

Monetary Policy Volatility Shocks in Brazil

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Non-technical Summary

This paper provides empirical evidence on the effects of an increase in the volatility of monetary policy on the Brazilian economy. It is important to understand the difference between the effects of monetary policy surprises (shocks) – in the sense of unexpected decisions about interest rates from the monetary policy authority – and changes in the volatility of such surprises – the dispersion around these unexpected decisions. An increase in the volatility of monetary policy surprises might result in an increase in overall economic uncertainty, with well-documented consequences on employment, investment and consumption. From this perspective, the methodology employed is capable to quantitatively separate the effects of both changes on the economy.

Empirical results show that a monetary policy shock – meaning, a surprise in the response of interest rates to the state of the economy – presents the usual responses described in the literature, where an unexpected increase in nominal interest rates temporarily reduces output and inflation. However, a monetary policy volatility shock – an increase in the dispersion of the surprise in the response of interest rates –, while reducing output, results in a small increase in inflation. This result is robust to several specifications of the model, including changes in the time span of data, the number of time series included in the model, among other features of the empirical exercise.

Despite robust in several dimensions, results on the effects of monetary policy volatility shocks are at odds with international literature, where an increase in volatility also results in a decrease in inflation. The last part of this paper argues that, among other possible factors, the pattern of the response of monetary policy to the state of the economy might play a significant role in explaining results. When the monetary authority puts a significant weight in output stabilization, agents in the economy are able to anticipate that the increase in volatility will be compensated with average lower interest rates over time, in order to keep output closer to its potential. As a consequence, aggregate demand remains stable and prices do not fall.

Sumário Não Técnico

Este trabalho oferece evidências empíricas sobre os efeitos de um aumento da volatilidade da política monetária na economia brasileira. É importante separar entre os efeitos de surpresas (choques) de política monetária – no sentido de decisões inesperadas da autoridade monetária sobre a taxa de juros – e mudanças na volatilidade destas surpresas – a dispersão ao redor destas decisões inesperadas. Um aumento na volatilidade das surpresas de política monetária pode resultar em um aumento na incerteza sobre a economia como um todo, com consequências bem documentadas em termos de emprego, investimento e consumo. Nessa perspectiva, a metodologia empregada é capaz de quantitativamente separar os efeitos de ambas as mudanças na economia.

Resultados empíricos mostram que um choque de política monetária – ou seja, uma surpresa na resposta das taxas de juros em relação ao estado da economia – apresenta as respostas usuais descritas na literatura, onde um aumento inesperado na taxa de juros nominal reduz temporariamente o produto e a inflação. Entretanto, um choque de volatilidade da política monetária – um aumento na dispersão da surpresa na resposta das taxas de juros –, enquanto reduz o produto, resulta em um pequeno aumento da inflação. Esse resultado é robusto a diversas variações do modelo, incluindo mudanças no período de cobertura da amostra, o número de séries de tempo incluídas no modelo, além de outras especificações do exercício empírico.

Apesar de robusto em várias dimensões, os resultados sobre os efeitos de choques de volatilidade da política monetária não estão de acordo com a literatura internacional, onde um aumento da volatilidade também resulta em uma queda da inflação. A última parte deste trabalho argumenta que, entre outras possibilidades, o padrão de respostas da política monetária ao estado da economia pode ter um papel importante para explicar os resultados. Quando a autoridade monetária atribui um peso significativo na estabilização do produto, os agentes econômicos são capazes de antecipar que o aumento de volatilidade será compensado com taxas médias de juros mais baixas ao longo do tempo, de tal forma que o produto se mantenha mais próximo do potencial. Como consequência, a demanda agregada permanece estável e os preços não caem.

Monetary Policy Volatility Shocks in Brazil

Angelo Marsiglia Fasolo^{*}

Abstract

This paper provides empirical evidence for the impact of changes in volatility of monetary policy in Brazil using a SVAR where the time-varying volatility of shocks directly affects the level of observed variables. Contrary to the literature, an increase in monetary policy volatility results in higher inflation, combined with reduction in output. The qualitative differences of impulse responses functions, compared to the literature for developed economies, are explained using a calibrated small-scale DSGE model with habit persistence in consumption and stochastic volatility shocks in the Taylor rule. The DSGE model is capable of explaining the increase of inflation in the medium term after a monetary policy volatility shock.

JEL Codes: C11, C13, C15, E30, E43, E52

Keywords: Time-varying volatility; DSGE models; Volatility shocks; Small Open Economies; Bayesian SVAR models

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1 Introduction

What are the effects of monetary policy volatility in an Emerging Economy like Brazil? Considering the case of developed economies, it seems that there is a consensus in the literature with respect to the impact of monetary policy volatility shocks on economic activity and prices. Mumtaz and Zanetti (2013)[23] provide evidence from a time-varying volatility structural vector autoregression (SVAR) estimated for the US that monetary policy volatility shocks have a similar effect of a demand shock, with a positive correlation between prices and economic activity. For a panel of open, developed economies, Benigno, Benigno and Nisticò (2012)[4] find similar results. Taking a step further in the open economy framework, they also provide evidence that monetary policy volatility shocks tend to generate an appreciation of domestic currency.

The literature has not fully explored the consequences of the volatility of monetary policy shocks, especially in Emerging Economies. Identifying and measuring the impact of unexpected changes in monetary policy have been a common exercise, with a wide range of techniques available to simulate the effects of unexpected changes in nominal interest rates over prices and economic activity. However, given the characteristics of the business cycle in Emerging Economies compared to developed economies¹, not only in terms of higher volatility but also with respect to correlations between consumption, investment and trade balance with output, it is worth investigating the contribution of unexpected monetary policy changes, both in terms of first and second moments, as a driving force of such economies.

This paper provides evidence on the impact of second moments of monetary policy shocks using an SVAR with time-varying volatility estimated for Brazil. Despite using the same framework as in Mumtaz and Zanetti (2013)[23], results show that prices tend to increase with the increase in the volatility of monetary policy shocks. Also, the exchange rate depreciates, both in nominal and in real terms, contrary to results in Benigno, Benigno and Nisticò (2012)[4]. After presenting the empirical evidence, the same theoretical model presented in Mumtaz and Zanetti (2013)[23] is modified in order to understand some possible causes for results from the SVAR. Two simple modifications in the model, combined with a more realistic calibration of the labor supply, are enough to generate inflation after a monetary policy volatility shock. The argument is based on the asymmetric shape of the firms' profit function with respect to price adjustments, in a similar argument presented in Fernández-Villaverde, Guerrón-Quintana, Kuester and Rubio-Ramírez (2015)[12].

Results in the empirical section show that responses of prices and interest rates presented in Mumtaz and Zanetti (2013)[23] should be re-evaluated when dealing with an Emerging Economy, such as Brazil. Besides the source of the dataset, one of the key differences in the procedure adopted here is data frequency. The use of monthly data results in smaller persistence of monetary policy volatility shocks. In the theoretical part, it is shown that persistence of shocks is one of the key elements to understand the transmission mechanism from volatility to prices. The other two are the design of the monetary policy rule in terms of response to output and the presence of habit persistence in consumption. These three elements combined are capable of generating positive inflation after an increase in volatility.

Brazil is an interesting case for the use of SVARs for measuring the effects of time-varying volatility in two important directions. First, the SVAR is estimated using monthly data in a period (2000-2016) when, despite the presence of significant shocks, monetary policy regime was

¹See Aguiar and Gopinath (2007)[1], table 1.

kept the same². As a consequence, Brazil is one of a few Emerging Economies that has a relatively large and stable sample for inference on the subject. The use of monthly data also allows for a richer description of monetary policy, including short-lived episodes of high volatility. Second, as a small open economy, business cycles in Brazil are also influenced by foreign shocks, requiring an appropriate empirical strategy in order to characterize monetary policy shocks. From this perspective, the SVAR is expanded beyond the simple three-variable system observed in the literature to include the nominal exchange rate.

As said before, the empirical section of this paper is an application of the SVAR used in Mumtaz and Zanetti (2013)[23], with contemporaneous and lagged time-varying volatility of structural shocks included as regressors in the measurement equation. Models with heteroskedastic errors became common in the literature, as recent hardware advances provided a computational structure that made feasible the estimation of these models using Bayesian methods. Examples include Cogley and Sargent (2005)[10], Primiceri (2005)[25], Sims and Zha (2006)[26], among others. Gambetti, Pappa and Canova (2008)[16] explore not only the impact of the volatility of shocks, but also changes in the parameters of the SVAR for the US. Nakajima (2011)[24] provides a review of the methodology with an application for Japanese data. Mumtaz and Zanetti (2013)[23] and Creal and Wu (2014)[11] provide a link between the volatility of shocks and the first moments of data. Early literature provided only the identification and the estimation of time-varying volatility in a SVAR, without testing for their effects on first moments of data. Creal and Wu (2014)[11] decompose the volatility of monetary policy into two components, with the long run volatility factor presenting a positive correlation between uncertainty and inflation.

This paper is also connected with a broader research agenda on the estimation of macroeconomic models to measure the effects of volatility on prices and the business cycle, as in Bloom (2009)[5] and Fernández-Villaverde and Rubio-Ramírez (2010)[15]. While this paper does not present an estimation of a structural macroeconomic model measuring how changes in volatility of monetary policy affect the economy, the estimated stochastic process for the volatility of interest rates, after a proper identification for monetary policy shocks, is used in a calibrated DSGE model to evaluate the ability of such model to replicate the empirical features found in the data, as in Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2011)[14] and Fernández-Villaverde, Guerrón-Quintana, Kuester and Rubio-Ramírez (2015)[12].

Beyond this introduction, this paper has three sections. The next section shows the procedure to estimate the SVAR and explores the empirical results found in data. Section 3 describes the small-scale DSGE model, trying to match moments with the SVAR and providing some rationale for results. Section 4 concludes.

2 Empirical analysis

The SVAR with stochastic volatility (SVAR-SV) is represented by a measurement equation of a vector of n endogenous demeaned observables y_t :

$$y_{t} = \sum_{k=1}^{K} B_{k} y_{t-k} + \sum_{l=0}^{L} C_{l} h_{t-l} + \Sigma_{t} u_{t}$$

 $^{^{2}}$ The absence of changes in monetary policy regime does not mean, however, that the relative weight of other variables compared to inflation in the monetary policy rule remained constant over time. It does mean that the framework and the implementation of monetary policy, including the monetary policy instrument, were the same during this period.

In the model, u_t is a vector of standard independent gaussian shocks, B_k is an $n \times n$ and C_l is an $n \times 1$ matrix of coefficients. In order to understand vector h_t , assume that covariance matrix $\Sigma_t \Sigma'_t$ might be decomposed in the following way:

$$\Sigma_t \Sigma'_t = A^{-1} H_t A^{-1'}$$

$$A = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \alpha_{21} & 1 & \dots & 0 \\ \dots & \dots & \dots & 0 \\ \alpha_{n1} & \dots & \alpha_{nn-1} & 1 \end{bmatrix} \quad H_t = \begin{bmatrix} \exp(h_{1,t}) & 0 & \dots & 0 \\ 0 & \exp(h_{2,t}) & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & \dots & 0 & \exp(h_{n,t}) \end{bmatrix}$$

From the decomposition, matrix A incorporates the structural information on the contemporaneous correlations between elements of y_t , while H_t matrix stacks the vector of the (log-) volatility of shocks of the model. The model is closed defining processes for the vector of states $h_t = [h_{1,t}, h_{2,t}, ..., h_{n,t}]$. Following Mumtaz and Zanetti (2013)[23], assume that the dynamics of h_t is given by a set of independent AR(1) processes:

$$h_t = c + \theta h_{t-1} + \eta_t$$
$$\eta_t \sim N(0, Q) \quad E(\eta_t, u_t) = 0$$

In this case, θ and Q are defined as diagonal matrices, while c is a vector of size $n \times 1$. Note that the stochastic process estimated in the SVAR-SV is later used to calibrate the DSGE model. One interesting feature of using the stochastic volatility estimated from the SVAR-SV in the DSGE model is the possibility of avoiding endogeneity issues generated from partial information estimations. As an example, Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2011)[14] use a simple stochastic volatility model to compute the second moments of the risk premium for some Emerging Economies. Implicit in the estimation is the hypothesis that the risk premium is exogenous to movements in the Emerging Economy. In Fernández-Villaverde, Guerrón-Quintana Kuester and Rubio-Ramírez (2015)[12], the authors estimate fiscal policy rules with the output gap as a regressor, assuming that the second-order effects from a change in volatility in the business cycle are negligible for the exercise.

The SVAR-SV is estimated with four observables for the period between 2000M01 to 2016M12: the log of end-of-period nominal exchange rate (domestic currency per dollar), the log of industrial production index, the log of the consumer price index (IPCA), the nominal end of period Selic rate – which is the monetary policy instrument in Brazil. Sample from 1995M01 to 1999M12 was used to build the priors for matrices A, B_k and C_l , for all k and l. Despite the adoption of the inflation targeting regime in June 1999, data for that period was not incorporated in the main estimation because it was necessary to incorporate in the priors some information for the period under free-floating exchange rate regime. Data on economic activity and inflation has its seasonal component removed using ARIMA X-12 procedure.

2.1 Identification strategy

A critical step in defining the impact of volatility on economic aggregates is the identification of structural shocks. The use of recursive identification schemes, based on a Choleski decomposition of the covariance matrix, brings a difficult task of choosing the order of variables between the monetary policy instrument and the nominal exchange rate. On the one hand, it is difficult to support the idea that monetary policy authority does not react immediately to changes in exchange rates; on the other hand, it is also hard to justify, at monthly frequency, that currency markets will not react to unexpected changes in monetary policy.

Given the issues above, the identification scheme adopted is based on sign restrictions for matrices A and B, as in Uhlig (2005)[27]. Instead of reducing the problem, simply identifying monetary policy shocks, as in Mumtaz and Zanetti (2013)[23], this paper proceeds on also identifying a supply shock and a demand shock. There are two reasons for working with additional structural shocks in the model: first, identification based on sign restrictions may be sensitive to the number of structural shocks imposed on the model or to the number of variables whose sign are restricted during the procedure³; second, the identification of supply and demand shocks allows for direct comparison of the effects of monetary policy volatility with other conventional shocks in the literature.

Sign restrictions are imposed on impact and for five lags, implying restrictions for a total of two quarters. Restrictions could be reduced to only one quarter, but at the cost of impulse response function of demand shocks generating implausible short-run dynamics, with GDP expanding on impact and contracting in the next quarter. However, every other identified shock and the responses to volatility shocks remained the same as in the baseline estimation⁴. Adding additional time periods to the set of restriction does not influence results, while, at the same time, reduces the number of draws in the Gibbs sampling accepted as valid for simulation. Low acceptance rates compromise the estimation of second moments of the simulations. Table 1 summarizes the identification scheme, describing the sign of correlations derived from matrices A and B.

Table 1: Sign Identification

	Monetary Policy	Supply Shock	Demand Shock
Exc.Rate	(-)		
Output	(-)	(+)	(+)
Inflation	(-)	(-)	(+)
Selic Rate	(+)	(-)	(+)

A few comments are worth mentioning on the set of identification restrictions. First, there are no restrictions imposed on the correlation between nominal exchange rates and both supply and demand shocks. This option is mostly due to the nature of the propagation of these shocks in an open economy. Transitory and permanent productivity shocks generate different responses of real exchange rates in estimated DSGE models. By the same token, demand from an exogenous increase in the demand for domestic goods from the rest of the world generates different exchange rate responses when compared to shocks from government spending shocks.

Second, the use of sign restrictions eliminates the need of additional variables to handle problems like the so-called "price puzzle" in VAR models. For Emerging Economies, the inclusion of an additional measure of commodity prices or risk premium is necessary to eliminate the result that prices increase after a contractionary monetary policy shock⁵. Sign restrictions use a subset of the parameter distribution, generating moments consistent with economic theory.

 $^{^{3}}$ Canova and Paustian (2011)[8] recommend that, given the significant flexibility provided by identification from sign restrictions, researchers should not be very agnostic with respect to both the number of shocks imposed for identification and the number of variables restricted in each shock.

 $^{^{4}}$ Section 2.3 and appendix B present other results in terms of robustness of the baseline estimation.

 $^{^{5}}$ Minella (2003)[21], estimating a VAR model for Brazil, includes the EMBI spread to correct the "price puzzle" using the IPCA to measure inflation.

2.2 Model estimation: priors and Gibbs sampling

The SVAR-SV is estimated using Bayesian techniques, based on a Gibbs sampler. As mentioned before, in order to set priors, a training sample with information between 1995M01 and 1999M12 (60 observations) is used. Priors for matrices c, θ and Q assume a low-persistence and high-variance process for stochastic volatility shocks. Both priors, however, are set with a relatively flat weight around the mean⁶.

The Gibbs sampler is based on the following steps:

- 1. Given the information set y_t and draws of H_t and A, the Kalman Filter is used to estimate matrices B_k and C_l , for all k and l, based on the procedure described in Carter and Kohn (1994)[9].
- 2. Given draws for B_k and C_l , for all k and l, and H_t , draws for A are based on a Normal distribution, since the elements of the matrix can be derived from a GLS transformation of a linear system of equations to make the errors homoskedastic. These draws, however, are accepted only if parameters in A and B respect the sign restrictions. In the case of rejection, the sampler takes new draws for A until acceptance.
- 3. Given draws B_k and C_l , for all k and l, A and H_t , parameters for matrices c, θ and Q are obtained using standard results for linear regressions.
- 4. Finally, given draws for B_k and C_l , for all k and l, and A, c, θ and Q, the multivariate non-linear state-space representation uses a modified particle filter to obtain draws of H_t .

Related to the last step of the Gibbs sampler, instead of the usual procedure in Primiceri (2005)[25] and Mumtaz and Zanetti (2013)[23], based on a Metropolis draw for each point of H_t , the particle filter proposed in Andrieu, Doucet and Holenstein (2010)[3], also used in Mumtaz and Theodoridis (2016)[22], is adopted in order to provide a better mixing of states. One drawback of the procedure is related to the number of particles. With a long and stable dataset at quarterly frequency, Mumtaz and Theodoridis (2016)[22] were capable of getting proper draws of the state vector using a swarm of only 50 particles. When using monthly data from a highly volatile economy like Brazil, a larger swarm of particles is needed in order to avoid degeneracy of states. Indeed, the particle filter here is set with a swarm of 10,000 particles, resulting in a significant computational cost. Statistics on the performance of the particle filter show that the number is, indeed, appropriate for the purposes of this paper⁷.

Parameter moments are extracted after 25,000 replications of the Gibbs sampler, with the first 5,000 draws discarded. Tests on the remaining 20,000 draws provide support for the convergence of the procedure, despite being a slow moving process characterizing the chains. Appendix A provides more information on the convergence of the algorithm.

2.3 Empirical results: time-varying volatility of monetary policy

The model is estimated setting k = 2 and l = 2, implying two autoregressive components and two lags of volatility of shocks, plus the contemporaneous effect, affecting the level of observed

⁶Priors for θ are set at 0.75, with variance of 0.16. Priors for Q are based on an Inverse-Gamma distribution with 5 degrees of freedom and scale parameter set at one.

⁷Based on Liu and Chen (1995)[19], the average number of effective particles over the sample size is estimated around 300, while a negligible number of draws at the Gibbs sampler collapsed because of the particle filter.

variables. While the literature adopts a larger number of lags when working with monthly data, the sample available for both estimation and setting priors does not allow for significant variation when setting both k and l^8 . Lower values of k could not properly describe the dynamics of the economy, as it was clear observing impulse response functions and the simulated paths of volatility. Moving k and l for values above the benchmark did not significantly alter the simulated paths of time-varying volatility or the impulse response functions from volatility shocks. It did change, however, the sign of impulse response functions from demand and supply shocks. In part, this result is a consequence of poorly specified priors, and it can not be properly addressed scaling the prior variance for larger values. Due to the loss in estimation precision and the excessive volatility in impulse response functions for output and interest rates generated by the use of monthly data, the option for the model with k = 2 seemed more appropriate.

The Gibbs sampling algorithm had a quick convergence and seems to properly explore the whole parameter space. Appendix A explores several statistics showing the convergence of the algorithm, based on the autocorrelation function of draws for stochastic volatility and the CUMSUM statistic for all parameters. There are signs of independence of draws after 30 lags, meaning that, with a total of 20,000 draws, there is a significant number of independent simulations. Inefficiency factors, also presented in appendix A, also signal to a proper coverage region of the posterior density. Figure 1 shows the median (red line), mode (blue line) and the distribution of matrices C, θ and Q.



Figure 1: Histogram of estimated parameters: stochastic volatility

⁸Sample available for estimation has 204 observations. It is necessary to estimate 22 parameters just to compute the variance of the system (10 parameters in matrix A, plus 12 parameters for the stochastic process of h_t). Considering that each lag in the VAR adds 16 parameters, and also the need to properly characterize the impact of volatility in the mean ($4 \times (l+1)$ parameters), the relation between sample size and number of estimated parameters quickly becomes unfavorable for statistical inference.

Figure 2 shows the median conditional standard deviation of each shock with a band of 68%. It is worth noting the high volatility of CPI, exchange rate and interest rates at the beginning of the sample, characterized by the transition to a new monetary regime. The figure also shows two short-lived increases in volatility: during the 2002-03 confidence crisis associated with the presidential election, and during the 2008-09 crisis, mostly associated with economic activity. The end of the sample shows an increase in inflation and exchange rate volatility. Despite the significant decline in output gap observed between 2014 and 2016, volatility of output actually remained below its historical level.



Figure 2: Conditional standard deviation of time series

The simulated volatility of Selic rate allows for a structural interpretation of the non-systematic changes in monetary policy. Evaluating the lower-right panel of figure 2, notice that periods of high volatility of monetary policy shocks are not directly associated with movements of raising nominal interest rates. Besides the beginning of the sample, characterized by the transition to the inflation targeting regime and the 2002-03 crisis, the simulated volatility of monetary policy shows three other peaks: i) at the 2008-09 crisis; ii) during the monetary policy easing episode started in August 2011, when interest rates reached 7.25% – at the time, the historical minimum observed; and iii) in early 2015, after a period of constant nominal interest rates. It is worth noting that, in the last two episodes, interest rates were not increased, while interest rates showed a significant decline during the 2008-09 crisis, as a response to the external shock.

Figure 3 shows the median impulse response function to a change in the volatility of Selic rate and the 68% band from the posterior of draws. As shown in the figure, a monetary policy volatility shock results in an immediate increase of inflation in the short run, with significant persistence up to one year after the shock. Despite significant, the increase in inflation does not seem to be quantitatively relevant. Most of the effects, however, are felt in the long run: there is

a significant exchange rate depreciation combined with a negative output gap. Both movements remain significant five years after the initial shock, and their magnitudes are relevant, with a devaluation of almost 2 percentage points two years after the shock and a fall of 0.3p.p. in the output gap. Evidence on the endogenous response of interest rate is mixed, with the confidence band including zero in the whole simulation, but with a small bias towards higher rates at the longer horizons.



Figure 3: Impulse response function: volatility of monetary policy

Qualitative results from impulse response functions are in sharp contrast with those in Mumtaz and Zanetti (2013)[23] and Benigno, Benigno and Nisticò (2012)[4]. The sequence of events in Mumtaz and Zanetti (2013)[23] suggest that a monetary policy volatility shock is similar to a negative demand shock: an increase in volatility of interest rates induces households to pay debt by reducing current consumption. Lower aggregate demand results in lower inflation and interest rates. From the SVAR-SV estimated for Brazil, an increase in volatility generates an increase in inflation while keeping nominal interest rates almost constant. At the same time, nominal exchange rate devaluates and output falls. Changes in opposite directions between economic activity and prices suggest that the view of volatility shocks as a negative demand shock does not hold for Brazil.

Two significant robustness checks were performed. First, the model was estimated using a partial sample, starting in January 2005, in order to eliminate the high volatility period from the beginning of the inflation targeting regime and the 2002-03 crisis. Then, back with the whole sample, the model was augmented using the country risk premium, measured by JP Morgan's EMBI+, and the log of CRB commodity index. In both cases, it should be noted that the alternative models are much less precisely estimated, when compared to the baseline estimation, either because of the shorter sample or because of the larger number of parameters in the model. Figure 4 compares the histogram of the kernel from estimates of the stochastic volatility process.

The six-variable VAR shows a very similar histogram for matrices c and θ , but smaller standard deviation of volatility, when compared to the baseline estimation. The short sample VAR, as expected, displays smaller values for c and θ , compared to both the baseline and the six-variable VAR. However, as expected, interest rate volatility still shows significant persistence, with a mode of the kernel around 0.7.



Figure 4: Kernel of histogram – Baseline, short sample and 6-variable SVAR-SV

Figure 5 adds to the simulated stochastic volatility in figure 2 the median path estimated in both alternative models. Overall, both alternative models do not significantly diverge from the baseline estimation, especially the model with a short sample. Most of the differences from the baseline estimation appear during stress periods, notably during the 2003-04 period for inflation in the 6-variable SVAR-SV and the 2008-10 period for output in both alternative models. Even with the discrepancy, both alternative simulations still move close to the baseline estimation.

Finally, figure 6 compares the IRFs of a monetary policy volatility shock with those generated in alternative estimations. Despite some differences in short-run dynamics and the magnitude of shock propagation, general results remain for three of the four endogenous variables of the baseline estimation remains: nominal exchange rate devaluates, output gap falls and inflation remains positive at least for some time. The major difference compared to the baseline estimation refers to interest rates, with both alternative estimations showing lower responses. In the estimation with a short sample, it is possible that the monetary policy reaction function is different from the early part of the sample, since the simulated path of stochastic volatility is very similar to the baseline estimation. For the case of the six-variable SVAR-SV, it is possible that the baseline estimation is missing on dynamics during the 2003-04 crisis, when there are significant differences in the path



Figure 5: Stochastic Volatility – Baseline, short sample and 6-variable SVAR-SV

of stochastic volatility of inflation.

For further checks, the SVAR-SV was also estimated: (i) using a larger particle swarm (30,000 particles); (ii) using a centered prior for the persistence of volatility shocks (prior for θ set at 0.5); (iii) using the real effective exchange rate, instead of the nominal exchange rate as an observed variable; (iv) reducing the number of periods sign restrictions are active to only one quarter; (v) removing the sign restriction on exchange rates after a monetary policy shock; (vi) increasing the number of lags of the impact of volatility on aggregate levels (l = 5). Results confirm that simulated stochastic volatility seems to be properly estimated in the model, and its impact in the economy is very similar to a supply shock, with a decline in output combined with an increase in prices. In these alternative estimations, interest rates followed closely the dynamics of the baseline estimation. These alternative estimations also show that the stochastic process for volatility is very well defined in terms of level and persistence.⁹.

3 The model

Results from the empirical section show that an increase in monetary policy volatility has the following consequences: i) as in the literature, output decreases; ii) contrary to the literature, inflation rises, at least in the short run; iii) there is no significant reaction from monetary policy over time; iv) exchange rate depreciates, probably in line with the argument in Benigno, Benigno and Nisticò (2012)[4] that exchange rate movements are a function of the hedging properties of the

⁹Appendix B show some results of these tests.



Figure 6: IRF monetary policy volatility – Baseline, short sample and 6-variable SVAR-SV

currency. In order to develop some intuition from results (i) to (iii), the DSGE model presented in Mumtaz and Zanetti (2013)[23] is generalized with (external) habit persistence in order to obtain the structural and parametric conditions under which a positive inflation follows an increase in monetary policy volatility. The main objective of this section is to show that it is possible to generate positive inflation after a monetary policy volatility shock, even without an adjustment of relative prices from real exchange rate.

3.1 Description of the model

Mumtaz and Zanetti (2013)[23] use a simplified version of the model in Ireland (2004)[17] to describe a closed economy with sticky prices based on quadratic adjustment costs and a linear technology using only labor as an input. The model is augmented with external habit persistence for households. Habit persistence is one of the simplest forms in DSGE models to deal with the equity premium puzzle¹⁰, where returns on risky assets are not consistent with the low volatility of consumption. Augmenting the model with external habit persistence is based only on convenience, since it is possible that other mechanisms affecting the stochastic discount factor are capable of qualitatively generating the same results presented in this section. As it will become clear, the main causes for an inflation increase after a monetary policy volatility shock are the shape of the (expected) profit function and the design of monetary policy. Therefore, the stochastic discount factor plays a critical role in the analysis.

Households choose, in the intertemporal problem, consumption (C_t) , the amount of debt to

 $^{^{10}\}mathrm{See}$ Mehra and Prescott (1985)[20].

hold (D_t) and the labor supply (H_t) given current nominal wage (W_t) , prices (P_t) and interest rates (R_t) . Households maximize the expected discounted value of utility, having as source of income wages and profits from the firms, Π_t . A lump-sum transfer from the monetary authority, T_t , closes the budget constraint. The problem is given by:

$$\max_{C_t, D_t, H_t} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t)$$

s.t. : $D_t + W_t H_t + T_t + \Pi_t \ge P_t C_t + R_{t-1} D_{t-1}$

Define $\pi_t = P_t/P_{t-1}$ as inflation, $r_{t,t+1}$ as the stochastic discount factor between periods t and t+1. The solution to the intertemporal problem is based on the set of first-order conditions:

$$\lambda_t = U_c(C_t, H_t) \tag{1}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left(\frac{R_t}{\pi_{t+1}} \right) \tag{2}$$

$$r_{t,t+1} = 1/R_t$$
 (3)

$$\lambda_t \frac{W_t}{P_t} = -U_h(C_t, H_t) \tag{4}$$

Household's utility function in Mumtaz and Zanetti (2013)[23] is augmented with external habit persistence, where the household keeps its consumption profile over time as close as possible to the average of the economy. In a symmetric competitive equilibrium, every household makes the same decisions in each period. As a consequence, $C_t = \overline{C}_t$:

$$U(C_t, H_t) = ln(C_t - \alpha \overline{C}_{t-1}) - (1/\eta)H_t^{\eta}$$
$$\implies U_c(C_t, H_t) = 1/(C_t - \alpha \overline{C}_{t-1})$$
$$\implies U_h(C_t, H_t) = -H_t^{\eta-1}$$

A firm producing an intermediate good *i* decides the amount of labor to hire, $H_t(i)$ and the price, $P_t(i)$. The production function is linear in terms of labor:

$$Y_t(i) = H_t(i)$$

Every firm *i* sets $P_t(i)$ and $H_t(i)$ in each period in order to maximize the present discount value of profits $\Pi_t(i)$, subject to a linear production function and the demand function for product *i* based on a CES aggregator across all intermediate goods. Firms face a quadratic adjustment cost in order to change prices in each period. Given $r_{0,t}$ – the stochastic discount factor between time 0 and *t* – the intertemporal problem of the firm is given by:

$$\max_{H_t(i), P_t(i)} E_0 \sum_{t=0}^{\infty} r_{0,t} P_t \left[\frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} H_t(i) - \frac{\phi}{2} Y_t \left(\frac{P_t(i)}{P_{t-1}(i)\pi} - 1 \right)^2 \right]$$

s.t.:
$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\theta} Y_t$$
$$Y_t(i) = H_t(i)$$

Replacing both restrictions in the objective function and using the definition of the stochastic discount factor in equations 2 and 3, the first-order condition in terms of $P_t(i)$ results in:

$$\begin{split} (\theta-1)\frac{Y_t}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-\theta} &= \theta\frac{W_t}{P_t}\frac{Y_t}{P_t}\left(\frac{P_t(i)}{P_t}\right)^{-(1+\theta)} - \phi\left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right)\left(\frac{Y_t}{\pi P_{t-1}(i)}\right) \\ &+ \beta\phi E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{P_{t+1}(i)}{\pi P_t(i)} - 1\right)\left(\frac{P_{t+1}(i)}{\pi P_t(i)}\right)\left(\frac{Y_{t+1}}{P_t(i)}\right)\right] \end{split}$$

The symmetric equilibrium of the model is characterized by all firms equally setting the demand for labor, prices and, as a consequence, having the same production level and profits in each period $t: P_t(i) = P_t, H_t(i) = H_t$ and $Y_t(i) = Y_t$. As a consequence, the first order condition for $P_t(i)$ reduces to:

$$\theta - 1 = \theta \frac{W_t}{P_t} - \phi \left(\frac{\pi_t}{\pi} - 1\right) \left(\frac{\pi_t}{\pi}\right) + \beta \phi E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}}{\pi} - 1\right) \left(\frac{\pi_{t+1}}{\pi}\right) \left(\frac{Y_{t+1}}{Y_t}\right)\right]$$
(5)

The aggregate resource constraint of the economy considers the equivalence between production and consumption, with a wedge given by the adjustment cost imposed on prices:

$$Y_t = C_t + \frac{\phi}{2} Y_t \left(\frac{\pi_t}{\pi} - 1\right)^2 \tag{6}$$

Monetary policy is set by a standard Taylor rule augmented with heteroskedastic shocks:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left(\frac{\pi_t}{\pi}\right)^{\rho_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\rho_y} e^{\sigma_{r,t}\epsilon_{r,t}} \quad \epsilon_{r,t} \sim N(0,1)$$
(7)

$$\sigma_{r,t} = \rho_{\sigma} \sigma_{r,t-1} + q_{\sigma} \epsilon_{\sigma,t} \quad \epsilon_{\sigma,t} \sim N(0,1) \tag{8}$$

A critical feature of the Taylor rule estimated in Ireland (2004)[17] is the response of interest rates to economic activity. The author estimates the Taylor rule with a response to both output growth and the output gap. In Mumtaz and Zanetti (2013)[23], the authors use only the response of interest rates to output growth. It is true that the estimated coefficient for the response to the output gap in Ireland (2004)[17] is significantly smaller, but the qualitative response of inflation after a monetary policy volatility shock is directly affected by the functional form and parametrization of the Taylor rule, as seen in section 3.3.

Now, it is time to define the competitive equilibrium of the economy. Given initial values D_0 and exogenous shocks $\{\epsilon_{r,t}, \epsilon_{\sigma,t}\}_{t=0}^{\infty}$, a (symmetric) competitive equilibrium of the economy is a set of allocations $\{Y_t, C_t, H_t, D_t\}_{t=0}^{\infty}$, interest rates $\{R_t\}_{t=0}^{\infty}$ and prices $\{\pi_t, W_t/P_t\}_{t=0}^{\infty}$ such that: i) given prices $\{\pi_t, W_t/P_t\}_{t=0}^{\infty}$, households solve the utility maximization problem; ii) given prices $\{\pi_t, W_t/P_t\}_{t=0}^{\infty}$, firms solve the profit maximization problem; iii) all markets clear.

3.2 Calibration and model solution

Calibration of the model is presented in table 2. Parameters describing the economy follow Mumtaz and Zanetti (2013)[23] and Ireland (2004)[17]. Despite presenting results for Brazil, the objective of this section is to provide qualitative evidence on the transmission mechanism capable of generating such results, instead of matching the empirical moments found in the SVAR-SV. From the perspective of highlighting the transmission mechanism, it is more useful to reset the baseline model with parameters from the literature. Matching specific moments from the SVAR-SV to obtain quantitative evidence could prove a difficult task, given that the DSGE model describes a closed economy, without a proper description of the non-linearities observed in Emerging Economies.

Evaluating the parameters for the exercise, habit persistence was set at 0.76, based on the estimation in Altig, Christiano, Eichenbaum and Linde (2005)[2]. Parameters calibrating the steady state of monetary policy volatility and the standard deviation of the stochastic volatility of interest rates were kept the same as in Mumtaz and Zanetti (2013)[23]. Despite significant diferences found in the estimation of the stochastic process for volatility, the fact that the SVAR-SV is estimated using data at monthly frequency would require additional changes in calibration to change the model to another frequency. However, the persistence parameter of stochastic volatility of interest rates is incorporated from the estimation of the SVAR-SV, set to match quarterly data.

Parameter	Value	Meaning		
β	0.99	Discount factor		
θ	11	Elasticity of substitution across goods		
ϕ	1620.7	Degree of price rigidity		
η	16.2	Inverse of Frisch elasticity		
α	0.76	Habit persistence		
π	1.005	Trend (gross) inflation rate		
$ ho_r$	1.0	Taylor rule: interest rate smoothness		
$ ho_{\pi}$	0.3597	Taylor rule: response to inflation		
$ ho_y$	0.2536	Taylor rule: response to output growth		
Exogenous shocks				
q_{σ}	$0.6880^{0.5}$	Standard deviation of interest rate volatility		
$ ho_{\sigma}$	0.731	Persistence of interest rate volatility		

Table 2: Baseline Calibration

The presence of stochastic volatility in the model demands a non-linear approximation of policy functions, since linearized models do not capture effects from changes in second moments. Thus, irrespective of the joint dynamics between shocks in the level and in second moment of interest rates¹¹, the model is solved using a third-order approximation of the policy functions. According to Caldara et al (2012)[7], third-order solutions provide a good approximation of models with stochastic volatility without a major compromise of computational time.

Impulse response functions were computed using the approach in Koop, Pesaran and Potter (1996)[18], where the generalized impulse response functions are simulated considering different paths for the set of shocks of the economy. A total of 5,000 paths were simulated and figures report the median impulse response function from those draws.

3.3 Model simulation

The baseline model from Mumtaz and Zanetti (2013)[23] with the calibration provided in table 2 and no habit persistence ($\alpha = 0$) is not capable of generating a positive response of inflation after a monetary policy volatility shock. However, the smaller persistence of volatility shocks generates an increase of nominal interest rates, suggesting that monetary policy response is a function of the persistence of the shock. Persistent increases in monetary policy volatility result in lower interest rates over time, as a response to lower output in the economy.

 $^{^{11}}$ A second-order approximation of models with stochastic volatility generates impulse responses from volatility shocks as long as shocks to the level of state variables are not equal to zero, as noted in Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez (2010)[13].

Figure 7 shows the IRFs of a monetary policy volatility shock, starting with the baseline model of Ireland (2004)[17], followed by cumulative changes in model structure and parameterization:

- 1. Habit persistence: $\alpha = 0.76$;
- 2. Frisch elasticity of labor supply: $\eta = 1.0$;
- 3. Taylor rule with response to output gap, instead of growth rate of GDP.

On the new calibration of the Frisch elasticity, it is worth mentioning that the value used in Ireland (2004)[17] implies a very low elasticity for the labor supply. According to the author, MLE estimations of the model using US data favored a specification where preference shocks have a significant role describing consumption dynamics at the expense of a lower elasticity of the labor supply. Calibration here is in line with standard values found in the literature.

Figure 7 highlights the role of the additional features incorporated in the model described in Mumtaz and Zanetti (2013)[23]. While the inclusion of habit persistence alone does not significantly changes the response to a monetary policy volatility shock, the combination with a more elastic labor supply significantly amplifies the impact on output, inflation and the feedback to interest rates. Figure 8, for the sake of the argument, provides impulse response functions for a traditional monetary policy shock, showing the same amplification mechanism from the combination of habit persistence and an elastic labor supply. In both shocks, the volatility of output and inflation only returns for lower levels when the Taylor rule with response to the growth rate of output is replaced by the rule with response to the output gap¹².



Figure 7: IRF DSGE: Shock to volatility of monetary policy

There are two important issues to discuss from the impulse response functions. First, why is the combination of an elastic labor supply and habit persistence in consumption important for firms'

 $^{^{12}}$ For convenience, the elasticity of the interest rates to output growth and to output gap was kept the same.

price setting? Second, how important is the design of monetary policy? For the first question, the combination of elastic labor supply and habit persistence directly affects the asymmetry of the profit function of the firm in terms of the relative price of good i. In order to find positive inflation after the increase in volatility, it is necessary that the model generates enough incentives for firms to alter their price decision, and the shape of the profit function is critical¹³. In figure 9, the instant profit function and its first derivative¹⁴, both evaluated in the steady state of the model with the calibration in Ireland (2004)[17], are compared under alternative calibrations. While, individually, changing the degree of habit persistence and the Frisch elasticity of labor supply does not really alter the asymmetry of the profit function, the combination of both changes in parameters shifts up the curve and increases asymmetry around price dispersion. Both effects generate a significant incentive for firms to have an upward bias in price formation, avoiding product prices below the average price level of the economy. Note, again, that results in figure 9 do not depend on the degree of price rigidity in the economy, since, in a model with quadratic adjustment costs, the steady state of the model is not affected by nominal rigidities.



Figure 8: IRF DSGE: monetary policy shock

Relative to the second issue, on the design of monetary policy, notice that results presented in figure 9 are not associated with the structure of the Taylor rule. However, back to figures 7 and 8, it is clear that the volatility of output and inflation are closely related to the dynamics resulting from the design of monetary policy. The move from a Taylor rule reacting to the growth rate of output to a reaction to the output gap is enough to generate some mild inflation after the monetary policy volatility shock. Figure 10 explores some other typical rules observed in

¹³The same argument here is also developed in Fernández-Villaverde, Guerrón-Quintana, Kuester and Rubio-Ramírez (2015)[12] and Born and Pfeifer (2017)[6]

¹⁴The instant profit function, after combining the constraints in the problem of the firm and considering the equilibrium and steady state of the economy, is given by $\Pi = Y\left(\frac{P_{(i)}}{P}\right)^{(1-\theta)} - \frac{W}{P}Y\left(\frac{P_{(i)}}{P}\right)^{(-\theta)}$



Figure 9: Period profits, habit persistence and labor supply

the literature, showing the impulse response function after a volatility shock. The green line is the same function presented in figure 7, with the standard calibration and a response to the output gap. The alternative rules explore three cases: i) the same baseline rule but with a stronger response to output; ii) a standard Taylor rule with no interest rate inertia; iii) the standard Taylor rule with interest rate inertia and the same long-run coefficients of case (ii). The figure shows that inflation will be higher as the response of the Taylor rule to the output gap increases. Also, the presence of interest rate inertia does not seem to significantly influence the qualitative response of inflation after the volatility shock: the inflation bias of the firm is confirmed, as long as there is a response of interest rates from the output gap.

4 Conclusion

This paper presented an estimation of a SVAR-SV for Brazil, including the nominal exchange rate as an additional variable to properly characterize the transmission mechanisms observed in Emerging Economies. The SVAR-SV was expanded in order to measure the effects of changes in volatility of monetary policy in the economy, providing an interesting empirical benchmark for DSGE models with stochastic volatility. In the case of Brazil, empirical findings did not match those observed in the US, especially in terms of inflation and interest rates. The DSGE model calibrated in this paper tried to clarify the sources of these differences, instead of providing a quantitative replication of the SVAR-SV.

Despite taking a first step in explaining the empirical findings, there is significant room to improve the analysis presented above. Notably, it is important to understand, in a formal frame-



Figure 10: IRF DSGE: volatility of monetary policy and Taylor rule

work, the transmission mechanism that generates exchange rate devaluations as a response of an increase in monetary policy volatility. It is also interesting to investigate in a more detailed model the quantitative implications of monetary policy volatility. The model presented here generated only an approximation of results in the empirical section. Further evaluating the relevance of the role of output stabilization in the Taylor rule as the source of this type of fluctuations is very important both from a theoretical and from a practical perspective for economic policy.

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Appendices

A Convergence of Gibbs sampler

The plot of autocorrelation functions show a fast decay rate, with negligible autocorrelation from draws of the Gibbs sampler after 30 to 40 iterations of the algorithm. Figure 11 shows the pattern of the function for stochastic volatility parameters. It is interesting to note the high first-order autocorrelation of draws, the significant decline in the tenth lag, and the relative stability of the function after 30 lags



Figure 11: Autocorrelation function: θ , C and Q

Indeed, in figure 12, the recursive values for the mean and standard deviation of parameters in the stochastic volatility functions become stable around the middle of the remaining 20,000 draws of the Gibbs sampler, with small fluctuations in the later draws, mainly in terms of second moments of the simulations.

However, despite relative stability after 10,000 remaining draws, a smaller number of simulations would not be ideal. Figures 13 to 15 show the CUMSUM statistic for all parameters of the model. It is possible to observe that the chains converge slowly, resulting in a significant amount of simulations in order to properly cover the whole space of parameter distribution.

Finally, inefficiency factors were computed using 5,000 lags of the autocorrelation function. Factors are in the range of 0.91 to 7.85, with a mean of 2.36. It implies that it is necessary to sample around 785 observations to obtain a precision equivalent to 100 draws from a theoretical iid sampler. Figure 16 show the inefficiency factors for all parameters of the SVAR-SV.



Figure 12: Recursive mean and standard deviation of stochastic volatility



Figure 13: CUMSUM: stochastic volatility



Figure 14: CUMSUM: covariance matrix A



Figure 15: CUMSUM: matrix B



Figure 16: Inefficiency factors

B Robustness Checks

This appendix compares results of the baseline model with six alternative formulations, namely: (i) larger particle swarm (30,000 particles); (ii) mean of prior for θ at 0.5; (iii) model with real effective exchange rate; (iv) sign restrictions during one quarter; (v) no sign restriction on exchange rates; (vi) more lags of volatility on levels (l = 5). Figures compare the histogram of estimated parameters, the historical path of simulated volatility and the impulse response functions to a shock in monetary policy volatility. More results are available upon request.



Figure 17: Histogram: θ , C and Q



Figure 18: Stochastic Volatility



Figure 19: Impulse response function: volatility of monetary policy

C Summary of the theoretical model

The model in is fully described by the following sets of variables:

- Prices: π_t , W_t/P_t , $r_{t,t+1}$
- Interest rates: R_t
- Allocations: C_t, Y_t, H_t, λ_t
- Shocks: $\sigma_{r,t}, \epsilon_{r,t}, \epsilon_{\sigma,t}$

Equations describing the model are given by:

$$\begin{split} \lambda_t &= 1/(C_t - \alpha C_{t-1}) \\ \lambda_t &= \beta E_t \lambda_{t+1} \left(\frac{R_t}{\pi_{t+1}}\right) r_{t,t+1} = 1/R_t \\ \lambda_t \frac{W_t}{P_t} &= H_t^{\eta-1} \\ Y_t &= H_t \\ \theta - 1 &= \theta \frac{W_t}{P_t} - \phi \left(\frac{\pi_t}{\pi} - 1\right) \left(\frac{\pi_t}{\pi}\right) + \beta \phi E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_{t+1}}{\pi} - 1\right) \left(\frac{\pi_{t+1}}{\pi}\right) \left(\frac{Y_{t+1}}{Y_t}\right)\right] \\ Y_t &= C_t + \frac{\phi}{2} Y_t \left(\frac{\pi_t}{\pi} - 1\right)^2 \\ \frac{R_t}{R} &= \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left(\frac{\pi_t}{\pi}\right)^{\rho_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\rho_y} e^{\sigma_{r,t}} e^{\epsilon_{r,t}} \\ \sigma_{r,t} &= \rho_\sigma \sigma_{r,t-1} + q_\sigma \epsilon_{\sigma,t} \\ \epsilon_{r,t} \sim N(0,1) \\ \epsilon_{\sigma,t} \sim N(0,1) \end{split}$$

The deterministic steady state, given parameters set in calibration, is described by:

$$\begin{aligned} \pi &= 1.005 \\ R &= \pi/\beta \\ r &= 1/R \\ \frac{W}{P} &= \frac{\theta-1}{\theta(1-\alpha)} \\ H &= \left(\frac{\theta-1}{\theta(1-\alpha)}\right)^{1/\eta} \\ Y &= \left(\frac{\theta-1}{\theta(1-\alpha)}\right)^{1/\eta} \\ C &= \left(\frac{\theta-1}{\theta(1-\alpha)}\right)^{1/\eta} \\ \lambda &= \left(\frac{\theta(1-\alpha)}{\theta-1}\right)^{1/\eta} \end{aligned}$$