frequency.

The inflation traded in the financial market is calculated from the difference between nominal and real interest rates of risk-free assets. This measure is called the breakeven inflation rate (BEIR). The nominal and real rates are traded in fixed-rate securities and derivatives.¹ The goal of this paper is to propose a method to estimate implied inflation from government fixed-income securities.²

Araujo and Vicente (2017) present an equation for the BEIR using assets traded in the financial market. The main contribution of that work is the treatment of the inflation lag that affects the inflation-indexed securities. The indexation lag is related to the fact that inflation does not index the bond up to the maturity day: inflation is always lagged for a certain period, since its calculation is not daily.³ Christensen (2018) analyses U.K. inflation-linked bonds with similar maturities but different indexation lags, and shows that the lag length is more relevant when the bond is close to maturity due to inflation seasonality. This lag makes the interest rate traded via inflation-indexed bonds not to exactly equal to the real interest rate.

¹In relation to securities, nominal rates are traded in fixed-rate securities and real rates are traded in inflation-indexed securities. Bernanke (2004) argues that inflation-indexed securities are a valuable source of information about inflation expectations.

²Vicente and Graminho (2015) show the main component of the difference between nominal and real interest rates is the inflation expectation.

³For example, in Brazil and in the U.S. the lags of inflation-indexed government bonds are 15 calendar days and 3 months, respectively. In the U.K., the first inflation-linked bonds issued have 8-months lag. Since 2005, the new issued bonds have 3-month lag. Therefore, bonds with both three-month and eight-month indexation lags have been traded simultaneously since then.

However, the equation presented by Araujo and Vicente (2017) only uses prices of financial assets that do not pay coupons (i.e., intermediate payments), which is not the case of inflation-indexed government bonds. This article presents an alternative method to estimate the BEIR based on inflation-indexed bonds.

In the Brazilian market, the most traded instruments indexed to inflation are government bonds (NTN-Bs) with coupons. Araujo and Vicente (2017) make use of a futures market instrument (DAP), which has the same underlying asset as NTN-Bs, in order to extract the intermediate rates of the bonds and construct synthetic bonds without coupons.⁴ However, according to Fleckenstein et al. (2014), in the U.S. market even securities of the same issuer, same indexation, same cash flows and maturity may show differences in pricing. In this way, it is not reasonable to combine two instruments that, despite having the same underlying asset, are traded in different markets.

When using these two instruments together, there are some factors that can affect the quality of the estimates of the BEIR:

i) The liquidity of the NTN-Bs is much higher than the liquidity of the DAP contracts. For example, on June 1, 2018, the DV01 of open positions in DAP amounted to less than 2% of the positions in NTN-B;⁵

ii) The nominal rates implied in the two assets are based on different financial instruments;⁶

iii) NTN-Bs are traded in the over-the-counter market, while DAP contracts are traded on the stock exchange;

iv) NTN-Bs are traded in the spot market, i.e., when buying NTN-Bs there is disbursement. The DAP is a future contract. Therefore, when buying or selling DAP contracts, there is no cash flow, but a margin of guarantee must be deposited;

⁴ Series B National Treasury Notes (NTN-B) are fixed-income securities issued by the Brazilian government whose face value is indexed to the Broad Consumer Price Index (IPCA), calculated by the Brazilian Institute of Geography and Statistics (IBGE). DAP is the code for the IPCA coupon futures contracts traded on the Brazilian stock exchange. The DAP contracts have the same characteristics as the NTN-Bs, i.e., both instruments have the same underlying asset, the interest rate called IPCA coupon, and are linked to inflation.

⁵DV01, in this case, is defined as the financial variation of market positions (excluding positions in the Central Bank of Brazil (BCB) portfolio) due to a 1 basis point change in the interest rate of the asset (DAP or NTN-B).

⁶ In DAP contracts, the nominal rate is based on the interbank deposits rate (DI rate), while in the NTN-B contracts the nominal rate is based on the basic interest rate set by the BCB (Selic rate).

v) The trading of each instrument is carried out on different bases: the NTN-Bs are traded based on the yield to maturity rate, and the DAP is traded based on the spot rate; and

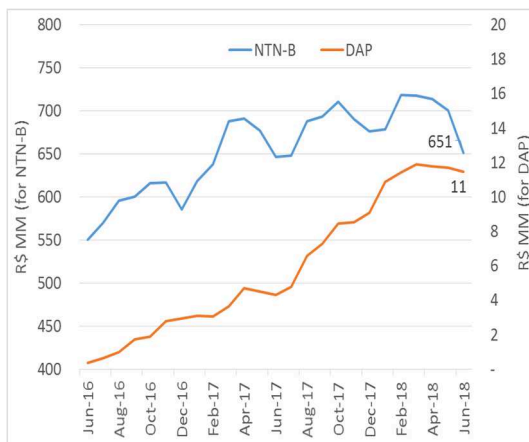
vi) DAP contracts are settled on a daily basis (as any other future contract), while NTN-Bs have cash flows only on specific dates.

It is appropriate here to outline some figures about market participants' exposure to these two instruments. Graphs 1.1 and 1.2 show the evolution of the DV01 of NTN-B and DAP positions held by market participants.

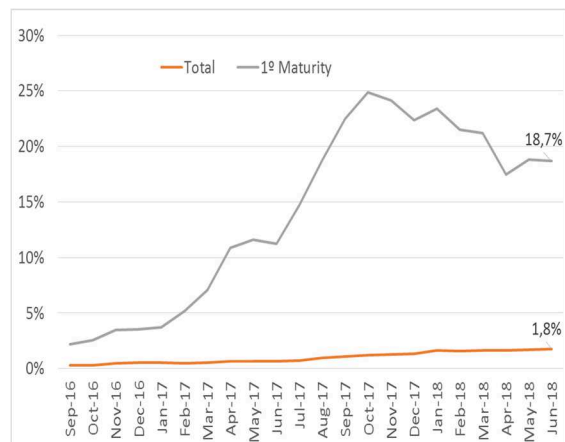
Between June 2016 and June 2018, the DV01 of the DAP contracts increased about 32 times, going from R\$340 thousand to R\$11.5 million, while the DV01 of NTN-Bs grew about 18% in the same period (see Graph 1.1).

From Graph 1.2, we can note the relative total exposure of the market in DAP contracts is still small, something around 1.8% over the NTN-B DV01. However, when we consider DV01 only for the first NTN-B maturity and its corresponding contract in the DAPs, we can see a higher and increasing market exposure in derivatives, indicating DAP contracts play an important role in the short-term strategies of market participants (the short-term DAP accounted for 18,7% of the DV01 held in NTN-Bs maturing in August 2018)

Graph 1.1 – DV01 Outstanding in NTN-B and DAP Contracts



Graph 1.2 – Relative DV01 Outstanding in DAP contracts versus NTN-B (DAP to NTN-B ratio)



Due to all these differences, it is not uncommon to have distinct BEIR estimates from these two instruments for the same period. For example, by comparing the BEIR for August 2018 estimated in the period from February 2018 to July 2018 for the two assets, there is an average daily difference of 3 basis points. The NTN-Bs in this period have no more intermediate payments.⁷ Therefore, the calculation of BEIR using only government securities is very important, since these assets are the inflation-indexed instruments with the highest liquidity. This article attempts to fill this gap.

The method developed here consists of pricing synthetic zero-coupon inflation-indexed bonds (synthetic zero-coupon NTN-Bs). The prices of these securities are necessary to estimate the BEIR with the equation presented by Araujo and Vicente (2017). These synthetic NTN-Bs mature on the same dates as the bonds traded in the market (NTN-Bs that pay coupons). In order to estimate the rates relative to synthetic bond prices, we construct the term structure with the Svensson (1995) interpolation method from coupon bonds.⁸

The Svensson (1995) interpolation does not fit well with inflation-indexed bonds, since this method allows only two curvatures for the real interest rate term structure, while its graph actually has several curvatures, as the inflation seasonality is high.⁹ However, the method of this article uses the Svensson interpolation in a different way. We use all the bonds traded in the market to estimate the parameters of the Svensson model but we do not use these values for all terms. We use only the parameters to estimate the interest rates for the time to maturity of the inflation-linked bonds, which are the contract lengths in which the largest cash flows of the securities are paid. After calculating the BEIR for a given date, we distribute the inflation rate across months, with weights proportional to the distribution of the Brazilian survey-based inflation expectations (the Focus Survey).¹⁰

⁷ In this case, we are not making approximations in the calculation of the BEIR because these NTN-Bs did not have further intermediate payments. Therefore, if the instruments were really the same, there should be no differences between the inflation rates.

⁸ Araujo and Vicente (2017), in addition to presenting a method that combines government bonds with coupons (NTN-Bs) and futures contracts (DAPs), also show how to extract the BEIR only from the futures market. However, the authors do not have an extraction method that uses only government bonds.

⁹ The real interest rate is the difference between the nominal rate and the BEIR. The nominal rate does not show seasonality. Therefore, just like inflation, the real interest rate also presents seasonality.

¹⁰ The Focus Report is a report released by the BCB containing projections on Brazilian economic indicators collected from market participants. These agents can update projections daily. See <http://www.bcb.gov.br/pec/GCI/PORT/readout/readout.asp>.

As an empirical exercise, we applied the method described above to estimate the BEIR between January 2010 and July 2018. We compared the BEIR generated by government bonds with the: (i) median forecast of the Focus survey; (ii) median of the forecasts of the short-term Top 5; (iii) method proposed in Araujo and Vicente (2017) to calculate the BEIR using futures contracts; and (iv) BEIR from government bonds that do not take into account the problems of lag-indexation and seasonality (the Naïve BEIR). The comparison was made only in the short term, with the hypothesis that in this case the risk premium is small. The results showed that the proposed BEIR is competitive in relation to the other measures: the median of the forecasts of the short-term Top 5 and the Proposed BEIR had the smallest forecast errors. We also concluded that the lag and seasonal adjustments are more important for short-term expectations.

The rest of this article is organized as follows. Section 2 presents the database. Section 3 shows the details of the method for estimating the BEIR from government securities. Section 4 presents the results of the empirical exercise that compares the proposed BEIR with the expectations of the Focus Report, with the BEIR using the futures market and with the Naïve BEIR. Finally, in Section 5, we offer our final remarks.

2. Database

In order to calculate the BEIR with the method proposed in this article and the Naïve BEIR, we use fixed-rate government bonds (the zero-coupon bonds - the LTNs, and the coupon bonds - the NTN-Fs) and coupon inflation-indexed government bonds (NTN-Bs). Besides LTNs and NTN-Fs, we also use the overnight Selic interest rate as the first maturity for the nominal interest rate curves estimations, since this rate can be considered as a one-day maturity fixed- rate bond.¹¹

LTNs and NTN-Fs are securities with face value of R\$1,000.00. NTN-Fs pay semiannual cash flows of 10% per year. NTN-Bs are inflation-indexed securities, which pays the interest rate agreed at the time of purchase (yield to maturity) plus the IPCA (Broad Consumer Price Index) variation. This security has semiannual payment flows corresponding to 6% per year of the face value, which is updated monthly by the IPCA change and is therefore called VNA (*Valor Nominal Atualizado* in Portuguese, or

¹¹ Selic rate is the BCB's policy interest rate.

Updated Nominal Value).¹² On the maturity date of the security, the VNA and the last coupon are paid.

The securities prices described above were calculated from the Brazilian Financial and Capital Markets Association (ANBIMA) indicative rates, which reflect the closing prices of the daily trading of these bonds in the government securities secondary market.¹³

We compare our BEIR measure (generated only with government bonds) with the BEIR extracted from future contracts (DAP), with inflation expectations published in the Focus Report and with the Naïve BEIR. The DAP, traded on the Brazilian stock exchange, has the same underlying asset of the NTN-B, i.e., the rate of an IPCA-indexed bond, but with the difference that this rate is a zero-coupon rate (spot rate) since it is traded in the futures market.

Regarding survey-based inflation expectations, we use the Focus median inflation expectations and Focus median Top 5 Short-Term expectations. Both are consensus series in the Brazilian market and constantly used by market agents, specialized media and the monetary authority itself to monitor inflation expectation. These series, available for a maximum forecast horizon of 18 months, were obtained from the Market Expectations System at the BCB website. The Focus Top 5 Short-Term inflation expectation is also used to distribute the monthly BEIR generated by government securities and by the DAP.

The comparison between inflation forecasts is made in relation to the realized inflation, the IPCA, obtained from the IBGE website.

The data cover the period between January 2010 and July 2018, except for the DAP, which has improved liquidity only from May 2016 onward, when the Brazilian stock exchange hired market makers to trade the contract for some maturities.

¹² The NTN-B base date is July 15, 2000, when the VNA was established at R\$ 1,000.00. Since then, this value has been updated monthly by the change of the IPCA, published before the 15th of each month by the Brazilian Institute of Geography and Statistics (IBGE).

¹³ ANBIMA is a Brazilian association of financial market institutions that provides various services, including daily prices of government securities obtained by survey with banks, brokers and fund managers.

3. Methodology

In this section, we discuss the difficulties of calculating BEIR, the general methodology that addresses the problem of indexation lag in respect to inflation, the methodology proposed in this paper that uses only government bonds, the methodology that uses only the futures market, and the incorporation of the monthly seasonality in the calculation of the BEIR.

a. Difficulties in calculating the BEIR

There are two difficulties in calculating the BEIR: the problem of the inflation lag embedded in inflation-indexed securities and the seasonality of inflation. We detail these issues below.

The lag problem exists because inflation is not disclosed daily, but monthly, and with a lag. Thus, inflation cannot index a bond all the way to the maturity date, since it is not known in the days approaching maturity. Bonds all over the world have a lag. In Brazil, the NTN-Bs have a delay of 15 calendar days.

This problem means that the rate traded via these securities is not the real interest rate. The following argument will show the problem for a zero-coupon bond, but the same reasoning holds true for coupon bonds.

For any zero-coupon fixed-income security,

$$P_{t,T} = \frac{VNA_T}{(1 + R_{t,T})}, \quad (1)$$

in which $P_{t,T}$ is the price of the security at date t with maturity at T , VNA_T is the nominal value on the maturity date T (for fixed-rate securities this value is constant), and $R_{t,T}$ is the nominal rate from t to T .

We will use a zero-coupon NTN-B to exemplify the problem. The traded rate on this bond is called the IPCA coupon. Suppose we are in a day when the VNA is known.¹⁴ The VNA at maturity (VNA_T) will be the VNA at date t (VNA_t), adjusted by the lagged inflation of 15 calendar days:

$$VNA_T = VNA_t (1 + \pi_{t-15, T-15}),$$

¹⁴ The NTN-B VNA of the 15th day of each month is known.

where $\pi_{t-15,T-15}$ is the inflation of 15 calendar days before the security is priced up to 15 calendar days before its maturity. We can also decompose the nominal rate into real rate plus inflation. In this way, equation (1) can be rewritten as:

$$P_{i,T} = \frac{VNA_t(1 + \pi_{t-15,T-15})}{(1 + r_{i,T})(1 + \pi_{i,T})},$$

where $r_{i,T}$ is the real interest rate from t to T . Note that because of the inflation lag in the VNA calculation, the inflation rates in the numerator and denominator do not correspond to the same period. Decomposing the inflation rates in the previous equation, we obtain:

$$P_{i,T} = \frac{VNA_t(1 + \pi_{t-15,t})(1 + \pi_{t,T-15})}{(1 + r_{i,T})(1 + \pi_{i,T-15})(1 + \pi_{T-15,T})}, \text{ or}$$

$$P_{i,T} = \frac{VNA_t(1 + \pi_{t-15,t})}{(1 + r_{i,T})(1 + \pi_{T-15,T})}.$$

The IPCA coupon (C_IPCA), the traded rate on NTN-Bs, is defined as the solution of the following equation:

$$P_{i,T} = \frac{VNA_t}{(1 + C_IPCA_{i,T})}. \quad (2)$$

Thus,

$$1 + C_IPCA_{i,T} = \frac{(1 + r_{i,T})(1 + \pi_{T-15,T})}{(1 + \pi_{t-15,t})}. \quad (3)$$

If we assume that

$$\pi_{t-15,t} \approx \pi_{T-15,T},$$

the IPCA coupon equals the real rate. For long terms, this approximation does not cause problems, even if the two inflation rates are very different, since the real interest rate will hold for a period much longer than 15 calendar days. On the other hand, for shorter periods this difference may not be negligible.

Since the traded rate of inflation-indexed securities is not the real interest rate, the difference between the nominal rate and this rate is not exactly the BEIR.

In short, we can say that the problem of the indexation lag of Brazilian government securities occurs because the NTN-Bs do not trade inflation between the moment of their purchase and their maturity, but rather the inflation accrued between 15 days before purchase and 15 days prior to maturity. This indexation lag is a characteristic present in several countries that have inflation-indexed government bonds. The United States and United Kingdom are the two countries with the highest stocks and liquidity of inflation-indexed securities, in which Treasury Inflation-Protected Securities (TIPS) and Indexed-linked Gilts (Linkers) are traded respectively. Linkers issued prior to 2005 had an 8-month lag to incorporate the Retail Price Index (RPI) into the face value, while those issued after 2005 have a 3-month lag, the same as the TIPS.

The second problem is related to the seasonality of inflation. Inflation indexes have seasonality, as is the case of the IPCA, the index used in calculating NTN-B prices. The intervals of the cash flows of inflation-linked securities are six months, while the rates negotiated in these securities have large monthly seasonality, because of the normal behavior of the nominal rates and the high monthly seasonality of inflation. Thus, when calculating BEIR monthly, this seasonality must be taken into account.

b. General Methodology for the BEIR Calculation

The difference between this and other methodologies lies in the fact that the BEIR is extracted not from the trading date of the securities, but from the month in which the inflation is not yet known.

Equation (1) states, for a zero-coupon fixed-income security, that

$$P_{t,T} = \frac{VNA_T}{(1+R_{t,T})} ,$$

where $P_{t,T}$ is the price of the security at date t with maturity at T , VNA_T is the updated nominal value on the maturity date T and $R_{t,T}$ is the nominal rate from t to T .

We again use a zero-coupon NTN-B. Let VNA_{t^*} be the last known VNA. The VNA_T can be described as the last known VNA adjusted by inflation to maturity, with a lag of 15 calendar days. Thus:

$$P_{t,T} = \frac{VNA_{t^*}(1 + II_{t^*-15,T-15})}{(1 + R_{t,T})}, \quad \text{or}$$

$$II_{t^*-15,T-15} = \frac{P_{t,T}(1 + R_{t,T})}{VNA_{t^*}} - 1 \quad (4)$$

in which $II_{t^*-15,T-15}$ is the implied inflation embedded in the bond with a 15-calendar-day lag between t^* and the maturity T . The BEIR traded via a zero-coupon bond is the implied inflation extracted from equation (4).

NTN-B maturities occur on May 15th or August 15th, and the NTN-B VNAs are known on the 15th of each month when the previous month's inflation is already disclosed.¹⁵ Therefore, the BEIR extracted comprises the period from the first day of the month of the last known VNA until the last day of the month preceding the maturity of the NTN-B (April or July).

c. Method using only Government Bonds

In this section, we show the method developed in this study to calculate the BEIR using only government securities. In practice, inflation-linked zero-coupon bonds have no liquidity and will not be used in this calculation. We use only coupon bonds (NTN-Bs with coupons).

There are two situations for these inflation-linked coupon bonds: when they do not have any further intermediary payments and when they have. When there are not intermediary payments, we can use directly equation (4), with a change in the VNA_{t^*} , since these bonds pay a coupon at maturity. As the semiannual payment flows correspond to 6% per year of the face value (in compound interest rate), the equation becomes:

¹⁵Specifically, NTN-Bs maturing in an odd year mature on May 15th, while those maturing in even years mature on August 15th.

$$H_{t^*-15,T-15} = \frac{P_{t,T}(1+R_{t,T})}{VNA_{t^*}(1+2,956301\%)} - 1.$$

When there are intermediary payments, the initial step is to estimate the nominal rate curves (traded rates via LTNs and NTN-Fs) and IPCA coupons (traded rates via NTN-Bs) using the method of Svensson (1995).

The model developed by Svensson (1995) is widely used by the market and is currently estimated by central banks in several countries, such as Belgium, France, Germany, Norway, Spain, Sweden, Switzerland and Brazil.¹⁶ This method seeks to estimate interest curves smoothly and flexibly by adjusting at a point in time t a discount function to the bond price and assuming the following equation for the spot interest rate:

$$s_m = \beta_0 + \beta_1 \left[\frac{1 - \exp(-\lambda_1 m)}{\lambda_1 m} \right] + \beta_2 \left[\frac{1 - \exp(-\lambda_1 m)}{\lambda_1 m} - \exp(-\lambda_1 m) \right] + \beta_3 \left[\frac{1 - \exp(-\lambda_2 m)}{\lambda_2 m} - \exp(-\lambda_2 m) \right], \quad (4)$$

where s is the spot interest rate; m is the maturity; and $\beta_0, \beta_1, \beta_2, \beta_3, \lambda_1$ and λ_2 are model parameters to be estimated. The parametric equation in this work is estimated by applying a conditioned nonlinear optimization, in which the parameters λ_1 and λ_2 are greater than zero.

Regarding the parameter estimation, we work with two different error functions to be minimized:

$$\min error = \sum_{i=1}^I \frac{1}{d_i} (PU_{i,t} - \widehat{PU}_{i,t})^2 \quad \text{and} \quad \min error = \sum_{i=1}^I \left[\frac{1}{d_i} (PU_{i,t} - \widehat{PU}_{i,t}) \right]^2,$$

in which d_i is the duration of bond i , $PU_{i,t}$ is the price of bond i (ANBIMA indicative price), and $\widehat{PU}_{i,t}$ is the estimated price of bond i , both on date t . The first function is more traditionally used, but the second one provides more weight to short-term bonds.

¹⁶In addition, central banks of Finland and Italy use the Nelson and Siegel model (1987), considered the one that gave rise to the Svensson model (the Nelson and Siegel model is a reduced version of the Svensson model).

After simulations with these two error functions, we chose to work with the second one to estimate inflation-linked bond curves, since it gives more importance to the shortest part of the interest curve, which is the part that causes more problems in calculating the BEIR due to the inflation lag. In addition, since we work with spot rates, this part of the curve has the greatest impact of seasonal factors.¹⁷

To compute the initial beta and lambda values of the Svensson model we follow Manousopoulos and Michalopoulos (2009). The estimation of these parameters is a nonlinear optimization problem that depends on the initial points, which can be crucial to the accuracy of the estimated curve. The fact that these parameters have economic interpretation (expressing the level, slope and curvature of the interest curve) allows us to select initial points close to the solution and close to the convergence of the optimization algorithm. Thus, we use as initial values:

$$\beta_0 = \frac{1}{M} \sum_{i=1}^m y_i, \quad \beta_1 = y_c - \beta_0, \quad \beta_2 = \beta_3 = 0, \quad \lambda_1 = \lambda_2 = 1,$$

in which y is the yield to maturity of bond i ; m refers to bonds with the longest duration (in this case we use $m = 2$); and c refers to the bond with the shortest duration.

Table A in the Appendix shows the absolute errors in the short-term NTN-Bs during the analyzed period for the two error functions. The quadratic term applied to duration reduced the median of these errors by 10 basis points.

It is important to observe that the above method has not yet solved the problems in calculating the BEIR mentioned in section 3.1, i.e., we cannot guarantee that: (i) the traded rate in the inflation-indexed bond, the IPCA coupon, is equal to the real interest rate; and (ii) the real interest rate reflects cyclical price fluctuations. In sum, the Svensson model is used only to estimate the rates embedded in the bonds, that is, it does not address, respectively, the problem of indexation lag of government bonds and the problem of seasonality.

The method to solve these problems is described as follows: (i) we estimate the nominal and IPCA coupon spot rates to the maturity date of each NTN-B with the Svensson model described above. Identifying the yields only to the maturity date of the

¹⁷ For more details on interest rate estimation models, see BIS (2005) and Svensson (1995).

bonds allows us to avoid distortions due to strong changes in curvature that occur in the interest curve, especially in the short term; and (ii) we find the price of a synthetic zero-coupon bond at the valuation date:

$$P_{t,T} = \frac{VNA_{t^*}}{(1+C_{IPCA_{t,T}})} \quad ,$$

where $P_{t,T}$ is the synthetic zero-coupon NTN-B price on t with maturity on T , estimated considering the VNA_t , which is the nominal value updated at date t and $C_{IPCA_{t,T}}$, which is the IPCA coupon from t to T estimated by the Svensson model.¹⁸

With the price of a synthetic zero-coupon NTN-B, we can calculate the spot rate of implied inflation (BEIR) with a lag of 15 days for each NTN-B maturity ($\Pi_{t^*-15,T-15}$). For this, we use equation (4), where t^* is the date of the last known VNA and $R_{t,T}$ is the nominal interest rate calculated by Svensson using nominal government securities:

$$\Pi_{t^*-15,T-15} = \frac{P_{t,T}(1 + R_{t,T})}{VNA_{t^*}} - 1$$

d. Method using only the IPCA Coupon Futures Market (DAP)

In the previous section, we used the government securities market to calculate the BEIR from NTN-Bs with intermediate payments. We now compare the method of the previous section with the method that calculates the BEIR from the IPCA coupon futures contracts, the DAP.¹⁹ This method was described in Araujo and Vicente (2017), but to our knowledge this is the first work in which it has been implemented.

For this future contract, which trades the spot IPCA coupon, we must solve equation (4) to find the BEIR:

$$\Pi_{t^*-15,T-15} = \frac{PU_{t,T}(1+R_{t,T})}{VNA_{t^*}} - 1,$$

where $\Pi_{t^*-15,T-15}$ is the BEIR embedded in DAP contracts with a 15-day lag between t^* (the date of the last known VNA) and maturity T , $R_{t,T}$ is the nominal rate from t to T , and

¹⁸ The VNA_t is reported by ANBIMA.

¹⁹For more information on DAP contracts, see Bertinat (2016).

$PU_{t,T}$ is the current unit price of the DAP contract maturing at T . If we were using a NTN-B, the VNA_{t^*} would be the last known VNA. However, for DAP contracts, this value is not immediate.

As in a T-maturity DAP contract, the PU on date t is:

$$PU_{t,T} = \frac{100,000}{(1 + C_{IPCA_{t,T}})} .$$

Then, comparing with equation (2), the VNA_t of the DAP is 100,000. Therefore, the VNA_{t^*} of the DAP contract is equal to 100,000 discounted by the inflation with lag of 15 days from the current date:

$$VNA_{t^*} = \frac{100,000}{1 + II_{t^*-15,t-15}} .$$

Inflation in the denominator of this equation, although having already occurred, is not yet known. However, since the daily settlements of the DAP future contract consider the projection of IPCA inflation published by ANBIMA, the IPCA coupon used in the calculation of its PU takes this expectation into account. In this way, this inflation can be calculated from the VNAs released by ANBIMA:

$$II_{t^*-15,t-15} = \frac{VNA_t^{Anbima}}{VNA_{t^*}^{Anbima}} - 1 .$$

From this inflation, we can calculate the VNA_{t^*} of the DAP contract and calculate the BEIR ($II_{t^*-15,T-15}$) using equation (4).

3.5 Incorporating Seasonality in the BEIR

The effect of the inflation seasonality on the pricing of fixed-income securities is a subject explored by several researchers; see Ejsing et al. (2007), Canty (2009) and Stein (2012). Even market participants are looking for a better way to incorporate this

seasonality into prices and expectations (see Committee on Normalisation Obligatoire, 2016).

The methods proposed for the BEIR calculation return estimates of spot rates that can be transformed into forward rates. The periods of these forward rates coincide with the NTN-B maturities considered in the estimates.

The method proposed in this section basically incorporates the monthly seasonality of inflation for each forward rate. In order to incorporate information that may affect short-term inflation and agents' expectations about future inflation, we use the forecasts of the Focus Report of the BCB. Thus, in addition to distributing implied inflation seasonally, current information about future price is incorporated in the estimation. Among the measures of inflation expectations provided by the Focus Report, we choose the median of the Top 5 Short-Term forecasts for the IPCA, which is based on the predictions of the participants who were most successful forecasting short-term inflation in the recent past.²⁰

In order to facilitate the distribution of the monthly seasonality in the forward rates estimated in sections 3.3 and 3.4, we transformed the discrete rates into continuous ones and distributed the monthly Focus Top 5 expectation percentage among the months of each estimated forward period. Therefore, we find the monthly BEIR already treated by seasonality.

3.6. A Brief Example

In May 2018, a trucking strike occurred in Brazil, practically stopping all national freight transportation, generating a short-term inflation spike and enormous inconveniences for the Brazilian economy. This movement obtained massive adhesion of truck drivers' unions, independent truckers and even transport companies. Not expected, this movement began on May 21st and ended on June 1st, after intense negotiations with the federal government. We estimate the June break-even inflation on the second critical date before the IPCA announcement of that month, on May 22nd, one day after the sudden start of the strike, and analyze the results after the end of the event, on June 8th, when the inflationary effects of the strike were already being perceived.

²⁰The medians of the Focus Report forecasts can be found at <http://www.bcb.gov.br>.

The IPCA coupon calculated from the parameters of the Svensson equation from May 22nd to August 15th (the maturity of the shortest NTN-B) was 0.4612% and the VNA on May 22nd, released by Anbima, was R\$3,075.65. Therefore, the Proposed BEIR estimated on May 22nd for the quarter between May and July is calculated by the following formulas:

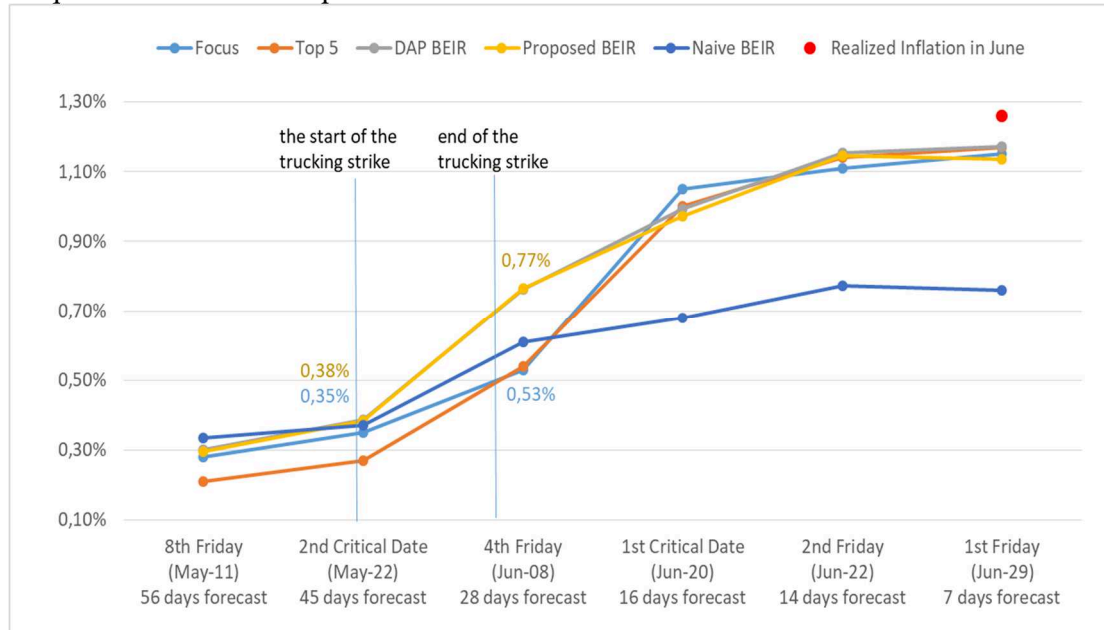
$$P_{t,T} = \frac{VNA_t}{(1 + C_{IPCA_{t,T}})} = \frac{3,075.65}{(1 + 0.4612\%)} = 3,061.53$$

$$II_{t^*-15,T-15} = \frac{P_{t,T}(1 + R_{t,T})^{du/252}}{VNA_{t^*}} - 1 = \frac{3,061.53(1 + 6.4375\%)^{60/252}}{3,073.07} - 1 = 1.1154\%$$

The Focus Top 5 inflation expectations were 0.26%, 0.27% and 0.25% for May, June and July, respectively, implying 0.78% in the quarter. Proportionally, we have 33.33%, 34.62% and 32.05% of the inflation rate allocated in each month. Multiplying the percentage of June, 34.62%, by the implied inflation in continuous rate, 1.12%, we obtain a continuous break-even inflation rate of 0.3840% for June. When discretizing this rate, we have the Proposed BEIR of 0.3847% for June. The BEIR for the other dates are calculated similarly.

Graph 2 shows the greater speed at which the DAP BEIR and the Proposed BEIR estimates adjust when compared to the others, incorporating the effect of the short-term inflationary shock generated by the strike. Between May 22nd and June 8th, the Proposed BEIR increased by 38 b.p. versus 18 b.p. in the Focus expectation.

Graph 2 – Evolution of Expectations and Realized Inflation for June 2018



4. Results

We compared the BEIR method for government bonds proposed in this article (section 3.c), called "Proposed BEIR" in tables and graphs, with (i) the median of expectations published in the Focus Report; (ii) the median of expectations for the Top 5 Short-Term published in the Focus Report ("Top 5"); (iii) a measure to extract the BEIR from government securities that does not take into account the problem of the inflation gap ("Naïve BEIR"); and (iv) the method of BEIR extraction by the futures market (section 3.d), named "DAP BEIR" in tables and graphs. The comparative measure is the absolute error between each measure and the actual IPCA.

The measure (iii), the Naïve BEIR, simply uses Svensson for nominal rates (from the LTNs and NTN-Fs) and for the IPCA coupon (from the NTN-Bs) and computes the BEIR from the difference between the factors of these rates. This method is the same used by ANBIMA and is a reference for financial institutions to mark several assets to the market.²¹ The comparison between the measures is carried out between January 2010 and July 2018, except for measure (iv), since the future DAP market only obtained reasonable

²¹ For the Naïve BEIR, we also worked with the quadratic term of duration in the error function as we did when estimating the Proposed BEIR.

liquidity from May 2016.²² The comparison is always made in the short term, between 4 and 57 calendar days, since BEIR measures have risk premium. In this way, we are assuming the inflation risk premium is small and negligible in the short run.

For comparison purposes, we consider some Fridays closer to the IPCA disclosure date and the IPCA-15 disclosure eve, which are called the first and second critical dates. The choice of these dates is due to the fact that most of the updates in the Focus Report are carried out on Fridays and analysts often improve their forecasts on the so-called critical dates used to calculate the Top 5 ranking of the Focus Report.²³

Thus, for each of the inflation measures we have six forecast horizons to compare the absolute errors: (i) the first Friday closest to the IPCA disclosure date; (ii) the second Friday closest to the IPCA disclosure date; (iii) the fourth Friday closest to the IPCA disclosure date; (iv) the eighth Friday closest to the IPCA disclosure date; (v) the first critical date closest to the IPCA disclosure date; and (vi) the second closest critical date of the IPCA disclosure date. Table B in the Appendix shows calendar days between the forecasted expectations and the IPCA disclosure date.

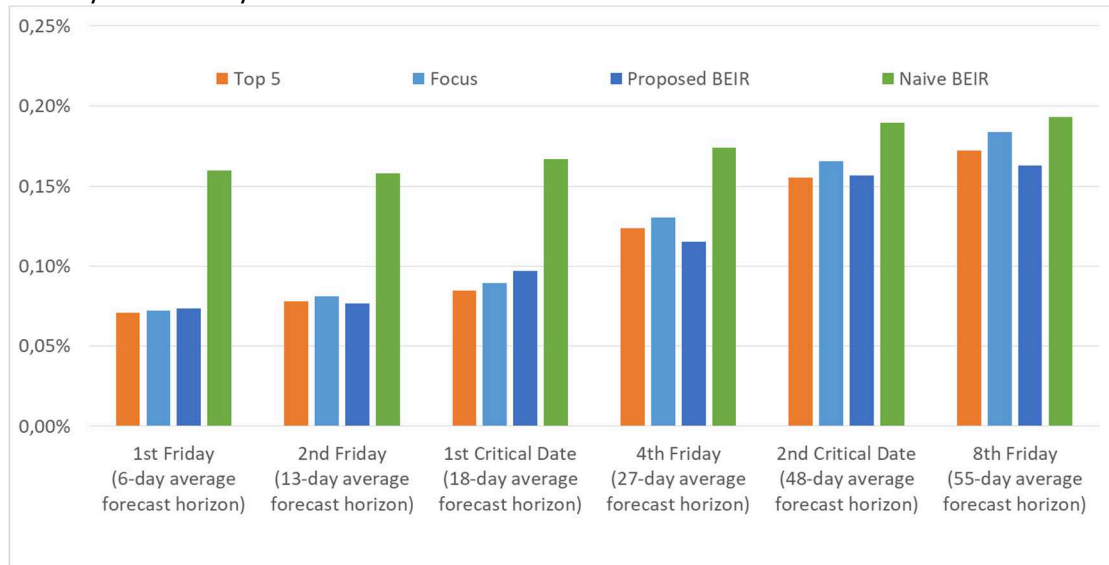
The Naïve BEIR measure shows the highest mean absolute errors and have statistically significant differences in relation to all measures for all six comparison dates, even when compared with the DAP in the smaller sample (there are only non-statistically significant differences of errors for the 8th Friday before the IPCA disclosure date), as can be seen in Tables C, D and E, presented in the Appendix .

Another noteworthy result observed by looking at the Naïve BEIR measure is that lag indexation and seasonal adjustments are more important for short-term expectations (see Graph 3 and Graph 4). As the forecast horizon increases, expectations without the lag and seasonal adjustments tend to have the same predictive error as the expectation with adjustment (the Proposed BEIR). In addition, Graph 3 and Graph 4 show a gradual increase in the mean absolute errors of Top 5, Focus, Implied DAP and Proposed BEIR to the level of Naïve BEIR when moving from the 1st Friday to the 8th Friday before the IPCA disclosure.

²² The liquidity of DAP increased from May 2016 onwards with the hiring of market makers by the Brazilian stock exchange. See Bertinat (2016).

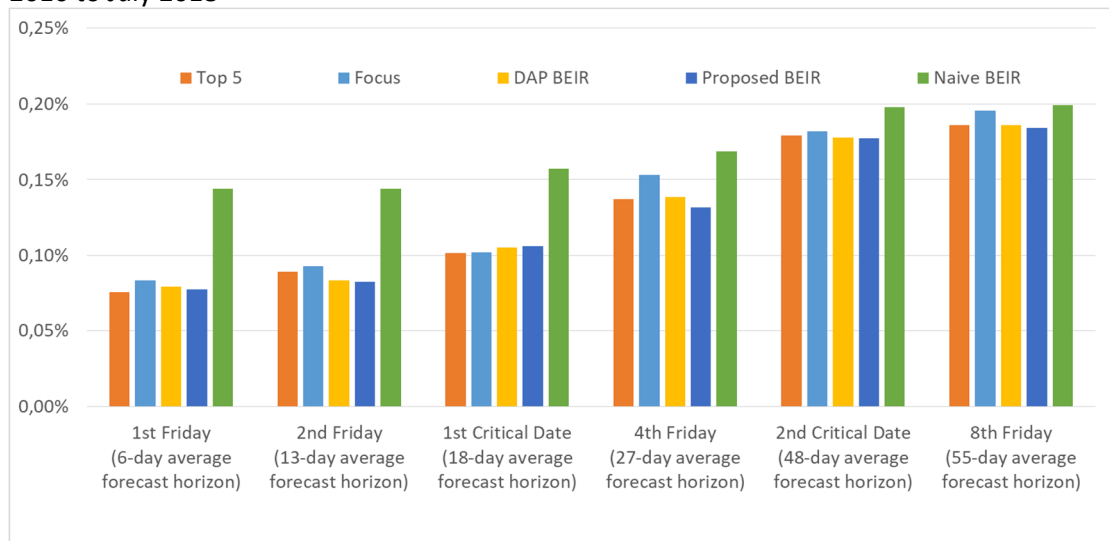
²³ The eve of the publication of the IPCA-15 by the IBGE is considered as the critical date for calculating the Top 5 ranking of the Focus Report.

Graph 3 – Mean Absolute Errors between the Expected Inflation and the Actual IPCA – January 2010 to July 2018



Note: IPCA is the Broad Consumer Price Index that remunerates the inflation-indexed Brazilian government bonds.

Graph 4 – Mean Absolute Errors between the Expected Inflation and the Actual IPCA – May 2016 to July 2018

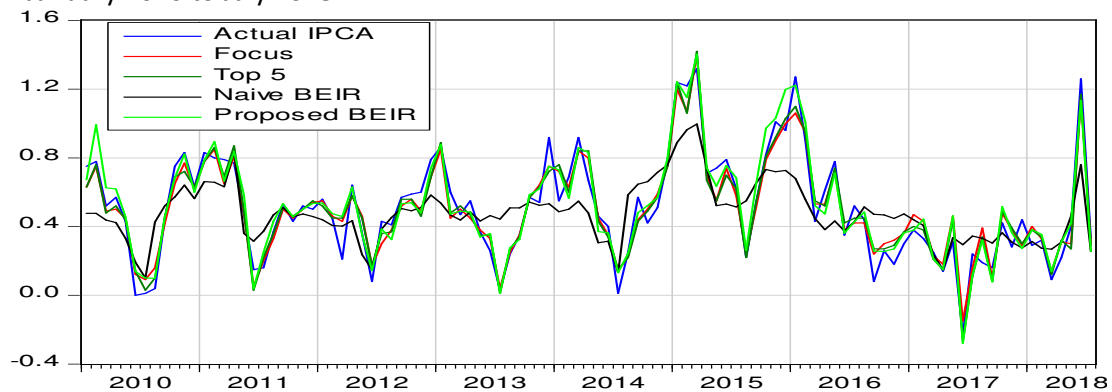


Note: IPCA is the Broad Consumer Price Index that remunerates the inflation-indexed Brazilian government bonds.

Graphs 5 and 6 validate the results of the previous paragraph. In Graph 5, the Proposed BEIR and the DAP BEIR are well fitted in the forecasts estimated on the 1st Fridays (forecasted 6 days, on average, before the IPCA disclosure), while Graph 6 shows

deterioration of expectations estimated on the 8th Fridays (forecasted 55 days, on average, before the IPCA disclosure).

Graph 5 – Realized Inflation and Forecasts on the 1st Fridays (6-day average forecast horizon) - January 2010 to July 2018



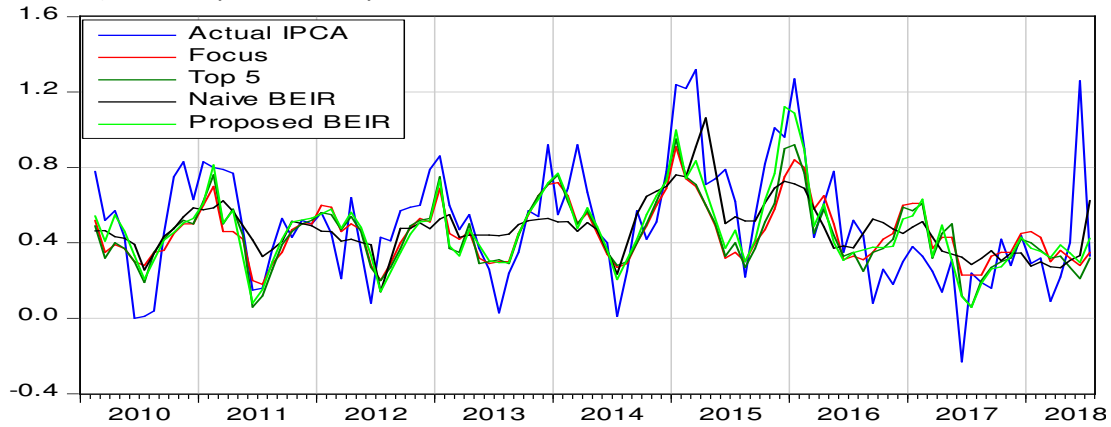
Note: IPCA is the Broad Consumer Price Index that remunerates the inflation-indexed Brazilian government bonds.

When we analyze the t-test results of the absolute errors differences between each of the inflation expectations and the actual IPCA (Table D), we conclude the Top 5 and the Proposed BEIR alternate in presenting the smallest forecast errors for the six periods analyzed. Specifically, for the period from January 2010 to July 2018 (the period without the DAP BEIR), we observe that:

- For the 1st and 2nd Friday before the IPCA disclosure date, there is no significant difference between the forecast errors of the three seasonally adjusted measures (Focus, Top 5 and Proposed BEIR);
- For the 4th Friday before the IPCA disclosure date, the Proposed BEIR returns the smallest forecast errors, with no significant differences in relation to the Top 5 errors but with significant differences in relation to Focus and the Naïve BEIR errors;
- For the 8th Friday before the IPCA disclosure date, the Proposed BEIR returns the smallest forecast errors, with significant differences in relation to the errors of the other measures (Top 5, Focus and Naïve NTN-B);
- For the first critical date before the IPCA disclosure date, Top 5 returns the smallest forecast errors, with significant differences in relation to the errors of the other measures; and For the second critical date before the IPCA disclosure date, Top 5 returns the smallest forecast errors, with no

significant differences in relation to the errors of the Proposed BEIR but with significant differences in relation to the other measures.

Graph 6 – Realized Inflation and Forecasts on the 8th Fridays (55-day average forecast horizon) – January 2010 to July 2018



Note: IPCA is the Broad Consumer Price Index that remunerates the inflation-indexed Brazilian government bonds.

Considering the period when we can obtain the BEIR from the DAP contracts (May 2016 to July 2018), the Proposed BEIR, DAP BEIR and Top 5 have the smallest forecast errors. Specifically, we note that:

- The Proposed BEIR has smaller forecast errors than the DAP BEIR for five of the six periods analyzed;
- Only for the 4th Friday before the IPCA disclosure date does the Proposed BEIR predict significantly better than the DAP BEIR;
- The DAP BEIR and Top 5 are never statistically different from each other;
- The DAP BEIR forecasts significantly better than Focus for the 2nd and the 4th Fridays before the IPCA disclosure date.

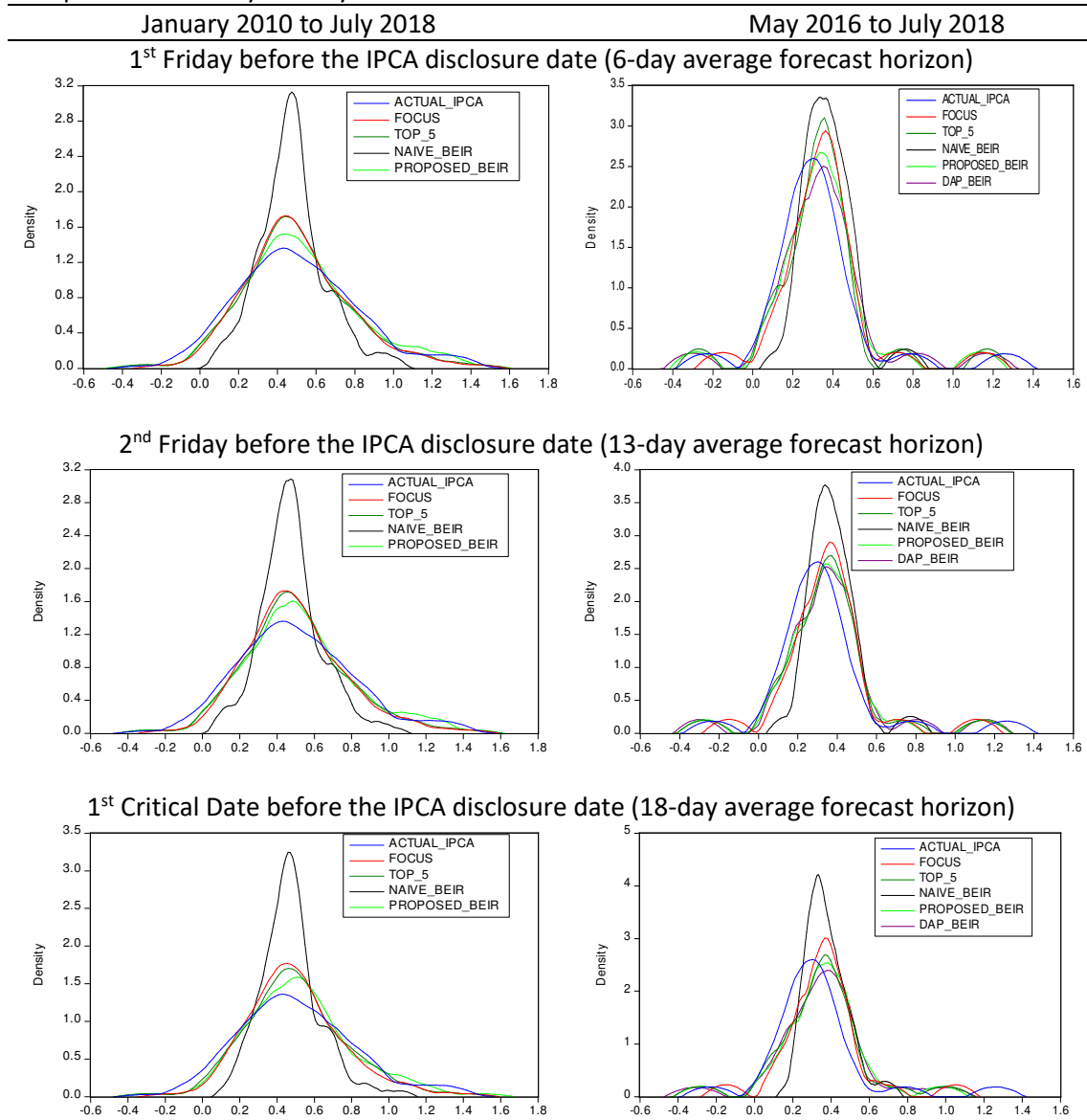
Graph 7 was generated by the kernel density estimator. Between May 2016 and July 2018, the densities show that expectations overestimated the realized inflation rate on a recurring basis, indicating the market may have underestimated the speed of

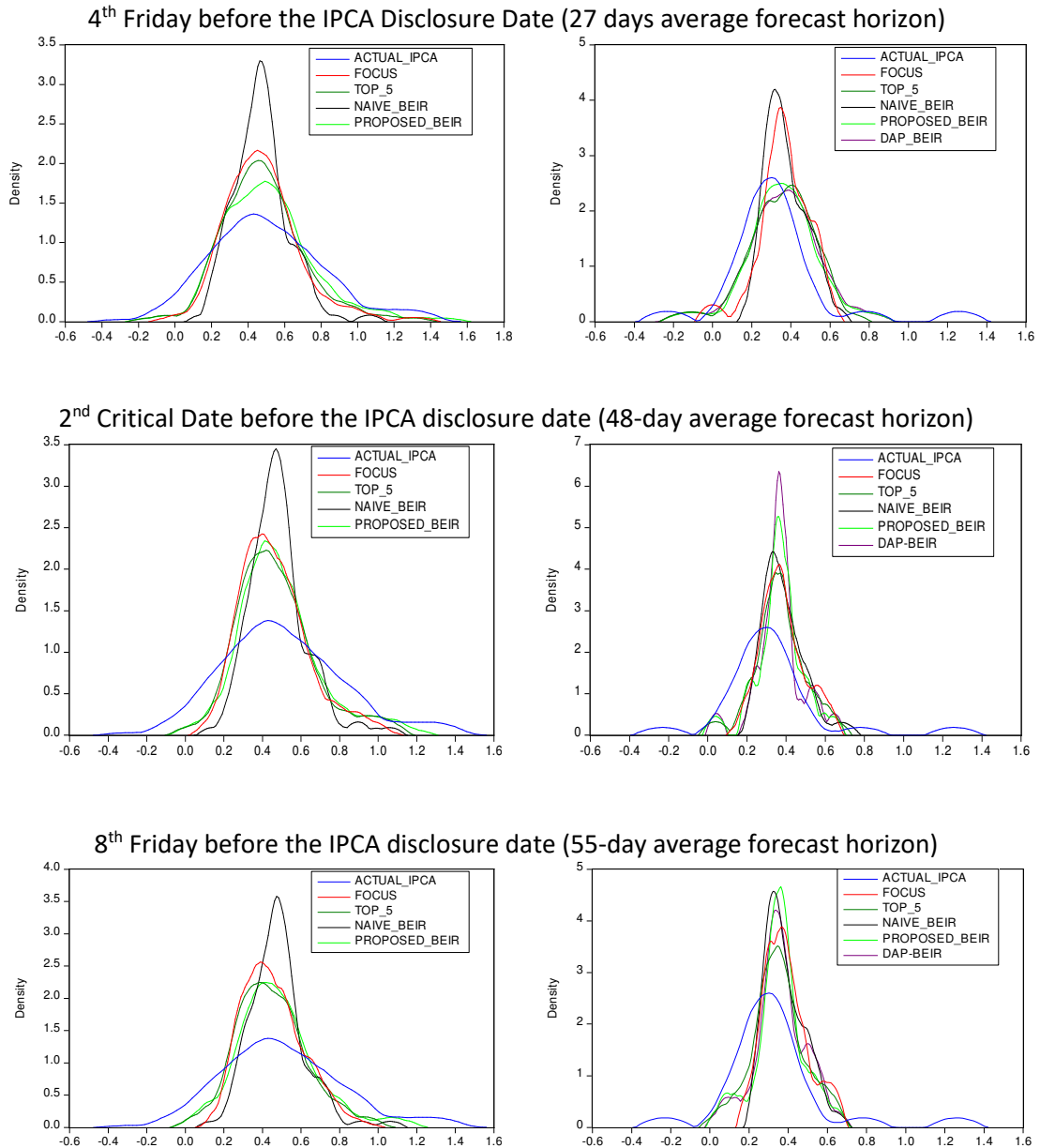
convergence of future inflation and the process of anchoring of expectations as of the beginning of 2016.

Between January 2010 and July 2018, the densities show better convergence between the estimated expectations and the realized inflation rate. The distributions of all measures (expectations and realized inflation) show positive asymmetry and leptokurtosis for all events.

In addition, the Naive BEIR has much lower volatility than the other measures in the two periods analyzed, which helped this measure to have the largest estimation error.

Graph 7 - Probability Density Functions





Note: IPCA is the Broad Consumer Price Index that remunerates the inflation-indexed Brazilian government bonds.

5. Final Remarks

The breakeven inflation rate, or BEIR, is strongly correlated with expected inflation, especially for horizons shorter than six months. However, two issues make the calculation of this measure a difficult task. The first concerns the design of inflation-indexed bonds in which the inflation has indexation lag. The second refers to the seasonality of inflation, which is not taken into account by the traditional models of interest rate term structure. These two problems are amplified in the short term. In this

paper we present a parsimonious method to extract the BEIR of government bonds that deals directly with these two problems.

The empirical results show the indexation lag and seasonal adjustments are more important for short-term expectations. As we increase the forecast horizon, expectations with and without these adjustments tend to have the same predictive error. In addition, the results show the BEIR proposed in this article – which uses government bonds indexed to inflation – has predictive power statistically equal to or better than survey-based inflation expectations.

These findings are relevant for at least two reasons: (i) outdoing survey-based inflation expectations is not a trivial task, especially in the short run; and (ii) the breakeven inflation rate is constantly updated, allowing real-time monitoring of market agents' expectations.

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Appendix

Table A – Mean Absolute Errors (in basis points) Generated by the two Error Functions used to Estimate the Parameters of the Svensson Model for Short-Term NTN-Bs: One with the Linear Term Applied to the Duration and Other with the Quadratic Term Applied to the Duration

Maturity	Linear Optimization	Quadratic Optimization	Reduction	Reduction (%)
15-Aug-10	37,7	2,2	-35,5	-94%
15-May-11	27,8	1,9	-25,9	-93%
15-Nov-11	44,3	0,9	-43,4	-98%
15-Aug-12	10,8	0,9	-9,9	-92%
15-May-13	10,8	0,4	-10,4	-97%
15-Aug-14	13,7	0,9	-12,9	-94%
15-May-15	8,3	0,4	-7,9	-95%
15-Aug-16	9,9	2,0	-7,9	-80%
15-May-17	9,8	0,8	-8,9	-92%
15-Aug-18	7,9	0,4	-7,5	-95%
Average	18,1	1,1	-17,0	-93%
Median	10,8	0,9	-10,2	-94%

Table B – Forecast Horizon by Event (in calendar days)

Statistic/Event	1st Friday (6-day average forecast horizon)	2nd Friday (13-day average forecast horizon)	1st Critical Date (18-day average forecast horizon)	4th Friday (27-day average forecast horizon)	2nd Critical Date (48-day average forecast horizon)	8th Friday (55-day average forecast horizon)
Minimum	4	11	11	25	38	53
Maximum	8	16	23	29	55	57
Average	6	13	18	27	48	55

Table C – Mean Absolute Errors between the Expected Inflation and the Actual IPCA – January 2010 to July 2018

Expectations / Day	1st Friday (6-day average forecast horizon)	2nd Friday (13-day average forecast horizon)	1st Critical Date (18-day average forecast horizon)	4th Friday (27-day average forecast horizon)	2nd Critical Date (48-day average forecast horizon)	8th Friday (55-day average forecast horizon)
Focus	0,072%	0,081%	0,090%	0,130%	0,165%	0,184%
Top 5	0,070%	0,077%	0,084%	0,124%	0,155%	0,172%
Proposed BEIR	0,073%	0,076%	0,097%	0,115%	0,157%	0,163%
Naive BEIR	0,160%	0,158%	0,167%	0,174%	0,189%	0,193%

Note: IPCA is the Broad Consumer Price Index that remunerates the inflation-indexed Brazilian government bonds.

Table D – Mean Absolute Errors between the Expected Inflation and the Actual IPCA – May 2016 to July 2018

Expectations / Day	1st Friday (6-day average forecast horizon)	2nd Friday (13-day average forecast horizon)	1st Critical Date (18-day average forecast horizon)	4th Friday (27-day average forecast horizon)	2nd Critical Date (48-day average forecast horizon)	8th Friday (55-day average forecast horizon)
Focus	0.083%	0.093%	0.102%	0.153%	0.182%	0.195%
Top 5	0.076%	0.089%	0.102%	0.137%	0.179%	0.186%
DAP BEIR	0.079%	0.083%	0.105%	0.139%	0.178%	0.186%
Proposed BEIR	0.077%	0.082%	0.106%	0.132%	0.177%	0.184%
Naive BEIR	0.144%	0.144%	0.157%	0.169%	0.198%	0.199%

Note: IPCA is the Broad Consumer Price Index that remunerates the inflation-indexed Brazilian government bonds.

Table E – T-statistics of the Difference between the Mean of the Absolute Errors of the Expected Inflation and the Actual IPCA (matched samples)

January 2010 to July 2018 May 2016 to July 2018

1st Friday before the IPCA disclosure date (6-day average forecast horizon)

Expectation	FOCUS	TOP 5	Proposed BEIR	Naive BEIR
FOCUS	N/A			
TOP5	0,57	N/A		
Proposed BEIR	-0,26	-0,59	N/A	
Naive BEIR	-7,11	-7,04	-6,66	N/A

Expectation	FOCUS	TOP 5	DAP BEIR	Proposed BEIR	Naive BEIR
FOCUS	N/A				
TOP5	2,49	N/A			
DAP BEIR	1,20	-0,92	N/A		
Proposed BEIR	1,63	-0,52	0,91	N/A	
Naive BEIR	-4,42	-4,75	-4,52	-4,82	N/A

2nd Friday before the IPCA disclosure date (13-day average forecast horizon)

Expectation	FOCUS	TOP 5	Proposed BEIR	Naive BEIR
FOCUS	N/A			
TOP5	1,23	N/A		
Proposed BEIR	0,87	0,23	N/A	
Naive BEIR	-6,26	-6,23	-6,27	N/A

Expectation	FOCUS	TOP 5	DAP BEIR	Proposed BEIR	Naive BEIR
FOCUS	N/A				
TOP5	1,29	N/A			
DAP BEIR	2,64	1,52	N/A		
Proposed BEIR	3,07	1,79	0,53	N/A	
Naive BEIR	-3,70	-3,68	-4,21	-4,42	N/A

1st Critical Date before the IPCA Disclosure Date (18-day average forecast horizon)

Expectation	FOCUS	TOP 5	Proposed BEIR	Naive BEIR
FOCUS	N/A			
TOP5	1,59	N/A		
Proposed BEIR	-1,41	-2,57	N/A	
Naive BEIR	-6,41	-6,59	-5,52	N/A

Expectation	FOCUS	TOP 5	DAP BEIR	Proposed BEIR	Naive BEIR
FOCUS	N/A				
TOP5	0,10	N/A			
DAP BEIR	-0,86	-0,96	N/A		
Proposed BEIR	-1,23	-1,25	-0,52	N/A	
Naive BEIR	-4,06	-3,89	-3,69	-3,75	N/A

4th Friday before the IPCA Disclosure Date (27-day average forecast horizon)

Expectation	FOCUS	TOP 5	Proposed BEIR	Naive BEIR
FOCUS	N/A			
TOP5	1,51	N/A		
Proposed BEIR	2,20	1,49	N/A	
Naive BEIR	-3,92	-4,28	-4,87	N/A

Expectation	FOCUS	TOP 5	DAP BEIR	Proposed BEIR	Naive BEIR
FOCUS	N/A				
TOP5	3,13	N/A			
DAP BEIR	2,10	-0,27	N/A		
Proposed BEIR	3,48	0,94	2,46	N/A	
Naive BEIR	-1,56	-2,63	-2,44	-3,18	N/A

2nd Critical Date before the IPCA Disclosure Date (48-day average forecast horizon)

Expectation	FOCUS	TOP 5	Proposed BEIR	Naive BEIR
FOCUS	N/A			
TOP5	2,05	N/A		
Proposed BEIR	1,60	-0,23	N/A	
Naive BEIR	-2,08	-2,88	-2,91	N/A

Expectation	FOCUS	TOP 5	DAP BEIR	Proposed BEIR	Naive BEIR
FOCUS	N/A				
TOP5	0,44	N/A			
DAP BEIR	0,74	0,44	N/A		
Proposed BEIR	0,82	0,56	0,26	N/A	
Naive BEIR	-1,45	-1,63	-1,84	-1,85	N/A

8th Friday before the IPCA Disclosure Date (55-day average forecast horizon)

Expectation	FOCUS	TOP 5	Proposed BEIR	Naive BEIR
FOCUS	N/A			
TOP5	2,37	N/A		
Proposed BEIR	3,05	1,76	N/A	
Naive BEIR	-0,82	-1,72	-2,54	N/A

Expectation	FOCUS	TOP 5	DAP BEIR	Proposed BEIR	Naive BEIR
FOCUS	N/A				
TOP5	1,69	N/A			
DAP BEIR	1,46	-0,02	N/A		
Proposed BEIR	1,73	0,44	1,34	N/A	
Naive BEIR	-0,36	-1,14	-1,30	-1,42	N/A

Note: IPCA is the Broad Consumer Price Index that remunerates the inflation-indexed Brazilian government bonds. Positive values of the *t*-statistic indicate the mean absolute error of the row inflation measure is lower than that of the column inflation measure and vice versa. For example, on the 8th Friday before the IPCA disclosure date, the *t*-statistic of the difference between mean errors of Proposed BEIR (row measure) and Top 5 (column measure) is 1.74, which means the mean absolute error of the Proposed BEIR is lower than that of the Top 5.