ISSN 1518-3548

BANCO CENTRAL DO BRASIL

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September 2018

Working Papers

482

ISSN 1518-3548
CGC 00.038.166/0001-05

Working Paper Series	Brasília	no. 482	August	2018	p. 1-32

Working Paper Series

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The Banco Central do Brasil Working Papers are all evaluated in double-blind refereeing process.

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Non-technical Summary

At the outset of the recent global financial crisis, spillovers in volatility across the international markets resulted in widespread losses. Such episodes have since been observed many times since. These characteristics of asset prices significantly deviate from log-normality and display time-varying stochastics, with ample evidence of jumps transmitting from one asset price or market to others. Traditional diffusion models are not able to capture this behavior, implying that risk arising from extreme variations and risk spillovers from one asset class or country to other asset classes or regions would get misevaluated.

We propose a multivariate jump diffusion model with Markovian contagion to capture these asset price dynamics, where the channel of contagion periodically switches from an active to an inactive state. We use a dynamic conditional correlation network approach to identify and estimate the Markovian contagion model. We apply the model to an international equity and currency portfolio allocation. The fat tail characteristics captured help evaluate the extent of model risk, intra-asset class, inter-asset and inter-region contagion.

Sumário Não Técnico

No início da recente crise financeira mundial, transmissão de volatilidade entre mercados resultou em perdas financeiras generalizadas. Tais episódios foram observados várias vezes desde então. Essas características de preços de ativos desviam significativamente das premissas de log-normalidade de preços e apresentam características estocásticas que variam no domínio do tempo, com evidência ampla de saltos sendo transmitidos dos preços de um ativo ou mercado para outro. Os modelos tradicionais de difusão podem não detectar esse comportamento, o que implica que risco originado por variações extremas e transmissão de risco excessivo de uma classe de ativos ou país para outras classes ou regiões pode ser mal estimado.

Este artigo propõe um modelo multivariado de difusão com saltos baseados em contágio Markoviano para incorporar essas características da dinâmica de preços de ativo. Nesse modelo, o canal de contágio muda periodicamente entre o estado ativo e o estado inativo. Para identificação e estimação do modelo, utilizamos uma abordagem de rede baseada em correlação condicional dinâmica. O artigo aplica esse modelo no contexto de uma carteira de índices de ações e moedas internacionais. As características de caudas gordas identificadas auxiliam na avaliação do risco de modelo e da extensão do contágio intra-ativo, inter-ativos e inter-região.

Multivariate Jump Diffusion Model with Markovian Contagion

Pablo Jose Campos de Carvalho[1] [2]

Aparna Gupta[3] [4]

Abstract

Asset prices exhibit significant deviation from log-normality, with time-varying stochastics and ample evidence of jumps transmitting from one asset or market to another. We propose a multivariate jump diffusion model with Markovian contagion to capture these asset price dynamics, where the contagion channel periodically switches from an active to an inactive state. Using a dynamic conditional correlation network approach to estimate the Markovian contagion model, we apply the model to an international equity and currency portfolio allocation. The fat tail characteristics captured help evaluate the extent of model risk, intra-asset class, inter-asset and inter-region contagion.

Keywords: asset prices; jump diffusion; jump correlation; contagion dynamics; network analysis.

JEL Classification Code: G12, G11, C22, C32, C38.

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1 Introduction

At the outset of the recent global financial crisis, US financial market collapse was accompanied by significant hike up in volatility across the globe. Spillovers in volatility across the international markets resulted in widespread losses. Such episodes have since been observed many times in the years following the outset of the global financial crisis. This characteristic of stochastic dynamics of asset prices would be hard to capture using traditional diffusion models, implying that risk arising from extreme variations and risk spillovers from one asset class or country to other asset classes or regions would get misevaluated.

It is widely known that equity returns (Ball and Torous, 1983), exchange rates (Jorion, 1988) and interest rates (Das, 2002) depart from the traditional diffusion model assumptions, exhibiting jumps in their paths. For purposes of portfolio allocation, these asset price jumps are often assumed to be uncorrelated with each other, thus ignoring their possible interrelation. Stylized facts, however, show that asset price jumps seem to be correlated (Das and Uppal, 2004) and that jumps in one region tend to increase the probability of jumps in other regions (Aït-Sahalia et al., 2015). In this paper, we develop a model for correlated jumps that incorporates spillover risk characteristics of asset price dynamics and comovement.

Although there is some disagreement regarding the precise definition of contagion, its most accepted key feature is an increase in comovement in asset prices after a shock to an entity or a set of entities (Forbes and Rigobon, 2002). Some of this escalated comovement may be explained by known common factors, which is often referred to as interdependence. Therefore, contagion is often associated with shocks in one asset that affect another asset in excess of what is explained by observable factors (Bekaert et al., 2005). Contagion and interdependence are both central to the non-stationary comovement in asset returns. A unique feature of our model is that the likelihood of contagion is not constant. Instead, the channel of contagion between assets may take two possible states, active or inactive, modeled as a Markovian process.

Empirical literature shows that comovement in asset prices is more pronounced during bear markets (Ang and Chen, 2002; Longin and Solnik, 2001). There is also evidence of clustering, i.e., increased comovement during high volatility periods (Boyer et al., 2006).

When these non-stationary features of asset dynamics are ignored, the missing traits of joint asset return distributions can be a source of significant model risk. A poor choice of the dependence structure can lead to an inferior estimation of diversification benefits, negatively impacting portfolio allocation and risk assessment. For instance, attempting to incorporate comovement using structures, such as, the Gaussian copula, ends up being too optimistic regarding the diversification effects of portfolios (Kole et al., 2007), thus underestimating the risk of extreme events (Poon et al., 2004).

Our model accounts for possible non-stationarity in interdependence between asset returns and common factors. We split asset returns into two components, one component that explains the interdependence with common factors, which is non-diversifiable, and another that is asset specific, or idiosyncratic. We use a dynamic beta model (Engle, 2014) to identify the time-varying interdependence between the assets and the factors. Considering the residuals as the remnant asset price risk after the influence of observable factors is filtered, comovements between idiosyncratic components can help isolate spillover risk.

Financial markets are highly interconnected with rather complex relationships between financial instruments. Complex systems are often described in hierarchical form, where stronger relationships are first filtered, and specific and minor relations surface later in the process (Simon, 1962). In many practically important areas, a network representation significantly aids the analysis of large datasets (Huang et al., 2009). Our framework relies on a minimum spanning tree (MST) representation for most significant, yet minimalist, identification of connections between asset prices and their temporal evolution. Connections arising from two idiosyncratic components suggest existence of similarities between two assets that are only weakly explained by the factors, where the relative and absolute importance of factors change over time.

The challenge of identifying occurrence of contagion is tackled using a network analysis approach. We use the decomposed asset returns and the common factors to build a layered minimum spanning tree (MST) network (De Carvalho and Gupta, 2014). A layered MST helps extract the most significant idiosyncratic connections that may be recruited for contagion when shocks are experienced by an asset. This topological space relies on a minimal number of links (or edges) to connect three distinct types of nodes: common factors nodes, pure nodes (or asset return nodes) and idiosyncratic nodes. Distance between two nodes is characterized by a norm transformation of the dynamic conditional correlation (Engle, 2002). Each node is connected in the minimum spanning tree (MST) through a node it is nearest to by the chosen distance metric. An asset return node is connected to another asset return node either through one of the explanatory common factors or through its idiosyncratic node. Therefore, this network structure helps identify the strongest sources of codependencies between assets, namely due either to their unique characteristics or a fundamental explanatory variable.

The MST is allowed to change with time, with the nodes remaining unchanged, while the links represent the changing strong correlations in node-pairs as idiosyncratic and systematic connections between asset return nodes. The model uses the dynamic network to characterize the two possible states for the channel of contagion. Specifically, when two idiosyncratic nodes are connected in the network, we consider the channel of contagion between them to be active, otherwise this contagion link is inactive. These two states for the channel of contagion extend the regime switching literature of non-stationary comovement in asset prices in a distinct way. Past models have focused on the change in the jump arrival rates (Eraker, 2004) or in the nature of the copula model (Garcia and Tsafack, 2011).

Regime switching in the channel of contagion is important for our proposed jump diffusion model, and distinguishes our work from the literature on correlated jumps. For instance, Das and Uppal (2004) recognize that jumps tend to happen simultaneously, however, they impose a perfect correlation between jumps, contrary to the possibility that jumps can be unique to a single asset at least some of the time. This approach potentially overstates the risk of jumps, thus producing a pessimistic estimate of the diversification benefits of a portfolio. In another closely related paper, Aït-Sahalia et al. (2015) use a Hawkes process to describe mutual excitation between assets, where intensity of jumps are time varying and mutually exciting. In their model, connections between jumps in one asset price and those in other assets' prices are always active. Thus, jumps in one asset price always increase the likelihood of jumps in other assets. In the terminology of our model, the channel of contagion is always active, regardless of the state of the economy.

These models fit some stylized facts of asset return data. For instance, markets in the same region tend to exhibit jump contagion (Bae et al., 2003). However, Pukthuanthong and Roll (2015) use modern jump detection measures to study 82 country equity indices and find that jumps are weakly correlated. They argue that jumps are idiosyncratic, and that variations of returns reflect global factors. Similarly, other papers show that the US securities markets explain a large part of jumps in local markets (Asgharian and Nossman, 2011) and most of the sovereign credit risk premium (Longstaff et al., 2011).

In our model, we account for the important role common factors play in capturing interdependence in asset returns. Thereafter, we focus on the residuals for the latent factors that may be driving comovement (Duffie et al., 2009). The idiosyncratic component of returns is subject to jumps, as well as spillovers from jumps in other actively connected idiosyncratic nodes. We argue that the reason for weak correlation in jumps, as found in Pukthuanthong and Roll (2015), is that the channel of contagion and transmission of jump shocks is not always active. Instead, the regime switch in the channel of contagion allows us to more accurately characterize the times when contagion, and not interdependence, is responsible for increased comovement.

We show the importance of evaluating the persistence of the contagion connection by analyzing the performance of diffusion-based optimal portfolio weights under three different assumptions of market conditions. We consider a risk averse US-based investor that maximizes her utility of terminal wealth. She does so by choosing to invest in two international equity indices of two different regions, Europe and Japan and, as such, holding exposure to the corresponding foreign exchange rate risk. Using returns in dollar terms, we estimate optimal portfolio weights under the pure diffusion model. We then use Monte Carlo simulations of a plain diffusion-based model, a model with perfect correlated jumps and the Markovian Contagion model to evaluate the expected utility, the Value-at-Risk and the expected shortfall of the portfolio. We observe that in the Markovian contagion model, the intensity of jumps in the residuals is smaller than the intensity of perfectly correlated jumps in the second model. Although the dispersion of portfolio returns using the Markovian contagion model is similar to that of the diffusion model, the Markovian contagion model based portfolio return scenarios display higher realizations in the tails.

We find that the choice of an inferior model for determining the optimal portfolio allocation is likely to result in loss of utility under a more realistic model. Depending on the specific performance measures chosen, the adverse impact would vary in severity. Under the Markovian contagion assumption, portfolio performance, measured by expected utility, is consistently worse than under the diffusion model. A fatter tail of the return distribution results in larger value at risk (VaR) and expected shortfall values. Markovian contagion also reduces the expected utility. We also find that in most cases, the portfolio performance under Markovian contagion is better than the case with perfectly correlated jumps.

We begin with describing the Markovian contagion asset price evolution dynamics in Section 2. As part of the model, we discuss the identification of the idiosyncratic component of asset price dynamics and Markov transition matrices for active and inactive states of idiosyncratic contagion links. Section 3 provides a detailed data and empirical estimation strategy, followed by studying the impact of Markovian contagion on portfolio diversification in Section 4. We conclude with some final remarks in Section 5.

2 Markovian Contagion Model

There are multiple definitions of contagion, all of whom involve the increase of comovement in the market after a shock to one or more assets (Forbes and Rigobon, 2002). The elevated comovement, however, could arise from interdependence of asset prices on the observable common factors or from shocks in one asset that is transmitted to other assets in excess of the explanatory power of the factors (Bekaert et al., 2005). From the development of the Capital Asset Pricing Model (CAPM) (Sharpe, 1964), factor models have been widely used to explain and predict asset returns, thus reducing the complexity and the randomness in the analysis of dependence structure between assets. In order to separate interdependence from contagion, we decompose asset returns ($y_{i,t} = \frac{dY_{i,t}}{Y_{i,t}}$) into a component explainable by factors ($x_t = \frac{dX_t}{X_t}$) and an idiosyncratic component ($z_{it} = \frac{dZ_{i,t}}{Z_{i,t}}$).

$$\frac{dY_{it}}{Y_{it}} = \beta^T \frac{dX_t}{X_t} + \frac{dZ_{it}}{Z_{it}}. \tag{1}$$

For simplicity, we consider that factor returns follow a geometric Brownian motion as follows:

$$\frac{dX_t}{X_t} = \alpha_F dt + \sigma_F dW_t^F, \tag{2}$$

where α_F is a column vector of drift coefficients for the f factors, σ_F is a $f \times f$ diagonal matrix of volatility coefficients and W^F is a $f \times 1$ vector of independent Wiener processes.

Variation of the residuals in the above decomposition of asset returns could be arising from temporary increase in correlation between asset returns and common factors due to market conditions, as documented in Ang and Chen (2002), Longin and Solnik (2001) and Boyer et al. (2006). To account for this source of variation in residuals, we use a dynamic beta model based on the conditional correlation of the common factors and the asset returns (Engle, 2014). We assume that asset log-return time series, $U_{i,t}$, with $U \in \{X, Y\}$, is normally distributed with mean, μ_{U_i}, and conditional variance, $h_{U_{i,t}}$.

$$U_{i,t} = \sqrt{h_{U_{i,t}}} u_{U_{i,t}}, \ \texttt{and} \ u_{U_{i,t}} \sim N(0,1), \tag{3}$$

Therefore,

$$U_{i,t} \sim N(0, h_{U_{i,t}}). \tag{4}$$

Conditional variances and covariances are modeled as the following GARCH(1,1) model.

$$\begin{aligned} h_{i,t} &= \omega_i + \alpha_i h_{i,t-1} + \gamma_i U_{i,t-1}, & (5)\\ h_{ij,t} &= \omega_{ij} + \alpha_{ij} h_{ij,t-1} + \gamma_{ij} U_{ij,t-1}, & (6)\\ U_{ij,t-1} &= U_{i,t} U_{j,t}, & (7)\\ w_{ij} &= 1 - \alpha_{ij} - \gamma_{ij}. & (8) \end{aligned}$$

We use the dynamic conditional covariance matrix of factors, H_{xx}, and the conditional covariance matrix of common factors and asset returns, H_{xy}, to estimate the dynamic betas as follows.

$$y_{it} = x_{it}\beta_{it} + z_{it}, \tag{9}$$

$$\beta_{it} = H_{xx,t}^{-1} H_{xy,t}. \tag{10}$$

Since there may be latent factors driving asset return comovement (Duffie et al., 2009), our model accounts for the importance of factors, but our focus henceforth will be on the residuals. The idiosyncratic component of asset returns is taken to follow a multivariate Markovian jump diffusion, where jumps in one asset price may cause jumps in other assets' prices when the channel of contagion between them is active. Therefore, the idiosyncratic component of asset returns is subject to jumps and also spillovers from jumps of other assets.

$$\frac{dZ_t}{Z_t} = \sigma_I dW_t^I + J_t dN_t, \tag{11}$$

where σ_I is a $n \times n$ diagonal matrix of volatility coefficients for the idiosyncratic returns, W^I is a $n \times 1$ column vector of independent Wiener processes driving the idiosyncratic returns, $\mathbf{N}_t$ is a $n \times 1$ vector of independent Poisson processes and J_t is a $n \times n$ time-varying matrix that incorporates jumps from each of the assets.

We argue that the weak correlation of jumps reported in Pukthuanthong and Roll (2015) is because the channel of contagion is not always active. There are periods when a jump in one asset price does spillover to other assets, and then there are periods when jumps in one asset are not transmitted to other assets. To capture this contagion characteristic, we enhance the jump process with a Markov chain to indicate the active and inactive states of inter-asset contagion. At each point in time, contagion has the same conditional probability of occurring given the state of the contagion channel in the previous period. Change of states for the contagion channel allows us to more accurately characterize the times when contagion, and not interdependence, is responsible for increased comovement.

It is important to highlight that we do not change the jump arrival rate for asset prices, such as in the models of Bates (2000), Pan (2002) and Eraker (2004). Instead of a regime switching model that changes the arrival rates of jumps, in our model the likelihood of contagion changes by regime. Each element of the jump matrix, $J_{ij}(t)$, is defined as a product of an indicator for asset-pair that is susceptible to contagion, I_{ij}, and a two-state Markov chain, $V_{ij}(t)$, that switches between inactive or '0'-state and active or '1'-state. Transition rates of the Markov chain are represented in a $[2 \times 2]$ transition matrix, T_{ij}, defined for each asset-pair susceptible to contagion. Finally, P_{ij}, is an independent identically log-normally distributed jump shock size that transmits from asset j to asset i. Therefore, $log(1 + P_{ij}) \sim N(\mu_{ij}, \delta_{ij}^2)$, and combining all the terms, the jump process is constructed as,

$$J_{ij}(t) = I_{ij} V_{ij}(t) P_{ij}. \tag{12}$$

For identification of the contagion network, we use a layered minimum spanning tree (MST) methodology developed in De Carvalho and Gupta (2014). The estimation of the transition matrix for the Markov chain, $V_{ij}(t)$, utilizes a time-varying MST structure. An MST is progressively built by minimally linking node-pairs characterized by the strongest link or smallest distance, until all nodes are connected without causing any cycles in the graph. The distance between nodes is measured by the following norm transformation of the conditional correlations.

$$d^z(i,j) = \sqrt{1 - |\rho_{z_i z_j,t}|}, \tag{13}$$

$$d^{yx}(i,j) = \sqrt{1 - |\rho_{y_i x_j,t}|}, \tag{14}$$

where z subscript refers to idiosyncratic returns, y subscript is for asset return and x represents the common factors. We identify the dynamic conditional correlations (Engle, 2002) by representing the conditional covariance matrix as follows:

$$H_t = D_t^{1/2} R_t D_t^{1/2}, \tag{15}$$

where D_t is the diagonal matrix of conditional variances,

$$D_t = \begin{bmatrix} h_{i,t} & 0 \\ 0 & h_{j,t} \end{bmatrix}, \tag{16}$$

and R_t is the matrix of conditional correlation,

$$R_t = \begin{bmatrix} 1 & \rho_{ij,t} \\ \rho_{ij,t} & 1 \end{bmatrix}, \tag{17}$$

where each pair of conditional correlation is estimated as follows.

$$\rho_{ij,t} = \frac{h_{ij,t}}{\sqrt{h_{i,t} h_{j,t}}}. \tag{18}$$

In the layered MST network with three types of nodes, factor nodes, asset return nodes and idiosyncratic nodes, only the most significant connections between nodes are enabled. As a result, the MST network of $3n$ nodes has $3n - 1$ links representing the largest absolute conditional correlations. We fix the asset return nodes to be always connected to their idiosyncratic return nodes, thereafter two asset return nodes are connected either through a common factor or through their idiosyncratic return nodes. This is illustrated in Figure 1. The network identification is complete when all nodes are connected.

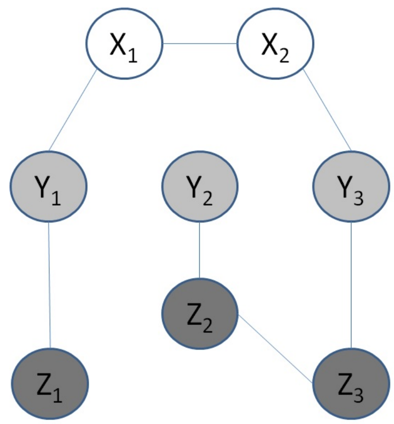

Figure 1: Schematic for the minimum spanning tree connectivity. The asset return nodes are always connected to their idiosyncratic return nodes.

We are particularly interested in contagion between assets, when there is an increased comovement between assets that is not explained by the common factors. The above network construction allows us to define the two distinct regimes for the contagion channel among assets by considering time-varying changes in the above layered MST network. As the layered MST network changes with time, when two idiosyncratic nodes are connected, this implies that this is the shortest link possessing strongest comovement relative to those with any of the common factors, and therefore the corresponding contagion channel is active. On the other hand, when the idiosyncratic nodes are disconnected with each other, the contagion channel is inactive. By observing these temporal changes in idiosyncratic connections, we estimate the transition probabilities, T_{ij}, of the contagion channel between assets i and j, $V_{ij}(t)$, as follows.

$$T_{ij} = \begin{bmatrix} P(V_{ij,t}=1|V_{ij,t-1}=1) & P(V_{ij,t}=0|V_{ij,t-1}=1) \\ P(V_{ij,t}=1|V_{ij,t-1}=0) & P(V_{ij,t}=0|V_{ij,t-1}=0) \end{bmatrix}. \tag{19}$$

We use the contagion channel transition matrices to implement the estimation of the jump diffusion model parameters. This particular network approach is important as it considers the active state of the contagion channel, when a jump shock experienced by one asset node is transmitted to other asset nodes through its active contagion channels. Inactive idiosyncratic links indicate a weaker linkage, and hence are not recruited for jump shock transmission. Taking the contagion channel transition matrix estimates into account and for a small value of Δt, it is reasonable to assume that in each time interval at most one jump shock occurs for an asset, as in Merton (1976). Under this assumption, we re-write Equation (11) in discrete form as,

$$Z_{i,t} \quad = \quad Z_{i,t-1} exp\Big[-\frac{1}{2}\sigma_{I_i}^2\Delta t + \sigma_{I_i}\Delta W_t\Big]\prod_{j=1}^{J}\Big(1 + P_{ij,s}V_{ij,s}I_{ij}\Big)\Delta N_{j,s}. \tag{20}$$

Although the assumption of one jump per period does not hold for lower frequency data, it provides a more stable estimation procedure. Since $\lambda\Delta t$ is small, a Poisson distribution with intensity λ can be approximated by the sum of n identically distributed independent Bernoulli random variables, as in Ball and Torous (1983). Using this approximation, we can re-write Equation (20) for discrete times, $t = k\Delta t$ for $k = 0, 1, \ldots, T$, as follows,

$$log\Big(\frac{Z_{i,t}}{Z_{i,t-1}}\Big) \quad = \quad \Big[-\frac{1}{2}\sigma_{I,i}^2\Delta t + \sigma_{I,i}\Delta W_t\Big] + \sum_{j=1}^{N}\Big(1 + P_{ij,t}V_{ij,t}I_{ij}\Delta t q_{j,t}\Big), \tag{21}$$

where

$$\Delta W_t \sim N(0, \Delta t), \tag{22}$$

$$log(1 + P_{ij,t}) \sim N(\mu, \delta), \tag{23}$$

$$\Delta q_{j,t} \sim b(1, \lambda_j\Delta t). \tag{24}$$

The unconditional density function of the log-returns $z_{i,t} = log(Z_{i,t}/Z_{i,t-1})$, with parameters $\Psi = (\sigma_i, \lambda_j, \mu_{ij}, \delta_{ij})$ is the sum of probability of no jumps occurring and the probability of jumps experienced by any of the assets and its impact on asset i.

$$\begin{aligned} p(z_i;\Psi) \quad &= \quad \Big(1 - \sum_{j=1}^{N}\lambda_j\Big)\phi\Big[z_i; \Big(-\frac{1}{2}\sigma_{I,i}^2\Delta t\Big); \sigma_{I,i}^2\Delta t\Big] \\ &+ \quad \sum_{j=1}^{N}\lambda_j\phi\Big[z_i; \Big(-\frac{1}{2}\sigma^2\Delta t\Big) + \mu_{ij}; \sigma_{I,i}^2\Delta t + \delta_{ij}^2\Big]V_{ij}I_{ij}. \end{aligned} \tag{25}$$

However, the information about the channel of contagion is not revealed ex-ante. For this reason, we identify the Markovian transition density matrix for the contagion channels and use the conditional density function of z_i defined as follows.

$$\begin{aligned} p(z_i;\Psi|V_{ij,t-1}) \quad &= \quad \Big(1 - \sum_{j=1}^{N}\lambda_j(1 - I_{ij}[P(V_{ij,t}|V_{ij,t-1}) = 1])\Big)\phi\Big[z_i; \Big(-\frac{1}{2}\sigma_{I,i}^2\Delta t\Big); \sigma_{I,i}^2\Delta t\Big], \\ &+ \quad \sum_{j=1}^{N}\lambda_j\phi\Big[z_i; \Big(-\frac{1}{2}\sigma^2\Delta t\Big) + \mu_{ij}; \sigma_{I,i}^2\Delta t + \delta_{ij}^2\Big]I_{ij}[P(V_{ij,t}|V_{ij,t-1}) = 1]. \end{aligned} \tag{26}$$

The parameters, Ψ, are obtained by maximizing the log-likelihood function,

$$\ell(z_{i,1}, z_{i,2}, ..., z_{i,T}, \Psi) = \sum_{t=1}^{T} log(p(z_{i,t}, \Psi|V_{ij,t-1})). \quad (27)$$

Once the parameters, Ψ, are estimated from the above log-likelihood maximization, combining Equations (11) and (2), the model for asset returns in Equation (1) can be constructed. We implement the calibration of the model in the next section, with first describing the data and discussing the empirical analysis.

3 Data and Empirical Estimates

We apply the Markovian contagion model to analyze international portfolio selection. An investor that allocates her capital in multiple equity markets to construct an internationally diversified portfolio would be concerned with the unfavorable comovement of these assets. However, the diversification benefits of investing in international markets is under scrutiny (Ang and Bekaert, 2002), because these benefits seem to diminish due to spillover risks precisely when an investor needs them the most.

We consider a US-based investor who chooses to invest in two international equity indices of two different regions, Europe and Japan. This asset allocation is also exposed to the corresponding foreign exchange rate risk. We pick the two international equity indices denominated in USD and the two corresponding foreign exchange rates, to account for the possibility that the investor would want to hedge the foreign exchange rate risk when investing in the two foreign equity markets. Additionally, in order to mitigate the adverse effects of different time zones of the two international equity markets, we use weekly returns. Therefore, $\Delta t = \frac{1}{52}$.

We use data from the Morgan Stanley Capital International (MSCI) equity indices and foreign exchange rates, from January 1st, 2001 to December 31st, 2015. The foreign exchange rates are given in terms of USD necessary to buy one unit of the foreign currency. Hence, when the USD is relatively stronger, the foreign exchange rate yields negative returns. Descriptive statistics for the data are given in Table 1. We notice that all return time series display negative skewness and excess kurtosis. We also observe that maximum and minimum of returns are many standard deviations from the mean.

	Mean	Std. Dev	Skewness	Kurtosis	Minimum	Maximum
Japanese Stocks	0.0003	0.0264	-0.2661	5.0081	-0.1513	0.1165
European Stocks	0.0008	0.0315	-0.7850	8.6589	-0.2297	0.1357
Dollar/Euro	0.0004	0.0138	-0.2922	4.0184	-0.0587	0.0512
Dollar/Yen	0.0002	0.0140	0.3573	4.4395	-0.0449	0.0787

Table 1: Descriptive statistics of the asset time series included in our study. The data consist of weekly dollar returns for time series from January 1, 2001 to December 31, 2015 for Japanese Stocks (MSCI Japan Index), European Stocks (MSCI European Index) and foreign exchange rates given by US Dollar over International currency.

We choose two relevant common factors for international portfolio allocation, namely US equity market index and the strength of US Dollar, as these two common factors are known to affect international equity returns (Bahmani-Oskooee and Sohrabian, 1992). It is reasonable to assume that US investors investing in international equity will benchmark against indices from US equity market, such as the S&P 500 index, to help explain comovements in these asset prices. Movements in international equity markets must be compared against a single numeraire, hence we include the US Dollar as a factor, proxied by the US Dollar index. The US Dollar index is calculated by the Intercontinental Exchange, which is available from Bloomberg Professional. It is calculated by averaging the exchange rates between the USD and major world currencies supplied by about 500 banks.

Figure 2 shows the nested dynamic beta estimated using an approach similar to Engle (2014). We notice that the dynamic beta significantly varies with time, particularly for the equities. Tables 2 and 3 present the descriptive statistics for the dynamic beta estimates, and compare it with OLS betas. We confirm that, although the OLS beta and the mean value of dynamic betas are generally close, an OLS beta would fail to accurately capture the changes in interdependence of asset returns on the common factors over time.

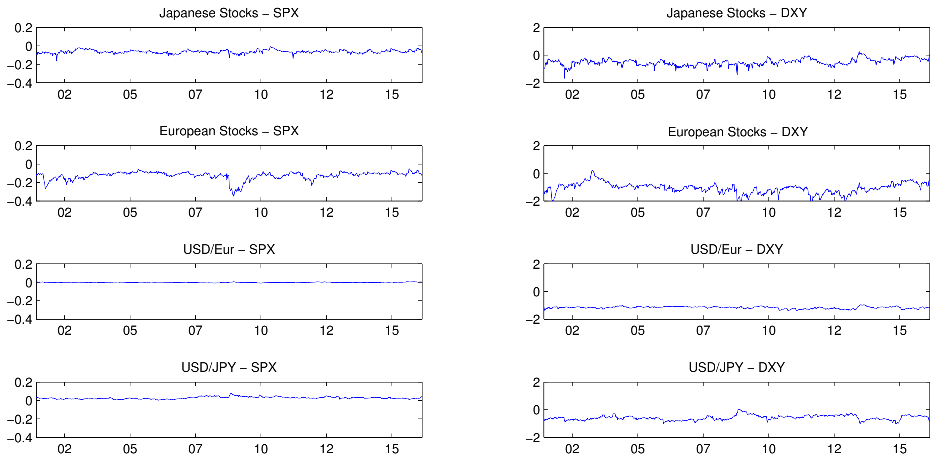

Figure 2: Figures show the time evolution of the Dynamic beta estimated in Engle (2014) from a two factor model $Y_{it} = \alpha + \beta_{1t} USStocks + \beta_{2t} DollarIndex + z_{it}$. The dependent variables are dollar returns for Japanese Stocks (MSCI Japan Index), European Stocks (MSCI European Index) and foreign exchange rates given by US Dollar over International currency. The independent variables are US Stock returns (S&P 500 index), and Dollar Strength calculated by the Intercontinental Exchange (DXY ticker available from Bloomberg Professional).

	Mean	Std. Dev	Minimum	Maximum	OLS
Japanese Stocks	-0.0631	0.0176	-0.1640	-0.0089	-0.0634
European Stocks	-0.1249	0.0443	-0.3495	-0.0509	-0.1247
Dollar/Euro	-0.0010	0.0027	-0.0087	0.0067	0.0001
Dollar/Yen	0.0275	0.0121	0.0013	0.0814	0.0301

Table 2: Descriptive statistics of the time evolution of the Dynamic beta on US stocks estimated in Engle (2014) from a two factor model $Y_{it} = \alpha + \beta_{1t} USStocks + \beta_{2t} DollarIndex + z_{it}$. The dependent variables are dollar returns for Japanese Stocks (MSCI Japan Index), European Stocks (MSCI European Index) and foreign exchange rates given by US Dollar over International currency. The independent variables are US Stock returns (S&P 500 index), and Dollar Strength calculated by the Intercontinental Exchange (DXY ticker available from Bloomberg Professional).

	Mean	Std. Dev	Minimum	Maximum	OLS
Japanese Stocks	-0.5056	0.2414	-1.6720	0.2642	-0.5423
European Stocks	-1.0714	0.3746	-2.4743	0.2131	-1.2057
Dollar/Euro	-1.1658	0.0770	-1.3789	-0.9589	-1.1607
Dollar/Yen	-0.5847	0.1732	-1.0276	0.0405	-0.5125

Table 3: Descriptive statistics of the time evolution of the Dynamic beta on Dollar Strength as estimated in Engle (2014) from a two factor model $Y_{it} = \alpha + \beta_{1t} USStocks + \beta_{2t} DollarIndex + z_{it}$. The dependent variables are dollar returns for Japanese Stocks (MSCI Japan Index), European Stocks (MSCI European Index) and foreign exchange rates given by US Dollar over International currency. The independent variables are US Stock returns (S&P 500 index), and Dollar Strength calculated by the Intercontinental Exchange (DXY ticker available from Bloomberg Professional).

We extract the residuals from the dynamic beta models to represent the idiosyncratic component of the asset returns. Table 4 shows the summary statistics for the idiosyncratic returns. We observe that skewness and excess kurtosis remain, suggesting that the idiosyncratic returns encompass jumps that occur in excess of the variance explained by the observable common factors.

	Mean	Std. Dev	Skewness	Kurtosis	Minimum	Maximum
Japanese Stocks	-0.0001	0.0138	0.0345	8.1855	-0.0997	0.1032
European Stocks	0.0000	0.0125	-0.1382	7.5187	-0.0841	0.0728
Dollar/Euro	0.0000	0.0062	-0.0626	4.3986	-0.0325	0.0265
Dollar/Yen	0.0000	0.0071	-0.3043	8.0325	-0.0667	0.0549

Table 4: Descriptive statistics of the residual z_{it} as estimated in Engle (2014) from a two factor model $Y_{it} = \alpha + \beta_{1t} USStocks + \beta_{2t} DollarIndex + z_{it}$. The dependent variables are dollar returns for Japanese Stocks (MSCI Japan Index), European Stocks (MSCI European Index) and foreign exchange rates given by US Dollar over International currency. The independent variables are US Stock returns (S&P 500 index), and Dollar Strength calculated by the Intercontinental Exchange (DXY ticker available from Bloomberg Professional).

We use the factors, asset returns and the idiosyncratic returns to construct a layered MST network (De Carvalho and Gupta, 2014) based on dynamic conditional correlations (Engle, 2002). The average distance between the nodes of the MST network is seen to decrease with time, suggesting an increased level of interdependence. The links of the layered MST change with time, with the idiosyncratic nodes getting periodically connected and disconnected. Using these networks, we estimate the transition matrices for the Markov chain, $V_{ij}t$, for each pair of idiosyncratic nodes i, j. The transition matrix of the Markov chains provides us the unconditional probability of link existence. We present these probabilities in Table 5. There is a high probability of connectivity between the equity indices and also between the currencies, suggesting strong intra-asset links. The likelihood of inter-asset or intra-region connectivity, however, is quite low.

Asset	Japanese Stocks	European Stocks	Dollar/Euro	Dollar/Yen
Japanese Stocks	100.00%	57.40%	4.03%	2.21%
European Stocks	57.40%	100.00%	0.39%	0.91%
Dollar/Euro	4.03%	0.39%	100.00%	40.65%
Dollar/Yen	2.21%	0.91%	40.65%	100.00%

Table 5: Unconditional probability of contagion channel being enabled.

4 Impact of Contagion in Portfolio Performance

We analyze and demonstrate the impact of contagion on the performance of a portfolio using the Markovian regime switching model developed in this paper. Comovement of financial time series is known to significantly influence portfolio investment decisions. Portfolio selection models that fail to capture the asymmetric dependence and regime switching of contagion can result in significant costs or losses for an investor (Chollete et al., 2009). A model lacking in the appropriate comovement structure of asset prices would lead to suboptimal portfolios and inaccurate assessment of risk exposures (Kole et al., 2007).

We consider a risk-averse investor that maximizes her utility of terminal wealth, W_T, using the following constant relative risk aversion (CRRA) utility with a risk-aversion parameter,

γ.

$$U = E\left[\frac{W_T^{1-\gamma}}{1-\gamma}\right]. \tag{28}$$

In order to diversify internationally, we assume that the investor chooses to invest in the European and the Japanese equity markets, or hold cash in domestic currency (USD). Using returns in dollar terms, we estimate optimal portfolio weights under the pure diffusion model for asset returns as follows,

$$\hat{w} = \frac{1}{\gamma}\hat{\Sigma}^{-1}\hat{R}. \tag{29}$$

Table 6 shows the portfolio weights in equities and currencies for different choices for the risk aversion levels. As expected, the allocation in risky assets is smaller for higher risk aversion levels. The largest allocation is in the Euro currency and there is a small short position in the Japanese equities.

Portfolio	1	2	3	4	5	6	7
γ	1.1	1.2	1.15	2	2.5	3	5
Japanese Stocks	-0.084	-0.077	-0.062	-0.046	-0.037	-0.031	-0.019
European Stocks	0.580	0.531	0.425	0.319	0.255	0.213	0.128
Dollar/Euro	1.201	1.101	0.881	0.661	0.529	0.440	0.264
Dollar/Yen	0.634	0.581	0.465	0.349	0.279	0.232	0.139

Table 6: Diffusion Model Based Portfolio Weights ($\hat{w}$) based on returns ($\hat{R}$) in dollar terms, covariance matrix ($\hat{\Sigma}$) and risk aversion coefficient (γ).

We evaluate the performance of these assets and portfolios under different market conditions, using Monte Carlos simulation for three different models. The three models incorporate asset comovement with different levels of sophistication. These simulations enable us to evaluate the implications of the choice of asset evolution dynamics for performance of the portfolios and estimate the impact of mis-specification of asset returns on the performance of the portfolios. The first model is a plain diffusion model for all assets, with a fixed covariance structure, given as,

$$\frac{dY}{Y_t} = \alpha_Y dt + \sigma_Y dW_t, \tag{30}$$

where Y and α_Y are column vectors and σ_Y^2 is the covariance matrix. This model is consistent with traditional diffusion models in the literature that assume jumps to be uncorrelated, and hence diversifiable. The parameter estimation for this model's specification and assets chosen for this study are shown in Table 7.

$\times 10^{-3}$	MSCI Japan Index	MSCI European Index	Dollar/Euro	Dollar/Yen
α	0.691	1.292	0.501	0.269
σ^2	0.698	0.454	0.073	0.030
	0.454	0.991	0.188	-0.061
	0.073	0.188	0.191	0.058
	0.030	-0.061	0.058	0.196

Table 7: Diffusion Model Parameters: parameters for the equation $\frac{dY}{Y_t} = \alpha_Y + \sigma_Y dW_t$. Variables $\frac{dY}{Y_t}$ are dollar returns for Japanese Stocks (MSCI Japan Index), European Stocks (MSCI European Index) and foreign exchange rates given by US Dollar over International currency.

The second model we consider is similar to the one developed by Das and Uppal (2004), where asset returns follow jump diffusion processes with perfectly correlated jumps.

$$\frac{dY}{Y_{it}} = \alpha_Y dt + \sigma_Y dW_t + (J_Y - 1)dQ(\lambda). \tag{31}$$

In above, in addition to the diffusion parameters of Equation (30), the asset price processes share a Poisson process, $Q\tilde{P}o(\lambda t)$, common to all assets to capture the perfectly correlated jumps. $log(J_Y) \sim N(\mu_Y, \delta_Y)$ represents the vector of jump shock sizes. Table 8 shows the estimates of these parameters using weekly asset return data.

$\times 10^{-3}$	Japanese Stocks	European Stocks	US Dollar/ Euro	US Dollar/ Yen
α	0.3422	0.7961	0.4058	0.1714
σ^2	0.4574	0.0838	-0.0329	-0.0848
	0.0838	0.4179	0.0258	-0.2352
	-0.0329	0.0258	0.1442	0.0071
	-0.0848	-0.2352	0.0071	0.1400
λ	176.7751	176.7751	176.7751	176.7751
μ	-5.3733	-13.4078	-1.8223	0.0163
δ	36.5158	55.3579	16.1455	17.8021

Table 8: Jump Diffusion with Perfectly Correlated Jumps: parameters for the equation $\frac{dY}{Y_t} = \alpha_Y + \sigma_Y dW_t$. Variables $\frac{dY}{Y_t}$ are dollar returns for Japanese Stocks (MSCI Japan Index), European Stocks (MSCI European Index) and foreign exchange rates given by US Dollar over International currency.

Parameters estimated for this specification point to considerably low diffusive correlation among assets. However, jumps have a much larger volatility, i.e., when the jumps occur, they augment the dispersion of the return paths and increase the kurtosis. For instance, our estimated parameters suggest that the contribution of correlated jumps, $\lambda(\sigma_{ij} + \delta_i\delta_j)$, is four times larger for the pair of equity indices and seven times larger for the pair of foreign exchange rates.

It is important to highlight, however, that in Das and Uppal (2004), these perfectly correlated jump processes are applied to lower frequency data in order to capture systemic risk. At a lower frequency of observation, jumps are noted to happen less often, as larger fluctuations at a higher frequency tend to offset one another. Perfectly correlated jumps may arise from interdependence, or systemically, rather than due to contagion between assets. As an enhancement for jump characteristics, we consider the case of Markovian contagion as described in Equation (11). The model allows us to separate the effect of contagion from interdependence through the common factors, by identifying the correlated jumps in the idiosyncratic component of asset returns.

We estimate the parameters for the Markovian contagion model for the case under study and present the point estimates in Table 9. Parameters μ_{ij} and δ_{ij} presented in row i and column j in the table should be read as the impact of a jump on asset j transmitted to asset i when the link between them is active. Examining the off-diagonal (covariance) elements of δ, we notice that when contagion links are active, the spillover risk is larger than individually disconnected jump risk. Spillovers are also seen to be asymmetric. For instance, jumps in the currencies seem to be more important to the equity indices than vice-versa.

$\times 10^{-3}$	Japanese Stocks	European Stocks	Euro	Yen
σ	23.5973	18.1859	3.4973	10.9918
λ	30.9916	145.1685	141.3558	49.9006
	-2.9846	0.3831	6.2488	4.4692
	-22.0989	6.2828	12.1399	-5.5802
μ	6.9469	1.1691	-1.1483	-0.3427
	14.4502	-5.2510	2.9515	-4.1593
	22.9407	27.2708	52.5801	73.5851
	42.5410	17.3902	84.5869	86.8295
δ	16.5380	21.1561	3.7369	7.8936
	54.5765	48.1131	19.0018	10.7588

Table 9: Parameters for Markovian Contagion Jumps: parameters for the equation $\frac{dZ_t}{Z_t} = \sigma_I dW_t^I + (J_t)dN_t$, where σ_I is an $n \times n$ diagonal matrix of volatility coefficients for the idiosyncratic returns, W^I is an $n \times 1$ column vector of independent Wiener processes driving the idiosyncratic returns, N_t is an $n \times 1$ vector of independent Poisson processes and J_t is an $n \times n$ time-varying matrix that incorporates jumps from each of the assets. Each element of the jump matrix, $J_{ij}(t) = I_{ij}V_{ij}(t)P_{ij}$, is defined as a product of an indicator for asset-pair that is susceptible to contagion, I_{ij} , and a two-state Markov chain, $V_{ij}(t)$, that switches between inactive or '0'-state and active or '1'-state. P_{ij} , is an independent identically log-normally distributed jump shock size that transmits from asset j to asset i, characterized by $log(1+P_{ij}) \sim N(\mu_{ij}, \delta_{ij}^2)$. Variables $\frac{dZ}{Z_t}$ are the idiosyncratic components of dollar returns for Japanese Stocks (MSCI Japan Index), European Stocks (MSCI European Index) and foreign exchange rates given by US Dollar over International currency.

The parameters of the Markovian contagion model cannot, unfortunately, be directly compared to the ones of the first two model specifications, as the parameters for the Markovian contagion model are determined for the residuals of the equity and currency returns. Nevertheless, we observe that in the Markovian contagion model, the intensity of jumps in the residuals is smaller than the intensity of perfectly correlated jumps in the second model. This lends support to our hypothesis that jumps in asset prices don't always transmit to other assets, and that the proportion of times when jumps do transmit to multiple assets is small.

Another important observation is that jumps are neither significantly positive nor significantly negative. This suggests that although jumps increase volatility, they do not necessarily lend a clear direction of asset price movement. We also note that when inter-asset links are active, the jump transmission has much higher intensity. Lastly, in this model, we incorporate interdependence in the asset prices, so that each asset price follows a process described in Equation (1). The parameters of the diffusion model for the evolution of common factors are reported in Table 10.

$\times 10^{-3}$	S&P 500	Dollar Index
α	19.4110	-0.1100
σ	20.6229	0.1314

Table 10: Diffusion Parameters for Factors: Parameters for the equation $\frac{dX_t}{X_t} = \alpha_F + \sigma_F dW_t^F$, where α_F is an $f \times 1$ column vector of drift coefficients and σ_F is an $f \times f$ diagonal matrix of volatility coefficients and W^F is an $f \times 1$ column vector of independent Wiener processes. Factors are S&P 500 index and Dollar index.

For evaluating the performance of the portfolios in Table 6, we use the long-term βs from Table 2 and the estimated parameters for the Markovian contagion model of Equation (2), shown in Table 9. We simulate the model for a duration of one year (50 weeks). Similarly, we simulate the other two models for asset returns in order to compare the performance of the portfolios under different model representations of market conditions.

Combining simulated asset price trajectories from the three models with portfolio weights, we generate scenarios for returns of the portfolios. The dispersion of returns for the perfectly correlated jumps model is much larger than for the other two models. Although the dispersion of portfolio returns using the Markovian contagion model is similar to that of the diffusion model, the Markovian contagion model based portfolio return scenarios display higher realizations in the tails. Therefore, the nature in which the jumps are realized in the Markovian contagion model captures the fatter tail characteristics of asset or portfolio returns.

We evaluate the expected utility function for each model corresponding to each of the portfolios in Table 6. The choice of an inferior model for determining the optimal portfolio allocation is likely to result in loss of utility under a more realistic model. The utility function evaluations under different model choices are presented in Table 11. For each portfolio, we show the expected utility and the standard deviation of the utility function under each model choice. For all levels of risk aversion, expected utility is seen to deteriorate in the two advanced models relative to the pure diffusion case. The standard deviation of utility is also higher for the two advanced models. Thus, portfolio allocations made by the diffusion model are significantly sub-optimal for cases where correlated jumps may not be ignored.

The manner in which correlated jumps are incorporated in a model is also important. For the two advanced model specifications, we see that expected utility is larger for the Markovian contagion model than for perfectly correlated jumps model. However, standard deviation of utility is also higher for the Markovian contagion model. This suggests that, although portfolio return scenarios under Markovian contagion are fairly concentrated around the mean, the heavier tails of the joint distribution increases the standard deviation of utility function. This observation has two implications. First, a higher standard deviation of utility function implies that the standard error for expected utility estimates is higher under the Markovian contagion model. Second, if there is a greater deviation from the diffusion model in terms of tail risk, performance measures that focus on tail risk must be utilized to evaluate the Markovian contagion model.

		Expected Utility			Standard Deviation Utility		
Portfolio	γ	Diffusion	Perfectly Correlated Jumps	Markovian Contagion	Diffusion	Perfectly Correlated Jumps	Markovian Contagion
1	1.1	-9.9452	-10.0208	-9.9630	0.1803	0.2277	0.2654
2	1.2	-4.9498	-5.0189	-4.9660	0.1650	0.2081	0.2422
3	1.5	-1.9600	-2.0149	-1.9727	0.1315	0.1654	0.1921
4	2	-0.9700	-1.0110	-0.9796	0.0983	0.1233	0.1430
5	2.5	-0.6427	-0.6755	-0.6504	0.0785	0.0983	0.1139
6	3	-0.4801	-0.5073	-0.4864	0.0653	0.0818	0.0946
7	5	-0.2381	-0.2544	-0.2419	0.0391	0.0489	0.0564

Table 11: Evaluation of the expected utility function for each model corresponding to each of the portfolios in Table 6. For each portfolio, we show the expected utility and the standard deviation of the utility function under each model choice.

We consider two alternative measures of performance for the portfolios of diversified assets that focus on portfolio tail risk, namely, value at risk (VaR) and expected shortfall. Value at risk measures the threshold loss value for a given time horizon that realized losses may exceed with p probability. We estimate this threshold value, labelled $(1-p)100\%$ confidence level value at risk, by sorting the terminal wealth values for each portfolio and selecting the $p-$th percentile value for corresponding terminal wealth realizations. Table 12 shows the estimated VaR values for three confidence level choices for a portfolio ($\gamma = 2$) under the three model choices. The portfolio has a much higher VaR value under the Markovian contagion model compared to the other two models. This is clearly due to the fatter tail of the distribution under this model choice, which is particularly pronounced in the $(1-0.005)100\% = 99.5\%$ confidence level VaR as a result of rarer and severe impact of contagion.

VaR	p = 0.5%	1%	5%
Diffusion	-0.2029	-0.1832	-0.1229
Perfectly Correlated Jumps	-0.2792	-0.2549	-0.1852
Markovian Contagion	-0.3000	-0.2685	-0.1870

Table 12: Estimated VaR value using Monte Carlo simulation for three confidence level choices for a portfolio ($\gamma = 2$) under the three model choices.

The expected shortfall measures the expected value of portfolio loss in the worst $p\%$ scenarios of the portfolio return. Table 13 presents the results for the expected shortfall for different choices of p. These results are consistent with those for value at risk. Expected shortfall under the Markovian contagion model is larger than that for the diffusion model and the perfectly correlated jump model. Since expected shortfall is a conditional expectation of the tail of the portfolio return distribution, for all choices of confidence levels, $(1 - p)100\%$, Markovian contagion model estimates a significantly higher value of this risk measure.

Expected Shortfall	0.05%	1%	5%
Diffusion	-0.2292	-0.2105	-0.1599
Perfectly Correlated Jumps	-0.3096	-0.2878	-0.2280
Markovian Contagion	-0.3330	-0.3079	-0.2374

Table 13: Estimated Expected Shortfall value using Monte Carlo simulation for three confidence level choices for a portfolio ($\gamma = 2$) under the three model choices.

The results of the comparison constructed show that if contagion risk persists in a class of assets, even if the assets are themselves well-diversified, it can result in adverse impact on the performance of the portfolio. Depending on the specific performance measures chosen, the adverse impact would vary in severity. The contagion channel being assumed to be perpetually active can create a mis-specification of the model with misleading consequences. While this mis-specification of contagion suggests a greater deterioration in utility, it fails to comprehend the impact of truly devastating portfolio outcomes. Besides providing more accurate risk assessment, a dynamic description of activeness of contagion channels provides dynamic hedging opportunities that would not be otherwise available in static descriptions of correlated jumps.

5 Discussion and Conclusion

This paper acknowledges that spillover risks of shocks to asset prices transmitted to other assets in excess of what is explained by observable common factors is rare, but significant. We proposed a model of Markovian contagion in asset returns to capture these asset return dynamics. The model relies on a network filtration to identify when jumps from one asset get transmitted to other assets. We account for interdependence on the common factors using a dynamic conditional beta factor model, and specify that contagion occurs only when conditional correlation between idiosyncratic asset returns is high.

We estimate the parameters of the Markovian contagion model in the context of international equity and currency prices. Instead of a persistently active contagion possibility, the contagion channels are episodic in transmitting shocks only when the channels are active. We show that in a Markovian contagion model jumps experienced by individual assets are less frequent, but stronger than a perfectly correlated jump model. We also show that when contagion occurs, albeit infrequently, the effects are severe without a specific direction.

The model is applied to evaluate the effect of the correlated jumps on portfolio performance. Our analysis shows that Markovian contagion model provides more accurate fat tail characteristics than pure diffusion or perfectly correlated jump models. A pure diffusion allocation is suboptimal under Markovian contagion, with reduced expected utility and increased standard deviation of utility. On the other hand, value at risk (VaR) and expected shortfall are significantly larger when assets follow a Markovian contagion model. Failing to capture fat tails can cause serious model risk, if tail characteristics are important in a context.

For the international equity and currency assets considered in this paper, we find low inter-asset and inter-region contagion, but high intra-asset class contagion. We also find stronger directional contagion from currencies to equities than the reverse. Other application cases for the Markovian contagion model may reveal other specific characteristics and asset dynamics behavior, which would be valuable insights for dynamic portfolio allocation, risk assessment, as well as developing dynamic hedging strategies.

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