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Exchange Rate Pass-Through in Brazil: a Markov switching estimation for the inflation targeting period (2000-2015)

Fabrizio Almeida Marodin and Marcelo Savino Portugal

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Sumário não técnico

O estudo da relação entre taxa de câmbio e inflação é tema recorrente na análise macroeconômica. Desvalorizações cambiais, quando a moeda externa fica mais cara em Reais, tendem a produzir efeitos de alta na inflação local. A principal causa é o aumento direto no preço dos insumos importados, e suas consequencias na cadeia produtiva e preços ao consumidor. A literatura reconhece que grandes desvalorizações produzem efeitos proporcionalmente maiores na alta da inflação, do que desvalorizações moderadas ou mesmo valorizações. Este fenômeno é chamado de mecanismo ou efeito de repasse não-linear.

A existência e a quantificação do efeito não-linear do câmbio na inflação é de suma importância para que os agentes econômicos, firmas, governos e a autoridade monetária realizem uma boa previsão de inflação durante períodos em que a economia experimenta choques cambiais. Por exemplo, se a previsão for realizada tomando-se o comportamento histórico (e corrente) de modo linear, estará sendo subestimado o efeito que uma desvalorização da moeda pode causar na inflação.

Este artigo revisita a relação entre a taxa de câmbio e a inflação brasileira durante o regime de metas utilizando metodologia inovadora.

As evidências empíricas encontradas reforçam a existência do mecanismo não-linear. Argumenta-se, no entanto, que este processo é disparado não somente por causa da magnitude da desvalorização, mas ocorre em períodos de alta incerteza e instabilidade econômica, em função da diferença nas expectativas dos agentes.

Ainda, o artigo permite identificar diferentes períodos de comportamento cíclico da relação câmbio-inflação, denominando-os de "ciclo normal" (2003-2014) e "ciclo crise" (2000-2003 e 2015). Espera-se que o artigo traga contribuições metodológicas para a evolução dos atuais modelos de previsão de inflação.

Non-technical Summary

The relationship between the exchange rate and inflation is a recurrent topic on macroeconomic analysis. Devaluations, that is when the external currency becomes more expensive in comparison to local currency, tend to produce higher local inflation. The main cause is the direct increase on import prices which consequently propagates to the production chain and consumer prices. The literature recognizes that large devaluations lead to proportionally higher increase on inflation than moderate devaluations or even valuations. This phenomena is denominated a non-linear "pass-through" mechanism.

The measurement of the non-linear effect of exchange rate on inflation is of great importance for a reliable forecasts of inflation for economic agents, such as firms, government and the monetary authority, during periods of shocks to the exchange rate. For instance, if the base forecast is make assuming a linear historical (and current) behaviour, the agent might be underestimating the effect that a devaluation of local currency can cause on inflation.

This paper explores the relationship between Brazilian exchange rate and inflation behaviour during the inflation targeting period using a novel methodology.

The empirical findings confirm the existence of a non-linear mechanism. We interpret, however, that the process is triggered not only as a function of the devaluation size, but occurs in periods of high economic uncertainty and instability due to differences in agents expectations.

Besides, the research allow us to identify cyclical periods of behaviour of the exchange rate-inflation relationship. We call them "normal cycle" (2003-2014) and "crisis cycle" (2000-2003 and 2015). The paper hopefully brings methodological contributions to current inflation forecasting models.

Exchange Rate Pass-Through in Brazil: a Markov switching estimation for the inflation targeting period (2000-2015)

Fabrizio Almeida Marodin[*]

Marcelo Savino Portugal[**]

Abstract

This paper investigates the nonlinearity of exchange rate pass-through in the Brazilian economy during the floating exchange rate period (2000-2015) using a Markov-switching semi-structural new Keynesian model. We apply the methods proposed by Baele et al. (2015) and a basic new Keynesian model, with the addition of new elements to the AS curve and a new equation for the exchange rate dynamics. We find evidence of two distinct regimes for the exchange rate pass-through and for the volatility of shocks to inflation. Under the so-called "normal" regime, the long-run pass-through to consumer prices inflation is estimated at near zero value, only 0.00057 percentage point given a 1% exchange rate shock. Comparatively, the expected pass-through under a "crisis" regime is of 0.1035 percentage point to inflation, for the same exchange rate shock. The Markov-switching (MS) model outperforms the fixed parameters model according to several comparison criteria. The results allowed us to identify the occurrence of three distinct cycles for the exchange rate pass-through during the inflation targeting period in Brazil.

Keywords: Exchange-rate pass-through, New Keynesian model, Regime Switching, Markov Chain
JEL codes: E31, F31, C3

[*] Graduate Program in Economics (PPGE), Universidade Federal do Rio Grande do Sul, Porto Alegre/RS, Brazil. Central Bank of Brazil, Porto Alegre/RS. Email: fabrizio.marodin@gmail.com
[**] Graduate Program in Economics (PPGE), Universidade Federal do Rio Grande do Sul, Porto Alegre/RS, Brazil.

1. Introduction

This paper assesses the exchange rate pass-through to inflation in the Brazilian economy during the floating exchange rate period using a Markov-switching (MS) new Keynesian model. Our aim is to check for nonlinear behaviour of the exchange rate pass-through, given the possibility it could further amplify inflation during external sector or currency crisis.

The research is aligned with the recent literature in structural parameter drifting, or nonlinear behavior of structural parameters. According to Hamilton (2014), nonlinear mechanisms that trigger macroeconomic regime shifts are some of the most noteworthy contemporary issues in macroeconomics. Current economies are subject to remarkable changes, recurrent crises, recessions, and financial stress. These events produce "dramatic breaks" in macroeconomic time series and, consequently, lead agents to create expectations under different regimes.

Our motivation derives from the risk of underestimating the effect of an exchange rate shock to inflation, specially under large devaluation events. The international literature has found some meaningful evidence of nonlinear exchange rate pass-through, as we will further explore. If that has been the case for the Brazilian economy, even after the adoption of the inflation targeting regime, the researchers or policymakers may be incurring in a greater than expected forecast error. Indeed, we sustain that the forecast error would be even greater during external sector crisis, where policy decisions are of the highest importance.

The traditional approach to analyze changes in structural parameters is based on Hamilton's (1989) business cycle model. In this method, some parameters chosen in each regression vary freely according to Markov processes. Sims & Zha (2006), for instance, conducted a seminal work on regime shifts in the U.S. monetary policy by proposing and estimating a structural MS-VAR model. However, the theoretical and empirical advances of new Keynesian models, as well as their broad use in the analysis of economic policies, naturally arouse interest in expanding the scope of these models so as to include regime-switching mechanisms.

There is a well developed international literature on Markov switching new Keynesian models. Justiniano & Primicieri (2008), for example, assess regime switching in the volatility of shocks whereas Fernández-Villaverde, Guerrón-Quintana & Rubio-

Ramírez (2010), Bianchi (2013), Baele et al. (2015), and Iboshi (2016) focus on changes in the Taylor rule parameters and their consequences for macroeconomic equilibrium. Authors are usually interested in identifying periods during which the U.S. monetary policy has an "active" vs. a "passive" behavior towards inflation.

Regarding solution methods for Markov-switching rational expectations (RE) models, major contributions are given by Farmer, Waggoner & Zha (2009, 2011). These authors develop a set of necessary and sufficient conditions for equilibria to be determinate, as well as an algorithm to check these conditions in practice. Liu & Mumtaz (2010), and later Choi & Hur, (2015) rely on the solution proposed by Farmer, Waggoner & Zha (2011) and utilize Bayesian estimation methods in empirical studies on regime switching in monetary policy rules for the UK and South Korea, respectively. More recently, Foerster et al. (2014) have proposed a new estimation method, which uses perturbations to approximate solutions to nonlinearized MS rational expectation models, for which Maih (2015) presents a practical implementation.

In Brazil, nearly all the estimated new Keynesian rational expectation models have constant parameters, as in Silveira (2008), Furlani, Portugal & Laurini (2010), De Castro et al. (2015), and Palma & Portugal (2014). An exception is Gonçalves, Portugal & Arágon (2016), who use the open-economy model proposed by Justiniano & Preston (2010), the solution methods of Farmer, Waggoner & Zha (2011), and a Bayesian estimation method similar to that of Liu & Mumtaz (2010). The authors find a superior fit of a switching RE model with changes in the Taylor rule parameters and in volatility of shocks, comparatively to fixed parameters models.

On the other hand, there are several studies that investigate changes in structural parameters of the Brazilian economy by applying conventional regime switching models such as Hamilton (1989). Fasolo & Portugal (2004), for instance, find changes in the Phillips curve parameters whereas Vieira & Pereira (2013) describe differences in the business cycle dynamics. More recently, Rodrigues & Mori (2015) have identified different monetary policy regimes using a model with changes in the Taylor rule parameters, and Oliveira & Feijó (2015) have investigated the nonlinearity between unemployment and inflation using a Phillips curve with Markov switching.

We therefore assume that the paucity of empirical studies on MS new Keynesian models in Brazil is due mainly to their recent development rather than to the belief that our economy is subject to fixed structural parameters. Hence, we understand that our

investigation into regime switching in the exchange rate pass-through parameter should be conducted by employing this class of models in order to contribute to their dissemination and improvement.

The variable, or nonlinear, behavior of the exchange rate pass-through is theoretically endorsed by arguments from Dixit (1989) and Taylor (2000). Dixit (1989) attributes the differences in pass-through to firms decision-making uncertainty. According to him, the more uncertain the steady state of the exchange rate, the greater the incentive for firms to adopt a waiting strategy before making the decision to adjust prices, given adjustment (menu) and reputation costs, if the firm needs to reverse its decision. Thus, if the exchange rate shock is seen as permanent, agents would respond with a higher pass-through to prices, compared to cases of temporary shocks. Dixit's (1989) assumption is part of an approach that considers the pass-through to be incomplete as a result of firms' behavior and of market prices denominated in local currency, which Razafindrabe (2016) calls "positive approach." Larue, Gervais & Rancourt (2010), for instance, provide microeconomic evidence in favor of this assumption, relating menu cost to different levels of incomplete pass-through.

Taylor (2000), however, sustain that differences in the level of pass-through are related to price rigidity. In periods of higher inflation, firms transfer their costs more frequently, including costs associated with imported inputs, as overall price rigidity is smaller. Razafindrabe (2016) clarifies that nominal price rigidity of imported goods is the main explanation to incomplete pass-through under the so-called normative approach. The author introduces a RE model in which the problem with optimal price adjustment by importing firms, through a Calvo mechanism, causes a deviation from the law of one price and, therefore, incomplete exchange rate pass-through to inflation. Figueiredo & Gouvea (2011) support this viewpoint by giving empirical evidence of heterogeneity in the pass-through between disaggregated prices negatively linked to the level of price rigidity. Also, the model proposed by Choudhri & Hakura (2015) is based on price rigidity to explain the differences in the level of pass-through between the prices of imported and exported goods. Besides providing a theoretically consistent explanation, the authors manage to reproduce some characteristics of time series observed for several countries.

From an empirical standpoint, variable or nonlinear exchange rate pass-through has been investigated by the literature, but, in the case of Brazil, not within the new Keynesian framework. For example, Goldfajn & Werlang (2000) confirm that the

intensity of pass-through in the cases of exchange rate depreciation is not fixed, but depends upon a series of economic state variables. The key factors would be the cyclical component of output, the initial overvaluation of the real exchange rate, the initial inflation rate, and the level of economic openness. In the Brazilian economy, Albuquerque & Portugal (2005) asseverate that the intensity of exchange rate pass-through varies over time and depends on macroeconomic factors, which is also supported by Tombini & Alves (2006). Minella et al. (2003) and Kohlscheen (2010) affirm that exchange rate volatility is associated with variance of inflation and with higher pass-through. Moreover, the nonlinear or asymmetric behavior of exchange rate pass-through is verified in the empirical studies undertaken by Correa & Minella (2006), Nogueira Jr (2010), and Pimentel, Modenesi & Luporini (2015).

In other countries, Holmes (2009) and Khemiri & Ali (2012) assess regime switching in exchange rate pass-through by means of regressions based on the Phillips curve for Tunisia and New Zealand, respectively. Donayre & Panovska (2016) gather strong evidence of nonlinear behavior between the pass-through and economic activity for Canada and Mexico in a Bayesian threshold VAR model. In particular, the authors find a higher pass-through in expansionary periods, corroborating again Goldfajn & Werlang (2000). The influence of the macroeconomic environment and of inflation stability on the observation of smaller pass-throughs is also advocated by Winkelried (2014) in an empirical study for Peru.

In this paper, our goal is to estimate a basic semi-structural new Keynesian model subject to regime switching in the exchange rate pass-through parameter and in the volatility of shocks to inflation by applying the methods developed by Baele et al. (2015). One of the peculiarities of this method is the use of survey data on market expectations, making the estimation of the model easier. At the same time, Baele et al. (2015) suggest Cho's (2014) recursive solution method for regime switching rational expectations models, which circumvents some problems of convergence observed in Farmer, Waggoner & Zha (2011). Our study differs from that of Baele et al. (2015), who investigate regime switching in the monetary policy rule, as our focus lies in the exchange rate pass-through. To achieve that, we expand the original model by adding new elements to the AS curve and a new equation for exchange rate dynamics.

The MS model estimation allows us to identify two possible regimes for the exchange rate pass-through and the volatility of shocks to inflation. During a "normal"

cycle, the expected long-run pass-through is very close to zero, only 0.00057 percentage point, to consumer prices inflation, given a 1% exchange rate shock. Comparatively, the expected effect during a "crisis" cycle is much higher, of 0.1035 percentage point to inflation, given the same exchange rate shock. In addition, we verified that the volatility of shocks to inflation is larger during the "crisis" period. The MS model outperforms the linear model according to several comparison criteria. We therefore understand that the results are useful to enrich models of inflation forecasting and economic policy analysis.

The paper is organized into five sections, apart from this introduction. Section 2 describes the basic new Keynesian model and its extensions, introduce regime switching, assess the equilibrium conditions and presents our identification strategy. Section 3 presents the data and the estimation method. Section 4 describes the results and their implications. Finally, Section 5 makes the concluding remarks.

2. The Model

This section describes the basic macroeconomic model, with the introduction of exogenous exchange rate shocks. We expand the model by adding a regime-switching mechanism. Then, we assess the rational expectations equilibrium and finally describe the strategy for including survey expectations.

2.1 The new Keynesian model

Let us first consider the following macroeconomic new Keynesian structural model with three variables and three equations, which is a benchmark for research in this area and was used by Baele et al. (2015).

$$\pi_t = \delta E_t \pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda y_t + \epsilon_{\pi,t} \qquad \epsilon_{\pi,t} \sim N(0, \sigma_{AS}^2) \qquad (1a)$$

$$y_t = \mu E_t y_{t+1} + (1-\mu)y_{t-1} - \phi(i_t - E_t \pi_{t+1}) + \epsilon_{y,t} \qquad \epsilon_{y,t} \sim N(0, \sigma_{IS}^2) \qquad (2)$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\beta E_t \pi_{t+1} + \gamma y_t] + \epsilon_{i,t} \qquad \epsilon_{i,t} \sim N(0, \sigma_{MP}^2) \qquad (3)$$

We follow the notation of Baele et al. (2015) where π_t is the inflation rate, y_t is the output gap and i_t is the nominal interest rate. The operator E_t refers to conditional expectations. Each equation is amenable to unexpected shocks, respectively: $\epsilon_{\pi,t}$ is the aggregate supply shock (AS shock); $\epsilon_{y,t}$ is the aggregate demand shock (IS shock); $\epsilon_{i,t}$ is the monetary policy shock (MP shock).

As to the structural parameters of the model, δ and μ stand for the forward-looking behavior of firms (AS curve) and consumers (IS curve), respectively. The model allows for endogenous persistence if these parameters are different from 1, with weight attached to the past values of each variable. Parameter λ is the response of inflation to the output gap whereas ϕ is the response of output to the real interest rate. The monetary authority's reaction function is a Taylor rule with smoothing parameter ρ_i, which reacts to inflation expectation with response β and to deviations in output gap with parameter γ. It is assumed that the monetary policy should not react to temporary shocks, which affect only the current inflation rate without affecting its future path.

The equations presented in this simple rational expectations (RE) model are derived from the first-order log-linearized conditions of the optimization problems of each representative agent: consumers, firms, and monetary authority. A thorough description of the microfoundations of the basic new Keynesian model can be seen in Galì (2008) or Romer (2011). The model describes the dynamics of endogenous macroeconomic variables, in which current decisions are a function of future expectations for these variables and their past values. As a closed economy model, it does not deal with exchange rate pass-through. We found two alternatives to circumvent this problem.

The first option would be to add the full dynamics of a small open economy to the model, as proposed by Adolfson et al. (2007), to estimate the incomplete pass-through. In this case, the model is expanded to include nominal rigidity in the import and export sectors, the Phillips curve is decomposed into domestic price and imported price curves, and capital, investment, government sector, in addition to prices, output, and external interest rate, are included. We would naturally also add the regime-switching parameters, and the transition probabilities. This results in a complex model with dozens of parameters to be estimated or calibrated.

Since our focus is on the effect of pass-through during the floating exchange rate period in Brazil, we have a relatively short time series and, therefore, we prefer to opt for

a less complex modeling strategy. We decided to model exchange rate shock as an observable "demand shock" to non-produced inputs. So, we were inspired by Blanchard & Galí (2007), who included a demand shock in the Phillips curve – called Δm.

We are aware that we are choosing to prioritize the direct effects on inflation of the adjustment of input prices, as a consequence of exchange rate fluctuations. Therefore, we are disregarding the indirect effects on aggregate demand, such as the change in the relative prices of domestic and imported goods, the effect on the domestic interest rate, and the possibility of a wealth effect. Our decision is justifiable for at least two reasons. First, exchange rate depreciations in relatively closed economies, just as Brazil, tend to cause a relatively smaller change in spending on domestic and imported goods. This argument is advocated by Albuquerque & Portugal (2005) and also discussed in the empirical findings of Goldfajn & Werlang (2000). Second, the estimation of a model with multiple regimes and small observed time series would be hindered if the number of parameters increased considerably. In what follows, we then describe the argument used by Blanchard & Galí (2007) to include a demand shock in the new Keynesian Phillips curve.

Blanchard & Galí (2007) demonstrate that the optimizing behavior of consumers and firms in an environment with real wage rigidity and price rigidity similar to that of the Calvo model implies the following equilibrium relationship between inflation and output gap. We keep the original notation used by the authors, which differs from that shown in our equations (1)-(3).

$$\pi_t = \delta E_t \pi_{t+1} + \Phi_1 x_{1t} - \Phi_2[\Delta m_t + (1 + \phi)^{-1}\Delta\xi]$$

In this equation, x_1 is a linear combination between output gap - relevant for the current welfare - and its lagged terms. The demand shock is represented by Δm while $\Delta\xi$ represents a preference shock. Parameter ϕ, in the authors' notation, stands for disutility of labor and is included in the equation for the marginal rate of substitution between labor and leisure. The operator Φ_2 is a nonlinear combination of structural parameters and the lag operator. The economic interpretation of this relationship is that inflation depends on its future expectation, on a combination between output gap and its lags, and on a combination of demand shocks and preference shocks and their own lags. The difference between the Phillips curve obtained by Blanchard & Galí (2007) and our equation (1) lies,

therefore, in the terms on the right-hand side, chiefly Δm and $\Delta \xi$. As we disregard preference shocks, the next step is to alter equation (1) to include Δm.

$$\pi_t = \delta E_t \pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda y_t - \Phi_2 \Delta m_t + \epsilon_{\pi,t} \qquad \epsilon_{\pi,t} \sim N(0, \sigma_{AS}^2)$$

Our pass-through modeling strategy, as described, takes into account the direct effect of the exchange rate shock on the price of imported goods, which leads us to replace the positive demand shock Δm with a negative shock at the nominal price of the foreign currency $-\Delta e$. Note that variable Δe corresponds, here, to exchange rate fluctuation or first difference between the price of foreign currency e in a given period.

$$\pi_t = \delta E_t \pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda y_t + \Phi_2 \Delta e_t + \epsilon_{\pi,t} \qquad \epsilon_{\pi,t} \sim N(0, \sigma_{AS}^2)$$

The last step consists in describing the operator Φ_2 and defining the scope and method for its measurement in our model. Blanchard & Galí (2007) set Φ_2 as a nonlinear combination of structural parameters associated with nominal price rigidity λ, with real wage rigidity γ, with the productivity of non-produced inputs α, combined with the lag operator of the variable of interest Δe. Note that we are using the original notation, which differs from that in our equations (1)-(3).

$$\Phi_2 = \frac{\lambda \gamma \alpha}{1 - \gamma L}$$

In order to verify the nonlinearity of exchange rate pass-through, we chose a simple empirical strategy, i.e., to estimate the aggregate parameter that represents the effect of the exchange rate shock on inflation, Φ_2, for several lags. Again, we put aside some details of the model for the sake of simplicity. In practice, we do not identify which structural parameter is subject to regime switching, but we observe its aggregate set. Theoretically, regime switching is expected to occur due to the variation in nominal price rigidity λ.

Hence, the Phillips curve is expressed in equation (1) below, taking into account, for instance, two lags that are relevant for the supply shock. The most appropriate number of lags to describe the dynamics of the variables will be checked empirically. In addition, it is assumed that the exchange rate shock follows a first-order autoregressive process, which is described by equation (4), where $\epsilon_{e,t}$ is an identically distributed exogenous

exchange rate shock. Together with equations (2) and (3), these make up the four equations of our empirical model.

$$\pi_t = \delta E_t \pi_{t+1} + (1-\delta)\pi_{t-1} + \lambda y_t + \kappa_0 \Delta e_t + \kappa_1 \Delta e_{t-1} + \epsilon_{\pi,t} \quad \epsilon_{\pi,t} \sim N(0, \sigma_{AS}^2) \quad (1)$$

$$y_t = \mu E_t y_{t+1} + (1-\mu)y_{t-1} - \phi(i_t - E_t \pi_{t+1}) + \epsilon_{y,t} \qquad \epsilon_{y,t} \sim N(0, \sigma_{IS}^2) \quad (2)$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)[\beta E_t \pi_{t+1} + \gamma y_t] + \epsilon_{i,t} \qquad \epsilon_{i,t} \sim N(0, \sigma_{MP}^2) \quad (3)$$

$$\Delta e_t = \rho_e \Delta e_t + \epsilon_{e,t} \qquad \epsilon_{e,t} \sim N(0, \sigma_e^2) \quad (4)$$

Our model differs from that of Baele et al. (2015) in equation (1) as we included the supply shock, and in equation (4) for the inclusion of the exchange rate path. Note that the Phillips curve represented by equation (1) is similar to the specifications used in previous studies on exchange rate pass-through in the Brazilian economy, such as Carneiro, Monteiro & Wu (2004), Correa & Minella (2006), Tombini & Alves (2006) and Nogueira Jr (2010). Our approach, however, differs as it considers structural and equilibrium constraints derived from model-consistent expectations[1].

In matrix notation, the model can be written as:

$$AX_t = BE_t X_{t+1} + DX_{t-1} + \epsilon_t \qquad \epsilon_t \sim N(0, \Sigma) \qquad (6)$$

Here, X_t is the vector of macroeconomic variables and ϵ_t is the vector of structural shocks. Matrices A, B, D contain the values of the structural parameters and Σ represents the diagonal matrix with the variances of ϵ_t. In our case, we have $X_t = [\pi_t \; y_t \; i_t \; \Delta e_t]'$.

We follow Baele et al. (2015) considering that the rational expectations equilibrium (REE) of the model is the one that depends solely on minimal state variables, also known as fundamental solution. The solution to model (6) follows the VAR(1) law

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[1] Some small open economy models include the exchange rate variation in the Taylor rule, as for example Furlani, Portugal & Laurini (2010). Other single equation estimations of the Central Bank of Brazil reaction function, such as Rodrigues & Mori (2015), even find statistical significance for the reaction to the exchange rate, during some periods of time. We have tried this type of specification for equation (3). However, our solution method was unable to find a stable solution to the rational expectations model in this case.

of motion, where matrices Ω and Γ are highly nonlinear functions of the structural parameters:

$$X_t = \Omega X_{t-1} + \Gamma \epsilon_t \qquad\qquad \epsilon_t \sim N(0, \Sigma) \qquad (7)$$

Baele et al. (2015) underscore that a model written in this format can be solved by several methods, such as the one described by Sims (2002) or Cho & Moreno (2011). The inclusion of regime shifts in the model, however, requires a new characterization of the rational expectations equilibrium, which will be dealt with further ahead.

It is widely known that log-linearized rational expectations models fail to reproduce some empirical characteristics of macroeconomic series. The first problem is intrinsic on the linearization method itself. As explained by Fernández-Villaverde (2009), linearization derives from first-order terms of a Taylor expansion, and when solved by conventional perturbation methods, it gives us an approximate, simpler solution to the original model. Perturbation, calculated around the steady state, will not predict stronger shocks that pull the system away from this state. It is possible to obtain higher-order expansions, but these require more complex solution and estimation methods. Fernández-Villaverde (2009) discusses the use of the particle filter, a simulation algorithm based on the Monte Carlo method, which allows exploring the likelihood function of nonlinear models, even those with non-Gaussian shocks. Second, there is ample evidence of instability in structural parameters, both in developed economies and in Brazil; for example, the works by Sims & Zha (2006), Fernández-Villaverde, Guerrón-Quintana & Rubio-Ramírez (2010), and Bianchi (2013).

Our study follows Gonçalves, Portugal & Arágon (2016), Liu & Mumtaz (2010), and Bianchi (2013) by estimating a linearized Markov-switching new Keynesian model. Nevertheless, we use a different identification and estimation strategy, proposed by Baele et al. (2015), which employs survey-based expectations instead of estimating state-space models, in which the expectations are unobserved variables. Also, our aim is to assess changes in the exchange rate pass-through to inflation whereas most studies focus on investigating the monetary policy dynamics.

The use of survey-based expectations for the estimation of new Keynesian models is rather uncommon, even though it is relatively simple. Admittedly, market surveys may contain missing information bias or reflect the opportunistic behavior from agents.

Notwithstanding, for Baele et al. (2015), survey-based expectations represent different perceptions of economic agents based on a potentially richer set of information, and hence they could be useful to improve estimation. The authors mention the high predictive power of surveys conducted with professional forecasters. In Brazil, the inflation forecast exercise of Altug & Çakmakli (2016) do confirm the high predictive power of survey-based expectations. On the practical side, as will be seen further ahead, the calculation of the likelihood function and the identification of regime shifts in the MS model become a lot easier, since only the state variable and transition probabilities will be regarded as unobserved.

2.2 Introducing regime switching

Our aim is to allow for two possible regimes for both the exchange rate effect on inflation and the volatility of structural shocks on the aggregate supply curve. Thus, we define the discrete unobserved variable S_t, which takes on two possible values $S_t^\pi = [0, 1]$ and serves as an indicator of the state of the economy in period t. The variable S_t evolves according to a first-order Markov process, where $P[S_t = 0 | S_{t-1} = 0] = p_{00}$; $P[S_t = 1 | S_{t-1} = 0] = p_{10} = (1 - p_{00})$; $P[S_t = 1 | S_{t-1} = 1] = p_{11}$; $P[S_t = 0 | S_{t-1} = 1] = p_{01} = (1 - p_{11})$. The model is called fixed transition probabilities, as proposed by Hamilton (1989) and discussed by Kim & Nelson (1999).

The MS model is then defined as:

$$\pi_t = \delta E_t \pi_{t+1} + (1 - \delta)\pi_{t-1} + \lambda y_t + \kappa_{1S_t}\Delta e_{t-1} + \epsilon_{\pi,t} \qquad \epsilon_{\pi,t} \sim N(0, \sigma_{AS}^2(S_t^\pi)) \ (8)$$

$$y_t = \mu E_t y_{t+1} + (1 - \mu)y_{t-1} - \phi(i_t - E_t \pi_{t+1}) + \epsilon_{y,t} \qquad \epsilon_{y,t} \sim N(0, \sigma_{IS}^2) \qquad (9)$$

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)[\beta E_t \pi_{t+1} + \gamma y_t] + \epsilon_{i,t} \qquad \epsilon_{i,t} \sim N(0, \sigma_{MP}^2) \ (10)$$

$$\Delta e_t = \rho_e \Delta e_{t-1} + \epsilon_{e,t} \qquad \epsilon_{e,t} \sim N(0, \sigma_e^2) \ (11)$$

Note that the regime shift is considered only in the first equation (aggregate supply) in parameters κ_{1S_t}, representing the exchange rate pass-through, and

$Var\left(\epsilon_{\pi,t} | X_{t-1}, S_t^\pi\right) = \sigma_{AS}^2(S_t^\pi)$. These two parameters jointly depend on the state of the economy S_t^π. We assume that the exchange rate shock has an impact on inflation in the following period, according to the following arguments. Goldfajn & Werlang (2000) empirically demonstrate that the effect of the pass-through is relatively small within the first months after an exchange rate depreciation (or appreciation). Theoretically, it should be recalled that the agents need some time to adjust their optimal prices due to the presence of price rigidity in the form of menu and reputation costs. Furthermore, as pointed out by Dixit (1989), uncertainty over the steady state of the exchange rate incentivizes firms to adopt a waiting strategy.

We assume that regime $S_t^\pi = 0$ will have the smallest volatility in aggregate supply curve shocks: $\sigma_{AS}^2(S_t^\pi = 0) < \sigma_{AS}^2(S_t^\pi = 1)$. The model includes the possibility that regimes may occur recurrently through transition probabilities. There is no ex-ante restriction to a higher pass-through period occurring on states $S_t^\pi = 0$ or $S_t^\pi = 1$.

The representation of the model in matrix notation, with the introduction of dependent variables, is as follows:

$$AX_t = BE_t X_{t+1} + D(S_t)X_{t-1} + \epsilon_t \qquad \epsilon_t \sim N\left(0, \Sigma(S_t)\right) \qquad (12)$$

Where matrix $D(S_t)$ takes on a different value in each regime, and so do the variance-covariance matrices between structural shocks $\Sigma(S_t)$. Note that the regime shift could also occur in matrices A and B; however, this will not be necessary in our case. Baele et al. (2015), for instance, assume regime switching in Taylor rule parameters, which are represented in matrices A and B.

One of the advantages of the representation method adopted by Baele et al. (2015) lies in its simplicity. Liu & Mumtaz (2010) and Gonçalves, Portugal & Arágon (2016), for example, follow the state-space representation to solve the regime-switching rational expectations model by the extended state vector method proposed by Farmer, Waggoner & Zha (2011). This yields a state-space MS-VAR model, which uses the algorithm of Kim & Nelson (1999) for its estimation. Later on, we will describe how the strategy adopted by Baele et al. (2015) allows solving and assessing the rational expectations equilibrium, in addition to estimating its parameters in a simpler way.

2.3 Assessing the rational expectations equilibrium

We follow the method proposed by Baele et al. (2015), which is based on Farmer, Waggoner & Zha (2009, 2010, 2011) and Cho (2014), to characterize the stability and determinacy of the rational expectations equilibrium of the model. A linear rational expectations (RE) model, as the one shown in equation (6), is considered to be determinate if it has a single and stable (non-explosive) equilibrium, which takes the form of a fundamental rational expectations equilibrium (REE) denoted by equation (7). The concept of stability should be formally established and checked so that we can eliminate or disregard unstable solutions and identify fundamental solutions. So, we adopt the concept of mean-square stability of Farmer, Waggoner & Zha (2010), which requires that the first and second moments of X_t be finite.

Therefore, by following Farmer, Waggoner & Zha (2009, 2011), the general solution to our Markov-switching model, in equation (12), is expressed as the sum of a fundamental solution plus a non-fundamental (sunspot) component:

$$X_t = \Omega(S_t)X_{t-1} + \Gamma(S_t)\epsilon_t + u_t \qquad (13)$$

$$s.t. \ u_t = F(S_t)E_t u_{t+1} \qquad (14)$$

Note that the first two components of (13) represent the fundamental solution given by equation (12) and u_t is the non-fundamental (sunspot) component. The state variables are the vector of lagged endogenous variables X_{t-1}, the vector of exogenous variables ϵ_t, and the current set of regimes S_t. The two necessary conditions for the determinacy of the model are uniqueness of the stable fundamental solution and the non-existence of a stable sunspot component.

To check the determinacy of the model we use the generalized forward method for the linear rational expectations models proposed by Cho & Moreno (2011) and Cho (2014). The forward solution to models of this type is the single fundamental solution that satisfies the transversality condition, i.e., the condition that makes the expectations about the current value of future endogenous variables converges to zero. Consequently, the forward solution selects an economically reasonable fundamental equilibrium and calculates its numerical solution in the same step. Cho (2014) demonstrates that the

rationale behind the forward solution also applies to Markov-switching models, providing easily treatable formal conditions using the mean-square stability concept.

We first consider the conditions for determinacy of a linear rational expectations model with n dimensions without regime switching, as described by Baele et al. (2015). It is widely known that this model has *2n* generalized eigenvalues and that it will be determinate if there are exactly n stable roots. It is possible to demonstrate that the n roots of Ω in equation (13) and the reciprocals of the roots of F in equation (14) constitute the *2n* generalized eigenvalues. By using this observation, we can say that the model is determinate based on the following conditions:

Conditions 1 and 2: The rational expectations model is determined if there is an Ω and its associated F such that $r(\Omega) < 1$ and $r(F) \leq 1$, where $r(.)$ is the spectral radius – the maximum absolute value among the eigenvalues of the argument matrix.

The second condition has a straightforward intuitive interpretation, stemming from $u_t = F(S_t)E_t u_{t+1}$. If $r(F) \leq 1$, then there is no stable sunspot component u_t, as the expected sunspot is explosively related to the current sunspot, given that the inverse of F has unstable eigenvalues. This condition, along with the first one related to Ω, guarantees the existence of a single stable fundamental solution and the model is therefore determinate.

The extension of these conclusions to regime-switching models should take into account that there exist state transitions and, hence, different coefficient matrices. Cho (2014) presents conditions that are analogous to our conditions 1 and 2 for general regime-switching models. Let $\bar{D}_\Omega$ and D_F be the matrices weighted by transition probabilities between states $S_t = 0$ and $S_t = 1$:

$$\bar{D}_\Omega = \begin{bmatrix} p_{00}\Omega(0) \otimes \Omega(0) & p_{10}\Omega(0) \otimes \Omega(0) \\ p_{01}\Omega(1) \otimes \Omega(1) & p_{11}\Omega(1) \otimes \Omega(1) \end{bmatrix}, \quad D_F = \begin{bmatrix} p_{00}F(0) \otimes F(0) & p_{01}F(0) \otimes F(0) \\ p_{10}F(1) \otimes F(1) & p_{11}F(1) \otimes F(1) \end{bmatrix}$$

Where $\Omega(i)$, $F(i)$, for $i = 1, 2$ denote coefficient matrices associated with each regime i; transition probabilities between regimes are given by $p_{ij} = P[S_t = i | S_{t-1} = j]$. According to Cho (2014), it is possible to affirm that:

Conditions 1-MS and 2-MS: The regime-switching rational expectations model, as the one in eq. (12), is determinate if there is a solution in the form of eq. (13) and (14), as well as their weighted matrices $\overline{D}_\Omega$ and D_F, such that $r(\overline{D}_\Omega) < 1$ and $r(D_F) \leq 1$.

Accordingly, in order to check the determinacy of the model, we have to calculate matrices $\overline{D}_\Omega$ and D_F, which can be done in a relatively easy way using the forward solution method proposed by Cho (2014). During the estimation process, described in the following section, each maximum likelihood solution will be tested for determinacy conditions. For further details, we refer the reader to Cho (2014) or to Appendix A in Baele et al. (2015).

2.4 Identifying the model by using survey expectations

As previously described, our identification and estimation strategy for the MS model follows Baele et al. (2015) and make use of survey-based market expectations. The authors assume that survey expectations for inflation and for output gap follow the law of motion below:

$$\pi_t^f = \alpha E_t \pi_{t+1} + (1 - \alpha)\pi_{t-1}^f + w_t^\pi \qquad\qquad w_t^\pi \sim N\left(0, \sigma_f^\pi\right) \qquad (15)$$

$$y_t^f = \alpha E_t y_{t+1} + (1 - \alpha)y_{t-1}^f + w_t^y \qquad\qquad w_t^y \sim N\left(0, \sigma_f^y\right) \qquad (16)$$

These two equations allow for a slow adjustment mechanism in expectations formation, in which survey expectations potentially react to rational expectations one to one only when parameter α is equal to 1. Otherwise, the adjustment of expectations is slower and depends on past values. The process is inspired in Mankiw & Reis's (2002) model of the Phillips curve in which the information disseminates slowly.

Baele et al. (2015) simplify the estimation mechanism by assuming that the volatility of shocks σ_f^π and σ_f^y in the equations for expectations movement is equal to zero. In this case, the survey-based expectations are the exact function of current rational expectations and of the past values from the survey. Substituting both equations above into our main model, we have:

$$\pi_t = \frac{\delta}{\alpha}\left(\pi_t^f - (1-\alpha)\pi_{t-1}^f\right) + (1-\delta)\pi_{t-1} + \lambda y_t + \kappa_{1S_t}\Delta e_{t-1} + \epsilon_{\pi,t} \qquad (17)$$

$$y_t = \frac{\mu}{\alpha}\left(y_t^f - (1-\alpha)y_{t-1}^f\right) + (1-\mu)y_{t-1} - \phi i_t + \frac{\phi}{\alpha}\left(\pi_t^f - (1-\alpha)\pi_{t-1}^f\right) + \epsilon_{y,t} \quad (18)$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\frac{\beta}{\alpha}\left(\pi_t^f - (1-\alpha)\pi_{t-1}^f\right) + \gamma y_t\right] + \epsilon_{i,t} \qquad (19)$$

$$\Delta e_t = \rho_e \Delta e_t + \epsilon_{e,t} \qquad (20)$$

Where $\epsilon_{\pi,t}\sim N(0,\sigma_{AS}^2(S_t))$, $\epsilon_{y,t}\sim N(0,\sigma_{IS}^2)$, $\epsilon_{i,t}\sim N(0,\sigma_{MP}^2)$, $\epsilon_{e,t}\sim N(0,\sigma_e^2)$. Note that when $\alpha = 1$, it is assumed that the rational expectations are equivalent to the survey expectations. Defining $X_t^f = [\pi_t^f \; y_t^f]'$, we can write the model in matrix form:

$$AX_t = BX_t^f + DX_{t-1}^f + G_{S_t}X_{t-1} + \epsilon_t \qquad\qquad \epsilon_t\sim N\big(0,\Sigma(S_t)\big) \qquad (21)$$

Where the matrices are specified as follows:

$$A = \begin{bmatrix} 1 & -\lambda & 0 & 0 \\ 0 & 1 & \phi & 0 \\ 0 & -(1-\rho_i)\gamma & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad G_{S_t} = \begin{bmatrix} (1-\delta) & 0 & 0 & \kappa_{1S_t} \\ 0 & (1-\mu) & 0 & 0 \\ 0 & 0 & \rho_i & 0 \\ 0 & 0 & 0 & \rho_e \end{bmatrix}$$

$$B = \begin{bmatrix} \dfrac{\delta}{\alpha} & 0 \\[2mm] \dfrac{\phi}{\alpha} & \dfrac{\mu}{\alpha} \\[2mm] \dfrac{(1-\rho_i)\beta}{\alpha} & 0 \\[2mm] 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} -\dfrac{\delta(1-\alpha)}{\alpha} & 0 \\[2mm] -\dfrac{\phi(1-\alpha)}{\alpha} & \dfrac{-\mu(1-\alpha)}{\alpha} \\[2mm] \dfrac{-(1-\rho_i)(1-\alpha)\beta}{\alpha} & 0 \\[2mm] 0 & 0 \end{bmatrix}$$

$$\Sigma(S_t) = \begin{bmatrix} \sigma_{AS}^2(S_t) & 0 & 0 & 0 \\ 0 & \sigma_{IS}^2 & 0 & 0 \\ 0 & 0 & \sigma_{MP}^2 & 0 \\ 0 & 0 & 0 & \sigma_e^2 \end{bmatrix}$$

If we establish the condition that $\alpha \neq 0$ and assure the invertibility of matrix A, we can multiply each side of the equation by A^{-1} and write the following reduced form, which will be used for the estimation:

$$X_t = \Omega_1 X_t^f + \Omega_2 X_{t-1}^f + \Omega_3(S_t)X_{t-1} + \Gamma\epsilon_t \qquad \epsilon_t \sim N\big(0, \Sigma(S_t)\big) \qquad (22)$$

In this equation, we have $\Omega_1 = A^{-1}B$, $\Omega_2 = A^{-1}D$, $\Omega_3(S_t) = A^{-1}G_{S_t}$ and $\Gamma = A^{-1}$. Baele et al. (2015) highlight that the biggest advantage of this approach is that the matrices that determine the law of motion of vector X_t are simple analytical functions of the structural parameters, which makes the calculation of the likelihood function relatively easy. Market expectations add new information, which is absent from the other variables and from the structure of the original model and which will contribute to estimation. It will not be necessary to compute the rational expectations equilibrium with multiple regimes, solving the model in each step of the likelihood optimization, as in Farmer, Waggoner & Zha (2011) and Liu & Mumtaz (2010). Otherwise, unobserved regimes will be inferred by the conventional multivariate methods proposed by Hamilton (1989) and Kim & Nelson (1999). More specifically, we will be maximizing the log-likelihood function of a structural VAR (SVAR) model with regime switching, in which structural restrictions stem from the new Keynesian model and are given by matrices $A, B, D, G_{S_t}, \Sigma(S_t)$. The estimation of the likelihood function of the regime-switching VAR model follows the description of Hamilton (1994), Bellone (2005), and Krolzig (1997), and the algorithm of inference about regimes is the conventional Hamilton filter, which was implemented according to the description provided by Kim & Nelson (1999).

3. Data and Estimation

This section presents the data series and basic descriptive statistics, as well as the estimation method used.

3.1 Descriptive statistics and stationarity tests

The estimation of the model requires six observed variables: inflation, output gap, interest rate, exchange rate movement, and the survey expectations for inflation and

output gap. Sixty-four quarterly observations – from the first quarter of 2000 to the fourth quarter of 2015 – were considered for the sample. We opted to leave the year 1999 out of the sample due to the large fluctuations observed shortly after the transition to the floating exchange rate regime, and also because data on survey expectations are not readily available.

The seasonally adjusted quarterly IPCA (%) was used for consumer price inflation. First, the monthly series[2] was accumulated quarterly and then we applied a multiplicative moving average seasonal adjustment. The output gap was obtained from the quarterly GDP logarithm at seasonally adjusted market values[3] and the trend was estimated by the Hodrick-Prescott filter. The remaining component (business cycle) was considered to be the output gap. A broader window, beginning in 1996, was used for extracting the gap so as to avoid the tail effect at the beginning of the period. The extraction of output gap by Beveridge-Nelson in the AR(1) model was also tested, but we did not obtained the desired statistical properties. In turn, the quarterly exchange rate movement is calculated as the first difference of the nominal exchange rate value, R$ vis-à-vis US$, at the end of the period.[4]

We choose to use as the quarterly interest rate i_t the nominal interest rate discounted for the long-run real interest rate. In this sense, we try to account for the fact that the Brazilian economy experienced a sharp reduction in its long-run real interest rate between 2005 and 2012. Thus, the data we are taking to the model is the nominal rate in excess of the long run interest rate, and we should obviously consider this characteristic when analysing our estimation results. In order to calculate our series i_t, we first took the quarterly equivalent of the monthly Selic Over rate (% p.a.)[5] at end of the period. The long-run real interest rate was built from the trend of an HP filter under the real interest rate, which is determined by the nominal rate minus the observed inflation. We then discount the long-run real interest rate from our quarterly nominal interest rate.

Finally, the Central Bank of Brazil's[6] survey-based market expectations were used to calculate inflation and output gap expectations for the subsequent quarter. Inflation expectation was measured as the median value of the survey for the consumer prices

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[2] Source: Series 433 (monthly IPCA). Central Bank of Brazil Time Series.
[3] Source: Series 22109 (seasonally adjusted GDP). Central Bank of Brazil Time Series.
[4] Source: Series 3696 (free exchange rate). Central Bank of Brazil Time Series.
[5] Source: Monthly Over/Selic interest rate (% p.a.) series. IPEA Data System.
[6] Source: Central Bank of Brazil Market Expectations System (Focus Report).

inflation (IPCA) for the three months of the subsequent quarter, observed on the first business day of the current quarter. The monthly values were accumulated to obtain the quarterly inflation expectation. Data observed on the first business day is used to circumvent the endogeneity problem between inflation in the current quarter and inflation expectations for the subsequent period, without having to rely on instrumental variables. In fact, one avoids the correlation between exogenous shock to inflation in the current period $\epsilon_{\pi,t}$ and future expectations $E_t\pi_{t+1}$ as basically information from the current period is not included in the measure.

On the other hand, it was necessary to use a calculation procedure to obtain the output gap expectations for the subsequent quarter. The variable observed by the market survey is the real growth of GDP (% p.a.) for the subsequent quarter. The first step consisted in extracting the equivalent quarterly growth rate, and then estimating the real domestic product expected for *t+1*. The seasonally adjusted series observed in the past was included up to period *t*, under the value expected for *t+1*, forming a new series. The log of the complete new series was extracted and its trend was estimated by the HP filter. The value of the cycle in period *t+1* thus corresponds to an estimate of the output gap expectation. Table 1 presents the descriptive statistics for each of the series.

Table 1: Time series descriptive statistics. Units are percentage points / 100. Source: Authors' calculations.

	π_t	y_t	i_t	Δe_t	$E_t\pi_{t+1}$	$E_t y_{t+1}$
Mean	0.0161	0.0002	0.0161	0.0122	0.0133	0.0096
Median	0.0145	0.0041	0.0164	-0.0066	0.0126	0.0103
Maximum	0.0551	0.0331	0.0327	0.3143	0.0305	0.0351
Minimum	0.0012	-0.0488	0.0044	-0.1708	0.0083	-0.0352
Std. dev.	0.0089	0.0164	0.0063	0.0961	0.0040	0.0159
Skewness	1.9608	-0.9021	0.3704	0.8913	1.9747	-0.5447
Kurtosis	8.2738	3.9325	2.8547	3.9075	8.7659	2.8346
Jarque-Bera	115.1797	10.9999	1.5198	10.6694	130.2482	3.2383
Probability	0.0000	0.0041	0.4677	0.0048	0.0000	0.1981

The six series are tested for stationarity. We provide the results for the conventional Augmented Dickey-Fuller (ADF) test, with intercept, and Phillips-Perron (PP) test, in Table 2. Of these six series, only one is in first difference (*Δe*), and the other ones are used in the level. As demonstrated, both the ADF and PP tests reject the presence of unit root at the 5% significance level for inflation, output gap, exchange rate movement, and inflation expectation. For the interest rate series, the ADF test rejects the

presence of unit root at 5%, whereas the PP test cannot reject it, not even at 10%. Nevertheless, we do not consider this evidence strong enough to invalidate the use of this series.

Table 2: Unit root tests (full sample). Source: Authors' calculations.

	Augmented Dickey-Fuller			Phillips-Perron		
	t-Statistic	Prob.	Lag length	Adj. t-Statistic	Prob.	Bandwidth
π_t	-4.8340	0.0002	0	-4.7683	0.0002	4
y_t	-4.2781	0.0011	1	-2.9916	0.0411	2
i_t	-3.1076	0.0311	1	-2.4491	0.1328	2
Δe_t	-6.8397	0.0000	0	-6.7941	0.0000	6
$E_t\pi_{t+1}$	-2.9610	0.0044	2	-4.6421	0.0003	9
$E_t y_{t+1}$	-2.5837	0.1017	1	-2.4118	0.1426	3

It is not possible to reject the null hypothesis for the output gap expectation. However, when the unit root tests are taken with a slightly reduced sample, without the last three quarters (2015q2 to 2015q4), we get far better results. The ADF test rejects the unit root hypothesis at 5% and the PP rejects at 10%. We can argue that the output expectations suffered a severe shock from the third quarter of 2015 onwards, which has not been totally reversed to its mean yet. Given the theoretical hypotheses of output gap stationarity and rational expectations, the reversion should take place in the long run. We conclude that the econometric estimation of the model can proceed without any restrictions. For the sake of illustration, Figure 1 displays the series used.

Figure 1: Observed series (from left to right, up to bottom): inflation, output gap, interest rate, exchange rate movement, inflation expectation, and output gap expectation. Units are percentage points / 100. Source: Authors' calculations.
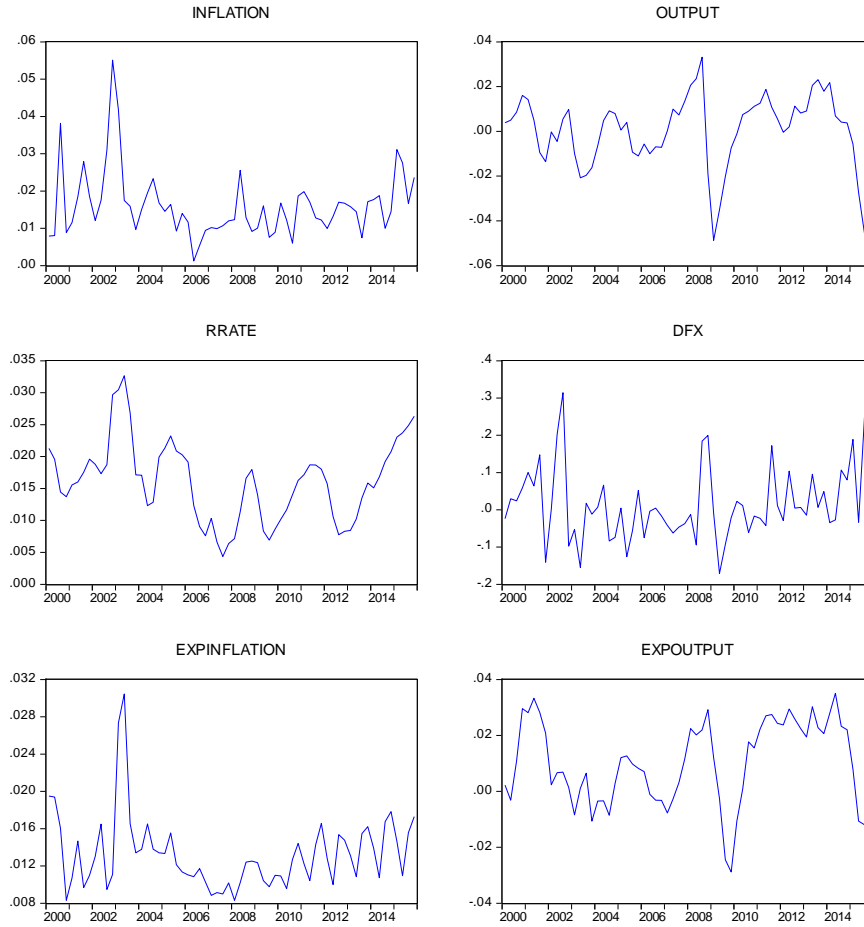
3.2 Estimation method

The reduced-form MS model in equation (22) will be estimated by maximum likelihood using the Hamilton filter and the likelihood function of a structural VAR model with Markov switching, following the methods described in Hamilton (1994), Kim & Nelson (1999), Bellone (2005), and Krolzig (1997). We follow the notation of Kim & Nelson (1999), where ψ_{t-1} is the set of information available at $t-1$, and the observed data yield $y_t = [X_t \; X_t^f]'$. The full set of parameters to be estimated is called θ, a line vector that contains the structural parameters of the MS model, including the volatilities of shocks and the transition probabilities between p, q. A total of 18 parameters need to be estimated:

$$\theta = \{\delta, \lambda, \kappa_{1S_t}(S_t = 0), \kappa_{1S_t}(S_t = 1), \mu, \phi, \rho_i, \beta, \gamma, \rho_e, \alpha, \sigma_{AS}(S_t = 0), \sigma_{AS}(S_t = 1), \sigma_{IS}, \sigma_{MP}, \sigma_e, p, q\}$$

The log-likelihood function is given by $\ln L = \sum_{t=1}^{T} \ln(f(y_t))$, where $f(y_t)$ is expressed in terms of its parameters θ. The aim is to maximize the density function $\ln L(y_t; \theta)$. In the case of regime-switching models, we do not observe regimes S_t, but we can infer about them in every time period. Kim & Nelson (1999) describe the following steps to determine the log-likelihood function of a general regime-switching model.

Step 1: First, one should consider the joint density of y_t and the unobserved variable S_t, based on the information up to $t - 1$, which is given by the product of conditional and marginal density:

$$f(y_t, S_t | \psi_{t-1}) = f(y_t | S_t, \psi_{t-1}) f(S_t | \psi_{t-1})$$

Step 2: Then, in order to obtain the marginal density of y_t, the variable S_t is included in the joint density by the sum of all the possible values for S_t:

$$f(y_t | \psi_{t-1}) = \sum_{S_t=0}^{1} f(y_t, S_t | \psi_{t-1}) = \sum_{S_t=0}^{1} f(y_t | S_t, \psi_{t-1}) f(S_t | \psi_{t-1})$$

In the case of only two regimes, we have $f(S_t = j | \psi_{t-1}) = Pr[S_t = j | \psi_{t-1}]$, and the log-likelihood function is given by:

$$\ln L = \sum_{t=1}^{T} \ln\{\sum_{S_t=0}^{1} f(y_t | S_t, \psi_{t-1}) \ Pr[S_t | \psi_{t-1}]\} \tag{23}$$

Kim & Nelson (1999) underscore that the marginal density above can be interpreted as a weighted average between conditional densities, in the cases where $S_t = 0$ and $S_t = 1$. To derive the marginal density and also the log-likelihood, it is necessary to calculate the weighting factors $Pr[S_t = 0 | \psi_{t-1}]$ and $Pr[S_t = 1 | \psi_{t-1}]$. At this moment, we assume that the discrete variable S_t follows a first-order Markov process, where its state at t depends only on its previous state S_{t-1}. We again follow Kim & Nelson

(1999) to define the transition probabilities p and q: $p = Pr[S_t = 1|S_{t-1} = 1]$, and $q = Pr[S_t = 0|S_{t-1} = 0]$.

In the case of the Markov process, we use a filter to calculate the weighting factors $Pr[S_t = j| \psi_{t-1}], j = 0, 1$, which takes into account the transition probabilities between states:

Step 1: Given $Pr[S_{t-1} = i| \psi_{t-1}]$, for $i = 0, 1$, at the beginning of period t, the weighting term is calculated as

$$Pr[S_t = j| \psi_{t-1}] = \sum_{i=0}^{1} Pr[S_t = j, S_{t-1} = i| \psi_{t-1}]$$

$$= \sum_{i=0}^{1} Pr[S_t = j|S_{t-1} = i]Pr[S_{t-1} = i| \psi_{t-1}]$$

Where $Pr[S_t = j|S_{t-1} = i]$ are the transition probabilities between states.

Step 2: Once y_t, is observed at the end of period t, we can update the probability term as follows

$$Pr[S_t = j| \psi_t] = Pr[S_t = j| \psi_{t-1}, y_t] = \frac{f(S_t = j, y_t|\psi_{t-1})}{f(y_t|\psi_{t-1})}$$

$$= \frac{f(y_t|S_t = j, \psi_{t-1})Pr[S_t = j| \psi_{t-1}]}{\sum_{j=0}^{1} f(y_t|S_t = j, \psi_{t-1}) Pr[S_t = j| \psi_{t-1}]}$$

Where $\psi_t = \{ \psi_{t-1}, y_t \}$. The steps above are performed iteratively to calculate $Pr[S_t = j| \psi_t], t = 1, 2, ..., T$, i.e., the filtered probabilities for the whole sampling period. To initiate the filter at $t = 1$, we assume unconditional probabilities, or steady state, at $t = 0$:

$$Pr[S_0 = 0| \psi_0] = \frac{1 - p}{2 - p - q} \qquad\qquad Pr[S_0 = 1| \psi_0] = \frac{1 - q}{2 - p - q}$$

The description of the steps above and of the probability update filter makes it clear that the marginal density $f(y_t|\psi_{t-1})$ is a function of parameters θ, which include the traditional likelihood parameters, the parameters that vary across states, in addition to the transition probabilities for state p, q. In the case of the Markov-switching model, the log-likelihood function of equation (23) is then given by:

$$\ln L = \sum_{t=1}^{T} \ln \left\{ \sum_{S_t=0}^{1} f(y_t|S_t, \psi_{t-1}) \, Pr[S_t| \psi_{t-1}] \right\} \qquad\qquad (24)$$

In what follows, we describe some peculiarities about the algorithm implemented for our estimation.[7] We begin by assigning an initial value to each parameter of vector θ_0. The initial values were chosen based on estimation results obtained by Baele et al. (2015) and are displayed in Table 3.

From the value of θ_0, we maximize the log-likelihood function with a numerical constraint optimization algorithm. In each optimization step, the parameters of the candidate vector θ_i are used for the construction of matrices $A, B, D, G_{S_t}, \Sigma(S_t)$, which represent the model in its structural form. We proceeded directly with the computation of matrices $\Omega_1, \Omega_2, \Omega_3(S_t), \Gamma$ to change the model into its reduced form. With the matrices in reduced form and transition probabilities, the log-likelihood calculation was then made using the Hamilton filter, described previously, and the sample likelihood function of an MS-VAR model, shown next (Hamilton, 1994; Bellone, 2005; Krolzig, 1997). At last, we check whether the determinacy conditions for the rational expectations equilibrium are met with each candidate solution vector θ_i, and we penalize the objective function if that is not the case. This procedure will guarantee that the search will be made along a stable solution path.

Let $n = 4$ be the number of endogenous variables, $m = 8$ the number of regressors of the reduced model, and $T = 64$ the number of observations. Following Hamilton's (1994) notation, consider:

- $y_t = [X_t]$ the vector of endogenous variables, $nx1$;
- $x_t = [X_t^f \ X_{t-1}^f \ X_{t-1}]$ the vector containing the grouped regressors of the reduced model, $mx1$;
- $\Omega_{Var}(S_t) = \Gamma \Sigma(S_t)\Gamma'$ the variance-covariance matrix of the reduced model for each state obtained from $\Sigma(S_t)$ and from Γ, nxn;
- $\Pi(S_t)' = [\Omega_1 \ \Omega_2 \ \Omega_3(S_t)]$ the state-dependent coefficient matrix of the reduced model, nxm.

The same reduced model in equation (22) can be written as a regime-switching VAR model:

$$y_t = \Pi(S_t)'x_t + u_t \qquad\qquad u_t \sim N\big(0, \Omega_{Var}(S_t)\big) \qquad (25)$$

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[7] The estimation algorithm was implemented using Matlab R2011 and is available upon request.

After defining this notation for each filtering step, the marginal density of the VAR model, given θ, S_t, ψ_{t-1}, is as follows:

$$f(y_t|\theta, S_t, \psi_{t-1}) = (2\pi)^{-n/2} \sqrt{\left|(\Omega_{Var}(S_t))^{-1}\right|} \exp\left\{-\frac{1}{2}[y_t - (\Pi(S_t)'x_t)]'(\Omega_{Var}(S_t))^{-1}[y_t - (\Pi(S_t)'x_t)]\right\}$$

The log-likelihood maximization yields a vector of optimal estimated parameters $\hat{\theta}$.

Table 3: Initial parameters and restrictions of the MS model. Source: Authors' calculations.

Parameter	Initial value	Minimum	Maximum
δ	0.425	0.00001	1
λ	0.102	0.00001	$+\infty$
$\kappa_{1S_t}(S_t = 0)$	0.005	$-\infty$	$+\infty$
$\kappa_{1S_t}(S_t = 1)$	0.09	$-\infty$	$+\infty$
μ	0.675	0.00001	1
ϕ	0.10	0.00001	$+\infty$
ρ_i	0.834	0.00001	0.99999
β	1.10	0.00001	$+\infty$
γ	0.80	0.00001	$+\infty$
ρ_e	0.16	$-\infty$	0.99999
α	0.90	0.00001	1
$\sigma_{AS}(S_t = 0)$	0.0038	0.00001	$+\infty$
$\sigma_{AS}(S_t = 1)$	0.0098	0.00001	$+\infty$
σ_{IS}	0.0108	0.00001	$+\infty$
σ_{MP}	0.0043	0.00001	$+\infty$
σ_e	0.0950	0.00001	$+\infty$
p	0.90	0	1
q	0.76	0	1

It should be underscored that the parameters are restricted to a domain of possible values, which are also shown in Table 3, and which stem from the theoretical constraints of the original RE model. Our constraints are similar, but lighter than those of Baele et al. (2015). The referenced authors initially allow the parameters to be free, but later present a set of values for each parameter (domain), calculated by grid search, for which the solution to the model is more likely. We opted to implement a simpler process by applying some basic theoretical constraints directly to the initial parameter domain, which will be used in the numerical constrained optimization. Note that the estimated vector $\hat{\theta}$ was

calculated by verifying the determinacy conditions for rational expectations solution at each step of the optimization in both cases.

4. Empirical results

In this section, we first discuss the estimation results for the MS model, which allows for joint Markov switching in the exchange rate pass-through coefficient and in the volatility of shocks to inflation, and then we compare them with the results obtained for the conventional fixed coefficients model. In what follows, we introduce some specification and linearity tests and we analyze the impulse response functions. Finally, we describe the regimes identified by the MS model and their relationship with economic periods.

4.1 Parameter estimation in the MS model

Table 4 shows the estimates for each parameter in the MS model, as well as their standard deviation and corresponding *p*-value obtained in the conventional *t* test. The variance-covariance matrix of the maximum likelihood estimates was calculated using the information matrix outer product method, as suggested by Hamilton (1994). The solution offered by the model characterizes a fundamental stable rational expectations equilibrium, as shown in detail in Section 2.3. The determinacy conditions for the regime-switching rational expectations model were assessed and confirmed: $r(\overline{D}_\Omega) < 1$ and $r(D_F) \leq 1$.

Most parameters are statistically significant. Recall that the signs of the parameters are guaranteed by the constraints imposed on the likelihood function optimization and that no parameter was calibrated. The parameters for which joint regime switching was allowed were the pass-through coefficient κ_{1S_t} and the volatility of shocks $\sigma_{AS}(S_t^\pi)$ to inflation.

In the aggregate supply (AS) equation of the MS model we estimated $\delta = 0.6971$, demonstrating a relatively heavier weight to inflation expectations in comparison to the endogenous persistence (backward-looking) term. This value is quite close to the estimates made by Silveira (2008), who found $\delta = 0.61$ in his model with price indexation. In the demand curve (IS), however, a lighter weight was attached to the

expectations element, with $\mu = 0.1234$, and a higher standard deviation. This finding suggests, on the one hand, higher output persistence and, on the other hand, smaller predictive power of market expectations about output performance in the subsequent periods perhaps as a result of the high volatility of shocks to demand (σ_{IS}). Silveira (2008), in his model with habit formation in consumption, found parameters that would correspond to $\mu = 0.26$, and a confidence interval that would include our value of $\mu = 0.12$. By and large, we may assume that the model provides evidence in favor of endogenous persistence of both output and inflation.

The response of inflation to the output gap is estimated at the value of $\lambda = 0.0722$, which is in line with Bayesian estimations of more complex new Keynesian models such as Gonçalves, Portugal & Arágon (2016) who yielded $\lambda = 0.0654$. Our result, however, does not displays statistical significance due to a relatively high standard deviation. Actually, several studies on the Phillips curve for the Brazilian economy do not demonstrate a statistically significant impact of the output gap, or of marginal cost, on inflation (Alves & Areosa, 2005; Areosa & Medeiros, 2007; Arruda, Ferreira & Castelar, 2008), prompting Sachsida (2013) to put the validity of this assumption into question. An exception is seen in Mazali & Divino (2010), who estimate the new Keynesian curve with GMM, controlling for the exchange rate pass-through and observe a significant effect of unemployment on inflation. Estimations that use other series to represent the output gap, such as the Beveridge-Nelson decomposition suggested by Tristão & Torrent (2015), were tested; however, none of them provided a better fit than the one introduced herein. Anyway, further investigation into this topic is not within the scope of this paper.

The MS model identifies two distinct regimes regarding the behavior of the exchange rate pass-through, confirming the major assumption of our paper. We refer to the regimes as $S_t = 0$ and $S_t = 1$, which correspond to low and high exchange rate pass-through periods, respectively. The value estimated in the AS curve for the pass-through in regime $S_t = 0$ is statistically zero, with $\kappa_{1S_t}(S_t = 0) = 0.0004$. The point estimate would correspond to a long-run effect of only 0.00057 percentage points on inflation, which is practically zero, considering a 1% exchange rate shock (depreciation of the domestic currency). On the other hand, the estimate for regime $S_t = 1$ is $\kappa_{1S_t}(S_t = 1) = 0.0722$, with strong statistical significance. The long-run effect, considering a 1% exchange rate shock during the high pass-through regime, is 0.1035 percentage points on

inflation. Note that the point estimate for the pass-through is several times higher during regime $S_t = 1$, compared to the other regime.

Table 4: Parameters estimated for the semi-structural MS model. Source: Authors' calculations.
Note: The first row contains the parameter estimation; the second row contains the standard deviation and *p*-values in brackets.

1. Parameters for the inflation curve

δ	λ	$\kappa_{1S_t}(S_t^\pi = 0)$	$\kappa_{1S_t}(S_t^\pi = 1)$
0.6971	0.0722	0.0004	0.0722
0.1189 (0.000)	0.0508 (0.145)	0.0153 (0.397)	0.0316 (0.026)

2. Parameters for the output gap curve

μ	ϕ
0.1234	0.6740
0.1056 (0.199)	0.3043 (0.037)

3. Monetary policy parameters

ρ_i	β	γ
0.2515	0.7852	0.0117
0.0711 (0.001)	0.1692 (0.000)	0.0680 (0.390)

4. Parameters for exchange rate dynamics	5. Expectations formation
ρ_e	α
0.1488	0.9999
0.1685 (0.267)	0.2583 (0.000)

6. Volatilities

$\sigma_{AS}(S_t^\pi = 0)$	$\sigma_{AS}(S_t^\pi = 1)$	σ_{IS}	σ_{MP}	σ_e
0.0043	0.0096	0.0108	0.0043	0.0950
0.0024 (0.002)	0.0063 (0.028)	0.0040 (0.000)	0.0023 (0.001)	0.0425 (0.000)

7. Transition probabilities

q	p
0.9583	0.9559
0.0608 (0.000)	0.0412 (0.000)

8. Statistics

R_{AS}^2	R_{IS}^2	R_{MP}^2	R_e^2	*Log-likelihood*
0.4728	0.5112	0.5384	0.0063	752.0262

In addition to a smaller pass-through regime $S_t = 0$ demonstrated smaller volatility in shocks to inflation, with a standard deviation estimated at $\sigma_{AS}(S_t^\pi = 0) = 0.0043$, compared to $\sigma_{AS}(S_t^\pi = 1) = 0.0096$. The transition probabilities reveal relatively high and very similar persistence for both regimes. Consequently, the economy is expected to remain for several quarters in one specific regime, once the transition occurs. Parameter $q = 0.9583$ corresponds to the probability of the economy remaining in regime $S_t = 0$ when it is already in it, i.e., $Pr[S_t = 0|S_{t-1} = 0]$. Additionally,

parameter $p = 0.9559$ is equivalent to the probability of remaining in regime $S_t = 1$, that is $Pr[S_t = 1 | S_{t-1} = 1]$. In brief, the model estimates that periods of high pass-through and high volatility in the shocks will be slightly shorter than periods of low pass-through and low volatility, an issue that will be dealt with in a forthcoming section of this paper. For the sake of simplicity, we will, henceforth, refer to regime $S_t = 1$ as "crisis" and to regime $S_t = 0$ as "normal."

Note that the MS model is superior to the fixed parameters model in terms of better fit (parameter R_{AS}^2), larger log-likelihood value, and higher value for the Schwartz criterion. Table 5 shows the comparison between the models, as suggested by Hamilton (2005). Moreover, by assuming regime switching in the volatility of shocks, we ran the Wald test on constraint $\kappa_{1S_{t=0}}(S_t = 0) = \kappa_{1S_t}(S_t = 1)$ and the result is the rejection at 5% significance. In other words, the test rejects the hypothesis of equal pass-through coefficients in both regimes. This results strengthens our argument for the superiority of a Markov switching representation.

Table 5: Comparison between selected models. Source: Authors' calculations. Note: Schwartz criterion calculated as $\mathcal{L} - (k/2)\log T$, where $\mathcal{L}$ is the log-likelihood, k is the number of parameters and T is the sample size (Hamilton, 2005).

Model	Number of parameters	Log-likelihood	Schwartz criterion
Markov switching (MS)	18	752.03	714.59
Linear (fixed parameters)	14	741.42	712.31

The values estimated for the exchange rate pass-through are consistent with earlier findings, although the difference across sample periods does not allow strict comparisons. In particular, numerous studies include the first stage of the Real Plan (1994-1999), prior to the implementation of the inflation-targeting and floating exchange rate regime. In that initial phase, the Brazilian foreign exchange rate was highly controlled by the Central Bank, working as an anchor to prices while most of the macroeconomic shocks were absorbed by sharp moves on the interest rate.

Pimentel, Modenesi & Luporini (2015), for instance, attempt to measure the exchange rate pass-through between 1999 and 2013 assuming an asymmetric effect during appreciations versus depreciations. Our estimate for the pass-through during the "crisis" regime is very close to the value these authors obtain for the pass-through in

depreciation events. In fact, they find a long run effect of 0.1138 percentage points in inflation, given a 1% exchange rate depreciation. Correa & Minella (2006) investigated the pass-through between 1995 and 2005, having estimated an effect of 0.20 percentage points in the long-run inflation for every 1% of depreciation, if that occurred within a period of large exchange rate movement. Conversely, the pass-through is statistically zero for periods with small exchange rate movements. Our findings are comparable to those of Correa & Minella (2006), but it should be recalled that the comparison is limited due to the large difference between sample periods. Carneiro, Monteiro & Wu (2004) analyzed the period from 1994 to 2001 and found a nonlinear effect of short-run pass-through ranging from 5.6% to 11%, whereas our results for the high pass-through period yielded 7.22%. Tombini & Alves (2006) presented a variable estimate for exchange rate pass-through between 2002 and 2006, which varied from zero to approximately 8%, which is again consistent with our findings. Finally, our results are in line with the exchange rate pass-through estimate published by the Central Bank of Brazil (2015) in its several small scale linear projection models.

Note that our long-run pass-through estimate (10.35%), even in a "crisis" period, is considered relatively low by the criteria established by Goldfajn & Werlang (2000) and Belaisch (2003), which implies that the economy has exhibited reasonable capacity to absorb exchange rate shocks without direct pass-through to consumer inflation.

In order to analyze the results of the aggregate demand (IS) and the monetary policy (MP) curves we should recall that our measure of the interest rate i_t is calculated as the nominal interest rate discounted for the long term real interest rate. In other words i_t is the interest rate "in excess" of the long term real rate. With that in mind, we find that the IS curve demonstrates a strong response of output to the interest rate "in excess", with parameter $\phi = 0.64740$. In fact, a relatively high value should be expected by theory. Our estimated parameter is much higher than the calibration of Baele et al. (2015), of $\phi = 0.1$, or the estimation of Gonçalves, Portugal & Arágon (2016) who obtain $\phi = 0.4063$, as both of these works use purely the nominal interest rate as input to their models. Our findings indicate a strong reaction of aggregate demand to the interest rate "in excess" of the long term real rate, which, in turn, shows an efficient channel for monetary policy transmission in Brazil.

Regarding the monetary policy rule, we obtain an interest rate smoothing value of $\rho_i = 0.2515$, which is relatively small in comparison to Bayesian estimations of new

Keynesian models such as Furlani, Portugal & Laurini (2010). Again, the difference in our interest rate input series could explain a much slower smoothing coefficient. The Central Bank would be aiming to smooth the nominal interest rate, which leads to a smaller smoothing of the interest rate that is "in excess" of the long term real rate.

Parameter β, which stands for the response of the interest rate to inflation expectation, was estimated at 0.7852 and indicates an activist response to inflation as it is significantly different from zero.[8] The interpretation that arises, in our case, is that the Central Bank would be willing to raise the interest rate above the long term real rate for every positive shock in inflation expectations. Anyway, our result is higher than the value estimated by Gonçalves, Portugal & Arágon (2016), who used the nominal interest rate as input and obtained $\beta = 0.56$ in their model with fixed coefficient in the Taylor rule. These authors only manage to identify an activist regime, with $\beta > 1$ in their case, using a regime-switching model in the parameter β itself.

The response of monetary policy to output is estimated at $\gamma = 0.0117$, and due to its high standard deviation, it is not statistically different from zero. In any case, a positive value would indicate that the Central Bank responds to output gap deviations, but that estimate should be smaller than those of Palma & Portugal (2014) and Gonçalves, Portugal & Arágon (2016), for example, again due to difference on the series for interest rates.

The exchange rate dynamics shows some positive autocorrelation in exchange rate movements, with $\rho_e = 0.1488$, but it is not significant. The volatility of shocks to the exchange rate equation is by far the largest and the fit of the curve is almost irrelevant.

Parameter α describes the law of motion of market expectations, and is very close to one, implying that the model disregards market expectations assessed in the previous period. According to Baele et al. (2015), this finding indicates that market expectations fully adjust to rational expectations, and the slow dissemination of information does not appear to be important in this process.

Table 6 shows the estimation results for the new Keynesian model without regime switching, for the sake of comparison. The exchange rate pass-through coefficient, estimated in the AS curve, is $\kappa_1 = 0.0419$, which corresponds to a long-run effect of 0.0762 percentage points on inflation, considering an exchange rate shock of 1%.

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[8] Note that, if we used the nominal interest rate as i_t instead of the rate "in excess" of the long term real rate, the activist regime would be characterized by $\beta > 1$.

Naturally, this value is within the interval between the smallest and largest pass-through values estimated in the two-regime model. The other AS curve parameters have similar values to those obtained for the MS model. The relative weight of endogenous persistence of inflation is a bit larger ($\delta = 0.5497$). In the meantime, the output gap parameter has a heavier weight ($\lambda = 0.0887$), but that is still nonsignificant. As observed earlier, the Markov-switching model has larger log-likelihood value, larger value for the Schwartz criterion, and better fit for the AS curve, indicating its superiority.

Table 6: Parameters estimated for the semi-structural model without regime switching. Source: Authors' calculations. Note: the first row contains the parameter estimation and the second row contains the standard deviation and *p*-values in brackets.

1. Parameters for the inflation curve

δ	λ	κ_1		
0.5497	0.0887	0.0419		
0.1029 (0.000)	0.0776 (0.206)	0.0158 (0.014)		

2. Parameters for the output gap curve

μ	ϕ			
0.1231	0.6718			
0.1015 (0.189)	0.3167 (0.044)			

3. Monetary policy parameters

ρ_i	β	γ		
0.2513	0.7857	0.0110		
0.0711 (0.001)	0.1614 (0.000)	0.0541 (0.389)		

4. Exchange rate dynamics parameters | | | 5. Expectations formation |

ρ_e			α	
0.1488			1	
0.1556 (0.250)			0.2819 (0.001)	

6. Volatilities

σ_{AS}	σ_{IS}	σ_{MP}	σ_e	
0.0073	0.0108	0.0043	0.0950	
0.0000 (0.000)	0.0000 (0.000)	0.0000 (0.003)	0.0017 (0.000)	

7. Statistics

R^2_{AS}	R^2_{IS}	R^2_{MP}	R^2_e	Log-likelihood
0.2688	0.5111	0.5383	0.0063	741.4227

4.2 Specification and linearity tests

We ran the basic univariate specification tests on the standardized residuals of each equation – serial autocorrelation, normality, and conditional variance – and linearity tests on the MS model. The results of the specification tests on standardized residuals are

shown in Table 7. Serial autocorrelation was assessed by the Ljung-Box Q test for 20 lags, using $\min(20, T-1)$ as standard, as suggested by Box, Jenkins & Reinsel (1994). Figure 2 also shows the sample autocorrelation for each series of standardized residuals with the corresponding confidence intervals.

Table 7: Specification tests on standardized residuals of the MS model (*p* values). Source: Authors' calculations. Note: *p* values in brackets.

Univariate statistical tests	MS model			
	Inflation	Output gap	Monetary policy	Exchange rate
Serial autocorrelation (*p* values) standard 20 lags	(0.196)	(0.696)	(0.000)	(0.999)
Skewness	0.304	-1.717	0.392	0.745
Kurtosis	3.178	8.283	3.209	3.629
Jarque-Bera test (*p* values)	(0.500)	(0.001)	(0.296)	(0.030)
Serial autocorrelation of squared residuals (*p* values) standard 20 lags	(0.904)	(0.754)	(0.923)	(0.431)

The weakness of the MS model seems to be its inability to eliminate serial autocorrelation in residuals, especially in the equation for monetary policy response. For the other equations, the lack of autocorrelation is not rejected, at least for 20 lags. Baele et al. (2015) admit that these statistics may be biased in small samples, especially when the data-generating process is nonlinear as in our model. In their empirical study, the authors cannot prevent the rejection of the hypothesis of no serial autocorrelation in the residuals of the output gap equation in the MS rational expectations and unrestricted MS-VAR models, even when using critical test values obtained from a Monte Carlo simulation with a small sample. Our analysis included some attempts to change the specification of the model, inserting a larger number of lagged endogenous variables as regressors in all equations $(X_{t-2}, X_{t-3}, X_{t-4})$. Yet, it was not possible to eliminate the signs of serial autocorrelation, so we opted to keep the model simpler. A possible way to circumvent this problem would be to model the shocks in each curve as autoregressive processes. However, that would require a more complex estimation method.

Figure 2: Sample autocorrelation of standardized residuals of the MS model for each equation. From left to right, top to bottm: inflation, output, interest rate, and exchange rate movement. Source: Authors' calculations. Note: The blue horizontal line indicates the confidence interval (two standard deviations).
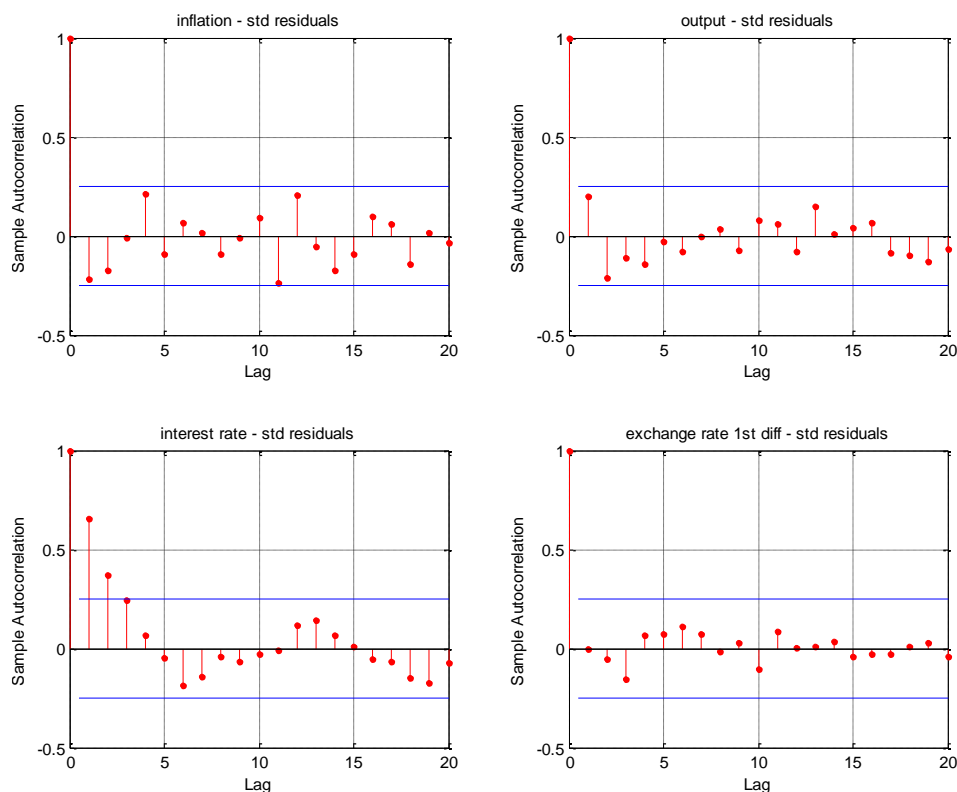
As to the other tests, normal distribution is rejected for the residuals of the output gap and exchange rate equations due to high kurtosis. We checked for the existence of conditional variance through a Ljung-Box Q test in the squared residuals. There was no evidence of conditional variance in any of the error terms of the equations.

Table 8: Specification test on the residuals of the semi-structural linear model (p values). Source: Authors' calculations. Note: p values in brackets.

Univariate statistical tests	Linear model (without regime switching)			
	Inflation	Output gap	Monetary policy	Exchange rate
Serial autocorrelation (p values) standard 20 lags	(0.854)	(0.694)	(0.000)	(0.999)
Skewness	0.390	-1.719	0.392	0.745
Kurtosis	4.187	8.286	3.209	3.629
Jarque-Bera test (p values)	(0.047)	(0.001)	(0.298)	(0.030)

Serial autocorrelation in squared residuals (*p* values) standard 20 lags	(0.848)	(0.755)	(0.925)	(0.431)

For the sake of comparison, Table 8 also shows the same tests performed on the model without regime switching, confirming the difficulty in eliminating serial autocorrelation in the residuals. It should be remarked that the regime-switching mechanism in the inflation curve reduced kurtosis of the distribution of standardized errors.

The literature recognizes the difficulty in testing for linearity in Markov-switching models, since usual regularity conditions for likelihood-based inference are violated (Hansen, 1992; Carrasco, Hu & Ploberger, 2014; Di Sanzo, 2009). Under the null hypothesis of linearity some parameters are not identified, such as transition probabilities. Di Sanzo (2009) clarifies that, in this case, the likelihood function is no longer quadratic, but rather flat at the optimal level and its scores are identically zero. Therefore, the asymptotic distribution of the test statistics of interest, such as LR, no longer has its conventional chi-square form. Hansen (1992), Carrasco, Hu & Ploberger (2014), and Di Sanzo (2009), among others, propose alternative tests to assess the stability of parameters and the validity of the linearity hypothesis. We decided to submit the MS model to Di Sanzo's (2009) test, which is based on a bootstrap distribution of the likelihood ratio under the null hypothesis. The aim of the test is to compare the likelihood ratio (LR) obtained from the linear (H_0) and the MS (H_1) models, with bootstrap distribution of a likelihood ratio calculated under the LR^* null in order to find the corresponding *p*-value. The author gathers evidence that the bootstrap-based test works well in small samples and may be superior to those of Hansen (1992) and Carrasco, Hu & Ploberger (2014) in terms of power and size, with much simpler computation requirements.

Di Sanzo's (2009) bootstrapping algorithm has the following steps. First, we estimated the model under H_0, obtaining the vector of parameters $\widehat{\theta_0}$ and the estimated structural residuals $\widehat{u_t}$. Note that, in our case of a SVAR model, we are interested in structural residuals, which theoretically are i.i.d and are calculated from the residuals estimated in reduced form. Second, we estimated the model under H_1 so as to calculate the following LR statistic:

$$LR = 2\big[L\big(\hat{\theta}|I_T\big) - L\big(\widehat{\theta_0}|I_T\big)\big]$$

Where $\hat{\theta}$ represents the estimate for the MS model, and $L(\theta|I_T)$ is the sample log-likelihood function conditional on the observed data I_T. The third step consists of the generation of bootstrap error series u_t^*, of size $T-1$, by sampling with replacement from the structural residuals $\widehat{u_t}$, and the calculation of the corresponding errors in reduced form $u_{RF,t}^*$. Afterwards, we built the bootstrap x_t^* sample, of size T, starting at $t = 2$, as:

$$x_t^* = \Omega_1 X_t^f + \Omega_2 X_{t-1}^f + \Omega_3 x_{t-1}^* + u_{RF,t}^*$$

The values of initial periods x_0^* and x_1^* were defined as the observed values for each series. Note that the expectations series X_t^f were considered to be strongly exogenous. Finally, in the fourth step, we used the bootstrap x_t^* sample as if it were observed data in order to calculate a new LR statistic. This value will be referred to as LR^*. The experiment consists in repeating through simulation the third and fourth steps for a large number B times, storing the distribution of the random variable LR^*. The bootstrapped p-value is then calculated as the fraction of LR^* values larger than the value initially observed for LR, i.e., $p_B = \sum_{i=1}^{B} Ind(LR_i^* \geq LR)/B$.

Figure 3: Bootstrap distribution of LR^*, based on Di Sanzo's (2009) method. Source: Authors' calculations. Note: Number of bootstrap samples generated=5000.
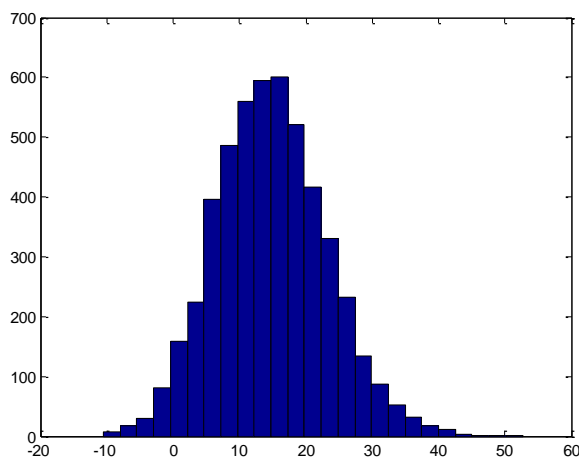

In our experiment, the likelihood ratio value between the two models is $LR = 21.2070$. We simulated the series x_t^* and calculated LR^* 5,000 times, i.e., for $B = 5000$. Figure 3 shows the bootstrap distribution obtained for LR^*. The result does not allow

rejecting the null hypothesis of linearity, as the calculated p-value is $p_B = 0.2136$, despite the better fit of the MS model.

4.3 Impulse response functions

The effect of independent structural shocks on endogenous variables can be represented by impulse response functions. Figure 4 displays the responses to inflation, output, interest rate, and exchange rate in each row for each structural shock.

Figure 4: Impulse response function for inflation, output, interest rate, and exchange rate shocks. Units are percentage points / 100. Source: Authors' calculations. Note: Shocks represent one standard deviation. Legend: A continuous blue line indicates the response in regime $S_t = 0$ ("Normal"); the dotted red line indicates the response in regime $S_t = 1$ ("Crisis").
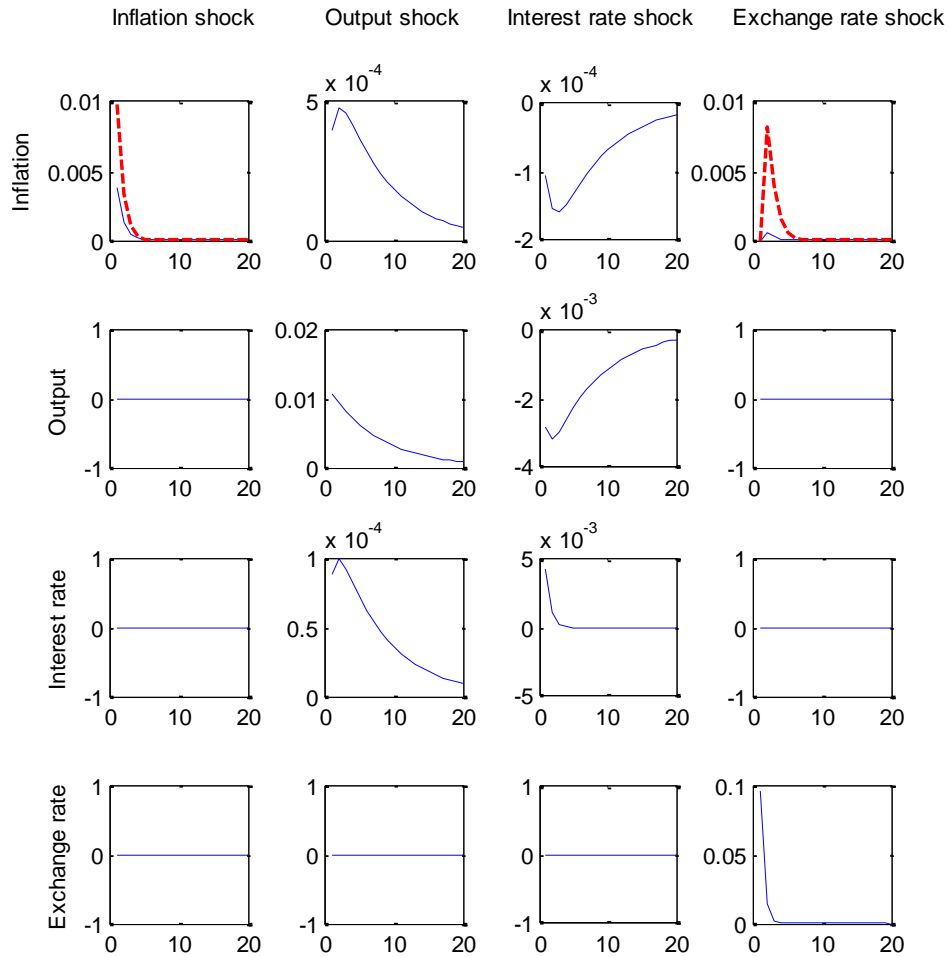
The responses, in general, are consistent with those expected from new Keynesian models. An unexpected shock to inflation does not cause reaction to the other variables, as it is assumed that both output and interest rate should react only to inflation expectations. Shock to output causes a rise in inflation, according to the Phillips curve,

and also a rise in the interest rate, according to the Taylor rule. There is high persistence, with an effect on the steady state even after 20 quarters. In turn, the unexpected shock to interest rate reduces output and inflation. Finally, exchange rate shock has an effect only on inflation.

As observed, regime switching causes different responses only to inflation, given the shocks on the AS curve and on the exchange rate. As regime $S_t = 1$ has a higher standard deviation of shocks on the AS curve, the effect on inflation is much larger in the first period. Likewise, the magnitude of exchange rate pass-through is much larger in regime $S_t = 1$; therefore, the effect on inflation is stronger and lasts for approximately twice as long. The behavior of each variable was calculated by assuming no regime switching after the shock.

4.4 Identification of high exchange rate pass-through regimes

One of the major results of the MS model is the identification of macroeconomic regimes, which, in our case, correspond either to the "normal" regime (low exchange rate pass-through and lower volatility of shocks to inflation) or the "crisis" regime (high exchange rate pass-through and larger volatility of shocks to inflation).

As previously commented, both regimes demonstrate strong persistence, with a slightly higher value attributed to the "normal" regime. In effect, the expected duration for the "normal" cycle is $E(D|S_t = 0) = 24.0$ quarters against $E(D|S_t = 1) = 22.7$ quarters for the "crisis" cycle.

The graphs in Figure 5 show filtered and smoothed probabilities for each regime throughout the period. Note that the probabilities tend to concentrate around 1 or zero most of the time, allowing for a plausible identification of regimes and confirming the usefulness of the model. We are able to clearly identify two periods of high exchange rate pass-through and high volatility of shocks to inflation, with duration between 6 and 14 quarters. Table 9 summarizes the information on the beginning and end of each period, as well as on exchange rate movements and accumulated inflation. The "crisis" regime totals 20 quarters, whereas the "normal" one extends for 44 consecutive quarters.

Table 9: Periods of high exchange rate pass-through and high volatility of shocks to inflation identified by the MS model. Source: Authors' calculations.

Beginning	End	Duration (quarters)	Identification	Larger exchange rate depreciation (in one quarter)	Inflation accumulated in the period (IPCA)
2000/q1	2003/q2	14	Internet bubble (USA) and domestic electoral crisis	31.4%	36.3%
2014/q3	2015/q4	6	Domestic political crisis	24.7%	12.9%

During each cycle of the "crisis" regime, we find events of large exchange rate depreciations, between 24.7% and 31.4%, in at least one of the quarters. Exchange rate shocks above certain limits is one of the arguments presented by Correa & Minella (2006) for the nonlinear behavior of the exchange rate pass-through, and this feature appears to be relevant here. Note that mean exchange rate depreciation per quarter in "crisis" regimes reaches 5.22% against a mean appreciation of 0.6% in "normal" regimes. Taken the whole sample, we observe a mean depreciation of 1.2% per quarter. Notwithstanding, we identified some quarters in which large exchange rate depreciations (above 10%) were not enough to characterize for a regime switching in the pass-through. In other words, in a few cases, the economy appears to be able to absorb the exchange rate shock without significant pass-through to inflation. For example, we mention quarters 2011/q3 and 2012/q2, which had exchange rate depreciations of 17.2% and 10.3%, respectively.

As expected owing to the volatility of shocks on the AS curve, the mean inflation per quarter during the "crisis" regime was 2.19%, a much higher value than the overall sample mean of 1.60%, or the "normal" regime mean of 1.34%. Note that quarterly inflation exceeded 2% in only one occasion (2004/q3) out of 44 in which the "normal" regime was active.

The first "crisis" cycle begins in 2000/q1 and lasts until mid-2003. The period is characterized by several events that affected confidence in the Brazilian economy. First, the transition to the floating exchange rate regime and the resulting sharp exchange rate depreciation of 1999 launched a sudden increase in inflation for the following quarters. Besides, the burst of the US stock market bubble for high-tech companies in 2000 triggered considerable uncertainty in international financial markets. Emerging market economies such as Brazil suffered capital outflows, exchange rate depreciations and increased country risk premium. Added to this, the political crisis in the neighbor country

Argentina, a major trade partner, was particularly severe in the years 2000-2001. To make things worse, the Brazilian economy was hit by a systemic shortage of energy between 2001-2002 which forced the government to cut up to 20% of consumption in some regions. All this facts affected the perception of country risk in Brazil. For instance, in the third quarter of 2000, the Brazilian exchange rate depreciation amounted to 8.42%, with a peak in inflation of 3.82%. In 2001/q3, we observe another strong exchange rate shock with further increase in country risk premium[9].

Over the year of 2002, the Brazilian presidential elections marked another round of harsh confidence crisis. Markets were skeptical about the economic policy intentions of the labour's party presidential candidate, which was winning by far at the opinion polls. The country risk premium increased considerably, reaching its peak in October 2002.[10] The domestic currency experienced more than 50% depreciation in 2002 whereas yearly inflation exceeded 12%. Indeed, inflation volatility only decreased at the end of 2003, as the new government's monetary and economic policies became consolidated as orthodox and adherent to the principles of inflation-targeting.

The following period, a long "normal" cycle, starts from mid-2003 and lasts until the end of 2014, totaling 44 quarters in a row. Note that the exchange rate had quite a significant appreciation from the beginning to the end of the period, 32% from 2003/q2 to 2014/q2, although we observe a few quarters with depreciations higher than 10%.

The fact that the exchange rate pass-through was low in that period is consistent with the assumption of Pimentel, Modenesi & Luporini (2015) about the asymmetric effect of exchange rate movements. These authors found that appreciations tend to exhibit a much lower pass-through level than depreciations of the same size. The total appreciation of 32% contributed to keep inflation in low levels during this period. What is a striking fact is how come depreciations greater than 10% in some quarters did not triggered a higher level of pass-through? For example, in the second and third quarters of 2008, during the onset of the international financial crisis, the MS model indicates an increase in the probability of a "crisis" cycle. The exchange rate depreciation reaches 42% between 2008/q3 and q4, even though inflation is kept at relatively low levels for both 2008 and the following year. As a result, the smoothed probabilities signal that the

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[9] The Embi+BR country risk premium index, measured by JP Morgan, had an average monthly value of 1,165 basis points in October 2001.

[10] The Embi+BR index had an average monthly value of 2,039 basis points in October 2002.

economy appears to continue under a "normal" cycle. Thus, according to the model, the sharp depreciation was not enough in order to substantially raise the level of pass-through. This finding is in contrast with Correa & Minella (2006) who argue that exchange rate shocks above a certain threshold would lead to a non-linear elevation on the level of pass-through. In our opinion, there are two possible explanations for this phenomena. First, the sudden negative shock in economic activity during 2008-2009 could be helping to alleviate inflation pressures, in line with the Phillips curve assumptions. Second, there appears to be a role for confidence. The "crisis" cycle is being signalled by the model during phases when the Brazilian economy appears to be in distress, as it was the case from 2000-2003 and during the final cycle described below.

Figure 5: Probabilities estimated for the MS model for Regime 0 (at the top) and Regime 1 (at the bottom). Source: Authors' calculations. Note: Filtered probabilities in bars and smoothed probabilities in lines.
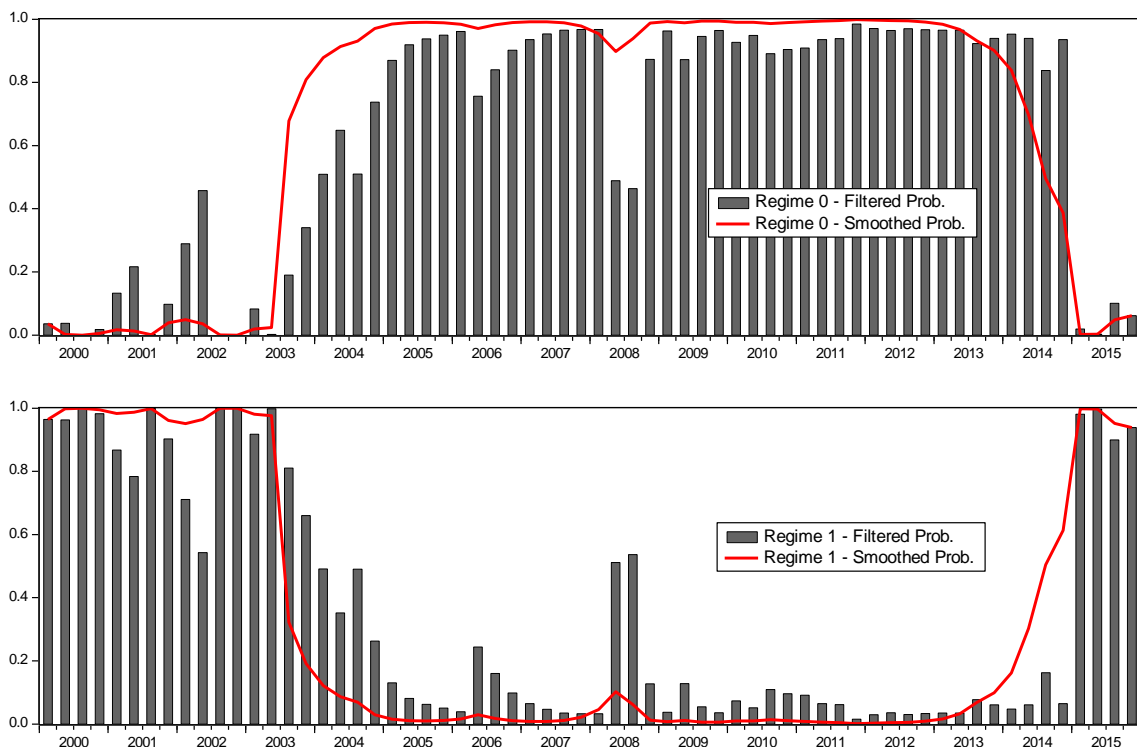
From the beginning of 2015, the MS model began to clearly indicate a new "crisis" cycle. We see a strong exchange rate depreciation (18.9%) in 2015/q1 and a constant rise in inflation levels. The period is characterized by a deep downturn on economic activity in Brazil, political crisis at the federal government level, and fiscal hardships. As a matter of fact, the high persistence of inflation combined with an expansionary government budget, though unsustainable, have hindered the actions of the monetary authority. Based

on model findings, one can expect that the 24.7% exchange rate shock of 2015/q3 will drag substantial effect on inflation during the following quarters.

5. Conclusion

The present paper investigates the exchange rate pass-through in the Brazilian economy during the floating exchange rate period using a semi-structural new Keynesian Markov-switching model. Our basic hypothesis is of nonlinear behavior in the pass-through coefficient, combined with regime switching in the volatility of shocks to the aggregate supply curve.

The estimated model consists of four basic equations: the aggregate supply (AS) curve or Phillips curve, the demand curve (IS), the reaction function of the monetary authority, and one equation to describe the exchange rate dynamics. We propose to model the exchange rate pass-through as a supply shock in the Phillips curve, following Blanchard & Galí (2007). Our specification and estimation strategy was based on Baele et al. (2015), although these authors analyse monetary policy regimes in the US in a closed economy setting. Hence, the original model was extended to account for pass-through effects.

In particular, the estimation approach of Baele et al. (2015) uses survey expectations for inflation and output, in addition to observed macroeconomic series. The authors argue that professional forecasters have a rich information set, which may be useful in improving the estimation. This reasoning is confirmed by Altug & Çakmakli (2016) who found a high predictive power of survey-based expectations for inflation in Brazil. The method proposed by Baele et al. (2015) allows us to transform the Markov-switching model in a structural MS-VAR model, which can thus be estimated through conventional multivariate methods such as Hamilton (1989, 1994) and Kim & Nelson (1999).

Our empirical results indicate that the exchange rate pass-through assumed two possible states, or regimes, during the period. In the first regime, conveniently referred to as "normal", the pass-through is very low and statistically nonsignificant while, at the same time, the volatility of shocks to inflation is also relatively low. On the other hand, in the second regime the pass-through is relevant and significant, of about 10.3% in the long run, while the volatility of shocks to inflation is also relatively higher. The high pass-

through regime was named "crisis" as it appears to occur in periods when the Brazilian economy was facing different types of distress, or confidence crisis. The MS model outperformed the linear specification by some usual econometric criteria, such as the Schwartz criterion.

More specifically, the long-run effect estimated for the pass-through during "crisis", amounts to 0.1035 percentage point for consumer inflation, given a 1% exchange rate shock, compared to near zero effect in the "normal" regime. The standard deviation of the shocks on the AS curve is estimated at 0.0096 for the "crisis" regime against 0.0043 in the normal regime. The transition probabilities of the Markov chain indicate high persistence of both regimes, with a slightly longer mean expected duration for "normal" cycles.

Our estimates for the nonlinear exchange rate pass-through in the Brazilian economy are consistent with those obtained by Carneiro, Monteiro & Wu (2004), Correa & Minella (2006), and Pimentel, Modenesi & Luporini (2015), taking into account the differences between the periods of interest and the estimation method. It should be highlighted that the long-run pass-through, of 10.3% even in the "crisis" regime, is relatively low according to the criteria set by Goldfajn & Werlang (2000) and Belaisch (2003), which implies some reasonable capacity of the economy to absorb exchange rate shocks without greater effect to consumer inflation.

The presence of regime switching in the volatility of shocks to inflation could be related to the heteroskedasticity of inflation itself (Engle, 1982; Brunner & Hess, 1993) or to theoretical arguments, such as Ball & Cecchetti's (1990) and Owyang's (2001). These authors point out that higher inflation levels lead to higher volatility and greater uncertainty over future inflation expectations. That is, unexpected inflation shocks increase uncertainty over future inflation and causes larger volatility in inflation in the subsequent periods. The system would tend to remain in a high volatility regime for some periods, which is well described in our findings.

The econometric specification tests indicate that the estimation method could not eliminate serial autocorrelation of standardized residuals, especially from the interest rate curve. We recognize that due to the simplicity of the method it cannot capture a possible autoregressive structure of the shocks in each equation. Even Baele et al. (2015) faced this problem in their empirical exercise. Moreover, the MS model was submitted to Di

Sanzo's (2009) bootstrapping linearity test, which could not reject the null hypothesis of linearity. In summary, we should be cautious when interpreting the results.

Most of the remaining structural parameters of the MS model yielded results in line with theory. The AS curve parameters confirm the relevance of inflation expectations and also indicate endogenous persistence. However, we cannot confirm a statistically significant influence of output gap on inflation. The IS curve shows a smaller effect of output gap expectations, and at the same time higher persistence of output. The response of output to the real interest rate is remarkable, as expected, indicating an efficient monetary policy transmission mechanism. The Taylor rule parameters reveal an active response of the interest rate to inflation expectations. We note that our estimates differ from the usual literature as we use an interest rate measure that is adjusted for the long term real interest rate.

The present study innovates in terms of methodology by using a semi-structural MS model for the identification of nonlinearity of exchange rate pass-through in the Brazilian economy. Previous studies usually sought to measure exchange rate pass-through directly in the Phillips curve (Carneiro, Monteiro & Wu, 2004; Correa & Minella, 2006; Nogueira Jr, 2010; Pimentel, Modenesi & Luporini, 2015) or in regressions specifically derived from microfoundations, such as in Albuquerque & Portugal (2005). From an econometric perspective, the literature utilizes nonlinear least squares (Carneiro, Monteiro & Wu, 2004), threshold models (Correa & Minella, 2006), smooth transition regression (Nogueira Jr, 2010), asymmetric SVAR models (Pimentel, Modenesi & Luporini, 2015), or models with variable parameters (Albuquerque & Portugal, 2005). Our review of the extant literature did not find any publication that has used regime switching models to assess exchange rate pass-through in the Brazilian economy, either for modeling of the Phillips curve only or in larger models. On the other hand, our study differs from that of Baele et al. (2015) as it assesses exchange rate pass-through instead of focusing on the monetary policy rule. To achieve that end, we extended the original model by adding new elements to the AS curve and a new equation to describe the exchange rate dynamics.

We make a novel contribution by identifying three phases, or cycles, for the pass-through behaviour in the Brazilian economy during the inflation targeting period. Under our interpretation, two "crisis" cycles, one at the beginning of the sample period (2000-2003) and the other at the end (2015), are separated by a long "normal" cycle (2003-

2014). The "crisis" cycles appear to occur during periods where the Brazilian economy is under different types of stress. In some cases, the economic strain was due to external factors, such as the emerging markets confidence runs of 2000-2001. In other cases, political facts such as the crisis of 2002, triggered by uncertainty during the presidential elections, or the political crisis of 2015, appear to be related to exchange rate shocks complemented with a higher level of pass-through.

We conclude that the overall results of the MS model are useful to the economic analysis and interpretation of the exchange rate pass-through dynamics and its nonlinear effects to a great extent. The model have shown to provide relevant information for inflation forecast, especially during large exchange rate shocks, when there is more uncertainty about the effect of the pass-through. For example, the econometrician forecasting inflation with a linear model would assume a 7.6% level of pass-through in the long run. On the other hand, if the econometrician is using a nonlinear model of our kind, there is room for a more subtle interpretation. If the economy is under a "crisis" regime, the MS model indicates that the expected pass-through is substantially higher, of 10.35%. However, under a "normal" regime, one could expect a long run pass-through roughly null, of 0.57%. The difference between the two model types carry obvious consequences for policy analysis and design.

Finally, our findings underscore the importance of assessing regime switching in certain structural parameters of the Brazilian economy in new Keynesian models, corroborating, to some extent, Gonçalves, Portugal & Arágon (2016). However, there exist some limitations concerning the estimation method and the simple basic model used, with just four equations. The extension to a more complete open economy model, such as proposed by Adolfson et al. (2007) or Choudhri & Hakura (2015), and the application of more sophisticated econometric techniques such as those of Foerster et al. (2014) would allow for more robust conclusions, and are thus suggestions for future research.

References

ADOLFSON, M.; LASÉEN, S.; LINDÉ, J.; VILLANI, M. (2007) Bayesian estimation of an open economy DSGE model with incomplete pass-through. *Journal of International Economics*, v.72, p. 481-455, 2007.

ALBUQUERQUE, C. R.; PORTUGAL, M. S. (2005) Pass-through from exchange rate to prices in Brazil: an analysis using time-varying parameters for the 1980-2002 period. *Revista de Economia*, vol 12, n. 1, Banco Central del Uruguay, Mai. 2005.

ALTUG, S.; ÇAKMAKLI, C. (2016) Forecasting inflation using survey expectations and target inflation: evidence for Brazil and Turkey. *International Journal of Forecasting*, v. 32, issue 1, pp. 138-153, Jan.-Mar. 2016.

ALVES, S. A. L.; AREOSA, W. D. (2005) Targets and inflation dynamics. *Banco Central do Brasil Working Papers Series,* n. 100, Oct. 2005.

AREOSA, W. D.; MEDEIROS, M. (2007). Inflation dynamics in Brazil: the case of a small open economy. *Brazilian Review of Econometrics*, Vol. 27, n. 1, 2007, pp.131-166.

ARRUDA, E.; FERREIRA, R.; CASTELAR, I. (2008) Modelos lineares e não lineares da curva de Phillips para previsão da taxa de inflação no Brasil. *Anais do XXXVI Encontro Nacional de Economia*, Salvador, BA, Brasil. Dez. 2008.

BAELE, L.; BEKAERT, G.; CHO, S.; INGHELBRECHT, K.; MORENO, A. (2015) Macroeconomic regimes. *Journal of Monetary Economics*, v. 70, Mar. 2015, pp. 51-71.

BALL, L.; CECCHETTI, S. (1990) Inflation and uncertainty at short and long horizons. *Brookings Papers on Economic Activity*, v. 1, pp. 215-54, 1990.

BELAISCH, A. (2003) Exchange rate pass-through in Brazil. *IMF Working Papers*, WP/03/141, Jul. 2003.

BELLONE, B. (2005) Classical estimation of multivariate Markov-switching models using MSVARlib. *EconWPA*, Econometrics 0508017, 2005.

BIANCHI, F. (2013). Regime switches, agents' beliefs, and post-World War II U.S. macroeconomic dynamics. *Review of Economic Studies*, 80 (2), pp. 463-490.

BLANCHARD, O.; GALÍ, J. (2007) Real wage rigidities and the New Keynesian model. *Journal of Money, Credit and Banking*, v. 39, pp. 35-65, 2007.

BOX, G. E. P.; JENKINS, G. M. ; REINSEL, G. C. (1994) *Time Series Analysis: forecasting and control*. 3rd ed. Englewood Cliffs, NJ: Prentice Hall, 1994.

BRUNNER, A. D.; HESS, G. D. (1993) Are Higher Levels of Inflation Less Predictable? A State-Dependent Conditional Heteroscedasticity Approach. *Journal of Business & Economic Statistics*, vol. 11, Issue. 2, 1993.

CARNEIRO, D. D.; MONTEIRO, A. M. D.; WU, T. Y. H. (2004) Mecanismos não-lineares de repasse cambial para o IPCA. *Revista de Economia e Administração*, v.3, n.1, pp.01-14, Jan-Mar 2004.

CARRASCO, M.; HU, L.; PLOBERGER, W. (2014) Optimal test for Markov switching parameters. *Econometrica*, v. 82, n. 2, Mar. 2014, pp. 765-784.

de CASTRO, M. R.; GOUVEA, S. N.; MINELLA, A.; SANTOS, R.; SOUZA-SOBRINHO, N. F. SAMBA: Stochastic Analytical Model with a Bayesian Approach. *Brazilian Review of Econometrics*, v. 35, n. 2, 2015.

CENTRAL BANK OF BRAZIL (2015). *Relatório de Inflação (Inflation Report)*. Brasilia: Banco Central do Brasil, v. 17, n. 2, Jun. 2015.

CHO, S. (2014) Characterizing Markov-switching rational expectation models. *University of Indiana Economics Department Spring Working Papers*. Available at: http://www.indiana.edu/~econdept/workshops/Spring_2013_Papers/SEONGHOON%20CHO.pdf.

CHO, S.; MORENO, A. (2011) The forward method as a solution refinement in rational expectations models. *Journal of Economic Dynamics and Control*, v. 25 (3), pp. 257-327.

CHOI, J.; HUR, J. (2015) An examination of macroeconomic fluctuations in Korea exploiting a Markov-switching DSGE approach. *Economic Modelling*, v. 51, Dec. 2015, pp. 183-199.

CHOUDHRI, E. U.; HAKURA, D. S. (2015) The exchange rate pass-through to import and export prices: the role of nominal rigidities and currency choice. *Journal of International Money and Finance*, v. 51, Mar. 2015, pp. 1-25.

CORREA, A. S.; MINELLA, A. (2006) Nonlinear mechanisms of the exchange rate pass-through: a Phillips Curve model with threshold for Brazil. *Banco Central do Brasil Working Papers Series,* n. 122, Nov. 2006.

DI SANZO, S. (2009) Testing for linearity in Markov switching models: a bootstrap approach. *Statistical Methods and Applications*, v. 18, issue 2, Jul. 2009, pp. 153-168.

DIXIT, A. (1989) Hysteresis, import penetration, and exchange rate pass-through. *The Quarterly Journal of Economics*, v. 104, n. 2, pp. 205-228, May 1989.

DONAYRE, L.; PANOVSKA, I. (2016) State-dependent exchange rate pass-through behavior. *Journal of International Money and Finance*, v. 64, Jun. 2016, pp. 170-195.

ENGLE, R. F. (1982) Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, vol. 50, no. 4, July, 1982, pp.987-1007.

FARMER, R. E. A.; WAGGONER, D. F.; ZHA, T. (2009) Understanding Markov-switching rational expectations models. *Journal of Economic Theory*, v. 144, pp. 1849-1867, 2009.

FARMER, R. E. A.; WAGGONER, D. F.; ZHA, T. (2010) Generalizing the Taylor principle: a comment. American Economic Review, 100 (1), pp. 608-617, 2010.

FARMER, R. E. A.; WAGGONER, D. F.; ZHA, T. (2011) Minimal state variable solutions to Markov-switching rational expectations models. *Journal of Economic Dynamics & Control*, v. 35, pp. 2150-2166, 2011.

FASOLO, A. M.; PORTUGAL, M. S. (2004) Imperfect rationality and inflationary inertia: a new estimation of the Phillips curve for Brazsil. *Estudos Econômicos*, v. 34, n. 4, pp. 725-779, Out-Dez, 2004.

FERNÁNDEZ-VILLAVERDE, J. (2009) The econometrics of DSGE models. *NBER Working Paper*, No. 14677, Jan. 2009.

FERNÁNDEZ-VILLAVERDE, J.; GUERRÓN-QUINTANA, P.; RUBIO-RAMÍREZ, J. F. (2010) Fortune or virtue: time-variant volatilities versus parameter drifting in U.S. data. *NBER Working Paper*, No. 15928, Apr. 2010.

FIGUEIREDO, F. M. R.; GOUVEA, S. (2011) Repasse Cambial para a Inflação: o papel da rigidez de preços. In: BANCO CENTRAL DO BRASIL (2011). *Dez Anos de Metas para Inflação 1999-2009*. Brasília: Banco Central do Brasil, 2011.

FOERSTER, A.; RUBIO-RAMÍREZ, J. F.; WAGGONER, D.; ZHA, T. (2014) Perturbation methods for Markov-switching models. *NBER Working Paper*, No. 20390, Aug. 2014.

FURLANI, L. G. C.; PORTUGAL, M. S.; LAURINI, M. P. (2010) Exchange rate movements and monetary policy in Brazil: econometric and simulation evidence. *Economic Modelling*, v. 7, issue 1, PP. 284-295, Jan. 2010.

GALÍ, J. (2008) *Monetary Policy, Inflation, and the Business Cycle: an introduction to the New Keynesian framework*. Princeton University Press: Princeton, NJ, 2008.

GOLDFAJN, I.; WERLANG, S. R. C. (2000) The pass-through from depreciation to inflation: a panel study. *Banco Central do Brasil Working Papers Series,* n. 5, Set. 2000.

GONÇALVES, C. C. S.; PORTUGAL, M. S.; ARÁGON, E. K. S. B. (2016) Assessing Brazilian macroeconomic dynamics using a Markov-switching DSGE model. *EconomiA*, Mar. 2016.

HAMILTON, J. D. (1989) A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, v. 57, n. 2, pp 357-384, Mar. 1989.

HAMILTON, J. D. (1994) *Time Series Analysis*. Princeton University Press, 1994.

HAMILTON, J. D. (2005) What's real about the Business Cycle? Federal Reserve Bank of St. Louis Review, 87 (4), pp. 435-452, Jul-Aug 2005.

HAMILTON, J. D. (2014) Macroeconomic regimes and regime shifts, in Taylor, J. B. & Uhlig, H. (Eds.) *Handbook of Macroeconomics*, Vol. 2.

HANSEN, B. E. (1992) The likelihood ratio test under nonstandard conditions: testing the Markov switching model of GNP. *Journal of Applied Econometrics*, v. 7, Dec. 1992, pp. S61-S82.

HOLMES, M. J. (2009) The response of exchange rate pass-through to the macroeconomic environment. *The Open Business Journal*, v. 2, pp. 1-6, 2009.

IBOSHI, H. (2016) Monetary policy regime shifts under the zero lower bound: an application of a stochastic rational expectations equilibrium to a Markov switching DSGE model. *Economic Modelling*, v. 52, part A, Jan. 2016, pp. 186-205.

KHEMIRI, R.; ALI, M. S. B. (2012) Exchange rate pass-through and inflation dynamics in Tunisia: a Markov-switching approach. *Economics E-journal Discussion Paper*, n. 2012-39, Aug., 2012.

KOHLSCHEEN, E. (2010) Emerging Floaters: pass-throughs and (some) new commodity currencies. *Banco Central do Brasil Working Papers Series,* n. 224, Nov. 2010.

KROLZIG, H.M. (1997) *Markov-Switching Vector Autoregressions: modelling, statistical inference and application to business cycle analysis*. Springer-Verlag: Berlin, 1997.

LARUE, B.; GERVAIS, J.; RANCOURT, Y. (2010) Exchange rate pass-through, menu costs and threshold cointegration. *Empirical Economics*, v. 38, issue 1, Feb. 2010, pp. 171-192.

LIU, P.; MUMTAZ, H. (2010) Evolving macroeconomic dynamics in a small open economy: an estimated Markov-switching DSGE model for the United Kingdom. *Bank of England Working Paper* No. 397.

MAIH, J. (2015) Efficient perturbation methods for solving regime-switching DSGE models. *Norges Bank Working Paper*, Jan. 2015.

MANKIW, N. G.; REIS, R. (2002) Sticky information versus sticky prices: a proposal to replace the new Keynesian Phillips curve. *Quarterly Journal of Economics*, 117 (4), pp. 251-301.

MAZALI, A. A.; DIVINO, J. A. (2010) Real wage rigidity and the new Phillips curve: the Brazilian case. *Revista Brasileira de Economia*, v.64, n.3, pp. 291-306, Jul-Sep 2010.

MINELLA, A.; DE FREITAS, P. S.; GOLDFAJN, I.; MUINHOS, M. K. (2003) Inflation Targeting in Brazil: Constructing Credibility under Exchange Rate Volatility. *Banco Central do Brasil Working Papers Series,* n. 77, Jul. 2003.

NOGUEIRA JR, R. P. (2010) Inflation environment and lower exchange rate pass-through in Brazil: is there a relationship? *Revista Brasileira de Economia*, v. 64, n. 1, pp. 49-56, Jan.-Mar. 2010.

OLIVEIRA, L.; FEIJÓ, F. T. (2015) Curva de Phillips com mudança de regime markoviano: uma análise da economia brasileira para o período 1995-2014. *Anais do XLIII Encontro Nacional de Economia da ANPEC*. Florianópolis, SC, Brasil. Dez., 2015.

OWYANG, M. T. (2001) Persistence, excess volatility, and volatility clusters in inflation. *Federal Reserve Bank of St. Louis Review*, v. 83, n. 6, pp. 41-52, Nov-Dec 2001.

PALMA, A. A.; PORTUGAL, M. S. (2014) Preferences of the Central Bank of Brazil under the inflation targeting regime: estimation using a DSGE model for a small open economy. *Journal of Policy Modeling*, v. 36, issue 5, pp. 824-839, Sep-Oct 2014.

PIMENTEL, D. M.; MODENESI, A. M.; LUPORINI, V. (2015) Assimetrias no repasse cambial para a inflação: uma análise empírica para o Brasil (1999 a 2013). *Anais do XVIII Encontro de Economia da Região Sul ANPEC Sul*. Porto Alegre, RS, Brasil. Jun., 2015.

RAZAFINDRABE, T. M. (2016) A multi-country DSGE model with incomplete exchange rate pass-through: an application for the Euro-area. *Economic Modelling*, v. 52, part A, Jan. 2016, pp. 78-100.

RODRIGUES, W. G.; MORI, R. (2015) Mudanças de regimes na função de reação do Banco Central do Brasil: uma abordagem utilizando Markov regime switching. *Anais do XLIII Encontro Nacional de Economia da ANPEC*. Florianópolis, SC, Brasil. Dez., 2015.

ROMER, D. (2011) *Advanced Macroeconomics*, 4th Edition. McGraw-Hill: New York, 2011.

SACHSIDA, A. (2013) Inflação, desemprego e choques cambiais: uma revisão da literatura sobre a Curva de Phillips no Brasil. *Revista Brasileira de Economia*, v. 67, n.4, pp. 521-531, Out.-Dez. 2013.

SILVEIRA, M. A. C. (2008) Using a Bayesian approach to estimate and compare New Keynesian DSGE models for the Brazilian economy: the role for endogenous persistence. *Revista Brasileira de Economia*, v. 62, n. 3, pp. 333-357, Jul-Sep 2008.

SIMS, C. (2002) Solving linear rational expectations models. *Computational Economics*, v. 20 (1-2), pp. 1-20.

SIMS, C.; ZHA, T. (2006) Were there regime switches in U.S. monetary policy? *American Economic Review*, 91(1), pp.54-81.

TAYLOR, J. (2000) Low inflation, pass-through and the pricing power of firms. *European Economic Review*, 44, pp. 1389-1408, 2000.

TOMBINI, A. A.; ALVES, S. A. L. (2006) The recent Brazilian disinflation process and costs. *Banco Central do Brasil Working Papers Series,* n. 109, Jun. 2006.

TRISTÃO, T. S.; TORRENT, H. S. (2015) Relações não lineares na curva de Phillips: uma abordagem semi-paramétrica. *Economia Aplicada*, v. 19, n. 4, 2015, pp. 679-703.

VIEIRA, H. P.; PEREIRA, P. V. (2013) A study of the Brazilian business cycles (1900-2012). *Brazilian Review of Econometrics*, v. 33, no 2, pp. 123-143, Nov. 2013.

WINKELRIED, D. (2014) Exchange rate pass-through and inflation targeting in Peru. *Empirical Economics*, v. 46, issue 4, Jun. 2014, pp. 1181-1196.