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Optimal Monetary Policy, Gains from Commitment, and Inflation Persistence*

André Minella†

Abstract

Using a New Keynesian framework, this paper compares the effects on the welfare of optimal monetary policies under commitment and discretion, and examines the consequences of the presence of inflation persistence. A policy under commitment generates a better weighted average of the variances of output and inflation ("dynamic gains"), and eliminates the inflationary bias. Commitment usually delivers a lower variance of inflation and a higher variance of output than those under discretion. The effect of the presence of inflation persistence on the dynamic gains from commitment is somehow surprising: the benefits are increasing in the degree of inflation persistence for moderate levels of persistence. On the other hand, inflation persistence reduces the inflationary bias. Furthermore, under "restricted commitment", where the solution is restricted to be within the same family of rules of the discretionary case, the gains are substantially inferior to those from commitment.

Keywords: optimal monetary policy, commitment, discretion, inflation persistence

JEL Classification: E31, E52

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1 Introduction

Using a New Keynesian framework, this paper compares optimal monetary policies conducted with and without commitment, and examines the effects of the presence of inflation persistence. Without commitment—also called the discretionary case—the central bank is allowed to reoptimize every period, whereas, under commitment, it optimizes once and for all. The paper addresses the effects of these policies on the welfare, which is measured by a loss function that penalizes deviations of output gap and inflation rate from their targets. The main results are as follows. A policy under commitment generates a superior welfare compared to that under discretion. It eliminates the inflationary bias, and results in a better weighted average of the variances of output and inflation. Commitment usually delivers a lower variance of inflation rate than that under discretion, but results in a higher variance of output for a large range of parameter values. The relative importance of the gains coming from the variability of output and inflation ("dynamic gains") in comparison to those from the elimination of the inflationary bias depends significantly on the parameter values.

The effect of the presence of inflation persistence on the gains from commitment is somehow surprising. Since the benefits under commitment stem from the presence of forward-looking variables, we could expect that the presence of inflation persistence would reduce those gains. Nevertheless, for moderate levels of inflation persistence, the dynamic gains from commitment are greater than those verified without inflation persistence. On the other hand, the presence of inflation persistence reduces the inflationary bias, decreasing the relative benefits from commitment.

Furthermore, the paper also considers optimal policies under "restricted commitment", where the solution is restricted to be within the same family of rules of the discretion solution. The gains are significantly inferior to those coming from commitment.

The benefits from policies under commitment have been explored since Kydland and Prescott (1977) and Barro and Gordon (1983). This literature generally uses a Lucas-type aggregate supply curve, in which prices are flexible and output is positively related to the difference between realized and expected current inflation rate. Under discretion the monetary authority attempts to expand output beyond its natural rate because of the presence of some distortions in the economy, such as taxes or imperfect competition. Nevertheless, the private agents recognize this incentive, incorporating it in their inflationary expectations, which leads to an equilibrium with output at its natural rate and the inflation rate above the target—the inflationary bias. In contrast, if the central bank can commit itself, the inflationary bias is eliminated and the society is better off. In this framework, however, policies under commitment do not yield additional gains.

On the other hand, in the absence of commitment, the society is better off appointing a central banker with more inflation aversion than that of the society (Rogoff, 1985). With a more conservative central banker, the inflationary bias is reduced, but at a cost of some stabilization bias.

Differently from these models, which assume price flexibility, this paper draws on a recent literature that has compared policies under commitment and discretion using a New Keynesian framework based on time-dependent sticky prices (Clarida, Galí, and Gertler, 1999; Svensson and Woodford, 1999; Woodford, 1999a, 1999c). According to the New Keynesian Phillips curve, the current inflation rate depends on the current output gap and expected inflation rate (Roberts, 1995).

Nevertheless, the empirical relevance of the presence of persistence in inflation has been emphasized by some authors, such as Fuhrer and Moore (1995) and Fuhrer (1997). They suggest a hybrid form for the aggregate supply curve, which, besides the term corresponding to the expected inflation, also contains one corresponding to the past inflation. Similar formulation is developed in Galí and Gertler (1999), where the presence of backward-looking firms generates inflation persistence and is tested for the U.S. economy. They have concluded that backward-looking price setting, although statistically significant, is not quantitatively important.[1] Nevertheless, as

[1] See Galí, Gertler, and López-Salido (2001) for similar estimation for the Euro area.

we will see, even a small share of backward-looking firms has important implications for the welfare.

Furthermore, the New Keynesian Phillips curve implies that announcements of disinflation, if perfectly credible, result in instantaneous adjustment of inflation expectations, generating a costless disinflation. This result seems to be in contrast to the empirical finding, documented in Ball (1994), that disinflations are commonly accompanied by a reduction in the detrended output and employment.

Initially, I work with an aggregate supply curve without inflation persistence, showing some results that do not depend on the inflation persistence assumption. Subsequently, I introduce some persistence in inflation and investigate its implications for the previous conclusions.

The paper is organized in the following manner. Section 2 presents the theoretical model. Section 3 solves the model with and without commitment. Section 4 defines the parameter values of the model, and evaluates the effects of the different policies on the welfare. The basic conclusions are summarized in the last section.

2 The Model

I use an optimizing AS-IS-LM model with monopolistically competitive firms and time-dependent sticky prices, based on Clarida, Galí, and Gertler (1999). The aggregate supply curve (AS) is given by:

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t, \tag{1}$$

where π_t is the rate of inflation—defined as $p_t - p_{t-1}$, where p_t is the price level—x_t is the output gap, β is a discount factor, E_t refers to expectations conditional upon information available at time t, and u_t represents a "cost-push shock". Throughout the paper, all the variables in lower case represent log-deviations from their steady-state values.[2] The output gap is $x_t \equiv y_t - y_t^*$, where y_t is the actual output and y_t^* is the potential output. The latter is defined as the output that would prevail if prices

[2] These are the values prevailing with fully flexible prices and no stochastic disturbances.

and wages were perfectly flexible. This Phillips curve is obtained from intertemporal optimization of monopolistically competitive firms under price rigidity. The derivation is in the appendix. It employs an assumption used by Calvo (1983), in which, every period, each firm has a fixed probability θ of not adjusting its price, independently of the last time the firm adjusted it. This leads to $\lambda = (1-\theta)(1-\beta\theta)\kappa\theta^{-1}$, where κ is the output elasticity of marginal cost.[3]

Nevertheless, this formulation of the Phillips curve delivers some counterfactual results, as stressed in Galí and Gertler (1999) and Fuhrer (1997). It implies that current changes in inflation are *negatively* related to the lagged output gap, that is, a positive output gap would lead to a reduction in the inflation rate in the following period. In contrast, the empirical evidence is that a positive output gap is followed by an increase in the inflation rate over the cycle. Moreover, as noted before, this formulation implies that, with perfectly credible announcements, a disinflation is costless.

These empirical results have motivated some authors to work with an aggregate supply curve that includes inflation persistence. Fuhrer and Moore (1995) have generated inflation persistence assuming that agents care about relative wages over the life of the wage contract. Roberts (1997, 1998) has found some empirical evidence that expectations are less than perfectly rational: a fraction of the agents would have adaptive expectations or there would be a partial adjustment of expectations (these would adjust only gradually to the fully rational value).

The paper focuses on the effects of inflation persistence itself, without a particular concern about the specific derivation for the persistence. The aggregate supply curve (AS) presents a hybrid form:

$$\pi_t = \lambda x_t + \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + u_t. \tag{2}$$

One possibility for the derivation of these parameters is the formulation built in Galí and Gertler (1999). There is a fraction ω of backward-looking firms that

[3] The optimization generates $\pi_t = \chi mc_t + \beta E_t \pi_{t+1}$, where $\chi \equiv (1-\theta)(1-\beta\theta)\theta^{-1}$ and mc_t is the real marginal cost. Assuming the latter is proportional to the output gap, $mc_t = \kappa x_t$, we obtain equation (1), where $\lambda \equiv \chi\kappa$. Then u_t represents deviations from this proportion.

determines prices according to the rule of thumb $p_t^b = p_{t-1}^* + \pi_{t-1}$, where p_t^b is the price charged by a backward-looking firm that is allowed to adjust price at time t, and p_{t-1}^* is the price of a forward-looking firm that adjusted its price in the previous period. In this formulation, the resulting parameters of the hybrid AS curve are $\lambda = (1-\omega)(1-\theta)(1-\beta\theta)\kappa\varrho^{-1}$, $\gamma_f = \beta\theta\varrho^{-1}$, $\gamma_b = \omega\varrho^{-1}$,with $\varrho \equiv \theta + \omega(1-\theta(1-\beta))$. If $\beta = 1$, then $\gamma_f + \gamma_b = 1$. When $\omega = 0$, equation (2) reduces to (1).

This formulation allows the presence of inflation persistence to affect the coefficient λ (a greater ω decreases the value of λ). Nonetheless, the specific definition of the rule of thumb or other mechanism that generates inflation persistence involves some arbitrariness. Therefore, the paper also works with the case where γ_b is not restricted by this structural form.

The aggregate demand curve (IS) is

$$x_t = -\sigma(i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t, \tag{3}$$

where i_t is the nominal interest rate, g_t stands for a demand shock, and σ is the intertemporal elasticity of substitution of consumption. This relation can be derived from intertemporal optimization by an infinitely-lived representative household in the presence of government consumption.[4] The derivation is in the appendix.

The interest rate is the policy instrument variable controlled by the central bank. The monetary authority adjusts the quantity of money in order to achieve the desired interest-rate level. The money market equilibrium is shown in the appendix.

I assume both shocks follow a first-order autoregressive process:

$$u_{t+1} = \rho u_t + \xi_{t+1}^u, \tag{4}$$

$$g_{t+1} = \tau g_t + \xi_{t+1}^g, \tag{5}$$

[4] $g_t = E_t\{(e_t - y_t^*) - (e_{t+1} - y_{t+1}^*)\}$, where $e_t \equiv -\log(1-\frac{G_t}{Y_t}) + \log\left(1-\frac{\bar{G}}{\bar{Y}}\right)$, G_t and Y_t stand for government consumption and output, and $\bar{G}$ and $\bar{Y}$ are their steady-state values. g_t can also reflect autonomous changes in consumption, as a result, for example, of variations in the taste. We can also define the "natural interest rate" as $r^n = \sigma^{-1}g_t$, and write (3) as $x_t = -\sigma(i_t - r^n - E_t\pi_{t+1}) + E_t x_{t+1}$.

where ξ_t^u and ξ_t^g are i.i.d. mean-zero random variables, and $0 \leq \rho, \tau < 1$. The current shocks are assumed to be observed by both the central bank and private agents when making their decisions.

The intertemporal social loss function is given by:

$$\frac{1}{2}E_t\{\sum_{i=0}^{\infty}\beta^i\left[\alpha(x_{t+i}-x_{t+i}^*)^2+(\pi_{t+i}-\pi_{t+i}^*)^2\right]\}, \tag{6}$$

where x_t^* and π_t^* represent output-gap and inflation-rate targets, respectively, α measures society's dislike of inflation variability (a smaller α represents a greater aversion to inflation variability), and the discount factor β is generally assumed to have the same value as that in the AS equation. Rotemberg and Woodford (1998) and Woodford (1999b) have derived similar objective function (with $\pi_t^* = 0$) from a household's utility function.[5]

The presence of a positive output-gap target may arise from the existence of distortions in the economy (taxes, imperfect competition) that prevent it from reaching the social optimum. A certain positive value for the inflation target, in turn, allows negative real interest rates, which can possibly be optimal in some circumstances, for example, with a negative demand shock.[6] Moreover, the verified empirical costs of reduction in the level of the inflation rate may work as deterrent to a zero-inflation target.

3 Optimal Monetary Policy

The central bank's preferences are assumed to be the same as those of the society. The monetary authority minimizes the social loss function (6), subject to equations (1), and (3)-(5), in the case without inflation persistence. The optimization problem can be solved in two stages. In the first, the central bank chooses $\{x_{t+i}, \pi_{t+i}\}_{i=0}^{\infty}$ to

[5] Steinsson (2000) has derived a loss function in the presence of inflation persistence. Besides the terms corresponding to the deviations of inflation rate and output gap, the objective function also includes a term that penalizes the variation of inflation rate $(\pi_t - \pi_{t-1})^2$. I use only the objective function usually employed in the literature in order to concentrate on some results and compare them with the case without inflation persistence.

[6] Fischer (1994) has also stressed that the measures of inflation are biased upwards, and, because of some downward price inflexibility, the output costs of negative inflation rates may be greater.

minimize (6) subject to (1) and (4). In the second stage, given the state-contingent paths for output gap and inflation, it is possible to find the corresponding path for the interest rate using equations (3) and (5).[7]

Using the Lagrangian, the central bank minimizes

$$\mathcal{L} \equiv E_t\{\sum_{i=0}^{\infty} \beta^i [\frac{1}{2}[\alpha(x_{t+i} - x^*_{t+i})^2 + (\pi_{t+i} - \pi^*_{t+i})^2] +$$

$$\phi_{t+i}(\lambda x_{t+i} + \beta \pi_{t+i+1} + u_{t+i} - \pi_{t+i})]\}, \tag{7}$$

where ϕ_t is a non-negative Lagrange multiplier, with initial condition $\phi_{t-1} = 0$. In the model with inflation persistence, equation (2) substitutes for the AS restriction included in the Lagrangian.

Without inflation persistence, the first-order conditions are

$$x_{t+i} = -\frac{\lambda}{\alpha}\phi_{t+i} + x^*_{t+i}, \tag{8}$$

$$\pi_{t+i} = \phi_{t+i} - \phi_{t+i-1} + \pi^*_{t+i}. \tag{9}$$

The basic difference between optimal monetary policies under commitment and discretion is the way the central bank takes into account the private agents' expectations when minimizing the loss function. With commitment, the monetary authority chooses once and for all the state-contingent paths of output gap, inflation rate and interest rate. Since its future actions will be committed to a certain plan previously defined at the moment of the optimization, the policymaker can exploit the effect of its decisions on the private agents' expectations. Therefore, the central bank takes into account the effect of its decisions on the private agents' expectations. Suppose the optimization takes place at $i = 0$. When future arrives, $i \geq 1$, the action of the central bank is conditioned by the optimal plan defined at $i = 0$, which guaranteed the optimization for the whole period, i.e., since $i = 0$.

[7] Although the paper presents the corresponding path for the interest rate, its focus is welfare, which, in the present model specification, does not depend on the behavior of the interest rate.

Under discretion, however, the central bank is allowed to reoptimize every period. The optimal response is not restricted by any behavior or rule assumed in the past. Therefore, the central bank cannot commit itself to any behavior in the future. As a result, the policymaker cannot exploit promises of future behavior in order to affect the private agents' expectations.

The solution in the discretionary case turns out to be time consistent, whereas in the commitment case it is time inconsistent. This difference is emphasized in Svensson and Woodford (1999) using the Lagrangian approach. Under either discretion or commitment, when $i = 0$, the expectations of current inflation formed in the previous period do not affect the social loss function to be minimized. Thus, the restriction of the previous period is not binding, which leads to $\phi_{t+i-1} = 0$. Since under discretion the central bank is allowed to reoptimize every period, $\phi_{t+i-1} = 0$ for every $i \geq 0$. This imparts time consistency to the solution: for any $i \geq 0$, the optimal response of the control variables, conditional on the state of the economy, will be unchanged. Note that $\phi_{t+i-1} = 0$ corresponds to taking $E_t\pi_{t+i+1}$ as given in the minimization of the social loss function.

Under commitment, in turn, ϕ_{t+i-1} is not generally equal to zero for every period $i \geq 1$. Consequently, the state-contingent optimal behavior for $i \geq 1$ defined at $i = 1$ may be different from the one obtained at $i = 0$. Therefore, the solution is not time consistent.

First, the paper deals with the solution in the case under discretion and then turns to the one under commitment, considering, in each case, two situations: the presence and the absence of inflation persistence.[8] In addition, I also deal with the commitment solution when the optimal output-gap response is restricted to be within a certain family of rules ("restricted commitment").

[8] Part of the derivation of the optimal solutions of this section follows Clarida, Galí, and Gertler (1999). In addition to the derivation in that paper, I generalize for the case where the inflation-rate target is greater than zero, and solve for the commitment case with inflation persistence, besides solving for discretion with inflation persistence and $\rho > 0$. Furthermore, I deal with the cases under discretion with inflation persistence and under restricted commitment when the output-gap target is greater than zero, and I solve numerically for the case under restricted commitment with inflation persistence. Svensson and Woodford (1999) and Woodford (1999a) have also worked with a similar model, but without including inflation persistence.

3.1 Optimal Monetary Policy without Commitment

Under discretion and without inflation persistence, the optimal response of the output gap—obtained by combining equations (8) and (9) and using $\phi_{t+i-1} = 0$ for any i—is given by:

$$x_t = -\frac{\lambda}{\alpha}(\pi_t - \pi^*) + x^*. \tag{10}$$

Substituting equation (10) into the AS curve, solving it forward with rational expectation assumption, and assuming $\beta = 1$, we obtain the equilibrium inflation:[9]

$$\pi_t = \pi^* + \frac{\alpha}{\lambda}x^* + \frac{\alpha}{\lambda^2 + \alpha(1-\beta\rho)}u_t. \tag{11}$$

Plugging this equation into (10), we obtain the optimal output gap, expressed in terms of the cost-push shock:

$$x_t = -\frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho)}u_t. \tag{12}$$

In equilibrium, therefore, a positive output-gap target leads to a greater inflation rate without affecting the optimal output gap. Furthermore, there is no link between the output gap and its past behavior. The monetary authority uses the interest rate as the policy variable to reach the optimal output gap.

Using (10), (12), and the aggregate demand curve (equation 3), we can express the interest rate as:

$$i_t = \pi^S + (1 + \frac{\lambda(1-\rho)}{\sigma\alpha\rho})\left(E_t\pi_{t+1} - \pi^S\right) + \frac{1}{\sigma}g_t, \tag{13}$$

where $\pi^S = \pi^* + \frac{\alpha}{\lambda}x^*$. The interest rate increases more than the rise in the expected inflation.

In the model with inflation persistence, the future inflation rate also depends directly on the current inflation rate. Therefore, the central bank takes into account

[9] When $\beta = 0.99$, as it is generally assumed in the literature for quarterly data, the coefficients on π^* and x^* are different from those in equation (11), but the difference is not quantitatively important. The same reasoning is valid for (12), which would contain quantitatively unimportant terms associated with π^* and x^*. I keep β in the cost-push shock term because the expression is still valid when $\pi^* = x^* = 0$. I also assume throughout the paper that $E_t x^*_{t+i} = x^*$ and $E_t\pi^*_{t+i} = \pi^*$ for any $i \geq 0$, i.e., there are no expectations of change in the output-gap and inflation-rate targets.

that current decisions affect future inflation by the lagged inflation term. Nonetheless, in the rational expectation equilibrium, the link between inflation rate and its lagged value depends on the expectations concerning future inflation, and hence depends on the expected future behavior of the central bank. Since the central bank is not committed, it cannot exploit how the private agents' expectations are affected by its current decisions. As a consequence, it takes as given the link between future and current inflation.

I use the method of undetermined coefficients. The postulated solution is that inflation follows

$$\pi_t - \pi^s = a_\pi(\pi_{t-1} - \pi^s) + a_u u_t, \tag{14}$$

where $\pi^s \equiv a_{\pi^*}\pi^* + a_{x^*}x^*$, which represents the steady-state inflation. In this case, the central bank takes as given the coefficients a_π, a_u, a_{π^*}, and a_{x^*}. First, I lead equation (14), take expectations, and substitute for the expected inflation term in (2). I substitute the resulting equation for the AS curve included in the Lagrangian (equation 7). The first-order conditions are the following:

$$x_{t+i} = -\frac{\lambda}{\alpha}\phi_{t+i} + x^*_{t+i}, \tag{15}$$

$$\pi_{t+i} = \left(1 - a_\pi\gamma_f\right)\phi_{t+i} - \beta\gamma_b\phi_{t+i+1} + \pi^*_{t+i}. \tag{16}$$

Solving (15) for ϕ_{t+i}, and substituting into (16), we obtain a first-order difference equation for x_{t+i}. Solving forward this equation, and using (14), we find the solution for the optimal output-gap response. Using this solution into the AS curve, and matching the coefficients with equation (14), it is possible to find the values for the unknown coefficients. Assuming $\gamma_f + \gamma_b = 1$, that is, there is no long-run trade-off between inflation rate and output gap for a stable inflation rate, then $a_{\pi^*} = 1$, and the optimal output-gap response is

$$x_t = -\frac{\lambda}{\alpha(1-\beta^* a_\pi)}[a_\pi(\pi_{t-1} - \pi^s) + \frac{\left(1 - \gamma_f a_\pi\right)a_u}{\left(1 - \gamma_f a_\pi - \beta\gamma_b\rho\right)}u_t], \tag{17}$$

where $\beta^* \equiv (\gamma_f + \beta\gamma_b)$, and $\pi^s = \pi^* + \alpha[1-\beta+(\beta-a_\pi)\gamma_f]\lambda^{-1}x^*$. Assuming also that $\beta = 1$, then $\pi^s = \pi^* + \alpha(1-a_\pi)\gamma_f\lambda^{-1}x^*$.[10] The path of the inflation rate is given by equation (14), and a_π is the stable root of a cubic equation ($0 \leq a_\pi < 1$), being a function of λ, β, α, γ_b and γ_f, and reflects the inverse of the speed of disinflation.[11] A higher degree of inflation persistence (γ_b) or a lower aversion to inflation variability (a greater α) raises a_π, i.e., the disinflation is slower.

With inflation persistence, the output gap responds not only to the cost-push shock, but also to changes in the inflation-rate target (π^*). For instance, if the central bank lowers the inflation-rate target, the current inflation will be different from the target because of the inflation persistence component, leading to a reaction of the optimum output gap. The output gap, however, is not bound by its past behavior, which only matters as it affects the lagged inflation.

In the case of $\rho = 0$, the optimum interest rate is

$$i_t = \pi^S + (1 + \frac{\lambda(1-a_\pi)}{\sigma\alpha(1-\beta^* a_\pi)a_\pi})\left(E_t\pi_{t+1} - \pi^S\right) + \frac{1}{\sigma}g_t. \tag{18}$$

The real interest rate is positive.

3.2 Optimal Monetary Policy with Commitment

Under commitment, the central bank chooses once and for all the state-contingent paths of the output gap and inflation rate. Combining the first-order conditions (equations 8 and 9), we obtain

$$x_t - x_{t-1} = -\frac{\lambda}{\alpha}(\pi_t - \pi^*). \tag{19}$$

Now it is the variation of output gap that responds to inflation. Solving (19) for π_t, and plugging it into the AS curve, we obtain a second-order difference equation for

[10] With $\gamma_f + \gamma_b = 1$, and $\beta = 1$, then $\beta^* = 1$. When $x^* = \pi^* = 0$, the dynamics of the output gap are still given by equation (17)—with $\pi^S = 0$—without any assumption about the values of β, γ_f, and γ_b.

[11] $a_\pi\left(1-\gamma_f a_\pi\right) + \frac{\lambda^2 a_\pi}{\alpha(1-\beta^* a_\pi)} - \gamma_b = 0$, and $a_u = [1-\gamma_f(a_\pi+\rho) + \frac{\lambda^2(1-\gamma_f a_\pi)}{\alpha(1-\beta^* a_\pi)(1-\gamma_f a_\pi - \beta\gamma_b\rho)}]^{-1}$. These expressions are independent of the values assumed for γ_f and γ_b.

the output gap. Solving it forward yields

$$x_t = \delta x_{t-1} - \frac{\lambda\delta}{\alpha(1-\beta\delta\rho)} u_t, \tag{20}$$

where δ is the stable root $(0 < \delta < 1)$.[12] Differently from the discretionary case, the output response contains a lagged term that generates a more persistent behavior. The presence of the coefficient δ reflects the dependence of the current behavior of the central bank on its past actions. Since the past output reacted to previous cost-push shocks, the current optimal output gap depends on the history of the shocks, as emphasized by Svensson and Woodford (1999).

Combining the first-order conditions, we also obtain the dynamics of the Lagrange multiplier:

$$\phi_t - \phi^* = \delta(\phi_{t-1} - \phi^*) + \frac{\delta}{(1-\beta\delta\rho)} u_t, \tag{21}$$

where $\phi^* = \alpha\lambda^{-1}x^*$ is the steady-state value for the Lagrange multiplier.[13] The initial optimal output gap, however, is affected by the positive output-gap target. For instance, in the first period $(i = 0)$, $x_t = \delta x^*$, which is obtained by substituting equation (21) into (8), and using the initial condition for ϕ_{t-1}. When ϕ_t reaches the steady state, by equation (8), $x_t = 0$. Svensson and Woodford (1999) have used the concept of optimization from a "timeless perspective", according to which the plan selected today should be equal to one determined far in the past. For example, a plan defined far in the past would generate an optimal current output gap that does not depend on x^*. On the other hand, this output gap would depend on the entire history of the shocks. When conducting the simulations, I assume that the lagged Lagrange multiplier is at its steady-state value, that is, the optimal output gap is insulated from x^* and there is a history of no shocks.

Plugging the first-order conditions into the aggregate supply curve, solving for

[12] $\delta = \frac{1-\sqrt{1-4\beta a^2}}{2a\beta}$, with $a \equiv \frac{\alpha}{\alpha(1+\beta)+\lambda^2}$. I assume $\beta = 1$ to avoid the presence of an unimportant extra term $(-\lambda(1-\beta)(\beta\alpha)^{-1}\pi^*)$ in equation (20).

[13] Without assuming $\beta = 1$, $\phi^* = \alpha\lambda^{-1}x^* + \alpha\lambda^{-2}(1-\beta)\pi^*$.

the inflation rate, and using equation (20), we obtain

$$\pi_t = \pi^* + \frac{\alpha(1-\delta)}{\lambda} x_{t-1} + \frac{\delta}{(1-\beta\delta\rho)} u_t. \tag{22}$$

Although there is no inflation persistence, the current inflation rate depends on the past actions of the central bank. Since the policymaker is committed, the lagged output gap is an indicator of its current and future path. The path for the inflation rate can also be expressed as (after combining equations 1, 8, 9, 20, and 22):

$$\pi_t = \pi^* + \delta(\pi_{t-1} - \pi^*) + \frac{\delta}{(1-\beta\delta\rho)}(u_t - u_{t-1}). \tag{23}$$

In the steady state, $\pi_t = \pi^*$, i.e., under commitment, a positive output-gap target does not lead to any inflationary bias.

Employing (19), and the aggregate demand curve, it is possible to express the optimum interest rate as:

$$i_t = \pi^* + (1 - \frac{\lambda}{\sigma\alpha})\,(E_t\pi_{t+1} - \pi^*) + \frac{1}{\sigma} g_t. \tag{24}$$

In contrast to the discretionary case, the interest rate responds less than the expected inflation (negative real interest rate).

In the model with inflation persistence, the first-order conditions are

$$x_{t+i} = -\frac{\lambda}{\alpha}\phi_{t+i} + x^*_{t+i}, \tag{25}$$

$$\pi_{t+i} = \phi_{t+i} - \beta\gamma_b E_t\phi_{t+i+1} - \beta^{-1}\gamma_f\phi_{t+i-1} + \pi^*_{t+i}. \tag{26}$$

Solving equation (25) for ϕ_{t+i}, substituting into (26), and then using the AS curve, we obtain a fourth-order difference equation for the output gap. Solving it forward, and again using (26), the optimal output gap follows

$$x_t = b_1 x_{t-1} - \frac{\lambda}{\alpha} b_2(\pi_{t-1} - \pi^*) - \frac{\lambda}{\alpha} b_3 u_t, \tag{27}$$

where the values of b_1, b_2 and b_3 depend on several parameters.[14] The optimal output gap depends on its past behavior ("commitment effect") and on the deviations of the past inflation from the inflation target.

[14] $b_1 \equiv [\gamma_f(\delta_1 + \delta_2) - \delta_1\delta_2\beta]/\varkappa$, $b_2 \equiv \delta_1\delta_2\beta/\varkappa$, and $b_3 \equiv 1/[\gamma_b\beta\varkappa(\delta_3 - \rho)(\delta_4 - \rho)]$, where $\varkappa \equiv \gamma_f - \delta_1\delta_2\beta^2\gamma_b$, δ_1 and δ_2 are the stable roots of the difference equation, and δ_3 and δ_4 are the unstable ones. As before, I assume $\beta = 1$ and $\gamma_b + \gamma_f = 1$ to avoid the presence of extra terms related to π^* and x^*.

The path for the Lagrange multiplier is obtained in a similar way to that of the output gap. It follows

$$\phi_t - \phi^* = b_1(\phi_{t-1} - \phi^*) + b_2(\pi_{t-1} - \pi^*) + b_3 u_t, \tag{28}$$

where $\phi^* = \alpha\lambda^{-1}x^*$ is the steady-state value for the Lagrange multiplier.

Solving (25) for the Lagrange multiplier, plugging it into (26), and using (27) to substitute for x_t and x_{t+1}, we obtain the path for the inflation rate:

$$\pi_t = \pi^* + c_1 x_{t-1} + c_2(\pi_{t-1} - \pi^*) + c_3 u_t, \tag{29}$$

where the values of c_1, c_2, and c_3 depend on several parameters.[15] Now the inflation rate depends on both the past behavior of the central bank and the lagged inflation.[16] As before, a positive output-gap target does not generate any inflationary bias.

3.3 Optimal Monetary Policy with "Restricted Commitment"

The distinction between discretion and commitment stems from the way the private agents' expectations are taken into consideration when the central bank optimizes. This subsection deals with the case where the central bank takes into account the private agents' expectations, but the output-gap response is restricted to have the same structure as in the discretionary case, i.e., it is a function only of the cost-push shock. Differently from the commitment case—which is also called the "global solution"—the optimal output gap is restricted to respond only to the state variable of the system. I call this case "restricted commitment". The optimal output-gap response is given by:

$$x_t = -\nu u_t, \tag{30}$$

where ν is a parameter whose value is to be determined when minimizing the loss function. Substituting equation (30) into the aggregate supply curve, and solving it

[15] $c_1 \equiv \alpha(b_1^2\beta^2\gamma_b - b_1\beta + \gamma_f)/(\lambda\beta(1 + \beta\gamma_b b_2))$, $c_2 \equiv b_2(1 - \beta\gamma_b(\delta_1 + \delta_2))$, $c_3 \equiv b_3(1 - (b_1 + \rho)\beta\gamma_b)/(1 + \beta\gamma_b b_2)$.

[16] The optimal interest rate (not shown) can be expressed as a function of current or expected inflation, and lagged output gap and inflation.

forward, we obtain

$$\pi_t = \frac{1-\lambda\nu}{1-\beta\rho}u_t. \tag{31}$$

Therefore, the expected inflation is given by

$$E_t\pi_{t+1} = \frac{1-\lambda\nu}{1-\beta\rho}\rho u_t. \tag{32}$$

To solve the optimization problem, we can substitute directly equations (30) and (31) for the output gap and inflation rate in the loss function. Alternatively, in order to follow the previous structure, I set up the Lagrangian, employing equations (30) and (32):

$$\mathcal{L} \equiv E_t\{\sum_{i=0}^{\infty}\beta^i[\frac{1}{2}[\alpha(-\nu u_{t+i} - x^*_{t+i})^2 + (\pi_{t+i} - \pi^*_{t+i})^2]$$

$$+\phi_{t+i}(-\lambda\nu u_{t+i} + \beta\frac{1-\lambda\nu}{1-\beta\rho}\rho u_{t+i} + u_{t+i} - \pi_{t+i})]\}, \tag{33}$$

where ϕ_t is a non-negative Lagrange multiplier.

The first-order conditions with respect to ν and π_{t+i} are the following:

$$x_{t+i} = -\frac{\lambda}{\alpha\,(1-\beta\rho)}\phi_{t+i} + x^*_{t+i}, \tag{34}$$

$$\pi_{t+i} = \phi_{t+i} + \pi^*_{t+i}. \tag{35}$$

Combining them yields

$$x_t = -\frac{\lambda}{\alpha\,(1-\beta\rho)}\left(\pi_t - \pi^*\right) + x^*. \tag{36}$$

Using equation (36) in the aggregate supply curve, and solving it forward, we obtain the equilibrium inflation:

$$\pi_t = \pi^* + \frac{\alpha\,(1-\beta\rho)}{\lambda}x^* + \frac{\alpha\,(1-\beta\rho)}{\lambda^2 + \alpha(1-\beta\rho)^2}u_t. \tag{37}$$

Substituting this last equation into (36) yields the optimal output-gap response:[17]

$$x_t = -\frac{\lambda}{\lambda^2 + \alpha(1-\beta\rho)^2}u_t. \tag{38}$$

[17]As before, I assume $\beta = 1$ to avoid the presence of unimportant extra terms in this equation and of slightly different coefficients on π^* and x^* in equation (37).

Compared to the discretionary case (equation 12), the response of the output gap is stronger (when $\rho > 0$). Note that in the absence of persistence in the shock process ($\rho = 0$), the solutions turn out to be identical. Since the expected value of the cost-push shock for the following periods is zero, the central bank cannot promise some future output behavior in order to affect current inflation. In contrast to the commitment solution, where the output-gap response is not restricted to be within a family of rules, under restricted commitment the output is allowed to respond only to the current shock.

Combining (36), (38), and the aggregate demand curve, we can express the path of interest rate as:

$$i_t = \pi^S + (1 + \frac{\lambda (1-\rho)}{\sigma \alpha \rho (1-\beta\rho)}) \left(E_t \pi_{t+1} - \pi^S \right) + \frac{1}{\sigma} g_t, \tag{39}$$

where $\pi^S = \pi^* + \frac{\alpha(1-\beta\rho)}{\lambda} x^*$. Comparing to (13), we can see that the response of the real interest rate under restricted commitment is higher than that under discretion.

For the model with inflation persistence, the output gap responds to the two state variables:

$$x_t = -\nu_1 (\pi_{t-1} - \pi^S) - \nu_2 u_t, \tag{40}$$

I use numerical methods to find the values of ν_1 and ν_2.

4 Welfare

This section compares the three solutions (discretion, commitment, and restricted commitment) in terms of their effects on the welfare, given by the loss function. To measure the value of the loss function, I use the unconditional expectation (denoted by the operator E) of the loss function defined in equation (6).[18] After multiplying by $2(1-\beta)$, the value of the loss function is calculated as:

$$L = E\{\alpha (x_t - x^*)^2 + (\pi_t - \pi^*)^2\}. \tag{41}$$

[18] See, for example, Woodford (1999c), and Rudebusch and Svensson (1999).

Since the unconditional expectation of the output gap is zero in any of the three solutions, and the unconditional expectation of inflation is $\pi^* + \pi^{ib}$, equation (41) can be rewritten as:

$$L = \alpha Var(x_t) + Var(\pi_t) + \alpha(x^*)^2 + (\pi^{ib})^2, \tag{42}$$

where Var stands for the unconditional variance, and π^{ib} for the inflationary bias, which is defined as the inflation rate resulting from the presence of a positive output-gap target ($x^* > 0$). Therefore, the loss function comprises two elements: the weighted unconditional variances of the output gap and inflation rate ("dynamic loss"), and two terms associated with the presence of a positive output-gap target.

4.1 Parameter Values

The choice of the parameter values is based on the empirical literature, specially Rotemberg and Woodford (1998), and Galí and Gertler (1999).[19] Nevertheless, I also analyze the consequences of assuming different parameter values not only for robustness reasons, but also to understand the effects of each parameter and its role in the differences across policy regimes. For instance, I show the effects of assuming different values for α (the relative output weight in the objective function), γ_b (degree of inflation persistence), and θ (degree of price rigidity).

The discount factor (β) is usually assumed to be 0.99. According to the Euler equation (equation 62 in the Appendix), this value implies a real interest rate of about 4% per year in the steady state.

For the degree of price rigidity, I use $\theta = 0.75$, which implies that firms adjust their prices once a year on average. Rotemberg and Woodford (1998) have used $\theta = 0.66$, based on the findings of Blinder (1994). Galí and Gertler (1999) have found approximately $\theta = 0.83$ or greater than that, but they have stressed that the estimation is likely to be upward biased.

[19] The aim of the paper, however, is not to reproduce the moments and dynamic paths of the variables verified empirically. For this purpose, see Rotemberg and Woodford (1998), whose estimation includes the reaction function of the Federal Reserve to shocks.

It is more difficult to find a value for the output elasticity of marginal cost (κ), which combined with θ and β define the value of λ (the coefficient on the output gap in the aggregate supply curve). I use $\kappa = 0.3$ as the benchmark case. Combined with the values assumed for θ and β, we obtain $\lambda = 0.0257$ (if using inflation rate measured at an annualized rate, $\lambda = 0.1028$), close to the value estimated in Rotemberg and Woodford (1998) ($\lambda = 0.024$). In the model used for the derivation of the aggregate supply curve (see Appendix), $\kappa = \gamma_c + \gamma_n$, where γ_c is the inverse of the intertemporal elasticity of substitution of consumption, and γ_n is the inverse of the elasticity of labor supply with respect to the real wage. In this model, κ represents the elasticity of the real wage with respect to output (equation 93). Rotemberg and Woodford (1998) have also used 0.3 as the value for this elasticity based on some empirical literature.[20] For γ_c, they have estimated a value of 0.16 (an intertemporal elasticity of substitution equal to 6.25), implying, in the present model, $\gamma_n = 0.14$ (a wage elasticity of labor supply equal to 7.14).[21] Galí (2000), on the other hand, in a similar model, has used the values $\gamma_c = 1$ (log-utility function for consumption) and $\gamma_n = 1$, resulting in $\kappa = 2$, and $\lambda = 0.1713$ ($\lambda = 0.6853$ using inflation rate measured at an annualized rate). As a robustness exercise, I also show some results with the values of 1 and 2 for κ.

When working with the aggregate supply curve that includes inflation persistence (equation 2), I use two different specifications. The first one is based on Galí and Gertler (1999), where the values of γ_b and γ_f depend on the share of backward-looking firms (ω). I consider two values for ω: 0.25 and 0.5, which reflect approximately the estimates in Galí and Gertler (1999) using two different methodological specifications. In their formulation, the value of λ is affected by ω.[22] Nevertheless, since the main interest is to investigate the effects of inflation persistence, independently of the specific derivation for the persistence, I also work with a model where the values for γ_b and γ_f are defined directly (with the restriction

[20] Their model, however, presents some differences in the theoretical specification for κ.

[21] King and Woolman (1999) have assumed $\gamma_n = 0.1$.

[22] With $\omega = 0.25$ and 0.5, the annualized values of λ amount to 0.058 and 0.031, and the values of γ_b are equal to 0.250 and 0.401, respectively (using $\beta = 0.99$, and $\theta = 0.75$).

$\gamma_b + \gamma_f = 1$), without affecting λ. For example, when investigating the effects of inflation persistence on the gains from commitment, I consider all possible values for γ_b, keeping λ fixed.

For the intertemporal elasticity of substitution ($\sigma \equiv \frac{1}{\gamma_c}$) in the IS curve, I use the mentioned value of 6.25 (or 1.56 for annualized rates of inflation and interest). This parameter in the IS curve affects only the magnitude of the response of the interest rate. It does not have effect on the value of the loss function.

The choice of the relative output weight in the objective function (α) differs significantly in the literature. According to the derivation in Rotemberg and Woodford (1998) and Woodford (1999b), $\alpha = \frac{\lambda}{\vartheta}$, where ϑ is the elasticity of demand for an individual good, which was estimated as 7.88. This leads to $\alpha = 0.003$, which, using annualized inflation rate, is equivalent to 16 times this, i.e., approximately $\alpha = 0.05$. As of now I refer to the values of α corresponding to the specification that uses the annualized inflation rate. On the other hand, Rudebusch and Svensson (1999) have evaluated alternative policies assuming different values for α, such as 0.2 or 5, but have worked mainly with $\alpha = 1$.

The paper focuses on evaluating the results of assuming different values for α, which has important effects in terms of inflationary bias and variability of output and inflation. When working with some basic cases, the choice is based on the resulting relative variances of output and inflation. Since the absolute values of the variability of output and inflation are very sensitive to the assumptions concerning the variance of the random cost-push shock (ξ_t^u), I search for values of α that result in a ratio of the standard deviation of output gap to that of inflation—"ratio of variability" for brevity—closer to those observed empirically. Using quarterly data for the U.S. economy, from 1960:1 to 2000:1, the standard deviation of the output gap is 2.61, measured employing the GDP, and the Potential GDP estimated by the Congressional Budget Office (CBO). For the annualized inflation rate (using GDP deflator), the standard deviation is 2.48, resulting in a ratio of variability of 1.05. Considering only a more recent period (1980:1 - 2000:1), the values of the standard deviations are 2.33 for the output gap, and 2.07 for inflation, resulting in a ratio of 1.13.[23]

The ratio of variability in the simulations, however, depends on the value assumed for the autoregressive coefficient of the cost-push equation (ρ). I use two values: $\rho = 0.6$ and 0.8. In the benchmark case, for a range of the ratio of variability between 0.9 and 1.5, α goes from 0.07 to 0.43 with $\rho = 0.6$, and from 0.07 to 0.71 with $\rho = 0.8$. With inflation persistence, using $\rho = 0.6$, the range is not very affected.[24] As a result, I choose $\alpha = 0.1$, 0.3, and 0.5 as the basic cases, besides showing some results for more extreme values ($\alpha = 0.04$ and 1).

The value of the standard deviation of the random cost-push shock (ξ_t^u) affects the absolute value of the unconditional variances of inflation and output, but not their relative values across regimes. It affects, however, the importance of the gains under commitment coming from the unconditional variances in comparison to those stemming from the absence of inflationary bias. I do the following exercise to find a reasonable range value for the variability of ξ_t^u. Initially, I find the values of α, in each regime, that result in a ratio of variability of 1.1. For the benchmark case, with $\rho = 0.6$, the calculated values of α are 0.094, 0.231, and 0.318 for discretion, restricted commitment, and commitment, respectively.[25] Then I find the values for the variability of the random cost-push shock that generate absolute values of standard deviation of output and inflation similar to those empirically observed. Assuming $\rho = 0.6$, an annualized standard deviation of ξ_t^u equal to 1.0%—corresponding to a standard deviation of 1.25% for u_t—generates values close to those empirically observed.[26] In the model with inflation persistence, the simulations indicate a stan-

[23] The estimates in the literature are close to these values. In McCallum and Nelson (1999), which use a different method for estimating the output gap, the estimates imply a ratio of 1.08 (1955-1996). The estimates in Rudebusch and Svensson (1999), which employ a method similar to that used by the CBO, and measure inflation as the four-quarter average, result in a ratio of 1.20 (1961:1 - 1996:2) .

[24] It goes from 0.07 to 0.44 with $\omega = 0.25$, and from 0.07 to 0.52 with $\omega = 0.5$. With inflation persistence term (γ_b) equal to 0.4 defined directly, however, the range goes from 0.19 to 1.16

[25] With $\rho = 0.8$, the values of α are 0.094, 0.450, and 0.543.

[26] Using $\rho = 0.8$, however, a more reasonable annualized standard deviation of ξ_t^u is 0.5%, corresponding to a standard deviation of 0.83% for u_t.

dard deviation of about 0.6% for ξ_t^u.[27] Unless otherwise noticed, I assume the value of 1.0%. When comparing the gains under commitment coming from the unconditional moments to those from the absence of inflationary bias, I also use the value of 0.6%.

Finally, to measure the consequences of imperfect competition on the potential output (equation 99), it is necessary to assume some values for the elasticity of substitution among alternative goods (ϑ). I use the mentioned value of 7.88, besides the value of 4, used by King and Wolman (1999), which imply steady-state mark-ups equal to 1.15 and 1.33, respectively. I also consider a mark-up equal to 1.1.

The simulations were conducted using the procedure in Söderlind (1999), and were compared with the analytical solutions when available. The unconditional variances were calculated using formula in Hamilton (1994, pp. 264-266).[28] In the case of restricted commitment, I employ the analytical results for the model without inflation persistence, and use numerical methods to find the optimal output-gap response with inflation persistence.

4.2 Unconditional Variances

The objective function includes neither interest-rate smoothing nor penalty for interest-rate variability. Consequently, any demand shock can be completely offset by the central bank by moving the interest rate accordingly. The source of movements of the output gap and inflation rate around the steady state is the cost-push shock. This subsection initially presents the results in the model without inflation persistence, and subsequently those with inflation persistence.

4.2.1 Model without Inflation Persistence

Table 1 presents the results for the benchmark case. It shows the values of the unconditional standard deviations of output gap and annualized inflation rate, and the value of the loss function associated with different policies. In order to concentrate

[27] With $\omega = 0.25$ and 0.5, the simultations indicate 0.6% and 0.4%, whereas with $\gamma_b = 0.4$ the value is 0.6% (all assuming $\rho = 0.6$).

[28] Using iteration to find the unconditional variances generates similar results.

on the effects in terms of stabilization, the value of the loss function reported in all tables and figures, except in Figure 13, is calculated assuming $x^* = 0$, i.e., the loss is the weighted sum of the unconditional variances. Table 1 shows the results for $\alpha = 0.04$, 0.1, 0.3, 0.5, and 1.0, considering $\rho = 0.6$, whereas Table 2 reports the values for the case with $\rho = 0.8$.

The solution under commitment is clearly superior to that under discretion.[29] The value of the loss function under commitment is substantially lower: it is between 53% and 74% of the value under discretion with $\rho = 0.6$, and between 40% and 55% with $\rho = 0.8$, for the values of α considered in Tables.1 and 2.[30] For instance, with $\alpha = 0.3$ in Table 1, the value of the loss function for commitment corresponds to 62% of that for discretion. Figure 1 shows the ratios of the value of loss function under discretion to those under commitment and restricted commitment as a function of α, assuming $\rho = 0.6$. For large values of α, the difference across regimes tends to be smaller.

Under restricted commitment, the welfare is superior to that under discretion, and inferior to that under commitment. The magnitude of the gains, however, depends highly on the values of ρ. With $\rho = 0.6$, the gains are not significative: the value of the loss function under restricted commitment is between 83% and 98% of the value under discretion, whereas with $\rho = 0.8$, it is between 57% and 88% (using Tables 1 and 2). Figure 2 shows the ratios of the value of loss function under discretion to those under commitment and restricted commitment as a function of ρ. A higher ρ increases the effect of a cost-push shock in the expected inflation. Consequently, it is more important to affect the private agents' expectations, resulting in more gains for any of the commitment solutions. Nevertheless, the gains from restricted commitment are significant only when ρ is extremely high. Employing (11), (12), (37), and (38), it can be shown that the ratio of the value of the loss

[29] Similar result has also been found by McCallum and Nelson (2000), who have quantified the value of the losses under commitment and discretion, including the inflationary bias, but without considering specifically the differences in terms of variability of output and inflation.

[30] See the last columns of the tables. The penultimate column records the inverse of the last one: it shows the ratio of the value of the loss under discretion to the loss under the regime in the corresponding line.

function under discretion to that under restricted commitment (with $x^* = 0$) is

$$\frac{L^D}{L^{RC}} = \frac{k_1 + (\beta\rho)^2}{k_1}, \tag{43}$$

where k_1 is decreasing in ρ[31]. Therefore, the loss under discretion is greater than that under restricted commitment, and the difference is increasing in ρ.

Considering the unconditional moments separately, the variance of inflation is lower under commitment than under discretion in the benchmark case. Figures 3 and 4 show the standard deviations of inflation and output under the three regimes as a function of α. As expected, a greater α leads to a lower output variance and a higher inflation variance. The variability of output, however, is higher under commitment than that under discretion for the benchmark case.

Nevertheless, the comparison of the variances depends ultimately on the parameter values. In particular, when $\rho = 0$ and $\beta = 0.99$, using equations (12) and (20), it is possible to find that, for the variance of output under commitment to be equal or greater than that under discretion, it is necessary that

$$\alpha \geq 4.7443\lambda^2. \tag{44}$$

Since in the benchmark case, $\lambda = 0.1028$, the condition is fulfilled for $\alpha \geq 0.050$. For $\rho > 0$, the requirement is verified even for lower values of α. Using numerical simulations, it is possible to find that with $\rho = 0.6$ and 0.8, the conditions are $\alpha \geq 1.715\lambda^2$ and $\alpha \geq 1.274\lambda^2$, which are reached for $\alpha \geq 0.018$ and $\alpha \geq 0.013$, respectively.

Table 3 shows some results for a larger value of the output elasticity of marginal cost (κ). For $\kappa = 1$ with $\alpha = 0.1$, and for $\kappa = 2$, condition (44) is not satisfied: the standard deviation of output under commitment is smaller than that under discretion.[32]

For the variance of inflation under commitment to be lower than that under

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[31] $k_1 = 2(1 - \beta\rho) + \frac{\lambda^2}{\alpha} + \frac{\alpha(1-\beta\rho)^2}{\lambda^2}$.

[32] In a numerical example, Galí (2000) has found a variance of output under commitment smaller than that under discretion because he has used $\kappa = 2$.

discretion, the condition, with $\rho = 0$ and $\beta = 0.99$, is

$$\alpha \geq 0.53993\lambda^2, \tag{45}$$

which can be found using (11), (20), and (19).

This condition is easily verified in the benchmark case (requires only $\alpha \geq 0.006$). In the case of $\rho = 0.6$ or 0.8, all combinations of values of α and λ imply a lower variance of inflation under commitment than under discretion.[33] In fact, all cases shown in Tables 1 to 3 present this feature. Therefore, we can conclude that, with the model specification of this paper, policies under commitment usually generate a lower inflation variability than that under discretion as well as result, for a large range of parameter values, in a higher output-gap variability.

Likewise the solution under restricted commitment delivers a lower inflation variability and a higher output variability than those under discretion when $\rho > 0$. Differently from the commitment case, this result is independent of the parameter values. Analytically, it can be seen comparing equations (37) with (11), and (38) with (12). The comparison of the inflation-rate and output-gap variabilities under restricted commitment with those under commitment, however, depends on the combination of the parameter values of α, ρ, and λ.

Figure 5 presents the trade-off between the variabilities of output gap and inflation for the three regimes (with and without inflation persistence). It shows the possible combinations of standard deviations of output gap and inflation associated with different values of α (not shown).[34] The three lines closer to the origin refer to the model without inflation persistence, whereas the other three are obtained from the case with inflation persistence. There exists an evident gain in the trade-off: for each combination of standard deviations of output and inflation under discretion, it is possible to find values of α that generate, under commitment, a lower variability for both variables.[35]

[33] Except when λ is close to zero (the variances of inflation under commitment and discretion tend to be equal to each other).

[34] The values employed for α are between 0.002 and 2. The part of the curve with high standard deviation of output corresponds to a few extremely low values of α.

[35] For instance, for a ratio of variability equal to 1.1, commitment results in standard deviations

Under discretion, since the central bank takes as given the private agents' expectations, it perceives the gains and costs for the loss function as if the aggregate supply were $\pi_t = \lambda x_t + u_t$. Variations in the output gap are perceived as generating only λ units of change in the inflation rate.

When the central bank can commit itself, however, there is an increment in the marginal gains from variations in the output gap. Solving forward the aggregate supply curve yields:

$$\pi_t = E_t \sum_{i=0}^{\infty} \beta^i (\lambda x_{t+i} + u_{t+i}). \tag{46}$$

Under restricted commitment, using (30), equation (46) can be rewritten as:

$$\pi_t = \frac{\lambda}{1-\beta\rho} x_t + \frac{1}{1-\beta\rho} u_t. \tag{47}$$

A unit of output-gap variation is perceived as producing $\lambda/(1-\beta\rho)$ units of inflation-rate variation. The trade-off between inflation and output is improved compared to the discretionary case. Under commitment, the improvement is still higher. Using equation (20), the aggregate supply curve can be written as:

$$\pi_t = \frac{\lambda}{(1-\beta\delta)(1-\beta\rho)} (x_t - \rho\beta\delta x_{t-1}) + \frac{1}{1-\beta\rho} u_t. \tag{48}$$

The term $\frac{1}{1-\beta\delta}$ represents the gain originated from the link, represented by the root δ, between expected and past output gaps.[36] Even when $\rho = 0$, output has a higher effect on inflation than that under discretion.

The gains from commitment are significantly affected by the degree of price rigidity. Figure 6 shows the ratio of the value of loss function under discretion to those under commitment and restricted commitment as a function of θ. The benefits from commitment are highly pronounced in the range of values of θ that is usually employed in the literature: between 0.55 and 0.85. The gains are virtually zero for a low price rigidity specification. The lower θ, the less important the expected value

of output and inflation that are about 81% of those under discretion (with $\rho = 0.6$ and $\omega = 0$).

[36] Equation (48) can also be expressed as a function only of x_t and u_t. In (48) the coefficient on u_t is the same as those under discretion and restricted commitment.

of marginal cost, decreasing the benefits from commitment. Note, however, that the gains are close to zero for values of θ near to 1 as well. With a high value of θ, the value of λ becomes very low. As a result, the expected future values of x_t have small effects on the current inflation. Thus, the benefits from commitment reduce rapidly.

The difference in the behavior between commitment and discretion can be seen more clearly in Figure 7, which shows the impulse-response functions of inflation rate, output gap and nominal interest rates to an annualized one-percentage-point cost-push shock when $\rho = 0$. In Figures 7, 8, and 10, the output gap is measured in percentage deviations from the steady state, and the inflation rate and the nominal interest rate are measured in annualized percentage points (expressed as deviations from a steady state with positive values). Under discretion or commitment, the shock leads to an increase in the inflation rate and nominal interest rate and to a reduction in the output gap (note that the scales of the graphs are different). The monetary authority raises the interest rate in order to lower the output gap, and thereby to reduce the inflationary pressure. Since the shock lasts for only one period, the output-gap response under discretion has the same duration. Under commitment, however, the output-gap reduction persists for a longer period. It is the "output cost" of commitment: even after the shock dies out, output falls because of its effects on the inflation rate in the initial period. It is evident the time inconsistency of the commitment solution. After collecting the gains in the first period ($i = 0$), a reoptimization at $i = 1$ generates $x_{t+1} = 0$ as the output-gap response.

Figure 8 shows the path of the variables when the shock presents some persistence ($\rho = 0.6$). As before, under commitment, the inflation rate is lower and the output-gap response is stronger than those under discretion. The output-gap response shows a hump-shaped form in the commitment case: although the value of the shock is decreasing, the response is increasing for some period. At the same time, the nominal interest rate is initially lower under commitment.

Under restricted commitment, the shape of the responses is similar to those with discretion. Nevertheless, the response of the output gap is stronger and inflation rate is lower than those under discretion.[37]

4.2.2 Model with Inflation Persistence

If inflation persistence is included, the effect on the loss function is very significative, even for low levels of persistence. Table 4 provides the results for the cases with $\omega = 0.25$ and 0.5, and Table 5 for the case with $\gamma_b = 0.4$ (degree of inflation persistence directly defined). Even for a small degree of inflation persistence ($\omega = 0.25$), the standard deviations of output and inflation are between 42% and 116% greater than those without inflation persistence. The effect on the value of the loss function is considerable: the loss is between 126% and 191% higher than that without inflation persistence. In Figure 5, it is evident the effect of the presence of inflation persistence on the feasible combinations of variabilities.[38]

Nevertheless, the effect of the presence of inflation persistence on the relative loss across regimes is somehow surprising. Figure 9 presents the ratios of the value of the loss function under discretion to those under commitment and restricted commitment as a function of the degree of inflation persistence (γ_b).[39] The relative gains from commitment are *increasing* in γ_b for some important range of values of γ_b. The benefits from commitment are increasing between $\gamma_b = 0$ and 0.32, and are still superior to those without inflation persistence for $\gamma_b \leq 0.46$.[40] The reason seems to be that a greater inflation persistence has two effects. On the one hand, since a higher γ_b implies a lower γ_f, the importance of the central bank to take into account the effect of its decisions on the private agents' expectations is smaller. Therefore, there exist less benefits from commitment. The solutions under discretion

[37] The difference of the pattern of the responses across the three solutions is robust to different values of α, which affect only the magnitude of the responses.

[38] Figure 5 assumes $\omega = 0.25$ for the inflation persistence case.

[39] Figure 9, which employs $\alpha = 0.3$ and $\rho = 0.6$, was constructed without any structural form assumption for γ_b, except for the condition $\gamma_b + \gamma_f = 1$.

[40] If we use a higher α, the increase in the gains is more pronounced. Similar result is obtained when using ω instead of γ_b: the benefits from commitment are increasing in ω for a range of ω between 0 and approximately 0.6. Using $\gamma_b = 0.5$ and different parameter values, McCallum and Nelson (2000) have also found an increment in the gains from commitment with inflation persistence.

and commitment tend to be close to each other. On the other hand, a higher γ_b also leads to a greater effect of the cost-push shock on the *value* of the expected inflation. Consequently, it is more important to act on the expectations. For some range of γ_b, this second effect dominates the first one. Note that a higher ρ has only the second effect. Under restricted commitment, the results are qualitatively similar to those under commitment.

The previous results concerning the comparison of the variabilities of output and inflation across regimes for the model without inflation persistence hold with inflation persistence.[41] Figure 10 shows the impulse-response functions to a cost-push shock in a model with inflation persistence ($\omega = 0.25$). Qualitatively, the paths of the variables are similar to those in the case without inflation persistence (with $\rho > 0$).[42]

4.3 Inflationary Bias

The central bank attempts to reach the output-gap target. The resulting inflationary pressure stems from two components: the current output gap, and the expected inflation. In the discretionary case, the policymaker takes as given the expected inflation. Therefore, the central bank incorporates only partially the cost of a positive output gap. Consequently, it perceives as optimal to have a positive output gap. Plugging equation (10) into (1), and assuming $\pi^* = 0$ and $\beta = 1$, we obtain:

$$\pi_t = \frac{\alpha\lambda}{\alpha + \lambda^2} x^* + \frac{\alpha}{\alpha + \lambda^2} E_t \pi_{t+1}. \tag{49}$$

This equation is represented in the top panel of Figure 11, which shows the values of inflation rate associated with different levels of the expected inflation rate. In the steady-state equilibrium with rational expectations, $\pi_t = E_t \pi_{t+1}$, which is given by the intersection of the 45^o-degree line with the line representing equation (49).

[41] In the cases $\omega = 0.25$, $\omega = 0.50$, and $\gamma_b = 0.4$, the conditions for a variance of output greater under commitment than under discretion are approximately the following: $\alpha \geq 1.625\lambda^2$, $\alpha \geq 3.603\lambda^2$, and $\alpha \geq 1.458\lambda^2$, respectively. The variance of inflation is similar across the two regimes only with λ close to zero.

[42] The main difference is in the solution under restricted commitment: the maximum response of the output gap occurs in the second period instead of the first one.

Plugging this equation into (10) yields

$$x_t = \frac{\alpha}{\alpha+\lambda^2}x^* - \frac{\lambda}{\alpha+\lambda^2}E_t\pi_{t+1}, \tag{50}$$

whose representation can be seen in the bottom panel of Figure 11. If $E_t\pi_{t+1}$ were zero, it would be optimum to have a positive output gap, which would not be equal to x^* because the central bank considers the effect of the *current* output gap on the inflation rate. Nevertheless, the private agents recognize the incentive of the monetary authority and incorporate it in the expected inflation rate. As a result, the cost of having a positive output gap increases dramatically. In fact, solving forward the aggregate supply curve, and assuming $\beta = 1$, a permanent positive output gap "explodes" the inflation rate. Therefore, the costs of a positive output gap exceed any gain obtained in the output-gap term in the loss function. As a consequence, the resulting optimum output gap is zero.[43] The equilibrium inflation rate, however, is greater than zero exactly because the private agents incorporate the incentives of the central bank into their expectations.

From equation (11) the inflationary bias under discretion is

$$\pi^{ib} = \frac{\alpha}{\lambda}x^*. \tag{51}$$

The smaller α, the lower the costs of having an output gap different from the target. Accordingly, the central bank has less incentive to achieve the output-gap target, resulting in less inflationary pressure. The value of the inflationary bias is very sensitive to the value of α. Table 6 reports the inflationary bias associated with different values of α for an output-gap target equal to 1%.[44] Without inflation persistence, values of α equal to 0.1, 0.3, and 0.5 generate an (annualized) inflationary bias of 1.0%, 3.0%, and 5.0%, respectively. If we use the value of α that generates the ratio of variability equal to 1.1 in the discretionary case ($\alpha = 0.094$), the inflationary bias is relatively low (0.9%).

[43] Note that when $\beta < 1$, the inflationary cost of $x_t > 0$ is lower because the effect of the expected inflation on the current one is smaller. Consequently, it may be optimal to have a slightly positive output gap.

[44] Table 1.6 assumes $\beta = 1$, $\theta = 0.75$, and $\kappa = 0.3$.

The specification of the value of the output-gap target could be based on the distortion generated by monopolist competition. Nonetheless, the estimation of the gap between the potential output with monopolist competition and the one with perfect competition is very sensitive to some parameter values. For instance, using the specification in the appendix (equation 99), with $\gamma_c + \gamma_n \equiv \kappa = 0.3$, the output-gap target would be 45.2% for a mark-up equal to 1.15. Using $\alpha = 0.094$, the inflationary bias would amount to 41.4%.[45] With $\kappa = 2$, the output-gap target would be 6.8%, implying an inflationary bias equal to 0.9% (using $\kappa = 2$ to calculate λ as well).

As previously observed, under commitment there is no inflationary bias. The reason is that the central bank takes into consideration the effect of trying to have a positive output gap on the expected inflation. Thus the monetary authority does not have the incentive of having a positive output gap. As the private agents perceive that the central bank does not have this incentive, they do not raise their prices. In the discretionary case, the private agents' perception of the central bank's incentives drives the inflation rate up, leading to an equilibrium with zero output gap. Under commitment, it is the central bank that at first incorporates the cost.

Under restricted commitment, however, the inflationary bias is not eliminated. From equation (37):

$$\pi^{ib} = \frac{\alpha\,(1-\beta\rho)}{\lambda} x^{*}. \tag{52}$$

The inflationary bias is positive because the central bank does not take into account the effect of a positive output-gap target on the inflationary expectations. By equation (32), the expected inflation, for the central bank, is restricted to be dependent only upon the cost-push shock and the associated reaction of the output gap. On the other hand, the inflationary bias is lower than that under discretion. Comparing equation (36) to (10), the reaction of the policymaker to any inflationary pressure is greater than in the discretionary case. Since the central bank recognizes that a

[45] With mark-ups equal to 1.1, and 1.33, the output-gap target would be 31.8%, and 95.9%, and the inflationary bias would amount to 29.1%, and 87.7%, respectively.

positive output gap generates some inflationary pressure by the current output gap, it has less incentive to achieve the output-gap target, resulting in less inflationary pressure. Using $\rho = 0.6$ and 0.8, the inflationary bias under restricted commitment is 60% and 80% lower, respectively, than those under discretion.

Finally, under discretion, but with inflation persistence, the inflationary bias is given by:[46]

$$\pi^{ib} = \frac{\alpha}{\lambda}(1 - a_\pi)\gamma_f x^*. \tag{53}$$

Since $0 \leq (1 - a_\pi)\gamma_f < 1$, the inflationary bias is smaller than that without inflation persistence, for a given λ. The reason is that the central bank takes into account that current decisions affect future inflation by the lagged inflation term (although it takes as given this link between inflation and lagged inflation, whose value depends on the rational expectation equilibrium). As a result, the costs perceived of a positive output gap are larger, decreasing the inflationary pressure.

Table 6 shows the inflationary bias for different cases (assuming $\beta = 1$). With $\gamma_b = 0.4$ (directly defined in the aggregate supply curve) the reduction in the inflationary bias is significant (between 66% and 74%). Figure 12 shows the inflationary bias as a function of γ_b.[47] The presence of inflation persistence decreases significantly the inflationary bias. In the case of $\omega = 0.25$ and 0.5, the reduction is less pronounced because the value of λ is decreasing in ω.

Under commitment, the comparison of the gains coming from the variances of output and inflation with those from the absence of inflationary bias is highly sensitive to the values attributed to the variance of the cost-push shock, α and x^*. Figure 13 shows the loss as a function of γ_b. It presents the total value of the loss function (equation 42), which comprises both the loss coming from a positive output-gap target and the dynamic loss, and the dynamic loss separately. The top panels assume $\alpha = 0.1$, and the bottom ones use $\alpha = 0.3$ (all assume $x^* = 1\%$). The

[46]From the equation for the steady-state inflation in Section 3.1 (assuming $\gamma_f + \gamma_b = 1$, and $\beta = 1$).

[47]It assumes $\alpha = 0.3$. The pace of the reduction in the inflationary bias as a result of the rise in the degree of inflation persistence is not significantly affected by the values of α.

standard deviation of the cost-push shock is assumed to be 1.0% for the left panels, and 0.6% for the right ones. For low values of degree of inflation persistence, the importance of the reduction in the inflationary bias in comparison to the dynamic gains depends on the parameter value specification. For example, in the upper left panel, inflationary bias is responsible for 11.4% of the greater loss under discretion (with $\gamma_b = 0$), whereas in the lower right panel the share is 71.0%. The importance of those gains, however, decreases rapidly as the degree of inflation persistence rises. For instance, with $\gamma_b = 0.3$, the importance of those gains is diminutive in the four panels.

5 Conclusions

The society is better off with a policy conducted under commitment, according to a welfare measure based on an object function that penalizes deviations of the inflation rate and output gap from their targets. The solution under commitment eliminates the inflationary bias and generates a lower inflation-rate variability than that under discretion, although usually leading to a higher output variance. The solution under restricted commitment, however, generates significative gains only when the cost-push shock has a high degree of persistence.

The presence of persistence in the inflation-rate dynamics increases significantly the output-gap and inflation-rate variabilities, even though it reduces the inflationary bias. The relative gains from commitment are increasing in the degree of inflation persistence for moderate levels of persistence.

Appendix

A Derivation of the Aggregate Supply and Demand Curves

The derivation is similar to those in the recent literature of sticky-price models, such as Yun (1996) and Rotemberg and Woodford (1998).

A.1 The Structure of the Economy

The economy consists of private agents—households and firms—and government. The firms produce differentiated consumption goods.

A.1.1 Households

There is a continuum of j infinitely-lived households, whose total is normalized to one. Households obtain utility from consumption, real money holdings and leisure. The utility function of the representative household is given by:

$$U = E_t\{\sum_{i=0}^{\infty}\beta^i[\frac{1}{1-\gamma_c}C_{t+i}^{1-\gamma_c} + \frac{a_m}{1-\gamma_m}\left(\frac{M_{t+i}}{P_{t+i}}\right)^{1-\gamma_m} - \frac{a_n}{1+\gamma_n}N_{t+i}^{1+\gamma_n}]\}, \tag{54}$$

where $\gamma_c, \gamma_m, \gamma_n$, $a_m, a_n > 0$, β is a discount factor, M_{t+i} are money balances, P_{t+i} is the level of prices, N_{t+i} is the labor supply, and C_{t+i} is a composite index good (to ease the notation, I omit the superscript j that should appear in the variables). C_{t+i} is a constant-elasticity-of-substitution (CES) aggregator over all the differentiated goods, which are indexed by $z \in [0, 1]$:

$$C_{t+i} = [\int_0^1 C_{t+i}(z)^{\frac{\vartheta-1}{\vartheta}}dz]^{\frac{\vartheta}{\vartheta-1}}, \tag{55}$$

where $\vartheta > 1$ is the constant elasticity of substitution among alternative goods, and $C_{t+i}(z)$ represents the consumption of each of the differentiated good z. The household's budget constraint is

$$C_{t+i} = \frac{W_{t+i}}{P_{t+i}}N_{t+i} + \quad {}_{t+i} - T_{t+i} - \frac{M_{t+i} - M_{t+i-1}}{P_{t+i}} - \frac{\left(\frac{1}{1+i_t}\right)B_{t+i} - B_{t+i-1}}{P_{t+i}}, \tag{56}$$

where B_{t+i} are private discount bonds (zero in net supply), i_t is the nominal interest rate, T_{t+i} are lump-sum net taxes (taxes minus transfers), W_{t+i} are nominal wages, and $\;_{t+i}$ are the profits received from the firms, which are transferred in a lump-sum way. The total expenditure in consumption for each household is

$$D_{t+i} = P_{t+i}C_{t+i}. \tag{57}$$

The household's problem is to choose the sequence of $C_{t+i}(z)$, C_{t+i}, $\frac{M_{t+i}}{P_{t+i}}$, $\frac{B_{t+i}}{P_{t+i}}$ and N_{t+i} to maximize the utility function subject to the budget constraint. The problem can be solved in two stages. In the first one, the household chooses $C_{t+i}(z)$ to maximize equation (55) subject to:

$$D_{t+i} = \int_0^1 P_{t+i}(z)C_{t+i}(z)dz, \tag{58}$$

given $\frac{P_{t+i}(z)}{P_{t+i}}$ and $\frac{D_{t+i}}{P_{t+i}}$, where $P_{t+i}(z)$ is the price of each individual good. The optimization generates

$$C_{t+i}(z) = \left(\frac{P_{t+i}(z)}{P_{t+i}}\right)^{-\vartheta} C_{t+i}. \tag{59}$$

It represents the demand of each individual j for each differentiated good as a function of the relative price of the good and of the total consumption of the individual.

Aggregating over all the individuals, we get the total household's demand for good z:

$$C^a_{t+i}(z) = \left(\frac{P_{t+i}(z)}{P_{t+i}}\right)^{-\vartheta} C^a_{t+i}, \tag{60}$$

where C^a_{t+i} is the total household's consumption in the economy.

In the second stage, the household chooses $\{C_{t+i}, \frac{M_{t+i}}{P_{t+i}}, \frac{B_{t+i}}{P_{t+i}}, N_{t+i}\}_{i=0}^{\infty}$ to maximize the utility function (equation 54) subject to the budget constraint (equation 56). I use dynamic programming to solve it. The Bellman equation is

$$\begin{aligned} V\left(\frac{M_{t-1}}{P_t}, \frac{B_{t-1}}{P_t}\right) &= \max_{\frac{M_t}{P_t}, \frac{B_t}{P_t}, N_t} \{\frac{1}{1-\gamma_c} C_t^{1-\gamma_c} + \frac{a_m}{1-\gamma_m}\left(\frac{M_t}{P_t}\right)^{1-\gamma_m} \\ &\quad - \frac{a_n}{1+\gamma_n} N_t^{1+\gamma_n} + \beta V\left(\frac{M_t}{P_{t+1}}, \frac{B_t}{P_{t+1}}\right)\}, \end{aligned} \tag{61}$$

where equation (56) substitutes for C_t. The first-order conditions are

$$C_t^{-\gamma_c} = (1+i_t)\frac{P_t}{P_{t+1}}\beta C_{t+1}^{-\gamma_c}, \tag{62}$$

$$C_t^{-\gamma_c} = a_m\left(\frac{M_t}{P_t}\right)^{-\gamma_m} + \frac{P_t}{P_{t+1}}\beta C_{t+1}^{-\gamma_c}, \tag{63}$$

$$C_t^{-\gamma_c}\frac{W_t}{P_t} = a_n N_t^{\gamma_n}. \tag{64}$$

Using equation (62), we can rewrite (63) as:

$$a_m\left(\frac{M_t}{P_t}\right)^{-\gamma_m} = \frac{i_t}{(1+i_t)}C_t^{-\gamma_c}. \tag{65}$$

Equation (62) represents the intertemporal optimal condition for consumption (Euler equation), (64) is the labor supply, and (65) represents the demand for money.

A.1.2 Government

The government consumption G_{t+i} is also a composite good:

$$G_{t+i} = [\int_0^1 G_{t+i}(z)^{\frac{\vartheta-1}{\vartheta}}dz]^{\frac{\vartheta}{\vartheta-1}}, \tag{66}$$

where $G_{t+i}(z)$ represents the government consumption of each of the differentiated good z. The government minimizes its expenditures given by:

$$Z_{t+i} = \int_0^1 P_{t+i}(z)G_{t+i}(z)dz, \tag{67}$$

subject to equation (66). Similarly to the household's problem, the optimization generates the government demand for good z:

$$G_{t+i}(z) = \left(\frac{P_{t+i}(z)}{P_{t+i}}\right)^{-\vartheta} G_{t+i}. \tag{68}$$

Therefore, the government purchases a proportion of each good equal to that purchased by households.

Government expenditures are financed by money creation and lump-sum net taxes:

$$G_{t+i} = \frac{M_{t+i} - M_{t+i-1}}{P_{t+i}} + T_{t+i}. \tag{69}$$

A.1.3 Firms

There is a continuum of monopolistically competitive firms owned by the households, whose total is normalized to one. Each firm produces a differentiated good z and uses the same technology. The production function is

$$Y_{t+i}(z) = A_{t+i}N_{t+i}(z), \tag{70}$$

where A_{t+i} is a stochastic technological factor.

The factor demand is derived by the minimization of the firm's cost. Firms choose N_{t+i} to minimize

$$\frac{W_{t+i}}{P_{t+i}}N_{t+i}(z) \tag{71}$$

subject to (70). Using the Lagrangian, we obtain:

$$\frac{W_{t+i}}{P_{t+i}} = \Pi_{t+i}A_{t+i}, \tag{72}$$

where Π_{t+i} is the Lagrangian multiplier and represents the real marginal cost.

According to the economy resource constraint:

$$Y_{t+i} = C^a_{t+i} + G_{t+i}, \tag{73}$$

where Y_{t+i} is the aggregate output.

The production of z—denoted by $Y_{t+i}(z)$—faces a demand schedule obtained by adding household's and government consumption (equations 60 and 68) and using equation (73):

$$Y_{t+i}(z) = \left(\frac{P_{t+i}(z)}{P_{t+i}}\right)^{-\vartheta} Y_{t+i}. \tag{74}$$

Following Calvo (1983), only a fraction of the firms is allowed to adjust prices each period. Every period, each firm faces a fixed probability θ of not adjusting its price, independently of the last time the firm adjusted it. Therefore, a fraction $1-\theta$ of the firms is allowed to change prices at each moment. In the average, firms adjust price every $\frac{1}{1-\theta}$ periods. As a consequence, firms face an intertemporal optimization.

They have to form expectations of the future behavior of their demand and costs. The firm that is allowed to adjust prices chooses the price $P^*_{t+i}(z)$ that maximizes

$$E_t\{\sum_{i=0}^{\infty}(\theta^i\beta^i)[\Psi_{t,i}\frac{P^*_{t+i}(z)-Q_{t+i}}{P_{t+i}}Y^*_{t+i}(z)]\}, \tag{75}$$

subject to

$$Y^*_{t+i}(z)=\left(\frac{P^*_t(z)}{P_{t+i}}\right)^{-\vartheta}Y_{t+i}, \tag{76}$$

where $\Psi_{t,i}\equiv\left(\frac{C_{t+i}}{C_t}\right)^{-\gamma_c}$, and Q_{t+i} is the nominal marginal cost. $\beta^i\Psi_{t,i}$ is the stochastic discount factor between t and i. Since the firms are owned by the households, the profits are discounted according to the intertemporal optimal condition for consumption (equation 62).

The first-order condition is

$$E_t\{\sum_{i=0}^{\infty}(\theta\beta)^i\Psi_{t,i}[\frac{Y^*_{t+i}(z)}{P_{t+i}}-\vartheta\left(\frac{P^*_{t+i}(z)-Q_{t+i}}{P_{t+i}}\right)\frac{Y^*_{t+i}(z)}{P^*_{t+i}(z)}]\}. \tag{77}$$

Rearranging terms, we can write the optimal price as:

$$P^*_{t+i}(z)=\varsigma\frac{E_t\{\sum_{i=0}^{\infty}(\theta\beta)^i\Psi_{t,i}Q_{t+i}Y^*_{t+i}(z)\frac{1}{P_{t+i}}\}}{E_t\{\sum_{i=0}^{\infty}(\theta\beta)^i\Psi_{t,i}Y^*_{t+i}(z)\frac{1}{P_{t+i}}\}}. \tag{78}$$

where $\varsigma\equiv\frac{\vartheta}{\vartheta-1}$ is the mark-up. Since in symmetric equilibrium:

$$P^*_t(z)=P^*_t, \tag{79}$$

we can write equation (78) as:

$$P^*_{t+i}=\varsigma\frac{E_t\{\sum_{i=0}^{\infty}(\theta\beta)^i\Psi_{t,i}Q_{t+i}Y_{t+i}\left(\frac{1}{P_{t+i}}\right)^{1-\vartheta}\}}{E_t\{\sum_{i=0}^{\infty}(\theta\beta)^i\Psi_{t,i}Y_{t+i}\left(\frac{1}{P_{t+i}}\right)^{1-\vartheta}\}}, \tag{80}$$

The price index is given by an aggregator over the prices of the differentiated goods:

$$P_{t+i}=[\int_0^1 P_{t+i}(z)^{1-\vartheta}dz]^{\frac{1}{1-\vartheta}}, \tag{81}$$

which represents the minimum expenditure required to buy one unit of the composite consumption good.

Since a fraction θ of the firms is not allowed to adjust prices at i, the price aggregator evolves according to:

$$P_{t+i} = [\theta P_{t+i-1}^{1-\vartheta} + (1-\theta) P_{t+i}^{*1-\vartheta}]^{\frac{1}{1-\vartheta}}. \tag{82}$$

A.2 Equilibrium and Approximation Around a Steady State

In the symmetric equilibrium, the following conditions hold:

$$\begin{aligned} C_{t+i} &= C_{t+i}^{a}, \\ N_{t+i}(z) &= N_{t+i}, \\ Y_{t+i}(z) &= Y_{t+i}, \\ P_{t+i}(z) &= P_{t+i}. \end{aligned} \tag{83}$$

The equilibrium in the economy is described by equations (62), (64), (65), (70), (72), (73), (80), and (82). I take log-approximations of the variables around the deterministic steady state, where there is no stochastic shock and the prices are fully flexible. All the lower case variables represent log-deviations from their steady-state values; for example:

$$y_t = \log (Y_t) - \log \left(\bar{Y}\right), \tag{84}$$

where $\bar{Y}$ is the steady-state value for the output. Since $\log(1+r) \approx r$, the lower case variables represent percentage deviations from the steady state.

A.2.1 Aggregate Demand Curve

I rewrite the economy resource constraint as $C_{t+i} = \left(1 - \frac{G_{t+i}}{Y_{t+i}}\right) Y_{t+i}$, and use it in the Euler equation (62). Defining $e \equiv -\log\left(1 - \frac{G_{t+i}}{Y_{t+i}}\right) + \log\left(1 - \frac{\bar{G}}{\bar{Y}}\right)$, where $\bar{G}$ is the steady-state government consumption, taking the log-approximation, and using the fact that $\log(1+i_t) \approx i_t$, we obtain

$$y_t - e_t = -\frac{1}{\gamma_c}\left(i_t - E_t \pi_{t+1}\right) + E_t\{y_{t+1} - e_{t+1}\}, \tag{85}$$

where i_t now stands for the log-deviation of the nominal interest rate from its steady-state value, $\pi_{t+1} = p_{t+1} - p_t$ represents the inflation rate, and $e_t = y_t - c_t$. The real interest rate is $r_t \approx i_t - E_t\pi_{t+1}$.

I define

$$x_t \equiv y_t - y_t^*, \tag{86}$$

where y_t^* is the potential output (log-deviation from its steady-state value) and corresponds to the output that would prevail if prices and wages were perfectly flexible. Combining equation (85) with (86), we obtain the aggregate demand curve or the "IS curve":

$$x_t = -\sigma(i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t, \tag{87}$$

where $\sigma \equiv \frac{1}{\gamma_c}$, and $g_t = E_t\{(e_t - y_t^*) - (e_{t+1} - y_{t+1}^*)\}$.

A.2.2 Money Market Equilibrium

The money market equilibrium (where money supply is equal to the demand M_{t+i}) is given by (65), which log-linearized around the steady state is expressed as:

$$m_t - p_t = \iota_1 c_t - \iota_2 i_t, \tag{88}$$

where $\iota_1 \equiv \frac{\gamma_c}{\gamma_m}, \iota_2 \equiv \frac{1}{\gamma_m \bar{\imath}}$, and $\bar{\imath}$ is the steady-state nominal interest rate. Using the expression for e_t, this equation can be written as:

$$m_t - p_t = \iota_1 y_t - \iota_1 e_t - \iota_2 i_t. \tag{89}$$

A.2.3 Aggregate Supply Curve

The log-linearization of equation (80) yields

$$p_t^* = (1 - \theta\beta)\, E_t\{\sum_{i=0}^{\infty} (\theta\beta)^i\, (q_{t+i} + p_{t+i})\}. \tag{90}$$

The price level (equation 82) in log-linear terms is

$$p_t = \theta p_{t-1} + (1 - \theta)\, p_t^*. \tag{91}$$

Combining (90) with (91), and using the fact that $p_t^* = (1-\theta\beta)(q_t + p_t) + \theta\beta p_{t+1}^*$, we obtain

$$\pi_t = \chi(q_t - p_t) + \beta E_t \pi_{t+1}, \tag{92}$$

where $\chi \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$, and $q_t - p_t$ is the real marginal cost.

Using the production function and the economy resource constraint in the labor supply equation (64), and log-linearizing it around the steady state, we obtain:

$$w_t - p_t = (\gamma_c + \gamma_n)y_t - \gamma_c e_t - \gamma_n a_t. \tag{93}$$

Log-linearizing equation (72) around the steady state, and combining with (93), we obtain an expression for the real marginal cost:

$$q_t - p_t = (\gamma_c + \gamma_n) y_t - \gamma_c e_t - (1 + \gamma_n) a_t. \tag{94}$$

Now I find an expression for the potential output (y_t^*). In flexible price equilibrium ($\theta = 0$):

$$\frac{P_{t+i}(z)}{P_{t+i}} = \frac{\vartheta}{\vartheta - 1} \frac{W_{t+i}}{P_{t+i}} \frac{1}{A_{t+i}}, \tag{95}$$

which can be seen as a special case of equation (78). Using the symmetric equilibrium condition $P_{t+i}(z) = P_{t+i}$ in equation (95), combining it with equations (64), (70) and (73), and log-linearizing it around the steady state, we obtain:

$$y_t^* = \frac{\gamma_c}{(\gamma_c + \gamma_n)} e_t^* + \frac{\gamma_n}{(\gamma_c + \gamma_n)} a_t, \tag{96}$$

where $e_t^* \equiv -\log\left(1 - \frac{G_{t+i}^*}{Y_{t+i}^*}\right) + \log\left(1 - \frac{\bar{G}}{\bar{Y}}\right)$, and $\frac{G_t^*}{Y_t^*}$ is the ratio of government purchases to output in flexible price equilibrium. Using equations (86) and (96) in the real marginal cost equation (94), and assuming a constant actual ratio $\frac{G_t}{Y_t}$ (equal to $\frac{G_t^*}{Y_t^*}$), which implies $e_t = e_t^*$, we obtain

$$q_t - p_t = \kappa x_t, \tag{97}$$

where $\kappa \equiv (\gamma_c + \gamma_n)$. Therefore, in this formulation, the real marginal cost is proportional to the output gap. Plugging into equation (92) yields

$$\pi_t = \lambda x_t + \beta E_t \pi_{t+1}, \tag{98}$$

where $\lambda = (1-\theta)(1-\beta\theta)\kappa\theta^{-1}$. The cost-push shock included in equation (1) represents a deviation of the proportion between real marginal cost and output gap.

To find the effect of imperfect competition in the level of potential output, I follow the same procedures used to obtain equation (96), but now the log-linearization is around the values in a perfect competition economy. In this case, we obtain

$$y^{**} = \frac{-\log(\frac{\vartheta}{\vartheta-1})}{(\gamma_c + \gamma_n)}, \tag{99}$$

where y^{**} is the deviation of potential output with monopolist competition from the one with perfect competition (both in logs). Note that $\frac{\vartheta}{\vartheta-1}$ is the mark-up (equal to one in perfect competition).

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Table 1

Unconditional Moments, and Value of the Loss Function, with rho = 0.6

Alpha	Monetary Policy	Standard Deviation of Output (%)	Standard Deviation of Inflation (%)	Ratio Stand. Dev. Output to Stand. Dev. Inflation	Value of the Loss Function (1)	Ratio Loss Function Discretion to Loss Function Regime	Ratio Loss Function Regime to Loss Function Discretion
0.04	Commitment	5.54	1.04	5.33	0.023	1.90	0.53
	Restricted Commitment	7.48	1.18	6.34	0.036	1.21	0.83
	Discretion	4.80	1.86	2.58	0.044	1.00	1.00
0.1	Commitment	3.81	1.45	2.64	0.036	1.86	0.54
	Restricted Commitment	4.75	1.87	2.54	0.058	1.14	0.88
	Discretion	2.51	2.44	1.03	0.066	1.00	1.00
0.3	Commitment	2.21	1.92	1.15	0.052	1.61	0.62
	Restricted Commitment	2.14	2.54	0.85	0.078	1.06	0.94
	Discretion	0.97	2.83	0.34	0.083	1.00	1.00
0.5	Commitment	1.66	2.12	0.78	0.059	1.49	0.67
	Restricted Commitment	1.38	2.73	0.51	0.084	1.04	0.96
	Discretion	0.60	2.93	0.21	0.087	1.00	1.00
1.0	Commitment	1.10	2.35	0.47	0.068	1.35	0.74
	Restricted Commitment	0.73	2.89	0.25	0.089	1.02	0.98
	Discretion	0.31	3.00	0.10	0.091	1.00	1.00

Notes: The annualized standard deviation of the random cost-push shock is assumed to be 1%. The output is measured in percentage deviations from the steady-state value, and the inflation rate is measured in annualized percentage points.

(1) Calculated as the weighted sum of the unconditional variances of output and inflation rate in percentage units.

Table 2

Unconditional Moments, and Value of the Loss Function, with rho = 0.8

Alpha	Monetary Policy	Standard Deviation of Output (%)	Standard Deviation of Inflation (%)	Ratio Stand. Dev. Output to Stand. Dev. Inflation	Value of the Loss Function (1)	Ratio Loss Function Discretion to Loss Function Regime	Ratio Loss Function Regime to Loss Function Discretion
0.04	Commitment	10.51	1.34	7.87	0.062	2.53	0.40
	Restricted Commitment	13.91	1.12	12.38	0.090	1.74	0.57
	Discretion	9.07	3.52	2.58	0.157	1.00	1.00
0.1	Commitment	8.28	2.12	3.91	0.114	2.75	0.36
	Restricted Commitment	11.49	2.32	4.95	0.186	1.67	0.60
	Discretion	5.47	5.31	1.03	0.311	1.00	1.00
0.3	Commitment	5.64	3.29	1.71	0.204	2.38	0.42
	Restricted Commitment	7.28	4.41	1.65	0.353	1.37	0.73
	Discretion	2.35	6.85	0.34	0.486	1.00	1.00
0.5	Commitment	4.54	3.88	1.17	0.254	2.13	0.47
	Restricted Commitment	5.32	5.38	0.99	0.431	1.25	0.80
	Discretion	1.50	7.27	0.21	0.540	1.00	1.00
1.0	Commitment	3.27	4.68	0.70	0.326	1.80	0.55
	Restricted Commitment	3.19	6.43	0.50	0.516	1.14	0.88
	Discretion	0.79	7.62	0.10	0.587	1.00	1.00

Notes: The annualized standard deviation of the random cost-push shock is assumed to be 1%. The output is measured in percentage deviations from the steady-state value, and the inflation rate is measured in annualized percentage points.

(1) Calculated as the weighted sum of the unconditional variances of output and inflation rate in percentage units.

Table 3

Unconditional Moments, and Value of the Loss Function: Different Output Elasticities of Marginal Cost (Kappa), with rho = 0.6

Kappa	Alpha	Monetary Policy	Standard Deviation of Output (%)	Standard Deviation of Inflation (%)	Ratio Stand. Dev. Output to Stand. Dev. Inflation	Value of the Loss Function (1)	Ratio Loss Function Discretion to Loss Function Regime	Ratio Loss Function Regime to Loss Function Discretion
1	0.1	Commitment	2.56	0.49	5.26	0.009	1.52	0.66
		Restricted Commitment	3.19	0.38	8.46	0.012	1.17	0.86
		Discretion	2.71	0.79	3.43	0.014	1.00	1.00
	0.3	Commitment	1.90	0.87	2.17	0.019	1.84	0.54
		Restricted Commitment	2.56	0.91	2.82	0.028	1.22	0.82
		Discretion	1.79	1.56	1.14	0.034	1.00	1.00
	0.5	Commitment	1.59	1.09	1.46	0.025	1.91	0.52
		Restricted Commitment	2.14	1.27	1.69	0.039	1.20	0.83
		Discretion	1.34	1.95	0.69	0.047	1.00	1.00
2	0.1	Commitment	1.60	0.18	8.71	0.003	1.18	0.85
		Restricted Commitment	1.76	0.10	16.91	0.003	1.06	0.94
		Discretion	1.68	0.24	6.87	0.003	1.00	1.00
	0.3	Commitment	1.36	0.41	3.35	0.007	1.43	0.70
		Restricted Commitment	1.65	0.29	5.64	0.009	1.14	0.88
		Discretion	1.45	0.63	2.29	0.010	1.00	1.00
	0.5	Commitment	1.22	0.56	2.19	0.011	1.59	0.63
		Restricted Commitment	1.55	0.46	3.38	0.014	1.18	0.85
		Discretion	1.27	0.93	1.37	0.017	1.00	1.00

Notes: The annualized standard deviation of the random cost-push shock is assumed to be 1%. The output is measured in percentage deviations from the steady-state value, and the inflation rate is measured in annualized percentage points.

(1) Calculated as the weighted sum of the unconditional variances of output and inflation rate in percentage units.

Table 4

Unconditional Moments, and Value of the Loss Function: Different Share of Backward-Looking Firms (Omega), with rho = 0.6

Omega	Alpha	Monetary Policy	Standard Deviation of Output (%)	Standard Deviation of Inflation (%)	Ratio Stand. Dev. Output to Stand. Dev. Inflation	Value of the Loss Function (1)	Ratio Loss Function Discretion to Loss Function Regime	Ratio Loss Function Regime to Loss Function Discretion
0.25	0.1	Commitment	6.41	2.37	2.71	0.097	1.88	0.53
		Restricted Commitment	7.72	2.66	2.90	0.130	1.40	0.71
		Discretion	4.02	4.08	0.98	0.183	1.00	1.00
	0.3	Commitment	3.80	3.17	1.20	0.144	1.66	0.60
		Restricted Commitment	4.24	3.75	1.13	0.195	1.23	0.81
		Discretion	1.62	4.81	0.34	0.239	1.00	1.00
	0.5	Commitment	2.89	3.52	0.82	0.166	1.54	0.65
		Restricted Commitment	2.99	4.18	0.71	0.220	1.16	0.86
		Discretion	1.01	4.99	0.20	0.254	1.00	1.00
0.5	0.1	Commitment	11.50	3.98	2.89	0.290	1.93	0.52
		Restricted Commitment	13.29	4.31	3.09	0.362	1.55	0.64
		Discretion	7.13	7.15	1.00	0.562	1.00	1.00
	0.3	Commitment	7.21	5.43	1.33	0.451	1.79	0.56
		Restricted Commitment	8.22	6.10	1.35	0.575	1.40	0.71
		Discretion	3.18	8.80	0.36	0.805	1.00	1.00
	0.5	Commitment	5.63	6.10	0.92	0.531	1.67	0.60
		Restricted Commitment	6.24	6.94	0.90	0.677	1.31	0.76
		Discretion	2.07	9.31	0.22	0.888	1.00	1.00

Notes: The annualized standard deviation of the random cost-push shock is assumed to be 1%. The output is measured in percentage deviations from the steady-state value, and the inflation rate is measured in annualized percentage points.

(1) Calculated as the weighted sum of the unconditional variances of output and inflation rate in percentage units.

Table 5

Unconditional Moments, and Value of the Loss Function: Degree of Inflation Persistence (Gamma Backwards) = 0.4, with rho = 0.6

Alpha	Monetary Policy	Standard Deviation of Output (%)	Standard Deviation of Inflation (%)	Ratio Stand. Dev. Output to Stand. Dev. Inflation	Value of the Loss Function (1)	Ratio Loss Function Discretion to Loss Function Regime	Ratio Loss Function Regime to Loss Function Discretion
0.1	Commitment	7.08	1.50	4.71	0.073	1.63	0.61
	Restricted Commitment	7.78	1.48	5.24	0.083	1.44	0.70
	Discretion	6.52	2.76	2.37	0.119	1.00	1.00
0.3	Commitment	5.36	2.46	2.18	0.147	1.88	0.53
	Restricted Commitment	6.09	2.53	2.41	0.176	1.57	0.64
	Discretion	4.39	4.67	0.94	0.276	1.00	1.00
0.5	Commitment	4.58	3.01	1.52	0.196	1.96	0.51
	Restricted Commitment	5.29	3.16	1.67	0.240	1.60	0.62
	Discretion	3.46	5.69	0.61	0.384	1.00	1.00

Notes: The annualized standard deviation of the random cost-push shock is assumed to be 1%. The output is measured in percentage deviations from the steady-state value, and the inflation rate is measured in annualized percentage points.

(1) Calculated as the weighted sum of the unconditional variances of output and inflation rate in percentage units.

Table 6

Inflationary Bias (%)

Model	Alpha		
	0.1	0.3	0.5
No Inflation Persistence	1.00	3.00	5.00
Inflation Persistence, omega = 0.25	0.92	2.71	4.49
Inflation Persistence, omega = 0.5	0.78	2.15	3.49
Inflation Persistence, gamma backwards = 0.4	0.34	0.84	1.30

Figure 1 Ratios of the Value of the Loss as a Function of α (Relative Output Weight in the Objective Function)

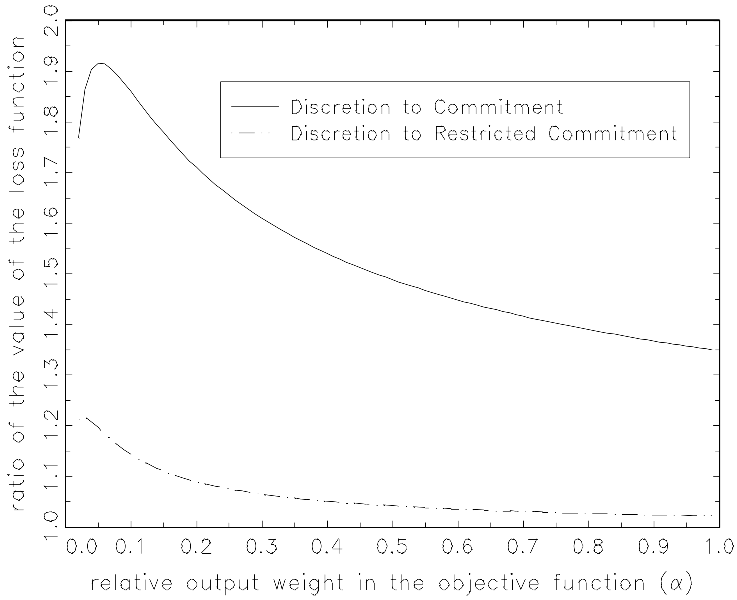


Figure 2 Ratios of the Value of the Loss as a Function of ρ (Autoregressive Coefficient of the Cost-Push Shock Process)

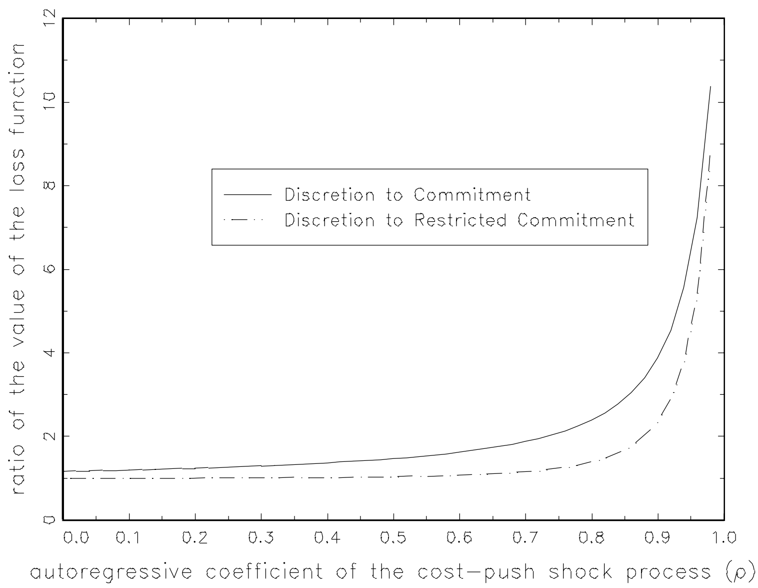


Figure 3 Unconditional Standard Deviation of Inflation Rate as a Function of α
(Relative Output Weight in the Objective Function)

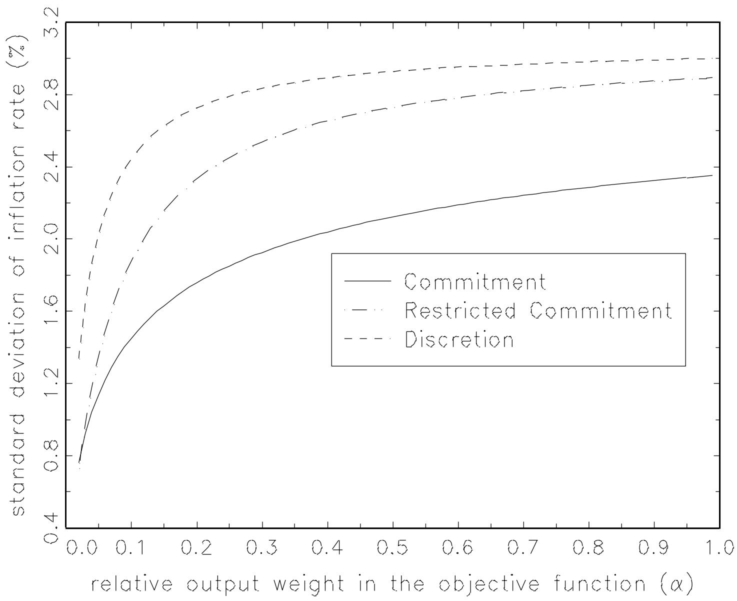


Figure 4 Unconditional Standard Deviation of Output Gap as a Function of α
(Relative Output Weight in the Objective Function)

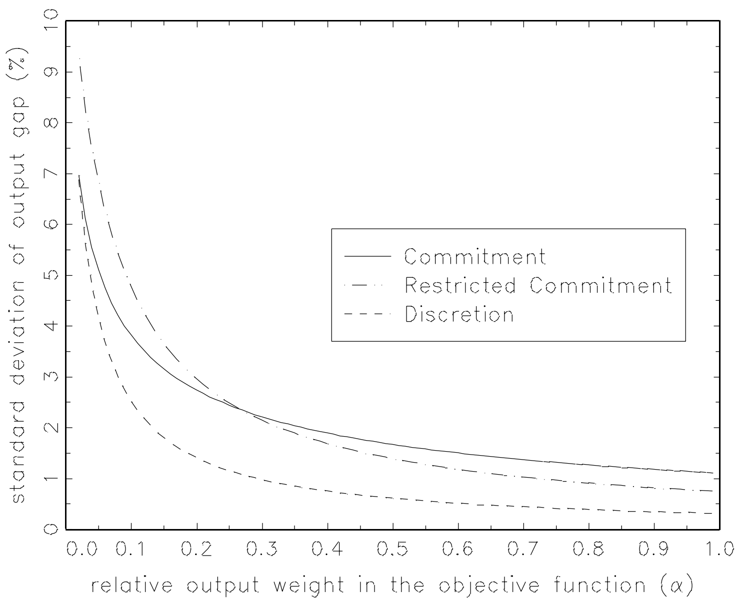


Figure 5 Trade-Off between Variabilities of Output Gap and Inflation

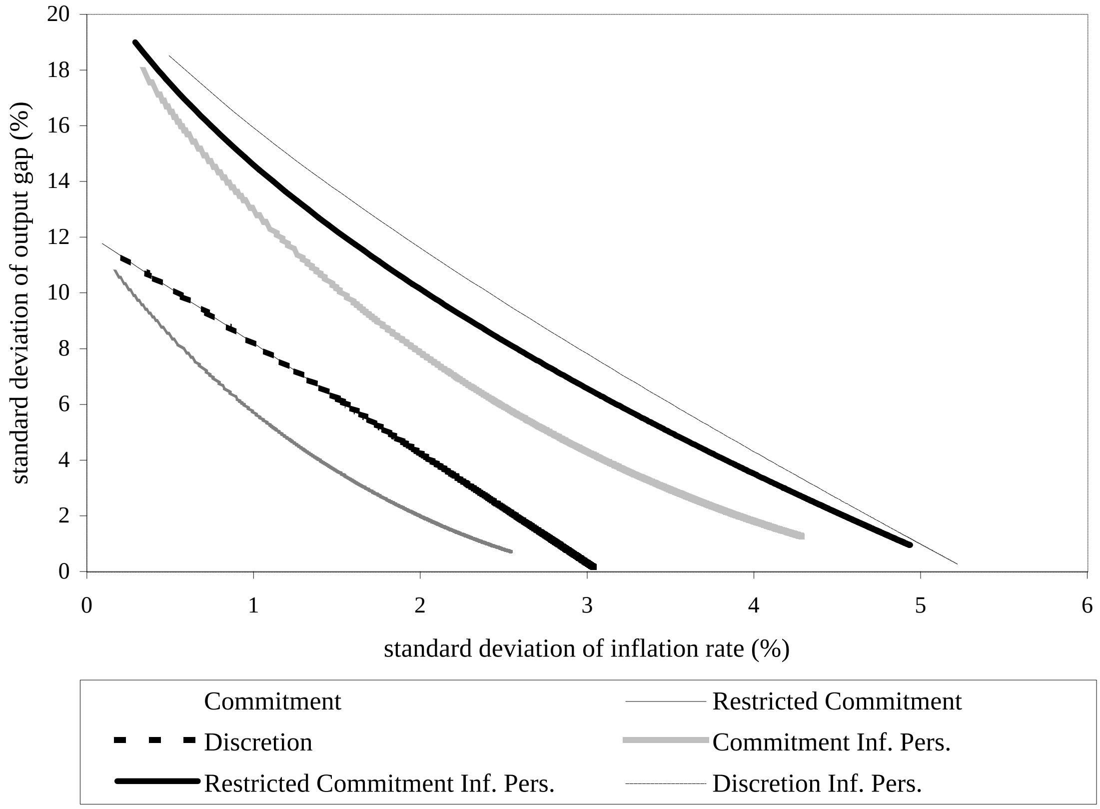


Figure 6 Ratios of the Value of the Loss as a Function of θ (Degree of Price Rigidity)

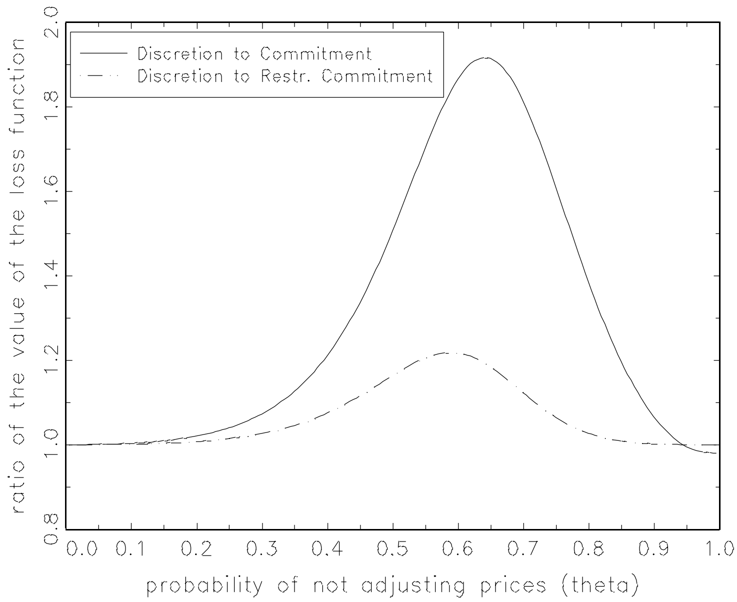


Figure 7 Impulse Responses to a One-Percentage-Point Cost-Push Shock, with $\rho = 0$

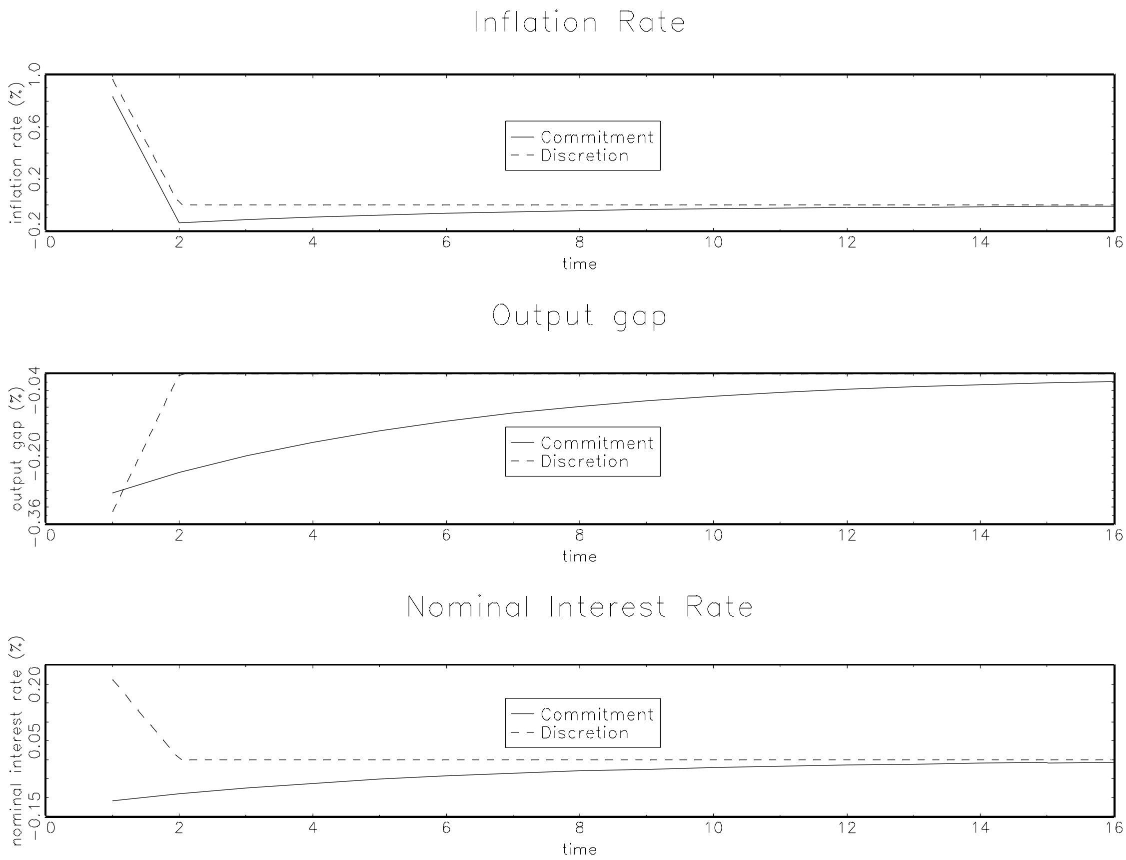


Figure 8 Impulse Responses to a One-Percentage-Point Cost-Push Shock, with $\rho = 0.6$

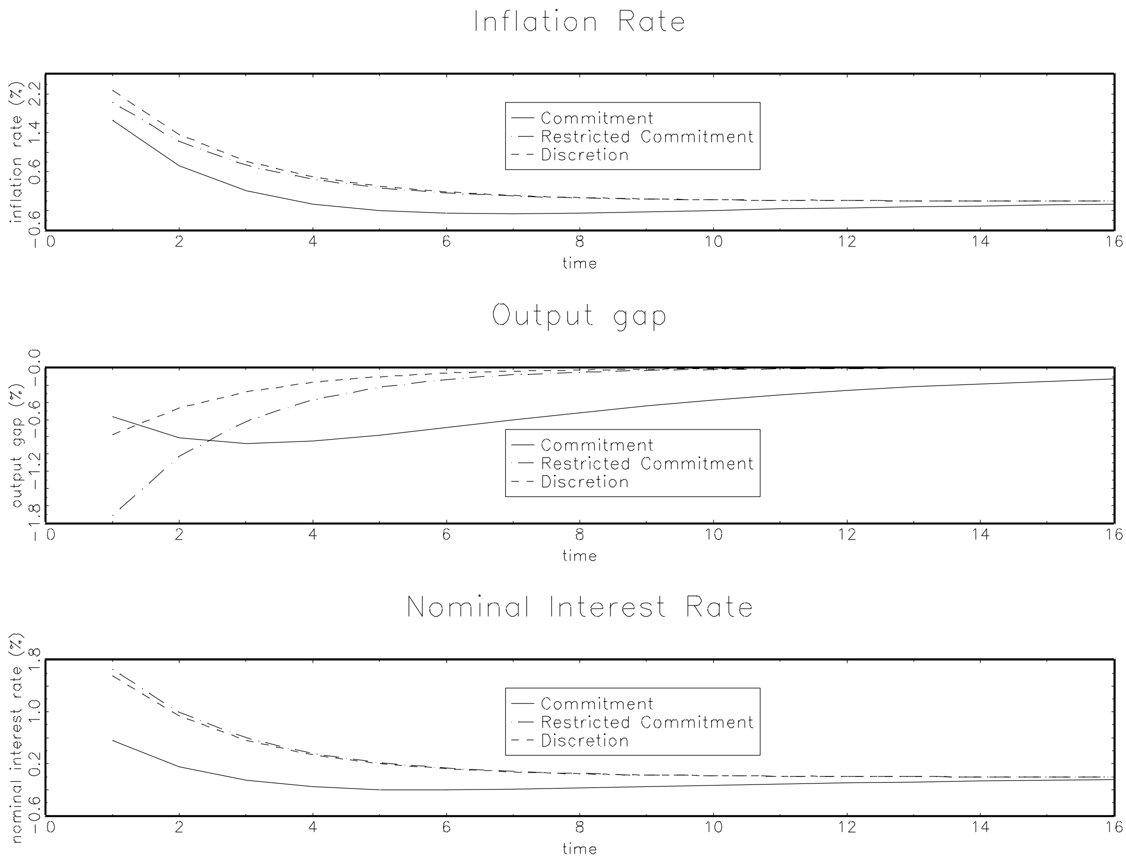


Figure 9 Ratios of the Value of the Loss as a Function of γ_b (Degree of Inflation Persistence)

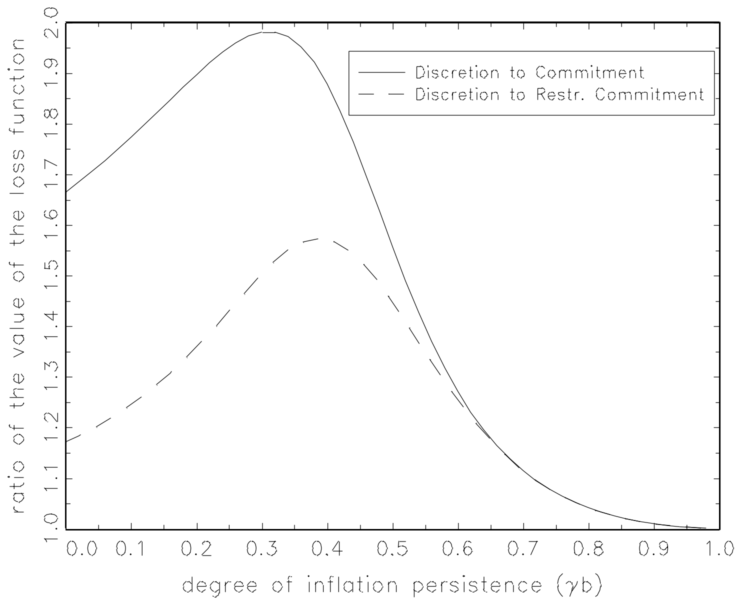


Figure 10 Impulse Responses to a One-Percentage-Point Cost-Push Shock, with Inflation Persistence ($\omega = 0.25$)

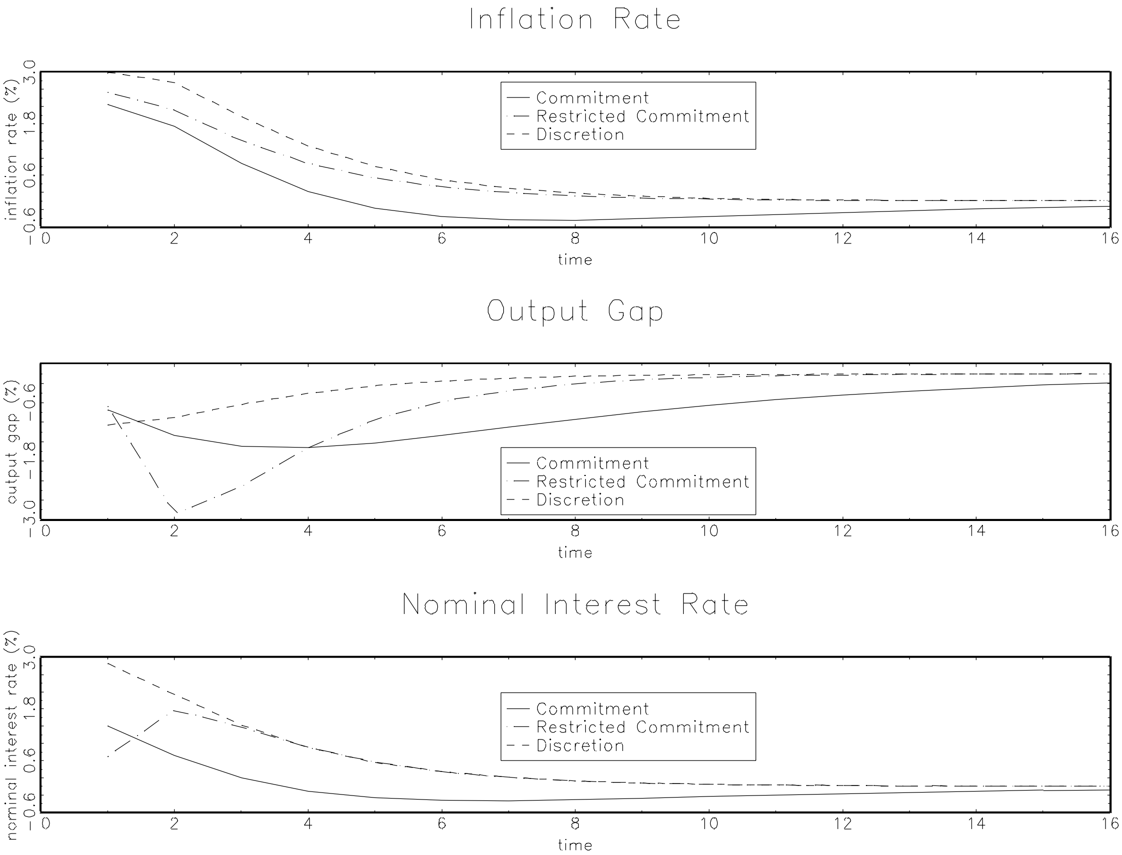


Figure 11 Inflationary Bias: Inflation Rate and Output Gap as a Function of the Expected Inflation Rate

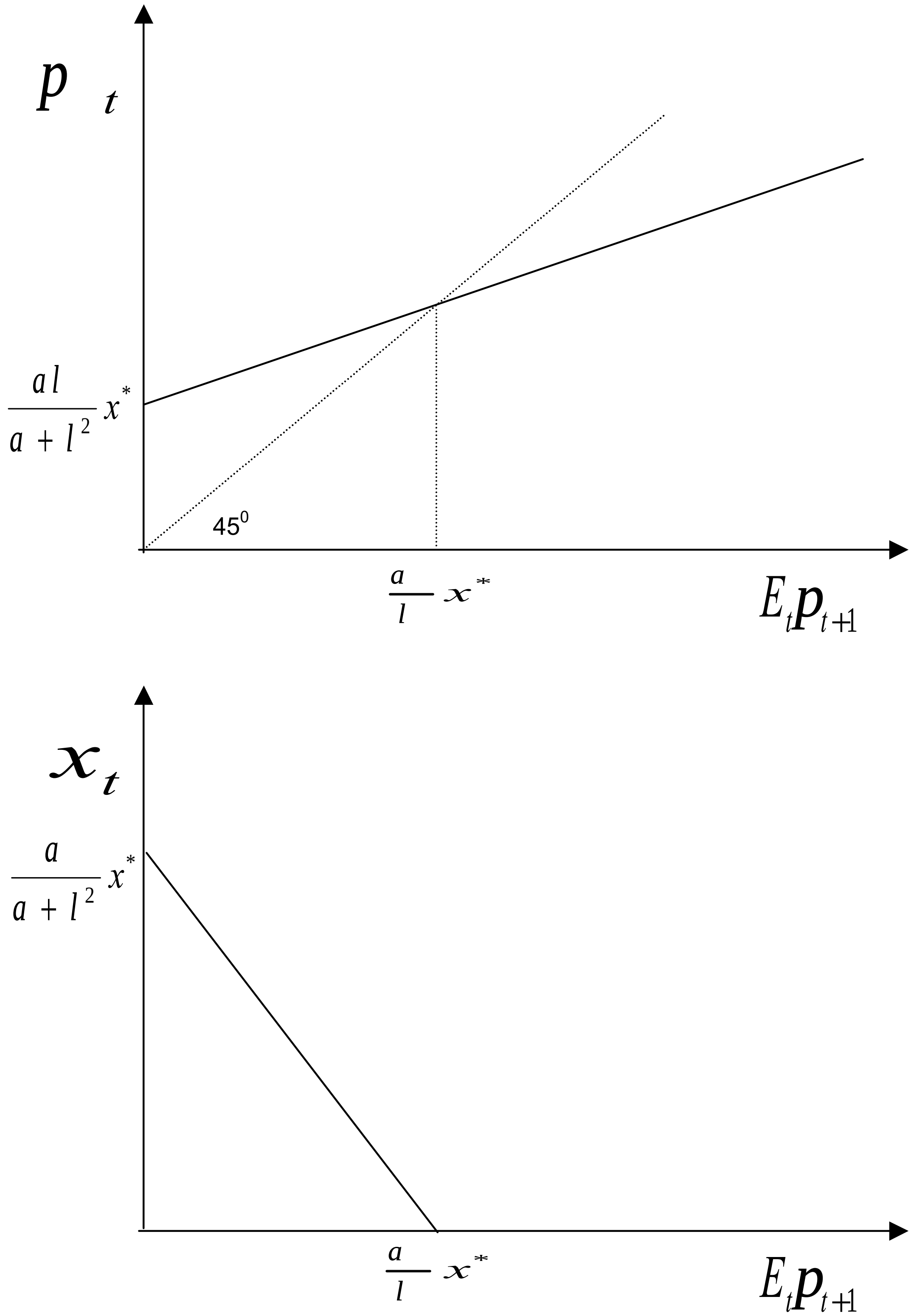


Figure 12 Inflationary Bias as a Function of γ_b (Degree of Inflation Persistence)

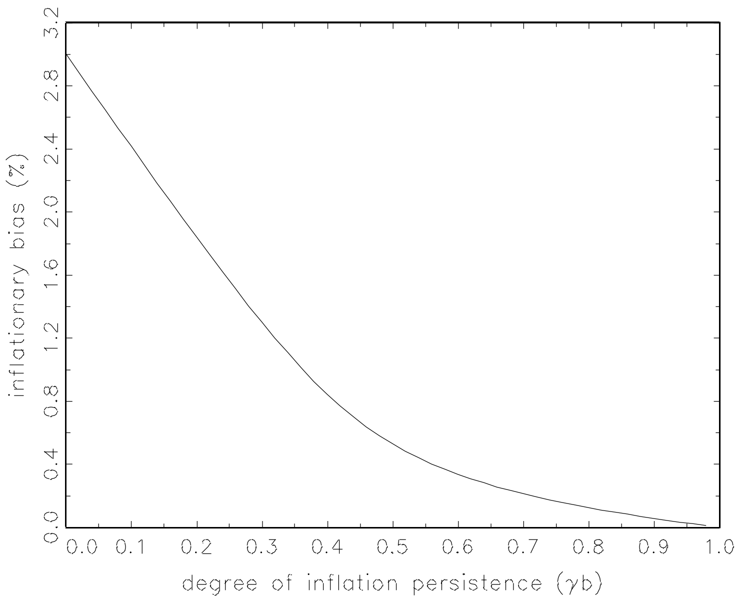
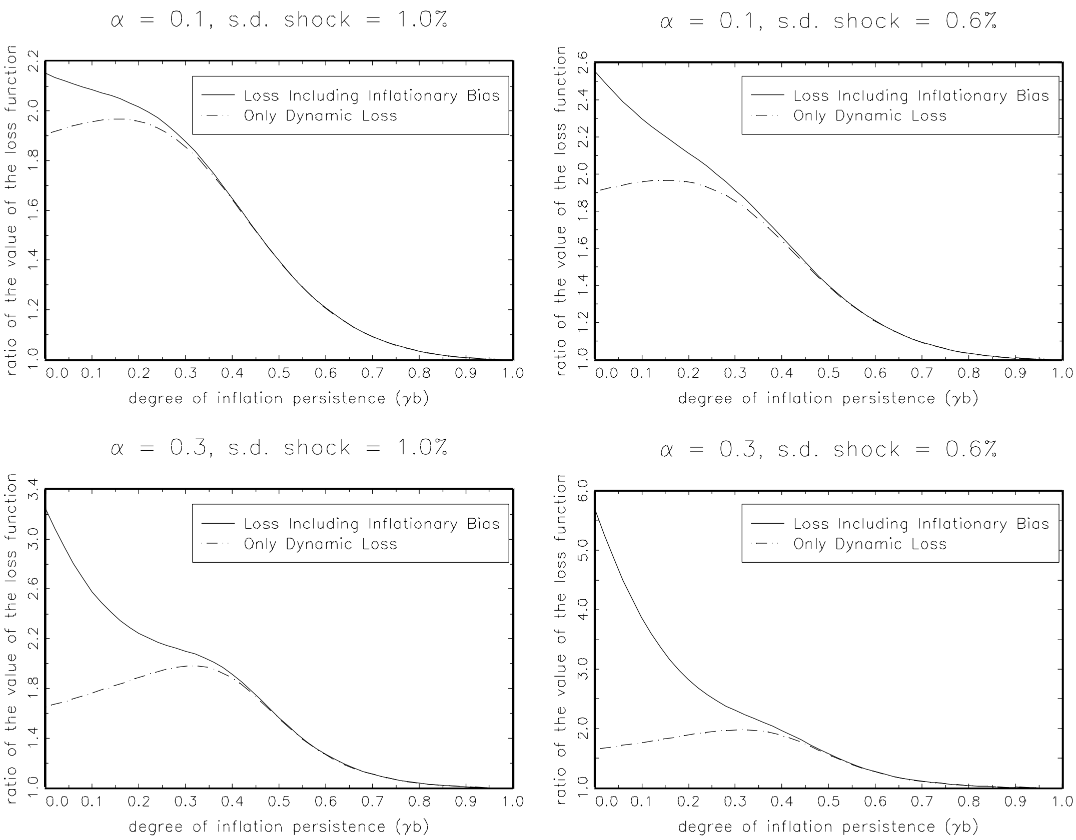


Figure 13 Ratios of the Value of the Total Loss and of the Dynamic Loss Under Discretion to Those Under Commitment as a Function of γ_b (Degree of Inflation Persistence)



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