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Predicting Exchange Rate Volatility in Brazil: An approach using quantile autoregression

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Non-technical Summary

The foreign exchange rate market is one of the most important in the global financial system nowadays, due to its huge trading volume and great importance to economic agents, in particular investors and policymakers.

Exchange rates allow investors, for instance, to design trading strategies to hedge against market risk. They also influence central banks' decisions on monetary policy, since movements of the exchange rate can affect future price dynamics.

Nonetheless, the potential variety of factors that can influence exchange rates (for instance, macroeconomic fundamentals, speculative transactions and currency interventions, among others) may bring to this market substantial volatility in periods of distress.

In this paper, we estimate 60 models to forecast the exchange rate volatility in Brazil, including some traditional approaches such as GARCH and EGARCH models. The main contribution of this study is to employ quantile regression, which is an econometric estimation technique, in some of its new formulations to build proxies for the exchange rate volatility based on the conditional autoregressive value at risk (CAViaR) model of Engle and Manganelli (2004). One of the advantages of the proposed framework is to allow for asymmetric dynamics in the distribution of returns of the exchange rate.

In order to analyze the relative forecasting performance of the investigated models, we apply the predictive ability test of Giacomini and White (2006). The results suggest that forecasts from the asymmetric CAViaR model are better than forecasts from great part of the models, thus corroborating the benefits of our proposed setup over traditional approaches.

Sumário Não Técnico

O mercado de câmbio é um dos mais importantes no sistema financeiro global hoje em dia, devido ao seu enorme volume de negócios e grande importância para os agentes econômicos, em particular, investidores e formuladores de políticas públicas.

As taxas de câmbio permitem aos investidores, por exemplo, elaborar estratégias de investimento para se protegerem contra o risco de mercado. Elas também influenciam as decisões dos bancos centrais sobre política monetária, uma vez que movimentos das taxas de câmbio podem afetar a dinâmica futura de preços ao consumidor.

No entanto, a grande variedade de fatores que podem influenciar as taxas de câmbio (por exemplo, fundamentos macroeconômicos, transações especulativas e intervenções monetárias, dentre outras) pode trazer para esse mercado volatilidade excessiva em períodos de estresse.

Neste artigo, estimamos 60 modelos para prever a volatilidade da taxa de câmbio no Brasil, incluindo algumas abordagens tradicionais, como os modelos GARCH e EGARCH. A principal contribuição deste estudo é utilizar a regressão quantílica, que é uma técnica econométrica, em algumas de suas novas formulações, para estimar a volatilidade da taxa de câmbio com base no modelo CAViaR de Engle e Manganelli (2004). Uma das vantagens da metodologia proposta é permitir dinâmicas assimétricas na distribuição dos retornos da taxa de câmbio.

Para analisar a capacidade preditiva dos modelos investigados, aplicamos o teste de Giacomini e White (2006). Os resultados sugerem que as previsões do modelo CAViaR assimétrico são melhores do que as previsões da maior parte dos modelos, corroborando assim os benefícios da metodologia proposta em relação às abordagens tradicionais.

Predicting Exchange Rate Volatility in Brazil: An approach using quantile autoregression

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Abstract

We apply quantile regression in some of its new formulations to analyze exchange rate volatility. We use the conditional autoregressive value at risk (CAViaR) model of Engle and Manganelli (2004), which applies autoregressive functions to quantile regression to estimate volatility. That model has proved effective when compared to others for various purposes. We not only compare the forecasting power of models based on quantile regression with some models of the GARCH family, but also examine the behavior of the exchange rate along its conditional distribution and its consequent volatility. When applying CAViaR in the whole distribution, our results show differentiation of the angular coefficients for each quantile interval of the distribution for the asymmetric CAViaR model. With respect to the exchange rate volatility, we build forecasts from 60 models and use two models as reference to apply the predictive ability test of Giacomini and White (2006). The results indicate that the prediction of the asymmetric CAViaR model with quantile interval of (1, 99) is better than (or equal to) 66% of the models and worse than 34%. In turn, the other benchmark model, the GARCH (1,1), is worse than 71% of the models, better than 13%, and equal in forecasting precision to 16% of the models.

Keywords: Government intervention; quantile regression; volatility; endogeneity; foreign exchange market; stock market; CAViaR; GARCH.
JEL Classification: C14; C22; C53; F31; G17.

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1. Introduction

Starting with his work in 1952, Markowitz laid the foundation for a new way of analyzing and comparing investments, by developing the mean-variance model: the mean is associated with the return and the variance with the risk of the investment. Hence, the volatility is calculated as the variance or square root of the variance of the return of the asset or portfolio of interest.

Abdalla (2012) recalled that while volatility is associated with risk, it is not exactly the same as risk. In reality, it is a measure of the uncertainty of both positive and negative movements in the price of an asset. Nevertheless, this measure has been widely used in the literature since the work by Markowitz, including theoretical and empirical investigations carried out by financial institutions, regulatory agencies and academics. Indeed, its importance is so great that, with the evolution of derivatives markets, various instruments have been created so that volatility can be traded through options.

In respect to exchange rate volatility, an ongoing question naturally arises: Is there a precise way to estimate it? Various approaches have been taken in the literature. Some researchers have sought to estimate this volatility based on macroeconomic variables and make predictions from the relations found with these variables.[1] In recent years, volatility has been predicted most often based on autoregressive time series models.[2]

For instance, Engle & Manganelli (2004) developed a semiparametric model of volatility called CAViaR (conditional autoregressive value at risk by regression quantiles) based on quantile autoregression. Taylor (2005) applied the CAViaR to a pair of symmetric quantiles from the tails of the distribution and predict the exchange rate volatility. Huang et al. (2011) adopted the same approach as Taylor, but for all percentiles.

The method applied here fits under the autoregressive models used for short-term prediction. Our objective is to study the exchange rate volatility in Brazil by applying the CAViaR technique and to predict the volatility by the quantile regression employed by Huang et al. (2011). We also compare it with the GARCH (1,1) and EGARCH (1,1) models and the prediction method investigated by Taylor (2005).

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[1] Huang et al. (2011) mentioned the comprehensive work of Engel & West (2005), which investigated the relation of some economic variables with exchange rate volatility, but did not find evidence of significant effects of those variables on this volatility. Bekiros (2014), also inspired by Engel & West (2005), studied foreign exchange volatility by considering macroeconomic fundamentals with linear and nonlinear models, but did not find a relationship, in particular for short-term forecasting.

[2] Many works can be cited in this area, among them Engle & Bollerslev (1986), Scott & Tucker (1989), Beine et al. (2007), Hansen & Lunde (2005) and Abdalla (2012).

Although the exchange rate is always a hot media topic in Brazil, be it due to external crises or the domestic political scenario, to the best or our knowledge there has been no study of the Brazilian exchange rate applying quantile regression and the CAViaR technique. Our contribution is to introduce a tool that can shed light on the behavior of the conditional distribution of the exchange rate. We not only compared the forecasting power of models based on quantile regression with some models of the GARCH family, but also examined the behavior of the exchange rate along its distribution and its consequent volatility.

The rest of this paper is structured in four sections. In the second section, we review the literature on quantile regression in general, with particular focus on the models used in the financial literature to measure volatility. In the third section, we present the methodology, and in the fourth, we present and discuss the empirical results. The fifth section summarizes the conclusions and makes some suggestions for future research.

2. Literature review

The vast literature on exchange rate volatility is divided into various specific currents, with two standing out. The first involves modeling the volatility using independent variables based on macroeconomic fundamentals, such as inflation or its volatility, international trade and aggregate supply. The second involves application of autoregressive models.

The effect of exchange rate variations on fixed capital and financial investments has attracted the attention of scholars and investors for decades. Exchange rate volatility took on new significance, at least in nominal terms, with the end of the Bretton Woods system in 1973 and the decision by most countries to allow their currencies to float. However, some theories at the time argued that, with the floating of currencies, the real exchange rate, which considers the fluctuation of price-level indexes between countries, would be more stable than under the pegged regime.

This idea was expressed by Flood and Rose (1999), who in their introduction argued that it was surprising for exchange rates to be volatile. In this respect, they cited Friedman (1953) that argued: *“[...] instability of exchange rates is a symptom of instability in the underlying economic structure [...] a flexible exchange rate need not to be an unstable* *exchange rate. If it is, it is primarily because there is underlying instability in the economic conditions [...]"* (Friedman, 1953, cited in Flood & Rose, 1999, p. F660).

Although this discussion is outside the scope of this study, it can be seen that even before 1973, exchange rate volatility was already a concern of academics and economic agents. Hence, a myriad of empirical and theoretical studies have been published investigating the variables that can influence and be influenced by fluctuations in the foreign exchange market.[3]

In turn, the volatility of exchange rates winds up influencing the prices of goods between countries and also the level of domestic consumption. In Brazil, for example, the export sector constantly lobbies the government to take measures to weaken the currency, to make Brazilian products more competitive abroad and imported products less attractive to local consumers. But policies in this direction involve complicated tradeoffs, especially because a weaker exchange rate increases inflationary pressure.

In the international arena, Abdalla (2012) studied the exchange rate volatility in 19 Arab countries and cited various studies relating this volatility with macroeconomic fundamentals (inflation, interest rates, GDP), such as Choi & Prasad (1995) and Pavasuthipaisit (2010). Exchange rate volatility and its influence on global trade has been widely studied, as reflected in Krugman (1986) and Hooper & Kohlhagen (1978), both of which demonstrated that the uncertainties associated with the exchange rate increase the prices in international trade. Also, Kliatskova (2013) studied whether the level of development plays a role in the relation between exchange rate volatility and international trade in a sample of countries.

Other problem caused by high foreign exchange volatility is that it reduces the mobility of capital, as demonstrated by Lai et al. (2008). Gonzaga & Terra (1997) reported significant effects of exchange rate volatility on inflation. In turn, Hausmann et al. (2006) showed that this volatility is higher in developing than in developed countries. Another important topic is the connection of the exchange rate with interest rate differentials, as studied by Menkhoff et al. (2012), who analyzed the variations in profits resulting from the carry trades between currencies. In turn, Huang et al. (2011) advocated

[3] The relationship of exchange rate volatility with aggregate supply (Hau, 2002), inflation volatility (Gonzaga & Terra, 1997) or business profitability (Baum, Caglayan & Barkoulas, 2001) are some examples of investigations of the impact of the exchange rate on macroeconomic variables.

the idea that predicting exchange rate volatility can improve the profitability of transactions involving exchange rates.[4]

As previously mentioned, other strand of the literature studies the exchange rate volatility by using time series models. In the autoregressive model proposed by Bollerslev (1986), the conditional variance is estimated based on the heteroskedasticity, time dependence and a moving average. This class of models, called the GARCH family, emerged in response to some stylized facts related to the behavior of the distribution of returns of a variable, for example, volatility clustering (Brooks, 2008) or asymmetry between positive and negative exchange rate return movements, addressed, e.g., by the EGARCH model proposed by Nelson (1991).[5]

Using the GARCH (1,1) model, Choudhry (2005) found evidence that the volatility both of the nominal and real exchange rates generates significant negative impacts on the exports of the United States to Canada and Japan.

Some of the studies mentioned previously analyzed the relation between the exchange rate and/or its volatility and macroeconomic variables, including the application of autoregressive models to examine the role of economic fundamentals. Among those that have obtained significant results, we can mention Bollerslev (1990), Jorion (1995), Andersen & Bollerslev (1998), Brooks & Burke (1998), Yoon & Lee (2008) and Musa et al. (2014). The last paper, in particular, applied the multivariate GARCH model to analyze the exchange rate of Nigeria's currency (the Naira) against the currencies of some developed countries.

Forecasting of volatility using the GARCH model can also be seen in Scott & Tucker (1989) and Hansen & Lunde (2005). The latter authors compared the GARCH (1,1) with various specifications (more than 300) of the GARCH family and evaluated the results by six different types of loss functions.

Huang et al. (2011) and Abdalla (2012), among others, developed the interesting idea that foreign exchange volatility, besides being a non-observable variable, is an aspect of uncertainty/risk associated with a determined asset, besides being very important.

[4] In particular, he noted that trading currencies is different than trading other financial assets. The latter are traded more freely between parties, while governments often impose controls on currency trading, seeking to stabilize the exchange rate or weaken or strengthen the currency depending on concerns over trade flows or inflation. Holmes (2008), for example, reported evidence of the non-stationarity of the real exchange rate under the Markov regime-switching framework.

[5] It should be noted, however, that these models are based on assumptions about the distribution and the parameters, such as the normal distribution or Student t-distribution. Besides this, the majority of works assume that the distribution tends to a constant value with time. Some authors, like Rapach & Strauss (2008), have shown that exchange rate series have structural breaks due to external interventions.

Specifically, that variable is subject to shocks that are not necessarily random, such as monetary shocks and government interventions (as in dirty float regimes, for instance). Foreign exchange volatility can be modeled by the GARCH family of models with conditional variance, methods that respond well to the stylized facts found during decades of empirical studies, such as long-term persistence (when the coefficients α and β are near 1 in the GARCH (1,1) model), time dependence, asymmetry and even regime change, such as changes in the Markov state.

However, as observed by several authors mentioned previously, exchange rates can be significantly affected by governmental actions, so the structure of the distribution of returns of this financial asset can vary substantially in time. In other words, the initial assumptions can change during the sample period as a result of government interventions. A way to address this problem was developed by Nikolaou (2008), who applied quantile regression to analyze real exchange rate quantiles and found an interesting result, namely that shocks cause different impacts depending on the magnitude of the shocks, and also lead to different dynamics, such as the mean reversion tendency.

Here we analyze volatility by the autoregressive model, using quantile regression to predict exchange rate volatility. Since this model has two stages (the first ascertaining the volatility by the CAViaR model, and the second predicting the volatility based on the quantiles determined by the CAViaR, using linear regression), after presenting the volatility models we provide a brief review of quantile regression and then return to the CAViaR model proposed by Engle & Manganelli (2004), before finally turning our attention to the models developed by Taylor (2005) and Huang et al. (2011) to forecast volatility.

3. Methodology

Here we use the method proposed by Taylor (2005), extended as advocated by Huang et al. (2011), who presented an alternative form between predicting volatility and estimating the quantiles. The method is carried out in two stages. In the first, estimates are generated from the distribution of the quantiles of the returns, without making assumptions about the characteristics of the distribution function. In this stage, the CAViaR method is used. In the second stage, the estimated quantiles can be applied directly in the predictions of volatility or by approximating the ratio between quantiles and the variance, or by regression models.

In order to present all the building blocks of the proposed methodology, this section is organized into five subsections: (i) exchange rate volatility; (ii) Value-at-Risk (VaR); (iii) quantile regression, which is one of the bases for calculating the CAViaR model; (iv) description of the CAViaR model, with the equations formulated in Engle & Manganelli (2004); and (v) presentation of the method of predicting volatilities proposed by Taylor (2005) and Huang et al. (2011).

3.1 Exchange rate volatility

In this paper, we work with log returns, implying a continuous rate, as is the practice in large part of the literature. Therefore, the variable under analysis is:

$$r_t = ln\left(\frac{S_t}{S_{t-1}}\right), \qquad \textbf{(1)}$$

where r_t is the continuous rate of return at time t, S_t is the exchange rate at t, and S_{t-1} is the exchange rate at t-1. As mentioned before, in analyzing investments according to the return and risk, Markowitz (1952) defined volatility (risk measure) as the standard deviation, σ, of the distribution of the calculated returns, according to equation (2). This form of calculation is widely used in the financial literature, and is defined by the following expression, given a sample with *n* observations:

$$\sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})^2, \qquad \textbf{(2)}$$

where

$\sigma = \sqrt{\sigma^2}$ is the volatility

r_t = return at t

$\bar{r}$ = unconditional mean return

Brooks (2008) and Hull (2009) mentioned some of the main volatility models developed in preceding decades. Among them are the historical volatility, by which weights can, if desired, be attributed to the sample periods; and implied volatility, calculated based on the option prices of a determined target asset, employing the option pricing formula of Black & Scholes (1973), given the price of a determined option and the other variables (e.g., maturity and price of the underlying asset, interest rate), among others to calculate the volatility projected by the market for the asset covered by the option. In this case, just as in the historical volatility model, it is assumed that the volatility is constant during the term to maturity, at all the asset's price levels. An advance to these models is provided by the exponential smoothing technique (exponentially weighted moving average – EWMA), which can be understood as a special case of the weighted average in which the weights decay exponentially, according to the expression below:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)u_{t-1}^2. \qquad \textbf{(3)}$$

The estimate σ_t^2 is calculated from the volatility at t-1 (σ_{t-1}^2) and the new information, given by the error u_{t-1}^2 (difference between the actual and expected returns). The coefficient of decay, λ, determines the weight to be attributed to the more recent observations.

On the other hand, many papers in the financial literature present empirical results showing heteroskedasticity of various assets, meaning their volatility is not constant. A series is heteroskedastic when its volatility is variable over time, so that it can present the ARCH effect. Engle (1982) proposed the model shown below, to express volatility as dependent on past returns:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2, \qquad \textbf{(4)}$$

where $E(u_t\ ) = 0$

$u_t = \sigma_t * \ \varepsilon_t$

$\varepsilon_t = i.i.d.(0,1)$

The equation above represents the ARCH (1) model, by expressing the dependence of volatility only on the error at t-1.

Bollerslev (1986) developed an autoregressive model that can deal with these characteristics of financial time series, besides another characteristic often observed, the clustering of volatility. In this later specification, the variance not only depends on past returns, but also depends on past variance. In the equation of the GARCH model below, it can be seen that the equation of the conditional variance, for the simplest case, is a generalization of the EWMA model, according to the following equation:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2. \tag{5}$$

This expression indicates the so-called GARCH (1,1) model, widely used in the literature. One of the variations of the GARCH model is the EGARCH model, developed by Nelson (1991), with the aim of reflecting some stylized facts found in studies of volatility, such as the asymmetry found in the stock market in general. One specification of the EGARCH, among others in the literature, is shown below:

$$\ln(\sigma_t^2) = a + \beta_1 \ln(\sigma_{t-1}^2) + \beta_2 \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \beta_3 \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right]. \tag{6}$$

This model has several advantages over the basic GARCH model. For example, in this specification the modeling is performed with the log of σ_t^2, so even if the estimated parameters are negative, σ_t^2 will be positive, and there will be no need for *a priori* restriction to positive parameters. Another advantage is the asymmetries found in the response to negative and positive returns of volatility. In the EGARCH, if the ratio between volatility and return is negative, β_2 will be negative. This often happens when calculating exchange rate volatility.

Although there are many variants in the GARCH family of models, in most of them the parameters are estimated by maximum likelihood, assuming the error term to have i.i.d. distribution, with a previously determined distribution.

3.2 Value-at-Risk (VaR)

Value-at-Risk is a risk measure widely used by investment firms and financial institutions. Its property of expressing risk in a single metric is one of the reasons for its popularity. It is defined as the maximum value that can be lost by holding an asset or portfolio during a previously established period, given a probability (confidence level). In other words, the VaR measure is the estimate of loss from holding an asset or portfolio over a determined period (generally one or ten days), which will only be exceeded with a small probability α (generally 1 or 5% - the Basel capital requirement rule is 1%).

From this definition, it can be seen that the VaR can be considered a "positional measure", i.e., the VaR is in a position such that, given a confidence level α, the value at risk will be greater than 1- α for all other values. This characteristic is similar to the quantiles of distributions. For example, for a period of one day, and α (confidence level) = 5%, the VaR shows the negative return that will not be exceeded on that day with probability of 95%. Mathematically, this is expressed as follows:

$$Prob[r_t < -VaR_t | I_{t-1}] = \alpha, \tag{7}$$

where I denotes the information set available at time t-1.

In statistical terms, the confidence level α is the quantile θ of the asset's probability density function. The basic VaR metric assumes normality of the asset's distribution function, so with quantile θ= 5%, the number of standard deviations (η) from the mean, with 95% confidence, is equal to 1.67. Under the normality hypothesis, the VaR would be:

$$VaR = \mu + \eta\sigma, \tag{8}$$

where μ = mean and σ = volatility.

According to Engle & Manganelli (2004), by whatever method the VaR is used, there is a common structure for the various types of calculation, namely: (1) mark the investment portfolio to the market; (2) estimate a distribution of the portfolio's returns, and (3) by applying the volatility found in (2), calculate the portfolio's VaR. Step (2) is what differs the most among the various models. In the case of parametric models, the distribution is pre-established, and is generally assumed to be constant over time. But semiparametric and nonparametric approaches also exist, such as the quantile regression, next described.

3.3 Quantile regression

The central idea of quantile regression, as formulated by Koenker & Bassett (1978), is that if the estimate of the mean can be obtained by minimizing the sum of the squared residuals, it is also possible to find an estimate of the median as a solution to minimize the sum of the absolute residuals. The authors expanded, then, the concept of minimizing the sum of the absolute deviations – with the solution being the median – to other quantiles.

They reported that the estimators calculated by quantile regression are just as efficient as those calculated by the least squares method in Gaussian distributions. In turn, the parameters calculated by the quantile regression model are more efficient for data with non-Gaussian distributions.

Koenker & Bassett (1978) developed a general formulation, described as follows: Given a sequence x_t, with t = 1,...,T, denote a sequence of K-vectors of a determined matrix. Also suppose that y_t, with t = 1,...,T, is a random sample of the process of the following regression: $\boldsymbol{u_t = y_t - x_t b}$**,** with a distribution function F. The θ-th quantile of the regression, $0 < \theta < 1$, is defined as any solution to the following minimization problem:

$$\min_{b \in R^k} \left[\sum_{t \in \{t: y_t \geq x_t b\}} \theta |y_t - x_t b| + \sum_{t \in \{t: y_t < x_t b\}} (1-\theta) |y_t - x_t b| \right]. \quad \textbf{(9)}$$

Finally, the estimated conditional quantile is linear and the conditional linear expression utilized for the case of one variable (K=1) is the following:

$$Y_{\theta,t} = b_{0,\theta} + b_{1,\theta} x_t. \quad \textbf{(10)}$$

Quantile regression was first widely used to analyze cross-sectional data, but it has been spreading to other areas, and is now applied in autoregressive models typical of time series, such as the CAViaR model, next described.

3.4 Conditional Autoregressive Value at Risk by Quantile Regression (CAViaR)

Stylized facts about volatility indicate its distribution is generally not normal, and it can also vary with time. Engle & Manganelli (2004) stated that, since the VaR is related to volatility, if the latter presents clustering, so should the VaR. And a statistical way to represent this grouping is by establishing an autoregression relation. Besides this, to relax the previous hypotheses about the distributions, they used quantile regression, and thus proposed the conditional autoregressive value at risk (CAViaR) model.

This formulation of the VaR, besides not establishing prior properties of the distribution, also allows variation in time of the probability density in the error and volatility terms. If the probability associated with the VaR is defined as θ and r_t is a vector of the observed returns of a portfolio at time t, the θ-th quantile (generally 1% or 5%) at period t of the portfolio's return generated at t-1 is denoted by:

$$Q_t(\theta) = Q(r_{t-1}, \beta_\theta) \ , \tag{11}$$

where β_θ is a p-vector of unknown parameters. The generic specification of the CAViaR model can be described as follows:

$$Q_t(\theta) = \gamma_\theta + \sum_{i=1}^{q} \gamma_i Q_{t-i}(\theta) + \sum_{i=1}^{p} \alpha_i l(r_{t-i}, \varphi) \quad , \tag{12}$$

where $\beta' = (\alpha', \gamma', \varphi')$ and l are functions of a finite number of lagged values of the observations. The autoregressive terms $\gamma_i \, Q_{t-i}(\theta)$ for i = 1,...,q permit smoothing the changes in the quantile over time.

The term $l(r_{t-i},\varphi)$ relates $Q_t(\theta)$ with the observable variables belonging to the information set, completing the function by the innovations of the market in the context of GARCH modeling.

The method of Engle & Manganelli (2004) directly models a given conditional quantile of the return instead of specifying an entire distribution of returns, as occurs, for example, in the GARCH model. Furthermore, the CAViaR does not pose time restrictions between the quantiles, tying them to the temporal dynamics between two quantiles, as happens in parametric models.

Engle & Manganelli (2004) presented four different conditional autoregressive specifications. The first of them is adaptive and is the simplest specification. In it, the VaR increases with the existence of innovation peak (*Innov*) in the last observation, and declines slightly otherwise.

Adaptive:

$$Q_t(\theta) = Q_{t-1}(\theta) + \beta_1\{[1 + \exp(\mathrm{G}[r_{t-1} - Q_{t-1}(\theta)])]^{-1} - \theta\}. \qquad \textbf{(13)}$$

$\mathrm{Q_t(\theta)}$ can be described as the VaR_t with confidence level θ %, so the notations $\mathrm{Q_t(\theta)}$ and $\mathrm{VaR_t}$ are equivalent in this paper. G is a positive real number. For $G \to \infty$ and considering that $Q_t(\theta)$ is the θ % VaR, equation (13) can be expressed more simply as:

$$VaR_t = VaR_{t-1} + \beta_1(Inov_{t-1}); \quad Innov_t = I(r_t < -VaR_t) - \theta \quad \textbf{(13a)}$$

where *I*(.) is an indicator function and θ is the probability of the VaR. The adaptive model only considers the rises, but not the returns that are near the VaR.

The second formulation, called the symmetric absolute value model, also considers the lagged return value, but now in modulus (absolute value). The parameter of the lagged VaR is estimated rather than being restricted to the value 1 as in the adaptive CAViaR.

Symmetric absolute value:

$$VaR_t = \beta_0 + \beta_1 VaR_{t-1} + \beta_2|r_{t-1}| \quad . \qquad \textbf{(14)}$$

The third model, called the asymmetric slope model, assumes that the past positive and negative returns affect the VaR differently. Its formulation, where $(r^+) = \max(r, 0); \quad (r^-) = -\min(r, 0)$, is as follows:

Asymmetric slope model:

$$VaR_t = \beta_0 + \beta_1 VaR_{t-1} + \beta_2({r_{t-1}}^+) + \beta_3({r_{t-1}}^-) \ . \qquad \textbf{(15)}$$

The fourth and last model is called the Indirect GARCH (1,1). In this specification, although it is assumed that the tails of the distribution have a different dynamic than the distribution as a whole, the GARCH is still applied, expressed as:

Indirect GARCH (1,1) model:

$$VaR_t = k\left(\beta_0 + \beta_1\left(\frac{VaR_{t-1}}{k}\right)^2 + \beta_2 r_{t-1}^2\right)^{1/2}, \qquad \textbf{(16)}$$

where *k* is usually defined as 1.

The vector of parameters β of the CAViaR model is estimated through quantile regression technique, as previously discussed, with minimization of the sum of the absolute errors weighted by the quantiles and their complements. Füss et al. (2010) commented that Koenker & Bassett (1978) showed that the variance of the mean is slightly smaller in ordinary linear regression with data having normal distribution, but is greater than the variance of the median by least absolute deviations in all other distributions. Engle & Manganelli (2004) demonstrated this result mathematically and showed that the quantile estimator $\hat{\beta}$ is consistent and asymptotically normal.

It is worthwhile stressing an important aspect of this quantile regression model: There is no parametric assumption about the distribution of exchange rate returns,[6] so the VaR estimates are generated by analyzing the serial dependence of the quantiles along the behavior of the returns, strongly reflecting more recent market returns.

3.5 Forecasting exchange rate volatility – Taylor (2005) and Huang et al. (2011)

The next stage of the proposed method is to forecast the volatility from estimated quantiles. Taylor (2005) built on a result of Pearson & Tukey (1965), according to which the ratio for the interval between symmetric quantiles, Q(θ) and Q(1-θ), in the tails of the distribution, is notably constant for various distributions. Taylor proposed some simple approximations of the standard deviation in terms of tail quantiles, namely:

$$\hat{\sigma} \cong \frac{\hat{Q}(0.99)-\hat{Q}(0.01)}{4.65} \qquad \textbf{(17)}$$

$$\hat{\sigma} \cong \frac{\hat{Q}(0.975)-\hat{Q}(0.025)}{3.92} \qquad \textbf{(18)}$$

$$\hat{\sigma} \cong \frac{\hat{Q}(0.95)-\hat{Q}(0.05)}{3.29} \qquad \textbf{(19)}$$

The above approximations supply the base to predict the volatility of financial returns by quantile estimates, produced by a VaR model, such as the CAViaR employed by Taylor (2005) and Huang et al. (2011).

[6] Although quantile regression does not assume a parametric distribution, the technique requires various regularity conditions (as noted by Engle & Manganelli (2004)) for the stochastic process, such as stationarity.

Taylor (2005) applied equations (17), (18) and (19) to estimate the forecast volatility using 98%, 95% and 90% quantile intervals. As it can be seen in the mentioned equations, the numerator is the difference between $\hat{Q}(1-\theta)$ and $\hat{Q}(\theta)$, which are the estimated quantiles for a cumulative probability θ. The denominators of equations (17) to (19) are based on the central distances between the estimated quantiles under the Pearson curves and are slightly different than the distances of the Gaussian distribution: 4.653 (=2.326×2), 3.92 (=1.96×2) and 3.29 (=1.645×2). Taylor extended the idea, proposing a regression model to establish a relation between the quantile intervals and the squared volatility, represented by the following function:

$$\hat{\sigma}_{t+1}^2 = \alpha_1 + \beta_1(\hat{Q}_{t+1}(1-\theta) - \hat{Q}_{t+1}(\theta))^2 \quad , \qquad \textbf{(20)}$$

where $\hat{\sigma}_{t+1}^2$ is the squared volatility forecast for t+1, and α_1 and β_1 are the parameters to be estimated from the sample data. Taylor (2005) reached the result that the forecast estimated by quantile regression exceeds the forecasts carried out by the GARCH and moving average models.

Huang et al. (2011) proposed using percentiles instead of only symmetric pairs. Their study used a series of uniform quantile spaces, more precisely the percentiles. He indicated that the movements in these quantiles reflect not only the tails, but rather the entire distribution. An important argument made by them refers to the question of the volatility's estimate being very conservative when using only quantile estimates based on the tails. To avoid this situation, he proposed using regressions in uniform intervals along the entire distribution. The new formulation of this setup is the following:

$$\hat{\sigma}_{t+1} = \alpha_1 + \beta_1 F\ (\hat{Q}_{t+1}(\theta)) \quad , \qquad \textbf{(21)}$$

where F(.) represents a given function of the estimated quantiles, $Q_{t+1}(\theta)$ is the vector of the quantiles $\{\theta, 2\theta, ..., m\theta\}$ to be estimated at t+1, and $\theta > 0$ and $m\theta < 1$. Huang et al. (2011) used $\theta = 0.01$. Here we also define $\theta = 0.01$. The functions F(.) utilized here were the following (for each case, $\bar{Q}$ is the conditional mean of all the quantile estimates):

$$Standard\ Deviation\ (SD):\ \ F\,(.) = \left(\frac{1}{m-1}\sum_{m=1}^{99}(Q(0.05m) - \bar{Q})^2\right)^{1/2} \qquad \textbf{(22)}$$

$$Weighted\ Standard\ Dev.(WSD):\ \ F\,(.) = \left(\frac{1}{m-1}\sum_{m=1}^{99} W(Q(0.05m) - \bar{Q})^2\right)^{1/2} \qquad \textbf{(23)}$$

$$Standard\ Dev.\ of\ Median\ (SDM):\ \ F\,(.) = \left(\frac{1}{m-1}\sum_{m=1}^{99}(Q(0.05m) - Q(0.5))^2\right)^{1/2} \qquad \textbf{(24)}$$

where *m* is the number of quantiles in which the regression will be calculated.

In equation (23), the weight parameter W is defined as θ/25, when θ<=0.5, and (1-θ)/25 otherwise. This assures that the central quantiles will have a greater impact on the forecast volatility. To estimate the series $\hat{\sigma}_{t+1}$ (i.e., the volatility), we used the squared return as a proxy for volatility at t, because its expectation tends to be the volatility for the case where the mean of the returns is equal to zero. For that purpose, the mean is subtracted from each return and we used the standardized squared return.

The quantile intervals used were the following symmetric pairs: (1, 99), (5, 95), (10, 90) and (25, 75), estimated using the four models proposed for the CAViaR method (Engle & Manganelli, 2004), by linear regression according to Taylor (2005) and Huang et al. (2011). The interquartile interval [Q(0.75) - Q(0.25)] was included as a measure of the dispersion of the samples, and can be a tool for further analysis.

Chart 1 - Models used and the respective basic characteristics and assumptions

Model	**Mathematical Specification**
GARCH	$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2$
EGARCH	$\ln(\sigma_t^2) = a + \beta_1 \ln(\sigma_{t-1}^2) + \beta_2 \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \beta_3 \left[\frac{\lvert u_{t-1}\rvert}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right]$
1. Estimate of the quantiles by CAViaR.	**1.** Generic CAViaR (estimate of the quantiles) $Q_t(\theta) = \gamma_\theta + \sum_{i=1}^{q}\gamma_i Q_{t-i}(\theta) + \sum_{i=1}^{p}\alpha_i l(r_{t-i}, \varphi)$ - <u>Adaptive model:</u> $Q_t(\theta) = Q_{t-1}(\theta) + \beta_1\{[1 + \exp(\mathrm{G}[r_{t-1} - Q_{t-1}(\theta)])]^{-1} - \theta\}$ - <u>Symmetric absolute value model:</u> $VaR_t = \beta_0 + \beta_1 VaR_{t-1} + \beta_2\lvert r_{t-1}\rvert$ - <u>Asymmetric slope model:</u> $VaR_t = \beta_0 + \beta_1 VaR_{t-1} + \beta_2({r_{t-1}}^{+}) + \beta_3({r_{t-1}}^{-})$

	- <u>Indirect GARCH (1,1) model:</u> $$VaR_t = k\left(\beta_0 + \beta_1\left(\frac{VaR_{t-1}}{k}\right)^2 + \beta_2 r_{t-1}^2\right)^{1/2}$$
2. Estimation of volatility based on the QIs estimated by CAViaR (Taylor, 2005) employing variance as a proxy.	**2.** Estimation of volatility by symmetric quantile intervals, employing the following equation (estimation of volatility based on the tails of the distribution, although here we decided to calculate QI Q(0.75-0.25): $$\hat{\sigma}_{t+1}^2 = \alpha_1 + \beta_1(\hat{Q}_{t+1}(1-\theta) - \hat{Q}_{t+1}(\theta))^2$$
3. Estimation of volatility based on the deviations of the quantiles estimated by CAViaR (Huang et al., 2011).	**3.** Estimation of volatility by functions representing the deviations, according to the following equations: $$\hat{\sigma}_{t+1} = \alpha_1 + \beta_1 F\,(\hat{Q}_{t+1}(\theta))$$ where F(.) takes the form of the following three equations: a. $Standard\ Deviation\ (SD)$: $F\,(.) = \left(\frac{1}{m-1}\sum_{m=1}^{99}(Q(0.05m) - \bar{Q})^2\right)^{1/2}$ b. $Weighted\ Standard\ Deviation\ (WSD)$: $$F\,(.) = \left(\frac{1}{m-1}\sum_{m=1}^{99} W(Q(0.05m) - \bar{Q})^2\right)^{1/2}$$ c. $SD\ of\ Median\ (SDM)$: $F\,(.) = \left(\frac{1}{m-1}\sum_{m=1}^{99}(Q(0.05m) - Q(0.5))^2\right)^{1/2}$
4. Estimation of volatility based on the QIs using volatility as a proxy (this study).	**4.** Estimation of volatility by symmetric quantile intervals, using the following equation (estimation of volatility based on the tails of the distribution, as well as the QI = Q(0.75) – Q(0.25): $$\hat{\sigma}_{t+1} = \alpha_1 + \beta_1\left(\hat{Q}_{t+1}(1-\theta) - \hat{Q}_{t+1}(\theta)\right)$$
5. Estimation of volatility based on the squared deviations of the quantiles estimated by CAViaR, using variance as a proxy (this study).	**5.** Estimation of volatility by functions representing the deviations, according to the following equations: $$\hat{\sigma}_{t+1}{}^2 = \alpha_1 + \beta_1 F\,(\hat{Q}_{t+1}(\theta))$$ where F(.) takes the form of the following three equations: a. $Variance\ (Var)$: $F\,(.) = \left(\frac{1}{m-1}\sum_{m=1}^{99}(Q(0.05m) - \bar{Q})^2\right)$ b. $Weighted\ variance\ (WVaR)$: $F\,(.) = \left(\frac{1}{m-1}\sum_{m=1}^{99} W(Q(0.05m) - \bar{Q})^2\right)$ c. $Var.of\ Median\ (VarM)$: $F\,(.) = \left(\frac{1}{m-1}\sum_{m=1}^{99}(Q(0.05m) - Q(0.5))^2\right)$

Note: QI denotes Quantile Interval.

The interval between quantile 75 and quantile 25 contains 50% of the observations. This range of values provides the dispersion of the distribution. In a Gaussian distribution, the quartiles 25 and 75 are located -0.67448 standard deviation and +0.67448 standard deviation from the mean (and median), respectively. It can be stated, then, that the interquartile interval (IQI) for a <u>Gaussian distribution</u> is:

$$IQI = Q_{0,75} - Q_{0,25} = 2 * (0{,}67448) * \sigma = 1{,}34896 * \sigma \quad , \qquad \textbf{(25)}$$

where

IQI = interquartile interval

σ = volatility (standard deviation) of the distribution of returns

To generalize, for the case of a lognormal distribution, the lognormal quantile function can be expressed in the following form:

$$F^{-1}(p) = e^{[\mu+\sigma\Phi^{-1}(p)]},\ p \in (0{,}1)\ , \qquad \textbf{(26)}$$

where Φ = Gaussian cumulative distribution function, and $F^{-1}(p)$ is the inverse of the lognormal cumulative distribution function (cdf) and can be interpreted as the quantile itself.

Therefore, a relation exists among the mean of the distribution, its standard deviation and the quantile. In the case where the shape of the sample distribution is not known in advance, the value of the angular coefficient of the standard deviation will no longer be 1.34896 and can be estimated by a linear regression between the interquartile difference calculated based on the sample and a proxy for volatility, or the observed volatility. The quantiles of the distribution are estimated here by the CAViaR method.

With this, when running the regression specified in this fashion, we expected the linear coefficient to be zero and the angular coefficient of the volatility to be statistically significant. With respect to the interquartile intervals, we also expected a proportional relation, according to the Pearson relation, as expressed in equations 17 to 19. The results of these regressions are analyzed in the next topics.

4. Empirical results

4.1 Data

Our sample is composed of daily closing quotations of the Brazilian Real (R$) *versus* the U.S. Dollar, the so-called PTAX800 rate, calculated and published by the Central Bank of Brazil. The data cover the period from July 6, 2001 to December 30, 2014 (3,392 working days). The variable utilized is the daily exchange rate return, given by expression (1). To estimate the models (rolling window estimation procedure), we used the first 3,140 observations; while the period used for the out-of-sample forecasting exercise was composed of 252 observations.

4.2 Models used and intermediate results

In estimating the volatility and its respective forecast, we used two models: GARCH (1,1) as in equation (5), and EGARCH (1,1) as in equation (6). These specifications are widely used in the academic literature and by financial market analysts. Along with these, we used the models proposed by Taylor (2005) and Huang et al. (2011), which estimate the volatility by using the quantiles estimated by the four autoregressive equations that compose the CAViaR method developed by Engle & Manganelli (2004). We also employed the formulation of Taylor (2005), but in linear form – relating the standard deviation and symmetric quantile intervals instead of linear regression.

4.3 Considerations on the results obtained by the CAViaR method

In the first step of this work, we estimated the 1% to 99% quantiles by the CAViaR method using equations (13) to (16). One of the intermediate results of this first step is the estimate calculated by the CAViaR model for the various quantiles. Table 1 below presents, for illustrative purposes, the selection of the coefficients estimated for the 1%, 5%, 10%, 25%, 50%, 75%, 95% and 99% quantiles calculates using equation (15) from the CAViaR model with asymmetric slope.[7] Figure 1 shows the graphs of the three angular coefficients estimated in each of the quantiles reported in Table 1.

[7] The estimates of the other CAViaR models are not reported here to save space but are available upon request.

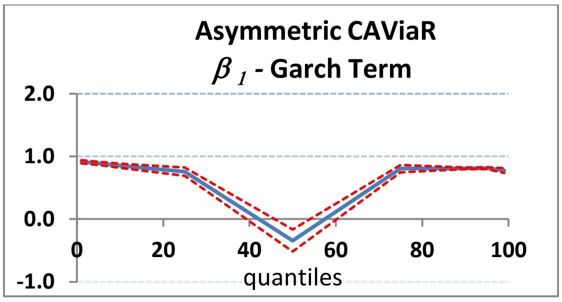
Table 1 – Angular coefficients of the asymmetric CAViaR model

	Asymmetric		
	β1	*β2*	*β3*
Quantile 1	0.9135	0.3039	0.1682
p-value	0.0000	0.0000	0.0045
Quantile 5	0.8899	0.2241	0.1276
p-value	0.0000	0.0000	0.0007
Quantile 10	0.8507	0.2216	0.1380
p-value	0.0000	0.0000	0.0001
Quantile 25	0.7564	0.1725	0.1453
p-value	0.0000	0.0000	0.0002
Quantile 50	-0.3436	-0.0800	0.1040
p-value	0.0254	0.1567	0.0016
Quantile 75	0.8057	-0.2250	-0.0467
p-value	0.0000	0.0000	0.1107
Quantile 90	0.8085	-0.3717	-0.1050
p-value	0.0000	0.0000	0.0000
Quantile 95	0.8112	-0.5123	-0.1338
p-value	0.0000	0.0000	0.0007
Quantile 99	0.7755	-0.7613	-0.1288
p-value	0.0000	0.0000	0.0335

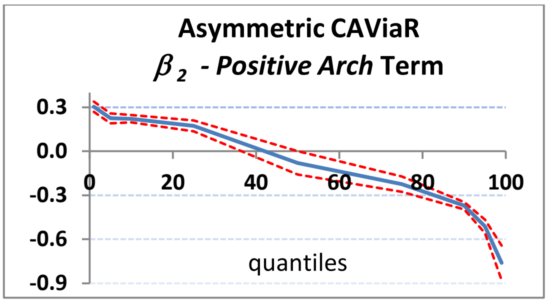
Figure 1 – Angular coefficients of the asymmetric CAViaR model

$$VaR_t = \beta_0 + \beta_1 VaR_{t-1} + \beta_2({r_{t-1}}^{+}) + \beta_3({r_{t-1}}^{-}).$$

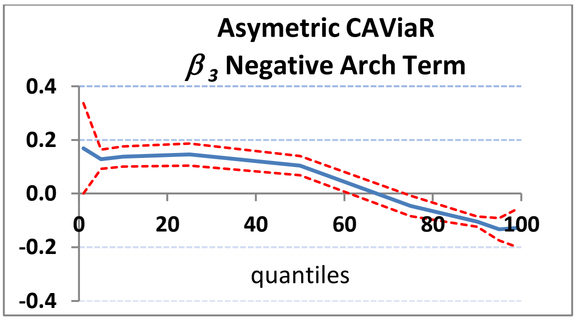
(a) Coefficient of the GARCH term



(b) Coefficient of the ARCH^{+} term



(c) Coefficient of the ARCH^{-} term



First, it should be noted that we are working with a time series of the R$ x US$ (Real x Dollar) exchange rate, where a positive variation in the exchange rate, i.e., a positive return in this series, meaning a devaluation of the Real (R$), while a negative variation corresponds to appreciation of the Real. Therefore, the $ARCH^{+}$ effect, i.e., a positive return in the series, corresponds to depreciation of the exchange rate, which we empirically found to increase the volatility of the series. Next, we analyze the behavior of the coefficients depicted in Figure 1, according to equation (15). The horizontal axis represents the percentiles and the vertical axis the values of the coefficients of each term in the equation. The dotted lines delineate the 95% confidence interval.

The evolution of the coefficient β_1 – related to the VaR (GARCH term) lagged by one time unit (VaR_{t-1}) along the quantiles – is depicted in Figure 1a. The typical agglomeration character of the heteroskedastic behavior can be seen, since the coefficient of the autoregressive term is positive. In other words, higher volatility is followed by a proportionally greater volatility. Note that the coefficient is smaller than 1, so the autoregressive process does not have a unit root, meaning the process is not explosive in any quantile.

With respect to the behavior along the distribution of the VaR, it can be seen that in the lower and higher quantiles, the coefficient β_1 presents similar modulus and direction, and when shifting to the central quantiles of the distribution, its modulus declines, reaching a value near zero for the 50% quantile and becoming non-significant there at the 1% level, while all the others show p-value practically equal to zero. In other words, it can be said that the VaR at the median follows a white noise pattern, meaning prediction is not possible.

Analysis of that behavior reveals that in the quantiles with lower VaR (greater appreciation of the exchange rate), the growth of the lagged volatility tends to add volatility, and in reality tends to bring the VaR to more central quantiles, since the VaR related to appreciation of the Real rises and shifts to positive returns. In counterpart, in the higher quantiles (depreciation of the R$), greater volatility increases the VaR, amplifying the value at risk in the direction of devaluation.

Another aspect that deserves attention is the absolute value of the coefficient associated with the GARCH term. That term's modulus is greater than the corresponding absolute values of the coefficients β_2 and β_3, both associated with the ARCH terms. Here the result is in line with the GARCH estimates of the exchange rate variation, whose results show the coefficient associated with the GARCH term near the interval between 0.75 and 0.90, and the coefficient associated with the ARCH term with value near the interval between 0.25 and 0.10, thus meaning the GARCH term has a greater impact on exchange rate volatility than do the $ARCH^+$ and $ARCH^-$ terms (positive and negative returns, respectively).

With respect to β_2, the angular coefficient of the $ARCH^+$ term (which indicates depreciation), the graph of the estimated quantiles shows it is not significant (this time at 10%) in Q(0.5). Regarding its behavior over the distribution of the VaR, the modulus of the $ARCH^+$ term makes a positive contribution in the lower quantiles (region of appreciation of the domestic currency), bringing the VaR to the more central quantiles of the distribution. In turn, when the VaR is in the depreciation region (higher quantiles), the coefficient β_2 reverses sign, so an increment in the exchange rate tends to shift the VaR to more central quantiles, as if the VaR of the exchange rate return were reverting to the mean, or more precisely, reverting to the central quantiles.

Regarding $\beta_{3,}$ the coefficient associated with the negative return – $ARCH^-$ (appreciation of the exchange rate) – it presents similar evolution to that of the coefficient β_2, i.e., in the lower quantiles of the VaR (appreciation region), the $ARCH^-$ term has a positive coefficient, which declines as it approaches the central region of the distribution, until reversing sign in the area where the VaR presents higher values (depreciation). Therefore, this coefficient acts as a force for reversion of the values to the central region of the VaR distribution, since in the region of appreciation of the Real, it has positive contribution, diminishing the appreciation or even increasing a slight depreciation, and contributes negatively in the depreciation region (VaR in the higher quantiles).

Finally, another noteworthy aspect is the intensity of the impacts of positive returns (depreciation of the Real) and negative ones (appreciation of the Real) on the series. The graphs show that depreciation generates a stronger impact on the volatility of the series than appreciation. These observations are valuable to shed light on a new way to analyze the distribution and the response in the conditional tails of the variables under analysis. These findings can help in the formulation/improvement of models related to the exchange rate.

4.4 Estimation of the proxy for the exchange rate volatility

In adopting the models proposed by Taylor (2005) and Huang et al. (2011), once again a series of information exists that can be compared to the theory of statistical distributions and that contributes to a better understanding of the exchange rate series studied. The comparison here provides a good indication of the relationship between the quantiles of the distribution and its volatility.

It is interesting to note that, by applying quantile intervals, as proposed by Taylor (2005), the quantile function is related to the volatility. It should be mentioned that Taylor used the variance (volatility squared) and its relation with the square of the interquartile difference, while Huang et al. (2011) used the volatility.

Here we used the specifications of both Taylor (2005) and Huang et al. (2011), but added to the former specifications the relation between volatility (square root of the proxy for variance) and the linear interquartile difference, and took the regressions of Huang et al. (2011) not only for the standard deviation, but also for the variance. The coefficients of the regressions shown in Chart 1 and the respective p-values and R^2 of the regressions are reported in Table 2.

Table 2 – Regressions of the proxy for volatility with the CAViaR estimates

Models	α_1	p-value	β_1	p-value	R^2
SM 1_99 SD	0.0035	0.0948	0.0143	0.0000	0.2830
SM 5_95 SD	0.0031	0.1375	0.0227	0.0000	0.2858
SM 10_90_SD	0.0001	0.9955	0.0320	0.0000	0.2841
SM 25_75_SD	0.0063	0.0015	0.0540	0.0000	0.2888
SM SD	0.0021	0.3262	0.0789	0.0000	0.2862
SM WSD	0.0025	0.2229	0.1282	0.0000	0.2876
SM SDM	0.0021	0.3029	0.0787	0.0000	0.2863
SM 1_99_Var	0.0017	0.0061	0.0303	0.0000	0.2183
SM 5_95_Var	0.0018	0.0033	0.0746	0.0000	0.2858
SM 10_90_Var	0.0015	0.0176	0.1448	0.0000	0.2233
SM 25_75_Var	0.0024	0.0001	0.4228	0.0000	0.2320
SM Var	0.0016	0.0073	0.8981	0.0000	0.2256
SM WSD_squared Var	0.0017	0.0043	2.3609	0.0000	0.2299
SM SDM_squared Var	0.0017	0.0069	0.8943	0.0000	0.2259
AS 1_99_SD	-0.0073	0.0010	0.0176	0.0000	0.3136
AS 5_95_SD	-0.0011	0.5969	0.0247	0.0000	0.3119
AS 10_90_SD	-0.0021	0.3124	0.0329	0.0000	0.3129
AS 25_75_SD	0.0021	0.2916	0.0576	0.0000	0.3142
AS SD	-0.0021	0.3002	0.0842	0.0000	0.3142
AS WSD	-0.0012	0.5354	0.1341	0.0000	0.3156
AS SDM	-0.0020	0.3299	0.0838	0.0000	0.3148
AS 1_99_Var	-0.0015	0.0164	0.0499	0.0000	0.2899
AS 5_95_Var	-0.0002	0.6988	0.0996	0.0000	0.2869
AS 10_90_Var	-0.0002	0.6996	0.1721	0.0000	0.2899
AS 25_75_Var	0.0005	0.3914	0.5351	0.0000	0.2985
AS Var	-0.0003	0.5476	1.1380	0.0000	0.2955
AS WSD_squared Var	-0.0000	0.8914	2.8614	0.0000	0.2986
AS SDM_squared Var	-0.0003	0.5812	1.1253	0.0000	0.2967
IG 1_99_SD	0.0004	0.8504	0.0335	0.0000	0.2829
IG 5_95_SD	-0.0015	0.5000	0.0015	0.0000	0.2760
IG 10_90_SD	-0.0034	0.1265	0.0680	0.0000	0.2870
IG 25_75_SD	-0.0022	0.3008	0.0033	0.0000	0.2941
IG SD	-0.0018	0.4075	0.0041	0.0000	0.2883
IG WSD	-0.0033	0.1234	0.2151	0.0000	0.2946
IG SDM	-0.0011	0.6101	0.1242	0.0000	0.2903
IG 1_99_Var	0.0054	0.0000	0.0863	0.0000	0.0548
IG 5_95_Var	0.0005	0.3875	0.4113	0.0000	0.2178
IG 10_90_Var	0.0005	0.4166	0.6759	0.0000	0.2367
IG 25_75_Var	0.0010	0.0759	1.9991	0.0000	0.2609
IG Var	0.0058	0.0000	1.4396	0.0000	0.0456
IG WSD_squared Var	0.0005	0.4285	6.7340	0.0000	0.2510
IG SDM_squared Var	0.0007	0.2408	2.3059	0.0000	0.2425
AD 1_99_SD	0.0038	0.2369	0.0110	0.0000	0.1231
AD 5_95_SD	0.0017	0.5066	0.0210	0.0000	0.1957
AD 10_90_SD	-0.0021	0.4523	0.0310	0.0000	0.1832
AD 25_75_SD	-0.0126	0.0001	0.0736	0.0000	0.1880
AD SD	-0.0089	0.0031	0.0795	0.0000	0.1865
AD WSD	-0.0118	0.0001	0.1471	0.0000	0.2078
AD SDM	-0.0100	0.0008	0.0798	0.0000	0.1956
AD 1_99_Var	-0.0005	0.5638	0.0279	0.0000	0.0696
AD 5_95_Var	-0.0007	0.3796	0.0900	0.0000	0.1135
AD 10_90_Var	-0.0009	0.2596	0.1851	0.0000	0.0926
AD 25_75_Var	-0.0029	0.0017	0.9629	0.0000	0.0940
AD Var	-0.0021	0.0152	1.4700	0.0000	0.1014
AD WSD_squared Var	-0.0027	0.0014	3.8136	0.0000	0.1141
AD SDM_squared Var	-0.0025	0.0039	1.1532	0.0000	0.1119

Notes: AD denotes the adaptive CAViaR (eq.13), SM denotes symmetric (eq.14), AS denotes asymmetric (eq.15) and IG denotes Indirect GARCH (eq.16). According to Chart 1, SD denotes standard deviation and indicates the estimates of the proxy for the volatility using the metrics (quantile deviations) of Huang et al. (2011), WSD denotes weighted standard deviation, and SDM standard deviation of median. Var indicates the estimate of the proxy for the squared volatility using the metrics (squared quantile deviations) of Taylor (2005).

It can first be observed that all the angular coefficients are significant, at the 1% level. With respect to the models used, for the quantiles estimated by the CAViaR (whether interquartile intervals or deviations calculated by the quantiles), all of the angular coefficients that associate those intervals and deviations with the proxies for variance and/or volatility are significant, even at 1%. It is interesting to note that, except in a few cases, the linear coefficients of the regressions performed based on the standard deviation as the dependent variable are not significant. For example, in the asymmetric and indirect GARCH models, the linear coefficient of the models estimated by OLS for the interquartile interval - IQI ($Q_{(0.75)}$ –$Q_{(0.25)}$) is not significant, even at the 10% level.

In the symmetric model, the linear coefficient is not significant in several cases. In other words, in a distribution with zero mean, the relation between volatility and the IQI is the coefficient $1/\beta$, approximately, an expected situation according to the considerations made about equation (25). One more consideration in this respect is that, by equation (25), for the cases studied, the value of 1.3489 is the value to be estimated. In the regressions carried out, it is given as $1/\beta$. In turn, the relation of variance with the squared quantile interval (IQ) – the regression proposed by Taylor (2005) – with some exceptions, results in significant angular coefficients and linear coefficients. This is because:

$$\sqrt{\alpha_1 \ + \beta_1(\hat{Q}_{t+1}(1-\theta) - \hat{Q}_{t+1}(\theta))^2} \neq \alpha_1 + \beta_1(\hat{Q}_{t+1}(1-\theta) - \hat{Q}_{t+1}(\theta)) \ . \quad \textbf{(27)}$$

Thus, the relation is not direct and the correction is partly due to the linear coefficient.

Table 3 – Estimated coefficients - GARCH (1,1) model

	α_o	*p-value*	α_1	*p-value*	β	*p-value*
GARCH normal	0.0128	0.0000	0.1737	0.0000	0.8220	0.0000
GARCH t-Student	0.0001	0.0000	0.1686	0.0000	0.8079	0.0000

Table 3 reports the results of the GARCH (1, 1) models, with the normal and t-Student distributions, for the sample under analysis. Table 4 presents the results obtained when applying the asymmetric GARCH model, also called the EGARCH (1, 1) with the same two options: assuming normal or t-Student distribution.

Table 4 – Estimated coefficients - EGARCH (1,1) model

	α	*p-value*	β_1	*p-value*	β_2	*p-value*	β_3	*p-value*
EGARCH normal	-0.2344	0.0000	0.2775	0.0000	0.0818	0.0000	0.9676	0.0000
EGARCH t-Student	-0.2368	0.0000	0.2821	0.0000	0.0778	0.0000	0.9705	0.0000

The estimate of the GARCH (1, 1) model based on the observed data resulted in coefficients in line with the theory, i.e., both for the normal and t-Student distributions, the sum of the coefficients of the GARCH term and ARCH term are near and smaller than one. It should be noted that the coefficient of the GARCH has greater weight than that of the ARCH term, indicating that the autoregressive term of the volatility has higher weight than the market innovation. This behavior can also be seen in the CAViaR models. For example, the coefficients of the asymmetric model reported previously in Table 1, whose values are associated with the volatility term (GARCH term), have greater modulus than the values associated with the returns (ARCH terms). Returning to equation (6) for the EGARCH model, it follows that:

$$\ln(\sigma_t^2) = a + \beta_1 \ln(\sigma_{t-1}^2) + \beta_2 \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \beta_3 \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right].$$

The parameter β_3 represents the magnitude of the effect of symmetry of the innovation on volatility, while the parameter β_1 represents the persistence of volatility. If β_1 is relatively large, the volatility is highly persistent, meaning that the volatility, without innovations, decays slowly. The coefficient β_2 allows asymmetries. This is possible due to the fact that β_2 will be negative if the relationship between volatility and returns is negative.

In the case of the stock market, the expectation is for β_2 to have a negative sign, since negative shocks increase the volatility of a share price or of the entire market index. However, the foreign exchange market has a different dynamic. A positive (negative) return in the Real x U.S. Dollar market means depreciation (appreciation) of Brazil’s currency (Real). A weak (strong) Real generates higher (lower) volatility. Hence, the sign is as expected.

4.5 Evaluation of the exchange rate volatility forecasts

One of the main reasons for modeling the price of a financial assets or its volatility is to forecast these assets' values. Here, the forecast for t+1 was static.[8] According to this model, the forecast values are based on observed values contained in the out-of-sample series (here 252 observations, from January 2, 2014 to December 30, 2014).

To assess the precision of a forecast, some measures of the error between the observed and predicted values are used. Among them, here we used the following (note that in all the formulations $\hat{y}_t$ is the forecasted value and y_t is the observed value).

a) RMSE (root mean squared error), given by the following formula:

$$\text{RMSE} = \sqrt{\sum(\hat{y}_t - y_t)^2 / N} \quad ; \qquad \textbf{(28)}$$

b) MAE (mean absolute error), according to the following formula:

$$\text{MAE} = \sum|\hat{y}_t - y_t| / N \quad ; \qquad \textbf{(29)}$$

c) Theil coefficient, expressed by the following formula:

$$\text{Theil} = \frac{\sqrt{\sum(\hat{y}_t - y_t)^2 / N}}{\sqrt{\sum \hat{y}_t^2 / N} + \sqrt{\sum y_t^2 / N}} \quad . \qquad \textbf{(30)}$$

Table 5 – Evaluation of the volatility forecasts

Models	RMSE	MAE	Theil coeff.	Models	RMSE	MAE	Theil coeff.
SM_1_99_SD	0.0503	0.0375	0.3712	IG_10_90_SD	0.0503	0.0374	0.3632
SM_5_95_SD	0.0504	0.0371	0.3705	IG_25_75_SD	0.0516	0.0385	0.3681
SM_10_90_SD	0.0504	0.0371	0.3698	IG_SD	0.0503	0.0374	0.3637
SM_25_75_SD	0.0507	0.0374	0.3706	IG_WSD	0.0507	0.0377	0.3666
SM_SD	0.0504	0.0372	0.3707	IG_SDM	0.0504	0.0375	0.3639
SM_WSD	0.0506	0.0373	0.3714	IG_1_99_Var	0.0542	0.0436	0.3439
SM_SDM	0.0504	0.0372	0.3705	IG_5_95_Var	0.0540	0.0435	0.3431
SM_1_99_Var	0.0510	0.0387	0.3538	IG_10_90_Var	0.0536	0.0425	0.3451
SM_5_95_Var	0.1849	0.1778	0.5883	IG_25_75_Var	0.0545	0.0427	0.3538
SM_10_90_Var	0.0544	0.0434	0.3457	IG_Var	0.0533	0.0421	0.3457
SM_25_75_Var	0.0513	0.0383	0.3622	IG_WSD_squared	0.0538	0.0425	0.3479
SM_Var	0.0513	0.0390	0.3529	IG_SDM_squared	0.0532	0.0418	0.3466
SM_WSD_squared	0.0544	0.0433	0.3466	AD_1_99_SD	0.0515	0.0364	0.4101
SM_SDM_squared	0.0513	0.0390	0.3531	AD_5_95_SD	0.0503	0.0373	0.3656
AS_1_99_SD	0.0499	0.0372	0.3535	AD_10_90_SD	0.0507	0.0378	0.3702
AS_5_95_SD	0.0500	0.0373	0.3535	AD_25_75_SD	0.0510	0.0388	0.3648

[8] To be comparable with the predictions carried out with Matlab in the CAViaR model, the forecast was for t+1.

AS_10_90_SD	0.0501	0.0373	0.3541	AD_SD	0.0506	0.0367	0.3855
AS_25_75_SD	0.0504	0.0376	0.3553	AD_WSD	0.0505	0.0375	0.3724
AS_SD	0.0501	0.0374	0.3540	AD_SDM	0.0505	0.0385	0.3851
AS_WSD	0.0503	0.0375	0.3557	AD_1_99_Var	0.0505	0.0382	0.3591
AS_SDM	0.0501	0.0374	0.3540	AD_5_95_Var	0.0545	0.0426	0.3482
AS_1_99_Var	0.0580	0.0472	0.3471	AD_10_90_Var	0.0573	0.0464	0.3519
AS_5_95_Var	0.0550	0.0438	0.3431	AD_25_75_Var	0.0571	0.0458	0.3550
AS_10_90_Var	0.0553	0.0441	0.3439	AD_Var	0.0521	0.0400	0.3530
AS_25_75_Var	0.0541	0.0423	0.3455	AD_WSD_squared	0.0540	0.0420	0.3525
AS_Var	0.0554	0.0443	0.3442	AD_SDM_squared	0.0521	0.0397	0.3553
AS_WSD_squared	0.0555	0.0444	0.3443	GARCH normal	0.0531	0.0377	0.4305
AS_SDM_squared	0.0550	0.0437	0.3448	GARCH t-Student	0.0544	0.0379	0.4593
IG_1_99_SD	0.0501	0.0372	0.3628	EGARCH normal	0.0531	0.0378	0.4294
IG_5_95_SD	0.0500	0.0371	0.3627	EGARCH t-Student	0.0544	0.0379	0.4596

Notes: SM denotes symmetric, AS asymmetric, IG indirect GARCH, AD adaptive, SD standard deviation, WSD, weighted standard deviation, SDM standard deviation of median.

Table 5 presents the values of the prediction error measures for each of the 60 models estimated. The first aspect to note is the fact that the root mean squared error (RMSE) is consistently greater than the mean absolute error (MAE), denoting a certain variability in the errors. Since the RMSE penalizes high deviations, if the main interest is in the tails, it is necessary to choose models with smaller RMSE. In this case, of all the models, the forecast made by the regression of volatility with the quantile interval ($Q_{(0.01)} - Q_{(0.99)}$) estimated based on the asymmetric CAViaR model by the regression of the standard deviation was the most precise.

When considering the mean absolute error, the regression of volatility with the quantile interval ($Q_{(0.01)} - Q_{(0.99)}$) estimated with the adaptive CAViaR model, also by regression with the standard deviation, was the most precise model. In turn, when considering the Theil coefficient as the criterion, the regression of variance with the quantile interval ($Q_{(0.05)} - Q_{(0.95)}$) estimated by the Indirect GARCH (CAViaR model), with regression of the variance, and the asymmetric CAViaR model with the quantile interval ($Q_{(0.05)} - Q_{(0.95)}$) and regression of the variance, were the most precise.

We should mention that, to enable comparison, it was necessary to recalculate the error measures for the regressions with variance as the dependent variable, since their dimensions differed from those of the error measures calculated when the dependent variable was the standard deviation. We performed an equal adjustment in the GARCH and EGARCH models.

To test the robustness of the comparison, we applied the Giacomini-White (2006) test of conditional predictive ability. While the metrics reported earlier, in Table 5, are based on the difference between the real and predicted observations in the period analyzed (here 252 days), the Giacomini-White test analyzes the distribution of the errors by testing a null hypothesis about the difference between two loss functions (identified as 1 and 2). Loss function 1 is the difference between the predicted results of the benchmark model and the observed results, while loss function 2 is the difference between the results predicted by the model being tested and the observed results. The null hypothesis of the Giacomini-White test is therefore:

$$E(Loss\ Function\ 1 - Loss\ Function\ 2) = 0 \qquad \textbf{(31)}$$

If loss function 2 is significantly greater, in statistical terms, than loss function 1, the null hypothesis is rejected, and in this case the t-statistic will be negative (even for calculations using absolute values). In this case, model 1, related to loss function 1, is the benchmark model and will be better. If on the other hand, loss function 1 is significantly greater than loss function 2, the t-statistic is positive and the model tested is worse than the benchmark model.

Table 6 shows the results of the tests performed for the 59 models, taking the asymmetric CAViaR model as the benchmark and using regression by the standard deviation for the (1, 99) interquantile difference, called AS_1_99_SD. Table 7 shows the results of the Giacomini-White (2006) test, taking the GARCH (1,1) model as the benchmark and assuming a normal distribution. In both tables, in the column B/W/NS, B means the benchmark model is better than the model in the respective line, W means the benchmark model is worse than the model tested, and NS means there is no significant difference between the two models (such that H_0 is not rejected).

The results indicate that the forecast of the asymmetric CAViaR model, with symmetric quantile interval of (1, 99), was better than (or equal to) 66% of the 59 models compared, and worse than 34%. In turn, the GARCH (1,1) model was worse than 71% of the 59 other models and better than 13%, while the precision was statistically equal for 16% of the models.

Table 6 – Results of the Giacomini-White test

(model AS_1_99_SD is the benchmark)

Models	p-value	t-stat.	B/W/NS	Models	p-value	t-stat.	B/W/NS
SM_1_99_SD	0.0231	5.1623	W	IG_10_90_SD	0.0095	6.7210	W
SM_5_95_SD	0.0496	3.8539	W	IG_25_75_SD	0.5982	0.2776	NS
SM_10_90_SD	0.0260	4.9573	W	IG_SD	0.0156	5.8466	W
SM_25_75_SD	0.2099	1.5720	NS	IG_WSD	0.1332	2.2547	NS
SM_SD	0.0547	3.6920	NS	IG_SDM	0.0314	4.6296	W
SM_WSD	0.1379	2.2010	NS	IG_1_99_Var	0.2589	1.2745	NS
SM_SDM	0.0498	3.8497	W	IG_5_95_Var	0.2168	1.5252	NS
SM_1_99_Var	0.0139	6.0527	W	IG_10_90_Var	0.6203	0.2454	NS
SM_5_95_Var	0.0559	3.6538	NS	IG_25_75_Var	0.8241	-0.0493	NS
SM_10_90_Var	0.0001	17.9691	W	IG_Var	0.8006	0.0638	NS
SM_25_75_Var	0.6953	-0.1533	NS	IG_WSD_squared	0.7984	0.0652	NS
SM_Var	0.0261	4.9474	W	IG_SDM_squared	0.9196	0.0101	NS
SM_WSD_squared	0.0000	18.5731	W	AD_1_99_SD	0.0000	-170.0684	B
SM_SDM_squared	0.0294	4.7464	W	AD_5_95_SD	0.0000	17.0076	W
AS_1_99_SD	***BENCHMARK***			AD_10_90_SD	0.0000	25.2098	W
AS_5_95_SD	0.0000	-16.2690	B	AD_25_75_SD	0.0000	-166.8857	B
AS_10_90_SD	0.3894	-0.7405	NS	AD_SD	0.0137	6.0728	W
AS_25_75_SD	0.2168	-1.5247	NS	AD_WSD	0.0000	20.9304	W
AS_SD	0.1434	-2.1407	NS	AD_SDM	0.0213	5.2987	W
AS_WSD	0.1836	-1.7680	NS	AD_1_99_Var	0.0000	61.6199	W
AS_SDM	0.1393	-2.1847	NS	AD_5_95_Var	0.1143	-2.4934	NS
AS_1_99_Var	0.0000	-16.3153	B	AD_10_90_Var	0.4241	-0.6389	NS
AS_5_95_Var	0.0001	-13.8419	B	AD_25_75_Var	0.0295	-4.7351	B
AS_10_90_Var	0.0005	-11.9098	B	AD_Var	0.6585	0.1952	NS
AS_25_75_Var	0.0001	-14.0366	B	AD_WSD_squared	0.1046	-2.6332	NS
AS_Var	0.0003	-13.0166	B	AD_SDM_squared	0.4026	-0.7004	NS
AS_WSD_squared	0.0005	-11.9476	B	GARCH normal	0.0129	-6.1810	B
AS_SDM_squared	0.0003	-12.9939	B	GARCH t-Student	0.0000	-168.9601	B
IG_1_99_SD	0.0022	9.3574	W	EGARCH normal	0.0064	-7.4312	B
IG_5_95_SD	0.0005	11.8442	W	EGARCH t-Student	0.0122	-6.2759	B

Notes: The sign of the t-statistic in this case reflects the precision of the forecast of the compared models. A negative t-statistic with p-value < 5% indicates the benchmark model is more precise than the tested model, while a positive t-statistic with p-value <5% indicates the benchmark model is less precise than the tested model. For p-values > 5%, the difference in the models' precision is considered to be not significant. The column B / W / NS shows if the precision of model in the respective line in comparison to the benchmark is better (B), worse (W) or not significantly different (NS).

Table 7 – Results of the Giacomini-White test

(GARCH (1,1) normal is the benchmark)

Models	**p-value**	**t-stat.**	**B/W/NS**	**Models**	**p-value**	**t-stat.**	**B/W/NS**
SM_1_99_DP	0.0026	9.0240	W	IG_10_90_DP	0.0004	12.2243	W
SM_5_95_DP	0.0028	8.8741	W	IG_25_75_DP	0.0023	9.2408	W
SM_10_90_DP	0.0022	9.3738	W	IG_DP	0.0006	11.7905	W
SM_25_75_DP	0.0041	8.2095	W	IG_WSD	0.0014	10.1029	W
SM_DP	0.0029	8.8080	W	IG_SDM	0.0007	11.4838	W
SM_WSD	0.0041	8.2193	W	IG_1_99_Var	0.0000	93.4756	W
SM_SDM	0.0028	8.9230	W	IG_5_95_Var	0.0000	74.0471	W
SM_1_99_Var	0.0000	17.3128	W	IG_10_90_Var	0.0000	79.7028	W
SM_5_95_Var	0.0000	17.0250	W	IG_25_75_Var	0.0000	21.9948	W
SM_10_90_Var	0.0000	100.4832	W	IG_Var	0.0000	61.7645	W
SM_25_75_Var	0.0073	7.1814	W	IG_WSD_squared	0.0000	69.6904	W
SM_Var	0.0000	20.3055	W	IG_SDM_squared	0.0000	53.5185	W
SM_WSD_squared	0.0000	104.3669	W	AD_1_99_DP	0.0000	-55.6337	B
SM_SDM_squared	0.0000	19.9849	W	AD_5_95_DP	0.0001	14.9322	W
AS_1_99_DP	0.0129	6.1810	W	AD_10_90_DP	0.0000	17.5370	W
AS_5_95_DP	0.0000	-51.2013	B	AD_25_75_DP	0.0000	-52.8897	B
AS_10_90_DP	0.0121	6.2874	W	AD_DP	0.0027	8.970304	W
AS_25_75_DP	0.0146	5.9536	W	AD_WSD	0.0001	15.068630	W
AS_DP	0.0140	6.0264	W	AD_SDM	0.0035	8.521854	W
AS_WSD	0.0160	5.7959	W	AD_1_99_Var	0.0000	33.920960	W
AS_SDM	0.0142	6.0028	W	AD_5_95_Var	0.1834	1.769674	NS
AS_1_99_Var	0.0010	-10.7200	B	AD_10_90_Var	0.0678	3.334533	NS
AS_5_95_Var	0.0490	-3.8722	B	AD_25_75_Var	0.6709	0.180441	NS
AS_10_90_Var	0.0823	-3.0172	NS	AD_Var	0.0031	8.728750	W
AS_25_75_Var	0.1262	-2.3382	NS	AD_WSD_squared	0.1232	2.375133	NS
AS_Var	0.0375	-4.3268	B	AD_SDM_squared	0.0241	5.081819	W
AS_WSD_squared	0.1037	-2.6461	NS	GARCH normal	***BENCHMARK***		
AS_SDM_squared	0.0391	-4.2554	B	GARCH t-Student	0.0000	-54.6699	B
IG_1_99_DP	0.0004	12.5312	W	EGARCH normal	0.9904	-0.0001	NS
IG_5_95_DP	0.0003	12.8718	W	EGARCH t-Student	0.6059	0.2661	NS

Notes: The sign of the t-statistic in this case reflects the precision of the forecast of the compared models. A negative t-statistic with p-value < 5% indicates the benchmark model is more precise than the tested model, while a positive t-statistic with p-value <5% indicates the benchmark model is less precise than the tested model. For p-values > 5%, the difference in the models' precision is considered to be not significant. The column B / W / NS shows if the precision of model in the respective line in comparison to the benchmark is better (B), worse (W) or not significantly different (NS).

5. Conclusion

The volatility of financial assets is of great importance in forecasting returns and investment decisions. In particular, the exchange rate volatility plays an important role in formulating economic policies. It affects a country's balance of payments, the general level of prices and arbitrage between interest rates of different countries, and is also affected by these same factors. The exchange rate and its volatility can be studied with focus on parity of interest rates or the real exchange rate, considering the fundamentals of the balance of payments.

As pointed out by Huang et al. (2011), some autoregressive models have been proposed and have achieved some success in estimating not only the returns, but also the volatility of the exchange rate. According to Hansen & Lude (2005), the GARCH (1,1) model is still the best for predicting the volatility of stock returns, but this does not extend to volatility of exchange rates. One of the reasons for this (including the Brazilian case) is government intervention, such as by currency swap auctions, which can include an asymmetry factor, for example in the movement of exchange rates and their consequent volatility.

Our aim is to contribute in this area by introducing a tool that enables more detailed analysis on the behavior of the distribution of the exchange rate. We not only compared the forecasting power of models based on quantile regression with some models of the GARCH family, but also examined the behavior of the exchange rate along its distribution and its consequent volatility. The results were enriching. The coefficients of the terms of the estimated autoregressive equations revealed variations along the distribution.

For example, in estimating the quantiles in the asymmetric CAViaR model, we observed how the coefficients of positive and negative returns behaved as mechanisms to revert the returns to the more central part of the distribution (reverting the volatility to a certain level). Calculating the volatility of the 99 quantiles estimated by the VaR model and the standard deviations of these quantiles revealed this method was effective, with some of the indexes being relatively better for the method considering the semiparametric model. With respect to forecasting exchange rate volatility, we took two models as benchmarks to apply the Giacomini and White (2006) test of precision.

The results for Brazil showed that the prediction of the asymmetric CAViaR model, with quantile interval of (1, 99), was better than (or equal to) 66% of the 59 compared models, and worse than 34%. The other reference model, the GARCH (1,1), was worse than 71% of the other models, better than 13% and statistically equal in forecasting precision to 16% of the models.

A good deal of research has been done in this area, such as the study of cointegrated series using quantile regression (Wang, 2012) and the use of GARCH models applying a loss function calculated based on the modulus of the deviations in relation to the quantiles (Xiao and Koenker, 2009). Applied studies in this area can generate additional information about the relationship between the variables, such as the relationship of interventions by the central bank and their influence on the more extreme quantiles of the distribution of the returns of the exchange rate, or the relationship between the exchange rate and macroeconomic variables, testing theories of international finance with this additional semiparametric tool.

Finally, we suggest as topics for future research the comparison between the prediction models using a longer horizon, or aggregating structural models that use quantile regressions.

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