

# Decomposition of Systemic Risk Drivers in Evolving Financial Networks

João Barata Ribeiro Blanco Barroso, Thiago Christiano Silva and  
Sergio Rubens Stancato de Souza

December, 2016

## Working Papers



# 448

ISSN 1518-3548  
CGC 00.038.166/0001-05

Working Paper Series	Brasília	n. 448	December	2016	p. 1-51
----------------------	----------	--------	----------	------	---------

# *Working Paper Series*

Edited by Research Department (Depep) – E-mail: [workingpaper@bcb.gov.br](mailto:workingpaper@bcb.gov.br)

Editor: Francisco Marcos Rodrigues Figueiredo – E-mail: [francisco-marcos.figueiredo@bcb.gov.br](mailto:francisco-marcos.figueiredo@bcb.gov.br)

Co-editor: João Barata Ribeiro Blanco Barroso – E-mail: [joao.barroso@bcb.gov.br](mailto:joao.barroso@bcb.gov.br)

Editorial Assistant: Jane Sofia Moita – E-mail: [jane.sofia@bcb.gov.br](mailto:jane.sofia@bcb.gov.br)

Head of Research Department: Eduardo José Araújo Lima – E-mail: [eduardo.lima@bcb.gov.br](mailto:eduardo.lima@bcb.gov.br)

The Banco Central do Brasil Working Papers are all evaluated in double blind referee process.

Reproduction is permitted only if source is stated as follows: Working Paper n. 448.

Authorized by Carlos Viana de Carvalho, Deputy Governor for Economic Policy.

## **General Control of Publications**

Banco Central do Brasil

Comun/Dipiv/Coivi

SBS – Quadra 3 – Bloco B – Edifício-Sede – 14º andar

Caixa Postal 8.670

70074-900 Brasília – DF – Brazil

Phones: +55 (61) 3414-3710 and 3414-3565

Fax: +55 (61) 3414-1898

E-mail: [editor@bcb.gov.br](mailto:editor@bcb.gov.br)

The views expressed in this work are those of the authors and do not necessarily reflect those of the Banco Central or its members.

Although these Working Papers often represent preliminary work, citation of source is required when used or reproduced.

*As opiniões expressas neste trabalho são exclusivamente do(s) autor(es) e não refletem, necessariamente, a visão do Banco Central do Brasil.*

*Ainda que este artigo represente trabalho preliminar, é requerida a citação da fonte, mesmo quando reproduzido parcialmente.*

## **Citizen Service Division**

Banco Central do Brasil

Deati/Diate

SBS – Quadra 3 – Bloco B – Edifício-Sede – 2º subsolo

70074-900 Brasília – DF – Brazil

Toll Free: 0800 9792345

Fax: +55 (61) 3414-2553

Internet: <http://www.bcb.gov.br/?CONTACTUS>

# Decomposition of Systemic Risk Drivers in Evolving Financial Networks

João Barata Ribeiro Blanco Barroso \*

Thiago Christiano Silva \*\*

Sergio Rubens Stancato de Souza \*\*\*

*The Working Papers should not be reported as representing the views of the Banco Central do Brasil. The views expressed in the papers are those of the authors and do not necessarily reflect those of the Banco Central do Brasil.*

## Abstract

In this paper, we propose a methodology to decompose drivers of systemic risk that arise due to insolvency contagion in evolving financial networks. There is an ongoing discussion on how network topology and capital buffer influence systemic risk. On the one hand, the network contagion literature tends to emphasize the influence of the network topology. On the other hand, policy works tend to discuss restrictions over the capital buffers of financial institutions. Systemic risk is usually a complex function of both risk drivers and thus isolating the contributive effects of each risk driver to systemic risk is not a trivial task. Our decomposition methodology identifies and isolates these effects. We apply our methodology to the global banking network and find that the network topology effect explains most of the systemic risk measure's volatility. Additionally, we show that the capital buffer effect explains the persistent reduction in systemic risk buildup with effects concentrated around the global financial crisis. Our results confirm the importance of both risk drivers to measuring systemic risk.

**Keywords:** network topology, capital buffer, systemic risk, decomposition, risk driver.

**JEL Classification:** G01, G21, G28, C63.

---

\* Research Department, Banco Central do Brasil, e-mail: joao.barroso@bcbr.gov.br

\*\* Research Department, Banco Central do Brasil, e-mail: thiago.silva@bcbr.gov.br

\*\*\* Research Department, Banco Central do Brasil, e-mail: sergio.souza@bcbr.gov.br

# 1 Introduction

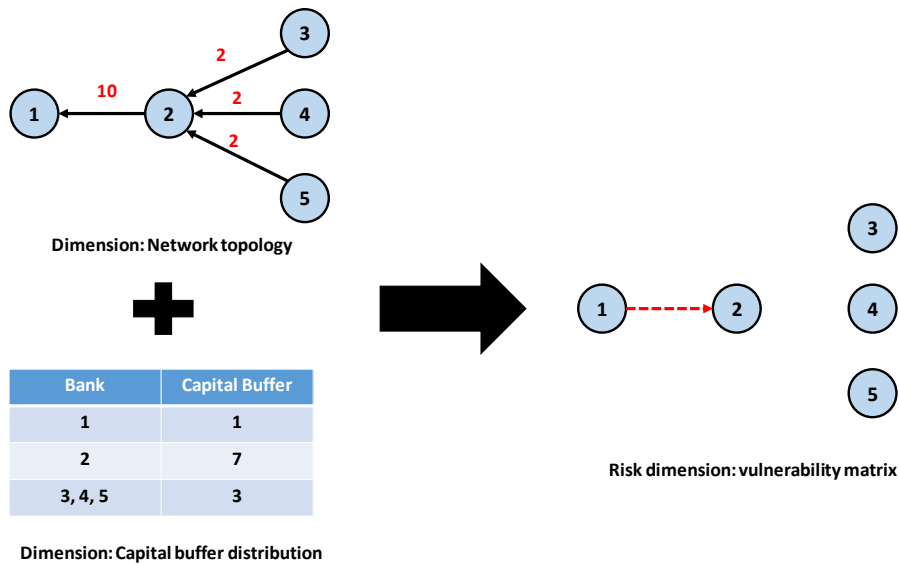
The proper monitoring of real financial networks requires the identification of the systemic risk drivers. The candidate drivers of this type of risk are network topology, capital buffers, market liquidity and the nature of shocks (Gai and Kapadia (2010); Haldane and May (2011); Loepfe et al. (2013); Roukny et al. (2013)). Yet, there is no methodology in the literature to identify the historical effects of such driving factors in actual evolving financial networks. This paper is the first one to propose such a methodology.

It is intuitive that the topology of the gross exposure network, allocation of capital buffers among network participants, current market liquidity and the nature and extent of shocks interact to create financial stability risks. The effect of capital buffers is the most evident. Sufficiently high capital buffers protect otherwise fragile networks from contagion risks, while sufficiently low buffers fail in preventing contagion even in robust networks. Capital buffers are thus a barrier against unexpected losses and should intuitively have an inverse relationship with systemic risk levels. The impact of the network topology is more subtle and is usually the subject of theoretical investigations that highlight specific features of this topology. For example, the relative completeness of the network may either contribute to risk diversification (Acemoglu et al. (2015); Allen and Gale (2000)), or facilitate propagation in the presence of amplification mechanisms such as asset fire sales and deposit runs (Battiston et al. (2012a)).

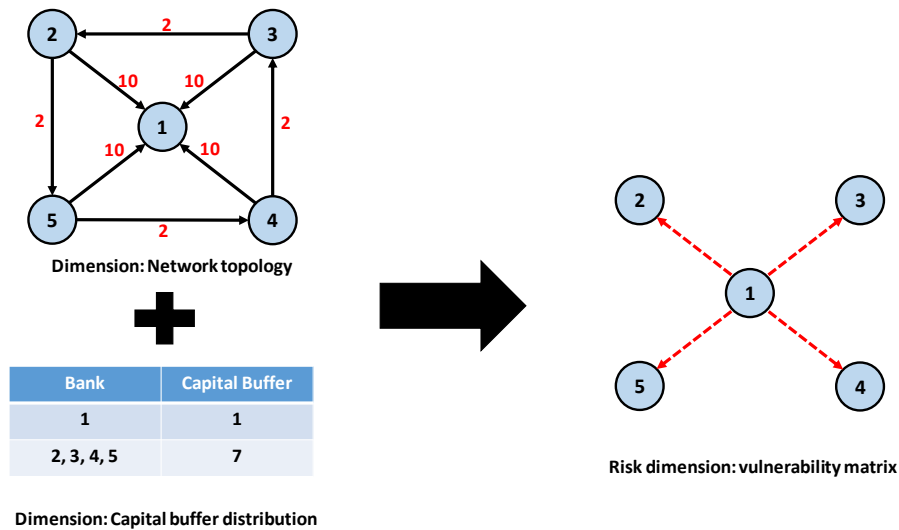
We focus on the risk drivers “network topology” and “capital buffer,” to which academic and policy works often refer as being contagion transmission channels or systemic risk factors. On the one hand, the network contagion literature tends to emphasize the influence of the topology of the exposures network. On the other hand, policy works tend to discuss restrictions over the capital buffers of financial institutions. Systemic risk is usually a complex function of both risk drivers and thus isolating the contributive effects of each risk driver to systemic risk is not a trivial task.

To illustrate the degree of complexity in which these risk drivers can interact, we can have well-known fragile network topologies, such as the star network, and still have a robust network in the sense of financial contagion. To see that, if the financial institution that plays the role of hub has a strong capital buffer, then it will essentially transform the network into a contagion-free system. Figure 1a illustrates this idea. Note that the star topology in the gross exposure network leads to an almost empty graph in the risk domain. In contrast, we see an opposite view in Fig. 1b, in which we have an exposure network that is apparently robust. However, if we take into account the capital buffer distribution, we get a star network in the risk domain, which is a higher risk-generating network disposition because the default of the hub can lead the entire system into default as well. This is because capital buffers interact in highly nonlinear ways with the network topology. Our

methodology tries to disentangle the intertwined effects of these risk drivers.



(a) Star topology of the exposure network leading (apparently fragile) to an almost empty topology of the vulnerability network (robust)



(b) Quasi-complete topology the of exposure network (apparently robust) leading to a star topology of the vulnerability network (fragile)

**Figure 1:** Schematic of how different combinations of network topology and capital buffer distribution can lead to different structural network organizations in the risk domain (vulnerability network). We construct the network topology using the gross exposure matrix, meaning that a link from bank  $i$  to  $j$  exists if  $i$  is exposed to  $j$ . The capital buffer list shows hypothetical capital buffer values for each of the banks in the network. Combining both components, we can construct the vulnerability network, which shows the potential contagion paths in the network. Most risk-related network measurements use this network to infer contagion risks. In this network, there is a link from bank  $i$  to  $j$  whenever the default of  $i$  leads  $j$  into default as well.

We propose a methodology to identify and isolate these effects that is applicable to systemic risk measures based on a vulnerability matrix, that is, the matrix represent-

ing cross exposures normalized by capital buffers (Souza et al. (2016)). Among these measures, we cite the computation of shock-related losses using the Eisenberg and Noe (2001)'s clearing algorithm, the DebtRank measure (Battiston et al. (2012b)) and its variations, for instance, the DebtRank with cycles measure (Bardoscia et al. (2015)), and the impact susceptibility and weighted impact diffusion influence measures, recently proposed by Silva et al. (2015). Vulnerability matrices depend on two risk drivers: network topology and capital buffer. To compute vulnerability matrix-based systemic risk measures it is enough to characterize a given financial system as the network of cross exposures of its components along with their capital buffer.

Our methodology aims at explaining variations of systemic risk measures that are evaluated over two network snapshots in terms of contributions of the capital buffer and network topology risk drivers. Fundamentally, the methodology takes as input the initial network snapshot and applies operations that gradually transform it into a reference network. These transformations operate exclusively either on the network topology or on the capital buffer risk driver. We define them in such a way that if we apply all of them consecutively, we obtain a financial network with the same vulnerability matrix as the reference network and thus the systemic risk estimation coincides. We show that our framework is robust to different orderings of these transformations. Given that there are several paths that can lead one initial network to a reference network, we define the capital buffer and network topology effects as the average values over all these possible paths. This approach allows us to summarize the effects of the nonlinear interactions between the risk drivers on which the systemic risk measures depend. The variability of the effects related to these different transformation paths is, in principle, lower for more similar networks.<sup>1</sup> Thus, we focus on comparing adjacent network snapshots in time during our empirical application.

We then apply the methodology to explain the systemic risk dynamics of the global banking network in the years surrounding the global financial crisis. Among the eligible systemic risk measures proposed in the literature, we select the impact susceptibility and impact diffusion influence that offer complementary views on systemic risk (Silva et al. (2015)). While the impact susceptibility measures how easy is that defaults inside the network reach and lead a particular country into default, the impact diffusion influence measures the financial system additional losses that arise from the exposures to defaults from a particular country. These are important measures because they focus on catastrophic events, i.e., they assume that defaulting economic agents do not make any debt repayment and yield non-zero results only if the default of an agent leads other agent(s) into default. For robustness, we also run simulations using DebtRank (Battiston et al.

---

<sup>1</sup>Except near boundary values that trigger jumps in the behavior of the network, which is likely to occur given that networks define complex systems.

(2012b)), which is a systemic risk measure that is also sensitive to the propagation of lower-level distress among economic agents.

We apply our methodology to the BIS CBS dataset, which provides information on gross cross-border exposures between countries. We find that both network topology and capital buffers risk drivers are relevant for the assessment of systemic risk buildup in the global banking network, confirming the importance of both factors. We find that network topology explains most of the volatility of contagion risk, including the large buildup in risk in the years preceding the global financial crisis. Moreover, we see that capital buffer can explain a persistent reduction in contagion risk, with effects concentrated around the global financial crisis.

We also identify countries that are sources or bearers of risk in the global banking network. Identifying risk bearers and sources in the global banking system is an important task as regulators may better outline cross-border risks that can impact their domestic market and hence possibly cause financial instability. We find that the UK and the USA are the countries bearing most of the cross-border risk. Additionally, they are risk sources that are able to inflict large losses on the global banking network.

Our results could be helpful in guiding policy making and modeling of contagion risk. While the network contagion literature tends to emphasize the influence of the network topology, policy works tend to discuss restrictions over the capital buffers of financial institutions. We find empirical evidence that justifies the emphasis on both factors when we deal with systemic risk, and hence, financial stability. Based on our results, we find that the large volatility of the network effects in the global banking network justifies the emphasis of the theoretical literature on the topological features of financial networks. In the literature related to capital buffer regulation, we find that the persistent effect of capital buffers justifies the emphasis this factor has received in policy circles.

A strong product of our decomposition methodology is that it allows the construction of counterfactual exercises in which we “hold fixed” all risk drivers except one in a set of evolving financial networks.<sup>2</sup> In this case, any remaining dynamics in the systemic risk estimates are due to variations on that single risk driver. Therefore, our framework may be useful to understand and quantify the impact and effectiveness of policies that aims at changing a single risk driver, because we can estimate the systemic risk change as if that policy requirement did not exist by systematically designing these counterfactual scenarios.

We also extend the model by showing how to compute the effect of changing network features in specific groups of vertices that form a partition of the network. For example, a partition into core-periphery vertices, into domestic and foreign vertices, or into

---

<sup>2</sup>We term “fixed” in the sense of being taken from a fixed reference network while the other comes from the network while it evolves along time

well-defined communities may be relevant in specific applications. This property may be useful for monitoring the financial system, as the methodology can identify contributions of specific banks or of communities to systemic risk as they change their exposures or capital buffers.

The remainder of the paper proceeds as follows. Section 2 presents the decomposition methodology of risk drivers in evolving financial networks. Section 3 applies our methodology to the global banking system data maintained by the BIS. Finally, Section 4 concludes the paper.

## 2 Methodology

The main goal of the paper is to explain the differences in risk measures over evolving financial networks using the network structure and the capital buffer as risk drivers. Let  $\mathbf{A} = (a_{ij})$  be the gross exposure matrix and  $\mathbf{E} = (e_i)$  be the strictly positive capital buffer vector, where  $i \in \mathcal{V}$ ,  $ij \in \mathcal{V} \times \mathcal{V}$ , and  $\mathcal{V} = \{1, 2, \dots, N\}$  is the vertex index set. Let  $\mathcal{N}_{\mathcal{V}^2 \times \mathcal{V}}$  denote the space of financial networks, that is, the space of pairs  $(\mathbf{A}, \mathbf{E})$ . Also denote  $v : (\mathbf{A}, \mathbf{E}) \rightarrow \mathbf{A}/\mathbf{E} = (a_{ij}/e_i)$ , with domain on financial networks and counter-domain on what we call vulnerability networks. We consider risk measurements  $m$  that depend only on the vulnerability network, i.e.,  $(\mathbf{A}, \mathbf{E}) \rightarrow m = m(\mathbf{A}/\mathbf{E})$ .

Our strategy is to define partial transformations that take an initial network in the direction of a reference network by varying one risk driver at a time. The idea is then to apply these partial transformations consecutively so that we arrive at the reference network by performing operations on each of these risk drivers at a time, so that we can decompose variations of the risk measures  $m$  that are due to each of these risk drivers. In these transformations, we first consider operators that work homogeneously on all of the network vertices in the next proposition. Later, we introduce operators that work on partitions of vertices.

**Definition 1.** Let  $(\mathbf{A}, \mathbf{E}), (\mathbf{A}', \mathbf{E}') \in \mathcal{N}$  be an initial and a reference financial network, respectively. Define the transformations:

$$t : (\mathbf{A}, \mathbf{E}) \rightarrow (t\mathbf{A}, \mathbf{E}) : t(a_{ij}) \rightarrow \frac{\sum_{\mathcal{V} \times \mathcal{V}} a_{ij} a'_{ij}}{\sum_{\mathcal{V} \times \mathcal{V}} a'_{ij}} a'_{ij} \quad (1)$$

$$r : (\mathbf{A}, \mathbf{E}) \rightarrow (\mathbf{A}, r_{\mathbf{A}}\mathbf{E}) : r_{\mathbf{A}}(e_i) \rightarrow \frac{\sum_{\mathcal{V} \times \mathcal{V}} a_{ij} e'_i}{\sum_{\mathcal{V} \times \mathcal{V}} a'_{ij}} e'_i \quad (2)$$

When convenient, we can make the reference network explicit in the notation, and write  $t_{(\mathbf{A}', \mathbf{E}')}(\cdot)$  and  $r_{(\mathbf{A}', \mathbf{E}')}(\cdot)$ . In particular, note that  $t_{(\mathbf{A}, \mathbf{E})} \circ t_{(\mathbf{A}', \mathbf{E}')} \mathbf{A} = \mathbf{A}$  and  $r_{(\mathbf{A}, \mathbf{E})} \circ$

$r_{(A', E')} \mathbf{E} = \mathbf{E}$ . Therefore, when investigating the drivers of risk differences between two network configurations, either one can be taken as the reference.

What do these transformations mean? Intuitively, they impose “counterfactual scenarios” that exactly replicate features of the reference network except for the sum of network assets in the exposure matrix. Each transformation operates on a different feature, either the gross exposure matrix, in the case of  $t(\cdot)$ , or the capital buffer distribution, in the case of  $r(\cdot)$ . In both cases, the scaling factor makes sure the proportionality between the capital buffer vector and the sum of network assets is not affected by the transformation.

Suppose we apply one of the transformations to an initial financial network. Since we are in fact changing a single feature of the initial network in the direction of the reference network, any corresponding variation of the risk measure is attributable to changes in that feature. That is, if we change the network structure with  $t(\cdot)$ , then we have a candidate measure for the effect of network structure. However, if we change the capital buffer and only then the network structure, we have another candidate that may differ. The difficulty then is dealing with the order in which transformations are applied and possible nonlinear interactions between network features. The proposition below shows that the order in which the transformations are applied does not matter. They all lead to the same overall effect on the risk measure, as long as they are defined on the vulnerability network.

**Proposition 1.** *Consecutive applications of the transformations  $t$  and  $r$  in Definition 1 on the initial network  $(\mathbf{A}, \mathbf{E})$  lead to the reference network  $(\mathbf{A}', \mathbf{E}')$ , irrespective to the applied order. Mathematically,*

$$v \circ (t \circ r) = v \circ (r \circ t) = \frac{\mathbf{A}'}{\mathbf{E}'} \quad (3)$$

holds for  $\forall (\mathbf{A}, \mathbf{E}), (\mathbf{A}', \mathbf{E}') \in \mathcal{N}$ .

*Proof.* Note that  $r \circ t(\mathbf{A}, \mathbf{E}) = (t\mathbf{A}, r_{t\mathbf{A}}\mathbf{E})$ , with:

$$r_{t\mathbf{A}}\mathbf{E} = \frac{\sum_{\mathcal{Y} \times \mathcal{Y}} t(a_{ij})}{\sum_{\mathcal{Y} \times \mathcal{Y}} a'_{ij}} e'_i, \quad (4)$$

but observe that:

$$\begin{aligned} \sum_{\mathcal{Y} \times \mathcal{Y}} t(a_{ij}) &= \frac{\sum_{\mathcal{Y} \times \mathcal{Y}} a_{ij}}{\sum_{\mathcal{Y} \times \mathcal{Y}} a'_{ij}} \sum_{\mathcal{Y} \times \mathcal{Y}} a'_{ij} \\ &= \sum_{\mathcal{Y} \times \mathcal{Y}} a_{ij}. \end{aligned} \quad (5)$$

Then, substituting (5) back into (4) results in:

$$r_{tA}E = \frac{\sum_{\mathcal{V} \times \mathcal{V}} a_{ij} e'_i}{\sum_{\mathcal{V} \times \mathcal{V}} a'_{ij}}. \quad (6)$$

Therefore,  $tA/r_{tA}E = A'/E'$ . Similarly,  $t \circ r(A, E) = (tEA, r_A E)$ , and, clearly  $tEA/r_A E = A'/E'$ . ■

In the case of two risk drivers, there are only two different trajectories that lead from  $(A, E)$  to  $(A', E')$  via partial transformations on each of the risk drivers. In a slight abuse of notation, let  $m(A', \cdot)$  denote  $m(tA, \cdot)$  and  $m(\cdot, E')$  denote  $m(\cdot, r_A E)$ . The possible trajectories are:

$$\begin{aligned} m(A, E) &\xleftrightarrow{A \text{ effect}} m(A', E) \xleftrightarrow{E \text{ effect}} m(A', E') \\ m(A, E) &\xleftrightarrow{E \text{ effect}} m(A, E') \xleftrightarrow{A \text{ effect}} m(A', E') \end{aligned} \quad (7)$$

Given that the order of the transformations does not matter, we can define the effect of network structure and capital buffer as their average effect over all possible different trajectories.

**Definition 2.** *The effect of the network topology on risk measure  $m$  when the financial network changes from  $(A, E)$  to  $(A', E')$  is:*

$$\Delta_A m = \frac{1}{2} [(m(A', E) - m(A, E)) + (m(A', E') - m(A, E'))], \quad (8)$$

and the effect of the capital buffer on risk measure  $m$  is:

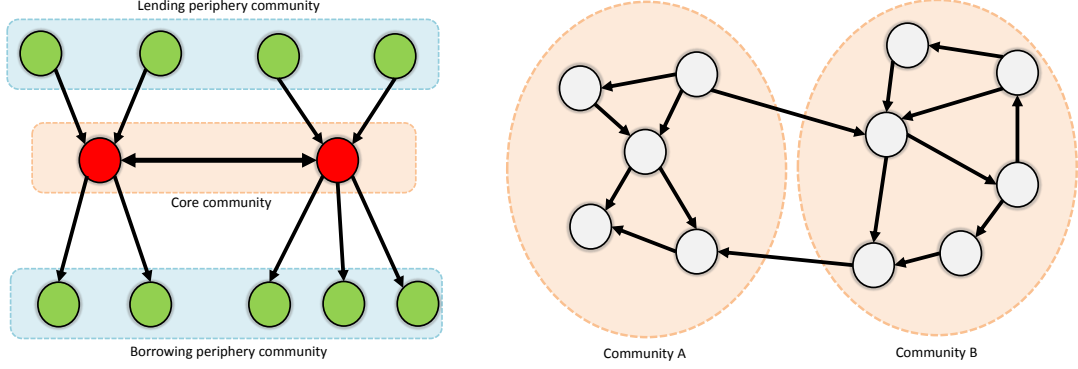
$$\Delta_E m = \frac{1}{2} [(m(A, E') - m(A, E)) + (m(A', E') - m(A', E))]. \quad (9)$$

Note that we completely characterize the variation of  $m$  from the initial to the reference network as Equation (10), once it is the sum of Equations (8) and (9):

$$m(A', E') - m(A, E) = \Delta_A m + \Delta_E m. \quad (10)$$

We now introduce partitions on vertices and, in the process, generalize the decomposition methodology to consider each of these partitions as a risk-driving feature. This

generalized methodology is particularly useful in the assessment of the risk contribution of changes in conditions (network topology and capitalization) of groups of vertices. These partitions can arise in networks on several conditions. For instance, Figures 2a and 2b show possible examples of network partitions in a core-periphery network and in a network with well-defined communities.



(a) Decomposition by core-periphery partitions (b) Decomposition by community partitions

**Figure 2:** Examples of decomposition strategies that operate on partition of vertices for different network topologies.

The intuition is that the transformations introduced before are linear in the input parameters. Linear transformations are additive in partial transformations that are taken on subsets of network members that form a partition. We may also refer to these subsets as different sectors of the financial network.

We note that although the transformations are linear, the decomposition itself is non-linear as the systemic risk measures decomposed are also non-linear.

**Definition 3.** Let  $(\mathbf{A}', \mathbf{E}') \in \mathcal{N}$ . Let  $\mathcal{V} = \cup_{k=1}^n \mathcal{V}_k$  denote a partition, that is a disjoint union of non-empty subsets in such a way that each of the elements is included in exactly one of the subsets. Define the transformations:

$$t_k : (\mathbf{A}, \mathbf{E}) \in \mathcal{N} \mapsto (t_k \mathbf{A}, \mathbf{E}) : t_k(a_{ij}) \mapsto \begin{cases} \frac{\sum_{(i,j) \in \mathcal{V}_k \times \mathcal{V}} a_{ij}}{\sum_{(i,j) \in \mathcal{V}_k \times \mathcal{V}} a'_{ij}} a'_{ij}, & (i, j) \in \mathcal{V}_k \times \mathcal{V} \\ a_{ij}, & (i, j) \notin \mathcal{V}_k \times \mathcal{V} \end{cases}, \quad (11)$$

$$u_k : (\mathbf{A}, \mathbf{E}) \in \mathcal{N} \mapsto (\mathbf{A}, u_{A,k} \mathbf{E}) : u_{A,k}(e_i) \mapsto \begin{cases} \frac{\sum_{(i,j) \in \mathcal{V}_k \times \mathcal{V}} a_{ij}}{\sum_{(i,j) \in \mathcal{V}_k \times \mathcal{V}} a'_{ij}} e'_i, & (i, j) \in \mathcal{V}_k \\ e_i, & (i, j) \notin \mathcal{V}_k \end{cases}. \quad (12)$$

where  $k \in \{1, \dots, n\}$ .

As before, each transformation impose “counterfactual scenarios” that exactly replicate features of a sector of the reference network, scaling them down to the level of

the total assets of the sector in the initial network. The scaling factor makes sure the proportionality between the capital buffer vector and the sum of network assets inside the sector in the reference network is not affected by the transformation. As before,  $t_{(\mathbf{A}, \mathbf{E}), k} \circ t_{(\mathbf{A}', \mathbf{E}'), k} \mathbf{A} = \mathbf{A}$  and  $u_{(\mathbf{A}, \mathbf{E}), k} \circ u_{(\mathbf{A}', \mathbf{E}'), k} \mathbf{E} = \mathbf{E}$ , in any order, since each transformation  $k$  operates on separate nodes. Moreover, the order in which the transformations are applied is irrelevant for the overall impact on a risk indicator that depends only on the vulnerability network.

**Proposition 2.** *The consecutive application of the partitioned partial transformations  $t_k$  and  $u_k$ :*

$$v \left( \bigcirc_{k=1}^N p_{(\mathbf{A}', \mathbf{E}'), k}(\mathbf{A}, \mathbf{E}) \right) = \frac{\mathbf{A}'}{\mathbf{E}'} \quad (13)$$

in which  $p_{(\mathbf{A}', \mathbf{E}'), k}(\mathbf{A}, \mathbf{E}) = t_{(\mathbf{A}', \mathbf{E}'), k} \circ u_{(\mathbf{A}', \mathbf{E}'), k}(\mathbf{A}, \mathbf{E}) = u_{(\mathbf{A}', \mathbf{E}'), k} \circ t_{(\mathbf{A}', \mathbf{E}'), k}(\mathbf{A}, \mathbf{E})$ , lead the initial network  $(\mathbf{A}, \mathbf{E})$  to the reference network  $(\mathbf{A}', \mathbf{E}')$ .

*Proof.* Each transformation  $t_k$  and  $u_k$  affects only the exposures  $a_{ij}$  and capital buffers  $e_i$  with  $i \in \mathcal{V}_k$ . Therefore, they operate on subsets or sectors of  $(\mathbf{A}, \mathbf{E})$ . The order of operations across subsets is irrelevant. The order of within subsets is also irrelevant by Proposition 1. The result follows. ■

**Remark 1.** *A similar result applies replacing  $\mathcal{V}_k \times \mathcal{V}$  by  $\mathcal{V} \times \mathcal{V}_k$ . In this alternative definition, we transform at each step the structure of links reaching vertices, while in the baseline we transform the structure of links leaving vertices.*

**Remark 2.** *A similar result applies as well changing  $\mathcal{V}$  by any of its subsets in the product  $\mathcal{V} \times \mathcal{V}_k$ . In this case, we are pivoting the transformation on this set of vertices.*

**Remark 3.** *The result does not apply when letting  $\mathcal{V} = \bigcup_{u=1}^K \mathcal{J}_u$  and defining transformations in  $\mathcal{V}_k \times \mathcal{J}_u$  because  $u_{\mathbf{A}, k}$  cannot be defined for all  $\mathcal{J}_u$ . Therefore, we cannot capture the effect of network structure within a group, only of the group with respect to other pivot group fixed in the decomposition.*

Here, we are using the same transformations that we defined previously to affect only the exposures  $a_{ij}$  or the capital buffer  $e_i$ . However, we apply them jointly using the transformations  $p_k$  aimed to affect only sector  $k$ . Above, we defined the individual effects of network structure and capital buffer as the average of their individual effects over all possible orderings of the transformations (only two orderings, in that case). We will apply the same principle to compute the effects of changes of exposures and capital buffers occurring within groups of vertices in partitions  $k$  over all possible orderings of

transformations  $p_k$ , which will require some combinatorial arguments, that we present in Appendix A.

Besides effect decomposition, the transformations may be used to isolate the effect of a particular feature, on a particular subset of nodes. As suggested in the introduction, we must keep all the remaining features fixed and transform the particular feature based on a reference financial network. Any difference among risk indicators of networks similarly transformed can in principle be attributable to changes in that isolated feature.

### 3 Application: Global banking system

In this section, we apply our decomposition methodology to the global banking system data maintained by the BIS. We start by introducing particularities about the employed data. Then, we discuss the risk-related network measurements that we use in our decomposition methodology. Finally, we report and discuss results we obtain using our methodology.

In the previous section, we have used the terms gross exposure matrix, vulnerability matrix, and capital buffer in an abstract manner. We now formally define them for our study case of the global banking system.

We represent the global banking network as a graph  $G = \langle \mathcal{V}, \mathcal{E} \rangle$  that we construct using international borrowing and lending relationships between countries that report cross-border banking statistics to the BIS. To build up such network, we represent the set of reporting countries  $\mathcal{V}$  as the vertices and the set of edges  $\mathcal{E}$  as the active operations. The matrix  $\mathbf{A} = (a_{ij}), ij \in \mathcal{V} \times \mathcal{V}$ , represents the gross exposure matrix (weighted adjacency matrix). In our analysis, there is no netting between countries  $i$  and  $j$ . As such, if an arbitrary pair of countries owe to each other, then two directed independent edges linking each other in opposed directions will emerge. An interesting property of maintaining the gross exposures in the network is that, if a country defaults, its debtors remain liable for their debts.

In terms of risk analysis, the gross exposure matrix  $\mathbf{A}$  is not a representative candidate for evaluating possible contagion routes among pairs of countries, as it does not convey the notion of countries' capability of absorbing impacts coming from their direct exposures. Rather, the exposure matrix  $\mathbf{A}$  only quantifies the pairwise exposures numerically, in an absolute manner. This is the main reason risk-related network measurements rely on the vulnerability network, which accounts for that shortcoming. The vulnerability of an institution  $j$  to defaults of an institution  $i$  is expressed as:

$$\mathbf{V}_{ij} \triangleq \begin{cases} 1, & \text{if } \frac{a_{ji}}{e_j} \geq 1, \\ 0, & \text{otherwise.} \end{cases} \quad (14)$$

in which  $a_{ij}$  represents the exposure of  $i$  to  $j$  and  $e_j$  is the available capital buffer of institution  $j$ .

Observe that  $\mathbf{V}_{ij} = 1$  only when country  $j$ 's exposure towards  $i$  surpasses its capital buffer. This configuration illustrates the situation in which the default of  $i$  can lead  $j$  into default as well. Conversely,  $\mathbf{V}_{ij} = 0$  whenever  $j$  can absorb the losses in case  $i$  defaults. Note now that the graph topology represented by the vulnerability matrix  $\mathbf{V}$  quantifies the possible contagion paths delineated by the current conditions and exposures of the global financial network participants.

### 3.1 Data

We use data on cross-border exposures of banks from the *Consolidated Banking Statistics* data set (CBS) that is publicly maintained by the BIS. The CBS holds data on consolidated positions of banks' worldwide offices, including the positions of banks' foreign subsidiaries and branches, but excluding inter-office activity. Such data are provided on a quarterly basis to the BIS by central banks and other official authorities, who collect them from individual consolidated banks headquartered in their jurisdiction. The institutions considered as banks in each country include commercial banks, savings banks, savings and loan associations, credit unions or cooperatives, building societies and post office savings banks or other government-controlled savings banks, but not central banks.

The CBS data set presents pairwise exposures data compiled according to the following criteria:

- *Immediate borrower basis*: Claims are allocated to the country of residence of the immediate counterparty. The data cover financial claims, risk transfers and certain liabilities reported by banks headquartered in the reporting country as well as selected affiliates of foreign banks.
- *Ultimate risk basis*: Claims are allocated to the country where the final risk lies. The data cover on-balance sheet claims as well as some off-balance sheet exposures of banks headquartered in the reporting country and provide a measure of country credit risk exposures related to banks' own risk management systems.

In this paper, we opt to use the immediate borrower basis perspective for conducting the contagion risk analysis. That choice is not perfectly accurate as in cases that there is

risk transfer, the risk exposures change from creditor-debtor to creditor-guarantor-debtor. In the case of the ultimate risk basis perspective, in which creditors are represented as exposed to guarantors, if any guarantor defaults, it is not liable for the guarantee if the debtor does not default. We choose the immediate borrower basis perspective, as the origin and destination of risk propagation paths are correctly represented for all exposures. In this perspective, if an exposure has a guarantor, the links that would divide the risk path from the debtor to the creditor are not represented within the network. However, the origin and the destination of the risk path would remain correct in the immediate borrower basis perspective.

There are 26 reporting countries that supply data to the BIS CBS, which are:

- *America*: USA, Canada, Brazil, Mexico, Chile, Panama;
- *Europe*: UK, Germany, France, Belgium, Austria, Denmark, Finland, Netherlands, Greece, Ireland, Italy, Portugal, Spain, Sweden, Switzerland;
- *Asia*: Japan, South Korea, Taiwan, Turkey; and
- *Oceania*: Australia.

In this paper, we use the same data and approach used in a previous work by Silva et al. (2016), to which we refer for further detail. Here, we investigate the evolution of this global banking network from 2005 to 2014. We analyze, for each date, the network of banks' cross-border exposures that are aggregated by country. We intend to assess banks' exposures to other countries in a comprehensive way by considering banks' exposures to banks, non-banks and to the public sector of other countries.

To compute risk-related network measurements, we would ideally need to have counterparty-level data (banks, private non-bank sector, public sector) of each country. The BIS CBS data set provides an aggregation of this information to a country-level, reporting exposures from the banks' point-of-view, corresponding to liabilities from any sector of the counterparty countries. The database does not inform non-banking sector exposures, only their liabilities, therefore, we aggregate the (partial) information we have to a country level, disregarding the economy sectors to which the data set information refers to. We present below the assumptions we need to make for representing individual countries as single banks. This approach enables us to compute a vulnerability matrix for the network, which is required by the methodologies we employ in this paper.

We start representing the global financial market as a hierarchical network with two levels. We consider the aggregate exposures of countries in the upper level, while we unfold each of these countries in their respective financial sector, and public and private non-financial sectors in the bottom level. We perform the evaluation of network-based

measures on the country-level network, i.e., the upper level perspective. By aggregating the financial sector, and public and private non-financial sectors into a single network vertex, we assume the following simplifications:

1. Shocks that originate from the non-payment of international obligations normally affect individual banks in the financial sector of a country, i.e., those that hold direct exposures to the defaulted debtor entities. However, due to the lack of data on domestic and international bank-level exposures, we aggregate the international assets and liabilities of banks of the same country's financial sector as a single representative bank, which is the vertex that represents the country in the upper level. We understand that the most representative large banks in the same sector normally have the same capitalization pattern, suggesting that shocks affect these banks in a uniform manner.
2. Due to external shocks, the financial sector may affect the private and public non-financial sectors. The aggregation of these sectors in a single representative country-level vertex implicitly supposes that the reactions of these sectors are transmitted to international creditors in a simultaneous manner.
3. The non-financial sectors depend on the financial sector to obtain credit, maturity transformation, and financial services. If the financial sector collapses due to a financial crisis, we expect a sudden increase in the default rates of the private non-financial sector. In contrast, the public non-financial sector may still honor its liabilities in a first moment, but that scenario may change in case the crisis lingers. In our model, we suppose that, once the financial sector collapses, both the private and public non-financial sectors collapse as well. This assumption yields very conservative results.

The three simplifications above only affect the evaluation of risk-related network measures. In this way, network structure measures are independent of such assumptions as they deal exclusively with the bilateral exposures between countries in the network.

In this way, we assume a representative bank for each country in the sample. To estimate the country's loss absorbing capabilities, we use the total Tier 1 capital of the country's largest banks.<sup>3</sup> We extract Tier 1 capital data of these representative financial institutions from Bureau Van Dijk's Bankscope and sum them up to obtain the aggregate Tier 1 capital at the country level on a yearly basis.<sup>4</sup> We preferably use consolidated accounting statements of all reporting financial institutions in Bankscope in these calculations. If a

---

<sup>3</sup>We have checked that other measures that could proxy countries' loss absorbing capabilities either are unavailable or are inconsistent with strong structural breaks inside the period that we perform the network analysis. The Tier 1 Capital measure, however, shows consistence and completeness during the period.

<sup>4</sup>We exclude central banks when estimating countries' Tier 1 capital.

consolidated statement is not available, then we use an unconsolidated/aggregate accounting statement, whatever is available. Similarly, if accounting statements are available on both IFRS (International Financial Reporting Standards) and Local GAAP (Generally Accepted Accounting Principles) reporting conventions, then we adopt the first convention.

We assume two hypotheses when we sum up the Tier 1 capital of the largest financial institutions to compose the country's capital buffer:

1. The first one is that cross-border exposures are only present in the largest financial institutions. That is, medium- and small-sized financial institutions do not hold foreign claims. In this way, we assume that the international banking market is a two-tier structure, in which only large financial institutions operate across borders in the interbank market and act as money centers for smaller domestic banks. This is consistent with evidences found by Gropp et al. (2009), who show that small financial institutions neither cause nor suffer from cross-border contagion events, irrespective to the fact that all institutions are equally prone to experience domestic contagion.
2. The second one is that the largest financial institutions of a same country share the country's proxy when they absorb cross-border losses. This is a strong assumption that we make because we do not have information on bank-level cross-border exposures. Rather, we only have country-level information.

Figure 3a presents the evolution of the total cross-border exposures (investments) of reporting countries' banks. In this chart, we only report exposures between pairs of reporting countries. The BIS CBS data set also presents information on exposures to non-reporting countries. In these operations, however, reporting countries are always investors in counterparties that may be either reporting or non-reporting countries. Assessing the representativeness of the network composed of only reporting countries, Silva et al. (2016) report that from 2006 to 2014, the share of exposures to reporting countries compared to total exposures decreased from 0.79 to 0.66.

In Figure 3a, we see a significant increase in total exposures from the beginning to March 2008 followed by a sharp decrease from March 2008 to June 2009 that is coherent with the volume of assets related to the growth and burst of the property market bubble, occurred mostly in the United States. Silva et al. (2016) state that along that period European countries kept about 60% of their cross-border investments within Europe and that most of the remaining 40% were directed to the USA. This explains the reason why in Figure 3b the volume of cross-border investments of the European countries group experiences the same increase and decrease from the beginning to June 2009.

Finally, Silva et al. (2016) analyze the average cross-border amounts per group reporting that the average European country is equivalent to the USA in the investment

perspective by December 2008, but that from that date to September 2012, the USA invested more than the average European country. They propose that the increases in the USA cross-border investments possibly reflect the country's quantitative easing programs QE 1 and QE 2.

Figures 3c and 3d portray the total and average estimated countries' capital buffers, respectively, for the studied period. We use the same groups of countries previously defined for better readability. We observe that, from the beginning of our sample until March 2009, capital buffers in European Countries and the USA roughly double, increasing even more afterwards. Latin America + Canada and Oceania + Asia also experience even steeper increases along the first period followed by stabilization (Latin America + Canada) or decrease (Oceania + Asia). This effect on Latin American, Canadian, Oceanian and Asian banks suggests that they have been less affected by the debt crisis outbreak in European countries than the other two country groups. The strategy of assuming higher and higher capitalization levels suggests that banks faced an increasing risk along the period of analysis.

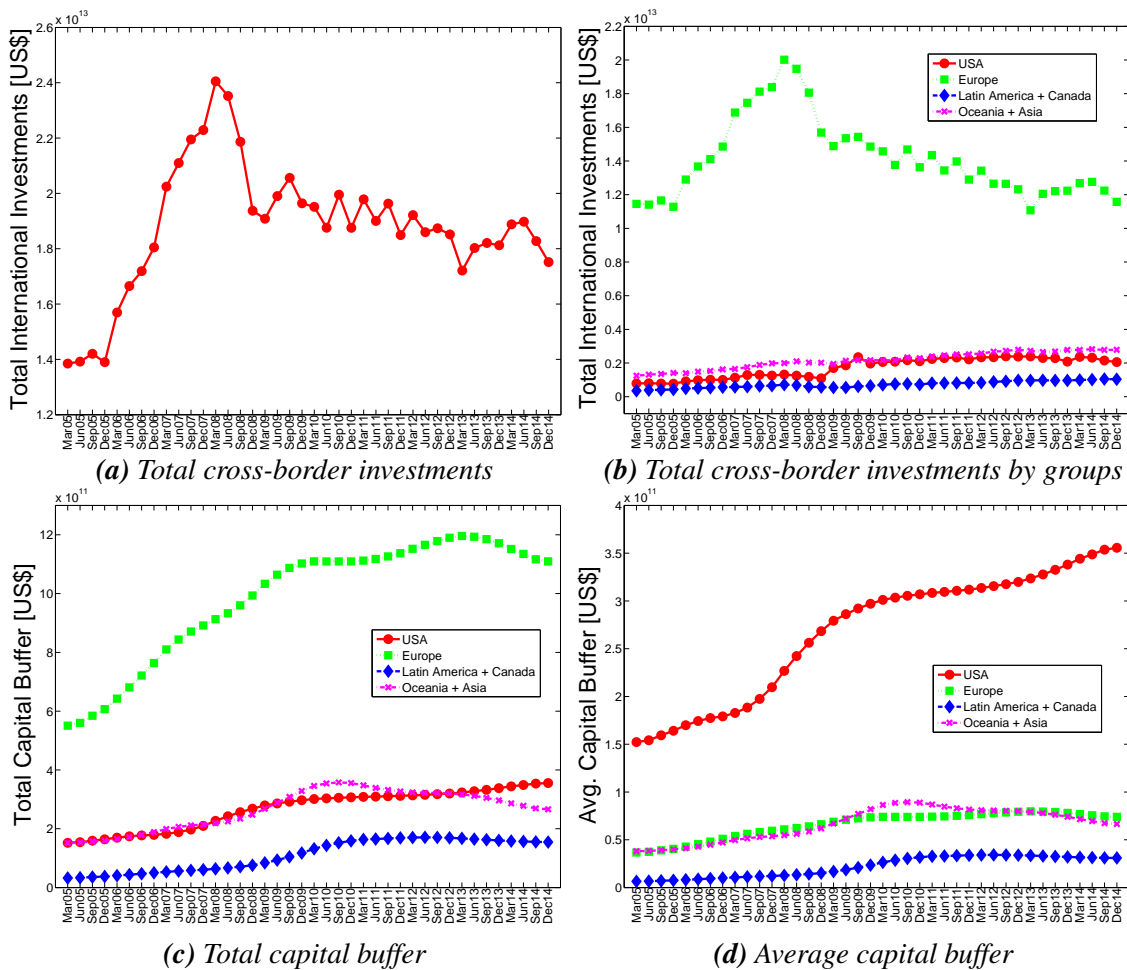


Figure 3: Cross-border investments and average estimated countries' capital buffers.

## 3.2 Risk-related network measurements

In this section, we show the network measurements we employ to extract the network structure and capital buffer effects according to our decomposition methodology. Recall that the methodology requires network measurements computed from the vulnerability matrix, rather than the gross exposure matrix. To our knowledge, all of the risk-related network measurements<sup>5</sup> proposed so far in the literature use a version of the vulnerability matrix.<sup>6</sup> Thus, our methodology is of wide applicability. The vulnerability matrix is commonly employed in risk analysis because, as opposed to the gross exposure matrix, it gives us a sense of how well economic agents can absorb losses from direct exposures to neighbors.

We start this section introducing the vulnerability network of the global banking system. Then, we analyze three recently risk-related network measurements proposed in the literature by Silva et al. (2015): impact susceptibility, network impact fluidity, and weighted impact diffusion influence. We review in detail these network measurements in Appendix B. These measures differ from the classical DebtRank in that they focus on catastrophic events, i.e., they yield non-zero results if there is a possibility of occurring a default cascade of any length. DebtRank, in turn, can be computed for a variety of initial shock scenarios and measures the stress they cause, being more related to the weighted impact diffusion influence. Thus, while the three recent measures can be associated with high level stress, DebtRank conveys the financial network stress of any magnitude, which can be useful in the identification of the build-up of stress when it is yet in a low level. The new measures identify more precisely destructive stress, therefore, these measures and DebtRank offer a complementary view of the evolution of systemic risk in financial networks. Therefore, for robustness, we present results using the DebtRank methodology.

### 3.2.1 Vulnerability network

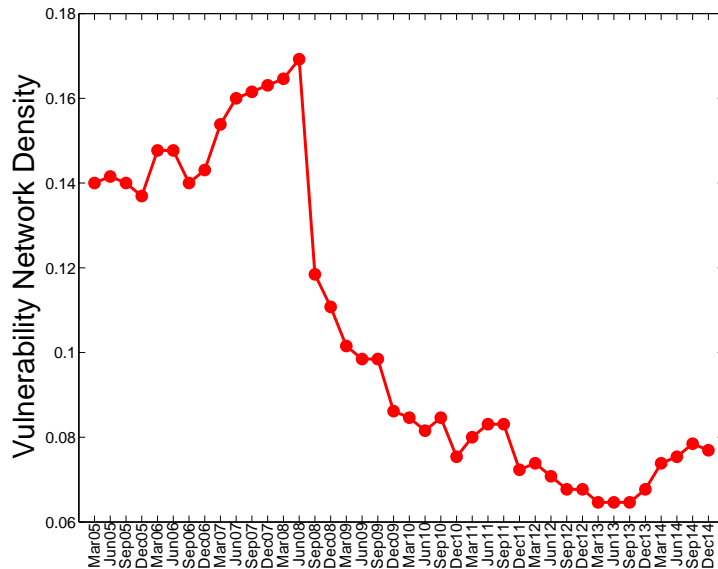
Figure 4 displays the density of the vulnerability network representing the global banking system. Despite the gross exposure network of the global banking system presents a very dense structure, the corresponding vulnerability network is perceptibly sparse. Note that, as the vulnerability network gets denser, it is an indicative of a potentially more fragile financial system, in that more exposures exist that may lead creditors of financial operations into default. We see that the network is potentially more fragile in-between 2007 and 2008, period that coincides with the outbreak of the global financial crisis. We

---

<sup>5</sup>We refer to measures that can be computed from a financial network, as defined in the beginning of Section 2.

<sup>6</sup>For instance, Battiston et al. (2012b)'s DebtRank is a well-known risk-related network measurement that makes use of a continuous version of the vulnerability matrix. So does the Eisenberg and Noe (2001) clearing algorithm, that can be used to quantify losses that stem from a given initial shock.

also observe that, from March 2005 to the onset of the global financial crisis in 2008, the trajectory of the vulnerability network density assumes a consistent upward trend. This suggests that economic agents were more willing to assume risks due to the characteristic expansionary period. After 2008, the density of the vulnerability network plunges to a quasi-plateau, fluctuating around the 7% level.

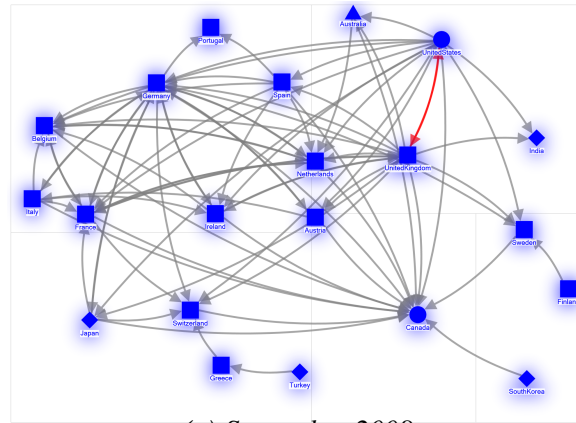


**Figure 4:** Evolution of the vulnerability network density in the international claims network constructed from the reported data taken from the BIS CBS.

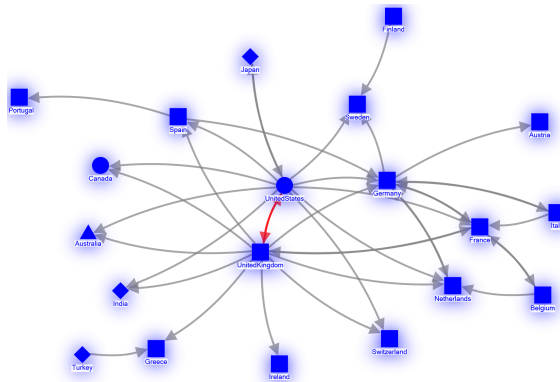
In order to understand how contagion routes are shaped from the international claims network and countries' capital buffers, Figs. 5a, 5b, and 5c depict the vulnerability network for September 2008 (verge of the global financial crisis), December 2011 (middle of the European sovereign debt crisis), and December 2014. One important feature of these three vulnerability networks is that the USA is vulnerable to UK in the three periods. Moreover, besides being the most representative players in terms of amounts of active operations, they are also vulnerable to each other. We highlight the corresponding vulnerability edge linking the USA and the UK in red. We can also observe the existence of more potential contagion paths at the verge of the global financial crisis, as the density of the vulnerability network is higher than in other periods.

### 3.2.2 Impact susceptibility

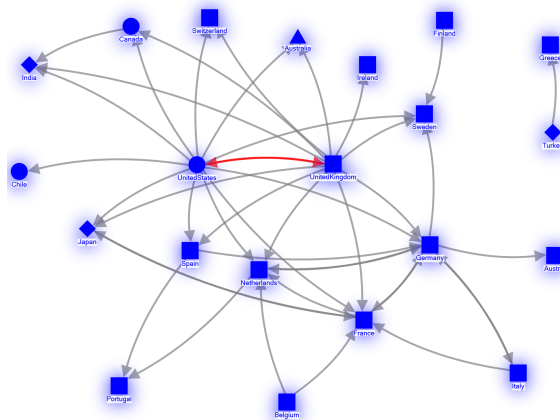
In general terms, the impact susceptibility of a country measures its likelihood of getting impacted from arbitrary shocks that occur inside the network. Appendix B.1 provides its mathematical features. The impact susceptibility is an indicator of potential fragility that takes into account not only the inherent capability of network members to absorb losses, but also accounts for the current network topology. Figure 6 shows the average impact susceptibility of country groups.



(a) September 2008



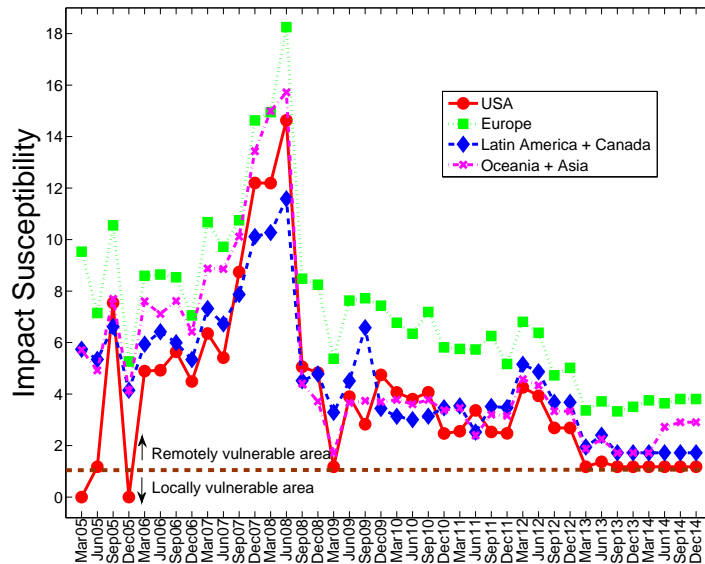
(b) December 2011



(c) December 2014

**Figure 5:** Vulnerability networks calculated from pairwise exposures between reporting countries. We highlight in red the mutual (bidirectional) vulnerability edge between the USA and UK. The vertex format corresponds to the continent to which a reporting country belongs. The circle format represents American countries, the square format indicates European countries, the diamond format depicts Asian countries, and the triangle format symbolizes Oceanian countries.

We first note that the impact susceptibility assumes its largest values, on average, in-between 2007-2008, interval that matches with the outbreak of the global financial crisis. The large values indicate a network in which members are highly susceptible of being impacted in case random shocks happen in the network. In the global crisis, we see that European countries are the most potentially susceptible to impacts, followed by Oceanian



**Figure 6:** Evolution of the average impact susceptibility of members of the international claims network constructed from the reported data taken from the BIS CBS.

and Asian countries, the USA, and finally Latin American countries and Canada. We also see that European countries shape the largest impact susceptibility values in the entire period. The USA, in contrast, has, on average, the smallest impact susceptibility values, except at the verge of the global financial crisis. Moreover, we see that all of the reporting countries are essentially remotely vulnerable, i.e., they are susceptible to impacts coming from their indirect neighborhood. This evidences the high complexity and entanglement levels of financial systems worldwide.

We also note that, during the expansionary period due to low interest rates in general, in which institutions were willing to take more risks due to the optimistic scenario, we see a consistent buildup of the computed impact susceptibility. This fact suggests that more and more potential contagion routes were being formed in that period.

When the risks materialized in 2007-2008, through the default of Lehman Brothers and other related institutions, agents were willing to reduce their exposures due to a global confidence crisis. In this turmoil, the impact susceptibility largely dropped, revealing the disappearance of several potential contagion paths.

In the beginning of 2009, we note that the global scenario changes again. The consistent downward trend of the impact susceptibility index ceases. The impact susceptibility reaches a plateau around which roughly stands still until the end of 2012 for all of the reporting countries. European countries, in contrast, are undergoing the European sovereign crisis, which reflects on their impact susceptibility values. In special, we see a local peak at March 2012, which captures the Greece’s default event, showing that the network became slightly more susceptible to impacts.

After the European sovereign debt crisis, the years of 2013 and 2014 are, in general, marked with low values for the impact susceptibility indices. This observation suggests

the existence of few potential contagion paths between countries that participate in the global financial network. Distinguishing from their peers, Oceanian and Asian countries show a rapid increase of the impact susceptibility index due to their more active international participation. As such, they become more exposed to the international network, which is translated by an increase on the number of potential contagion paths that can ultimately reach them. Furthermore, using our assumptions to estimate countries' capital buffer, we observe that all countries are remotely vulnerable, except the USA in some periods. The USA stays locally vulnerable after 2013 as its impact susceptibility remains stacked at one due to its direct exposure to the UK.

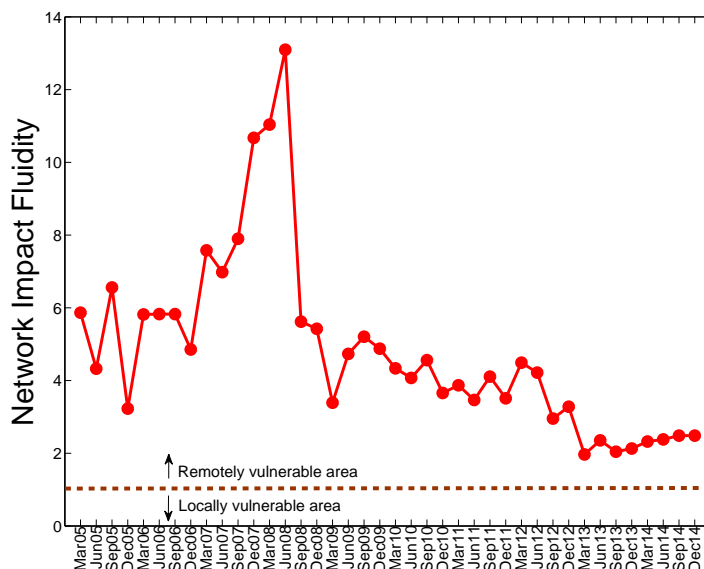
Finally, we mention a caveat about the data nature that we use. We must be aware that the USA has the unique power to emit dollars, which is the current international reserves currency. As such, if the USA is undergoing a severe economic crisis, the government possibly could print money and pay abroad to honor its debts. In essence, this prerogative means that the USA is virtually invulnerable to any other participant in the network. We can mimic this behavior in our model by simply endowing it with infinite capital buffer. In this way, no matter how much its international exposures are, they will be always smaller than the current absorbing capabilities of the USA. We recompute the impact susceptibility using this interesting caveat in Appendix C. Therein, we conclude that the theoretical "invulnerability" of the USA does not modify the conclusions presented in this section.

### **3.2.3 Network impact fluidity**

Figure 7 shows the network impact fluidity of the exposures network. Appendix B.2 provides technical details of the impact fluidity computation. It is computed as the average impact susceptibility indices of the network members and serves as a tool to evaluate the overall potential network fragility. We see that it captures well the global financial crisis as well as the European sovereign debt crisis. We also see a consistent buildup of the network impact fluidity in the pre-crisis period (2006-2008). Two plateaus after the global crisis are noticeable: one with a significant positive value that corresponds to the European sovereign debt crisis and another one after 2013, with smaller values. Comparatively, we also see that the pre-crisis potential risk buildup in the international network was much more severe than the potential risk over the European sovereign debt crisis.

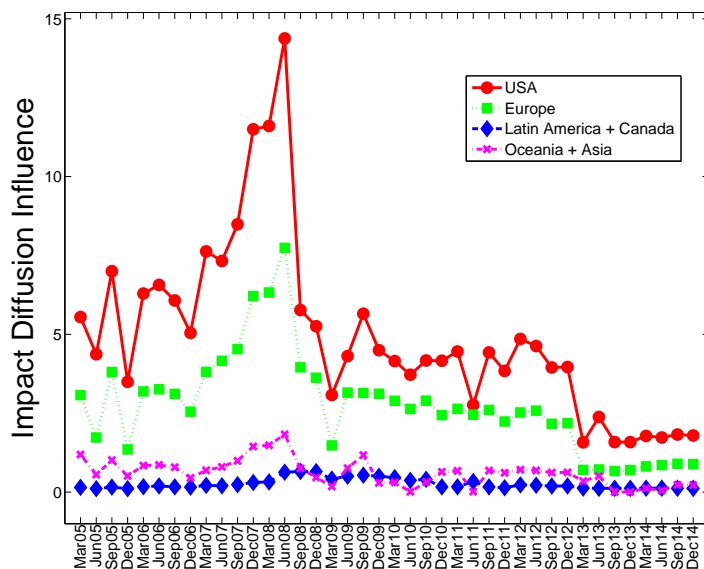
### **3.2.4 Weighted impact diffusion influence**

In contrast to the impact susceptibility that measures how likely one country is to receive random impacts within the network, the weighted impact diffusion influence gauges



**Figure 7:** Evolution of the network impact fluidity in the international claims network constructed from the reported data taken from the BIS CBS.

how harmful that country can be as a potential impact diffuser. Appendix B.3 provides the mathematical formulation to evaluate the weighted impact diffusion influence. Figure 8 portrays the weighted impact diffusion influence of the exposures network.



**Figure 8:** Evolution of the average weighted impact diffusion influence of members of the international claims network built from the reported data taken from the BIS CBS.

We see that the USA is the most potentially harmful country among the reporting countries of the BIS CBS, followed by European countries, Oceanian and Asian countries, and finally Latin American countries and Canada. Note that Latin American countries and Canada have a weighted impact diffusion essentially stacked at zero, i.e., no other country is vulnerable to them. Interestingly, we see that the weighted impact diffusion of Oceanian and Asian countries has a downward trend after the global crisis. In special,

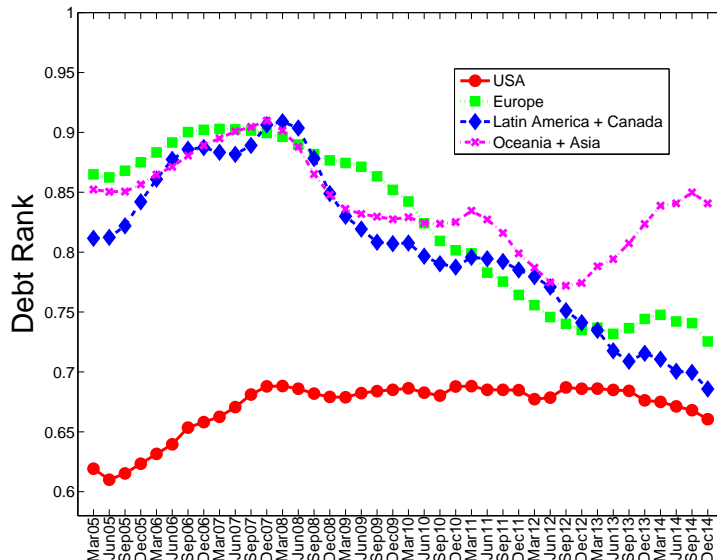
after September 2013, it remains at zero, meaning that they cannot cause default to other countries. The trajectories of the USA and European countries are very similar and they capture well the outbreak of the global financial crisis and the European sovereign debt crisis. Note that the USA becomes very harmful at the verge of the global financial crisis, distancing from the average weighted impact diffusion inflicted by European countries. Observe that the distance after this period, is roughly constant.

Again, we can repeat the exercise in which the USA is forced to be invulnerable in the international claims network by the same previously stated reasons. The re-evaluation of the weighted impact diffusion influence indices considering the caveat mentioned in Section 3.2.2 is provided in Appendix C. Again, the results do not alter significantly.

### 3.2.5 DebtRank

In this section, we apply the well-known feedback measure DebtRank introduced by Battiston et al. (2012b) to gauge network stress. We describe the computation of the DebtRank measure in Appendix B.4. The DebtRank methodology depends on the initial stress scenarios that are employed to estimate the relative loss of the financial system. Here, we assume that the initial shock is always a single default on a specific country. For instance, the DebtRank of country  $i$  is assumed to be the potential loss that it inflicts in the network if it defaults. The economic importance of a country  $i$  is given by the total intra-network liabilities that it holds towards other reporting countries.

Figure 9 presents the average DebtRank measures for the country groups. We see that the DebtRank methodology is able to identify the increase of risk in the international claims network prior to the global financial crisis, serving as an effective early warning for this particular crisis. This identification is clear for the USA and Latin American countries, as well as for Canada. The average DebtRank of European countries, however, does not seem to change significantly prior to the global financial crisis. In turn, the USA's DebtRank value reached when the Lehman Brothers defaulted remains at the same level until the end of our sample. The average DebtRank of European countries seems to decrease steadily after the end of 2008. This indicates that European countries are gradually originating less stress over time. Similarly, the weighted impact diffusion influence measure shows a gradual decreasing trend from 2009 followed by a step down to a plateau from the beginning of 2013. Interestingly, the DebtRank values of Oceanian and Asian countries are among the highest in the sample. The USA, in the other extreme, strictly shows the smallest DebtRank values during the period under study.



**Figure 9:** Evolution of the average DebtRank of members of the international claims network constructed from the reported data taken from the BIS CBS.

### 3.3 Counterfactual scenarios from varying risk drivers

In Section 3.2, we have performed a risk-related network analysis on the international claims network supplied by the BIS CBS. Therein, we have found that both the impact susceptibility and the weighted impact diffusion influence have consistently identified fragility and contagion potential in the recent financial crises. In fact, the comparison between the impact susceptibility in Fig. 6 and the diffusion potential in Fig. 8 suggests, in the sense of these two indicators, that the global financial crisis put the global financial network in a more fragile state than the European sovereign debt crisis did. A question that emerges from this digression is: which was the main factor driving the financial network to a more fragile state: the network topology or the not-so-strong capital buffers held by countries? Putting it in another way, had countries had stronger capital buffers, would they have been able to withstand the ongoing crisis?

In this section, we try to estimate the role that the network topology and the capital buffer distribution play as risk sources. Here, risk is represented by our two risk-related network measurements previously discussed: impact susceptibility and weighted impact diffusion influence. Analyzing Fig. 7, we see four distinct regions:

- March 2005 to December 2006: the vulnerability network density in Fig. 4 shows a slight increase while the network impact fluidity indicates that the overall network fragility is roughly constant and volatile.
- March 2007 to December 2009: period of great turmoil in view of the financial crisis. The impact fluidity trajectory assumes the form of a concave parabola, increasing fast and plummeting.

- March 2010 to December 2012: outbreak of the European sovereign debt crisis. The network impact fluidity continues to decrease, but to a lesser extent, while the countries under study recover gradually from the financial crisis. There are fluctuations around the trend.
- March 2013 to December 2014: the network impact fluidity reaches its smallest values in this period, suggesting that the international claims network is potentially safer in relation to the other periods.

It is not clear whether the variations of the network impact fluidity of these regions are due to the network topology, capital buffer distributions, or cross-term interactions among these two factors. To investigate this issue, we use the procedure proposed in Section 2 to build up counterfactual scenarios and identify the contribution of the network topology and the capital buffer distribution to the international claims network's systemic risk measures.

To assess these contributions along the period of analysis, we need firstly set a common ground, i.e., to define a reference network that will be the basis for the generation of a sequence of counterfactual networks in which only the risk driver which effect is under study evolves. We build these counterfactual networks using, as initial networks, the financial networks in each date of analysis and Equations (1) and (2). To isolate the effect of the network topology, we use Equation (2), generating counterfactual networks with capital levels of the reference network (scaled down to the initial network levels) and topology from the initial networks<sup>7</sup>. We use the same principle and Equation (1) to build counterfactual networks for analyses of the evolution of capital buffers along the period.

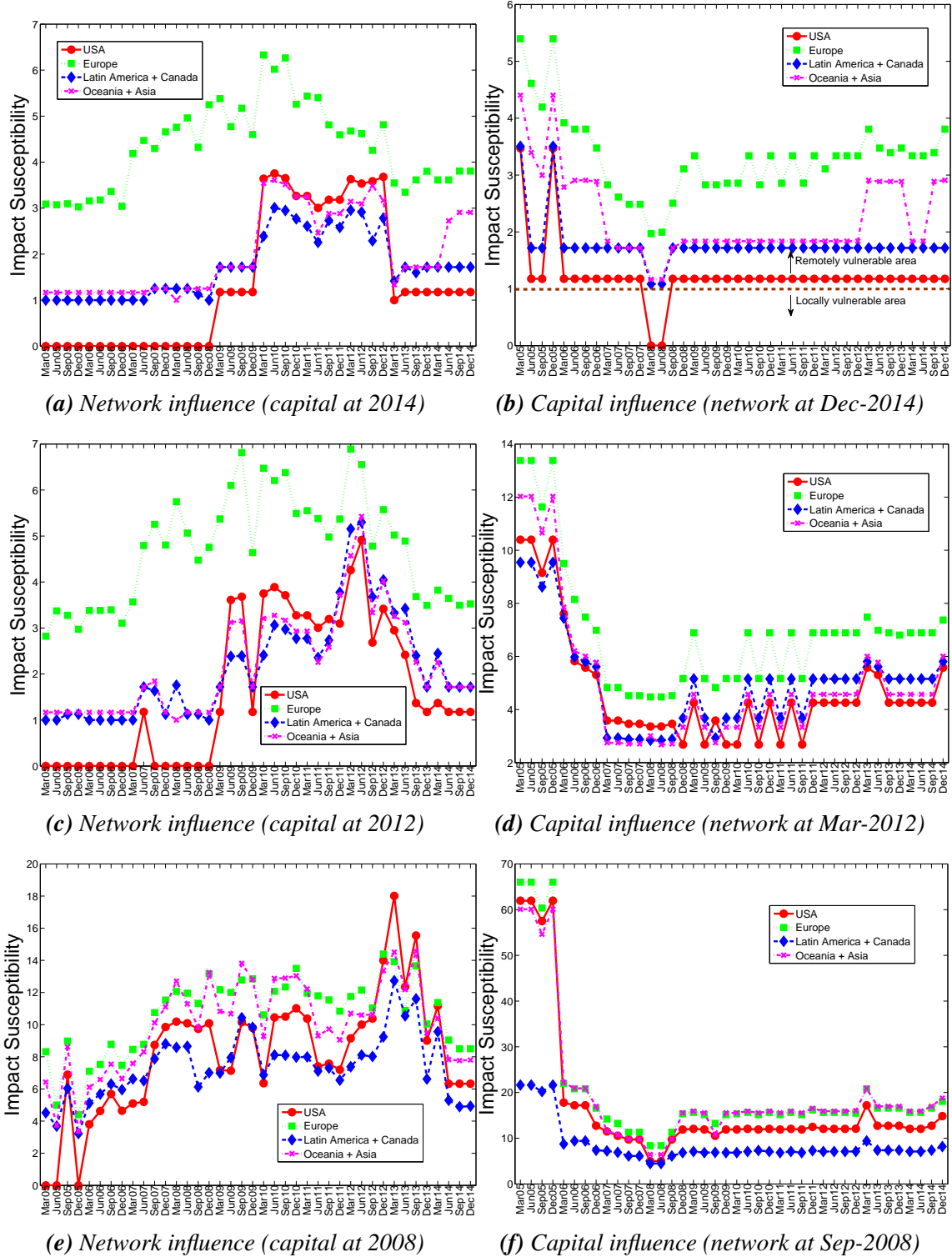
Figure 10 reports the impact susceptibility measures computed for counterfactual scenarios considering as reference networks the financial networks from three periods: September 2008 (Lehman Brothers' default and global financial crisis outbreak), March 2012 (Greece's default), and December 2014 (recent date).

One noticeable feature in Fig. 10 is that the impact susceptibility levels increase significantly as we go from the reference dates December 2014, March 2012, to September 2008. We also note that even though we proportionally fix, over the entire studied period, the capital buffers at the levels held by the reporting countries in December 2014, the network topology influence continues to show the onset of a break after September 2008 for the USA, right at Lehman Brothers' default. In this counterfactual scenario, due to the network topology influence, we also see a time shift of the most fragile period of the international claims network: it moves from September 2008 to September 2010. After the peak, while European countries, on average, have their impact susceptibility on a

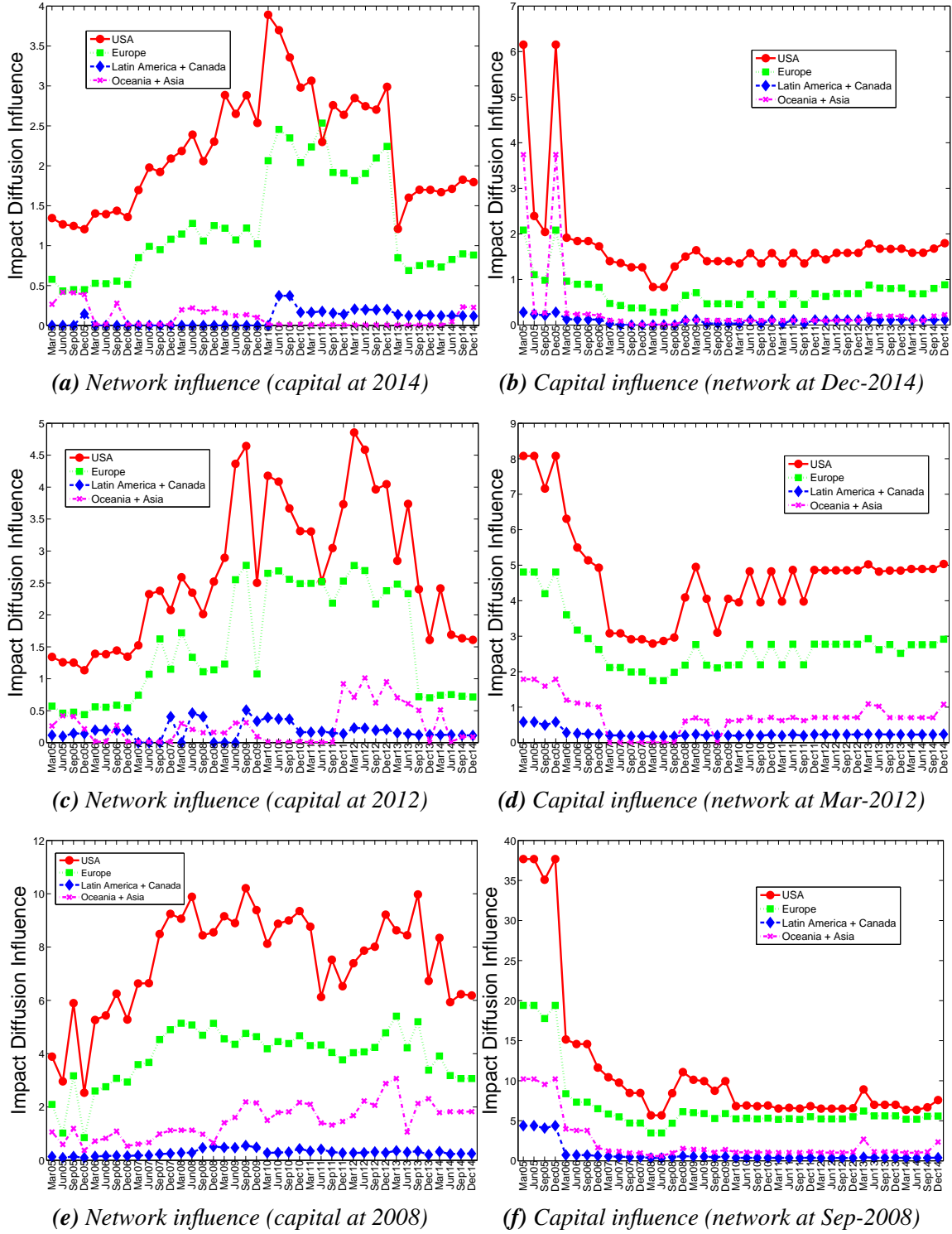
---

<sup>7</sup>Note that we could use Equation (1) with capital levels from a fixed-date initial network and topology from varying-date reference networks (scaled down to the initial networks levels,) arriving at the same vulnerability matrix.

downward trend, all of the other reporting countries show a roughly constant impact susceptibility. Interestingly, after the European crisis turmoil, in this counterfactual scenario, we see a sudden increasing impact susceptibility of Oceanian and Asian countries.



**Figure 10:** Influence of the network topology and capital buffer distribution on the impact susceptibility using the proposed counterfactual scenarios procedure at three different reference periods.



**Figure 11:** Influence of the network topology and capital buffer distribution on the weighted impact diffusion influence using the proposed counterfactual scenarios procedure at three different reference periods.

When we analyze the network topology influence using capital buffers of 2012, the break in the impact susceptibility of the USA also appears. We also see a peak in the impact susceptibility of all of the members except European ones in 2012. European countries show a similar impact susceptibility trajectory in relation to the capital buffer fixed

at 2014. The sudden increase that we previously verified for Oceanian and Asian countries disappears, suggesting that the increasing impact susceptibility is due to the network topology changes. When we analyze the network topology influence using counterfactual scenarios for the capital buffer fixed at 2008, we note that the impact susceptibility of the USA starts to grow from the start of the sample period. Moreover, we see a global peak occurring from the second half of 2012 to the first half of 2013. Additionally, the impact susceptibility indices are higher in the three counterfactual scenarios built to analyze the network topology influence.

Now, when we look at the capital buffer influence fixing the network topology at September 2008, March 2012, and December 2014, we verify that the impact susceptibility is high for periods prior to the global financial crisis. Interestingly, the impact susceptibility yields its lowest values in 2008 for the counterfactual scenarios that inspect the capital buffer influence. After the onset of the crisis, the fictitious scenarios show a steady increase of the impact susceptibility for all of the reporting countries.

For completeness, Figure 11 shows a similar investigation but employing as the risk-based network measurement the weighted impact diffusion influence. The same caveats that were analyzed using the impact susceptibility as the risk-related network measurement fully apply for the weighted impact diffusion influence index.

### **3.4 Identifying the network topology and capital buffer effects using the decomposition methodology**

In this section, we apply our methodology to disentangle the effects that the network topology and capital buffer have in the global banking network systemic risk measures. For each player or continent in our sample, our strategy is to understand how these two risk drivers influence the systemic risk perspectives of being susceptible to or harmful for other participants in the network. We select a series of network risk measures to understand the systemic risk buildup in the global financial system. The general idea is to build counterfactual networks for each period, using Equations (1) and (2), by fixing as initial and reference networks those in two adjacent dates. Then, we compute the influences of the network topology and capital buffer risk drivers in these network risk measures. We use quarterly data, therefore, our methodology is able to explain variations on network risk measures on a quarterly basis.

We start this section by computing global or network-level risk measures. They provide information on how systemic risk is building up in the network. Then, we take a closer look at continents and at each country in the global banking system. This analysis not only allows us to identify those countries that are sources and bearers of cross-border risk but also enables us to see how network topology and capital buffer influence these

risks. We show that the profile of countries that are risk sources or bearers in the network is significantly heterogeneous in the network.

### **3.4.1 A global view: susceptibility to receiving and propensity to diffusing shocks of the network**

Here, we decompose quarterly variations of the network impact fluidity and network weighted impact diffusion influence<sup>8</sup> in terms of the risk drivers network topology and capital buffer. Those two network measures convey the notion of the susceptibility to receiving and propensity to diffusing, respectively, shocks in the network.

Figure 12a exhibits the pointwise contribution of the network topology and capital buffer risk drivers to quarter over quarter changes in the network impact fluidity. Figure 12b reports the same information but using the accumulated effects of these two risk drivers for the network impact fluidity. Looking at the incremental changes of the network impact susceptibility decomposition, we see that the network topology is the most important contributor to the observed volatility in the network impact fluidity. Network topology is the main driver behind the increase in impact fluidity in the years preceding the global financial crisis. During the most critical period of the global financial crisis, in the third quarter of 2008, there are important changes in the network topology that reduce the network risks in the sense of the impact susceptibility measure.<sup>9</sup> After that, volatility in network topology explains peaks in impact fluidity in the third quarter of 2009, third quarter of 2010 and first quarter of 2012, as well as the plunge in the first quarter of 2013.

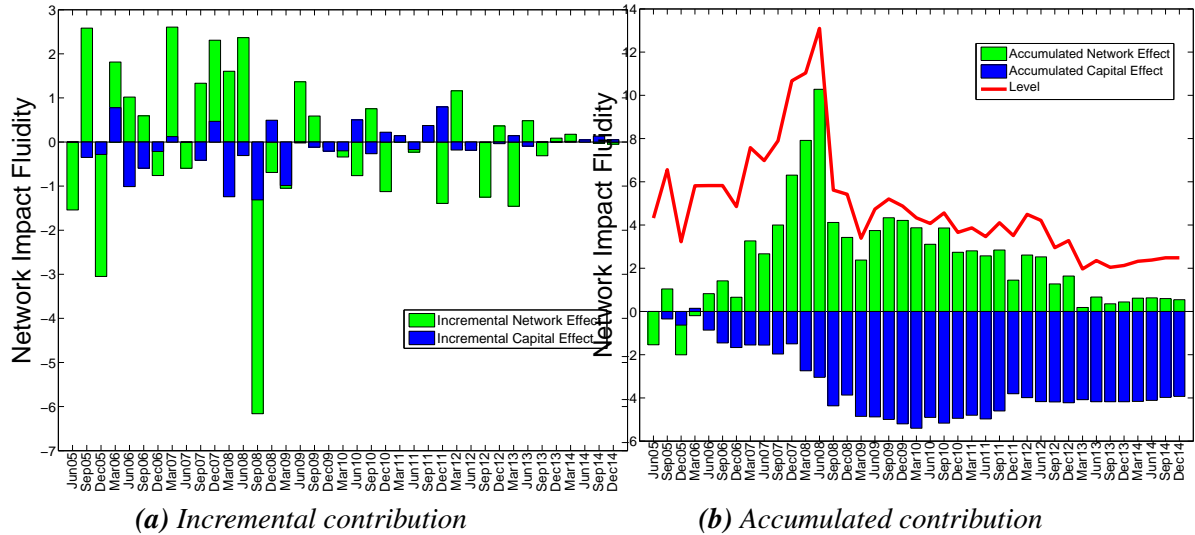
Analyzing the perspective of the accumulated changes of the network impact susceptibility decomposition, we conclude that capital buffer contributes to a persistent drop in impact fluidity, with most of the effects running from the first quarter of 2006 to the first quarter of 2010. After that, capital buffers mostly pressure impact fluidity to higher levels, so that network topology is the main factor behind the decrease in contagion risks towards the end of the sample. The plateau in impact fluidity observed from 2010 to 2011 is the result of these two opposites forces canceling out.

Figure 13a portrays the contribution of the network topology and capital buffer risk drivers to quarter over quarter changes in network weighted impact diffusion influence. Figure 13b reports the corresponding accumulated effects on the same indicator. The overall picture is similar to the case of impact fluidity. Network effects capture most of the period-by-period volatility, with a strong contribution to the buildup of diffusion risk just before the global financial crisis. Capital buffer effects contribute to lowering

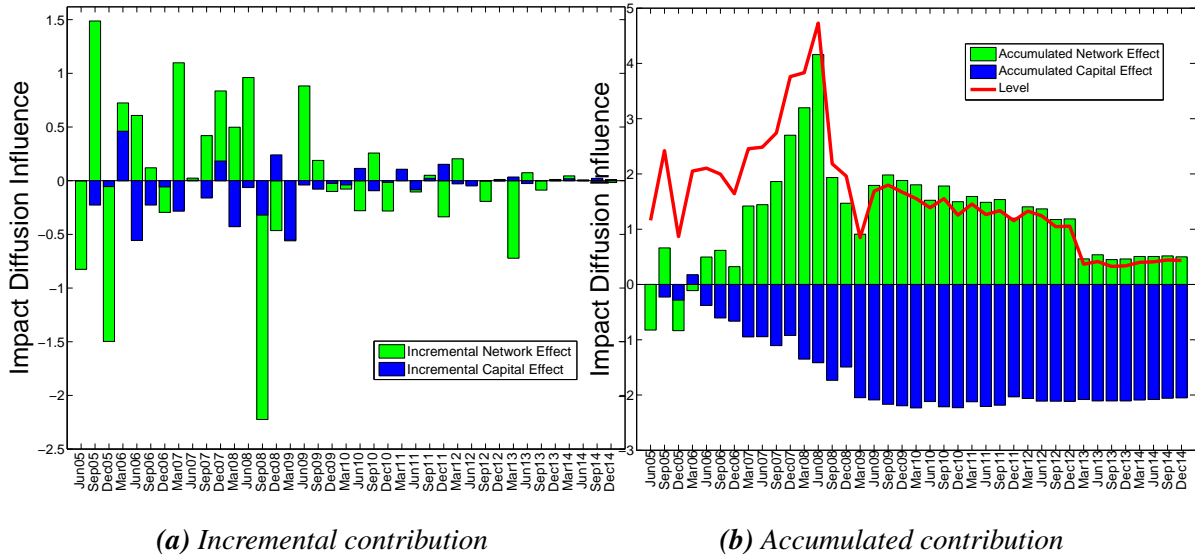
---

<sup>8</sup>The network weighted impact diffusion influence is computed by simply averaging the weighted impact diffusion influence of all of the network members.

<sup>9</sup>The BIS consolidated banking statistics does not report significant structural breaks in this period. Thus, this phenomenon can reflect risk transfers to financial institutions that are not covered by the BIS statistics.



**Figure 12:** Contributions of capital buffer and network topology factors to the network impact fluidity along time.



**Figure 13:** Contributions of capital buffer and network topology factors to the network weighted impact diffusion influence along time.

risks from 2006 to 2010, being the effect permanent thereafter. Unlike in the case of susceptibility risks, there is no reversal in the gains from capital buffers after 2010. In addition, the size of the effect relative to the size of the diffusion risk is much higher.

### 3.4.2 A personalized view: countries and continents that are sources and bearers of cross-border risk

In this section, we take a deeper look at the cross-border exposure data and identify countries and continents that are sources and bearers of cross-border risks.

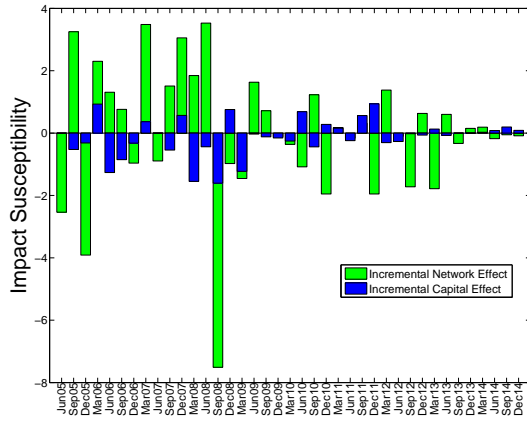
Countries that are risk sources in the international context represent players that can

propagate default throughout the global banking system and thus lead countries exposed to them into default. Thus, risk sources are countries to which other foreign countries have a level of exposure larger than their own capital buffers. In contrast, countries that are risk bearers market players that can default as a consequence of arbitrary impacts that can arise inside the global banking system. This happens as risk bearers are countries that invest or get overly exposed in the global banking system.

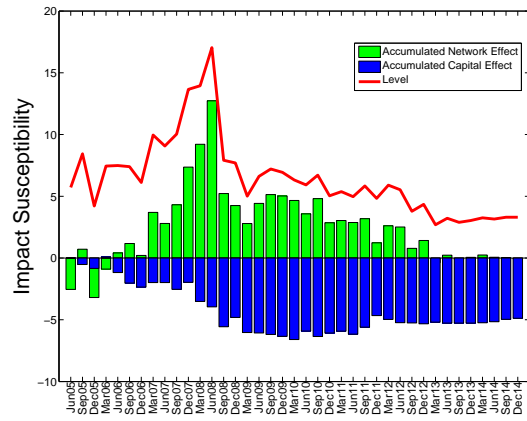
Identifying risk bearers and sources in the global banking system is an important task as regulators may better outline cross-border risks that can impact their domestic market and hence possibly cause financial instability. For instance, domestic sectors that are overly exposed to one or more cross-border risk sources may propagate losses to the financial system in case of instability of these international risk sources. In addition, using our risk driver decomposition methodology, we can better understand how the capital buffer levels and the network topology influence in amplifying or attenuating cross-border impacts between the domestic sector and foreign risk source counterparties.

We first focus on identifying countries and groups of countries that are risk bearers in the global banking system. We use the impact susceptibility to single out countries that are risk bearers because this measure retrieves the countries' likelihood of receiving impact in the global banking system. Then, we decompose variations of the impact susceptibility in terms of the capital buffer and network topology effects. Figure 14 exhibits the profiles of the cross-border risk bearers from the viewpoint of continents and groups of countries, as well as the associated contributions of the capital buffer and the network topology risk drivers in explaining variations of their impact susceptibility values. In turn, Figure 15 provides similar information on a countrywise level for the USA, the UK, and Greece.

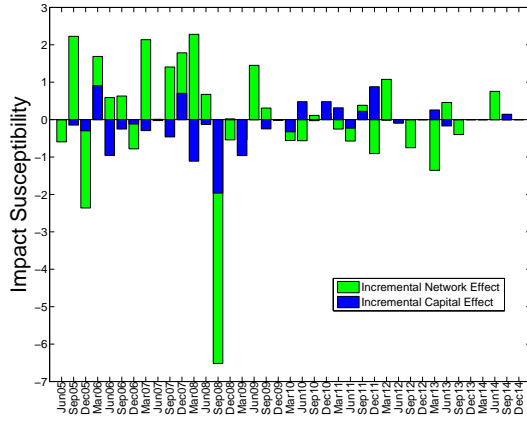
We see that the network topology effect is the main driver of the impact susceptibility buildup in the pre-crisis period for all groups of countries. Moreover, that effect is much stronger on the UK than on the USA and other European countries. Yet, the large noticeable decrease in the impact susceptibility in September 2008 is mostly due to the network topology effect. In turn, the capital buffer effect is responsible for a fraction of the impact susceptibility decrease mainly in the USA, European, Oceanian, and Asian countries. The impact susceptibility of Latin American countries and Canada, in contrast, is not affected by the capital buffer factor at the verge of the global crisis, suggesting that the large decrease on the impact susceptibility could be caused by a change on their positions in the network. In this sense, they seem to position themselves in more peripheral places in the global banking network such that they are less likely to be hit by foreign shocks. After the global financial crisis, the network and capital buffer effects contribute alternatively to the increase and decrease of the impact susceptibility of the USA. After 2013, the USA becomes locally vulnerable and the impact susceptibility stops varying.



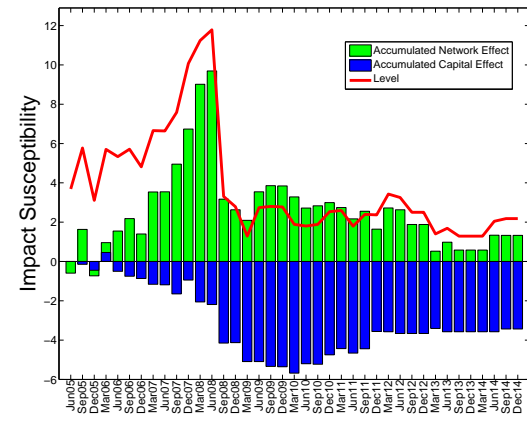
(a) Europe: Incremental contribution



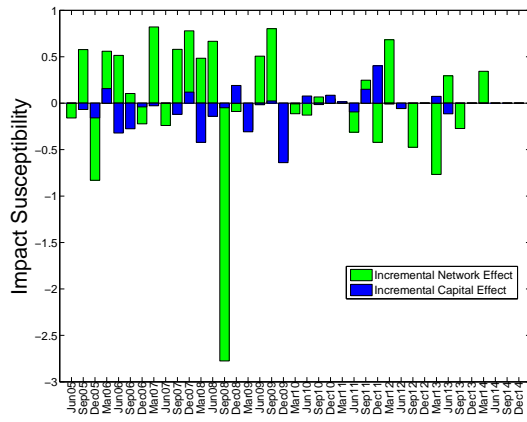
(b) Europe: Accumulated contribution



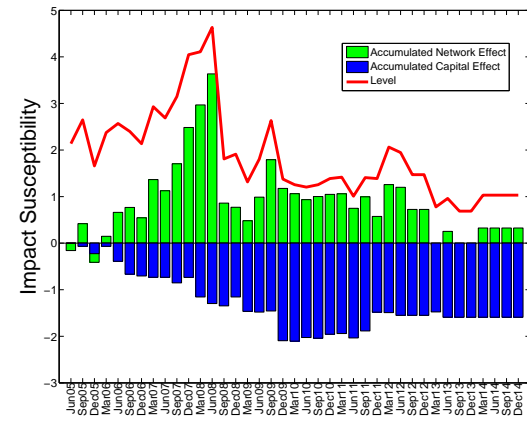
(c) Oceania + Asia: Incremental contribution



(d) Oceania + Asia: Accumulated contribution



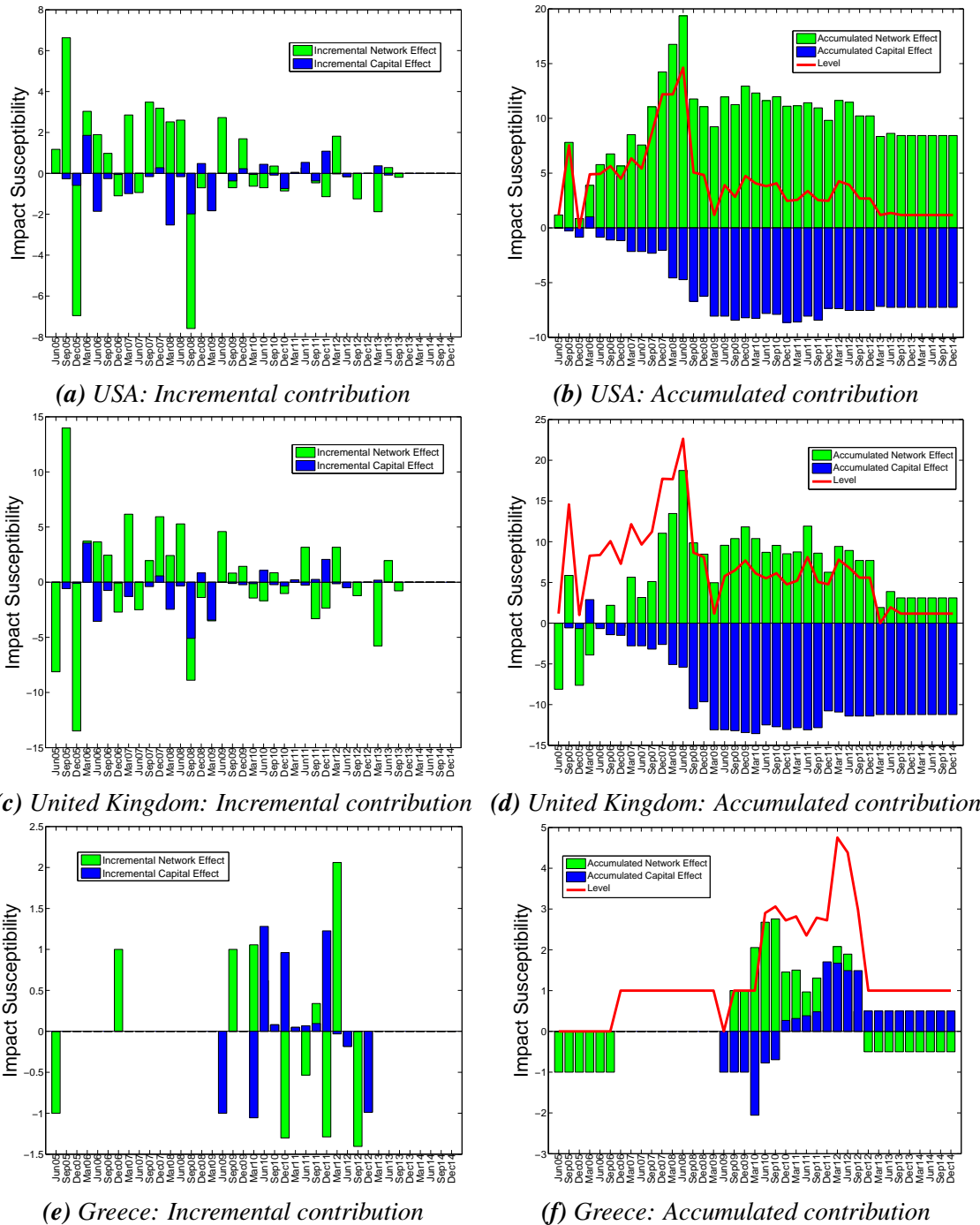
(e) Latin America + Canada: Incremental contribution



(f) Latin America + Canada: Accumulated contribution

**Figure 14:** Profile of the cross-border risk bearers that we estimate using the impact susceptibility of individual countries. We also report the contributions of capital buffer and network topology risk drivers for variations of the impact susceptibility risk measure by increments (left panel) and by the accumulated effects (right panel).

The capital buffer effect presents similar behaviors for all groups of countries. From the beginning of the sample to the start of the European sovereign debt crisis in the early 2010, the capital buffer driver contributes to the decrease of the impact susceptibility. This fact may be associated with the increasing capitalization of banks from 2005 to 2010 that



**Figure 15:** Profile of the cross-border risk bearers that we estimate using the impact susceptibility of individual countries. We also report the contributions of capital buffer and network topology risk drivers for variations of the impact susceptibility risk measure by increments (left panel) and by the accumulated effects (right panel).

we observe in Fig. 3c. After 2010, however, the capital buffer effect mostly pressures the impact susceptibility to higher levels until the end of 2012. After that, the capital buffer effect becomes mostly insignificant.

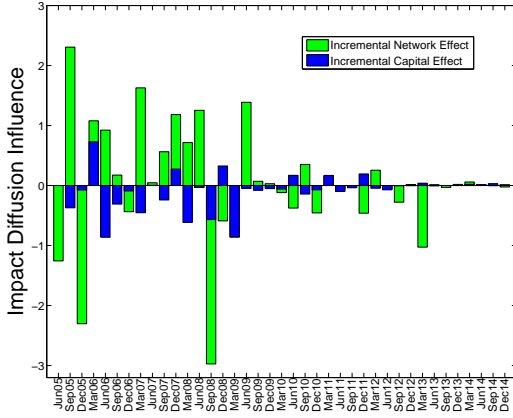
Looking at the individual contributions of the UK, the USA, and Greece, we verify that the UK and the USA have very similar profiles in terms of impact susceptibility. In

addition, they are the most representative risk bearers of cross-border risks in the global banking network. The UK seems to be consistently more exposed in the network than the USA. Greece in turn has a peculiar impact susceptibility curve. Greece's risk-bearing profile is not affected by the global financial crisis of 2008. However, Greece's impact susceptibility largely increases in the surroundings of its default. Before the default, the low capitalization of its large banks seems to be the main driver of increases in its likelihood of receiving impacts. After Greece's default, the large decrease in the impact susceptibility occurs due to its repositioning inside the global banking system followed by the recapitalization of its banks.

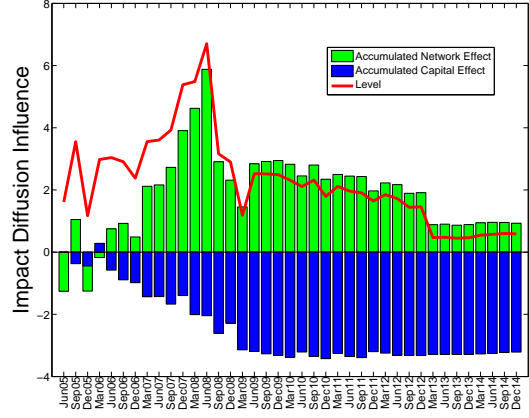
Now we shift our analysis to identifying countries and groups of countries that are sources of cross-border risks in the global banking system. We use the impact diffusion influence to classify those countries that are sources of cross-border risks as such measure indicates the ability of countries to diffuse and amplify shocks in the global banking system. As in the previous case, we also decompose variations of the impact diffusion influence in terms of the capital buffer and network topology effects. Figure 16 portrays the profiles of the cross-border risk sources from the viewpoint of continents and groups of countries, as well as the associated contributions of the capital buffer and the network topology risk drivers in explaining variations of their impact diffusion influence values. Figure 17 provides similar information on a countrywise level for the USA, the UK, and Greece.

We see that the network topology effect is the main risk driver behind the impact diffusion influence increase just before the onset of the global financial crisis. Moreover, that effect is much stronger on the UK than on the USA and other European countries. The large noticeable decrease in the impact diffusion in September 2008 is also due to the network effect in great part. The capital buffer effect, in addition, is responsible for a fraction of the impact diffusion decrease mainly in the USA, Oceanian, Asian, and European countries.

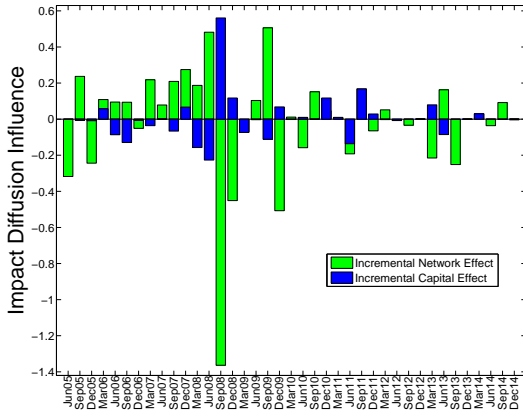
Looking at the individual contributions of the UK, the USA, and Greece, we verify that the UK and the USA have again very similar profiles in terms of impact diffusion influence, which is a consequence of their similar positioning and capital buffer levels in the network. In addition, they are the most representative risk sources of cross-border risks in the global banking network, because several countries invest in them and therefore are susceptible to shocks from these countries. Greece does not seem to be a country that can effectively diffuse impact in the global banking system mainly because no other country is overly exposed to it.



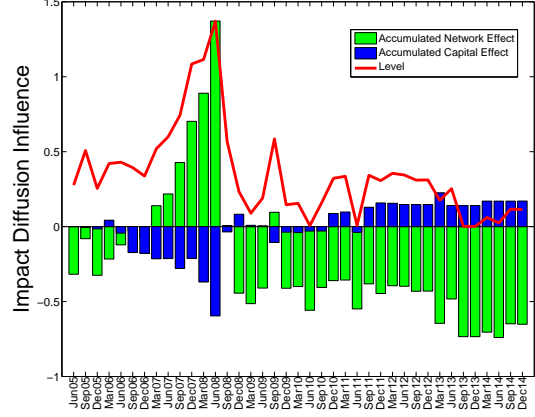
(a) Europe: Incremental contribution



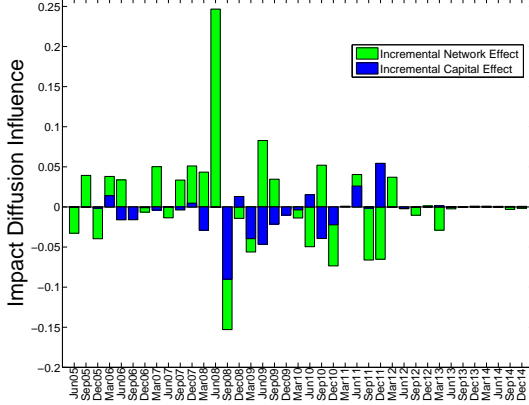
(b) Europe: Accumulated contribution



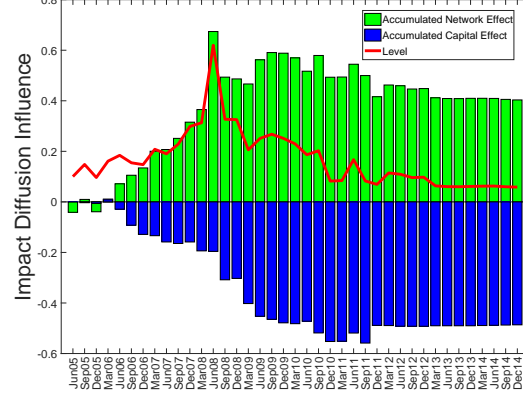
(c) Oceania + Asia: Incremental contribution



(d) Oceania + Asia: Accumulated contribution



(e) Latin America + Canada: Incremental contribution

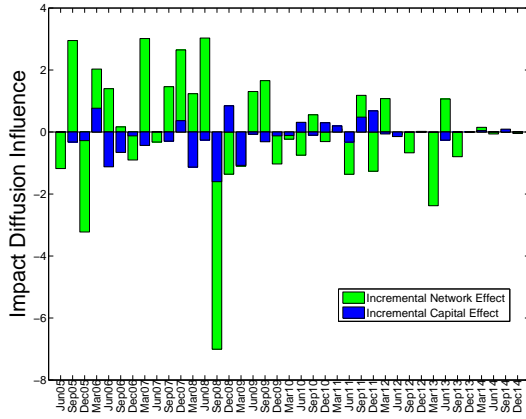


(f) Latin America + Canada: Accumulated contribution

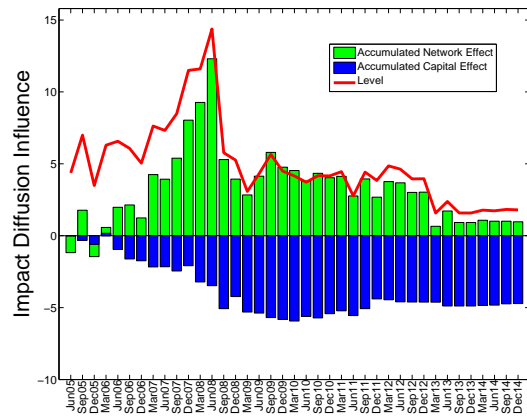
**Figure 16:** Profile of the cross-border risk sources that we estimate using the impact diffusion profiles of continents and/or groups of countries. We also report the contributions of capital buffer and network topology risk drivers for variations of the impact diffusion risk measure by increments (left panel) and by the accumulated effects (right panel).

## 4 Conclusion

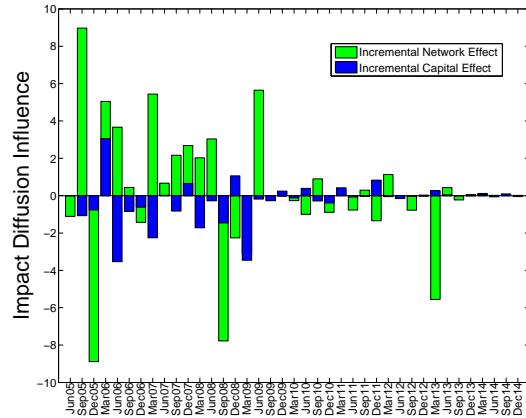
In this paper, we propose a general methodology to decompose systemic risk drivers in evolving financial networks. We focus on the risk drivers network topology and capital



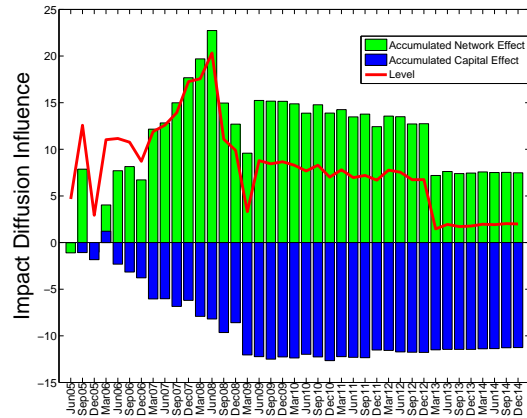
(a) USA: Incremental contribution



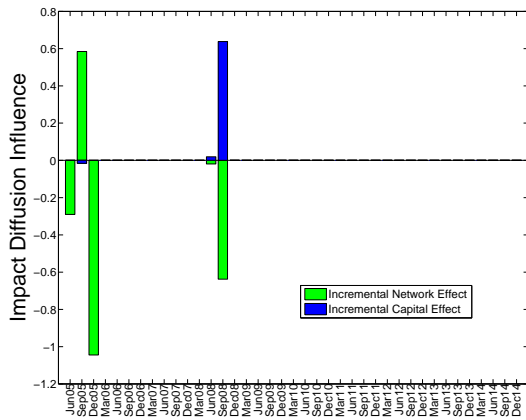
(b) USA: Accumulated contribution



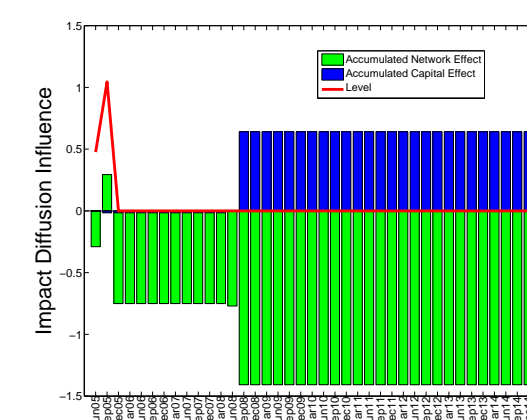
(c) United Kingdom: Incremental contribution



(d) United Kingdom: Accumulated contribution



(e) Greece: Incremental contribution



(f) Greece: Accumulated contribution

**Figure 17:** Profile of the cross-border risk sources that we estimate using the impact diffusion profiles of individual countries. We also report the contributions of capital buffer and network topology risk drivers for variations of the impact diffusion risk measure by increments (left panel) and by the accumulated effects (right panel).

buffers, which are often highlighted in academic and policy works to explain systemic risk and contagion transmission channels. Our methodology identifies and isolates their effects and is applicable to systemic risk measures whose computation relies on the vulnerability matrix, which encodes the web of pairwise vulnerabilities between network members. We show that several systemic risk measures fall into such category and hence

the application of the proposed method is broad.

We show that our methodology supports decomposition strategies that work both on the entire set or on subsets of vertices or countries in the network. Depending on the application, we also indicate several possible partitions in the network, such as into core and periphery partitions, into domestic and foreign financial institutions, or even into well-defined communities. We also provide mathematical underpinnings to designing counterfactual scenarios that we use to analyze the influence of a single risk driver at a time. The idea here is to vary only that risk driver while keeping constant the other risk drivers. We then attribute the resulting dynamics to the influence of the analyzed risk driver. This framework may be useful to understand and quantify the impact and effectiveness of policies aimed at changing a single risk driver, because we are able to estimate the systemic risk change as if that policy requirement did not exist.

To operationalize our framework, we use the impact susceptibility and impact diffusion influence as systemic risk measures, which depend on a functional form of the vulnerability matrix and hence are eligible for the methodology. These two measures offer complementary perspectives with regard to systemic risk and, unlike most systemic risk measures in the literature, are especially designed to capture catastrophic events in the network. In essence, the impact susceptibility provides a measure of how easy is that defaults inside the network severely hit a particular country and the impact diffusion influence estimates the extent that the financial system can suffer additional severe losses in view of the default of a particular country.

We apply our methodology to the BIS CBS dataset, which compiles gross cross-border exposures between countries. We show that the impact susceptibility and impact diffusion influence are able to capture the consistent systemic risk buildup before the global financial crisis. Using our decomposition methodology, we show that the main reason for this upward trend on systemic risk is the network topology. We also find that large drops on both systemic risk indicators just after the collapse of the Lehman Brothers are explained by a large change on the network topology effect, suggesting an adjustment on the risk appetite of countries that participate in the global banking network. In the surroundings of the global financial crisis, we find that the capital buffer serves as a mean to reduce the systemic risk indicators.

We use the impact susceptibility measure as a proxy to identify countries that are bearers of cross-border risks in the global banking network and the impact diffusion influence to single out countries that are sources of cross-border risks. Identifying countries that are bearers or sources of cross-border risks is essential to maintain financial stability in domestic markets with representative cross-border assets, as they are subject to shocks originating abroad that can potentially propagate via the web of vulnerabilities in the global banking network.

Our methodology could be a helpful tool for guiding policy making and modeling of systemic risk in financial networks. While the network contagion literature emphasizes the role of interconnectedness among different financial institutions to explain systemic risk, policy-guided researches are concerned in regulating and establishing suitable capital buffers that both (i) provide resilience to the financial system and (ii) do not financially hinder the ability of banks to service operations to the real and banking sectors. Our work supplies empirical evidence corroborating the importance of considering both risk drivers—the network topology and the capital buffer—when dealing with systemic risk.

The methodology also can be a useful tool for the financial system monitoring. It can provide information for the identification of the contributions from specific banks or communities to systemic risk as they change their risk drivers.

## References

## References

- Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2015). Systemic risk and stability in financial networks. *American Economic Review*, 105(2):564–608.
- Allen, F. and Gale, D. (2000). Financial Contagion. *Journal of Political Economy*, 108:1–33.
- Bardoscia, M., Battiston, S., Caccioli, F., and Caldarelli, G. (2015). Debtrank: A microscopic foundation for shock propagation. *PloS one*, 10(6):e0130406.
- Battiston, S., Gatti, D. D., Gallegati, M., Greenwald, B., and Stiglitz, J. E. (2012a). Default cascades: When does risk diversification increase stability? *Journal of Financial Stability*, 8(3):138–149.
- Battiston, S., Puliga, M., Kaushik, R., Tasca, P., and Caldarelli, G. (2012b). DebtRank: Too central to fail? Financial networks, the FED and systemic risk. *Scientific reports*, 2:541.
- Eisenberg, L. and Noe, T. H. (2001). Systemic risk in financial systems. *Management Science*, 47(2):236–249.
- Estrada, E. and Hatano, N. (2008). Communicability in complex networks. *Physical Review E*, 77:036111.
- Gai, P. and Kapadia, S. (2010). Contagion in financial networks. *Proceedings of the Royal Society*, 4666(2120):2401–2424.

- Gropp, R., Duca, M. L., and Vesala, J. (2009). Cross-border bank contagion in Europe. *International Journal of Central Banking*, 5(1):97–139.
- Haldane, A. G. and May, R. M. (2011). Systemic risk in banking ecosystems. *Nature*, 469(7330):351–355.
- Loepfe, L., Cabrales, A., and Sánchez, A. (2013). Towards a proper assignment of systemic risk: The combined roles of network topology and shock characteristics. *PLoS ONE*, 8(10):e77526.
- Roukny, T., Bersini, H., Pirotte, H., Caldarelli, G., and Battiston, S. (2013). Default cascades in complex networks: Topology and systemic risk. *Scientific Reports*, 3(2759).
- Silva, T. C., Souza, S. R. S., and Tabak, B. M. (2015). Monitoring vulnerability and impact diffusion in financial networks. Working Paper Series 392, Central Bank of Brazil.
- Silva, T. C., Souza, S. R. S., and Tabak, B. M. (2016). Structure and dynamics of the global financial network. *Chaos, Solitons and Fractals*, 88:218–234.
- Souza, S. R. S., Silva, T. C., Tabak, B. M., and Guerra, S. M. (2016). Evaluating systemic risk using bank default probabilities in financial networks. *Journal of Economic Dynamics and Control*, 66:54–75.

# Appendices

## A General decomposition methodology

When we partition subsets of vertices, the mapping of all of the possible trajectories that lead one initial network configuration to a reference network is not straightforward. In this section, we deal with the decomposition methodology for an arbitrary number of vertex partitions. Taking into account that we can use this methodology to assess risk contributions of the groups of vertices defined by each partition, we consider each of them as a risk driver. Therefore, vertex partition and risk driver are terms we use interchangeably throughout this section. Additionally, recall that the risk indicator  $m$  must be evaluated in the vulnerability matrix domain.

Let  $m = m(x_1, \dots, x_n)$  be a network measurement that depends on a set of  $n$  risk drivers  $x_1, \dots, x_n$  that we are interested in decomposing. For notational convenience,  $x_i$  and  $x'_i$  indicate the state of the  $i$ -th risk driver in relation to the initial and the reference networks, respectively.

We wish to decompose the contribution of each risk driver to variations in  $m$ . In order to get a better gist of how the decomposition methodology is applied, we first illustrate the three-variable case. (We discuss the two-variable case in the main text.) Afterwards, we generalize the proposed methodology to an arbitrary number of risk drivers. The original time-varying trajectory of  $m$  is assumed to be known.

### A.1 Three-variable decomposition case

Assume that we have an initial and a reference network that are characterized by the risk drivers  $(x_1, x_2, x_3)$  and  $(x'_1, x'_2, x'_3)$ . Say that we want to decompose the variation from the indicator  $m$ , i.e.,  $m(x'_1, x'_2, x'_3) - m(x_1, x_2, x_3)$ , in terms of the risk drivers  $x_1, x_2, x_3$ . Before defining the risk driver effects, we first need to map out all of the transitions that are possible from the initial to the reference network using only marginal movements in the direction of each of the risk drivers. They are listed in the following:

$$\begin{aligned}
& m(x_1, x_2, x_3) \xrightarrow{x_1 \text{ effect}} m(x'_1, x_2, x_3) \xrightarrow{x_2 \text{ effect}} m(x'_1, x'_2, x_3) \xrightarrow{x_3 \text{ effect}} m(x'_1, x'_2, x'_3) \\
& m(x_1, x_2, x_3) \xrightarrow{x_1 \text{ effect}} m(x'_1, x_2, x_3) \xrightarrow{x_3 \text{ effect}} m(x'_1, x_2, x'_3) \xrightarrow{x_2 \text{ effect}} m(x'_1, x'_2, x'_3) \\
& m(x_1, x_2, x_3) \xrightarrow{x_2 \text{ effect}} m(x_1, x'_2, x_3) \xrightarrow{x_1 \text{ effect}} m(x'_1, x'_2, x_3) \xrightarrow{x_3 \text{ effect}} m(x'_1, x'_2, x'_3) \\
& m(x_1, x_2, x_3) \xrightarrow{x_2 \text{ effect}} m(x_1, x'_2, x_3) \xrightarrow{x_3 \text{ effect}} m(x_1, x'_2, x'_3) \xrightarrow{x_1 \text{ effect}} m(x'_1, x'_2, x'_3) \\
& m(x_1, x_2, x_3) \xrightarrow{x_3 \text{ effect}} m(x_1, x_2, x'_3) \xrightarrow{x_1 \text{ effect}} m(x'_1, x_2, x'_3) \xrightarrow{x_2 \text{ effect}} m(x'_1, x'_2, x'_3) \\
& m(x_1, x_2, x_3) \xrightarrow{x_3 \text{ effect}} m(x_1, x_2, x'_3) \xrightarrow{x_2 \text{ effect}} m(x_1, x'_2, x'_3) \xrightarrow{x_1 \text{ effect}} m(x'_1, x'_2, x'_3)
\end{aligned} \tag{15}$$

Inspecting (15), we note that each risk driver appears just once in any of the six trajectories. Following the same strategy for the two-variable case, the effect of each risk driver is given by its average contribution on all of the feasible trajectories. Looking again at (15), they are:

$$\begin{aligned}
\Delta_{x_1} m &= \frac{1}{6} \left[ 2 \cdot (m(x'_1, x_2, x_3) - m(x_1, x_2, x_3)) + (m(x'_1, x'_2, x_3) - m(x_1, x'_2, x_3)) \right. \\
&\quad \left. + 2 \cdot (m(x'_1, x'_2, x'_3) - m(x_1, x'_2, x'_3)) + (m(x'_1, x_2, x'_3) - m(x_1, x_2, x'_3)) \right],
\end{aligned} \tag{16}$$

$$\begin{aligned}
\Delta_{x_2} m &= \frac{1}{6} \left[ (m(x'_1, x'_2, x_3) - m(x'_1, x_2, x_3)) + 2 \cdot (m(x'_1, x'_2, x'_3) - m(x'_1, x_2, x'_3)) \right. \\
&\quad \left. + 2 \cdot (m(x_1, x'_2, x_3) - m(x_1, x_2, x_3)) + (m(x_1, x'_2, x'_3) - m(x_1, x_2, x'_3)) \right],
\end{aligned} \tag{17}$$

$$\begin{aligned}
\Delta_{x_3} m &= \frac{1}{6} \left[ 2 \cdot (m(x'_1, x'_2, x'_3) - m(x'_1, x'_2, x_3)) + (m(x'_1, x_2, x'_3) - m(x'_1, x_2, x_3)) \right. \\
&\quad \left. + (m(x_1, x'_2, x'_3) - m(x_1, x'_2, x_3)) + 2 \cdot (m(x_1, x_2, x'_3) - m(x_1, x_2, x_3)) \right].
\end{aligned} \tag{18}$$

Summing up all of the differences due to the possible transitions and averaging the result over the total number of trajectories delineated in (15), we can relate the effects  $\Delta_{x_1} m$ ,  $\Delta_{x_2} m$ , and  $\Delta_{x_3} m$  to  $m' - m = m(x'_1, x'_2, x'_3) - m(x_1, x_2, x_3)$ , as follows:

$$m(x'_1, x'_2, x'_3) - m(x_1, x_2, x_3) = \Delta_{x_1} m + \Delta_{x_2} m + \Delta_{x_3} m. \tag{19}$$

## A.2 Arbitrary number of decomposable factors

Assume now that we have  $n$  risk drivers whose contributions to variations of  $m$  we want to decompose. Again, we fix an initial and a reference network with risk drivers  $(x_1, x_2, \dots, x_n)$  and  $(x'_1, x'_2, \dots, x'_n)$ , respectively. As in the previous section, we want to find a complete decomposition of  $m(x'_1, x'_2, \dots, x'_n) - m(x_1, x_2, \dots, x_n)$  in terms of the  $n$  risk drivers.

From the two- and three-variable cases, it is clear that the total number of possible trajectories that start in  $(x_1, x_2, \dots, x_n)$  and finish in  $(x'_1, x'_2, \dots, x'_n)$  is  $n!$  if the transitions are marginal with respect to each of the risk drivers. Denote as  $\mathcal{T}$  the set of these possible trajectories, in which the  $j$ -th trajectory we index as  $\mathcal{T}_j$ . Clearly,  $|\mathcal{T}| = n!$ .

Before deriving the general formula for the decomposition method, some extra notation is introduced. Suppose  $\mathbb{S}(l)$ ,  $l \geq 0$ , is a combinatorial mapper that outputs all ordered binary sequences of size  $n$ .<sup>10</sup> For example,  $\mathbb{S}(0) = \emptyset$ ,  $\mathbb{S}(1) = \{0, 1\}$ , and  $\mathbb{S}(2) = \{00, 01, 10, 11\}$ . An arbitrary ordered sequence  $s \in \mathbb{S}(l)$  is indexed in the form  $s = s_1 s_2 \dots s_l$ , where by construction  $s_i \in \{0, 1\}$ ,  $i \in \{1, 2, \dots, l\}$ .

Define the useful fixed ordered sequences that are feasible realizations of  $\mathbb{S}(l)$ :

$$s^{(\text{ones})}(l) = \overbrace{11 \dots 1}^{\text{n times}} \in \mathbb{S}(l), \quad (20)$$

$$s^{(\text{zeros})}(l) = \overbrace{00 \dots 0}^{\text{n times}} \in \mathbb{S}(l), \quad (21)$$

$$s_{-i}^{(\text{ones})}(l) = 11 \dots \overbrace{0}^{i\text{-th position}} \dots 1 \in \mathbb{S}(l), \quad (22)$$

$$s_{-i}^{(\text{zeros})}(l) = 00 \dots \overbrace{1}^{i\text{-th position}} \dots 0 \in \mathbb{S}(l), \quad (23)$$

where  $i \in \{1, \dots, l\}$ . Consider also the combinatorics operator  $\mathbb{S}^*(l) \subseteq \mathbb{S}(l)$  defined as follows:

$$\mathbb{S}^*(l) = \begin{cases} \mathbb{S}(l) - s^{(\text{ones})}(l) - s^{(\text{zeros})}(l), & \text{if } l > 2 \\ \emptyset, & \text{otherwise} \end{cases}, \quad (24)$$

i.e.  $\mathbb{S}^*(l)$  supplies the same grammar as  $\mathbb{S}(l)$ , but with at least one 0 and 1 in the ordered sequence. For instance,  $\mathbb{S}^*(3) = \{100, 010, 001, 110, 101\}$ . Consider that  $\mathbb{S}_{+i}^*(l)$  corresponds to all ordered proper subsets of  $\mathbb{S}^*(l)$  such that in the  $i$ -th position there is a 1. For example,  $\mathbb{S}_{+1}^*(3) = \{100, 110\}$ ,  $\mathbb{S}_{+2}^*(3) = \{010, 110\}$ .

Now we translate the ordered binary sequence  $s$  to a parametric list so as to retrieve the risk drivers  $(x_1, x_2, \dots, x_n)$  that are employed to evaluate  $m$ . For that end, we map:

$$s \mapsto (g(s_1), g(s_2), \dots, g(s_l)), \quad (25)$$

<sup>10</sup>A more precise definition of  $\mathbb{S}(n)$  can be borrowed from the Theory of Computation. It corresponds to a deterministic automaton whose grammar is all binary strings of arbitrary lengths.

where  $l = n$  and:

$$s_i = [x'_i]^{s_i} \cdot [x_i]^{1-s_i}, \quad (26)$$

which yields  $x'_i$  if the  $i$ -th element of  $s$  is 1 and produces  $x_i$ , otherwise. For instance, the binary string  $s = \{0101\}$  maps to  $(x_1, x'_2, x_3, x'_4)$ .

By induction, we can see that the effect of  $x_i$  on  $m$  is given by:

$$\begin{aligned} \Delta_{x_i} m = & \frac{1}{n!} \left[ (n-1)! \left( m(s^{(\text{ones})}(n)) - m(s_{-i}^{(\text{ones})}(n)) \right) \right. \\ & + (n-1)! \left( m(s_{-i}^{(\text{zeros})}(n)) - m(s^{(\text{zeros})}(n)) \right) \\ & \left. + (n-2)! \sum_{s \in \mathbb{S}_{+i}^*(n)} m(s) - m(s_{-i}) \right], \quad (27) \end{aligned}$$

in which  $|\mathbb{S}_{+i}^*(n)| = (n-2)(n-1)$  and  $n$  is the number of risk drivers. For each binary string, we apply the mapper as defined in (25). If we sum up the number of factors inside the outer brackets, we see that there is a total of  $(n-1)! + (n-1)! + (n-2)!(n-2)(n-1)$ . Noting that  $(n-2)!(n-2)(n-1) = (n-2)(n-1)!$ , we see that the number of factors sum up to  $(n-1)! + (n-1)! + (n-2)!(n-2)(n-1) = 2(n-1)! + (n-2)(n-1)! = (2+n-2)(n-1)! = n(n-1)! = n!$ . Since we divide the RHS of (27) by  $n!$ , we are really taking the average effect of all of the possible transitions of  $x_i$ .

Finally, the variation of  $m$  to  $m'$  is computed as:

$$m(x'_1, x'_2, \dots, x'_n) - m(x_1, x_2, \dots, x_n) = \sum_{i=1}^n \Delta_{x_i} m. \quad (28)$$

## B Risk-related network analysis

In this section, we review three risk-related network measurements that have recently been introduced by Silva et al. (2015): impact susceptibility, network impact fluidity, and the weighted impact diffusion influence. In this paper, we apply these measures to the international capital flow network of the BIS reporting countries and show that they are consistent as early-warning indicators for both the global financial crisis in 2007-2008 and the European sovereign debt crisis. We also review the classical Battiston et al. (2012b)'s DebtRank.

## B.1 Impact susceptibility

The impact susceptibility quantifies how susceptible is one country to be lead into default due to potentially receiving destructive random impacts that originated inside the global banking network.

The impact susceptibility measure relies on the vulnerability matrix to be computed. The vulnerability matrix  $\mathbf{V}$  defined in (14) not only provides information of direct contagion, but indirect contagion once we take higher powers of  $\mathbf{V}$ . For instance, the entry  $(\mathbf{V}^k)_{ij} \in \mathbb{N}$  indicates the quantity of contagion paths of length  $k$  that starts from  $i$  and are transmitted to  $j$  due to high vulnerabilities of other countries in the path.

We define the country  $q$ 's impact susceptibility, here symbolized as  $S_q$ , as:

$$S_q(\mathbf{V}) \triangleq \begin{cases} \frac{1}{k_q^{(\text{in})}(\mathbf{V})} \sum_{\substack{p \in \mathcal{V} \\ p \neq q}} G_{pq}(\mathbf{V}), & \text{when } k_q^{(\text{in})}(\mathbf{V}) > 0 \\ 0, & \text{otherwise} \end{cases}, \quad (29)$$

in which  $k_q^{(\text{in})}(\mathbf{V})$  is the number of counterparties to which  $q$  is directly exposed in the vulnerability matrix. Thus,  $k_q^{(\text{in})}(\mathbf{V})$  is the number of neighbors that can lead  $q$  into default. The term  $G_{pq}(\mathbf{V})$  is the communicability from  $p$  to  $q$  that we evaluate from the vulnerability matrix  $\mathbf{V}$  as follows (Estrada and Hatano (2008)):

$$G_{pq}(\mathbf{V}) \triangleq \frac{1}{s!} P_{pq} + \sum_{k>s} \frac{1}{k!} (\mathbf{V}^k)_{pq} = (e^{\mathbf{V}})_{pq}, \quad (30)$$

in which  $P_{pq}$  denotes the number of paths with the shortest length from  $p$  to  $q$ , and  $s$  is the length of such paths. The term  $(\mathbf{V}^k)_{pq}$  is the  $(p, q)$ -th element of the  $k^{\text{th}}$  power of matrix  $\mathbf{V}$ , which gives the number of walks of length  $k$  from  $p$  to  $q$  along the vulnerability matrix  $\mathbf{V}$ , with  $k > s$ . Note here that we are quantifying not only shortest paths between pairs of countries, but also longer paths that contagion routes can materialize from due to lower capital buffers of countries along these paths. In any case, we are always attenuating the influence of these walks in accordance with their lengths, so as to prefer direct to indirect contagion.<sup>11</sup>

Equation (29) provides important indicative information as to whether country  $q$  may be impacted in the event of the default of any countries in the network. If  $S_q(\mathbf{V})$  is large, the probability that a default of a randomly chosen country  $p$  reaches country  $q$  is high. Conversely, that probability is comparatively lower when  $S_q(\mathbf{V})$  is low. The impact susceptibility  $S_q(\mathbf{V})$  alone does not determine contagion risk, but rather provides

---

<sup>11</sup>The attenuating factor guarantees the convergence of the infinite series in (30).

subsidies to systematically identify whether the presence of vulnerable countries may render the system more or less risky.

We now retrieve the concept of remote vulnerability and also an important property of the impact susceptibility index.

**Definition 4. Remote vulnerability (Silva et al. (2015)):** we say that institution  $q$  is remotely vulnerable when  $\exists p \in \mathcal{V}$  and  $k > 1 : (\mathbf{V}^k)_{pq} > 0$ .

**Theorem 1.**  $S_q > 1$  if and only if  $q$  is remotely vulnerable.

*Proof.* The proof of Theorem 1 can be found in Silva et al. (2015). ■

Theorem 1 provides important information to global regulators in what concerns identifying countries that are susceptible to indirect shocks (remotely vulnerable). If country  $q$  is susceptible to indirect impacts, then, necessarily,  $S_q > 1$ . The financial health of these countries cannot be evaluated locally, as they are likely to be hit indirectly if random shocks occur in the network. Therefore, monitoring schemes over these remotely vulnerable countries must be applied in a more global form. In contrast, countries that are not remotely vulnerable, i.e.,  $S_q \leq 1$ , can have their financial health evaluated by looking at their local neighborhoods.

## B.2 Network impact fluidity

The fluidity of an impact is a measure of length and multiplicity of default cascades in the network, when they occur. The potential fluidity of an impact in the network  $F(\mathbf{V})$  is given by:

$$F(\mathbf{V}) \triangleq \frac{1}{N} \sum_{q \in \mathcal{V}} S_q(\mathbf{V}), \quad (31)$$

in which  $N$  is the number of countries in the global banking network. Note that we measure the potential fluidity in terms of average impact susceptibility in the network. When several countries have large impact susceptibility indices, random shocks can easily propagate to other countries along the network. In other terms, the vulnerability network formed by the countries favors the fluidity of impacts inside the network and hence  $F(\mathbf{V})$  is large. In contrast, when most countries have small impact susceptibility, few potential contagion routes exist that can ultimately reach them. In this case, the network retains most of the impact at the vicinity of where the default or original impact happens. As such, the impact fluidity of the network is small. As Silva et al. (2015) draws attention to, two main factors influence the network impact fluidity: capital buffer and network

topology. In this work, we try to disentangle the contribution of these two factors using a novel decomposition methodology.

### B.3 Weighted impact diffusion influence

Up to now, we have introduced risk-related network measurements that gauge how susceptible one country is to receiving impacts occurring on other countries that are members of the global financial network. Now, we take the opposed perspective and measure the potential influence exercised by a country on the diffusion or propagation of impacts in the network. While the susceptibility conveys the concept of how exposed one member of the network is in relation to the remainder of the participants, the influence on the impact diffusion process suggests how harmful is one member of the network to others. We conceive the impact diffusion influence as a feedback centrality measure of the country in the network or similarly the overall dependence of other countries in the network to that country. The weighted impact diffusion influence, in turn, takes into account the importance or value of each of the countries that are potentially led into default due to impacts from a given country.

We can understand the diffusion influence of country  $q$  in terms of the variation it causes on the communicabilities between all of the participants when  $q$ 's power of diffusing impacts is removed from the network. We can perform that by deleting all out-edges originating from  $q$ . This type of filtering transforms  $q$  in a sink vertex in the network, for every path that reaches  $q$  must end in there.

The reasoning behind that procedure is as follows. If  $q$  is responsible for diffusing a significant portion of impact throughout the network, then its removal will reduce the overall network impact fluidity, causing the impact susceptibility and communicability of all of the countries to diminish considerably. In contrast, if  $q$  does not potentially diffuse impact to the network, then the network impact fluidity will remain unaltered or slightly altered.

In light of that, we define the potential influence that  $q$  exerts on diffusing impact to the network, here denoted as  $I_q(\mathbf{V})$ , as:

$$I_q^{(w)}(\mathbf{V}, P^{(\text{value})}) \triangleq \frac{1}{k_q^{(\text{out})}(\mathbf{V})} \sum_{p \in \mathcal{V}} \sum_{\substack{r \in \mathcal{V} \\ r \neq p}} \left[ G_{pr}(\mathbf{V}) - G_{pr}(\mathbf{V}^{(q-)}) \right] \cdot P_r^{(\text{value})}, \quad (32)$$

in which  $k_q^{(\text{out})}(\mathbf{V})$  is the number of counterparties exposed directly to  $q$ . Thus,  $k_q^{(\text{out})}(\mathbf{V})$  quantifies the number of counterparties that  $q$  can lead into default. The term  $\mathbf{V}^{(q-)}$  denotes the modified vulnerability matrix, in which all out-edges that originate from  $q$  are removed.  $P_r^{(\text{value})}$  is a proxy for the importance of all of countries in the market. In

this paper, we attribute it to the last country reached by the impact paths in Equation (32), and define it as the country's total liabilities, computed from the exposures matrix. The factor  $\left[ G_{pr}(\mathbf{V}) - G_{pr}(\mathbf{V}^{(q-)}) \right]$  indicates the communicability index of walks from  $p$  to  $r$  that visit  $q$ . This term is evaluated by initially computing the communicability index of  $p$  to  $r$  in the original vulnerability network. From this, we subtract the fraction of that communicability that is not due to a path that has  $q$  along the way. Consequently, Equation (32) effectively quantifies the network communicability's decrease, which is weighted by the country's importance, occurred in the network when  $q$ 's power of diffusing impacts is disabled.

## B.4 DebtRank

DebtRank is a measure of the stress of each bank that participates in a network of exposures subject to a shock. The intuition associated with that measure is as follows. Suppose that, in a network of mutually exposed banks, one of them defaults. That bank will not pay its liabilities towards its creditors. Those creditors will suffer losses that depend on the amount invested in the defaulting bank. Each of them, by its turn, having suffered a loss that puts it under stress will propagate that stress, reducing the payments of its liabilities by an amount proportional to that stress. That propagation continues until the banking system achieves equilibrium. Then, the stress related to a shock (in this paper, the default of a single bank) will be the sum of the stress of each bank after the finish of the propagation process. Each bank's stress is the ratio between the amount of assets it did not receive and its capital buffer, multiplied by its relative economic value.

The DebtRank methodology, proposed by Battiston et al. (2012b), models the interbank market as a directed network, in which banks are vertices and the exposures between them are links. These links are represented by a weighted adjacency matrix, whose elements  $A_{ij}$  are amounts lent by bank  $i$  to bank  $j$ . The total liabilities of  $i$  are given by  $L_i = \sum_j A_{ji}$  and the relative economic value<sup>12</sup> of a bank  $i$  is given by  $v_i = L_i / \sum_i A_i$ , which is the ratio of  $i$ 's assets over the total assets in the interbank market. Each bank  $i$  has a capital buffer against shocks,  $E_i$ , which is the amount of its capital that exceeds a positive threshold  $\gamma$ . If a bank suffers a loss that depletes  $E_i$ , it defaults. If bank  $i$  defaults, all neighboring banks  $j$  suffer losses amounting to  $A_{ji}$ . These losses cause an impact on bank  $j$ , given by  $W_{ij} = \min(1, A_{ij}/E_i)$ , which is the vulnerability of bank  $j$  to shocks from bank  $i$ .

The presence in the network of cycles inflates the computed impacts by counting

---

<sup>12</sup>Battiston et al. (2012b) define the relative economic value of an institution as the share of  $i$ 's assets to the total assets in the network. However, we use the liabilities share because, once a bank defaults, the losses that other members in the network have correspond to the liabilities of that defaulted bank towards them.

the impact of a vertex onto another more than once. To avoid the distortion caused by this double-counting, Battiston et al. (2012b) present an algorithm that allows a vertex to propagate impact only once, which we outline next.

Let the state of bank  $i$  be composed of the following dynamical variables at time  $t$ :

- $h_i(t) \in [0, 1]$ , which accounts for the stress level of  $i$ . If  $h_i(t) = 0$ ,  $i$  is undistressed; when  $h_i(t) = 1$ ,  $i$  is on default.
- $s_i(t) \in \{U, D, I\}$ , which is a discrete variable that assumes one of the following values: undistressed (U), distressed (D) and inactive (I).

The initial conditions for the simulation are set when  $t = 1$ . The banks with initial stress level  $h_i(1) = 0$  are undistressed, i.e.,  $s_i(1) = U$ ; if  $h_i(1) > 0$ , the banks are distressed  $s_i(1) = D$ ; and if  $s_i(1) = I$ , they are initially on default. The dynamics for each time step, starting from  $t = 2$ , is given by:

$$h_i(t) = \min \left\{ 1, h_i(t-1) + \sum_j \mathbf{W}_{ji} h_j(t-1) \right\}, \text{ where } j \mid s_j(t-1) = D,$$

$$s_i(t) = \begin{cases} D & \text{if } h_i(t) > 0; s_i(t-1) \neq I, \\ I & \text{if } s_i(t-1) = D, \\ s_i(t-1) & \text{otherwise.} \end{cases} \quad (33)$$

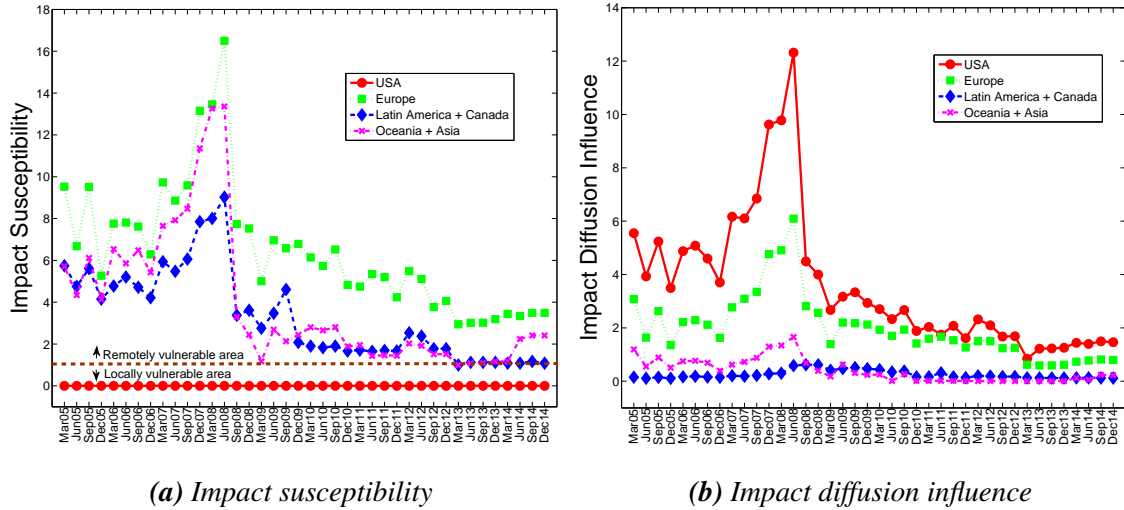
After a finite number of steps  $T$ , the dynamics stops and the DebtRank ( $DR$ ) is given by the difference between the final and the initial systemic stresses, i.e.:

$$DR = \sum_j h_j(T) v_j - \sum_j h_j(1) v_j. \quad (34)$$

## C Robustness test: checking when the USA is invulnerable

The cross-border exposures consolidated in the BIS CBS consists of aggregate amounts of banks' exposures to the banking, public, and non-bank private sectors of other countries. In Section 3.1, we assume that if a country's banking sector defaults, due to defaults from its cross-border exposures, this leads the other sectors of the economy into default or distress. We now focus on the USA as a large open economy, whose currency is a major international reserves currency. That very fact enables the USA to use monetary policy instruments to prevent the public sector's default. Consequently, it creates conditions for the recovery of other sectors. As such, this possibility would make the USA practically invulnerable to defaults on their cross-border investments.

In this section, we assess the effects on the impact susceptibility and impact diffusion influence when we consider the USA invulnerable to defaults on their banks' cross-border exposures. To make the USA invulnerable to defaults on its cross-border exposures, we fix the USA's capital buffer with a very high value (virtually, infinite).



**Figure 18:** Evaluation of risk-related measurements assuming that the USA has an infinite aggregate capital buffer. Note that its impact susceptibility always remains stacked at zero, as it is artificially invulnerable in the network.

Figure 18a presents the average impact susceptibility indices for the case in which the USA is invulnerable. The average impact susceptibility of the country group that do not include the USA remain almost the same as that for the case in which we consider the USA with its own capital buffer, as in Figure 6. The USA's impact susceptibility remains equal to zero, as it is artificially invulnerable in the network. Turning to the potential impacts diffusion, we see the same situation. Comparing the average potential impacts diffusion of the country group that do not include the USA, shown in Fig. 18b, with that computed for the case in which we consider the USA with its own capital buffer, as in Fig. 8, we see almost the same trajectories for all country group for both cases.

These facts indicate that the USA, with its own capital buffer, is barely vulnerable to defaults in most of its significant exposures.