Do Central Bank Actions Reduce Interest Rate Volatility?

Jaqueline Terra Moura Marins and José Valentim Machado Vicente

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Editor: Francisco Marcos Rodrigues Figueiredo – E-mail: francisco-marcos.figueiredo@bcb.gov.br

Co-editor: José Valentim Machado Vicente – E-mail: jose.valentim@bcb.gov.br

Editorial Assistant: Jane Sofia Moita – E-mail: jane.sofia@bcb.gov.br

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Comun/Divip
SBS – Quadra 3 – Bloco B – Edifício-Sede – 2º subsolo
Caixa Postal 8.670
70074-900 Brasília – DF – Brazil
Phones: +55 (61) 3414-3710 and 3414-3565
Fax: +55 (61) 3414-1898
E-mail: identidadevisual.comun@bcb.gov.br

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Non-technical Summary

The main goal of this paper is to analyze the influence of actions of the Central Bank of Brazil (CBB) on market uncertainty. Unlike related papers, we measure market uncertainty by the volatility extracted from interest rate options. Options are derivatives contracts whose price is directly connected to the future volatility of the underlying. In the financial literature, volatility extracted from option prices is known as implied volatility. Implied volatility has some advantages over the volatility estimated by traditional time series models. First, implied volatility is forward-looking. Second, it can be obtained without any assumption about the price dynamics of the underlying asset.

More specifically, we investigate what happens to the interest rate volatility around two events: the basic interest rate target decision and the release of the minutes of the corresponding meeting. CBB’s Monetary Policy Committee (Copom) sets the interest rate target in regular meetings. Shortly after each of these meetings, Copom issues a statement where the target rate is revealed. One week later, the minutes are released, which include the Copom’s assessment of the economic outlook. As the dates of these events are known in advance, but not the Copom’s decisions, we expect interest rate uncertainty to be affected by them.

Our sample period begins in January 2006 and ends in March 2015. During these years, we observed episodes of monetary policy expansion, as well as of monetary policy contraction. The resulting effect was a period of decreasing interest rates. Our results show that Copom’s decisions significantly reduce interest rate volatility around the dates of meetings. Moreover, this effect holds until the release dates of the minutes.
Sumário Não Técnico

O objetivo principal deste artigo é analisar a influência das atuações do Banco Central do Brasil (BCB) sobre a incerteza do mercado. Diferentemente de artigos correlatos, medimos a incerteza do mercado pela volatilidade extraída das opções de taxa de juros. Opções são contratos derivativos cujo preço está diretamente conectado com a volatilidade do ativo objeto. Na literatura de finanças, a volatilidade extraída de preços de opções é conhecida como volatilidade implícita. A volatilidade implícita tem vantagens sobre aquelas estimadas por modelos tradicionais de séries de tempo. Primeiro, ela é forward-looking. Segundo, ela pode ser obtida sem qualquer hipótese sobre a dinâmica do preço do ativo objeto.

Mais especificamente, investigamos o que acontece com a volatilidade da taxa de juros em torno de dois eventos: a decisão acerca da meta da taxa básica de juros e a publicação das notas da reunião correspondente. O Comitê de Política Monetária do BCB (Copom) estabelece a meta da taxa básica de juros em reuniões regulares. Pouco depois de cada uma dessas reuniões, o Copom emite um comunicado em que a meta da taxa é revelada. Uma semana depois, as notas da reunião são divulgadas, as quais incluem uma avaliação do Copom sobre as perspectivas econômicas. Como as datas desses eventos são conhecidas antecipadamente, mas não as decisões do Copom, esperamos que a incerteza da taxa de juros seja afetada por eles.

Nosso período de amostragem começa em janeiro de 2006 e termina em março de 2015. Durante esses anos, observamos tanto episódios de política monetária expansionista quanto de contracionista. O efeito resultante foi um período de taxas de juros decrescentes. Nossos resultados mostram que decisões do Copom reduzem significativamente a volatilidade da taxa de juros em torno das datas das reuniões. Além disso, este efeito é válido até as datas de divulgação das notas das reuniões.
Do Central Bank Actions Reduce Interest Rate Volatility?

Jaqueline Terra Moura Marins*

José Valentim Machado Vicente†

Abstract

This paper investigates how Central Bank of Brazil (CBB) actions influence market uncertainty. We consider two kinds of actions: the monetary policy decision about the interest rate target and the pure communication event of minutes release one week later. Unlike related papers, we measure market uncertainty by the implied volatility extracted from interest rate options. Implied volatility is more suitable than physical volatility to assess economic effects since it encompasses market beliefs adjusted by risk. We use an event study approach to evaluate the impact of CBB actions. The results show that both decisions about the target rate and communication event reduce interest rate volatility.

Keywords: Copom meeting, uncertainty, risk-neutral density, interest rate option.


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*Central Bank of Brazil e-mail: jaqueline.terra@bcb.gov.br
†Central Bank of Brazil e-mail: jose.valentim@bcb.gov.br
1 Introduction

The main goal of this paper is to analyze the influence of actions of the Central Bank of Brazil (CBB) on market uncertainty. We investigate what happens to the interest rate volatility around two events: the basic interest rate target decision and the release of the minutes of the corresponding meeting. CBB’s Monetary Policy Committee (Copom) sets the interest rate target in regular meetings. Shortly after each of these meetings, Copom issues a statement where the target rate is revealed. One week later, the minutes are released, which include the Copom’s assessment of the economic outlook. As the dates of these events are known in advance, but not the contents, we expect yield curve uncertainty to be affected by them.

There are a number of models devoted to gauging volatility of financial variables. Many of them rely on time series data. In this paper, we follow a different approach and use implied volatility as a measure of uncertainty. Implied volatility has many advantages over physical/historical volatility estimated by time series models. First, it is forward-looking, reflecting the aggregate opinion about the dispersion of future prices. However, implied volatility is connected with the risk-neutral distribution, which encompasses market beliefs (likelihood of the states of nature) adjusted by risk aversion. Therefore, it differs from physical volatility by the risk premium. Although the physical distribution is useful in forecasting exercises, according to Aït-Sahalia and Lo (2000) the risk-neutral probabilities are more suitable to evaluate economic impacts, since they contain fundamental information about time preferences. Moreover, as pointed out by Bali, Cakici and Chabi-Yo (2011), it is very difficult to estimate physical probabilities. On the other hand, the risk-neutral density can be easily obtained from option prices, as demonstrated by Breeden and Litzenberger (1978).\footnote{We emphasize that the effects of CBB actions on interest rate volatility measured in this paper do not allow us to judge the quality of the Central Bank’s communication or of the conduct of monetary policy. To do this, it would be necessary to differentiate volatility changes derived from fundamental factors, which central banks should not struggle against, from volatility changes generated by policy, which should be considered unnecessary and therefore minimized. Here we will only try to show if some significant change happens to the interest rate volatility when Central Bank actions related to Copom decisions occur.}

Besides the second moment of the interest rate risk-neutral distribution, we also analyze the behavior of the mean around CBB actions. There are two reasons to consider the first moment. The mean is directly observed from interest rate future contracts, which are more liquid than options. Moreover, the mean is a central measure, so its values are connected to next steps of monetary policy. An expectation of increase in the basic interest rate (which means that the first moment is greater than the current basic rate) is related to a monetary policy tightening while an expectation of reduction indicates an monetary policy easing.

We use event study approach as the statistical technique to evaluate the impact of CBB’s statement and minutes’ release on the first two moments of the interest rate risk-neutral distribution. Our sample period begins in January 2006 and ends in March 2015. During...
these years, we observed episodes of monetary policy expansion, as well as of monetary policy contraction. The resulting effect was a period of decreasing interest rates.

Our results show that Copom’s decisions significantly reduce the level and the time dynamics of interest rate volatility around the dates of meetings. Moreover, this effect holds until the release dates of the minutes. Regarding interest rate mean, we find that around statement dates in which the Copom increases the target rate, the one-year yield rises. For the case of target reduction decisions, the statement is more effective to alter the level than the time dynamics of the interest rate. However, one week later, when minutes are released, we do not note a persistence of the effects of the target rate decisions on the mean of the interest rate risk-neutral distribution.

Literature about the effects of monetary policy actions on financial market is large.\(^2\) However, fewer papers have examined the reaction of options market to central bank announcements. Bhar and Chiarella (2000) investigate the behavior of option-implied distributions of short-term interest rates around four interest rate reductions conducted by the Reserve Bank of Australia. They find that the probability of a decline in interest rates increases before the central bank rate reductions, suggesting that market participants anticipate the forthcoming interest rate cut. Mandler (2003) extracts risk-neutral probability density functions from LIFFE-Euribor futures options and looks for characteristic differences in market expectations related to meetings of the European Central Bank (ECB). He also shows that monetary policy meetings of the ECB tend to decrease uncertainty of market expectations. Vahamaa (2005) focuses on the skewness of option-implied distributions around ECB monetary policy actions. He finds that asymmetries in market expectations tend to increase before changes in monetary policy stance and to decrease afterwards. Breeden and Litzenberger (2014) estimate state prices from 3-month interest rate options around major actions of the Federal Reserve Board and the European Central Bank in the 2008-2013 period, a time of historic levels of central bank interventions. They show that these policy actions do affect the probability distribution of future interest rates. Sinha (2015) uses options data to extract state-price densities of investor beliefs. He finds that announcements about extension of the zero-lower bound policy in U.S. during 2012-13 reduced expectations about crash risk, but increased the uncertainty about future long-term yields.

Although all of these works investigate the impacts of central bank communication on the interest rate risk-neutral distribution, there are at least two new aspects which we analyze here. To our knowledge, we are the first to study the effects on yield curve risk-neutral distribution in an emerging economy. According to Papadamou et al (2015), central bank’s transparency is more relevant for monetary policy transmission in emerging markets than in the developed world. Moreover, as pointed out by Mendonça and Faria (2013), Brazil is a

\(^2\)Birru and Figlewski (2010) review this literature, focusing on Federal Open Market Committee’s (FOMC) decisions.
preferred laboratory experiment as an emerging economy since it has a developed financial market and successful inflation targeting regime. Second, we extract the implied volatility from Asian interest rate options instead of swaptions. This is an interesting feature since we can directly associate this volatility with the volatility of the yield with the same maturity of the option. ³

Regarding the Brazilian case, some papers study the influence of CBB decisions on financial markets. Costa Filho and Rocha (2010) find that interest rates increase while volatility decreases after the minutes’ release. Mendonça and Faria (2013) show that an increase of CBB’s transparency improves the efficiency of the expectation hypothesis and the anticipation of changes in the interest rate target. Carvalho, Cordeiro and Vargas (2013) point out that CBB’s statements have meaningful effects on yields at short-to-medium maturities. However, this effect stopped from 2011 to 2015. Cabral and Guimarães (2015) show that CBB’s statements influence the yield curve and the stock market index. In addition, they help to predict the content of the minutes. Notwithstanding the fact that all of these papers are devoted to the Brazilian case and share similar conclusions, unlike ours they do not analyze the impact of CBB’s actions on risk-neutral distribution, in particular on interest rate volatility. Moreover, we take into account two central bank actions: a monetary policy decision (statements of Copom meetings) and a pure communication event (minutes’ release). Therefore, we can compare the market reactions to these two kinds of actions. Finally, we use an event study approach with different window sizes, which allows investigating persistency of the effects of CBB’s actions on yield curve. According to Campbell, Lo and Mackinlay (1997), event study is an appropriate method to analyze announcements of macroeconomic variables.

The reminder of this paper is organized as follow. The next section presents the dataset considered in this paper, followed by a brief description of the methods used to calculate model-free implied volatilities of interest rate. It also describes the event study approach, the technique employed to test the impact of Copom decisions on implied volatilities. Section 3 presents results and Section 4 provides concluding remarks.

2 Methodology

In this section, we first describe the database we use to calculate model-free risk-neutral volatilities of interest rate distribution. Then we discuss how we extract these volatilities from the price of interest rate options. Finally, we present the statistical technique used to test the impact of Copom decisions on the moments of this distribution.

³Let t be the present date. The underlying asset of an interest rate swaption maturing at u > t is the yield between u and T (T > u). On the other hand, the underlying asset of the main Brazilian interest rate option maturing at u is the cumulative one-day rate between t and u. To the best of our knowledge, this kind of option is only available in the Brazilian financial market.
2.1 Data description

Copom sets the Selic interest rate target. The Selic rate is the interest rate for overnight inter-
bank loans collateralized by government bonds traded in the Selic system (Special Settlement
and Custody System). It is the basic rate used as reference for the monetary policy conducted
by CBB. To obtain the risk-neutral probability distribution of the interest rate, we use option
contracts whose underlying asset is another interest rate, known as DI rate, since there are
no option contracts defined on the Selic rate. The DI rate is an interbank deposit rate repre-
sented by the average of one-day loan rates for transactions between financial institutions. It
is calculated by Cetip (Center for Custody and Financial Settlement of Securities), a company
that offers services related to registration, deposit, trading and settlement of bonds and other
securities in the Brazilian market. The Selic and the DI rates are closely related, since both
are overnight interest rates, but the DI rate reflects interbank transactions not guaranteed by
government bonds.

The one-day interbank deposit future contract (DI future for short) with maturity $T$ is a
future contract whose underlying asset is the cumulative DI daily rate between $t$ and $T$. This
contract is very similar to a zero-coupon bond, except that investors’ positions are updated
daily. Each daily cash flow is the difference between the settlement price on the current day
and the settlement price on the day before, corrected by the DI rate of the day before. The
settlement price at $t$ is the amount of R$ 100,000.00 discounted by the closing interest rate of
the contract. The DI future is offered by BM&FBovespa exchange and is the main derivative
in the Brazilian market.\footnote{According to BM&FBovespa website, DI future contracts corresponded to 45% of the total financial volume negotiated in derivative contracts at BM&FBovespa in March 2015, followed by far of US Dollar future contracts in 25% of share.} We denote by $r_{t,T}$ the DI future rate at $t$ of the DI future contract expiring at $T$ and associate it with the risk-free rate of time to maturity $T - t$.

The DI index (IDI) is defined as the cumulative DI daily rate. Let $IDI_n$ be the IDI at the
end of day $n$ and $DI_I$ the DI annually compounded rate. Then

$$IDI_n = IDI_0 \cdot \prod_{i=0}^{n-1} (1 + DI_i)^{1/252}. \quad (1)$$

This index, computed every business day by BM&FBovespa, has been adjusted to a value
of R$ 100,000 on January 2, 1997, and was reset to its initial value on January 2, 2003 and
January 2, 2009. On March 31, 2015, the IDI amounted to R$ 179,508.81.

An IDI option with maturity $T$ is a European option where the underlying asset is the
IDI and whose payoff depends on $IDI_T$. If the strike is $K$, the payoff of an IDI option is
$(IDI_T - K)^+$ for a call and $(K - IDI_T)^+$ for a put. IDI options are the main interest rate
options of the Brazilian market. They are often used in strategies associated with expectations
related to the trajectory of the interest rate set by Copom. Note that the IDI option can be seen as an Asian option in which the underlying asset is the DI rate. This exotic feature makes its pricing and hedging strategies more complex than vanilla options. For a discussion about the pricing of IDI options, see for instance Almeida and Vicente (2009).

We used IDI option data collected from BM&FBovespa website in this study. Strike prices, option prices for IDI calls and puts, expiration dates, future DI rate and IDI spot prices were selected for all Copom meetings and their corresponding minutes releases from January 2006 to March 2015. We applied some filters to this dataset. We first selected option series maturing between seven and fifteen months. We used these medium-term maturing options because for the ones shorter than seven months uncertainty is lower and for those longer than fifteen months there is not much liquidity. After that, and to reduce low liquidity distortions, we identified the ones with moneyness between 0.01 and 0.99. Among those, we selected the series which had the greatest availability of strike prices for each maturity date. For example, on January 15, 2014, at the 180th meeting, 64 options were initially extracted from the BM&FBovespa dataset, with strike prices ranging from R$ 160,000 to R$ 190,000 and with five different maturity dates. After applying the mentioned filters, there were 19 options left maturing on May 2, 2015, with strike prices ranging from R$ 169,000 to R$ 177,500. Table 1 provides a clearer idea of the impact of these filters on our database. The table presents some statistics about how strikes and maturities of the options series have changed. Considering only Copom dates, the average number of strike prices initially extracted reduced around 75% after the two filters, while the average maturity became concentrated around 10 months after the filters, as desired.

<table>
<thead>
<tr>
<th></th>
<th>Initial database</th>
<th>After maturity filter</th>
<th>After liquidity filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average number of strikes</td>
<td>89</td>
<td>31</td>
<td>21</td>
</tr>
<tr>
<td>Average maturity (working days)</td>
<td>134</td>
<td>206</td>
<td>213</td>
</tr>
</tbody>
</table>

Notes: This table presents the evolution of the average number of strike prices and the average maturity of the option series extracted for all Copom meetings across the filters applied to the initial BM&FBovespa database.

2.2 Implied volatility

Options are derivative instruments whose price is connected with the volatility of the underlying asset. Therefore, we can extract from options prices the volatility of the underlying asset, which is called implied volatility. This is an appealing idea since options are forward-looking instruments. Thus, implied volatilities represent estimates of the degree of future movements of an asset not calculated from past returns.
Options (and any security) are priced using the risk-neutral distribution. Therefore, implied volatility is a measure of the spread of the underlying asset in the risk-neutral world. Let $Q$ be the risk-neutral measure. Then, the implied volatility of the log return $R_{t,T} = \ln \frac{IDI_T}{IDI_t}$ is the square root of the risk-neutral variance of the underlying asset return:\footnote{Note that $R_{t,T}$ is the cumulative one-day rate (DI rate) between $t$ and $T$. In the empirical exercise (Section 3) we set $T - t$ approximately equal to one year.}

$$\sigma_{t,T} = \sqrt{E^Q \left[ \left( \ln \frac{IDI_T}{IDI_t} - E^Q \left[ \ln \frac{IDI_T}{IDI_t} \right] \right)^2 \right]}.$$ 

There are some strategies to compute implied volatilities. The simplest method is to invert a formula, such as the Black and Scholes (1973) model, which gives a theoretical estimate of the price of an option as a function of the volatility of the underlying asset. However, this procedure has some caveats. First, it is model-dependent. Second, options with different strikes yield different implied volatilities. In this paper we implement two methods to estimate implied volatility that are model-free. This means that we do not assume a specific probability distribution of the future interest rate. The first approach was proposed by Bakshi, Kapadia and Madan (2003). In this approach, the implied volatility is extracted directly from option prices. The second method has an intermediate step. First we compute the risk-neutral probability density using the results of Breeden and Litzenberger (1978). Next, we obtain the implied volatility by integration. In the following, we explain each of these two methods in detail.

2.2.1 BKM implied volatility

Bakshi, Kapadia and Madan (2003), BKM hereafter, derive a model-free method to extract volatility, skewness and kurtosis of the risk-neutral distribution from a cross-section of calls and puts. They generalize the work of Britten-Jones and Neuberger (2000), who outline a similar procedure to compute implied volatility.

Let $c_t(T, K)$ and $p_t(T, K)$ be the prices of a call and a put with strike $K$ and maturity $T$ on date $t$. Applying the results of BKM, we have that the implied risk-neutral volatility of the log return $\ln \frac{IDI_T}{IDI_t}$ is given by

$$\sigma_{t,T} = \sqrt{e^{r_{t,T}(T-t)}V(t, T) - \mu(t, T)^2},$$

(2)
where
\begin{align*}
V(t, T) &= \int_{\text{IDI}_t}^{\infty} 2 \left( 1 - \ln \left[ \frac{K}{\text{IDI}_t} \right] \right) \frac{c_t(T, K)}{K^2} dK + \\
&\quad \int_{0}^{\text{IDI}_t} 2 \left( 1 - \ln \left[ \frac{K}{\text{IDI}_t} \right] \right) \frac{p_t(T, K)}{K^2} dK,
\end{align*}
\begin{align*}
\mu(t, T) &= e^{r_t(T-t)} - 1 - \frac{e^{r_t(T-t)}}{2} V(t, T) - \frac{e^{r_t(T-t)}}{6} W(t, T) - \frac{e^{r_t(T-t)}}{24} X(t, T),
\end{align*}
\begin{align*}
W(t, T) &= \int_{\text{IDI}_t}^{\infty} 6 \ln \left[ \frac{K}{\text{IDI}_t} \right] - 3 \left( \ln \left[ \frac{K}{\text{IDI}_t} \right] \right)^2 \frac{c_t(T, K)}{K^2} dK + \\
&\quad \int_{0}^{\text{IDI}_t} 6 \ln \left[ \frac{K}{\text{IDI}_t} \right] - 3 \left( \ln \left[ \frac{K}{\text{IDI}_t} \right] \right)^2 \frac{p_t(T, K)}{K^2} dK,
\end{align*}
\begin{align*}
X(t, T) &= \int_{\text{IDI}_t}^{\infty} 12 \left( \ln \left[ \frac{K}{\text{IDI}_t} \right] \right)^2 - 4 \left( \ln \left[ \frac{K}{\text{IDI}_t} \right] \right)^3 \frac{c_t(T, K)}{K^2} dK + \\
&\quad \int_{0}^{\text{IDI}_t} 12 \left( \ln \left[ \frac{K}{\text{IDI}_t} \right] \right)^2 - 4 \left( \ln \left[ \frac{K}{\text{IDI}_t} \right] \right)^3 \frac{p_t(T, K)}{K^2} dK.
\end{align*}

2.2.2 BL risk-neutral probability density

Breeden and Litzenberger (1978), BL for short, show that the risk-neutral probability density of an asset is the second derivative of the price of a call on this asset with respect to its exercise price. Applying this result to the IDI market, we have
\begin{equation}
f_t^Q(\text{IDI}_T) = e^{r_t(T-t)} \left. \frac{\partial^2 c_t(T, K)}{\partial K^2} \right|_{K=\text{IDI}_T},
\end{equation}
where \(f_t^Q(\text{IDI}_T)\) is the risk-neutral probability density of IDI\(_T\). Therefore, the volatility of the log return \(\ln \frac{\text{IDI}_T}{\text{IDI}_t}\) is
\begin{equation}
\sigma_{t,T} = \sqrt{\int_{\text{IDI}_t}^{\infty} \left[ \ln \frac{x}{\text{IDI}_t} \right]^2 f_t^Q(x) dx - \left[ \int_{\text{IDI}_t}^{\infty} \ln \frac{x}{\text{IDI}_t} f_t^Q(x) dx \right]^2}.
\end{equation}
2.2.3 Implementation

Some problems arise when computing model-free implied volatility. The available options prices do not provide us with a continuous function \( c_t(T, K) \) both on \( K \) and \( T \). IDI options are traded for a specific set of strike prices and maturities. The time to maturity gap is much higher than the strike gap. IDI options are usually issued with expiration in January, April, July and October. That is, the gap between two options with consecutive maturities is 0.25 year. On the other hand, the strike gap is R$ 100 which is equivalent to 0.05% of the IDI in March 2015. Thus, to circumvent this problem we use a mixed strategy.

For smoothing over time, instead of fixing a specific time to maturity and interpolating each date to obtain options prices with this time to maturity, we first collect options data expiring between seven and fifteen months for each date \( t \). Among them, we select the maturity of those that have the highest availability of strike prices. Notice that the time to maturity of options in our dataset varies through the sample. We opted for this strategy instead of keeping the time to maturity fixed at exactly one year in order to avoid interpolation errors arising from the procedure of volatility surface construction. Due to the large gap between two consecutives maturities, this approach introduces lower errors than the interpolation process.\(^6\) Over the strike dimension, we use a simple solution. That is, we linearly interpolate for the strike of interest.

There are also numerical issues in computing the integrals. We follow recommendations of Jiang and Tian (2005) to deal with the discretization of call prices and truncation of the integration domain. We transform put and call prices into implied volatilities according to Black and Scholes (1973) model. Then, we extrapolate the implied volatility to obtain the domain of integration. For strike prices below (above) the available strike price, we use the Black and Scholes implied volatility of the lowest (highest) available strike price. We test the trapezoidal and the Simpson rules to approximate the definite integral. We do not observe differences between these two traditional methods, thus we adopt the former since it is less computacional expensive. The length of the discretization sub-interval is defined as the half-width of the strike price range relative to the spot price.

2.3 Event study

We employ an event study approach to analyze the effect of Copom decisions on the moments of the risk-neutral distribution. We follow the event study method employed by Morel and Teilletche (2008). The starting point is to define what the event is and to identify its observation period, called the event window.

\(^6\)As pointed out by Christensen and Prabahala (1998), errors introduced by small variations in time to expiration of the options are lower than interpolation errors generated by the volatility surface construction.
Since 2006, Copom has held eight regular meetings per year, which last two days. The meeting begins on Tuesday and continues on the following day. Copom decision is announced to the public only at the end of the second day, therefore after options market closing. This is the statement date. In the sample period considered, CBB released the meeting minutes six working days after the interest rate target decision at 8:30 a.m. (before the opening of options market). This is the minutes date. Therefore, our event definitions are the day on which Copom announced the Selic rate target for the next 45 days approximately and the day before the minutes are released, called from now on the minutes date. Thus, we can compare the behavior of the risk-neutral moments in two distinct periods: pre-event and post-event. The final step of the event definition is to set the size of the pre and post-event windows. We choose one, two and five working days as event windows.

A well-known limitation of event study is the issue of endogeneity. For example, in studies that deal with foreign exchange intervention, not only exchange rate distribution depend on the event, but events also respond to movements in the exchange rate distribution. We argue that simultaneity is not an important issue in the case of interest rate target decisions regularly taken by central banks. Blinder et al. (2008) recognize that communication may be endogenous since the central bank can choose to communicate because of a sudden change in economic outlook or some other unexpected news. However, such endogeneity is a minor problem when the dates of major communications are known in advance, such as policy decisions about interest rate targets. They complete the argument by mentioning that this simultaneity problem can be especially problematic for speeches or interviews of committee members, which are flexible in both timing and content.

Figure 1 illustrates the definition of events. $t_i$ is the day of the $i^{th}$ event (the statement date or the minutes date), $h$ is the size of pre and post-event windows ($h = 1, 2$ and 5 days), and pre-event and post-event periods are defined as $[t_i - h, t_i]$ and $[t_i, t_i + h]$, respectively.

Figure 1: Events diagram

Notes: This figure presents the pre-event and post-event periods. We define two kinds of event: the statement date and the minutes date. Pre and post-event windows are sized as 1, 2 and 5 working days.

We use a mean difference t-test to evaluate the significance of changes in risk-neutral

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7Since April 2017, the minutes are released on the first tuesday after the interest rate target decision at 8:00 a.m.
moments between pre and post-event periods. We examine two criteria. In the direction criterion, we check if change occurs in the moment level after the event. In the smoothing criterion, we evaluate the occurrence of change in the dynamics of the moments, comparing their variation between pre and post-event periods. As Morel and Teilletche (2008) point out, the first criterion represents the strongest form of the test by seeking to ascertain whether the occurrence of the event is reflected by a significant change in the moment in the following period. The second criterion compares post-event changes with pre-event changes and it is weaker because it does not necessarily require the moment to change following the event.

<table>
<thead>
<tr>
<th>Table 2: Null and Alternative Hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direction Criterion (level change) - $H_0: E(\Delta M^{pos}) = 0$</td>
</tr>
<tr>
<td>Target Increase</td>
</tr>
<tr>
<td>$H_a: E(\Delta M^{pos}) &gt; 0$</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>$H_a: E(\Delta M^{pos}) &lt; 0$</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
<tr>
<td>$H_a: E(\Delta M^{pos}) &lt; 0$</td>
</tr>
<tr>
<td>Smoothing Criterion (dynamics change) - $H_0: E(\Delta M^{pos}_t - \Delta M^{pre}_t) = 0$</td>
</tr>
<tr>
<td>Target Increase</td>
</tr>
<tr>
<td>$H_a: E(\Delta M^{pos}_t - \Delta M^{pre}_t) &gt; 0$</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>$H_a: E(\Delta M^{pos}_t - \Delta M^{pre}_t) &lt; 0$</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
</tbody>
</table>

Notes: This table presents the null and the alternative hypothesis specifications for the direction and smoothing criteria and each risk-neutral moment. It takes into account if the Copom decision is to hike or cut. In the direction criterion, we check if change occurs in the moment level after the event. In the smoothing criterion, we evaluate the occurrence of change in the dynamics of the moments.

Let $M_t$ be a moment (mean or volatility) on day $t$ of the IDI risk-neutral density one-year ahead. Denote by $\Delta M^{pre}_t = M_{t_i} - M_{t_i-h}$ and $\Delta M^{pos}_t = M_{t_i+h} - M_{t_i}$ the moments’ changes in the pre-event and post-event intervals, respectively. Null and alternative hypothesis specifications for each criterion and each moment are defined in Table 2, according to whether the decision is a target increase or decrease.

The alternative hypothesis of each test represents the expected effects of Copom decisions on the risk-neutral moments if Central Bank’s policy is believed by the market. Regarding the direction criterion test, the expected effect on the one-year ahead IDI return (risk-neutral mean) is a positive change in the case of a target increase decision and a negative change otherwise. For the volatility of the one-year IDI return (risk-neutral standard deviation), we expect that any decision leads to a negative change. Regarding the smoothing criterion test, we expect the effects on the moments to be the same as in the first criterion, but in this case we compare the moments’ changes before and after intervention.
3 Results

3.1 Implied volatility

Figure 2 shows the time series of the implied volatility estimated by BKM and BL between January 2006 and March 2015 one day around the Copom meeting and minutes release dates. In both cases, the correlation between the volatility of the BKM and BL is very strong (about 70%). The difference is related to numerical errors and illiquidity problems of the IDI option market. In order to minimize these errors, we adopt as implied volatility the first principal component of BKM and BL volatilities.

![Figure 2: Implied Volatility](image)

Notes: This figure presents the implied volatility computed by BKM and BL methods for one-day window around the statement date (left-hand graph) and the minutes date (right-hand graph).

As a robustness check of our implied volatility measure, we compare it with the price of a straddle strategy. Market participants use strategies with options in order to trade volatility of future distribution of the underlying asset. There are many alternatives to do this. The simplest strategy is the straddle portfolio formed by purchasing an at-the-money call and an at-the-money put with the same strike. The price of the straddle is:

\[ \Pi_t(T, K) = c_t(T, K) + p_t(T, K). \]

Note that straddle price is not an implied volatility. However, the holder of a straddle position profits based on how much the price of the underlying asset changes, regardless of

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8For the windows of two and five days around Copom meeting and minutes release dates, the shapes of the graphs are very similar to the one day window. To save some space, we do not present these graphs.

9Much of the difference between the two implied volatilities is due to a few peaks in BL volatility. When we exclude these peaks, the correlation increases to more than 80%.

10This strategy can be made delta neutral by controlling the ratio of calls to puts. However, delta neutral strategy introduces model dependence since the calculation of the Greek letters is model dependent. So, we opt to use the unhedged straddle.
the direction of price movement. Thus a long straddle position is a strategy highly correlated with future volatility.

To implement straddle strategy, we have to define what an at-the-money option is. Traditionally, moneyness is measured by the delta of the option, computed using the Black and Scholes (1973) model. In this framework, the at-the-money option is the one with delta equal to 0.5. In this work, we do not adopt this rule. The IDI path does not follow geometric Brownian motion. It is increasing over time. Therefore, the option with Black and Scholes delta of 0.5 has risk-adjusted probability of exercise lower than 50%. Thus, we define an at-the-money option as the one whose delta is 0.65 rather than 0.5.

The correlation between our implied volatility measure and straddle price is greater than 60%. Note that we do not expect a perfect correlation since straddle strategy includes other kinds of risk. Therefore, we conclude that our implied volatility measure is able to capture risk-neutral volatility. In the next section, we study the effects on implied volatility of Copom meetings and minutes releases.

### 3.2 Graphical analysis

As a preliminary analysis, we investigate the behavior of the moments after the announcement of the target rate using a simple graphical procedure. Figure 3 presents variations of the expected value of the risk-neutral distribution of $R_{t,T}$ one, two and five days after the statement date as a function of changes in Selic rate. Figure 4 is very similar to Figure 3 but reflects variations of the second moment estimated by BKM method.\(^{11}\) In each graph, a circle marker represents a variation of the moment related to a specific statement decision. A cross marker indicates the average of moments variations observed for a given statement decision. Note that the variation of the mean of $R_{t,T}$ is in the same direction as the statement decision, that is, tighter monetary policy leads to a positive variation of the mean of $R_{t,T}$, while a drop in the Selic rate target yields a decline of the mean of $R_{t,T}$. Regarding the second moment, the variation is almost always negative. The patterns of these graphs also suggest that the absolute value of the variation of the first and second moments is independent of the size of the change in the Selic rate. Note that the positions of the cross markers are almost parallel to the horizontal axis.

### 3.3 Statistical analysis

Our results focus on event study tests of the first two moments of the IDI risk-neutral distribution one-year ahead. We report these results in terms of significance of changes in the

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\(^{11}\) We also plotted the same graph with BL implied volatility as dependent variable. Since the correlation between BKM and BL implied volatility is very high, we omit the BL plot for sake of space.
moments around the dates of Copom meetings and around the disclosure of its minutes, released a week after the decision. We distinguish moment’s changes between variation in its level and variation in its dynamics, as defined in the last section. The results of the event study, based on t-tests, are shown in Tables 3 to 8. We implement the event study tests for three different sizes of the event window: 1, 2 and 5 days.

Regarding the first moment of the one-year interest rate, the event study tests revealed that Copom decisions do not significantly affect the risk-neutral return either around the dates of the meetings or around the disclosure of the minutes (Table 3). This is an expected result, as the number of Copom decisions for interest rate hikes and cuts was practically the same during the considered period, as pointed out in Section 1.

Nevertheless, when we discriminate the decisions between the ones that resulted in a target rate increase from those resulting in a decrease, we can note more intuitive findings. In fact, around the dates of Copom meetings that resulted in a higher target, the level and the dynamics of returns rose significantly 2 and 5 days after the decision was taken (Table 4 - part 1). On average, a decision to increase the Selic rate is followed by an increase in the level and in the dynamics of the one-year interest rate ranging from 0.07% two days after the
Figure 4: Variation of the second moment

Notes: This figure presents the variation of the volatility of the risk-neutral distribution of the one-day rate accumulated over one-year ahead for one, two and five days after the statement date, as a function of changes in Selic rate.

Table 3: Effect of Copom decision on risk-neutral expected return on the statement and the minutes dates

<table>
<thead>
<tr>
<th></th>
<th>1-day window return</th>
<th>2-day window return</th>
<th>5-day window return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>Dynamics</td>
<td>Level</td>
</tr>
<tr>
<td>Copom</td>
<td>Mean</td>
<td>p-value</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td>-0.01</td>
<td>0.35</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>-0.01</td>
<td>0.56</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>0.24</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Notes: This table shows the results for the event study of the effects of Copom decision about interest rate target on the changes in the first moment (mean) of the one-year ahead IDI risk-neutral distribution. We analyze the effects around two dates: the statement date and the minutes release date. A t-test for the paired mean differences is applied. We analyze changes in level and in the dynamics. The sizes of pre and post-event windows are 1, 2 and 5 working days. There are 67 observations in the experiment.

decision to 0.10% 5 days after it.

In the case of Copom meetings that resulted in a lower target, decisions seem more effective to alter the level than the dynamics of the one-year interest rate. The significant decreases
Table 4: Effect of Copom discriminating decision on the risk-neutral expected return on the statement date

<table>
<thead>
<tr>
<th>Target increase</th>
<th>1-day window return</th>
<th>2-day window return</th>
<th>5-day window return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level Dynamics</td>
<td>Level Dynamics</td>
<td>Level Dynamics</td>
</tr>
<tr>
<td>Mean</td>
<td>0.04</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>p-value</td>
<td>0.13</td>
<td>0.03</td>
<td>0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Target decrease</th>
<th>1-day window return</th>
<th>2-day window return</th>
<th>5-day window return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level Dynamics</td>
<td>Level Dynamics</td>
<td>Level Dynamics</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.06</td>
<td>-0.06</td>
<td>-0.10</td>
</tr>
<tr>
<td>p-value</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: This table shows the results for the event study of the effects of Copom decision about interest rate target on the changes in the first moment (mean) of the one-year ahead IDI risk-neutral distribution. The study is implemented around the statement date. We discriminate the decisions between those that resulted in a target increase (22) from those that resulted in a target decrease (25). A t-test for the paired mean differences is applied. We analyze changes in level and in the dynamics. The sizes of pre and post-event windows are 1, 2 and 5 working days.

in returns are present in their levels for all the three event window sizes (Table 4 - part 2). Decisions to decrease the target rate reduce the level of the one-year interest rate by, on average, 0.06%. two days immediately after the decision is taken and this persists until five days afterward, reaching 0.10%. The effect on the dynamics of the long-term interest rate change is restricted to only the one-day event window case, where the average decrease is 0.04%.

Just a few Copom decisions are not anticipated by the market. We compared the prediction provide by the Focus report one day before the Copom meeting to the target decision. They are different only in 16 of the 67 meetings in our database. Therefore, the finding that anticipated shocks drive the mean of the risk-neutral distribution according to the nature of the decision can be seen as a puzzle. However, we can easily propose some intuition that explains this result. First, note that anticipated decisions concern to the one-day interest rate (Selic rate) while our event study focuses on the one-year interest rate. Although these two rates are correlated, there is information on the latter that is not explained by the former. Therefore anticipated changes in the Selic rate are not necessarily fully transmitted to the one-year rate. Second, information revealed by Copom meeting is not only about the level of Selic rate. The minutes of Copom meetings contain other important issues. For example, market participants are also interested in the bias of the monetary policy and in the votes of each member of Copom. This information is provided by the minutes and can help them to forecast future decisions of the committee. Depending on the content of Copom minutes, the future distribution of the one-year interest rate can shift to the left or to the right even if

\[ \text{Focus is a survey carried out by the Central Bank of Brazil. Focus is the main source of information regarding expectations for the Brazilian economy. It provides expectations about several economic variables, based on responses from more than 100 market participants, most of them financial institutions.} \]
the Copom decision is anticipated. Finally, note that we study the risk-neutral distribution. However, anticipated changes in Selic rate refer to the physical distribution. When we say that a decision is anticipated it means that the physical expected value of the Selic rate (extracted from the Focus survey) is equal to the decision of the target rate. Thus, we can observed changes in the first moment of the risk-neutral distribution that are not shared by changes of the Selic rate. Of course, in this case, we are facing a variation on the risk premium.\(^\text{13}\)

Around the minutes release dates, there were no significant results when the decision taken one week before was to increase the target rate (Table 5). For lower target decisions, we find a significant drop in the first moment only for the 5-day window size, where the level of the one-year interest rate fell by 0.05% on average. Therefore, we find different market reactions according to the kind of actions of CBB. While the decision regarding the target affects the first moment of the one-year interest rate, the minutes release has little impact on it.

### Table 5: Effect of Copom discriminating decision on the risk-neutral expected return on the minutes date

<table>
<thead>
<tr>
<th></th>
<th>1-day window return</th>
<th>2-day window return</th>
<th>5-day window return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>Dynamics</td>
<td>Level</td>
</tr>
<tr>
<td><strong>Target increase</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>p-value</td>
<td>0.09</td>
<td>0.09</td>
<td>0.29</td>
</tr>
<tr>
<td><strong>Target decrease</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.00</td>
<td>0.03</td>
<td>-0.02</td>
</tr>
<tr>
<td>p-value</td>
<td>0.54</td>
<td>0.93</td>
<td>0.21</td>
</tr>
</tbody>
</table>

*Notes:* This table shows the results for the event study of the effects of Copom decision about interest rate target on the changes in the first moment (mean) of the one-year ahead IDI risk-neutral distribution. The study is implemented around the minutes release date. We discriminate the decisions between those that resulted in a target increase (22) from those that resulted in a target decrease (25). A t-test for the paired mean differences is applied. We analyze changes in level and in the dynamics. The sizes of pre and post-event windows are 1, 2 and 5 working days.

Regarding the risk-neutral volatility, the results are also intuitive. Copom decisions significantly reduce the level and the dynamics of the volatility of the one-year interest rate around the dates of meetings and this effect persists for windows around the minutes release dates (Table 6). The results also show that market uncertainty adjustment to the news is not instantaneous; it continues beyond the announcement of the decision. The interest rate volatility reduction after Copom meetings and after the minutes release dates are respectively around 36% and 44% for the three considered event windows.\(^\text{14}\)

\(^{13}\)Understanding more deeply this finding is a very interesting question. However, it is not easy to study this phenomenon since we have in our database a small number of decisions not anticipated. So, we do not address this point in this study.

\(^{14}\)We highlight that these values refers to the variation of the first principal component of the standardized volatility. Therefore, the variations presented in tables 6, 7 and 8 are not directly translated to the real variation in volatility.
Table 6: Effect of Copom decision on volatility on the statement and the minutes dates

<table>
<thead>
<tr>
<th></th>
<th>1-day window return</th>
<th>2-day window return</th>
<th>5-day window return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>Dynamics</td>
<td>Level</td>
</tr>
<tr>
<td>Copom</td>
<td>Mean</td>
<td>-0.43</td>
<td>-0.77</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Minutes</td>
<td>Mean</td>
<td>-0.43</td>
<td>-0.86</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: This table shows the results for the event study of the effects of Copom decision about interest rate target on the changes in the second moment (standard deviation) of the one-year ahead IDI risk-neutral distribution. We analyze the effects around two dates: the statement date and the minutes release date. A t-test for the paired mean differences is applied. We analyze changes in level and in the dynamics. The sizes of pre and post-event windows are 1, 2 and 5 working days. There are 67 observations in the experiment.

The significant statistical decrease of the volatility is also verified whether Copom decides to increase or decrease the target rate (Tables 7 and 8). The only exceptions are the 2-day and 5-day windows for Selic decreases after Copom meetings, where the volatility decreases but not at significant levels. We also note a stronger impact on the dynamics of the volatility than on its level, meaning that the volatility following the events is not only reduced but also its reduction is accelerated.15

Table 7: Effect of Copom discriminating decision on volatility on the statement date

<table>
<thead>
<tr>
<th></th>
<th>1-day window return</th>
<th>2-day window return</th>
<th>5-day window return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>Dynamics</td>
<td>Level</td>
</tr>
<tr>
<td>Target increase</td>
<td>Mean</td>
<td>-0.53</td>
<td>-1.05</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Target decrease</td>
<td>Mean</td>
<td>-0.21</td>
<td>-0.32</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>0.02</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Notes: This table shows the results for the event study of the effects of Copom decision about interest rate target on the changes in the second moment (standard deviation) of the one-year ahead IDI risk-neutral distribution. The study is implemented around the statement date. We discriminate the decisions between those that resulted in a target increase (22) from those that resulted in a target decrease (25). A t-test for the paired mean differences is applied. We analyze changes in level and in the dynamics. The sizes of pre and post-event windows are 1, 2 and 5 working days.

15In order to provide robustness for our results, we also implement the nonparametric Wilcoxon test for the significance of the changes in the risk-neutral moments. We do not find differences between the parametric and the nonparametric approaches. The results of the Wilcoxon test are available upon request.
Table 8: Effect of Copom discriminating decision on volatility on the minutes date

<table>
<thead>
<tr>
<th></th>
<th>1-day window return</th>
<th>2-day window return</th>
<th>5-day window return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>Dynamics</td>
<td>Level</td>
</tr>
<tr>
<td>Target increase</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.38</td>
<td>-0.80</td>
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</tr>
<tr>
<td>p-value</td>
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<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>Target decrease</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.28</td>
<td>-0.51</td>
<td>-0.31</td>
</tr>
<tr>
<td>p-value</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Notes: This table shows the results for the event study of the effects of Copom decision about interest rate target on the changes in the second moment (standard deviation) of the one-year ahead IDI risk-neutral distribution. The study is implemented around the minutes release date. We discriminate the decisions between those that resulted in a target increase (22) from those that resulted in a target decrease (25). A t-test for the paired mean differences is applied. We analyze changes in level and in the dynamics. The sizes of pre and post-event windows are 1, 2 and 5 working days.

4 Conclusion

In this paper we examine the behavior of market uncertainty about Brazilian interest rates around Copom meetings and around corresponding minutes release dates. Uncertainty is defined as the standard deviation of option-implied probability distributions of the interest rate. The event study method is employed to evaluate the impact of interest rate target decisions on the first two moments of the interest rate risk-neutral distribution.

Our results show that risk-neutral distribution of future interest rates are significantly altered around monetary policy actions of Central Bank of Brazil. In general, the risk-neutral expected return consistently responds to Copom decision about the interest rate target. However, a week later, around the minutes release dates, we do not note effects on the risk-neutral expected returns. Regarding market uncertainty, we note a significant decrease after Copom decisions and around the minutes releases, no matter if it is a monetary tightening or loosening.
5 References


