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Neo-Fisherianism in a small open-economy New Keynesian model

Eurilton Araújo

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Phones: +55 (61) 3414-3710 and 3414-3565
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Non-technical Summary

This paper focuses on the relationship between nominal interest rates, inflation expectations and real interest rates in a small-open economy in which prices adjust in a sluggish way. Conventional wisdom suggests that, in the short run, after the emergence of shocks that cause unexpected expansionary movements in the nominal interest rate, this variable increase and inflation decreases. In other words, these variables tend to walk in opposite directions, at least in the short run.

Economists tend to believe that the real interest rate tend to be irresponsive to changes in nominal forces that influence inflation expectations in the long run. Hence, nominal interest rates follow the movements in expected inflation one to one. This pattern acts according to the Fisher identity, which states that nominal interest rates are the sum of two components: real interest rates and expected inflation. The Neo Fisherian hypothesis challenges the conventional wisdom by suggesting that in some situations, even in the short run, nominal interest rates and inflation move together in the same direction, since inflation expectations influence current inflation directly. Therefore, even in the short run, under certain circumstances that lead to a relative stability in real rates, nominal interest rates and inflation tend to co-move positively.

In fact, in the basic New Keynesian model there is no restrictions leading to the negative co-movement between nominal rates and inflation in the short run, as suggested by the conventional view on the relationship between interest rates and inflation. Indeed, this co-movement depends on the sensitivity of the real interest rates to nominal shocks. It is worth noticing that, in this basic model, prices adjust gradually and, therefore, unexpected oscillations is nominal variables affect current economic activity. In a closed economy, how fast prices adjust and the persistency of unexpected nominal movements can lead the canonical New Keynesian model to behave according to the Neo Fisherian hypothesis.

This paper investigates the conditions, under which the Neo Fisherian hypothesis may arise in a simple small open-economy New Keynesian model. The analysis concentrates on the role of trade openness for the emergence of a positive short run co-movement between nominal interest rates and inflation after a monetary policy shock. In addition, the paper discusses the relationship between the likelihood of Neo-Fisherian features in the model and the choice between domestic and consumer price (CPI) inflation as the relevant target for monetary policy. The results suggest that, under relatively high substitutability between domestic and foreign goods, the model is more likely to display the Neo-Fisherian behavior if the degree of openness to trade rises. In this situation, the real exchange rate responds more aggressively to monetary policy shocks, reducing the movements in the real interest rates necessary to adjust the economy to the effects of these shocks. Concerning the choice between alternative inflation measures as the relevant target for monetary policy, targeting CPI makes the model more likely to display Neo-Fisherian behavior, since this regime restricts the real exchange rate, stabilizing aggregate demand and inducing small variations in the real rates.

These findings remind monetary authorities that the conventional interest rate transmission channel of monetary policy has limits. Indeed, under the Neo Fisherian hypothesis, the real interest rates do not move much in response to nominal disturbances under certain circumstances.

Sumário Não Técnico

O artigo estuda a relação entre taxas de juros nominais, expectativas de inflação e taxas de juros reais em uma economia aberta na qual os preços não se ajustam instantaneamente. A visão convencional sustenta que, após choques positivos de política monetária, as taxas de juros nominais aumentam e a inflação se reduz no curto prazo. Ou seja, essas variáveis tendem a caminhar em direções opostas no curto prazo.

No longo prazo, os economistas acreditam que as taxas de juros reais são totalmente insensíveis a essas alterações advindas de choques nominais de modo que as taxas de juros nominais se movimentam *pari passu* com as expectativas de inflação. Este padrão se baseia na identidade de Fisher que estabelece que taxas nominais resultam da soma das taxas reais com as expectativas de inflação futura. A hipótese Novo Fisheriana desafia a visão convencional e atesta que existem situações nas quais as taxas de juros nominais e a inflação caminham na mesma direção mesmo no curto prazo. Assim, mesmo no curto prazo, sob certas condições que garantam estabilidade das taxas reais, as taxas de juros nominais e a inflação tenderiam a se mover na mesma direção.

Com efeito, na versão mais básica do modelo Novo Keynesiano, não existem restrições que levem necessariamente a movimentos em direções opostas para taxas de juros nominais e a inflação no curto prazo. De fato, esse padrão depende fundamentalmente da sensibilidade das taxas de juros reais a choques nominais, como, por exemplo, choques de política monetária. Cumpre lembrar que, neste modelo, o ajuste de preços é gradual, possibilitando, portanto, que choques de política monetária tenham efeitos na atividade econômica. Em uma economia fechada, a velocidade de ajuste de preços elevada e a alta persistência desses choques podem levar o modelo Novo Keynesiano a se comportar de acordo com a hipótese Novo Fisheriana.

Este artigo examina em que condições a hipótese Novo Fisheriana tende a valer no modelo Novo Keynesiano básico para uma economia aberta, focando no papel da abertura comercial e na escolha da medida de inflação relevante para a execução da política monetária (inflação doméstica versus inflação ao nível do consumo de bens finais). O artigo mostra que a abertura comercial pode levar a um comportamento compatível com a hipótese Novo Fisheriana se o grau de substituição entre bens domésticos e importados for relativamente elevada. Neste caso, a taxa de câmbio real responde bastante a choques de política monetária, reduzindo as variações nas taxas de juros reais necessárias para o ajuste da economia. No que tange a escolha da inflação a ser perseguida pela autoridade monetária, o modelo fica mais propenso a satisfazer a hipótese Novo Fisheriana se a inflação relevante é àquela medida a nível do consumidor e não do produtor (inflação doméstica), pois este regime restringe movimentos do câmbio real e isto tende a estabilizar a demanda agregada, induzindo variações menos contundentes na taxa real de juros.

Os resultados acima alertam formuladores de política monetária sobre os limites do canal de transmissão mais convencional da política monetária, pois a hipótese Novo Fisheriana mostra, sob certas circunstâncias, a possibilidade de que haja pouca sensibilidade das taxas de juros reais a choques nominais, como, por exemplo, choques de política monetária.

Neo-Fisherianism in a small open-economy New Keynesian model*

Eurilton Araújo**

Abstract

In this study, the Neo-Fisherian hypothesis denotes the positive short-run co-movement between the nominal interest rate and inflation conditional on a monetary policy shock. To investigate the situations in which this hypothesis may arise in a simple small open-economy New Keynesian model, I extend the analysis of Garín et al. (2018) to the framework in Galí and Monacelli (2005). This paper shows that, under relatively high substitutability between domestic and foreign goods, this hypothesis most likely emerges in economies that are more open. Furthermore, targeting CPI inflation accentuates the forces leading to a Neo-Fisherian behavior.

Keywords: small open-economy, monetary policy, Neo-Fisherianism
JEL Classification: E31, E52, F41

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** Research Department, Banco Central do Brasil. Email: eurilton.araujo@bcb.gov.br.

1 Introduction

In the short run, under certain circumstances, nominal shocks that lead to an increment in the nominal interest rate may increase inflation in models with sticky prices. According to Garín et al. (2018), the Neo-Fisherian hypothesis refers to this positive short-run co-movement between the nominal interest rate and inflation conditional on a nominal shock. In fact, for the sake of concreteness, these authors only consider the case of a monetary policy shock because the New Keynesian paradigm has a long tradition of studying the short-run implications of this type of nominal disturbance for the macroeconomy.

In the literature, the Neo-Fisherianism has been framed in somewhat different terms as the proposition asserting that to raise inflation the central bank has to increase the nominal interest rate rather than decrease it. This definition implies a causal effect running from the nominal interest rate to inflation. If the central bank pegs the nominal interest rate at a higher level, this measure leads to multiple equilibria. In this situation, fiscal policy selects an equilibrium with high inflation in order to satisfy the intertemporal government budget constraint without adjusting revenues and expenditures. Cochrane (2018) discussed extensively this formulation of the Neo-Fisherian hypothesis.

Indeed, economists believe that a positive co-movement between the nominal interest rate and inflation likely holds in the long run because the real interest rate tends to be irresponsive to changes in nominal forces that influence inflation expectations. This behavior follows from the Fisher relationship: $i_t = r_t + E_t(\pi_{t+1})$, where i_t denotes the nominal interest rate, r_t stands for the real interest rate, and $E_t(\pi_{t+1})$ is expected inflation.

In the short run, however, the real interest rate may respond to nominal variables due to nominal rigidities. Conventional wisdom suggests that this reaction leads to a negative short-run co-movement between the nominal interest rate and inflation conditional on a monetary policy shock. Nevertheless, in certain circumstances, New Keynesian models may display Neo-Fisherian features. Thus, a relevant research question is the following: under which conditions the Neo-Fisherian hypothesis may hold in models with nominal rigidities?

Cochrane (2018) asked how inflation would react to a permanent increase in nominal interest rates and explored situations engendering a positive co-movement. He highlighted the role of the fiscal theory of the price level to remove equilibria indeterminacy under a stable interest rate peg, allowing for the emergence of Neo-Fisherian characteristics in a simple model with price rigidity.

Garín et al. (2018) studied the textbook New Keynesian model, examining the co-movement between the nominal interest rate and inflation after an inflation-target shock. Under persistent changes in the target and small degrees of nominal rigidities, they showed that this closed-economy model was prone to Neo-Fisherian behavior. On the other hand, the model likely followed the conventional wisdom under a hybrid New Keynesian Phillips curve.

This paper extends the analysis of Garín et al. (2018) to a simple small open-economy New Keynesian (SOE-NK) model, developed in Galí and Monacelli (2005). Thus, I characterize the short-run co-movement between the interest rate and inflation after a monetary policy shock in a situation in which the central bank has perfect control over inflation and precisely hits an exogenous inflation target process. Hence, in this case of a strict inflation target, the monetary policy shock corresponds to an unexpected change in the target. In appendix A, I consider the case of a standard Taylor rule. Under this specification, the shock coincides with a surprise deviation from the interest rate that agrees with the systematic response of the central bank to inflation and output gaps.

In contrast to Cochrane (2018), equilibria are always determinate, and there is no attempt to peg the nominal rate. This paper does not argue in favor of any causation going from nominal interest rates to inflation; it rather explores the short-run positive correlation between these variables conditional on a monetary policy shock.

The analysis concentrates on the role of trade openness for the emergence of a positive short-run co-movement between the interest rate and inflation conditional on a monetary policy shock. In addition, I also discuss the relationship between the likelihood of Neo-Fisherian features in the model and the choice between domestic and consumer price (CPI) inflation as the relevant target for monetary policy.

Indeed, under relatively high substitutability between domestic and foreign goods, more openness to trade raises the chance that the SOE-NK model exhibits these features. In addition, targeting CPI inflation strengthens the mechanism that increases the likelihood of the Neo-Fisherian hypothesis.

The paper proceeds as follows. Section 2 sets out the model. Section 3 presents the case of an exogenous inflation target process. The last section concludes. Finally, appendix A discusses monetary policy under a Taylor rule.

2 A small open-economy model

After the log-linearization around the steady state of the equilibrium conditions, the following expressions represent the small open economy:[1]

$$x_t = E_t\left(x_{t+1}\right) - \frac{1}{\sigma_\alpha}\left[i_t - E_t\left(\pi_{H,t+1}\right)\right] \tag{1}$$

$$\pi_{H,t} = \beta E_t(\pi_{H,t+1}) + \kappa_\alpha x_t \tag{2}$$

$$\pi_t = \pi_{H,t} + \alpha(s_t - s_{t-1}) \tag{3}$$

$$x_t = \frac{1}{\sigma_\alpha} s_t \tag{4}$$

The endogenous variables x_t, $\pi_{H,t}$, i_t, π_t and s_t are the domestic output gap, domestic inflation, the domestic interest rate, CPI inflation and the terms of trade.[2] The exogenous foreign variables remain constant at their steady state levels. Furthermore, $E_t\left(x_{t+1}\right)$ and $E_t\left(\pi_{H,t+1}\right)$ denote expected values for the output gap and domestic inflation.

The Euler equation is the first expression, and it summarizes households' intertemporal consumption decisions. The second is the New Keynesian Phillips curve, and it condenses the price-setting behavior of monopolistic firms. The third computes CPI inflation, and the fourth combines the assumption of complete markets and market clearing conditions in the domestic economy.

[1] I follow Llosa and Tuesta (2008) and present the log-linearized system of equations. For more details, the reader should refer to Galí and Monacelli (2005).

[2] Galí and Monacelli (2005) defined the terms of trade as the ratio between foreign and domestic prices.

The third expression merges the following original equations:

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\Delta e_t \text{ and } s_t - s_{t-1} = \Delta e_t - \pi_{H,t}.$$

Moreover, under complete markets and for a given monetary policy strategy, the uncovered interest parity (UIP) condition $i_t = E_t(\Delta e_{t+1})$ is redundant to solve the system. In the UIP, $E_t(\Delta e_{t+1})$ denotes the expected variation in the nominal exchange rate.

The parameter β is the discount factor, and α measures the degree of trade openness, denoting the share of foreign goods in total consumption. The symbols σ_α and κ_α are convolutions of deep parameters such that: $\sigma_\alpha = \frac{\sigma}{(1-\alpha)+\alpha\Omega}$ and $\kappa_\alpha = (\sigma_\alpha + \varphi)\lambda$, with the ancillary variables $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ and $\Omega = \gamma\sigma + (1-\alpha)(\sigma\eta - 1)$.

In the preceding expressions, σ symbolizes the degree of relative risk aversion, φ is the inverse of the labor supply elasticity, γ measures the elasticity of substitution between imported goods, η is the elasticity of substitution between domestic and foreign goods, and θ denotes the degree of price stickiness.

3 The Case of an exogenous inflation target

3.1 Targeting Domestic Inflation

In every point in time, the central bank adjusts the nominal interest rate in order to exactly hit its target, i.e., $\pi_{H,t} = \pi_t^*$. Following Garín et al. (2018), the target π_t^* is a stationary AR(1) process, according to the equation $\pi_t^* = \rho_\pi \pi_{t-1}^* + \varepsilon_t$. The error ε_t is normally distributed with zero mean and unit variance. Given this specification of monetary policy, I solve the model analytically by the method of undetermined coefficients. Since I use only equations (1) and (2), the solution resembles the case of a closed-economy.

In equilibrium, x_t and i_t are functions of π_t^* and the expressions below describe their evolution:

$$x_t = \frac{1-\beta\rho_\pi}{\kappa_\alpha}\pi_t^* \text{ and } i_t = \left[\rho_\pi - \frac{\sigma_\alpha}{\kappa_\alpha}(1-\rho_\pi)(1-\beta\rho_\pi)\right]\pi_t^*$$

The computation of the relevant real interest rate in equation (1) leads to the following formula: $r_t = i_t - E_t\left(\pi_{H,t+1}\right) = -\frac{\sigma_\alpha}{\kappa_\alpha}(1-\rho_\pi)(1-\beta\rho_\pi)\pi_t^*$.

This real rate is the one that influences the behavior of the output gap in equation (1); therefore, there is no need to look at the real rate based on CPI inflation.

If r_t is less responsive to π_t^*, the likelihood of the Neo-Fisherian hypothesis increases. As in a closed economy, the derivative $\frac{dr_t}{d\pi_t^*}$ is negative, but its magnitude is small if changes in the target are persistent and prices are more flexible. These situations arise for large magnitudes of ρ_π and κ_α. Indeed, this last parameter increases with smaller degrees of nominal rigidity θ. As $\frac{dr_t}{d\pi_t^*}$ becomes flatter, i_t most likely increases with π_t^*.

I will not explore these results because they are analogous to the ones reported in Garín et al. (2018) for the closed-economy case. Therefore, I refer the reader to that paper for more clarification.

I focus now on the role of trade openness in increasing the likelihood of Neo-Fisherian features in the model. This issue is the main contribution of this investigation and highlights how open-economy considerations affect the responsiveness of the real interest rate to nominal forces.

As σ_α depends on the parameter α, trade openness influences the ratio $\frac{\sigma_\alpha}{\kappa_\alpha} = \frac{1}{\lambda(1+\frac{\varphi}{\sigma_\alpha})}$. Though $\frac{\sigma_\alpha}{\kappa_\alpha}$ increases with σ_α, the relationship between σ_α and α hinges, however, on the elasticities γ and η.

In this paper, I consider the case in which $\gamma = \eta$. This is a common assumption in the literature and simplifies the computation of the derivative $\frac{d\sigma_\alpha}{d\alpha}$. Indeed, one can show that σ_α falls with openness if $\sigma\eta - 1 > 0$ and rises if $\sigma\eta - 1 < 0$. Moreover, the model nests the closed-economy case because the parameterization $\sigma = \gamma = \eta = 1$ implies $\sigma_\alpha = \sigma = 1$, with no role for α in changing the coefficients in equations (1) and (2). Standard calibrations choose the empirically plausible restriction $\sigma\eta - 1 > 0$.[3] Thus, in these cases, more open economies lead to a less responsive r_t, increasing the likelihood of a short-run positive co-movement between i_t and π_t^*.

Indeed, in equation (1), the sensitivity of x_t to r_t takes into account the effects of movements in the terms of trade (s_t) on the demand for domestic goods. These movements also influence real marginal costs through the wealth effect on labor supply, which comes from induced changes in consumption. Therefore, in equation (2), κ_α also depends on σ_α.

A canonical representation, similar to its closed-economy counterpart, comprises equations (1) and (2). Since α controls the effects of the terms of trade on domestic demand and on real marginal costs, the parameters σ_α and κ_α depend upon α. After considering these effects on the canonical representation, the logic described in Garín et al. (2018) for the closed economy also applies to the SOE-NK with $\pi_{H,t}$ as the targeted inflation.

In fact, according to Garín et al. (2018), after the inflation-target shock, from equation (2), higher inflation requires an increase in the output gap. The size of this increment depends on how $E_t(\pi_{H,t+1})$ responds to the shock. Indeed, a larger response of $E_t(\pi_{H,t+1})$ leads to a smaller increase in x_t. In equation (1), given $E_t(x_{t+1})$, r_t needs to decrease, but its magnitude may be

[3] For calibrated models, see Llosa and Tuesta (2008) and Alba et al. (2011). Besides, estimation results in Nimark (2009), Justiniano (2010) and Paez-Farrell (2015) support this restriction.

very small as long as $E_t\left(\pi_{H,t+1}\right)$ is very large. Thus, in this case, there are instances in which a rise in i_t is compatible with this slight drop in r_t.

These situations correspond to high values for ρ_π and small magnitudes for the probability θ. Furthermore, in a SOE-NK, openness to trade may exacerbate or attenuate this trend. For instance, if $\sigma\eta - 1 > 0$, in more open economies, this Neo-Fisherian logic most likely emerges. This fact happens because an induced real depreciation, leading to the switching of expenditures towards domestic goods, amplifies the direct positive effect of a decrease in the real rate on the output gap.

Figure 1 shows the solution coefficients for i_t and r_t as a function of α under alternative values for ρ_π and θ. From the top to the bottom of Figure 1, each row corresponds to an increment in the auto-regressive parameter ρ_π. Analogously, from the left to the right of Figure 1, each column represents an increase in the degree of price rigidity θ. The baseline calibration assumes that $\sigma = 5$, $\varphi = 0.5$, $\gamma = \eta = 1.5$ and $\beta = 0.99$.

Figure 1 corroborates the analytical results discussed in this sub-section. Indeed, for each combination of ρ_π and θ, the solution coefficient for r_t increases with α and is always negative. Moreover, the solution coefficient for i_t also increases with α, i.e., the negative short-run co-movement between the nominal interest rate and inflation weakens with α and may become positive if $\theta = 0.25$ and $\rho_\pi \geq 0.23$. In this case, the positive co-movement becomes stronger with trade openness. Neo-Fisherian characteristics also emerge if $\rho_\pi = 0.7$ and $\theta = 0.5$. Nevertheless, for the same ρ_π and $\theta = 0.75$, these features materialize only for sufficiently high degrees of trade openness ($\alpha \geq 0.43$).

As highlighted in Garín et al. (2018), if nominal shocks are very persistent and prices are relatively flexible ($\rho_\pi = 0.7$ and $\theta = 0.25$), the solution coefficient for r_t is almost zero and varies little with α. In this situation, i_t is smaller than $E_t(\pi_{H,t+1}) = \rho_\pi \pi_t^*$ by a small amount, remaining very close to this magnitude.

Figure 2 presents impulse response functions concerning home inflation targeting and CPI inflation targeting. In simulations, in addition to the baseline parameters, I set $\alpha = 0.4$, $\theta = 0.75$ and $\rho_\pi = 0.4$, a moderate value for the persistence of the shock.[4] Here I discuss only the case of home inflation targeting. Later in this paper, I compare this situation with CPI inflation targeting.

Inspecting the case of home inflation targeting in Figure 2, the nominal shock is expansionary with a rise in the output gap that a drop in the real interest rate must endorse. Since $E_t(\pi_{H,t+1})$ does not jump aggressively due to the modest magnitude of ρ_π, the increase in x_t is significant. Therefore, the nominal interest rate has to decline to validate the reduction in the real rate consistent with the expansion in the domestic economy.

Indeed, the increase in the supply of domestic goods leads to a fall in their relative prices causing a rise in the terms of trade. This increase is consistent with a nominal depreciation in the exchange rate that, under complete markets, supports financial flows towards the rest of the world to fulfill the efficient risk-sharing arrangement, allowing consumption smoothing by foreigners that do not experience an economic expansion.

For the baseline calibration, a central bank that targets home inflation engenders a negative short-run co-movement between the nominal interest rate and inflation. This feature is compatible with Figure 1 because θ is high and ρ_π is moderate, which constitutes a calibration that contradicts the Neo-Fisherian hypothesis.

[4] According to Lolsa and Tuesta (2008), the chosen value for α corresponds approximately to the import to GDP ratio in Canada.

3.2 Targeting CPI Inflation

In an open economy, domestic and CPI inflation are two different measures that could be the target variable in a given monetary policy strategy. In the context of optimal policy design, Rhee and Turdaliev (2013) and Campolmi (2014) compared the welfare performance of these alternative inflation indicators. In this paper, I consider how CPI inflation targeting can possibly change the dynamics of macroeconomic variables and the potential effects of this choice on the likelihood of Neo-Fisherian behavior.

In the SOE-NK model, the choice of CPI inflation as the central bank's target induces backward-looking behavior in the main macroeconomic variables of the model. This statement follows from the solution for $\pi_{H,t}$ and x_t, which uses equations (2) and (3) after I manipulate (4) and replace s_t with $\sigma_\alpha\ x_t$ in (3). Now, x_{t-1} becomes a state variable and affects the dynamics of the model.

From the method of undetermined coefficients:

$$\pi_{H,t} = a_1\pi_t^* + a_2 x_{t-1} \text{ and } x_t = b_1\pi_t^* + b_2 x_{t-1}$$

The restrictions on the coefficients are:

$$P(b_2) = -\beta\sigma_\alpha\alpha(b_2)^2 + (\kappa_\alpha + \beta\sigma_\alpha\alpha + \sigma_\alpha\alpha)b_2 - \sigma_\alpha\alpha = 0$$
$$(1 - \sigma_\alpha\alpha b_1) = (1 - \sigma_\alpha\alpha b_1)\beta\rho_\pi + \kappa_\alpha b_1 + \beta\sigma_\alpha\alpha(1 - b_2)b_1$$
$$a_1 = 1 - \sigma_\alpha\alpha b_1$$
$$a_2 = (1 - b_2)\sigma_\alpha\alpha$$

From the first expression, which is a quadratic equation with a positive discriminant, one can pin down a stable solution.[5] Indeed, the roots of $P(b_2)$

[5] The discriminant is $(k_\alpha)^2 + 2k_\alpha(1+\beta)\sigma_\alpha\alpha + [(1+\beta)\sigma_\alpha\alpha]^2 - 4\beta(\sigma_\alpha\alpha)^2$, which simplifies to the positive number $(k_\alpha)^2 + 2k_\alpha(1+\beta)\sigma_\alpha\alpha + [(1-\beta)\sigma_\alpha\alpha]^2$.

are real and positive because their sum $1 + \frac{1}{\beta} + \frac{\kappa_\alpha}{\beta\sigma_\alpha\alpha}$ and their product $\frac{1}{\beta}$ are positive. Since the x-coordinate of the vertex of $P(b_2)$ is bigger than one, the root that lies on its right is also bigger than one.[6] The other root is, however, inside the unit circle because $\frac{1}{\beta}$ is less than one.

After finding the value for b_2, the remaining equations recursively determine b_1, a_1 and a_2. The analytical expressions for these coefficients are cumbersome; therefore, I resort to numerical simulations to show the effects of CPI targeting on the forces leading to Neo-Fisherian behavior.

Besides impulse responses for the case of home inflation targeting, Figure 2 displays impulse responses for CPI inflation targeting under two different degrees of openness ($\alpha = 0.4$ and $\alpha = 0.1$). In any situation, I choose $\rho_\pi = 0.4$, a moderate persistence level. Next, I compare CPI inflation targeting and home inflation targeting.

Regarding the case of CPI inflation targeting, Figure 2 shows that the nominal interest rate displays a Neo-Fisherian behavior for the same calibration considered in sub-section 3.1, which analyzes domestic inflation targeting. Therefore, targeting CPI inflation increases the likelihood of the Neo-Fisherian hypothesis.[7]

The economic mechanism following the expansionary inflation-target shock is similar to the one I described for the case of home inflation targeting, but the size of the domestic expansion is much smaller, which requires a very modest decline in the real interest rate, leading to a positive short-run comovement between the nominal interest rate and inflation.

In the case of home inflation targeting, the inflation-target shock directly

[6] The coordinate of the vertex is $\frac{k_\alpha + (1+\beta)\sigma_\alpha\alpha}{2\beta\sigma_\alpha\alpha}$. If this number is less than one then $k_\alpha < (\beta - 1)\sigma_\alpha\alpha < 0$, contradicting the fact that $k_\alpha > 0$.

[7] The same result obtains for $\sigma = 1$ and $\eta = 0.5$, i.e, whenever $\sigma\eta - 1 < 0$. In this case, more closed economies are prone to exhibit Neo-Fisherian features.

affects $\pi_{H,t}$, which is the relevant inflation for producers, and the expansion is, therefore, stronger in this circumstance. Indeed, since the producer can charge higher prices, the firm increases wages and hires more people.

By contrast, inspecting equation (3), targeting CPI inflation implies a persistent terms of trade depreciation that helps the central bank to hit the target, thereby inducing a smaller increase in $\pi_{H,t}$. According to the second equation, this relatively feeble domestic inflation leads to a weaker economic expansion compared to the regime of home inflation targeting.

Due to its persistence, the terms of trade (the intratemporal price) impart inertia on x_t, leading to an expected future expansion with a size close to the current one. From equation (1), given the small positive magnitude of $x_t - E_t(x_{t+1})$, r_t needs to decrease mildly to corroborate the rise in x_t. Therefore, there is less room for the real rate (the intertemporal price) to catalyze the economic boom.

In a more closed economy, with $\alpha = 0.1$, Figure 2 shows that the real interest rate drops more, and the nominal rate contradicts the Neo-Fisherian hypothesis. In more open economies, since the baseline calibration assumes $\sigma\eta - 1 > 0$, r_t is less responsive to π_t^* in order to corroborate movements in x_t. In contrast, in a more closed economy with $\alpha = 0.1$, r_t reacts more to nominal shocks, allowing the nominal interest rate to decrease.

In fact, with a smaller degree of trade openness, the real interest rate affects domestic demand for home goods with more vigor because expenditure switching effects are quantitatively weak due to the weight of foreign goods in consumption. Furthermore, terms of trade movements need to be substantial to intensify this switching effect. Hence, a more pronounced decrease in r_t is necessary for stimulating economic activity.

4 Conclusion

In this paper, the Neo-Fisherian hypothesis refers to a positive short-run co-movement between the nominal interest rate and inflation conditional on a monetatary policy shock. In the context of a small open-economy New Keynesian model, this study has investigated conditions that validate this hypothesis, emphasizing the potential role of trade openness and the choice between domestic and consumer price inflation as the target variable in the monetary policy strategy.

Summing up the main results, this paper has shown that, under relatively high substitutability between domestic and foreign goods, a SOE-NK model is more likely to display a Neo-Fisherian behavior if the degree of openness to trade rises. Moreover, targeting CPI inflation increases this likelihood.

These results depend on the forward-looking nature of domestic inflation and the assumption of complete markets. Hence, the introduction of backward-looking components in domestic inflation and the incorporation of incomplete international asset markets are directions for future research. Moreover, in the New Keynesian framework, models with informational frictions may also mitigate the forces leading to Neo-Fisherian behavior because these frictions can limit the response of inflation expectations to nominal shocks.

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Appendix A: Monetary Policy shocks under a Taylor Rule

1. Responding to Domestic Inflation

The interest rate rule that governs monetary policy is $i_t = \phi_\pi \pi_{H,t} + \phi_x x_t + u_t$ and $u_t = \rho_u u_{t-1} + \varepsilon_t$, which is a stationary AR(1) with ε_t being normally distributed with zero mean and unit variance. Under this monetary policy strategy, I use the method of undetermined coefficients to obtain the expressions for the main variables as a function of u_t, which I list below.[8]

Define $\Delta = (1-\beta\rho_u)(1-\rho_u+\frac{\phi_x}{\sigma_\alpha})+\frac{\kappa_\alpha}{\sigma_\alpha}(\phi_\pi-\rho_u)$; therefore:

$$\pi_{H,t} = -\frac{\kappa_\alpha}{\sigma_\alpha \Delta} u_t \text{ and } x_t = -\frac{1-\beta\rho_u}{\sigma_\alpha \Delta} u_t$$

The expressions for nominal and real interest rates are:

$$i_t = \left[\frac{(1-\rho_u)(1-\beta\rho_u)-\rho_u\frac{\kappa_\alpha}{\sigma_\alpha}}{\Delta}\right] u_t \text{ and } r_t = i_t - E_t\left(\pi_{H,t+1}\right) = \left[\frac{(1-\rho_u)(1-\beta\rho_u)}{\Delta}\right] u_t$$

I impose the Taylor principle ($\phi_\pi > 1$); this choice leads to $\Delta > 0$. Given this assumption, on impact, a positive policy shock decreases both domestic inflation and the output gap. The model implies a positive response of r_t to u_t. As the monetary policy shock becomes more persistent, the magnitude of r_t decreases and the likelihood of a Neo-Fisherian behavior increases, i.e., the nominal interest rate tends to move in the same direction as the induced change in domestic inflation.[9] Moreover, since κ_α increases with smaller degrees of nominal rigidity θ, Δ also increases. This pattern leads to a less responsive real interest rate, which raises the likelihood of a decrease in nominal rates and a positive short-run co-movement between this variable

[8] Galí (2015) carried out a similar analysis in its chapter 8.

[9] Computing $\frac{dr_t}{d\rho_u}$, one can show that, for $0 \le \rho_u \le 1$, r_t decreases with ρ_u.

and domestic inflation. Therefore, the results associated with the closed-economy model remain.

Concerning the role of openness, $\frac{\phi_x}{\sigma_\alpha}$ and $\frac{\kappa_\alpha}{\sigma_\alpha}$ move in opposition to σ_α. Therefore, if σ_α decreases, Δ increases and the real interest rate becomes less responsive. Again, the effects of trade openness depend on the relationship between σ_α and α. For instance, if $\sigma\eta - 1 > 0$, more open economies are more likely to exhibit Neo-Fisherian features.

Figure 3 shows the solution coefficients for i_t and r_t as a function of α under alternative values for ρ_u and θ. From the top to the bottom of Figure 3, each row corresponds to an increment in the auto-regressive parameter ρ_u. Analogously, from the left to the right of Figure 3, each column represents an increase in the degree of price rigidity θ. I adopt the same parameterization used in sub-section 3.1.

Since u_t is a contractionary shock, the solution coefficient for r_t is always positive and, assuming the Taylor principle ($\phi_\pi > 1$), it decreases with α. Moreover, the solution coefficient for i_t also decreases with α, i.e., the negative short-run co-movement between the nominal interest rate and inflation weakens with α and becomes positive whenever this solution coefficient becomes negative. In this situation, the positive co-movement increases with α. This pattern arises because the solution coefficient for $\pi_{H,t}$ is always negative under the Taylor principle.

The behavior displayed in Figure 3 is analogous to that presented in Figure 1, with high values of ρ_u and θ leading to Neo-Fisherian characteristics. Again, if $\rho_\pi = 0.7$ and $\theta = 0.25$, the solution coefficient for r_t is almost zero and varies little with α. In this case, i_t is bigger than $E_t\left(\pi_{H,t+1}\right) = -\frac{\kappa_\alpha}{\sigma_\alpha \Delta}\rho_u u_t$ by a small amount, remaining very close to this magnitude.

In correspondence with Figure 2, Figure 4 presents impulse response functions after a decline in u_t, characterizing an expansionary monetary policy shock. In this exercise, to complement the baseline calibration, I set $\alpha = 0.4$, $\theta = 0.75$, $\phi_\pi = 1.5$, $\phi_x = 0.5$ and $\rho_u = 0.6$. I choose ρ_u in order to generate Neo-Fisherian features analogous to the ones shown in Figure 2 for a similar case. I address here the responses related to the rule that reacts to $\pi_{H,t}$. Next, I contrast this situation with the rule that responds to π_t.

In Figure 4, the shock in u_t stimulates aggregate demand in the home country. The initial exogenous decline of i_t dominates the systematic response that reacts positively to increasing inflation and economic activity. Smaller real rates induce an outflow of capital; this behavior is compatible with efficient risk-sharing in complete markets. This movement in financial flows causes a nominal depreciation, which motivates an increase in the terms of trade.

In fact, the effects on macroeconomic variables are similar to the ones reported in Figure 2, and the Neo-Fisherian hypothesis does not hold under this particular calibration.

2. Responding to CPI Inflation

The Taylor rule is $i_t = \phi_\pi \pi_t + \phi_x x_t + u_t$, and I keep the previous assumptions about the shock u_t. From the method of undetermined coefficients:

$$\pi_{H,t} = a_1 \pi_t^* + a_2 x_{t-1} \text{ and } x_t = b_1 \pi_t^* + b_2 x_{t-1}$$

The restrictions on the coefficients are:

$$\begin{gathered} b_2 = (b_2)^2 - \tfrac{\phi_\pi}{\sigma_\alpha} a_2 - (\tfrac{\phi_x}{\sigma_\alpha} + \phi_\pi \alpha) b_2 + \tfrac{1}{\sigma_\alpha} a_2 b_2 + \phi_\pi \alpha \\ a_2 = \beta a_2 b_2 + \kappa_\alpha b_2 \\ b_1 = b_1 \rho_u + b_2 b_1 - \tfrac{\phi_\pi}{\sigma_\alpha} a_1 - (\tfrac{\phi_x}{\sigma_\alpha} + \phi_\pi \alpha) b_1 + \tfrac{1}{\sigma_\alpha} (\rho_u a_1 + a_2 b_1 - 1) \\ a_1 = \beta \rho_u a_1 + \beta a_2 b_1 + \kappa_\alpha b_1 \end{gathered}$$

The combination of the first and second expressions leads to the following equation:

$(b_2)^3 - [1 + \frac{\phi_x}{\sigma_\alpha} + \phi_\pi \alpha + \frac{1}{\beta}(1 + \frac{\kappa_\alpha}{\sigma_\alpha})](b_2)^2 + [\frac{1}{\beta} + \frac{\phi_x}{\beta\sigma_\alpha} + \frac{\phi_\pi \alpha}{\beta} + \frac{\phi_\pi \kappa_\alpha}{\beta\sigma_\alpha} + \phi_\pi \alpha]b_2 - \frac{\phi_\pi \alpha}{\beta} = 0$

To show that a stable solution exists, I need to analyze the roots of the following polynomial:

$$Q(\lambda) = \lambda^3 + A_2\lambda^2 + A_1\lambda + A_0$$

where $A_2 = -[1 + \frac{\phi_x}{\sigma_\alpha} + \phi_\pi \alpha + \frac{1}{\beta}(1 + \frac{\kappa_\alpha}{\sigma_\alpha})]$, $A_1 = \frac{1}{\beta} + \frac{\phi_x}{\beta\sigma_\alpha} + \frac{\phi_\pi \alpha}{\beta} + \frac{\phi_\pi \kappa_\alpha}{\beta\sigma_\alpha} + \phi_\pi \alpha$ and $A_0 = -\frac{\phi_\pi \alpha}{\beta}$.

A value for b_2 that induces a stable solution arises if one root of $Q(\lambda)$ is inside the unit circle and the two other roots are outside. According to Woodford (2003), this situation occurs if and only if either of the following cases happens:[10]

1. $1 + A_2 + A_1 + A_0 < 0$ and $-1 + A_2 - A_1 + A_0 > 0$
2. $1 + A_2 + A_1 + A_0 > 0$, $-1 + A_2 - A_1 + A_0 > 0$ and $(A_0)^2 - A_0A_2 + A_1 - 1 > 0$
3. $1 + A_2 + A_1 + A_0 > 0$, $-1 + A_2 - A_1 + A_0 > 0$, $(A_0)^2 - A_0A_2 + A_1 - 1 < 0$ and $|A_2| > 3$

The first case applies to $Q(\lambda)$ because this polynomial satisfies its requirements. Indeed, one can show that:

- $1 + A_2 + A_1 + A_0 =$

$-\frac{\phi_x}{\sigma_\alpha} - \phi_\pi \alpha - \frac{1}{\beta}(1 + \frac{\kappa_\alpha}{\sigma_\alpha}) - \frac{1}{\beta} - \frac{\phi_x}{\beta\sigma_\alpha} - \frac{\phi_\pi \alpha}{\beta} - \frac{\phi_\pi \kappa_\alpha}{\beta\sigma_\alpha} - \phi_\pi \alpha - \frac{\phi_\pi \alpha}{\beta} < 0$

- $-1 + A_2 - A_1 + A_0 = \frac{\phi_x}{\sigma_\alpha}(\frac{1}{\beta} - 1) + \frac{\kappa_\alpha}{\beta\sigma_\alpha}(\phi_\pi - 1) > 0$ if $\phi_\pi > 1$

[10] See proposition C.2 in appendix C, p. 672.

The second condition of the first case holds under the Taylor principle ($\phi_\pi > 1$). In other words, $\phi_\pi > 1$ guarantees a determinate equilibrium.

After finding the magnitude of b_2, the last three restrictions on the solution coefficients for $\pi_{H,t}$ and x_t determine a_2, a_1 and b_1.

As in the case of an exogenous CPI inflation target, I employ numerical simulations. The goal is to illustrate how a Taylor rule that responds to π_t affects the short-run co-movement between the nominal interest rate and inflation conditional on u_t.

Figure 4 displays impulse responses associated with a Taylor rule that responds to CPI inflation under two different degrees of openness ($\alpha = 0.4$ and $\alpha = 0.1$). The figure also shows results related to a Taylor rule that reacts to home inflation. In any situation, I choose $\rho_u = 0.6$ in order to replicate the Neo-Fisherian behavior exhibited in Figure 2 with $\alpha = 0.4$ and the CPI as the relevant inflation index. Next, I compare the responses of macroeconomic variables under these alternative specifications.

Concerning the Taylor rule that reacts to CPI inflation, Figure 4 shows that, on impact, the nominal interest rate agrees with the Neo-Fisherian hypothesis for the same calibration specified in the first section of this appendix, dedicated to the situation in which the monetary authority cares about domestic inflation.[11] Consequently, Neo-Fisherian features most likely emerge if the central bank responds to π_t.

Inspecting Figure 4, the main qualitative difference regarding the monetary policy rule that reacts to CPI inflation lies in the nominal interest rate response under alternative degrees of openness, with Neo-Fisherian characteristics arising in the more open economy ($\alpha = 0.4$).

On impact, the domestic currency depreciates due to risk-sharing arrange-

[11] If $\sigma = 1$ and $\eta = 0.5$, this result still holds. Moreover, if $\sigma\eta - 1 < 0$, the likelihood of the Neo-Fisherian hypothesis increases in more closed economies.

ments, implying an increase in the terms of trade that engenders a strong expenditure switching effect in a more open economy. Therefore, with $\alpha = 0.4$, movements in the terms of trade can stimulate aggregate demand a great deal, and the real interest rate declines mildly. The magnitude of this mechanism is much weaker with $\alpha = 0.1$, and the real rate drops a lot. In this situation, though stronger, the terms of trade depreciation is less effective in spurring economic activity because the share of consumption expenditures allocated to domestic goods is already considerable.

In a more open economy, according to the systematic component of the policy rule, the central bank raises the nominal rate to mitigate the effects of inflation caused by firms that have to meet a substantial increase in demand compared with the case in which the degree of openness is smaller. These producers expand supply and, consequently, their demand for labor, leading to higher wages and prices. This hike in the nominal interest rate is consistent with a small decline in the real rate because inflation expectations tend to be higher due to the persistence of u_t.

In the specification in which $\alpha = 0.4$, as inflation and the output gap return to their steady states, the nominal interest rate quickly drops because the persistent effect of the shock on the interest rate dominates the systematic component of monetary policy. With more persistent shocks, this steady-state reversion would be slower than the one shown in Figure 4, causing a more prolonged increase in nominal rates.

FIGURES

Figure 1. Solution Coefficients of Interest Rates: Domestic Inflation Targeting

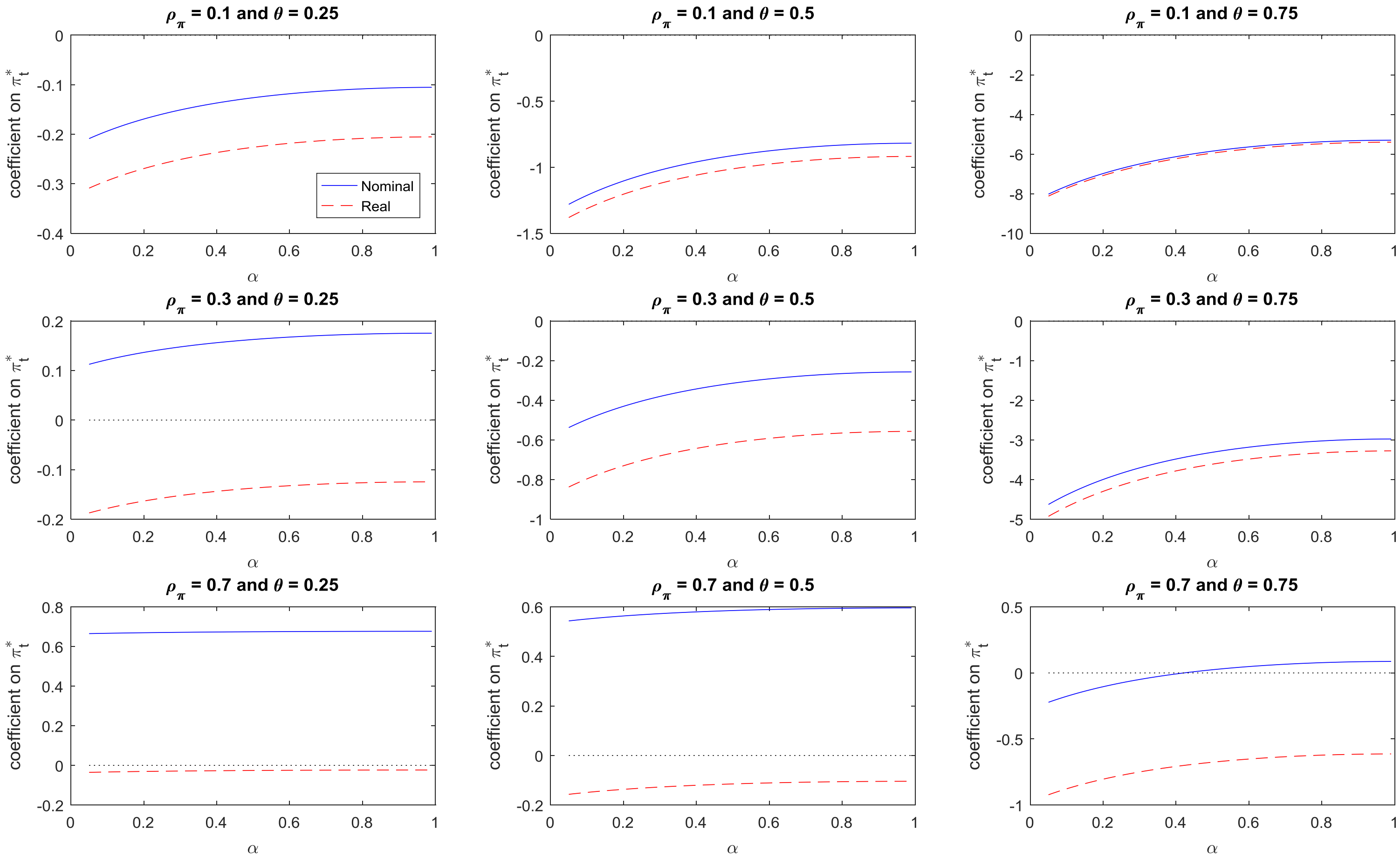

Figure 2. Impulse Responses under Inflation Targeting

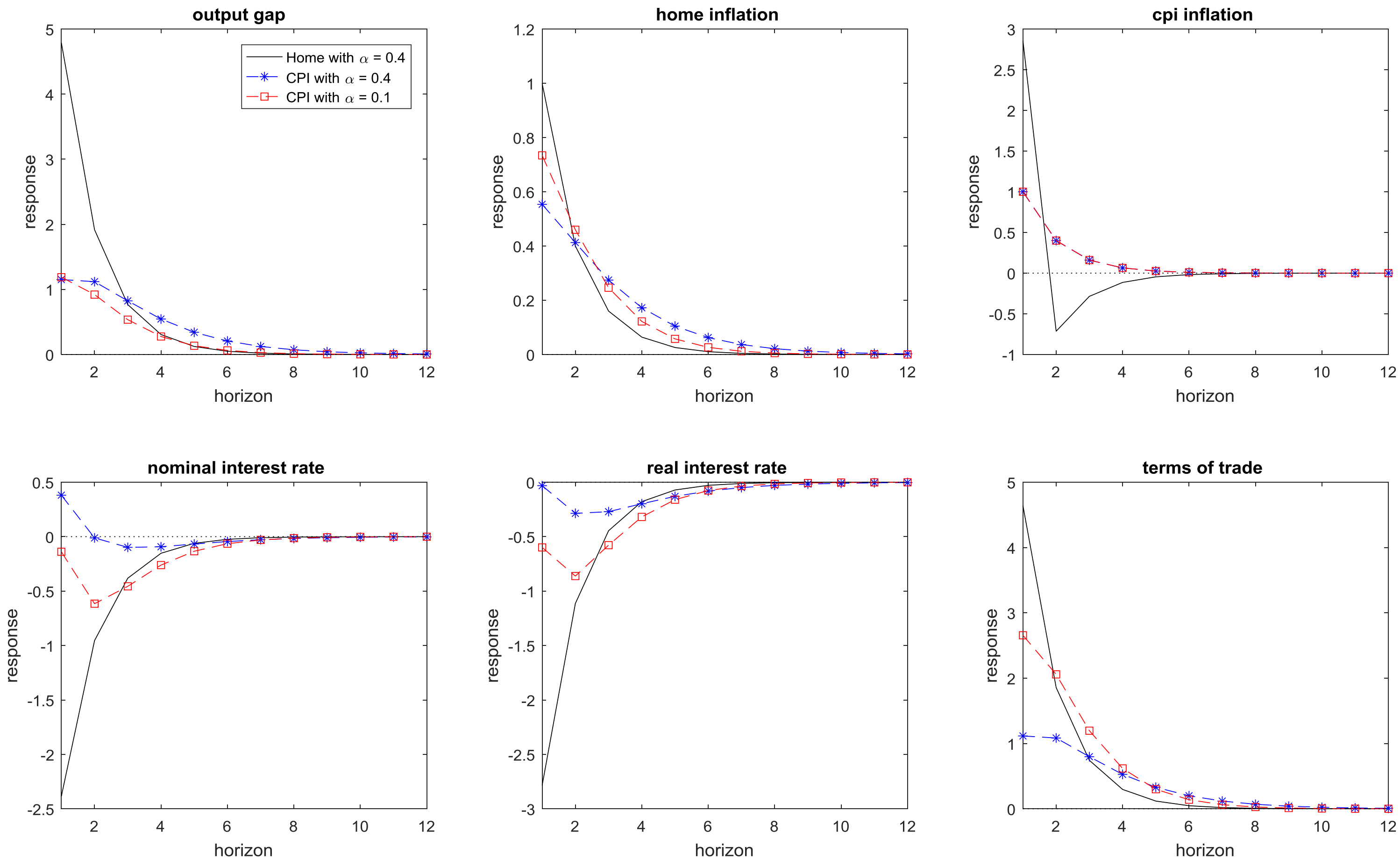

Figure 3. Solution Coefficients of Interest Rates: Taylor Rule with Domestic Inflation

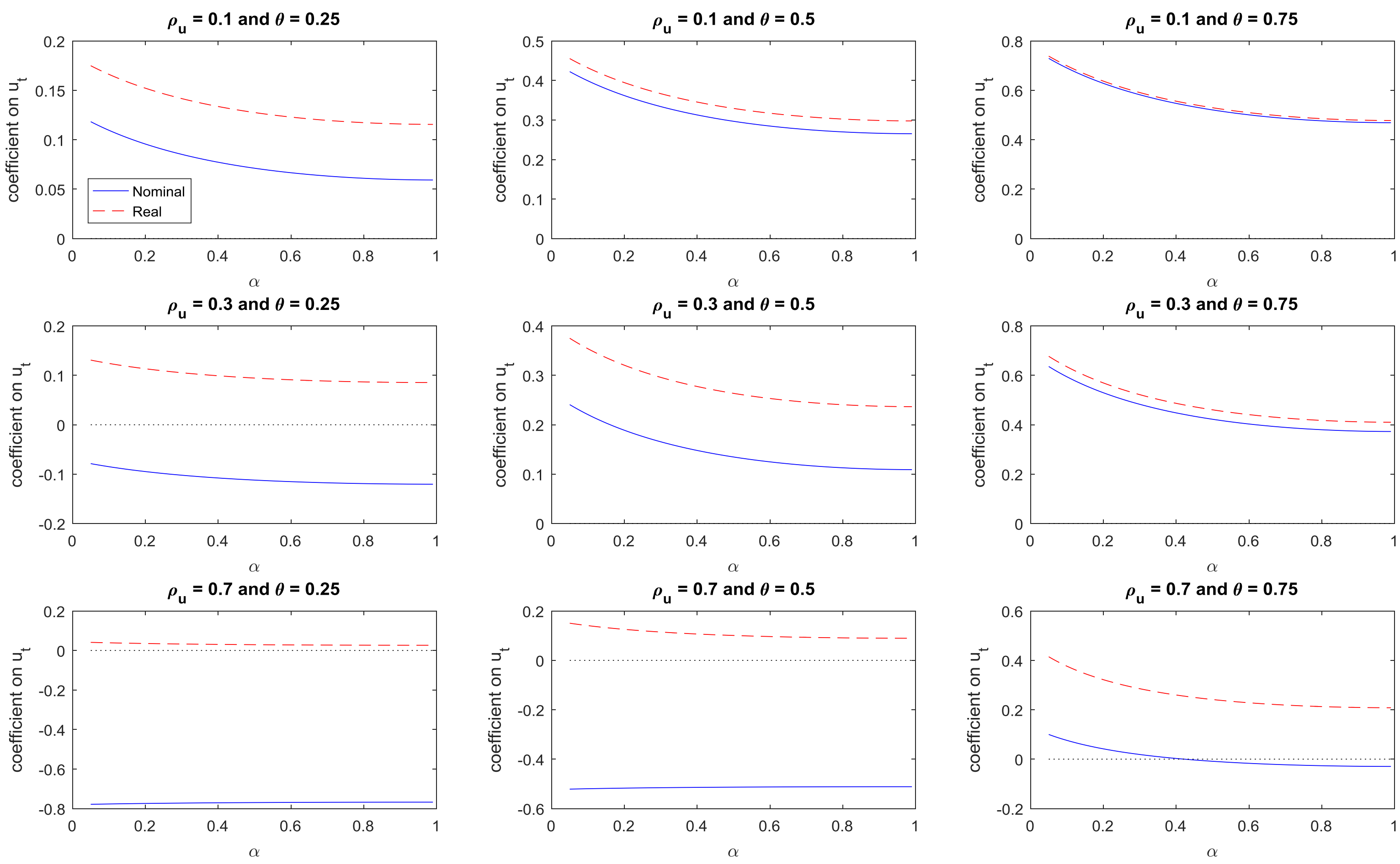

Figure 4. Impulse Responses under Taylor Rules