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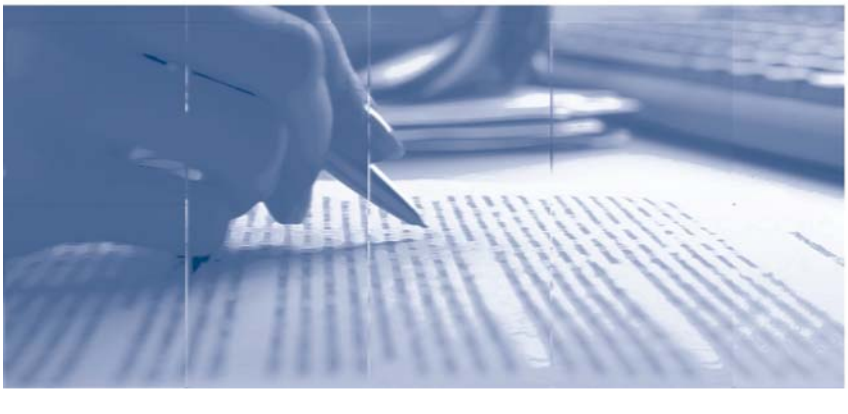

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Structural Trends and Cycles in a DSGE Model for Brazil*

Silvio Michael de Azevedo Costa**

Abstract

The Working Papers should not be reported as representing the views of the Banco Central do Brasil. The views expressed in the papers are those of the author(s) and not necessarily reflect those of the Banco Central do Brasil.

This paper builds and estimates a structural growth model with micro-founded specifications of trends and cycles to allow a consistent multivariate filtering of key macroeconomic variables. Emerging countries like Brazil show expressive trend dynamics that can blur the vectors determining real business cycles, but most models cannot consider this additional complexity, and so they usually throw out meaningful dynamics of scarce data. Our basic DSGE model cares for the raw data dynamics and aims to disentangle endogenously trends and cycles and unveil the underlying growth vectors. Thereby, historical interpretation and forecasts can hold internal consistency, which improves storytelling and calls for an in-depth forecasting. We report model evaluation summaries to support fully integrated models as a highly valuable tool for applied macro exercises and policy advising. Yet, we present an application for the measurement of potential output and the output gap.

Keywords: potential output, output gap, DSGE, detrending

JEL Classification: C54, E32, E37

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1 Introduction

Disentangling trends and cycles has always challenged economists involved in applied macroeconomic research. Obstacles are essentially technical, and to start with a theoretical point, models usually do not take into account the required structure to tackle simultaneously trend shifts and short run dynamics. For the sake of simplicity, univariate filtering or even more basic methods like demeaning have been widely used[1] to pretreat data and produce mean-zero covariance stationary processes, which are assumed to be thenceforth the relevant gap measure of the real business cycles for many types of applications. Two immediate problems of univariate methods concern exogeneity and lack of consistency in both time series and cross-section dimensions, thus placing several drawbacks of their practice in policy analysis and forecasting.

Emerging countries have special interest in this aspect because are always experiencing structural shifts motivated by either ongoing reforms, policy regime switches, high exposure to more prolonged external cycles, or home political turnovers. Demeaned data can put too much of trend shifts to be forcedly explained by a gap model, whereas time-varying trends from univariate filters can simply waste data by throwing out part of the real cycles.

Brazilian macro series can deepen the question of extricating trends and cycles since data samples are inevitably short. Breaks related to policy regimes and also methodological changes have entailed discontinued data series. Short samples are likely to miss most of the long run behavior and carry additional challenges to identify deep parameters. Furthermore, a closer look into Brazilian data can reveal a bunch of economic questions to consider regarding the recent slopes in the time series, which claim for a richer modelling approach and extended data sets to reconcile rightly theoretical cointegration of key variables and the empirical evidence.

Multivariate filtering methods have provided a reaction to issues like exogeneity and consistency. Their common application is for the output gap measurement, either using a production function approach or model-based gaps relations[2]. By including some theoretical constraints in the filtering mechanisms, multivariate filtering steps forward but not enough, insofar as some constraints are founded on a single equation or partial equilibrium, which potentially miss out the valuable contribution of all other variables from a general equilibrium perspective.

DSGE models supply the general equilibrium conditions and can be combined with a multivariate filtering to include all available information in a collaborative and systemic manner. Hence, a DSGE-based multivariate filtering approach can account for fluctuations of many economic variables in a structural perspective. To do so, Kalman filter is a handful procedure already included in filtering and smoothing steps within the DSGE

[1] See DeJong and Dave (2007).

[2] See Areosa (2008); Muinhos and Alves (2003); Alves and Correa (2013).

estimation procedures, in order to recover non-observable states from a set of observable variables and structural relationships. Once the underlying model represents a description of short run interactions, some structural shocks can be conveniently positioned into the equation system to allow data fitting.

Nonetheless, most DSGE models are built to explain only the cyclical fluctuations and are yet solved around an idle steady state, meaning that observed fluctuations are supposed to be stationary, and so requiring a preliminary data transformation before estimation. Therefore, data pretreatment discards the part of information that the model cannot deal with. From a theoretical point of view, the literature has long discussed about procedures to remove trends since they are crucial to determine filtering and estimation outputs, as argued in Gorodnichenko and Ng (2010); Fukač and Pagan (2010); Canova (2014); Canova and Ferroni (2011).

Some models introduce a unit root process in production technology in order to obtain cointegrated variables simulations and a corresponding balanced growth path. Ferroni (2011) claims that trends incorporated into the model might not reflect the real nature and the number of shocks driving non-cyclical fluctuations in data. For example, great ratios are often supposedly stationary, but neither nominal nor real stationarity can be found in US data (see Canova (2014)). Hence, when trends are misspecified, the structural model is also misspecified and parameter estimation is biased[3]. Lafourcade and de Wind (2012) exemplify this issue in Smets and Wouters (2007). Since they imposed a deterministic common growth rate for all real variables, all the dynamics are forced into the cyclical DSGE representation and yet the real variables turn out to be pairwise cointegrated, although US data shows strong evidence of the contrary. Their estimation results likely undergo a misspecification bias.

Notwithstanding, the usual combination of pretreated data and a purely cyclical DSGE model produces likewise biased estimates, given the empirical evidence from data and the arbitrariness of univariate filters. Either by using univariate filtering devices or estimating trends and cycles jointly with a DSGE model, both methodologies of extracting the cyclical component rely on assumptions about trend processes that can cause mismeasurement of cyclical components and bias the estimation of deep parameters, as highlighted in Canova (2014); Ferroni (2011); Canova and Ferroni (2011); Gorodnichenko and Ng (2010). A true step forward is to look into the data and understand which trends and shifts are important for selected macro variables, and incorporate them into a model by using parameter shifts or carefully-placed unit roots, as advocated in Lafourcade and de Wind (2012)[4]. Being aware of the theoretical implications of misplacement of model

[3] Ferroni (2011) advocates for a joint estimation of cyclical and non-cyclical components, using a DSGE model for the cyclical part and a flexible reduced-form specification to describe the non-cyclical part, with the purpose of building an estimation of the parameters related to the cyclical side that can be robust to the weighted-average results for the non-cyclical components. Despite long run is not his research focus, consistency issues apply in that case, as long as deep parameters driving both short and long run might not be reduced and apart.

[4] Lafourcade and de Wind (2012) support that misspecification issues may require reverse-engineering

trends, we draw attention to the fact that usual DSGE procedures are far from meeting conditions for unbiased and consistent estimates.

As far as policy is concerned here, not theoretical issues, we argue that the real-time understanding of economic trends is a decisive aspect to consider from the point of view of forecasting and policy analysis in at least four aspects. The first is about the stance of economic trends in itself. Figure 1 plots real-time calculations of the last value of GDP growth trend produced by three univariate methods: demeaning, one-sided Hodrick-Prescott filtering, and the usual two-sided HP filter. The headline variable is the 12-month accumulated real GDP growth, which is intrinsically a long memory measure. The graph clearly enlightens two periods in special. The period 2002-2005 is characterized by small disagreement between univariate methods, with differences below 0.5 percentage points. On the other hand, the period from late 2008 to 2013 shows very big disagreement, around 2-3 percentage points, and can be associated with data observation of domestic consequences of the turmoil following the international financial crisis. As widely known, Brazil quickly recovered from the first wave related to financial linkages, but experienced troubles in a scenario following the unfolding European debt crisis, the slowdown in advanced economies, and the falling of Chinese demand for commodities. This is a real policy challenge. Policymakers need to react accurately each period to dampen the relevant fluctuations, but the methods to circumscribe the relevant business cycles bring additional uncertainty in critical periods.

High uncertainty about the GDP trend in unconventional times is accompanied by silence to explain the shifts, and some of the methods have no commitment with their previous outputs. This is a second aspect to consider. Real-time revisions[5] of trends brought forth by methodological procedures, either for current or past periods, are highly inappropriate and can hazard the policy calibration. Assume that relative stability of trends in the short run is a highly desirable property. Figures 9 and 10, which are further discussed in details, show real-time revisions of the Brazilian potential output produced by four concurrent detrending method, just to give numbers to the problem. Although all the methods appear to be not biased over the horizon, the bounds reveal that demeaning reviewed its trend up to 1 percentage point from one period to the following, and along all the past. In its turn, HP filter reviewed its trend curve less than demeaning does in the short run (0.5 p.p. at most, but symmetrically). Standard deviations of revisions confirm that demeaning and HP filter are two of usual methods presenting the highest uncertainties in the short run.

A third aspect to be considered concerning policy-making is the technical content

whenever possible to place trends into the model and also careful consideration of which aspects of a model's balanced growth path are borne out in the data.

[5] We do not refer to either revisions of the previously GDP data series released by the official statistics bureau (IBGE in Brazil), or even changes in their methodologies. In this paper, we only concern the revisions in GDP trends caused by detrending methods working on the last released GDP time series. Real-time calculations in this context mean a rolling window in which new data becomes available each time.

provided by the methods to trace and explain the revisions. Trend shifts will happen for sure, and they need to be structurally consistent and integrated with real business cycles, as well as fully explained by the vectors driving the long run behavior. Story telling sustains the leading role of structural models in the suite of models for policy guidance, since it is the base upon which meaningful structural forecasts and accurate policy advising are built. The ability to forecast trends in the near horizon is also a hot topic for policy-making. It cannot be done properly without a consistent knowledge of the trend dynamics that sustain their procedures, especially in critical periods. Furthermore, featuring cyclical and non-cyclical structures along the same margin within the model boosts the scope for forecasting scenarios and counterfactuals exercises, which indeed leverages policy advising.

A fourth relevant aspect yet to consider is the ability to learn about the current stance of economic activity from the available information set. Once GDP data series are calculated on a quarterly basis and the release is delayed by months, univariate methods cannot work until a new data series becomes available, although current information set contains useful assistance to outline stances for trends and cycles. Structural detrending methods can present yet comparative advantages at this point.

This paper is policy-oriented and intends primarily to provide a internally-consistent trend-cycle decomposition within the DSGE model by using an one-step filtering approach that cares about the nonstationary in raw data, like Ferroni (2011) suggests[6]. At this point, it is important to note that there is no unique trend-cycle decomposition in the DSGE approach, and results are model-dependent by definition. Micro-foundations have been the road map to qualify the structural short-run connections, so they should be yet taken into account when filtering non-cyclical behavior, and thus partially mitigating the criticism related to model-based results. We setup and estimate a standard NNS DSGE model[7] with a balanced growth path built on theoretically founded growth vectors, attempting to draw a growing steady state as a result of consistent long run equilibrium conditions.

This version is a simple close-economy model that likely undervalue some of relevant mechanisms determining Brazilian prices, but its parsimony is required to allow dealing with the dimensionality problem in an augmented model with trends. On the other hand, we needed to generalize utility and production functions by taking functional forms that are not usual in DSGE literature but are highly valuable to improve balanced growth path decomposition and multivariate detrending.

Our methodology of filtering and estimation follows the one-step approach, as in Lafourcade and de Wind (2012) and Ferroni (2011). We estimate the parameters governing the filtering of excess trends, i.e. parameters of the theoretically founded stochastic trends, together with the DSGE model's structural parameters. According to Canova (2014),

6 Though, concerns about robustness and unbiased estimations are not the primary focus here.
7 NNS: Neo Neoclassical Synthesis. DSGE: Dynamic Stochastic General Equilibrium.

Ferroni (2011) and Cogley (2001), a joint estimation is always preferable to a two-step approach inasmuch as it avoids problems ranging from trend misspecification to wrong cross-frequency correlations[8]. We estimate the model using Bayesian techniques and a mixed-frequency data set with relevant macroeconomic variables.

We assess the properties of the estimated model in several dimensions to argue that even a simple close-economy model in a balanced growth path can reproduce traditional transmission channels and present competitive results in forecasting performance. Real-time revisions of the GDP growth trend provided by the multivariate filter can be compared to the concurrent detrending methodologies, while many outcomes of the DSGE filtering can clearly demonstrate its theoretical consistency and prove to be superior in terms of storytelling and value for policy analysis. Later, we introduce an application of the model to measure potential output in Brazil and illustrate the power of structural devices to tackle also theoretical issues, which contributes to enrich the debate about potential output and the implementation of monetary policy.

The road map of this paper is as follows. Section 2 outlines our model and defines the equilibrium conditions. Calibration, prior assumptions, and many of the model properties are finely discussed in Section 3. Section 4 discusses model-based definitions of potential output and shows our results for potential output dynamics in Brazil. A final section concludes.

2 Model Overview

This section outlines the general features of the model. According to the paper's purpose, the model is built to combine short run specifications and structural changes in the balanced growth path to allow for a consistent raw data fitting. The central hypothesis here is that the long run growth is driven solely by the structure of the real economy, so many features usually included into modern DSGE models should play no role to understand trends' behavior. However, as they can yet be helpful for identification and filtering of business cycles, some features are not entirely neglected and left out, despite the additional complexity involved. Nevertheless, the idea is to keep the framework as simple as possible.

The general equilibrium model is a simple, closed economy, comprising three types of economic agents: households, firms and the government. The long run setup formally includes four elementary trends in the scope of the production function: population,

[8] Ferroni (2011) suggests agnostic beliefs on trends specification even in the joint estimation approach, considering the previously referred problems of bias estimations brought up from wrong specifications. Lafourcade and de Wind (2012) consider using model-based restrictions on stochastic trend specifications instead of Ferroni's purely statistical specifications. Both methods ensure flexibility and hold the consistency of the one-step estimation. However, the theoretically founded stochastic trends specifications can deliver full-consistent multivariate trend-cycle decomposition and also affect the properties related to both high and low frequencies throughout the model.

technology, and labor and capital-specific factor productivities. We assume these four long run growth rates as parameters shifting over time, which generates a growing steady state. Output growth rate is determined by the production factors' growth rates, and then the economy is assumed to move along a consistent balanced growth path.

The one-step approach containing theoretically founded trends requires additional steps of increasing complexity towards the state-space linear solution, since trends are spread throughout the model and need to be solved jointly. As Lafourcade and de Wind (2012) states, the payoff is that the stochastic specification can capture data cointegration properties without which any long run analysis would likely be misspecified, and also the model is ready to be applied in exercises requiring short and long run dimensions.

2.1 Supply

The aggregate production function is the key specification to describe growth rates and states the growth vectors. We choose a more general specification that accounts for a time-varying elasticity of substitution[9], and also permits different productivity levels of capital and labor. A generalized production function broadens the possibilities of the model to catch some of the recent economic developments in Brazil still keeping the standard setup. Capital utilization is introduced for data fitting.

$$F(K_t, L_t) = A_t \left[\alpha (u_t K_t A_{Kt})^{\sigma} + (1-\alpha)(L_t A_{Lt})^{\sigma} \Big(\frac{K_t}{L_t} \Big)^{\mu\sigma} \right]^{\frac{1}{\sigma}} \tag{1}$$

where it apply the convectional assumptions of positive but diminishing marginal products, that is $F_{K,t} > 0, F_{L,t} > 0, F_{u,t} > 0, F_{KK,t} < 0, F_{LL,t} < 0, F_{uu,t} < 0$, and the Inada conditions. K_t and L_t are respectively the input factors capital and labor, u_t is the capital utilization level, A_{Kt} and A_{Lt} are specific time-varying productivity levels of capital and labor, and A_t is the general technology level of production. The parameters $\alpha > 0$, $\sigma > 0$ and $\mu \geq 0$ control the factors share, the substitution among factors, and the time changing in technical substitution, respectively. The elasticity of substitution collapses to a generalized CES function when $\mu = 0$, but varies over the time otherwise. When both $\mu = \sigma = 0$, the function converges to the traditional Cobb-Douglas specification.

The model is set to follow a balanced growth path. Since production grows at a rate g_t^Y, this rate can be easily disentangled into growth factors under the scope of the production function. Technology grows at a rate g_t^A, and capital productivity and labor productivity at rates $g_t^{A_K}$ and $g_t^{A_L}$, respectively, so as the population grows at a rate g_t^N. The variable elasticity of substitution also implies an additional vector improving growth that is related to the substitution of factors. Once capital grows at a higher rate than labor, then abundant capital can help to improve the marginal product of labor and increase the production. In another perspective, when comparing to the usual CES specification, the

[9] We follow the proposals in Lu and Fletcher (1968) and Revankar (1971) to set up a VES production function.

VES specification offers a nice decomposition of the labor-augmenting productivity into a substitution vector and a purely factor-specific productivity, the first being well-defined and controlled by the estimated parameter μ.

A competitive, representative firm that takes output and input prices as given produces the homogeneous good. Firm's problem is a one-period decision process to choose capital, labor, and capital utilization level. The first-order conditions are

$$\begin{aligned} p_t^Y F_{K,t} &= R_t^k + C(u_t)u_t && (2)\\ p_t^Y F_{L,t} &= W_t && (3)\\ p_t^Y F_{u,t} &= [C'(u_t)u_t + C(u_t)]K_t && (4) \end{aligned}$$

Capacity utilization function shows the usual properties at the steady state $C_u(\bar{u}) = 0, C_u'(\bar{u}) > 0$. The following functional form is assumed to represent the adjustment in use, where a is conveniently set to meet steady state conditions.

$$C(u_t) = a\frac{\varpi_C}{2}\Big(\frac{u_t}{\bar{u}}\Big)^2 + a(1-\varpi_C)\Big(\frac{u_t}{\bar{u}}\Big) + a\Big(\frac{\varpi_C}{2} - 1\Big) \qquad (5)$$

The marginal rate of technical substitution (therefore MRTS) is given by $\Psi_t = \dfrac{F_{K,t}}{F_{L,t}}\dfrac{K_t}{L_t}$ and varies over the time if $\mu > 0$, depending on the instantaneous ratio between effective capital and labor. The expression for the variable elasticity of substitution ϵ_t between capital and labor can be found after some algebra and, with $\mu = 0$, obviously the elasticity $\epsilon = 1/\sigma$ becomes constant over time.

$$\begin{aligned} \Psi_t &= \frac{1}{1-\mu}\left[\frac{\alpha}{1-\alpha}\Big(\frac{u_t A_t^K K_t}{e_t A_t^L L_t}\Big)^{\rho(1-\mu)} + \mu\right]\\ \epsilon_t &= \frac{1}{\sigma}\Big[\frac{\Psi_t}{\Psi_t(1-\mu)-\mu}\Big] \end{aligned}$$

2.2 Demand

The economy has a population N_t that grows at the rate g_t^N and comprisesa very large number of identical infinite-lived households who can choose consumption $\{c_t\}_{t=0}^{\infty}$ and supply labor l_t according to

$$\sum_{t=0}^{\infty} \beta_t U(c_t, l_t) \qquad (6)$$

with $\beta_t \in (0,1)$ being the discount factor parameter, which is allowed to drift over time. The time-varying β_t intends to catch permanent changes in the long run interest rate, as often argued in Brazil. So the model can provide a compatible long run interest rate path considering the structural behavior associated to permanent shifts in the discount factor and other determinants. The instantaneous utility is assumed to be a King-Plosser-

Rebelo[10] function, which ensures full compatibility with a balanced growth path.

$$U(c_t, l_t|c_{t-1}) = \nu_t^B \frac{[c_t - hc_{t-1}]^{(1-\phi)}}{(1-\phi)} \exp\Big[\eta(\phi - 1)\frac{l_t^{(1+\phi_L)}}{(1+\phi_L)}\nu_t^L\Big] \tag{7}$$

Each household derives utility from consumption of goods c_t relative to an external habit formation that depends on its past consumption with persistence h, and also perceives disutility from supplying labor services l_t. Utility also incorporates a consumption preference shock ν_t^B and a labor supply shock ν_t^L. Parameters $\phi > 0$ and $\phi_L > 0$ stand for elasticities of substitutions related to consumption and leisure, respectively, and η is the weight parameter. Households also choose capital stock k_t and investment i_t taking into account the following capital accumulation law of motion:

$$k_t = (1-\delta)k_{t-1} + [1 - \mathcal{S}(i_t|i_{t-1})]i_t \tag{8}$$

where $\delta \in (0, 1)$ is the depreciation rate and $\mathcal{S}$ is a non-negative adjustment cost function whose functional form is given by

$$\mathcal{S}(i_t|i_{t-1}) = \frac{\varpi}{2}\Big(\frac{i_t}{\nu_t^I i_{t-1}} - \bar{g}_i\Big)^2 \tag{9}$$

where ν_t^I is the usual adjustment cost shock in capital accumulation placed to fit short run movements in investment, and g_i is the steady-state growth rate of *per capita* investment.

The households' problem is to maximize the expected discounted utility flow subjected to the following budget constraint, in real terms:

$$c_t + i_t + b_t \leq w_t l_t + R_t^k k_t + R_{t-1}b_{t-1} \tag{10}$$

and considering the capital law of motion. R_t means the real interest rate. The first-order conditions with respect to $c_t, l_t, b_t, k_{t+1}, i_t$ are respectively,

$$\begin{aligned}
\lambda_t &= U_{c,t} - \beta_t h U_{c,t+1} && (11)\\
w_t &= -\frac{U_{l,t}}{\lambda_t} && (12)\\
\lambda_t &= \beta_t E_t[R_t \lambda_{t+1}] && (13)\\
q_t &= \beta_t E_t\{\frac{\lambda_{t+1}}{\lambda_t}[R_{t+1}^k + q_{t+1}(1-\delta)]\} && (14)\\
1 &= q_t\Big(1 - \mathcal{S}(i_t) - \mathcal{S}'(i_t)\frac{i_t}{\nu_t^I i_{t-1}}\Big) + \dots \\
&\quad \beta_t E_t\Big[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1}\mathcal{S}'(i_{t+1})\Big(\frac{i_t}{i_{t-1}}\Big)^2 \frac{1}{\nu_{t+1}^I}\Big] && (15)
\end{aligned}$$

[10] King et al. (1988).

where λ_t is the marginal utility of consumption (MUC) under habit persistence, and q_t is the real shadow price of capital.

2.3 Aggregation

Aggregation for most of the *per capita* variables over the population N_t is quite trivial:

$$
\begin{aligned}
C_t &= N_t \, c_t && (16)\\
I_t &= N_t \, i_t && (17)\\
K_t &= N_t \, k_t && (18)
\end{aligned}
$$

We want to recognize that part of population is not directly involved in labor activities and somehow take into account changes in Brazilian labor market suggested by data. In order to enrich labor market dynamics without incurring in additional modelling costs, we propose single linear specifications to account for short run changes in participation and employment rate.

Define labor force LF_t as a fraction of the population, also known as the labor force participation rate θ_t. Then, define employed population EP_t as a fraction of the labor force, which also determines the time-varying unemployment rate ϑ_t.

$$
\begin{aligned}
LF_t &= N_t \, \theta_t && (19)\\
EP_t &= LF_t \, (1 - \vartheta_t) && (20)
\end{aligned}
$$

Finally, we aggregate labor by summing up *per capita* labor supply over the employed population

$$\tilde{L}_t = EP_t \, l_t \tag{21}$$

2.4 Price and Wage Setting

Following the NNS tradition, we introduce a role for monetary policy by assuming nominal rigidities in prices and wages.

Intermediaries operating in monopolistic competition distribute goods from the representative firm to the households. There is a continuum of intermediaries indexed by j over the unit interval. They buy the homogeneous good produced by the representative firm at the same price p_t^Y, differentiate it and sell the varieties to the households. For convenience, we assume there is a perfectly competitive assembler that composes the varieties y_{jt} into a final good Y_t through a standard Dixit-Stiglitz aggregator:

$$Y_t = \Big(\int_0^1 y_{jt}^{\frac{\epsilon_t - 1}{\epsilon_t}} \Big)^{\frac{\epsilon_t}{\epsilon_t - 1}} dj \tag{22}$$

ϵ_t is the time-varying elasticity of substitution between the varieties that implies a time-varying markup in the market. Profit maximization by the final goods assembler implies a downward sloping demand curve for each variety of the final good and an aggregate price index:

$$y_{jt} = \Big(\frac{P_{it}}{P_t}\Big)^{-\epsilon_t} Y_t \tag{23}$$

$$P_t = \Big(\int_0^1 P_{jt}^{1-\epsilon_t} dj\Big)^{\frac{1}{1-\epsilon_t}} \tag{24}$$

Intermediaries face nominal rigidity *a la* Calvo to set prices. Intermediaries face each period a constant probability $1-\theta$ of setting prices in accordance to market conditions and a probability $\theta \in [0,1]$ of not doing so. In the last case, it can update the price considering past inflations and the long run inflation with a indexation weight $\chi \in [0,1]$. The long run inflation is assumed to be a simple mean between the inflation target and expected inflation two years ahead. So, the intermediate firm maximizes the real value of the expected discounted flow of profits considering all the probabilities of non optimal adjustment along the future paths, but subject to the demand curve for its variety and the indexation scheme.

A quite similar Calvo mechanism is assumed to operate in the labor market. We skip equations for briefness as the algebra involved are pretty straightforward in the DSGE literature[11].

2.5 Government

There is a central bank that changes the current nominal interest rate according to a Taylor-type rule. Monetary policy rule describes the partial adjustment in nominal interest rates to deviations of the CPI inflation from its explicit target, and to the model-based output gap.

$$\frac{R_t^{yrl}}{R_{LR}^{yrl}} = \Big(\frac{R_{t-1}^{yrl}}{R_{LR}^{yrl}}\Big)^{\gamma_R}\Big[\Big(\frac{\pi_{t+3}^{yoy}}{\vec{\pi}_t^{yoy}}\Big)^{\gamma_\pi} \tilde{y}_t{}^{\gamma_y}\Big]^{(1-\gamma_R)} z_R \tag{25}$$

where R_t^{yrl} is the yearly nominal interest rate at time t, R_{LR}^{yrl} is the yearly nominal interest rate in the long-run, π_{t+3}^{yoy} is the expected year-over-year CPI inflation one quarter ahead, $\vec{\pi}_t^{yoy}$ is the inflation target for that time, $\tilde{y}_t$ is the model-based output gap, and z_R is an AR process standing for actual deviations of the rule.

On the fiscal side, the government does three things: consumes a fraction of the private output, levies *lump sum* taxes on households, and issues debt b_t paying interest. We define primary surplus to GDP ratio τ_t as the difference between *lump sum* revenues to GDP ratio t_t and the government spending to GDP ratio g_t. The government budget

[11] Take samples in Smets and Wouters (2007); Castro et al. (2011); Lafourcade and de Wind (2012); Burgess et al. (2013).

constraint below says that primary surplus must equal the payment of interest on existing debt each period. R_t is the real interest rate at time t.

$$b_t = R_{t-1}b_{t-1} - \tau_t \tag{26}$$

The model turns out to be Ricardian, in the sense that it does not matter how the government finances its spending between taxes and bonds. Government spending moves procyclically, that means the short run deviations of the actual share g_t are fitted through an autoregressive process. The same way is done for public revenues, but the tax burden t_t is required to account for the public debt consolidation to close the model.

$$\Big(\frac{t_t}{\vec{t}}\Big) = \Big(\frac{t_{t-1}}{\vec{t}}\Big)^{\rho_\tau}\Big(\frac{b_{t-1}}{\vec{b}}\Big)^{(1-\rho_\tau)\gamma_B} \exp \epsilon_t^\tau \tag{27}$$

where $\vec{t}, \vec{b}, \vec{\tau}$ are the all-consistent long run targets for tax burden, debt-to-GDP ratio and primary surplus, respectively.

2.6 Market Clearing

Define aggregate supply Y_t as the output out of the implied real losses, that is:

$$Y_t\Omega_{Yt} = F(K_t, L_t) - \Delta_t - I_t\mathcal{S}(I_t|I_{t-1}) \tag{28}$$

where Ω_{Yt} is the dispersion associated to the price rigidity. As usual in the DSGE setup, we model a fixed cost Δ_t in production to favor calibration at the growing steady state. It has no relevant dynamics associated, but grows along the balanced growth path. Symmetrically, let Ω_{Lt} be the wage dispersion defined by the Calvo pricing setup, and define aggregate labor supply as

$$L_t^{supply}\Omega_{Lt} = \tilde{L}_t \tag{29}$$

On the demand side, the gross domestic product (GDP) is defined by the sum of the modelled components

$$GDP_t = C_t + I_t + G_t \tag{30}$$

Market clearing conditions for goods and labor are:

$$GDP_t = Y_t \tag{31}$$

$$L_t = L_t^{supply} \tag{32}$$

2.7 Exogenous Processes

Trends can be easily modeled within a DSGE model in many ways. However, formally including theoretically founded trends in a general equilibrium framework generates some cross-equation restrictions that connect tightly state and observation equations, which delivers a richer model-based structure describing low and high frequencies in data.

The balanced growth path model is defined by multiple stochastic trends in its growth vectors, as long as the production function and the utility function specifications allow the model to be carefully stationarized around the growing steady state. The solution can be seen as a approximation around a snapshot of growing steady state at an arbitrary date.

Generally speaking, there are two types of exogenous processes in the model: drifting and autoregressive processes. A generic exogenous variable X can be decomposed into two components:

$$X_t = X_t^{trend} X_t^{cycle} \tag{33}$$

The cyclical part is modelled as a mean-zero covariance stationary AR process that converges to one, that means:

$$X_t^{cycle} = (X_{t-1}^{cycle})^{\rho^X} \exp \epsilon_t^X \tag{34}$$

where $\rho^X \in [0, 1]$ is the autoregressive parameter and ϵ_t^X is the short run innovation to the variable X. The component X_t^{trend} represents the long run specification of the variable X, and is modelled as a integrated process as following:

$$X_t^{trend} = \bar{X}^{trend} \exp \epsilon_t^X \tag{35}$$

where $\bar{X}^{trend}$ is the value of X_t in a snapshot of the balanced growth path at an arbitrary date, and ϵ_t^X represents a permanent shock that drifts the long run steady state for the variable X. This general specification grants flexibility to model the exogenous variables and drifting parameters of the model. For example, the time-varying discount factor β_t can be defined such as β_t^{trend} means permanent changes in the discount factor and the cyclical part β_t^{cycle} is always 1, $\forall t$. Note carefully that the drifting parameter specification contains an important property, which is highly desired when taking expectations into the model:

$$E_t \log X_{t+s} = E_{t-1} \log X_{t+s} + \epsilon_t^X \tag{36}$$

So, the discount factor β_t is modeled as a parameter that drifts over time to help a consistent identification of permanent changes in the long run interest rate, as many Brazilian economists have argued in recent years. Also the inflation target is modeled as a parameter shifting over time.

Some exogenous variables contain both cyclical and trend components. The theoretical stochastic trends are placed in the growth factors $g_t^{A_K}$, $g_t^{A_L}$, g_t^N. All model variables can be easily related to those basic trends, according to endogenous consistency of equilibrium conditions. For instance, the equilibrium conditions determine uniquely the long run growth rate associated with technology g_t^A and the rate g_t^{VES} related to the factor substitution, and also the balanced growth path rate g_t^Y at which most of the aggregate variables grow. Of course, their cyclical components can show independent dynamics. Another example of endogenous consistency is on the *per capita* variables, which are required to grow at the TFP growth rate.

A remarkable setting is on the unemployment and the participation rates. Their cyclical part has nothing special, but the trend specification presents a noted variation:

$$\begin{aligned} \vartheta_t^{trend} &= \Big[\frac{g_t^{A_L}}{(g^{A_L})_t^{trend}}\Big]^{-\gamma_\vartheta} \exp \epsilon_t^{\vartheta} \\ \theta_t^{trend} &= \Big[\frac{(g^{A_L})_t^{trend}}{(g^{A_L})_{t-1}^{trend}}\Big]^{-\gamma_\theta} \exp \epsilon_t^{\theta} \end{aligned}$$

This exogenous setup is proposed aiming to link the two shares of the labor supply to the dynamics of labor productivity in the long run, that is, permanent changes in labor earnings and consumption patterns can modify the availability of workers in these two dimensions independently. It is noteworthy that these shares do not directly affect the parameters governing the balanced growth rates, but affect the growing steady state itself. The goal here is to improve data fitting and allow the model to consider an often-asserted long run shifting in Brazilian labor market. Our modelling choices differ from Alichi (2015) in several dimensions, but we share concerns about low frequency movements in production factors utilization driving the measurement of potential output.

Lastly, the remaining exogenous shocks are the preference shocks (ν_t^B, ν_t^L) and the adjustment cost in investment (ν_t^I). For these shocks, we suppose that only the cyclical specification is present, following the autoregressive specification above cited.

3 Model Estimation and Evaluation

Since the model contains structurally consistent specifications of trends and cycles, we can solve around the analytical steady state and find the reduced form of the model, such as the state-space representation that enables Kalman filter to compute likelihoods. Henceforth, the usual procedures of filtering, likelihood evaluation and the Bayesian estimation using Brazilian data can be applied. This section briefly describes the aspects concerning estimation, presents some basic outputs, and looks into selected properties of the model in order to evaluate accuracy and economic consistency.

3.1 Data

Some macroeconomic series available in Brazil are indeed quite short for reasons ranging from methodological changes to abrupt regime switching. The common way out is to shorten the samples to avoid spurious estimation and very poor fitting, as it is impossible for one model to care well of all the occurrences. However, some drawbacks of shortening the series are the lack of identification of parameters, and large variations in real-time filtering, which is already argued to be an unpleasant aspect for forecasting and policy analysis. It can be even worse if short series are previously treated by a external procedure that implies significant data wasting. Anyone can be rightly skeptical about the assumption of this survival residual data as being the real business cycles relevant for policy-making.

A large set of observable variables can improve identification. The strategy here takes advantage of the multivariate filtering process to observe data in mixed frequencies, in order to stretch the sample range and avoid data wasting. Data set is built on a monthly basis, and the values of quarterly series are placed in the last month of the quarter. The estimation runs over the sample from July 1999 to March 2015. All the observables are taken in levels immediately after going by a X13-ARIMA-SEATS seasonal adjustment and without any external detrending procedures.

National accounts are published by the Instituto Brasileiro de Geografia e Estatística (hereafter IBGE), and provide some of the quarterly series we use as observables, namely, Gross Domestic Product, private consumption, government consumption, and investment. Real changes indexes are provided with seasonal adjustment. To account for discrepancies between the real GDP calculated by the model as being the weighted sum of all the observed components and the observed GDP index, we add a measurement-type component in the aggregate demand equation 30, and do not discard the actual GDP index. Since the model cannot interpret the observables of external sector, but their dynamics can significantly explain most of the residuals, we define a leak variable that follows an autoregressive process.

Prices are all in a monthly frequency. The official inflation target index is the broad consumer price index named IPCA, also provided by IBGE. Aiming to assist the model to get closer to actual pressures in marginal costs, we use the wholesale price index produced by the Fundação Getúlio Vargas (hereafter FGV), named IPA-Market, as observable as well.

The Selic rate is the nominal interest rate that the Central Bank of Brazil (hereafter BCB) has chosen to operate the inflation-targeting regime since July 1999. We take monthly Selic as observable and calculate it as being the effective daily rate accumulated over the reference month and measured in yearly terms. Last, the monthly inflation target series is built by setting to each month the official target prevailing for the corresponding year. The inflation target is broadly informed by the BCB at least two years in advance.

Quarterly data series of use of installed capacity in industrial sector is provided by FGV, and we use it to position the levels of capital utilization. On the fiscal side, we observe the ratio of public sector borrowing requirements to GDP, released monthly by BCB/Economic Department, aiming to provide to the model some worthy information about effective fiscal conditions in Brazil.

Finally, four series related to labor market come from the monthly employment survey (PME) produced by IBGE, namely, population in active age, labor force, employed people, and average real wages earned by employed people.

Our data set of observables has been chosen considering data availability, macroeconomic relevance, and the closest links to the theoretical measures of the DSGE model.

3.2 Calibration, Priors, and Posterior Mode

The analytical calculation of the stationarized steady state of the model is surely one of the most relevant tasks in modelling, and additional attention has been taken to do it, seeking to simultaneously preserve economic meaning and ensure computational performance. The analytic steady state describes explicitly the long run growth of all variables, the steady relations between the aggregates, and makes their determinants all clear.

Suppose we take a snapshot of the growing steady state at January 2005[12]. Some key steady state relations and levels can be conveniently associated to this particular time frame in a parametric way, such that the Kalman filter can learn about the levels of variables containing unit roots and then anchor the growing steady state. Those anchoring parameters can be estimated as well, and this procedure delegates to Kalman filter all the handles of the steady state.

We estimate the model using Bayesian Maximum Likelihood (Ljung (1999)). Table 1 shows the parameter calibration. Both calibration and prior setting were done by following previous findings for the Brazilian economy, in special Castro et al. (2011), and also aiming to ensure sound parameter regions with economic meaning and reliable model properties. Trend components are expected to be smoother than cyclical components, thus priors on standard deviations of stochastic trends and parameter shifts are set to a smaller magnitude than those related to cyclical innovations of the related variable. Table 2 presents the estimation results.

3.3 Properties

In this section we explore model properties for selected shocks and check if results can reproduce the usual dynamics found for quarterly DSGE models. Our references are Castro et al. (2011) and Minella and Souza-Sobrinho (2009), two well-known working

[12] We have chosen this specific date based on uncertainty about GDP growth trend when using apolitical univariate methods. Figure 1 shows root squared pairwise distances between real-time trends produced by three univariate methods: demeaning, one-sided HP filter and two-sided HP filter. 2005M1 is in the first highlighted area characterized by relatively small methodological uncertainty. At 2005M1, the real-time assessment for GDP trend showed one of the lowest methodological uncertainty along the sample. Therefore, we estimate the DSGE multivariate trend at that moment, and set its initial value in the neighbourhood.

papers specially focused on recent developments of the Brazilian economy.

Impulse response functions can provide a standardized form to learn about model properties. Since their calculations are clearly defined for a large number of models that define structural shocks, impulse response functions can establish a baseline to compare behavior from very different modelling approaches. We measure our responses relative to the growing steady state.

Figure 2 shows the impulse responses to a monetary policy shock equivalent to 100 basis points in the yearly rate. Yet the model is monthly based, the responses present the expected hump shape and fit pretty well timing and persistence expected patterns. Effects on inflation reach a maximum about 4 quarters after the impulse, whereas GDP peaks earlier, mainly due to the impacts on consumption and government spending. The transmission mechanism is straightforward as follows. Real interest rate rises because nominal rigidities impose partial adjustment in prices, such that the return of savings increases and private consumption drops. Fall in domestic demand leads to weakened demand for production factors and fall in relative prices too, which ultimately lowers marginal costs and reduces inflation. Most variables are back to steady state before 24 months. Magnitudes on CPI inflation and GDP are all in accordance with the intervals in Castro et al. (2011) and the mean values in Minella and Souza-Sobrinho (2009), despite all the differences in frequency and transmission channels between the models.

Figure 3 shows how the economy responds to a temporary 1% technology shock. The transmission is in line with New Keynesian theory and has no new insights. The technological shock implies more output being produced for a given amount of inputs, which directly explains GDP increasing. Nominal rigidities prevent a prompt adjustment in prices, so that labor market adjusts reducing worked hours and increasing real wages, which then lowers marginal costs and consumer prices.

Monetary policy is able to react and does it immediately by cutting rates down as inflation is running low, seeking to recover real interest rates. However, as long as aggregate demand shows an excessive exuberance, central bank hikes the policy rate and real interest rates aiming to realign demand conditions. Note that the temporary technology shock stimulates aggregate demand across all its components. For instance, government spending exhibits a highly persistent dynamics to the shock.

Figure 4 shows responses to a government spending shock equivalent to 1% of GDP, which means about 5% increase of government spending. The nature of government demand shock implies a direct increase in GDP and can stimulate production to meet demand. Since demand for production factors increases, labor increases and also wages increase on impact, which leads to a pressure on marginal costs and consumer prices. In our model setup, augmenting government spending generates higher debt and a fiscal consolidation through *lump sum* taxes. Since all consumers are Ricardian, they reduce consumption responding to higher real interest rates instead of higher real wages, then the resulting effect is negative. The fiscal impulse causes shrinkage of private consumption and investment, so the fiscal multiplier is always below one over the horizon.

Monetary policy reacts to realign aggregate demand, but once it cannot target directly on the source of inflationary pressures, it has an auxiliary role in adjustment. The estimated parameters governing the fiscal structure have revealed a highly persistent nature of fiscal shocks in Brazil, which indeed masters the dynamics of the government spending shock.

Now, we look closer into the dynamics related to a parameter shifting and a permanent labor productivity shock. As discussed earlier, discount factor is allowed to shift in the model to capture commonly alleged permanent changes in the long run interest rate. Figure 5 shows how the β shifting works. Units in vertical axes are the gross levels of the growing steady state, and the economy is supposed to have followed the growth steady state up to time zero, when the shift occurs.

Long run interest rate drops when discount factor shifts up as expected, and we calibrate the parameter shifting to produce about 25 basis points drop in the long run interest rate. At time zero, a short run dynamics take place as long as the interest rate gap becomes strictly positive at the nominal policy rate prevailing before the shift. Tight monetary conditions discourage private consumption in favor of savings, and investment to GDP rises up. Higher β also fosters investment because it can smooth adjustment costs over time. The overall effect on GDP is positive and the output gap reaches its maximum about 18 months after the shifting.

Consumer prices fall in sequence of the shift because higher discount factor means that optimizing firms will weight by more a price adjustment in the future, and so they can set lower optimal price today. This deflationary effect is reversed few periods later, when a stronger aggregate demand can produce inflation. At that point, Central bank reacts to all positive gaps and starts a policy rate hiking. Despite the one-off shifting in β, the effects are highly persistent throughout the model. The set of short and long run responses outlines the theoretical structural requirements to fit in data an actual change in long run interest rate.

In a similar way, we explore the effects of a permanent labor-augmenting productivity shock. Some DSGE models present already a simplified growth structure that includes one growth vector, usually a labor-augmenting productivity (Castro et al. (2011), Burgess et al. (2013)). Figure 6 plots the dynamics for a permanent shift in the long run labor productivity growth rate. Once long run output growth increases and capital and GDP grow at the same rate in a balanced growth path, the K/L ratio is obviously higher, which means that capital grows faster than labor. Some input substitution takes place according to the VES production function. The VES growth vector also contributes to increase total-factor productivity. Because factor-specific productivities change, a higher rental rate of capital can drive an increase of long run interest rate consistent with the arbitrage between bonds and capital. Yet note that the estimated link between long run unemployment rate and long run labor productivity growth can produce a long run unemployment shifting[13].

Short run dynamics following the permanent shock in growth rates indicate a GDP increasing and peaking instantly. This is because labor income grows at once improving consumption at the same time as marginal costs peak down and drop consumer prices. Nominal rigidities are there to smooth the effects on wages and prices. Our estimates indicate that a stronger aggregate demand does not induce inflation all over the horizon, as found in BoE-Compass[14], but contrary to BCB-Samba[15], in which the policy rate is expected yet to hike to curb pressures on aggregate demand.

3.4 Filtering

This section presents the results of data filtering and smoothing using Kalman filter. For a given parametric version of the state-space representation, conveniently calibrated with estimated values of parameters, and using time series of observable variables, we can obtain smoothed estimates for all non-observable variables and innovations of the model. Smoothed time series are always model-dependent and also linked to a specific calibration set of parameters.

Missing values in observed time series are also calculated by Kalman filter. In this case, the filter matches all existing actual values and uses the information set available at that point in time to fill the gaps in observed time series. This Kalman filter built-in feature allows us to mix time series frequencies in a single data set. To do so, we assume the highest frequency as the base frequency of data set, and then assign missing values for the series that are only available in lower frequencies.

In practical terms, we have observed raw indexes for aggregate demand provided by IBGE only on a quarterly basis. So we assumed that the index values indeed correspond to the value of the end-quarter month. Missing values for intermediary months within the quarter are filled via the smoothing step of Kalman filter. The smoothed monthly IBGE index is shown in figure 7. Remind that the smoothing procedure is parameter- and model-specific, thus Kalman filter will calculate all the missing values again when time goes forward, as more information on the states becomes available. Minor revisions also can happen[16].

Figure 8 presents the GDP trend produced by the DSGE multivariate filtering approach when applied today to current Brazilian data. The bottom panel plots the corre-

[13] Remind that model's unemployment is exogenous in the short run, but in the long run we put a fancy link trying to capture some data comovements. Even so, we emphasize the model is not robust to explore deep questions on unemployment and labor market developments

[14] Burgess et al. (2013)

[15] Castro et al. (2011)

[16] As pointed earlier, the magnitude of revisions for the smoothed monthly GDP growth series are responsible for the disturbances in one-sided Hodrick-Prescott filtering reported in figure 9. The smoothed monthly IBGE index is the measure of GDP growth used as input for all detrending methods reported.

sponding GDP gaps. A simple visual inspection suggests a time-varying dynamics for the multivariate trend that demeaning fails to catch, for example. Comparing with Hodrick-Prescott filters that also produce time-varying trends, we find sensitive differences between the dynamics. While the multivariate trend looks smoother, but still recognizing changes over time, the usual trend from two-sided HP filter displays greater volatility, which means the corresponding HP gap is the smoothest between the selected methods. A practical implication is that the good part of variation in data is simply thrown out without any economic explanation, and a reduced dynamics is thenceforward assumed to be the real business cycles for model use. That is definitely an economically meaningless procedure to generate well-behaved gaps, and its main advantage is practicality.

One of the cons of the usual HP filter is the lack of robustness in the time dimension, since the filter generates big revisions as more information discloses. End-point revisions are clearly a problem for policy-making, and the one-sided HP filter is a statistical workaround to avoid revisions and stick to a previous path. Though this alternative can overcome the problem of end-point revisions, it is still empty of economic meaning. The reported output from the one-sided HP filter is a time-varying trend whose dynamics appears to be very lagged in relation to the contemporaneous dynamics of headline GDP growth. It seems that the approach waives temporal accuracy in exchange for a blind commitment to its previous output. This cannot be virtuous for economic analysis and policy-making. Robustness to time revisions as a built-in property of the one-sided HP filter should be carefully weighted in applied macroeconomics.

We can compare the detrending methods in terms of real-time outputs and analyse the revisions in GDP trend curve as time goes. See in figure 9 the graphs that present mean, maximum and minimum bounds, and selected quantiles for the revisions in real-time GDP growth trend. In general, all the methods appear to be not biased over the horizon, however bounds reveal that demeaning can review the trend up to 1 percentage point from one period to the following, and along all the past. In its turn, HP filter reviews its trends curve less than demeaning does in the short run (about 0.5 p.p., but symmetrically), and the magnitude of revisions declines far back. One-sided HP filter does not produce trend revisions by construction[17]. Note the DSGE multivariate filter produced lower bounds of real-time revisions than demeaning and HP filter. Figure 10 put all the methods in one single graph. Standard deviations of revisions over the horizon reported by detrending methods make clear that the DSGE multivariate filter and one-sided HP filter present very competitive outcomes. Yet, even though the one-sided HP filter is an open-and-shut parsimonious univariate method that ties naively to their past results, DSGE multivariate filter can produce low real-time revisions as well, without keeping a blind commitment to the past, besides it comes along with a fully-structural meaning.

[17] Revisions of one-sided HP filter in figures 9 and 10 refer to changes in filtered monthly GDP series, whose missing values in interim months can be slightly reviewed by the Kalman filter as time goes.

Besides the statistical benefits of multivariate filtering, its most interesting feature is the model-consistent decomposition of the filtered GDP trend. Since GDP growth can be disentangled in vectors under the scope of the production function, the multivariate approach provides a detailed economic interpretation of the growth process. Our generalized production function can yet account for an endogenous growth vector related to a particular labor-augmenting productivity driven by time-varying capital-labor elasticity of substitution.

Figure 11 shows the growth decomposition according to our estimated DSGE model. The solid line in black is the filtered monthly GDP growth trend measured as the accumulated rate in 12 months. The four-vector decomposition tells that population growth responds for the largest share in the GDP growth since 2000, but it has softly decreasing its relative importance in the last years[18].

The three other components refer to productivity and its model decomposition. They all have in common signals flipping between 2002 and 2005, from negative to positive growth rates. The traditional labor-augmenting productivity can be decomposed into two factors, one endogenous and another exogenous. Multivariate filtering tells that the productivity growth directly associated with labor factor increased in 2000-2005, remained almost constant for a while, and has decreased as of 2014. In its turn, the endogenous labor-augmenting productivity growth responds for a relatively small share and has kept its contribution over the years. An economic reasoning of the VES productivity is as following. Capital has grown more than population since 2004, so that there is relatively more capital involved in production, and part of the abundant capital can improve workers' productivity, generating an increase in the elasticity of substitution between factors. This Keynesian argument saying that there is an endogenous acceleration in labor productivity as long as the economy gets richer is weakly supported by our results, since VES productivity cannot account for more than a half percentage point in yearly rate of the long run GDP growth in 2000-2015.

The third vector in importance is the technology growth. In the scope of the production function, technology is a labor-augmenting productivity factor that allows labor to catch up output growth in a balanced growth path. Empirically speaking, it is the corresponding residual dynamics of the equation in which capital and labor factors and the output are measured by very close observables. So technological progress is the rationale behind the portion of output growth that cannot be directly explained via growth factors. Not surprisingly, our decomposition shows that the long run technology growth is a component presenting the richer dynamics in the considered time frame.

We can identify three phases in the historical panel plotted at figure 11. From 2000 to 2003, technology growth was steady and negative, meaning technological destruction. This is possibly the final stage of a cleansing period started in mid-1990 and

[18] This fact is strongly supported by detailed labor market data. Population growth variable is an observable that maps directly to the Population in Active Age time series from the PME survey (IBGE).

followed by monetary, fiscal, trade and banking reforms that inflicted great technological transformation in macro and micro levels. The second phase ranging from 2004 to 2009 was a fast-growing technology period, typically characterized by the commodity boom that boosted Brazilian exports. Considering the limitations of this version, we highlight that the model can still understand that actual GDP growth rates were stronger than expected, as long as the economy presented a high-level production without a corresponding growth in labor force. Thus, the model realizes that there was a reversal in the growth rates of the three labor-augmenting productivities. Finally, since 2009, Brazil has experienced an activity slowdown driven by both external and domestic situations. What the model can tell[19] is that the recent slowdown does not appear a purely cyclical phase, since it can be traced and explained by the deterioration in pace of all the vectors determining the long run growth rate.

The model-filtered Total-Factor Productivity growth is shown in the figure 12, as well as the filtered TFP growth long run trend. Generally speaking, our measure of TFP is filtered from the aggregated production function, and we should remark that none of the production factors is directly observable, as well as the total production. They are likely inferred from close observables such as GDP growth, labor force and unemployment rate. Alternative measures of TFP in Brazil are extensively calculated from available time series, and shown in figure 12. One common method calculates total industrial production over worked hours in industry [20], and of course drawbacks are in two dimensions: industrial production is not a complete measure of production, as well as worked hours, and worked hours also cannot be taken as a reliable reliable measure, considering the way the time series is calculated in Brazil. A second method consists of evaluating GDP growth per worker[21], and again one can point problems in two dimensions: GDP is an overall measure of activity, but does not reflect the gross production, and workers time series is not a satisfactory measure for effective labor.

All the measures have their shortcomings, but they appear to agree about the general level of TFP over the period 2000-2015. In particular, the periods presenting higher volatilities and discrepancies are associated to the effects of the 2002-2003 confidence crisis and the 2009-2010 financial crisis. Broadly speaking, Brazilian TFP has grown over that period. Our estimates for the long run TFP growth tell that the TFP had an upward trend since 2000 that decelerated as of the international financial crisis, and has been softly decreasing since 2010, accounting today for around 1.5 p.p. in yearly terms.

Long run trends are all consistent and based on the DSGE model growing steady state. The balanced growth path and its determinants change over time and some key parameters governing the steady state are allowed to shift as well. Hence, the resulting balanced growth path includes structural changes and Kalman filter can endogenously

[19] We remind the limits of this version to deal with external channels.
[20] Industrial Production: PIM-PF (IBGE). Worked Hours: PIMES (IBGE).
[21] GDP growth: National Accounts (IBGE). Workers: PIMES(IBGE)

decide between structural and cyclical dynamics to improve data fitting. Notice that any structural change has well-defined model response functions and accounts for different transmission channels, which means the filter cannot stack unfit dynamics so easily.

Back to our results of multivariate filtering, figure 13 shows the actual data and corresponding model-based long run trends for most relevant macro variables. Long run inflation in our model is a mix of inflation target and expected inflation 24-months ahead. The ratio is estimated about 0.90/0.10 and expectations can explain thin disturbances in long run inflation over the horizon, whereas the Brazilian inflation target has been stable since 2004. Further, figure 13 plots the filtered long run nominal interest rate which is model-consistent with growth rates and parameter shifts. Despite the downward dynamics in data, long run interest rate has not fallen below 10 percentage points in Brazil in recent years, as often argued. That means the structural channels of the model cannot support the argument of decreasing neutral interest rates, even considering discount factor shifting and changes in the balanced growth path. To put it in another way, the real interest rate in Brazil has been around 5.5 p.p. in yearly terms at least since 2000.

Another controversial debate involving Brazilian trends is about the long run unemployment rate. Actual data shows a downward pattern, but our filtered NAIRU is fairly steady over the reference period, which means the structural channels also cannot support such a decreasing trend in labor market. Remind that the trends of participation rate and unemployment rate are built to follow an exogenous process, combined with an endogenous channel that links permanent shocks in labor productivity to permanent changes in labor supply. This mechanism could not find corresponding permanent changes in labor productivity supporting a sharply downward unemployment trend, as often suggested. In other words, the model has chosen to interpret the observed sinking in unemployment time-series as a purely cyclical movement, which therefore requires a further adjustment to the regular conditions.

3.5 Forecasting performance

Now we evaluate some of the forecast properties of the model. To do so, we calculate root mean squared errors (RMSE) of the unconditional forecasts considering two time spans: the first ranges from January 2001 to March 2015, and the second considers only the forecasts reported in the last five years, i.e., from March 2010 to March 2015. A usual benchmark is to take naive forecasts, which means to assume the last observed value as the forecast for the following months. From each date in the time span, we produce forecasts for one to twenty four months ahead, and compare them to the naive forecasts.

Figure 14 shows these results for four key macroeconomic variables. The further, the higher the forecast error, and of course this is because the information set is kept unchanged over the entire horizon.

Generally speaking, forecasts results overcome the naive approach all over the horizon and considering both time windows. That is a nice performance, except for the year-over-year inflation in farther horizons. It is somehow expected once this version of the structural model does not feature important known vectors driving the inflation dynamics in Brazil, such as the exchange rate channel and the missing setup for administrated prices. These channels are successfully integrated in the SAMBA model's price setup, as described in Castro et al. (2011). Notwithstanding, we can compare the forecasting performance of our model and SAMBA for year-over-year inflation. SAMBA RMSEs are reported in the original paper (pages 62-64) and as being 1.74 p.p., 1.67 p.p., and 1.63 p.p., respectively for 12, 18 and 24 months ahead. Our RMSEs are respectively 1.78 p.p., 2.10 p.p., and 2.05 p.p., considering the general time span, and 0.35 p.p., 0.47 p.p., and 0.49 p.p., respectively, considering only the forecasts in the last five years. Thereby, we conclude that our model is indeed very competitive, despite its stylized price setup. Yet, in terms of numerical comparison, our forecast performance for the GDP growth and the nominal interest rate is sharply superior.

In comparison to the general time frame, the superior performance of the last five years time span means that model-based forecasting has becoming better than it was in the past. It can be analysed in several dimensions: i) data availability is higher in the recent time span, whereas missing values and early points of time series can hazard filtering of economic states; ii) larger samples can help Kalman filter to initialize levels rightly and reduce the relative relevance of particular time series; iii) economic predictability could have improved over the time, following the macroeconomic policy reforms and the institutional consolidation in the last two decades, so that economic fundamentals are more easily acknowledged even by a basic modelling outline.

Next, we present two forecasting exercises for selected unusual episodes of the Brazilian economy. The first episode refers to the developments of the global financial crisis within the domestic economy. As previously noted, this non-regular period placed policy challenges in many directions, including higher uncertainty about trends and cycles measurement. Figure 15 shows the unconditional forecasts run at January 2009 and the corresponding actual values for the year-over-year CPI inflation, the nominal interest rate in yearly terms, the 12-month accumulated GDP growth, and the average 12-month unemployment rate. Plots also include confidence intervals. Results show interesting short-term performance in terms of direction, timing and persistence, despite some actual magnitudes deviated from the forecast mean, albeit within the confidence bands. At that time, the model forecasted a long-lasting deflationary pressure with low output growth, slight unemployment increase, and smooth monetary policy reaction. However, actual monetary policy deviated fairly away from the conventional reaction to curb deflationary pressures, and that movement can explain much of the inflation forecast error and the earlier GDP recovery. SAMBA model[22] provided quite similar forecasts for this episode . Markets expectations and BCB Inflation Report forecasts presented superior overall

[22] Castro et al. (2011), page 114.

performance for their CPI inflation projections, however market expectations failed to forecast economic activity and monetary policy reactions at that time.

Two remarks should be made: first, our model does not have direct channels to describe in full the transmission of external episodes like the 2008 financial crisis to the domestic economy. Nevertheless, the exercise shows that the model could yet recognize resulting effects on the domestic economy coming from observed monthly data, since some deterioration started to appear in higher frequencies data. Secondly, results reported here are purely unconditional forecasts, which means only observed data were available and neither external projections nor judgment were joined in this procedure. As widely acknowledged today, pieces of external information, as well as technical judgment, can substantially improve DSGE forecasting in the short run.

The second exercise in figure 16 refers to a second phase of international deceleration marked by the deepening of sovereign debt crisis in Europe. Trade flows were largely affected throughout the world, at the same time as monetary policy responses abroad stimulated capital flows and exchange rate appreciation to emerging markets. None of these channels exist in this version of our model. However, unconditional forecasts in January 2012 accounted for a deflationary wave hitting the domestic economy, slowing GDP growth, increasing unemployment and asking for a mild monetary policy loosening cycle. GDP growth projections turn out to be close to actual data, although the actual reactions to this cycle showed sensitive differences from the projected path. In particular, model forecasts did not ask for such a loose monetary policy to balance the weakening in aggregate demand. All the previous remarks about model structure apply. Market expectations also could not catch some of developments in GDP growth and monetary policy.

Lastly, we present the recent performance in figure 17. The unconditional model forecasts are done in April 2015, therefore it is an out-of-sample forecast exercise. Our forecasts for the CPI inflation were quite close to market expectations in April 2015 and the BCB Inflation Report' forecasts in March 2015, for all the relevant horizon. We remind external sources take into consideration much more information than our model does. Short run forecasts showed to be close to the actual values, whereas the projections from 6-months ahead and on performed poorly. On the other hand, model forecasts for GDP growth and policy rate turn out to be good approximations to the actual data.

In general, we argue that our simple model can produce competitive forecasts for GDP growth and explain its determinants, even in exceptional periods of high uncertainty. However, a better overall performance depends strongly on the channels the model has to deal with relevant mechanisms of price setting in Brazil, namely the exchange rate pass-through and administrated prices.

4 Measuring the Output Gap

Many economic policy institutions around the world have put effort into research about measurement of potential output and the output gap. Output gap estimates have relevance to macroeconomic forecast but especially to real-time monetary policy-making, since credibility requires time-consistent decisions and precise calibration of instruments. We discussed earlier the drawbacks of usual univariate methods to produce a real-time measure of the output gap.

DSGE models have been contributing to empirical research about potential output. Once potential output is not directly observed in data, the methodology behind solution and estimation of general equilibrium models combines observable and non-observable variables using theoretically founded economic links and then it can be helpful to infer the economic slackness from close observables. Bayesian econometrics and Kalman filtering bring forward these technical advantages when produce consistent paths for non-observables and additionally provide a detailed structural decomposition of multiple shocks. Structural interpretation of historical data and disaggregation of transmission channels are such a rich output of DSGE models that allows exploring even more theoretical foundations, such as potential output estimates.

At this point, we might define potential output in the context of modern general equilibrium models. There are three conceptual definitions to be explored within the traditional framework built on the NNS theoretical background. From now on, we follow the potential output notions described in Vetlov et al. (2011):

1. *Trend Output* is the level of output resulting from the sequence of permanent stochastic shocks that characterizes the stochastic balanced growth path of the model; so as, it is affected only by unit roots that can permanently shift growth vectors (population, productivities, technology) and then change the growing steady state.

2. *Natural Output* is the level of output under flexible prices and wages, but still working on imperfectly competitive markets; more specifically, this is the output level when nominal rigidities are absent, but steady-state markups and markup shocks remain on the framework;

3. *Efficient Output* is the level of output under flexible prices and wages and under perfectly competitive markets, which means markups and markup shocks are zero all the times. Nominal rigidities and competition failures are absent, and pure business cycles and real rigidities explain short run fluctuations.

The corresponding gaps produced by these three notions of potential output hold a theoretical relationship. The output gap obtained from the DSGE long run output (namely, the model output gap) measures all the temporary fluctuations around the growing steady state. It is a gross measurement of the output slackness that can be qualified by using counterfactuals exercises to calculate underlying natural and efficient outputs.

The natural output gap is a refined measure obtained from the natural output. The underlying natural output is the result of a counterfactual exercise in which the state initialization and the sequence of shocks are the same as filtered before, but running an equivalent model without the nominal rigidities blocks. Thus, the difference between model output gap and natural output gap quantifies the overall effects of nominal rigidities on the economy over time.

In its turn, the efficient output gap is a refined slackness measure derived from the efficient output. The underlying efficient output is calculated in the same way as the natural output, but now running a modified version of model with perfectly competitive markets. Steady state markups are zero and markup shocks do not exist in this version, prices fluctuate freely and obviously there is no room for monetary policy in that scope. Therefore, the difference between the efficient output gap and the natural output gap quantifies the overall effects of competitive failures for the short run adjustment.

Figure 18 presents the measures of the output gap for the Brazilian economy using our DSGE model. We note that both model output gap and efficient output gap show higher volatility than natural output gap, as expected. Natural output gap identifies how much of economic slackness is specifically related to the nominal rigidities adjustment processes that monetary policy usually takes as a reference to stabilize prices. Thus, the natural output gap dynamics seem to have not claimed much monetary policy actions to respond to the implications of price sluggishness on the real economy. An alternative interpretation is that monetary policy could have been effective in closing the natural gap over the considered horizon.

Given the theoretical relationship between the three model-based measures of output gap, we can try to disaggregate the gross model output gap dynamics into three components: i) the real business cycles that push the economy away from the steady state; ii) the slackness caused by dynamic changes in the degree of price discrimination, i.e, changes in demand curves faced by firms operating in monopolistic competition; and iii) the dynamics purely associated with nominal rigidities and augmented deadweight losses. Figure 19 shows the decomposition.

Nominal rigidities account for a relatively thin fraction of the gross output gap dynamics, whereas real business cycles seem to explain the largest portion. Yet, observe that monopolistic competition seems to have had a balancing role to curb business cycles volatility, which means that prices have fluctuated a lot less than they would if firms were in a perfectly competitive environment. Hence, from 2004-2010, market power dynamics worked mostly absorbing business cycles volatility and it resulted in lowered output slackness. However, since 2013 it seems that the production sector can no longer absorb negative business cycles, and the contribution of markups to the output gap has been negative too.

Efficient gap dynamics shown in figure 18 appears to be not proportional to the natural gap all the time. This is expected since the model features a bunch of real rigidities and shocks producing inefficiency beyond sticky prices. Divine coincidence does not hold in our model, thus stabilizing inflation and natural output does not imply an output stabilization around the efficient rate. Saying in other words, there are trade-offs between stabilizing inflation and stabilizing the efficient output gap, and the central bank must be aware of them in order to make welfare improving policies, taking into special consideration the moments in which the distance between the two gaps increases. Though we acknowledge these trade-offs, optimal monetary policy is beyond the scope of this paper.

Potential output measures from DSGE multivariate filtering are an alternative to usual univariate procedures, but with bonuses of model decompositions and structural interpretation. Model potential output can be compared to the univariate estimates as a potential output measure driven only by permanent shocks shifting production factors. Refined measures like natural output and efficient output can also recognize temporary shocks hitting the economy as a relevant source of slackness for policy objectives.

Dynamics of potential output can be useful for policy-making, though the research is preliminary. Measures carry uncertainty, and so their practical use is still hazy. Qualified measures can step forward to guide policy analysis and decision-making in real-time, even if results are model-based. It seems clear that structural interpretation depends tightly on system properties like identification of key parameters and shocks and specification, notwithstanding the model must able to produce intelligible forecasts and present a reliable storytelling. Therefore, the most valuable DSGE feature is its internal consistency, which implies consistent forecasts and consistent potential output measurement.

Nevertheless, the DSGE approach shares with other methods many issues and have their own drawbacks, but also have the advantage of facing issues directly in the model and the ability to overcome some of the problems by changing specification or using strategies improving identification. We support DSGE multivariate filtering results as an additional, but strong contribution to the debate about potential output and output gap.

Despite the improvements in the measurement of potential output provided by our model, further advances in specification should tackle some of the identification issues in two branches: i) by elaborating missing blocks that are clearly related to business cycles in Brazil, which can help disentangling channels, enriching storytelling and improving forecast performance; and ii) by enlarging the data set to include other relevant observable variables, which can help to qualify transmission channels and quantify their relative importance, besides improving filter learnability.

5 Final Remarks

This paper describes a basic dynamic stochastic general equilibrium (DSGE) model with a detailed growth setup to explore multivariate filtering using raw data. Our model features the very basic NNS channels to describe business cycles, whereas a balanced growth path is derived from a generalized production function with variable elasticity of substitution. The stochastic specification of the balanced growth path allows the model to capture data relations and disentangle them into concurrent permanent and transitory disturbances.

Despite its light structure, the model exhibits properties quite close to those of central banks' applied DSGEs, in particular the Brazilian SAMBA model. Forecasts perform well relative to a random walk, and also are very competitive with SAMBA outputs. Using multivariate filtering, real-time revisions of estimates of the model trends are substantially lower than those produced by other univariate methods like demeaning or HP filter. These findings are in line with Benes et al. (2010). We argue there are yet other advantages in endogenous detrending, in particular the internal full-consistency and storytelling benefits. Forecasts include simultaneously business cycles and trend projections and policy advising can therefore be done on more solid foundations.

Counterfactual exercises are an advantage of the multivariate approach, and alternative theoretical measures of potential output for the Brazilian economy were calculated as of 2000. Corresponding estimates of output gap dynamics show that the central bank has faced trade-offs between stabilizing inflation and stabilizing output, mainly due to sources of inefficiencies other than price rigidities. An advanced discussion on this topic may require a richer model structure.

Future work should focus on augmenting our base model with relevant missing blocks, in order to have a better understanding of inflation dynamics in Brazil. Moreover, a more complete description of real business cycles can likely improve identification of endogenous trends and their determinants.

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A Tables and Figures

Table 1: Calibrated relations and parameters

Description		Value
Steady State Relations and Parameters		
G/GDP	Government Spending to GDP ratio	0.201
I/GDP	Investment to GDP ratio	0.185
τ/GDP	Primary Surplus to GDP (%)	2.0
u	Capital Utilization (%)	82.571
π_C	Inflation (% yearly)	4.5
δ	Capital depreciation (% yearly)	6

Table 2: Estimated Parameters and Shocks

	Description	Prior Distribution			Posterior
		Distribution	Mean	Std Dev	Mode
	Preference and Technology				
h	Habit persistence	Beta	0.80	0.05	0.691
ϕ	Inverse of intertemporal elasticity of substitution	Gamma	1.00	0.20	0.899
ϕ_L	Inverse of the elasticity of labor supply	Gamma	2.00	0.20	0.639
σ	Elasticity of Technical Substitution	Gamma	0.50	0.20	1.302
μ	Time-varying elasticity of technical substitution	Gamma	0.10	0.05	0.228
α	Capital share	Normal	0.35	0.01	0.360
ϖ	Adjustment cost in Investment	Gamma	0.10	1.00	20.877
ϖ_C	Adjustment cost in capital utilization	Gamma	0.10	1.00	1.093
	Nominal Rigidities				
α_Y	Calvo parameter for prices	Beta	0.85	0.10	0.815
α_L	Calvo parameter for wages	Beta	0.85	0.10	0.707
χ_Y	Price indexation	Beta	0.50	0.15	0.034
χ_L	Wage indexation	Beta	0.50	0.15	0.514
η_Y	EoS of intermediate goods	Gamma	11.00	1.00	8.813
η_L	EoS of differentiated labor	Gamma	8.00	1.00	8.152
	Policy rules				
γ_R	Interest rate smoothing	Beta	0.60	0.15	0.894
γ_π	Inflation coefficient	Gamma	3.00	0.50	1.291
γ_y	Output gap coefficient	Inverse-Gamma	0.05	Inf	0.018
$\gamma_{\vec{\pi}}$	Inflation targeting anchoring	Beta	0.85	0.10	0.892
γ_B	Government debt coefficient	Inverse-Gamma	0.50	Inf	1.622
	Growing Steady State Relations				
γ_ϑ	Long run decay in unemployment	Normal	0.00	0.50	0.830

Continued in next page

Table 2 – (cont.)

Description		Prior Distribution			Posterior
		Distribution	Mean	Std Dev	Mode
γ_θ	Long run decay in labor participation	Normal	0.00	0.50	-0.995
$g_{GDP}^{(2005M1)}$	Long run GDP growth at 2005M1	Normal	2.50	0.50	3.456
$g_{A}^{(2005M1)}$	Long run Tech growth at 2005M1	Normal	-0.50	0.50	0.259
$g_{N}^{(2005M1)}$	Long run Population growth at 2005M1	Normal	1.00	0.20	1.843
$R_{LR}^{(2005M1)}$	Long run Interest Rate at 2005M1	Normal	10.00	0.50	10.559
$\vartheta_{LR}^{(2005M1)}$	Long run Unemployment Rate at 2005M1	Gamma	8.00	1.00	10.019
$\theta_{LR}^{(2005M1)}$	Long run Participation Rate at 2005M1	Gamma	56.00	3.00	56.022
Autoregressive shocks					
ρ_G	Government consumption	Beta	0.60	0.15	0.986
ρ_T	Lump sum taxation	Beta	0.60	0.15	0.990
ρ_R	Monetary policy	Beta	0.60	0.15	0.719
ρ_{η_Y}	Price markup	Beta	0.60	0.15	0.985
ρ_{η_L}	Wage markup	Beta	0.60	0.15	0.892
ρ_{A_K}	Capital productivity	Beta	0.60	0.15	0.260
ρ_{A_L}	Labor productivity	Beta	0.60	0.15	0.844
ρ_A	Temporary technology	Beta	0.60	0.15	0.376
ρ_N	Population	Beta	0.60	0.15	0.016
ρ_ϑ	Unemployment	Beta	0.60	0.15	0.985
ρ_θ	Labor participation	Beta	0.60	0.15	0.906
$\rho_{\mathcal{S}}$	Adjustment cost in Investment	Beta	0.60	0.15	0.755
ρ_{GDP}	GDP residuals	Beta	0.60	0.15	0.985
ρ_{ν^B}	Preferences	Beta	0.60	0.15	0.387
ρ_{ν^L}	Labor supply	Beta	0.60	0.15	0.987
Transitory shocks					
ϵ_G	Government spending shock	Inverse-Gamma	10.00	Inf	9.997

Continued in next page

Table 2 – (cont.)

Description		Prior Distribution			Posterior
		Distribution	Mean	Std Dev	Mode
ϵ_T	Lump sum taxation shock	Inverse-Gamma	10.00	Inf	12.795
ϵ_R	Monetary policy shock	Inverse-Gamma	10.00	Inf	0.212
ϵ_Y	Price markup shock	Inverse-Gamma	10.00	Inf	12.221
ϵ_L	Labor markup shock	Inverse-Gamma	10.00	Inf	18.534
ϵ_{A_K}	Capital productivity shock	Inverse-Gamma	10.00	Inf	28.930
ϵ_{A_L}	Labor productivity shock	Inverse-Gamma	10.00	Inf	2.047
ϵ_A	Temporary technology shock	Inverse-Gamma	10.00	Inf	3.793
ϵ_N	Population shock	Inverse-Gamma	10.00	Inf	1.579
ϵ_ϑ	Unemployment shock	Inverse-Gamma	10.00	Inf	34.220
ϵ_θ	Labor participation shock	Inverse-Gamma	10.00	Inf	3.707
$\epsilon_{\mathcal{S}}$	Adjustment cost shock	Inverse-Gamma	10.00	Inf	13.718
ϵ_{GDP}	GDP residuals shock	Inverse-Gamma	10.00	Inf	5.241
ϵ_{ν^B}	Preferences shock	Inverse-Gamma	10.00	Inf	3.896
ϵ_{ν^L}	Labor supply shock	Inverse-Gamma	10.00	Inf	21.070
Trend shocks					
ζ_{A_K}	Capital productivity trend shock	Inverse-Gamma	1.00	Inf	10.573
ζ_{A_L}	Labor productivity trend shock	Inverse-Gamma	1.00	Inf	5.230
ζ_N	Population trend shock	Inverse-Gamma	1.00	Inf	4.884
ζ_ϑ	Unemployment permanent shock	Inverse-Gamma	1.00	Inf	0.000
ζ_θ	Labor population permanent shock	Inverse-Gamma	1.00	Inf	0.000
$\zeta_{\vec{\pi}}$	Inflation targeting shock	Inverse-Gamma	1.00	Inf	7.149
ζ_β	Intertemporal discount factor shift	Inverse-Gamma	1.00	Inf	4.175

Figure 1: Current value of GDP growth trend reported by detrending methods (real-time calculations)[23]

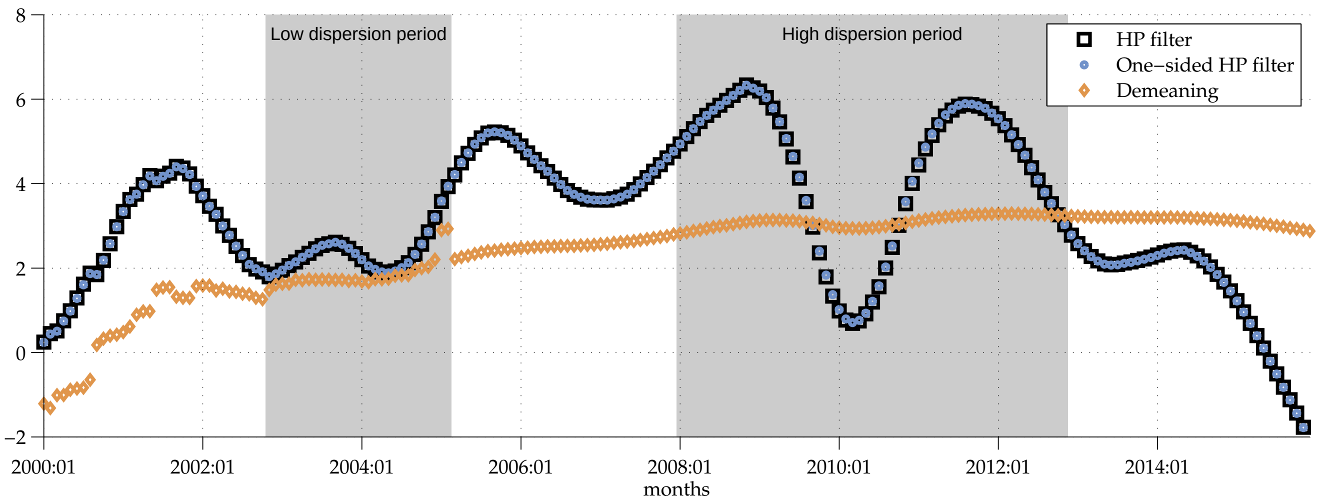


Figure 2: IRF to a monetary policy shock (100 b.p. yearly)

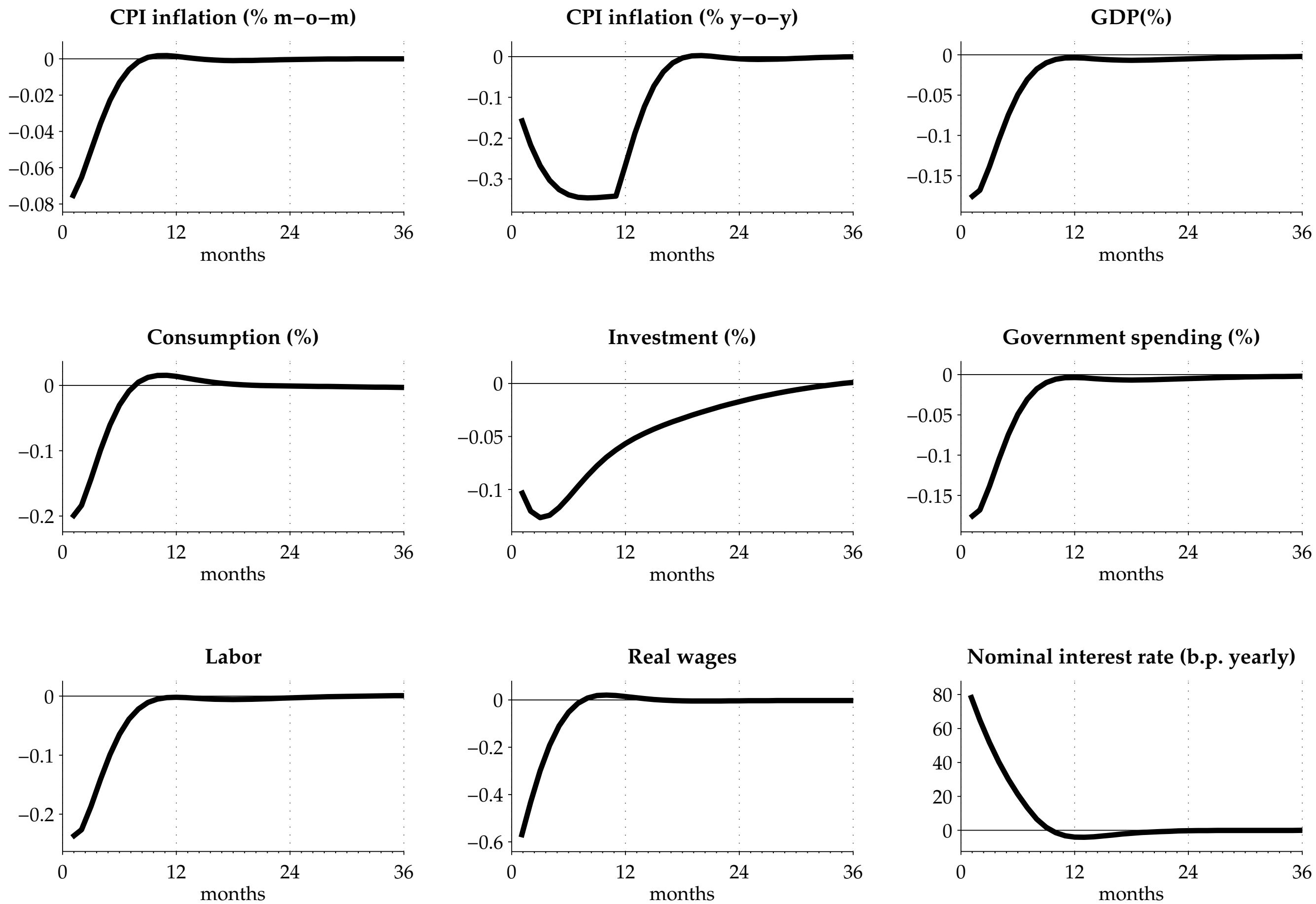


Figure 3: IRF to a 1% temporary technology shock

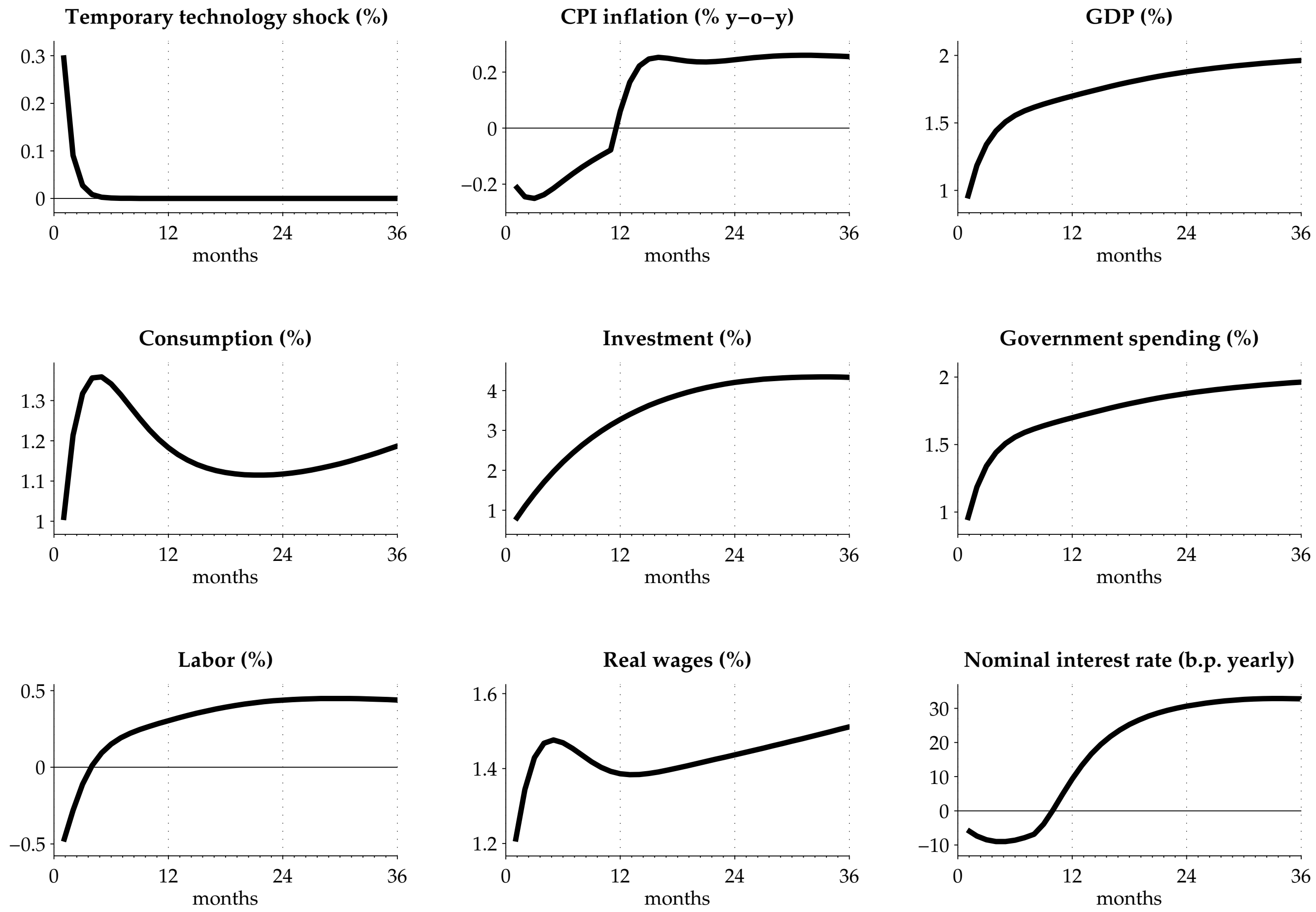


Figure 4: IRF to a government spending shock (equivalent to 1% of GDP)

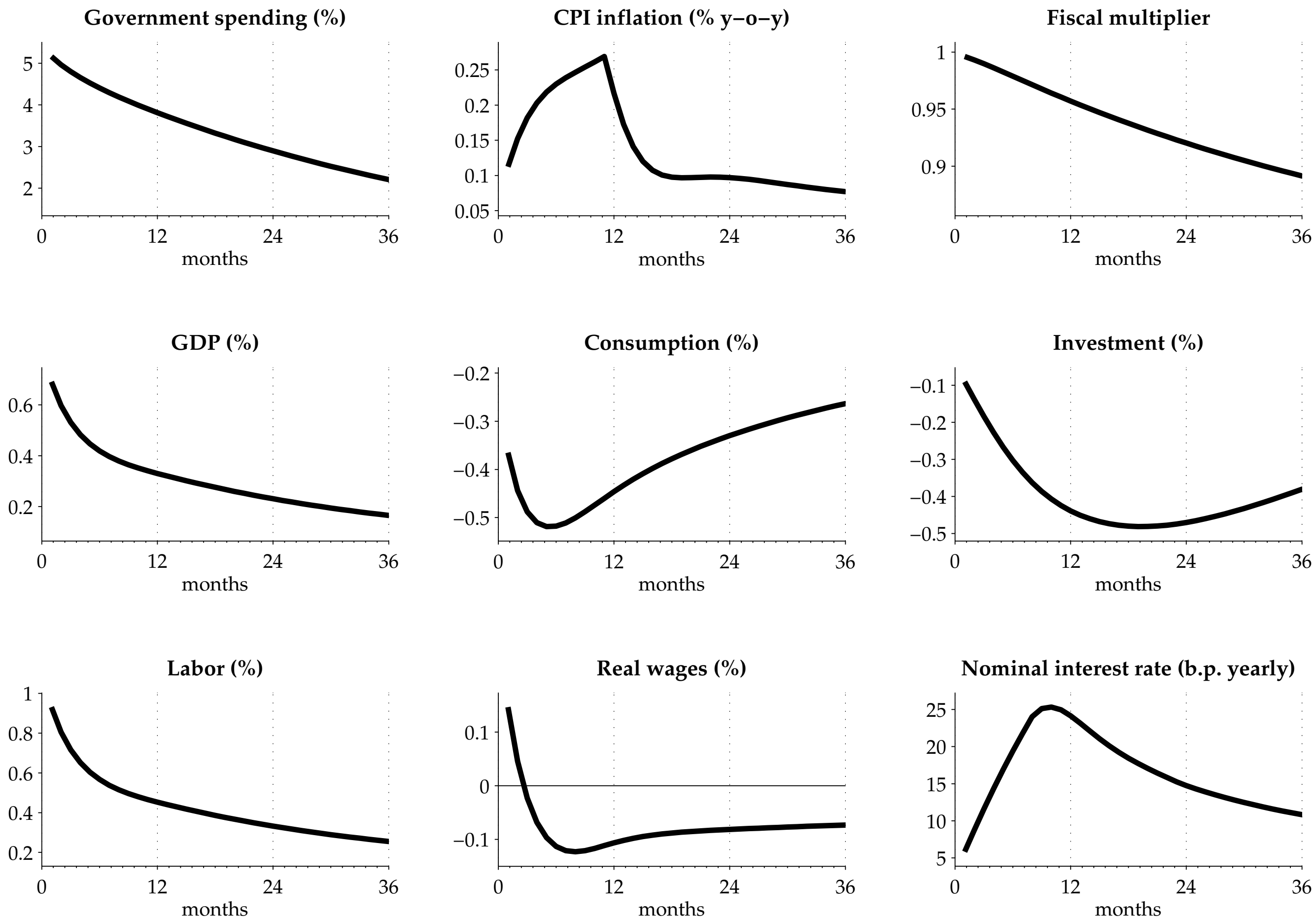


Figure 5: IRF to a shift in discount factor (equivalent to 25 b.p. drop in LR interest rate)

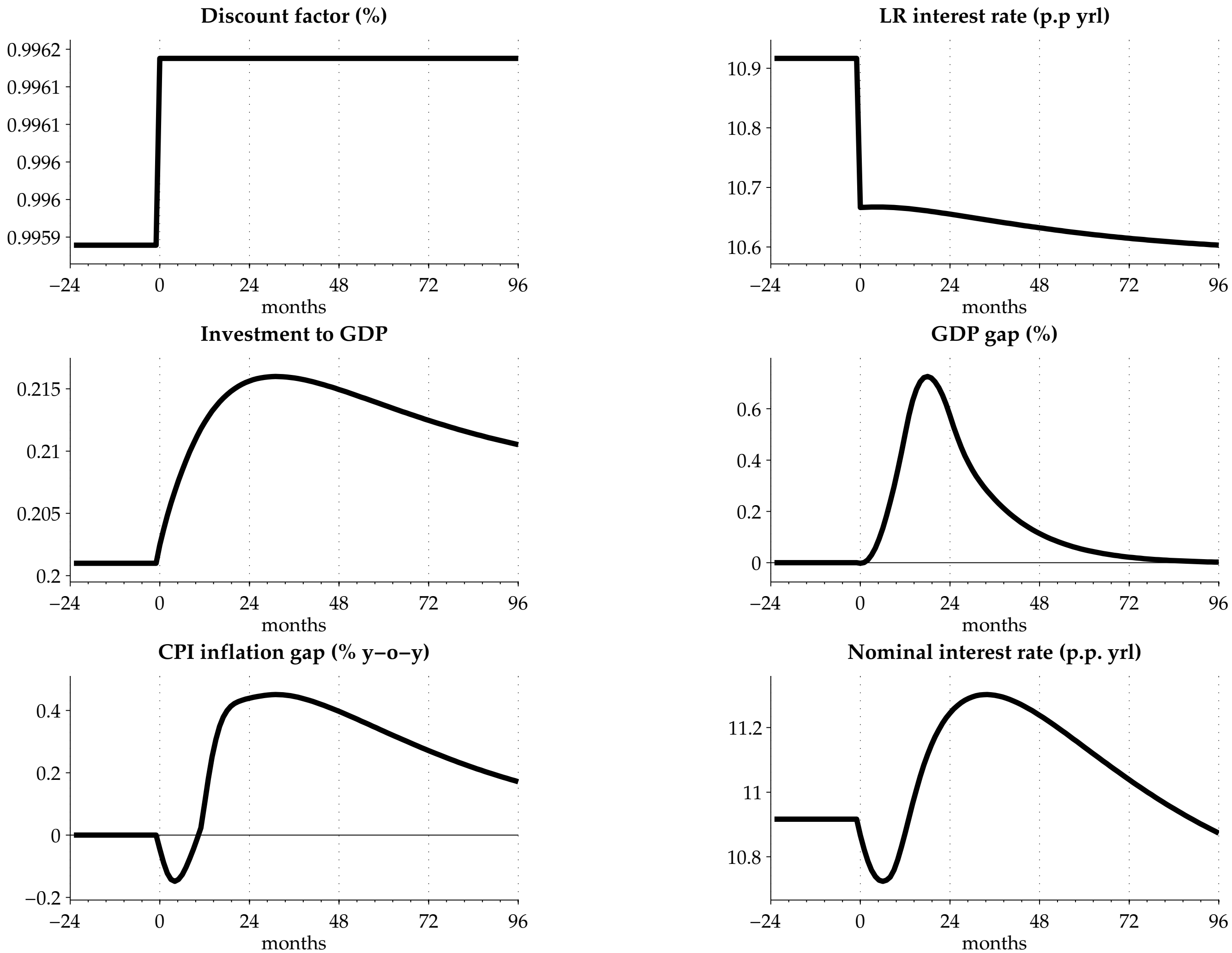


Figure 6: IRF to a permanent 1% labor productivity growth shock

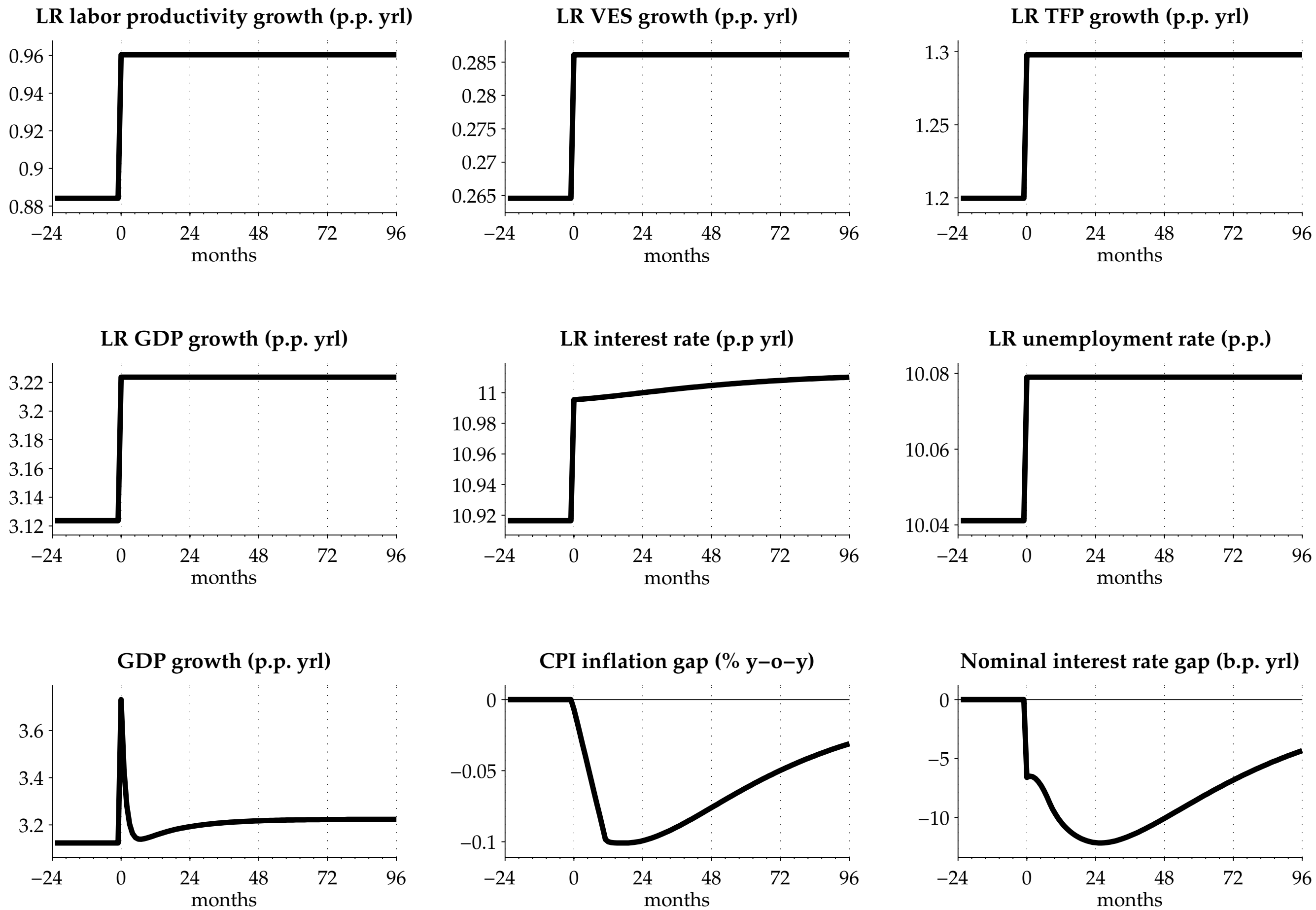


Figure 7: Filtered official GDP growth index

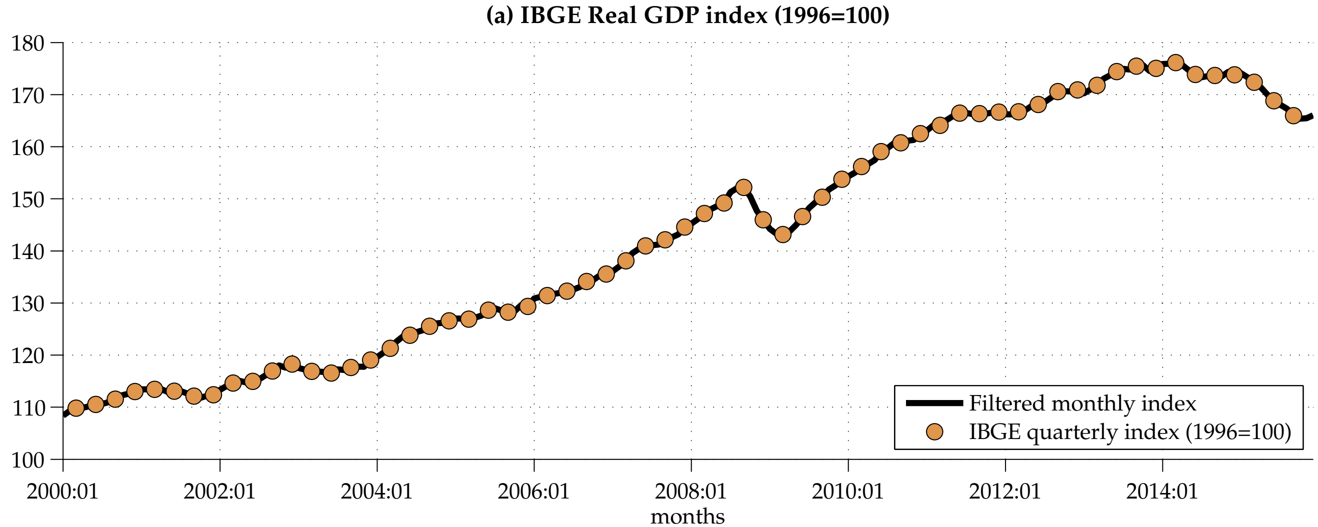


Figure 8: Trends and corresponding output gaps reported by detrending methods in December 2015

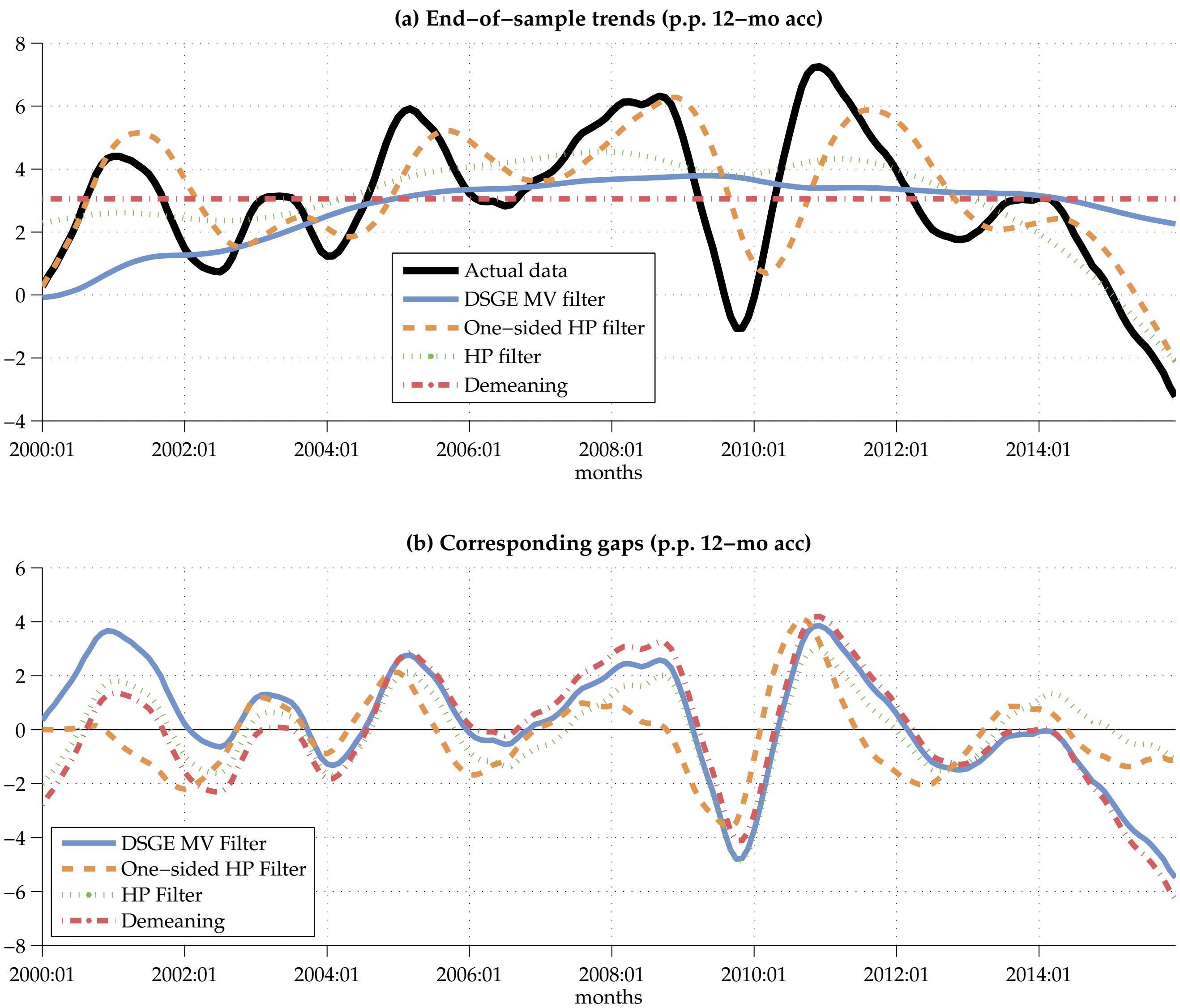


Figure 9: Real-time revisions of GDP growth trend

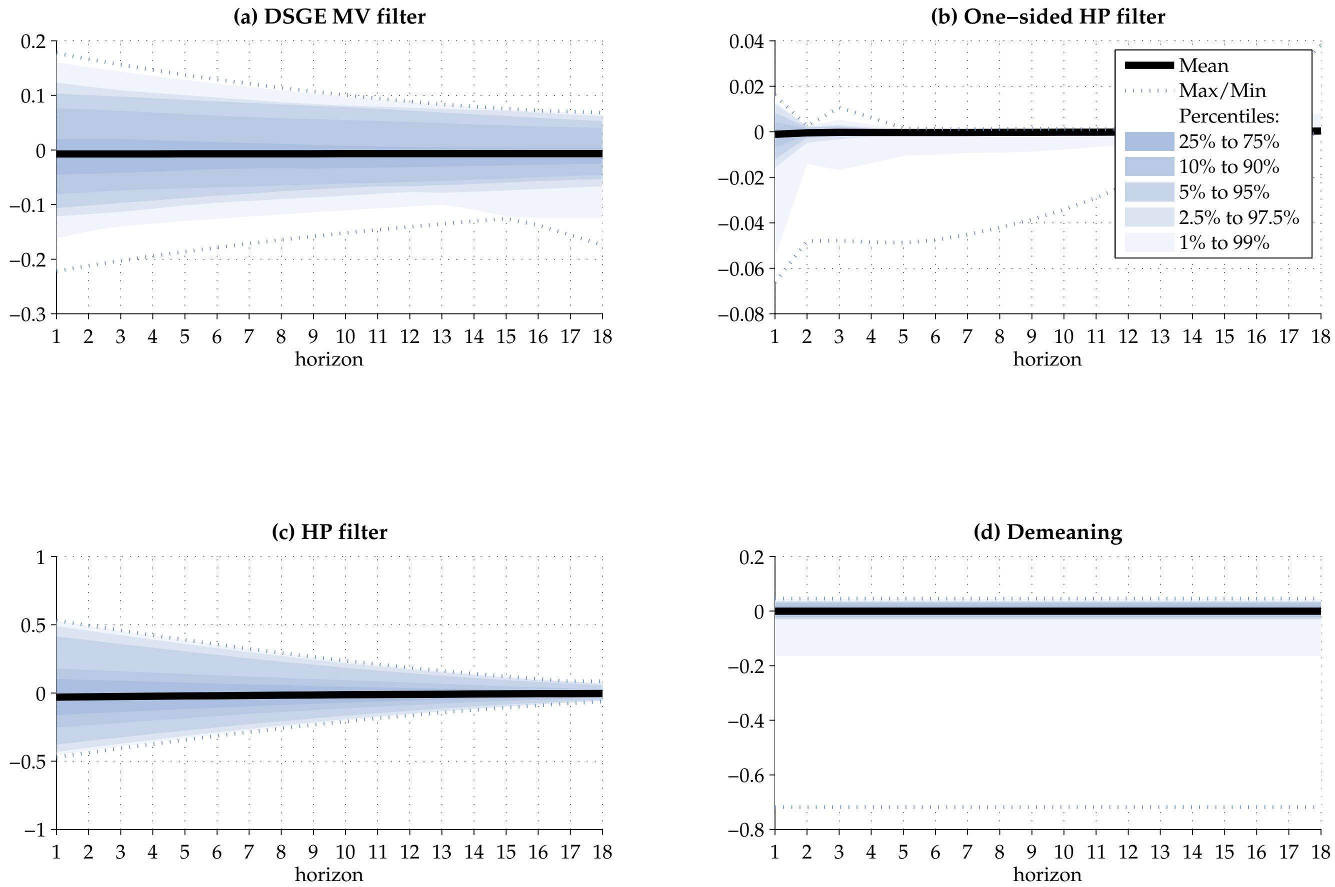


Figure 10: Standard deviations of trend revisions

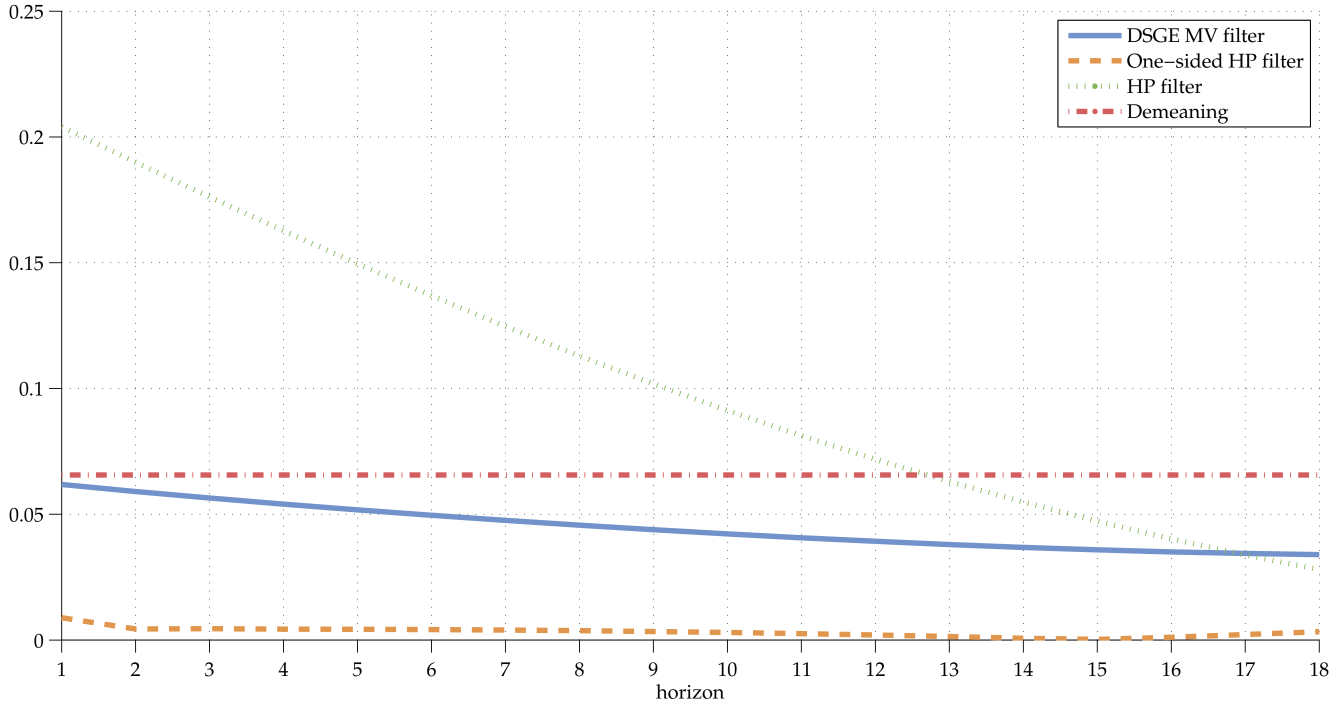


Figure 11: Long run GDP growth trend reported by the DSGE multivariate filter

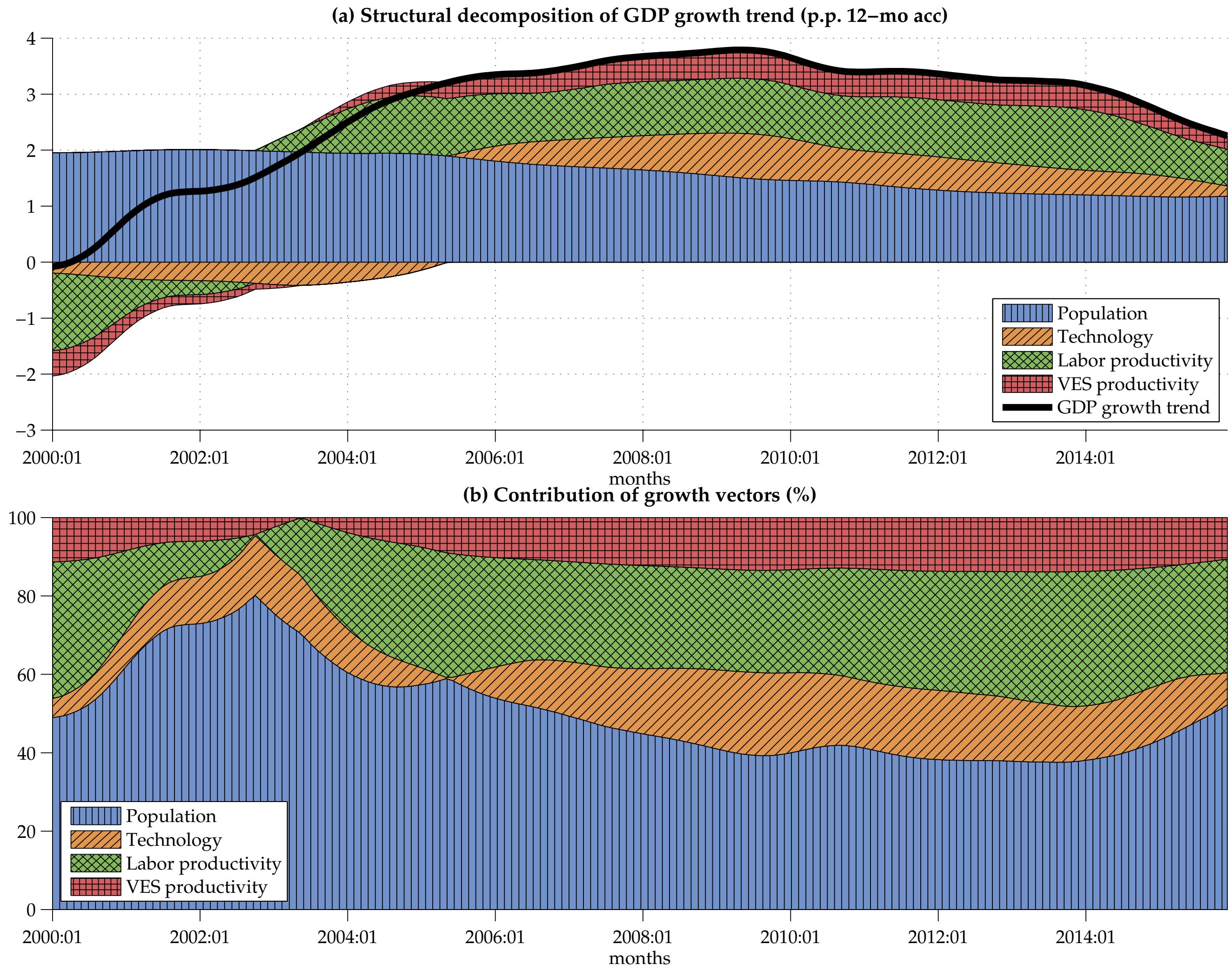


Figure 12: Comparison of measures of Total-Factor Productivity growth in Brazil

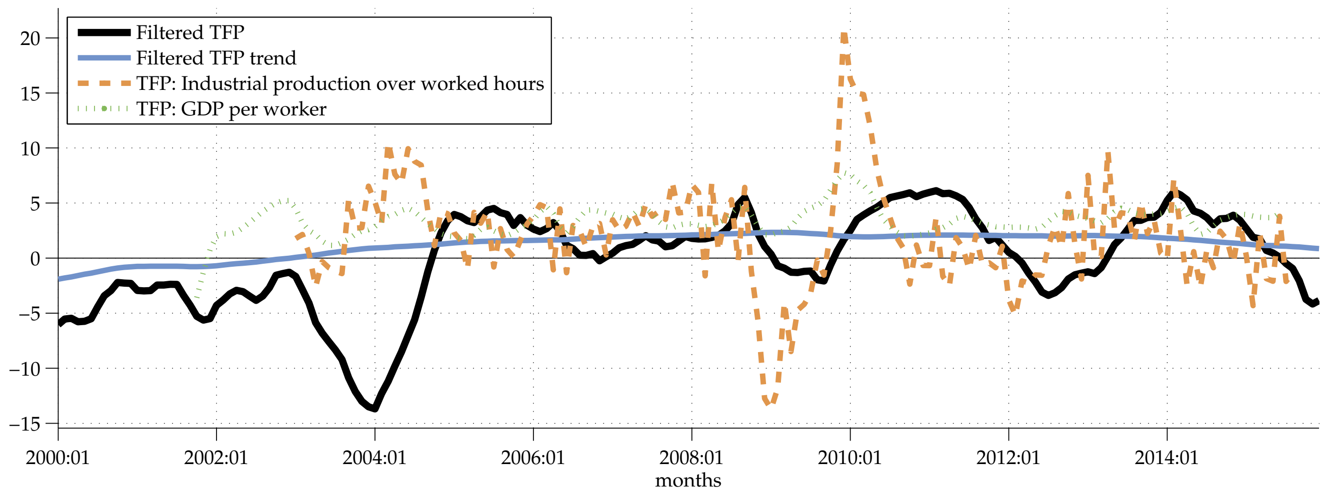


Figure 13: Main variables and their corresponding DSGE multivariate filtered trends

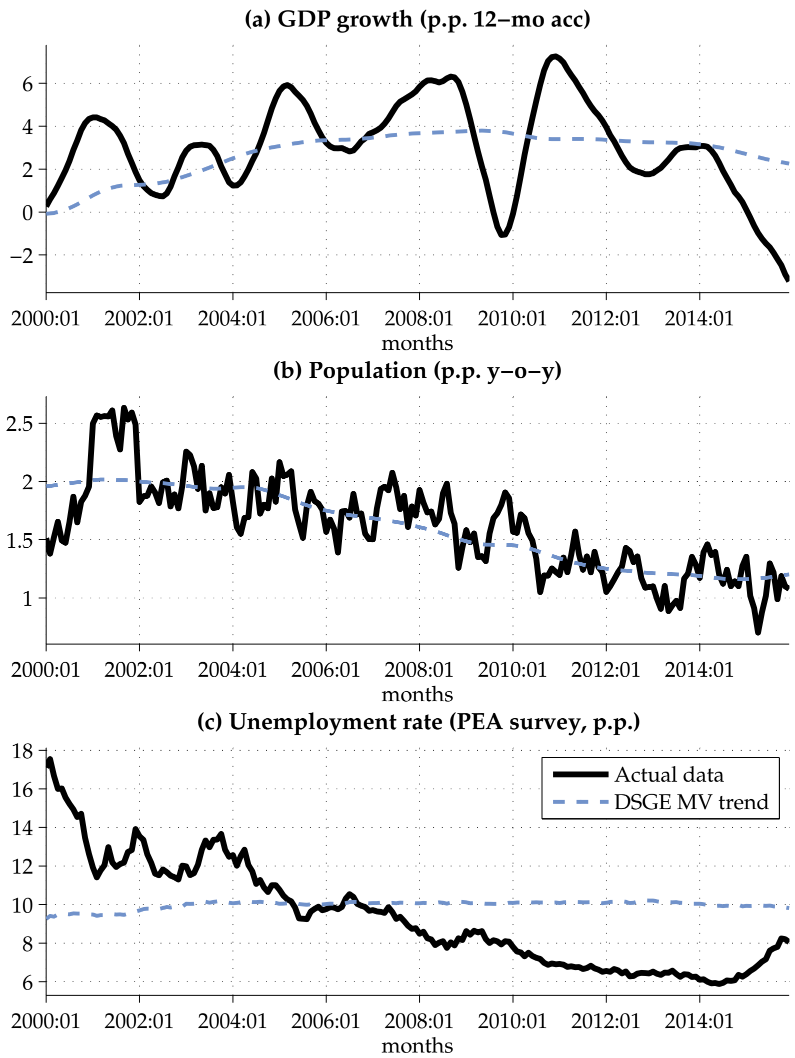
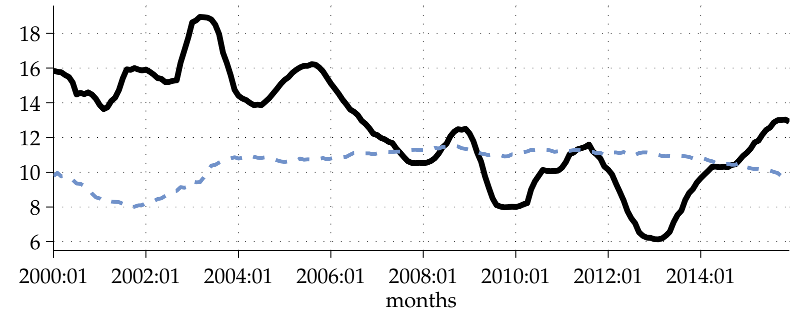
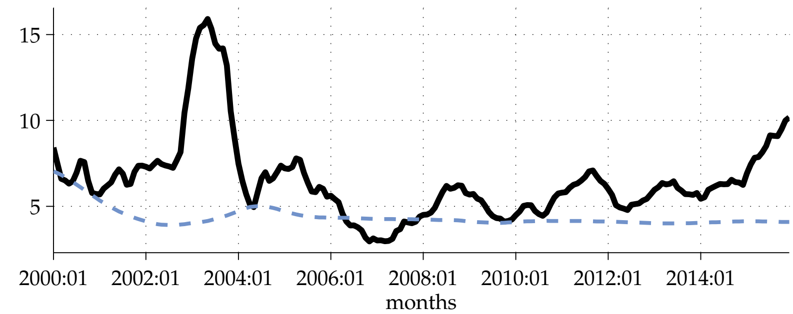


Figure 13: (continued)

(d) Nominal interest rate (p.p. yearly)



(e) CPI inflation (p.p. y–o–y)



(f) Real interest rate (p.p. yearly)

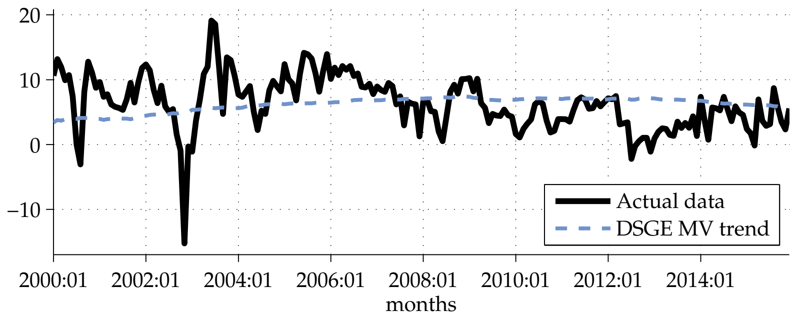


Figure 14: Root Mean Squared Errors for selected variables (real-time simulations)

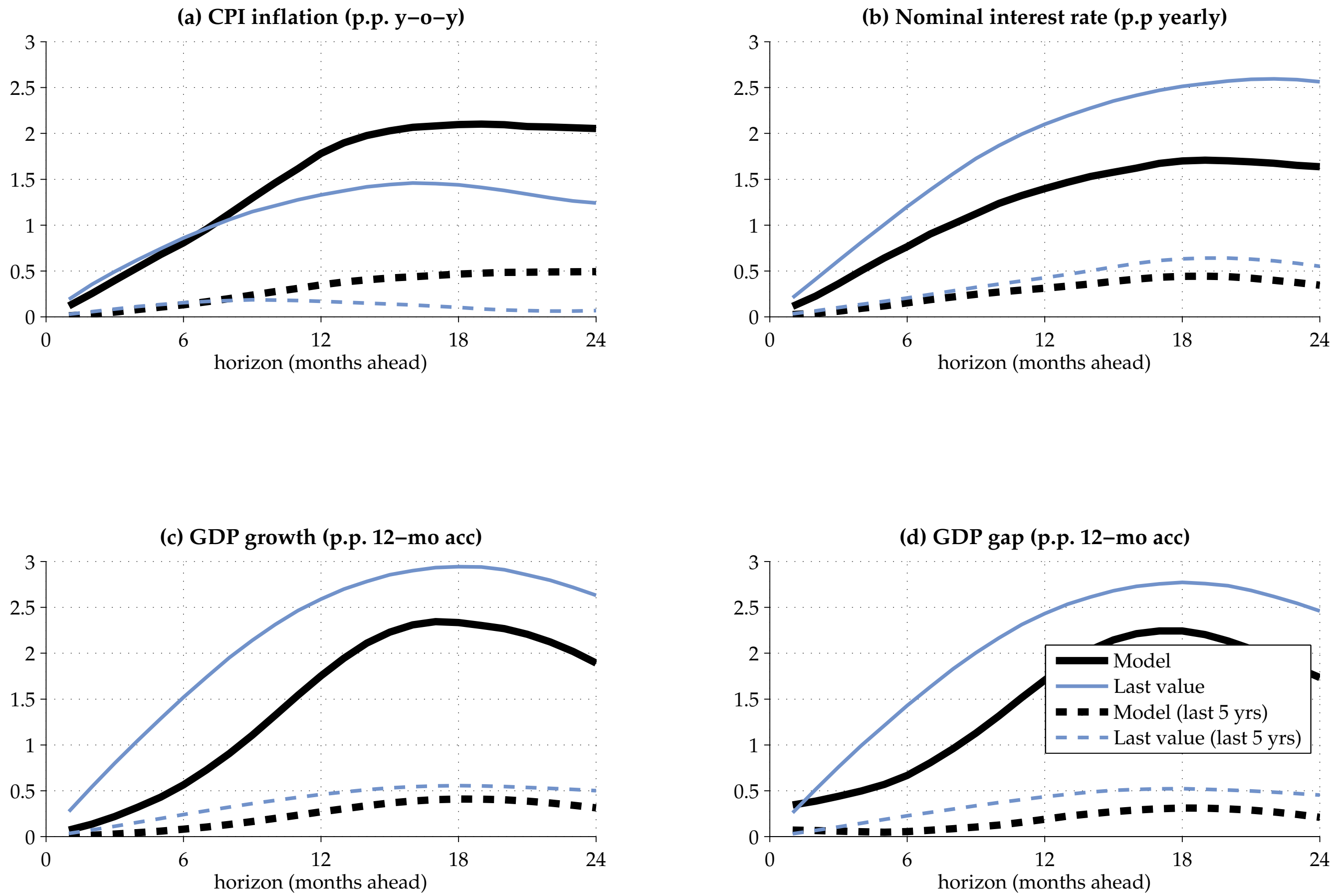


Figure 15: Unconditional forecasts in January 2009 (international financial crisis)

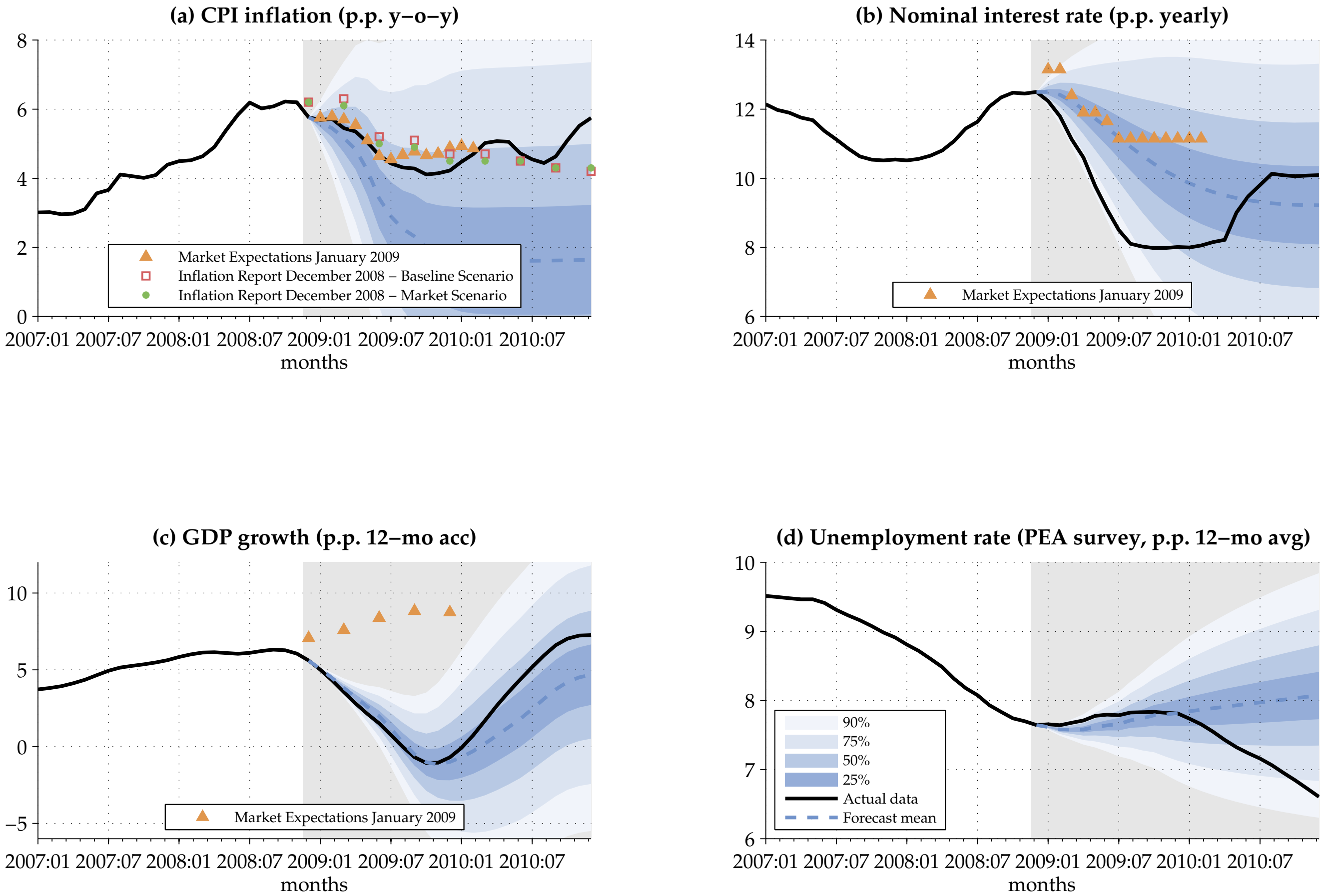


Figure 16: Unconditional forecasts in January 2012 (international scenario deterioration)

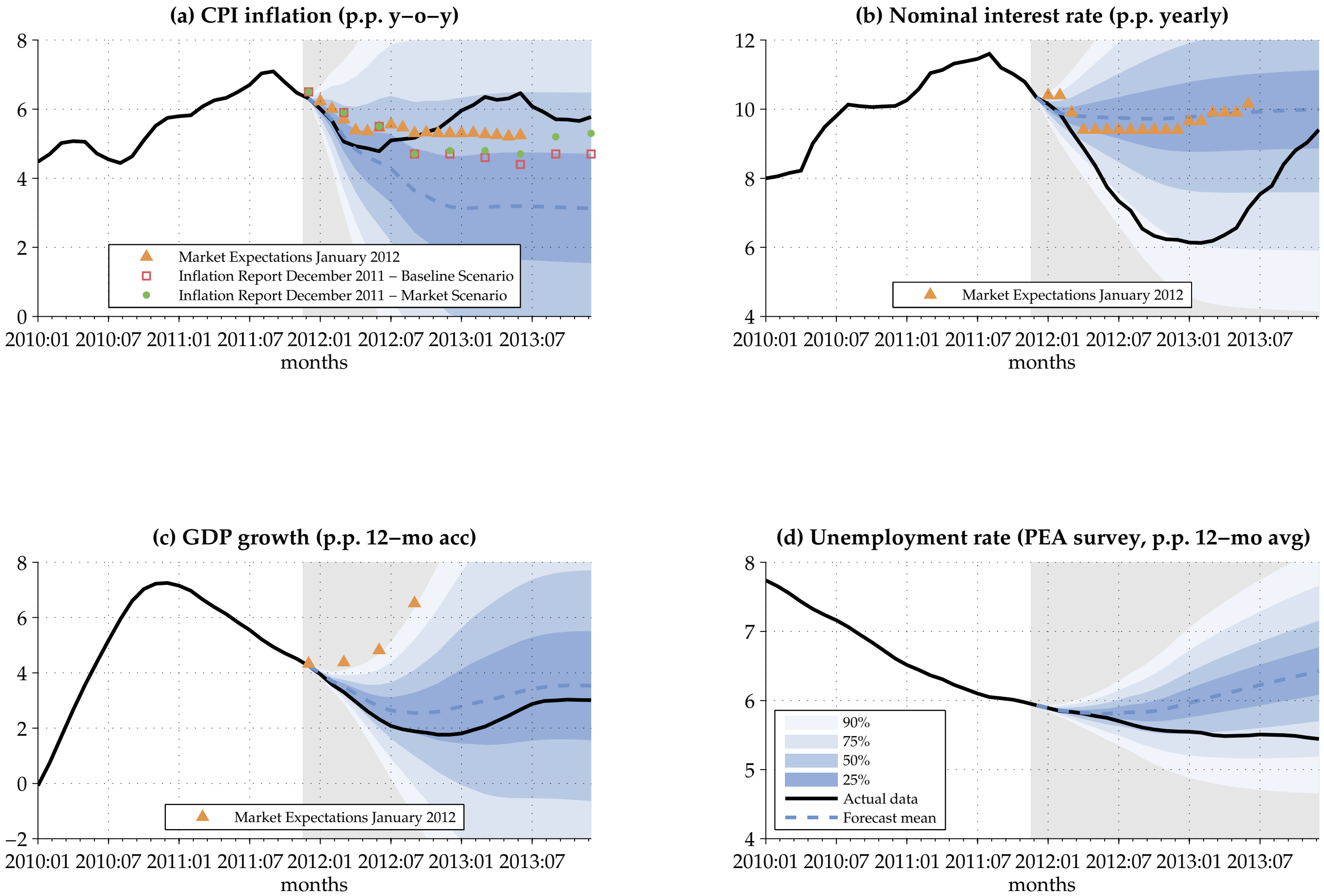


Figure 17: Unconditional forecasts in April 2015 (out-of-sample performance evaluation)

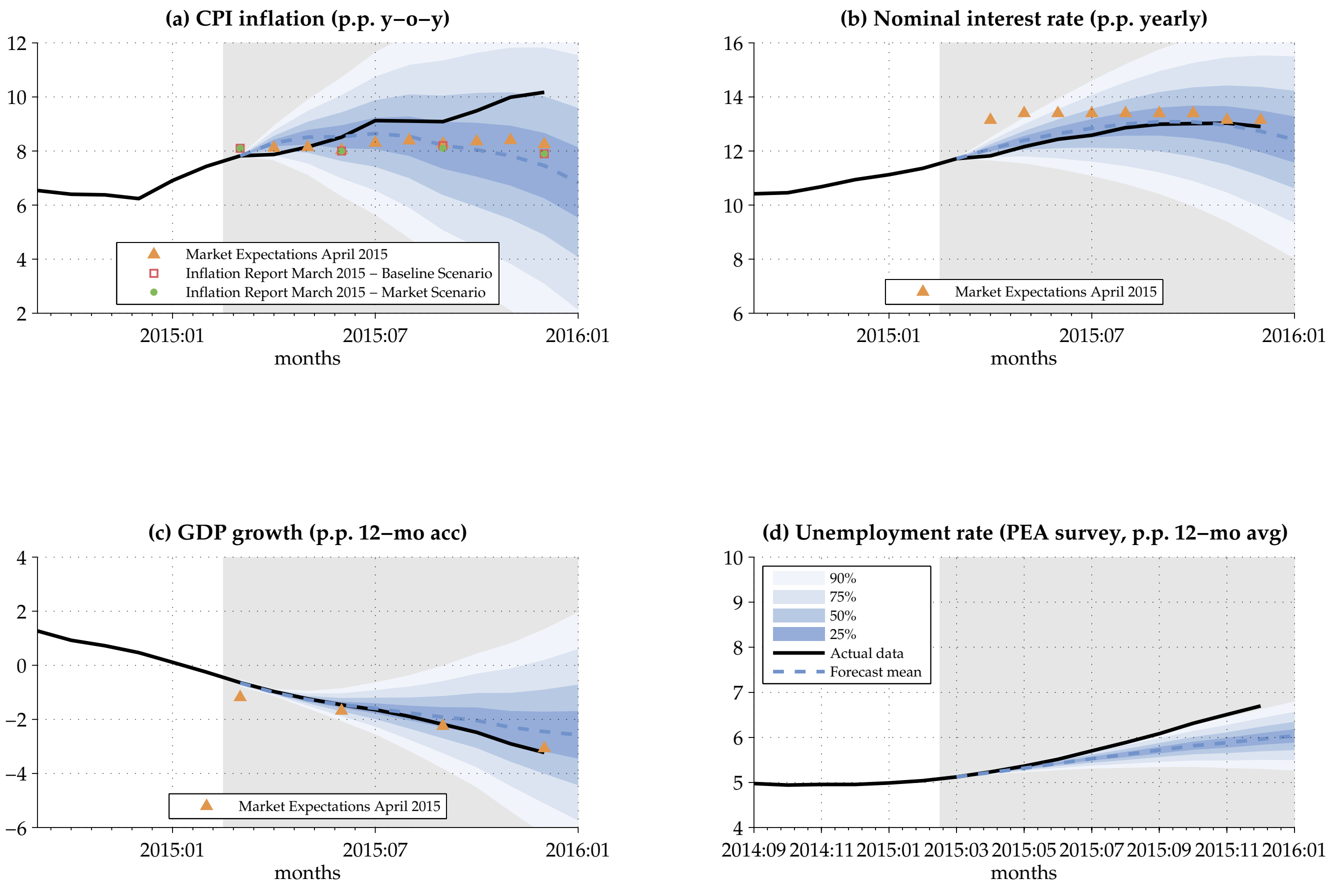
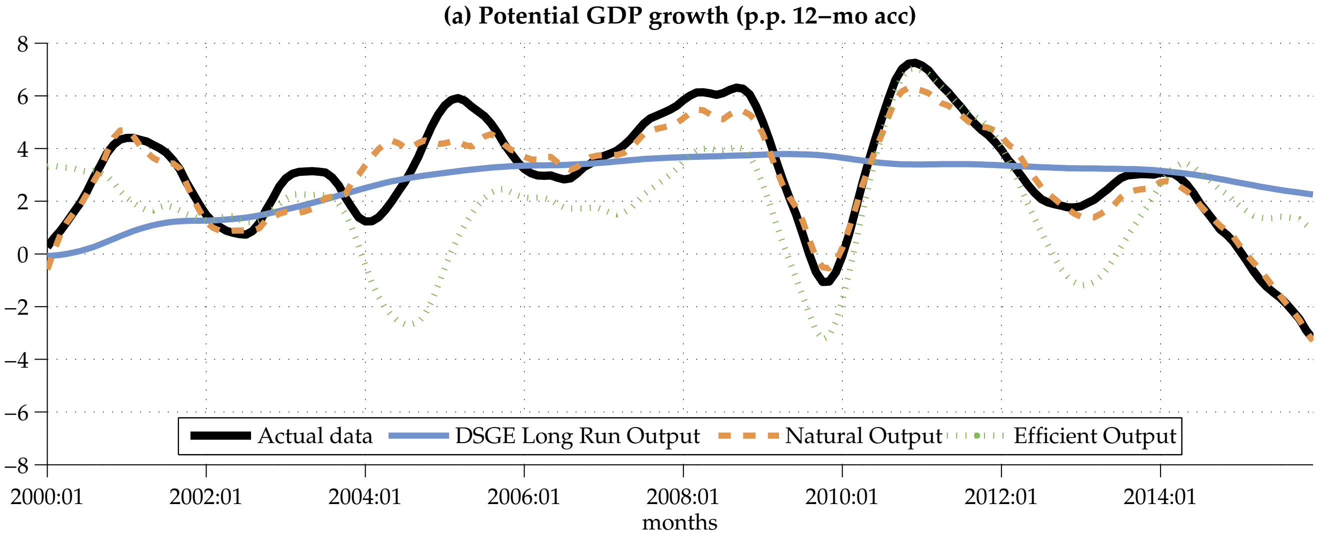


Figure 18: Model-based measures of potential output



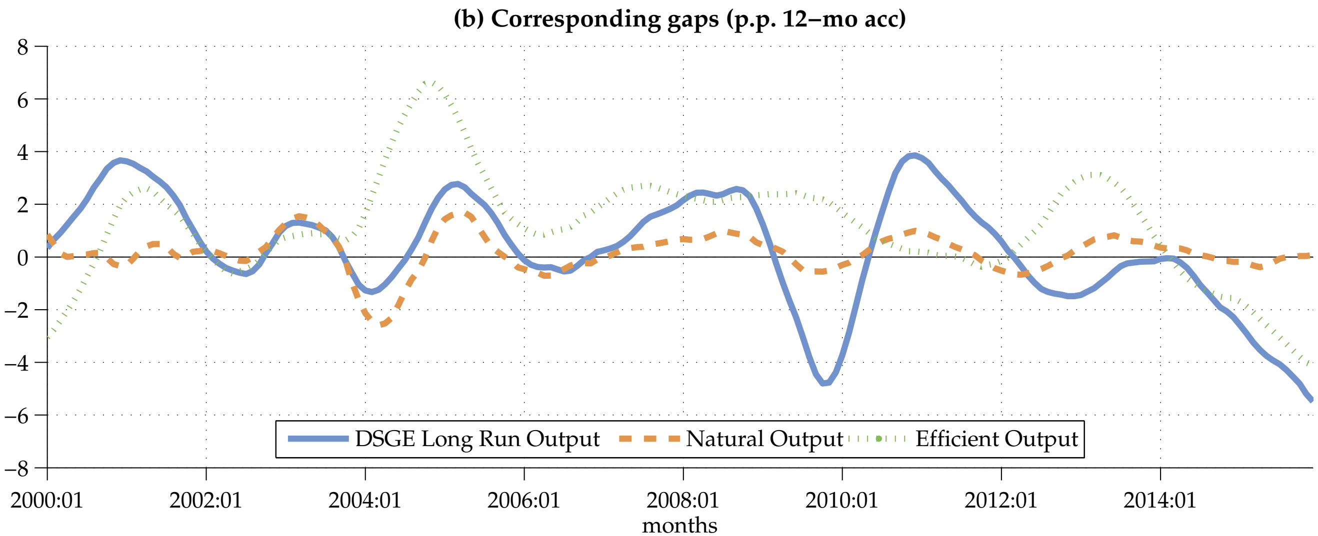


Figure 19: Theoretical decomposition of model-based output gap