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Assessing the Fit of a Small Open-Economy DSGE Model for the Brazilian Economy*

Fernando de Menezes Linardi**

Abstract

This paper estimates a small open-economy dynamic stochastic general equilibrium (DSGE) model using Brazil´s economy data for the inflation-targeting period. The model includes a number of shocks that are important to explain the macroeconomic fluctuations of emerging markets economies. Then the empirical fit of different specifications of the model is tested in a Bayesian framework. The potential model misspecification is also assessed by comparing it to a more general reference model using the DSGE-VAR approach. The results show that the model with no price indexation fits the data better than the fully specified one. The DSGE-VAR approach indicated some degree of misspecification in the stylized small open-economy model.

Keywords: Bayesian estimation, DSGE-VAR approach, model misspecification, small open-economy DSGE models.
JEL Classification: C11, C51, E52, F41.

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1 Introduction

In this paper we estimate a Dynamic Stochastic General Equilibrium (DSGE) model using Brazilian economy data for the period of inflation targeting regime followed by the Central Bank of Brazil since 1999. The model includes shocks to terms of trade, risk premium and world interest rates that are important to explain the macroeconomic volatility of emerging market economies (EMEs). The main objective is to assess in a Bayesian framework the empirical fit of different specifications of the small open economy model.

Currently, estimated DSGE models are widely used for empirical macroeconomics research and policy analysis. Following the influential papers of Christiano et al. (2005) and Smets and Wouters (2003) many works have developed DSGE models including a number of nominal and real rigidities that have proved to be crucial in fitting models to data. In general, they show that estimated DSGE models are able to perform better than standard VARs or to match the economic dynamics.

Despite the advances in economic modeling, the evaluation of model´s fit is still an important issue in the econometric analysis of DSGE models. Although they may provide a multivariate stochastic process representation for the data which is theoretically coherent, some models impose very strong restrictions on observed time series which may be rejected against less restrictive specifications such as a VAR (An and Schorfheide, 2007).

Significant contributions to the development of general equilibrium models for open economies were made by, among others, Kollmann (2002), Schmitt-Grohé and Uribe (2001) and Galí and Monacelli (2005). Kollmann (2002) develops a small open economy model assuming monopolistic competition and staggered price setting to compute the effect of the monetary policy regime on welfare and business cycles. Using a small open economy model with sticky prices calibrated to the Mexican economy, Schmitt-Grohé and Uribe (2001) compare the wealfare costs of business cycles in a dollarized economy to three alternative monetary policy regimes.

Galí and Monacelli (2005) present a calibrated DSGE model for a small open economy with Calvo type staggered price-setting, and use it as a framework for analyzing the properties and macroeconomic implications of three alternative monetary policy rules. Two of the monetary policy regimes considered are stylized Taylor-type rules and the other one is a nominal exchange rate pegging. They model the world economy as a continuum of small open economies represented by the unit interval and domestic policy decisions of each economy do not have any impact on the rest of the world. While different economies are subject to imperfectly correlated productivity shocks, the authors assume that they share identical preferences, technology, and market structure.

There is a vast literature on estimating small open-economy models using Bayesian methods. For instance, Adalfson et al. (2007) develop a DSGE model for an open economy, and estimate it on Euro area data using Bayesian estimation techniques. Following Christiano et al. (2005), a number of nominal and real frictions such as sticky prices, sticky wages, variable capital utilization, capital adjustment costs and habit persistence are included in the theoretical model. Following Smets and Wouters (2002) they also allow for incomplete exchange rate pass-through in both the import and export sectors by including nominal price rigidities.

Lubik and Schorfheide (2007) try to answer the question of to what extent central banks include exchange rates in the process of formulating monetary policy. They apply Bayesian estimation techniques to four small open economies (Australia, Canada, New Zealand and the U.K) using a simplified version of Galí and Monacelli (2005) model. The main finding of the paper is that the central banks of Australia, New Zealand, and the U.K. did not explicitly respond to exchange rates over the last two decades. However, the Bank of Canada did. According to the authors, this finding is robust over different specifications of the monetary policy reaction function, such as expected inflation targeting.

Justiniano and Preston (2010b) analyze optimal policy design in an estimated small open economy for Australia, Canada and New Zealand taking also in account the consequences of model uncertainty. Their aim is to verify whether policies in a class of generalized Taylor rules optimally respond to exchange rate variations as predicted by theory. Their analysis is done using extensions of the small open-economy framework proposed by Gali and Monacelli (2005) and Monacelli (2005), and considering a small–large country pair, rather than a continuum of small open economies. In this case, a small and large country each specializes in the production of a continuum of goods subject to imperfect competition and price rigidities. Imports are subject to local currency pricing giving rise to deviations from the law of one price. Differently from Gali and Monacelli (2005) they consider incomplete asset markets in addition to other rigidities such as indexation and habit formation as well as a large set of disturbances to fit the model to the data.

Finally, applying Bayesian methods Kam et al. (2009) also estimate the macroeconomic policy objectives of the central banks of Australia, Canada, and New Zealand within the context of a small open economy DSGE model. Using a stylized economy model similar to Justiniano and Preston (2010b) they find that none of the central banks is concerned for stabilizing the real exchange rate. Although all of them are concerned for minimizing the volatility in the change of the nominal interest rate. These results are contrary to Lubik and Schorfheide (2007) who find some evidence that Canada does respond to changes in the exchange rate. According to them, the difference is explained by the lack of an endogenous terms of trade specification and the lack of imperfect pass-through of nominal exchange rate changes into domestic import prices.

In the case of Brazil, Silveira (2008) used a Bayesian approach to estimate and compare alternative model specifications for the Brazilian economy with respect to two endogenous persistence mechanisms: habit formation and price indexation. Estimating a model based in Galí and Monacelli (2005) he concluded that both mechanisms are relevant to explain the dynamic of the economy during the period analyzed, although the evidence is less robust to price indexation.

Castro et al. (2011) developed and estimated a DSGE model for the Brazilian economy which is used as part of the macroeconomic modeling framework at the Central Bank of Brazil. They model a small open economy combining standard features of DSGE models as wage and price rigidities with specific features of the Brazilian economy. They concluded based on model evaluation techniques that the model can be used as a tool for policy analysis and forecasting.

Palma and Portugal (2014) based on the work of Kam et al. (2009) estimated the preferences of the Central Bank of Brazil after the inflation targeting regime. The authors considered that the central bank minimizes a loss function, taking into account deviation of inflation from its target, output stabilization, interest rate smoothing and the exchange rate. They concluded that the monetary authority in the period was concerned with stabilization of inflation, interest rate smoothing, stabilization of output and exchange rate, following this order of importance.

We present a model for a small open economy based on Kam et al. (2009) and estimate it on Brazilian data using Bayesian methods. Differently from Palma and Portugal (2014) who investigate the policy objectives of the Central Bank of Brazil we are mainly interested in assessing the fit of the model. The model includes a number of nominal and real frictions as habit persistence, price indexation and deviations from the law of one price. It nests a number of different specifications of the small open economy model, therefore it is possible to examine the relative fit across a range of models using posterior odds ratios. In addition, the ability of the fully specified model to fit particular second order moments of the data is discussed.

Then we investigate the potential model misspecification implied by invalid cross-coefficient restrictions on the time series representation generated by the DSGE model. These invalid restrictions manifest in poor out-of-sample fit relative to more densely parameterized reference models such as VARs (An and Schorfheide, 2007).

The misspecification of the small open economy model was analyzed by Justiniano and Preston (2010b). They found that the estimated model of the Canadian economy cannot account for the significant influence of the U.S. shocks on economic fluctuations of Canada that have been verified by many empirical works. Del Negro et al. (2007) found evidence of misspecification in a large-scale new Keynesian model. However, accounting for misspecification by relaxing the DSGE model restrictions does not significantly alter the responses to monetary policy and technology shocks. Despite its deficiencies, they concluded that the DSGE model can generate realistic predictions of the effects of unanticipated changes in monetary policy and technology shocks.

In our work the model misspecification is assessed using the DSGE-VAR approach developed by Del Negro and Schorfheide (2004) and extended in Del Negro et al. (2007). In this approach a VAR is used as an approximating model for the DSGE model. The cross-coefficient restrictions implied by the DSGE model are systematically relaxed and the model´s fit is assessed. Deviations of the VAR parameters from the restrictions are interpreted as DSGE model misspecification. This approach provide a reliable benchmark to evaluate the model´s misspecification given that posterior odds of a DSGE model versus a VAR with a fairly diffuse prior do not provide a robust assessment of fit. A VAR with a diffuse prior is likely to be rejected even against a misspecified DSGE model (An and Schorfheide, 2007).

Del Negro and Schorfheide (2008) use the DSGE-VAR approach to assess the model´s robustness to the presence of model misspecification in the study of the policy rule followed by the Central Bank of Chile. They analyze the dynamic response of inflation to domestic and external shocks and the change in the response of inflation under different policy parameters. Bäurle and Menz (2008) use the same framework to study the transmission of monetary shocks in Switzerland. They assess whether the central bank reacts to changes in the nominal exchange rate.

Gupta and Steinbach (2013) develop a small open economy DSGE-VAR model of the South Africa economy and use it to forecast output growth, inflation and short term interest rates. Forecast performance of the model is compared to an independently estimated DSGE model and to VAR and BVAR models. They conclude that the BVAR model provides the best forecasts of the three variables. Using data from the New Zealand economy Lees et al. (2011) also found that the BVAR out-performs both the DSGE-VAR and the central bank´s own forecasts. Finally, Marcellino and Rychalovska (2014) develop a two-region DSGE model of an open economy within the European Monetary Union and apply the DSGE-VAR approach to study the empirical validity of the restrictions implied by the DSGE model.

The paper is organized as follows. Section 2 discusses some issues in estimating small open economy models for emerging market economies. In section 3 the theoretical model is presented. In section 4 and 5 it is outlined the estimation methodology and the dataset used for the estimation. In section 6, the main results are discussed. Section 7 concludes.

2 Estimation of DSGE models for emerging market economies (EMEs)

Estimation of DSGE models for EMEs is challenge considering that their business cycles are characterized by large volatility and current account reversals ("sudden stops"). Aguiar and Gopinath (2007) explain that their trade balance is strongly countercyclical as compared to developed markets and consumption is much more volatile than income. In addition, income growth and net exports are twice as volatile in EMEs. Despite their peculiarities, the authors show that a standard real business cycle model (RBC) can reproduce to a large extent the business cycle features of EMEs. Their model is driven by a transitory and a permanent shock to the trend growth rate.

On the other hand, García-Cicco et al. (2010) find that, when estimated over a very long sample (1900-2005), the RBC model driven by permanent and transitory productivity shocks could not explain adequately observed business cycles in Argentina and Mexico along a number of dimensions such as trade balance-to-output ratio or the observed excess volatility in consumption. They further estimate an augmented model that incorporates debt elasticity of the country interest rate, country-premium shocks, preference shocks and domestic spending shocks. They conclude that the augmented model provides an adequate explanation of the Argentine business cycle.

Other works have included a number of disturbances that are important to explain the business cycle variations of EMEs. Given that many of them are exporters of a few primary commodities, shocks to the terms of trade − ratio of the price of imports to exports − may play an important role in explaining output fluctuations. Hove et al. (2015) evaluate the optimal monetary policy responses to terms of trade shocks in commodity dependent EMEs. Devereux et al. (2006) compare alternative monetary policy rules in a model of an EME that experiences external shocks to world interest rates and the terms of trade.

Furthermore, there are evidences that changes in the cost of borrowing affect significantly EMEs. Shocks could be caused by changes in world interest rates or country risk premiums. Many studies have emphasized the role of movements in U.S. interest rates and country spreads in driving business cycles in EMEs. Uribe and Yue (2006) conclude based on a panel VAR of seven EMEs that U.S. interest rate shocks explain 20% of the output variance and about 12% can be explained by country premium shocks.

Considering this, in addition to monetary policy and technology shocks, our model includes shocks to the terms of trade, risk premium and world interest rates. We estimate the model for the Brazilian economy using data on output growth, inflation, exchange rate and interest rates. After the launch of the *Plano Real* in 1994 which changed the currency and ended a period of high inflation in Brazil the monetary policy was based on exchange rate peg. In 1999 the floating exchange rate regime was adopted and in the same year the inflation targeting regime was officially implemented. Hence, we decided to use data only for the inflation targeting period which unfortunately it is not a long sample.

3 A simple small open economy model

The stylized economy is analogous to the open economy model presented in Kam et al. (2009). We consider the case of a small open economy where households are assumed to maximize a utility function over an infinite life horizon. The household derives utility from leisure and consumption relative to an external habit parameter. There are two types of firms in the economy: domestic goods firms and importing firms. The continuum of monopolistically competitive domestic goods firms produce differentiated goods and operate a linear production technology. The continuum of import retail firms add markups to goods imported at world prices.

Sticky price is introduced to the model following Calvo, where some of the firms choose their prices optimally and the other part chooses according to past inflation. Furthermore, in determining the domestic currency price of the imported good, retail firms are assumed to be monopolistically competitive giving rise to deviations from the law of one price. Finally, monetary policy is assumed to be conducted according to a Taylor-type rule and fiscal policy is specified as a zero debt policy.

The foreign economy is exogenous to the domestic economy and for simplicity it is assumed that output, inflation and real interest rate of the foreign economy are given by uncorrelated AR(1) processes.

3.1. Household sector

The economy is inhabited by a representative household who maximizes an intertemporal utility function given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1-\varphi}}{1-\varphi} \right), (1)$$

where β^t is the discount factor, σ and φ are the inverse elasticities of intertemporal substitution and labor supply, respectively. N_t is the labor supply and $H_t = hC_{t-1}$ is an external habit which is assumed to be proportional to aggregate past consumption. C_t is a composite consumption index given by a constant elasticity of substitution (CES) function:

$$\boldsymbol{C_t} = \left[(\mathbf{1}-\boldsymbol{\alpha})^{\frac{1}{\eta}}\boldsymbol{C}_{\boldsymbol{H},\boldsymbol{t}}^{\frac{\eta-1}{\eta}} + \boldsymbol{\alpha}^{\frac{1}{\eta}}\boldsymbol{C}_{\boldsymbol{F},\boldsymbol{t}}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \quad (2)$$

where α is the share of foreign goods in the domestic consumption bundle. $C_{H,t}$ and $C_{F,t}$ are the usual Dixit–Stiglitz aggregates of the domestic and foreign produced goods:

$$\boldsymbol{C_{H,t}} = \left[\int_0^1 \boldsymbol{C_{H,t}(i)}^{\frac{\varepsilon-1}{\varepsilon}}\boldsymbol{di}\right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ and } \boldsymbol{C_{F,t}} = \left[\int_0^1 \boldsymbol{C_{F,t}(i)}^{\frac{\varepsilon-1}{\varepsilon}}\boldsymbol{di}\right]^{\frac{\varepsilon}{\varepsilon-1}}. \quad (3)$$

Optimal allocation of the household expenditure across each good type gives rise to the demand functions:

$$\boldsymbol{C_{H,t}(i)} = \left(\boldsymbol{P_{H,t}(i)}/\boldsymbol{P_{H,t}}\right)^{-\boldsymbol{\varepsilon}}\boldsymbol{C_{H,t}} \text{ and } \boldsymbol{C_{F,t}(i)} = \left(\boldsymbol{P_{F,t}(i)}/\boldsymbol{P_{F,t}}\right)^{-\boldsymbol{\varepsilon}}\boldsymbol{C_{F,t}} \quad (4)$$

for all $i \in [0, 1]$ where the aggregate price levels are defined as:

$$\boldsymbol{P_{H,t}} = \left[\int_0^1 \boldsymbol{P_{H,t}(i)}^{\mathbf{1}-\boldsymbol{\varepsilon}}\boldsymbol{di}\right]^{\frac{1}{1-\varepsilon}} \text{ and } \boldsymbol{P_{F,t}} = \left[\int_0^1 \boldsymbol{P_{F,t}(i)}^{\mathbf{1}-\boldsymbol{\varepsilon}}\boldsymbol{di}\right]^{\frac{1}{1-\varepsilon}}. \quad (5)$$

The optimal consumption demand of home and foreign goods can be derived as:

$$C_{H,t} = (1-\alpha)\left(P_{H,t}/P_t\right)^{-\eta}C_t \text{ and } C_{F,t} = \alpha\left(P_{F,t}/P_t\right)^{-\eta}C_t \quad (6)$$

where P_t is the domestic price consumer price index (CPI):

$$\boldsymbol{P_t} = \left[(\mathbf{1}-\boldsymbol{\alpha})\boldsymbol{P}_{\boldsymbol{H},\boldsymbol{t}}^{\mathbf{1}-\boldsymbol{\eta}} + \boldsymbol{\alpha}\boldsymbol{P}_{\boldsymbol{F},\boldsymbol{t}}^{\mathbf{1}-\boldsymbol{\eta}}\right]^{\frac{1}{1-\eta}}. \quad (7)$$

The total consumption expenditures by domestic households are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$. Hence, the representative household intertemporal budget constraint, for all $t > 0$, is:

$$P_tC_t + \mathbb{E}_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_tN_t + T_t \quad (8)$$

where D_{t+1} is the nominal pay-off in period $t + 1$ of the portfolio held at the end of period t. $Q_{t,t+s}$ is the stochastic discount factor. The stochastic discount factor will be inversely related to the gross return on a nominal risk-less one-period bond $\mathbb{E}_tQ_{t,t+1} = R^{-1}$. W_t are wages earned on labor supplied N_t. Finally, T_t denotes government lump-sum taxes and transfers.

The intratemporal condition relating labor supply to the real wage must satisfy:

$$(\boldsymbol{C_t} - \boldsymbol{H_t})^{\boldsymbol{\sigma}}\boldsymbol{N_t^{\varphi}} = \frac{\boldsymbol{W_t}}{\boldsymbol{P_t}}. \quad (9)$$

The intertemporal optimality for the household decision problem is:

$$\boldsymbol{\beta}\left(\frac{\boldsymbol{C_{t+1}-H_{t+1}}}{\boldsymbol{C_t-H_t}}\right)^{-\boldsymbol{\sigma}}\left(\frac{\boldsymbol{P_t}}{\boldsymbol{P_{t+1}}}\right)=\boldsymbol{Q_{t,t+1}} \qquad (10)$$

and taking expectations yields the stochastic Euler equation:

$$\boldsymbol{\beta R_t}\mathbb{E}_{\boldsymbol{t}}\left\{\left(\frac{\boldsymbol{C_{t+1}-H_{t+1}}}{\boldsymbol{C_t-H_t}}\right)^{-\boldsymbol{\sigma}}\left(\frac{\boldsymbol{P_t}}{\boldsymbol{P_{t+1}}}\right)\right\}=\mathbf{1} \quad (11)$$

3.2. Domestic goods firms

There is a continuum of monopolistically competitive domestic firms $i \in [0, 1]$ producing differentiated goods. Calvo-style price setting is assumed which allows inflation to be partly a jump variable and also partially backward looking (Kam et al., 2009).

Goods are produced by firms using a linear production technology $Y_{H,t}(i) = \epsilon_{a,t}N_t(i)$ where $\epsilon_{a,t}$ is exogenous domestic technology shock. In each period t, a fraction $1 - \theta_H$ of firms set prices optimally and the remaining fraction of firms ($0 < \theta_H < 1$) partially index their price according to:

$$\boldsymbol{P_{H,t}(i)} \;=\; \boldsymbol{P_{H,t-1}(i)}\left(\frac{\boldsymbol{P_{H,t-1}}}{\boldsymbol{P_{H,t-2}}}\right)^{\boldsymbol{\delta_H}} \qquad (12)$$

where $\delta_H \in [0,1]$ measures the degree of inflation indexation. All firms that set prices optimally in period t face the same decision problem and consequently they will set the same price $P_{H,t}^{new}$. Given Calvo price setting, the evolution of the aggregate domestic goods price index is given by:

$$\boldsymbol{P_{H,t}}=\left\{(\mathbf{1}-\boldsymbol{\theta_H})(\boldsymbol{P_{H,t}^{new}})^{\mathbf{1}-\boldsymbol{\varepsilon}}+\boldsymbol{\theta_H}\left[\boldsymbol{P_{H,t-1}}\left(\frac{\boldsymbol{P_{H,t-1}}}{\boldsymbol{P_{H,t-2}}}\right)^{\boldsymbol{\delta_H}}\right]^{\mathbf{1}-\boldsymbol{\varepsilon}}\right\}^{\frac{\mathbf{1}}{\mathbf{1}-\boldsymbol{\varepsilon}}}. \qquad (13)$$

Firms setting prices in period t face a demand curve given by the demand constraint:

$$\boldsymbol{Y_{H,t+s}(i)}=\left[\frac{\boldsymbol{P_{H,t}(i)}}{\boldsymbol{P_{H,t+s}}}\left(\frac{\boldsymbol{P_{H,t+s-1}}}{\boldsymbol{P_{H,t-1}}}\right)^{\boldsymbol{\delta_H}}\right]^{-\boldsymbol{\varepsilon}}\left(\boldsymbol{C_{H,t+s}}+\boldsymbol{C_{H,t+s}^{*}}\right) \qquad (14)$$

where * denotes parameters and variables of the rest of the world.

The firm's price-setting problem in period t is to maximize the expected present discounted value of profits:

$$\max_{P_{H,t}(i)} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s}\theta_H^s Y_{H,t+s}(i)\left[P_{H,t}(i)\left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\delta_H} - P_{H,t+s}MC_{H,t+s})\right], \quad (15)$$

$$s.t. (14)\ for\ t,s\ \in\ \mathbb{N}$$

where the real marginal cost is:

$$MC_{H,t+s} = \frac{W_{t+s}}{\epsilon_{a,t+s}P_{H,t+s}}. \quad (16)$$

The factor θ_H^s in the equation (15) is the probability that the firm will not allowed to adjust its price in the next *s* periods.

The firm's optimization problem implies the first-order condition:

$$\mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s}\theta_H^s Y_{H,t+s}(i)\left[P_{H,t}(i)\left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\delta_H} - \left(\frac{\varepsilon}{\varepsilon-1}\right)P_{H,t+s}(i)MC_{H,t+s}\right] = 0.$$

(17)

Let the home goods inflation rate be $\pi_{H,t} \coloneqq ln(P_{H,t}/P_{H,t-1})$ and $y_t \coloneqq ln(Y_t/Y_{ss})$ be the percentage deviation of home output from steady state. The log-linear approximation of the optimal pricing decision rule gives the following Phillips curve for domestic goods inflation:

$$\pi_{H,t} - \delta_H\pi_{H,t-1} = \beta\mathbb{E}_t(\pi_{H,t+1} - \delta_H\pi_{H,t}) + \lambda_H mc_{H,t} \quad (18)$$

where $\lambda_H = (1-\beta\theta_H)(1-\theta_H)\theta_H^{-1}$ and

$$mc_{H,t} = \varphi y_t - (1+\varphi)\epsilon_{a,t} + \alpha s_t + \frac{\sigma}{1-h}(y_t^* - hy_{t-1}^*) + q_t \quad (19)$$

3.3. Importing firms

Importing firms are assumed to buy imported goods at competitive prices. However, in determining the domestic price of the imported good, firms are assumed to be monopolistically competitive. This small degree of pricing power leads to a violation of the law of one price in the short run (Justiniano and Preston, 2010b). The law of one price (LOP) gap, that is, the difference between the price of imported goods in domestic currency terms and the domestic retail price of imported goods, in log-linear terms is defined as:

$$\psi_{F,t} = e_t + p_t^* - p_{F,t} \quad (20)$$

where e_t is the percentage deviation of nominal exchange rate from its steady state.

Importing firms also face a Calvo-style price setting problem. In any period t, an fraction $1 - \theta_F$ of firms set prices optimally while the other fraction of firms adjusts goods price according to an indexation rule as described in the last section. Hence, the evolution of the imports price index is given by:

$$\boldsymbol{P_{F,t}} = \left\{ (\boldsymbol{1} - \boldsymbol{\theta_F})(\boldsymbol{P_{F,t}^{new}})^{\boldsymbol{1-\varepsilon}} + \boldsymbol{\theta_F} \left[\boldsymbol{P_{F,t-1}} \left(\frac{\boldsymbol{P_{F,t-1}}}{\boldsymbol{P_{F,t-2}}} \right)^{\boldsymbol{\delta_F}} \right]^{\boldsymbol{1-\varepsilon}} \right\}^{\frac{\boldsymbol{1}}{\boldsymbol{1-\varepsilon}}}. \quad (21)$$

Firms setting prices in period t face a demand curve given by the demand constraint:

$$\boldsymbol{Y_{F,t+s}(i)} = \left[\frac{\boldsymbol{P_{F,t}(i)}}{\boldsymbol{P_{F,t+s}}} \left(\frac{\boldsymbol{P_{F,t+s-1}}}{\boldsymbol{P_{F,t-1}}} \right)^{\boldsymbol{\delta_F}} \right]^{-\boldsymbol{\varepsilon}} \boldsymbol{C_{F,t+s}}. \quad (22)$$

The firm's price-setting problem in period t is to maximize the expected present discounted value of profits:

$$\max_{\boldsymbol{P_{F,t}(i)}} \mathbb{E}_{\boldsymbol{t}} \sum_{\boldsymbol{s=0}}^{\infty} \boldsymbol{Q_{t,t+s}} \boldsymbol{\theta_F^s} \boldsymbol{Y_{F,t+s}(i)} \left[\boldsymbol{P_{F,t}(i)} \left(\frac{\boldsymbol{P_{F,t+s-1}}}{\boldsymbol{P_{F,t-1}}} \right)^{\boldsymbol{\delta_F}} - \tilde{\boldsymbol{e}}_{\boldsymbol{t+s}} \boldsymbol{P^*_{F,t+s}(i)} \right], \quad (23)$$

$$s.t. (22)\ for\ t, s \in \mathbb{N}$$

The firm's optimization problem implies the first order condition:

$$\mathbb{E}_{\boldsymbol{t}} \sum_{\boldsymbol{s=0}}^{\infty} \boldsymbol{Q_{t,t+s}} \boldsymbol{\theta_F^s} \boldsymbol{Y_{F,t+s}(i)} \left[\boldsymbol{P_{F,t}(i)} \left(\frac{\boldsymbol{P_{F,t+s-1}}}{\boldsymbol{P_{F,t-1}}} \right)^{\boldsymbol{\delta_F}} - \left(\frac{\boldsymbol{\varepsilon}}{\boldsymbol{\varepsilon-1}} \right) \tilde{\boldsymbol{e}}_{\boldsymbol{t+s}} \boldsymbol{P^*_{F,t+s}(i)} \right] = \boldsymbol{0}. \quad (24)$$

The log-linearization of this condition gives:

$$\boldsymbol{\pi_{F,t}} = \boldsymbol{\beta} \mathbb{E}_{\boldsymbol{t}} (\boldsymbol{\pi_{F,t+1}} - \boldsymbol{\delta_F \pi_{F,t}}) + \boldsymbol{\delta_F \pi_{F,t-1}} + \boldsymbol{\lambda_F \psi_{F,t}} \quad (25)$$

where $\lambda_F = (1 - \beta\theta_F)(1 - \theta_F)\theta_F^{-1}$.

3.4. Monetary policy

As in Kam et al. (2009) it is assumed that the monetary policy is conducted according to the simple Taylor type rule in log-linear terms:

$$\boldsymbol{r_t} = \boldsymbol{\rho_r r_{t-1}} + (\boldsymbol{1} - \boldsymbol{\rho_r})(\boldsymbol{\psi_\pi \pi_t} + \boldsymbol{\psi_y y_t} + \boldsymbol{\psi_{\Delta e} \Delta e_t}) + \boldsymbol{\varepsilon_{r,t}} \quad (26)$$

where $\rho_r, \psi_\pi, \psi_y, \psi_{\Delta e}$ are the policy responses to the lag of the nominal interest rate, inflation, output growth and the change in the nominal exchange rate, respectively.

$\varepsilon_{r,t} \sim i.i.d.(0, \sigma_r^2)$ is an exogenous monetary policy shock or implementation error in the conduct of policy.

The change in the nominal exchange rate was included in the equation despite the low evidence that central banks in many countries explicitly respond to it (Justiniano and Preston, 2010b, Palma and Portugal, 2014 and Lubik and Schorfheide, 2007).

3.5. International risk sharing, terms of trade and equilibrium

The rest of the world solves a similar problem to the small open economy. Therefore, similar first-order conditions for optimal labor supply and consumption also hold. From equation (10) it is possible to derive the uncovered interest parity condition or the no-arbitrage condition for exchange rates:

$$\boldsymbol{R_t - R_t^* \frac{\tilde{e}_t}{\tilde{e}_{t+1}} = 0}. \qquad (27)$$

A log-linear approximation of this equation, and taking expectations with respect to the time t, yields the nominal interest parity condition:

$$\mathbb{E}_t \boldsymbol{e_{t+1} - e_t = r_t - r_t^*}, \qquad (28)$$

where $e_t = ln(\tilde{e_t}/e_{ss})$ and domestic and foreign interest rates are $r_t = R_t - 1$ and $r^*_t = R^*_t - 1$, respectively.

The real exchange rate is defined as:

$$\boldsymbol{Q_t = \tilde{e}_t \frac{P_t^*}{P_t}}. \qquad (29)$$

Log-linearizing around the deterministic steady state yields:

$$\boldsymbol{q_t = e_t + p_t^* - p_t}. \qquad (30)$$

The terms of trade (ToT) is the ratio of the foreign goods price index to the home goods price index. In log-linear terms it is:

$$s_t = p_{F,t} + p_{H,t}. \qquad (31)$$

Goods market clearing condition in the domestic economy requires for all t that domestic output equals total domestic and foreign demand for home produced goods:

$$\boldsymbol{Y_{H,t} = C_{H,t} + C_{H,t}^*}. \qquad (32)$$

The demand for home and foreign consumption goods can be written in log-linear form as:

$$c_{H,t} = (1-\alpha)[\alpha\eta s_t + c_t] \text{ and } c_{H,t}^* = \alpha[\eta(s_t + \psi_{F,t}) + y_t^*].$$

Therefore, (32) can be written as:

$$\boldsymbol{y_t = (2-\alpha)\alpha\eta s_t + (1-\alpha)c_t + \alpha\eta\psi_{F,t} + \alpha y_t^*} \quad (33)$$

3.6. The log-linearized model

A summary of the log-linear approximations of the model´s first order conditions around a non-stochastic steady state are presented below. Let $c_t \coloneqq ln(C_t/C_{ss})$, $y_t \coloneqq ln(Y_t/Y_{ss})$, $q_t \coloneqq ln(Q_t/Q_{ss})$ denote the percentage deviation of home consumption, output and real exchange rate from their respective steady states, where X_{ss} is the deterministic steady state value of a variable X_t, and $\pi_t \coloneqq ln(P_t/P_{t-1})$ is the inflation rate.

The consumption Euler equation is obtained by log-linearizing (10) and taking the expectations on the time *t*:

$$\boldsymbol{c_t - hc_{t-1} = \mathbb{E}_t(c_{t+1} + hc_t) - \frac{1-h}{\sigma}(r_t - \mathbb{E}_t\pi_{t+1})}. \qquad (34)$$

Domestic goods inflation is given by (18):

$$\pi_{H,t} = \beta\mathbb{E}_t(\pi_{H,t+1} - \delta_H\pi_{H,t}) + \delta_H\pi_{H,t-1} + \lambda_H\left[\varphi y_t - (1+\varphi)\epsilon_{a,t} + \alpha s_t + \frac{\sigma}{1-h}(c_t - hc_{t-1})\right]. \qquad (35)$$

Imports inflation is given by (25):

$$\pi_{F,t} = \beta\mathbb{E}_t(\pi_{F,t+1} - \delta_F\pi_{F,t}) + \delta_F\pi_{F,t-1} + \lambda_F[q_t - (1-\alpha)s_t]. \quad (36)$$

Combining (30) and (28) yields the real interest parity condition:

$$\mathbb{E}_t(q_{t+1} - q_t) = (r_t - \mathbb{E}_t\pi_{t+1}) - (r_t^* - \mathbb{E}_t\pi_{t+1}^*) + \epsilon_{q,t} \qquad (37)$$

where $\epsilon_{q,t}$ is a real interest parity shock. This shock is introduced to capture deviations from the uncovered interest parity condition resulting from risk premium shocks (Matheson, 2010).

First-differencing the terms of trade equation (31):

$$s_t - s_{t-1} = \pi_{F,t} - \pi_{H,t} + \epsilon_{s,t} \qquad (38)$$

where $\epsilon_{s,t}$ is terms of trade (ToT) shock.

Goods market clearing condition (32) in combination with the LOP gap (20) yields:

$$y_t = (1-\alpha)c_t + \alpha\eta q_t + \alpha\eta s_t + \alpha y_t^*. \qquad (39)$$

The first-difference of CPI definition gives CPI inflation:

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t}. \quad (40)$$

Exogenous stochastic processes for ToT, technology, and real-interest parity shocks are given by:

$$\epsilon_{j,t} = \rho_j \epsilon_{j,t-1} + \varepsilon_{j,t};\ \varepsilon_j \sim i.i.d.(0, \sigma_j^2)$$

for $j = s,\ a,\ q$. Finally, as in Kam et al. (2009), it is assumed for simplicity that the foreign processes $\{\pi^*, y^*, r^*\}$ are given by uncorrelated AR(1) processes:

$$\pi_t^* = a_1 \pi_{t-1}^* + \varepsilon_{\pi^*,t}\ ,$$

$$y_t^* = b_1 y_{t-1}^* + \varepsilon_{y^*,t}\ ,$$

$$r_t^* = c_1 r_{t-1}^* + \varepsilon_{r^*,t}\ .$$

where $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$, for $i = \pi_t^*$, y_t^* and r_t^*.

The model is solved using the set of equations (34) - (40) that describe the domestic economy and monetary policy rule (26) in the variables $\{c_t, y_t, r_t, q_t, s_t, \pi_t, \pi_{H,t}, \pi_{F,t}\}$, combined with the processes for the exogenous disturbances $\{\epsilon_{s,t}, \epsilon_{a,t}, \epsilon_{q,t}\}$ and the foreign economy $\{\pi_t^*, y_t^*, r_t^*\}$.

4 Data

In estimation, we use seven observable series to match the same number of structural shocks in the model. We collect quarterly time series of the variables $\{q_t, y_t, \pi_t, r_t, \pi_t^*, y_t^*, r_t^*\}$, for the period of inflation targeting regime (2000Q1–2014Q4), totaling 60 observations[1]. We use the index of real exchange rate, q_t; Home output (real GDP per capita), y_t; Home CPI inflation (*IPCA* index), π_t; Home nominal interest rate (*Selic* rate), r_t; U.S. inflation (CPI), π_t^*; U.S. output (real GDP per capita), y_t^*; and U.S. interest rates (FED fund rates), r_t^*.

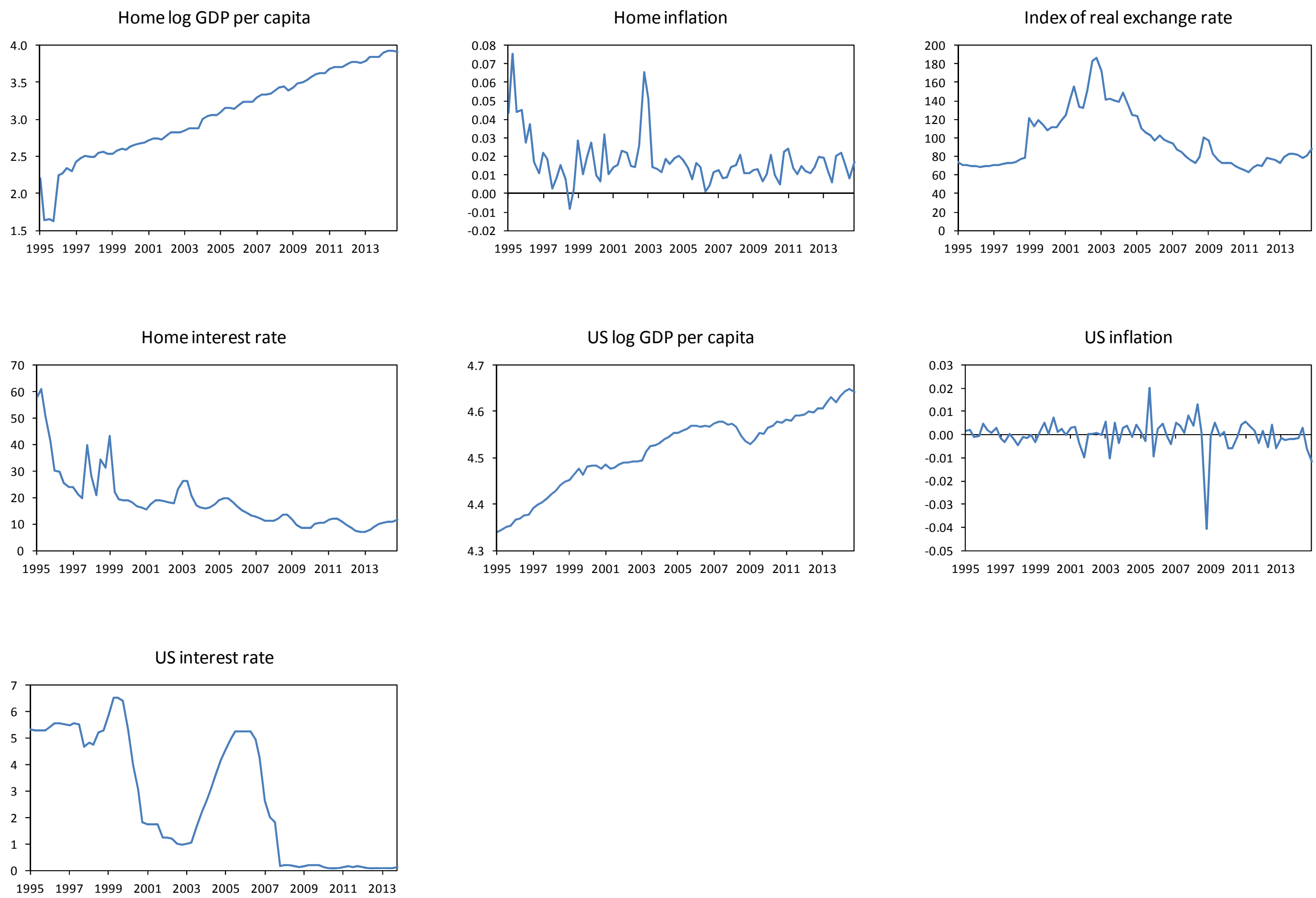
[1] Inflation targeting was adopted in June 1999 but I dropped the first observations.

Figure 1 presents the quarterly data for the time series over the period 1995-2014. The data were obtained from the Central Bank of Brazil (http://www.bcb.gov.br) and Ipea (http://www.ipeadata.gov.br). The data from U.S. were obtained from the Federal Reserve Bank of St Louis (http://research. stlouisfed.org/fred2).

In the estimation all observables are measured in log percentage deviations from the steady state which we assumed to be the mean from 1995 to 2014. In the case of the home output the GDP per capita was constructed dividing the quarterly, seasonally adjusted, real GDP by the labor force. We use the GDP per capita in log deviations from a linear trend for the estimation.

Home inflation data correspond to the quarterly log percentage change in the Brazilian consumer price index (*Índice Nacional de Preços ao Consumidor Amplo - IPCA*). Home nominal interest rate corresponds to the 3-month average rates of the reference interest rate of government bonds (*Selic*). For the real exchange rate we use the index of real exchange rate of the Central Bank of Brazil based on a currency basket of the main trading partners.

Figure 1: Quarterly data for home and foreign economy from 1995 to 2014.



For the specification of the foreign economy we assume it to be proxied by U.S. data. Foreign output is the GDP per capita constructed as the quarterly, seasonally adjusted, real GDP (FRED: GDPC96) divided by the civilian labor force (FRED: CLF16OV). For the estimation we use the GDP per capita in log deviations from a linear trend. Foreign inflation is the quarterly log percentage change in the CPI. Foreign interest rate is the 3-month average of the effective federal funds rate.

Table 1 presents the variance, cross-correlation and autocorrelation of the following observables: output (y), interest rate (r), real exchange rate (q) and inflation (π). In section 6 the first order autocorrelations and the variances for these variables and the corresponding statistics implied by the estimated model are compared.

Table 1: Variance, cross-correlation and autocorrelation of output, interest rate, real exchange rate and inflation.

	Variance ($\times 10^{-3}$)	Cross-correlation				Autocorrelation (*lag*)			
		y	r	q	π	1	2	3	4
y	0.8592	1	-0.2975	0.0847	-0.2708	0.5475	0.1037	0.2254	0.3943
r	0.1235		1	-0.0953	0.3900	0.9373	0.8197	0.7123	0.6455
q	6.3832			1	-0.054	0.2467	-0.2006	-0.2248	-0.1148
π	0.0928				1	0.4423	-0.0332	-0.0111	0.1684

5 Estimation methodology

The structural or deep parameters of the model are estimated using Bayesian methods. First, the second order moments of key observables are compared to those implied by the estimated model. Then we proceed to the estimation of four versions of the model. Using Bayesian posterior odds comparisons we test which model is more probable, all others things been equal. Finally, the comparisons of the DSGE model to VAR is done using the DSGE-VAR approach of Del Negro et al. (2007).

The set of parameters to be estimated are the central bank preferences parameters, $\{\rho_r, \psi_\pi, \psi_y, \psi_{\Delta e}\}$, the structural parameters, $\{h, \sigma, \phi, \eta, \delta_H, \delta_F, \theta_H, \theta_F\}$ and the parameters for exogenous processes $\{a_1, b_1, c_1, \rho_a, \rho_q, \rho_s, \sigma_a, \sigma_q, \sigma_s, \sigma_r, \sigma_{\pi*}, \sigma_{y*}, \sigma_{r*}\}$.

The estimation procedure uses the random-walk Metropolis Markov Chain Monte Carlo (MCMC) method as explained in Del Negro and Schorfheide (2011). The prior is described by a density function $p(\theta_i|\mathcal{M}_i)$, where $\mathcal{M}_i$ is a specific model and θ_i are the parameters of the model. The posterior density of the model parameters θ_i is proportional to the likelihood of the sample data Y multiplied by the prior:

$$p(\theta_i|Y) \propto \mathcal{L}(\theta_i|Y)p(\theta_i|\mathcal{M}_i).$$

The Kalman filter is used to evaluate the likelihood function associated with the linear state-space model. More specifically, the solution for the log-linearized equilibrium conditions of the model takes the form of:

$$s_t = \Phi_1(\theta)s_{t-1} + \Phi_\epsilon(\theta)\epsilon_t,$$

where the matrices Φ_1 and Φ_ϵ are functions of the DSGE model parameters θ, s_t is a state vector of model variables and ϵ_t are Gaussian innovations. The model is augmented by defining a set of measurement equations that relate the elements of s_t to a set of observables. Then the Kalman filter is used to estimate the likelihood function which is combined with the prior distributions for the parameters of the model to form the posterior density function.

Posterior distribution of the parameters were obtained through the Metropolis-Hastings sampling algorithm[2]. We made 100,000 draws from the posterior distribution using two parallel chains, discarding the first half of the draws to remove any effect of the initial condition. Then we perform some diagnostic tests to check whether the MCMC procedure has converged.

After estimation the posterior distribution of competing models we compute the posterior odds ratio or the Bayes Factor between model *i* and *j*:

$$BF_{ij} = \frac{p(Y|\mathcal{M}_i)}{p(Y|\mathcal{M}_j)}$$

where $p(Y|\mathcal{M}_i)$ is the marginal likelihood of model $\mathcal{M}_i$:

$$p(Y|\mathcal{M}_i) = \int p(Y|\theta_i)p(\theta_i)d\theta_i.$$

We use the modified harmonic mean estimator method proposed by Geweke (1999) to obtain numerical approximations of the marginal likelihood.

The potential misspecification of the DSGE model is assessed using the DSGE-VAR approach proposed by Del Negro and Schorfheide (2004). They showed how to combine a DSGE model and a VAR to provide a hybrid model that can be used for model evaluation, forecasting and monetary policy analysis (Lees et al., 2011).

In the DSGE-VAR approach the priors for the autoregressive matrices of the VAR are defined by the DSGE model. However, the cross-coefficient restrictions implied by the DSGE model on the VAR are not strictly imposed. Deviations from these restrictions are allowed and they are controlled by the hyperparameter λ. If $\lambda = \infty$ the restrictions are strictly enforced. If $\lambda = 0$ the restrictions are completely ignored in the estimation of the VAR parameters.

This hyperparameter λ scales the covariance matrix of the prior. If λ is large the variance is small, and most of the prior mass on the VAR coefficients concentrates near

[2] The estimates were done using Dynare version 4.4.3.

the DSGE model restrictions. The prior is combined with the likelihood function to form the posterior distribution of the VAR parameters. For large λ the posterior move toward the DSGE model restrictions and the less the restrictions are relaxed in the estimation (Del Negro and Schorfheide, 2011). If λ is small (the variance is big) the prior on the VAR coefficients is diffuse and the opposite occurs.

The hyperparameter λ is chosen to maximise the marginal data density. High posterior probabilities for large values of lambda indicate that the model is well specified. Del Negro and Schorfheide (2006) showed that the shape of the posterior distribution of λ is robust to the change in the sample and it has an inverse U-shape, indicating that the fit of the VAR can be improved by relaxing the DSGE model restrictions.

6 Empirical results

Many empirical studies have assessed which nominal or real frictions are important to improve the fit of DSGE models. As in Del Negro et al. (2007) we assess the importance of two particular features of DSGE models: price indexation and habit formation. They are usually added to models to account for the observed inertia in inflation and persistence in output. Thus, we run two alternative specifications of the *benchmark* model which excludes these two source of frictions. The model without price indexation is referred as *no indexation* model and the model without habit formation as the *no habit* model. As in Kam et al. (2009) we also test the case where the central bank is restricted to put no weight on exchange rate variability in its monetary policy function. This model is referred as the *no exchange rate* model. In total we have four different specifications to compare.

Table 2 summarizes the assumptions regarding the prior distribution of each estimated parameter in the model. We adopt fairly loose priors and their means are mostly derived from previous studies. We use the beta distribution for all parameters bounded between 0 and 1. For parameters assumed to be positive, we use the gamma distribution. Finally, the priors for the standard deviations of the shocks are assumed to be Inverse Gamma distributed. As in Justiniano and Preston (2010b), the discount factor β (equal to 0.99) and the share of imports in domestic consumption α (equal to 0.19) were calibrated. They imply a riskless annual rate of about 4% and the average share of exports and imports to GDP in Brazil, respectively.

6.1. Parameter estimates

Estimated parameters for each model are showed in Table 3. The table reports the mean and 95% confidence intervals for the *benchmark*, *no indexation*, *no habit* and *no exchange rate* models. Univariate convergence diagnostics were performed based on the ratio of between- and within-chain variances as in Brooks and Gelman (1998). In general, the tests indicate that the Markov chains have converged to their stationary distribution.

Analyzing the posterior distributions it is possible to see that a small number of parameters appear to be weakly identified by the data (Figure 2). It is common in Bayesian estimation to have some of them with prior similar to posterior distributions meaning that posterior estimates and log marginal data densities will be sensitive to the choice of priors (Matheson, 2010).

There are not large variations in the structural parameter estimates across different model specifications. Nonetheless, the exclusions of habit formation and price indexation from the model have some effect on the structural parameters as it can be seen for the inverse elasticities of intertemporal substitution and labor supply. Putting no weight on exchange rate variability in the monetary policy function of the central bank affects the other coefficients of the Taylor rule and the persistence parameters of foreign processes to some degree. For instance, the elasticity of substitution between home and foreign goods changes from 0.70 to 0.42 in the *no exchange* model.

The degree of habit persistence (h = 0.79) is similar to the values found in other studies for Brazil (Palma and Portugal, 2014 and Castro et al., 2011) although it is higher than the values estimated by Justiniano and Preston (2010b). The (inverse) intertemporal elasticity of substitution in consumption (σ = 0.96) is relatively lower than the values reported in the literature implying that the consumption is very sensitive to real interest rate changes. The inverse elasticity of labor supply is a parameter poorly identified in DSGE models (Justiniano and Preston, 2010b). In our estimates it is near to 0.5 ($\phi \approx 2.05$) which is close to the value obtained by Palma and Portugal (2014).

Tabel 2: Priors for model parameters

Parameter		Distribution (Mean,SD)	Domain
Habit persistence	h	Beta (0.7, 0.1)	[0, 1)
Inverse elasticity of substitution	σ	Gamma (1.0, 0.2)	$[0, \infty)$
Inverse elasticity of labor supply	ϕ	Gamma (2, 0.35)	$[0, \infty)$
Elasticity H-F goods	η	Gamma (1.0, 0.25)	$[0, \infty)$
Domestic inflation indexation	δ_H	Beta (0.8, 0.1)	[0, 1)
Foreign inflation indexation	δ_F	Beta (0.8, 0.1)	[0, 1)
Calvo domestic prices	θ_H	Beta (0.5, 0.25)	[0, 1)
Calvo foreign prices	θ_F	Beta (0.5, 0.25)	[0, 1)
Coef. AR(1) of foreign inflation	a_1	Beta (0.5, 025)	[0, 1)
Coef. AR(1) of external output	b_1	Beta (0.5, 0.25)	[0, 1)
Coef. AR(1) of foreign int. rate	c_1	Beta (0.5, 0.2)	[0, 1)
Inertia of technological shock	ρ_a	Beta (0.8, 0.1)	[0, 1)
Inertia of exchange rate shock	ρ_q	Beta (0.8, 0.1)	[0, 1)
Inertia of ToT shock	ρ_s	Beta (0.8, 0.1)	[0, 1)
Taylor rule - smoothing	ρ_r	Gamma (0.5, 0.15)	$[0, \infty)$
Taylor rule - inflation	ψ_π	Gamma (0.5, 0.15)	$[0, \infty)$
Taylor rule - output growth	ψ_y	Gamma (0.5, 0.15)	$[0, \infty)$
Taylor rule - exchange rate	$\psi_{\Delta e}$	Gamma (0.5, 0.15)	$[0, \infty)$
sd technology shock	σ_a	Inverse Gamma (0.5, Inf.)	$[0, \infty)$
sd exchange rate shock	σ_q	Inverse Gamma (0.5, Inf.)	$[0, \infty)$
sd ToT shock	σ_s	Inverse Gamma (0.5, Inf.)	$[0, \infty)$
sd domestic interest rate shock	σ_r	Inverse Gamma (0.5, Inf.)	$[0, \infty)$
sd foreign inflation shock	σ_{π^*}	Inverse Gamma (0.5, Inf.)	$[0, \infty)$
sd foreign output shock	σ_{y^*}	Inverse Gamma (0.5, Inf.)	$[0, \infty)$
sd foreign interest rate shock	σ_{r^*}	Inverse Gamma (0.5, Inf.)	$[0, \infty)$

The median estimates of the elasticity of substitution between domestic and foreign goods are between 0.42 and 0.70. The estimates are relatively higher than the values reported in the literature which implies that the domestic inflation or the output gap inflation are more sensitive to terms of trade changes. Nonetheless, inference on this parameter has tended to produce either very small elasticities or apparently implausibly large values (Justiniano and Preston, 2010b).

The posterior mean estimates of the frequency of price changes are between 0.87 and 0.91 in the home goods sector and between 0.86 and 0.92 in the imported goods sector. These values imply that the prices remain fixed for at least 8 quarters in the home goods sector. The frequency of changes in the imported goods sector is higher, being reoptimized on average every 7 quarters. The frequencies of price changes are lower than the values reported by Castro et al. (2011) for Brazil.

The estimates of the backward-looking components of the Phillips curve for the home goods and the imported goods sectors were higher than those found by Kam et al.

(2009), Justiniano and Preston (2010b) and by Palma and Portugal (2014) for the Brazilian economy. The exogenous processes for the home economy shocks are in general persistent, in particular the technology rate shock presents a high degree of inertia.

Finally, evaluating the monetary policy parameters we conclude that the central bank is more concerned with stabilization of inflation followed by interest rate smoothing and output growth. The weight attached to exchange rate variation is not high although we could not conclude that is zero.

Table 3: Posterior estimates

	Benchmark		*No habit*		*No indexation*		*No exchange rate*	
Par.	Mean	95%CI	Mean	95%CI	Mean	95%CI	Mean	95%CI
h	0.7937	[0.6837,0.8884]	-	-	0.789	[0.6841,0.8748]	0.8015	[0.6992,0.8976]
σ	0.9614	[0.6204,1.2975]	1.4144	[0.8823,1.9571]	0.8164	[0.5509,1.0319]	0.9598	[0.6087,1.2838]
ϕ	2.0551	[1.4846,2.6232]	1.8608	[1.2602,2.4433]	1.8948	[1.3565,2.5858]	1.8947	[1.2756,2.5267]
η	0.7041	[0.3992,1.0288]	0.7547	[0.5013,1.0391]	0.646	[0.3774,0.8827]	0.4209	[0.2711,0.577]
δ_H	0.5285	[0.3089,0.761]	0.4447	[0.2546,0.704]	-	-	0.4778	[0.3188,0.6606]
δ_F	0.8191	[0.6825,0.9702]	0.8599	[0.7282,0.9712]	-	-	0.7543	[0.5836,0.9247]
θ_H	0.8833	[0.8771,0.9429]	0.8725	[0.8687,0.9409]	0.9067	[0.8927,0.9437]	0.9128	[0.9061,0.967]
θ_F	0.8611	[0.8501,0.9249]	0.8816	[0.8729,0.9565]	0.9274	[0.9115,0.9671]	0.9201	[0.9186,0.963]
a_1	0.3615	[0.0141,0.6951]	0.3257	[0.0204,0.7189]	0.3474	[0.0264,0.6579]	0.3699	[0.0064,0.7301]
b_1	0.5685	[0.2307,0.9122]	0.6216	[0.2779,0.9594]	0.627	[0.2809,0.9644]	0.4924	[0.0466,0.8637]
c_1	0.4519	[0.0634,0.8508]	0.4866	[0.0598,0.8668]	0.5000	[0.0745,0.9082]	0.4354	[0.0299,0.8364]
ρ_r	0.5217	[0.3187,0.7293]	0.4394	[0.2588,0.6262]	0.5627	[0.3794,0.7454]	0.3597	[0.2115,0.5423]
ψ_y	0.4601	[0.1934,0.8014]	0.4926	[0.1914,0.8987]	0.5354	[0.2757,0.843]	0.5013	[0.2344,0.7982]
ψ_π	0.956	[0.5329,1.2946]	1.2613	[0.7052,1.7095]	0.952	[0.5964,1.3368]	1.1739	[0.6582,1.7885]
$\psi_{\Delta e}$	0.271	[0.133,0.447]	0.2652	[0.1025,0.4402]	0.2883	[0.141,0.4388]	-	-
ρ_s	0.3797	[0.2444,0.5325]	0.2089	[0.1318,0.3015]	0.3607	[0.2182,0.5036]	0.4201	[0.2326,0.605]
ρ_a	0.4756	[0.2805,0.6656]	0.4535	[0.284,0.6169]	0.5806	[0.4151,0.7567]	0.4707	[0.3162,0.6257]
ρ_q	0.3694	[0.2428,0.5313]	0.3696	[0.1807,0.5121]	0.3995	[0.2701,0.5388]	0.2384	[0.1255,0.3627]
σ_s	0.3436	[0.2229,0.4804]	0.5834	[0.3351,0.8313]	0.4050	[0.2668,0.6354]	0.5436	[0.3621,0.7315]
σ_a	0.233	[0.1127,0.3619]	0.2745	[0.1276,0.3834]	0.2181	[0.1187,0.3611]	0.8029	[0.4411,1.1418]
σ_q	0.0712	[0.059,0.0831]	0.0636	[0.0589,0.0701]	0.0694	[0.0593,0.0813]	0.2494	[0.2353,0.2731]
σ_r	0.0619	[0.0588,0.0677]	0.0633	[0.0588,0.0719]	0.0614	[0.0588,0.066]	0.2469	[0.2352,0.2691]
σ_{π^*}	0.0616	[0.0588,0.0667]	0.0617	[0.0588,0.0675]	0.0615	[0.0588,0.0666]	0.2469	[0.2352,0.2692]
σ_{y^*}	0.0616	[0.0588,0.0669]	0.0617	[0.0588,0.0668]	0.0615	[0.0588,0.0667]	0.2464	[0.2352,0.2653]
σ_{r^*}	0.0612	[0.0588,0.0655]	0.0616	[0.0588,0.0671]	0.0613	[0.0588,0.066]	0.2455	[0.2352,0.266]

Figure 2: Posterior distribution of key parameters. Prior (gray) and posterior (black).

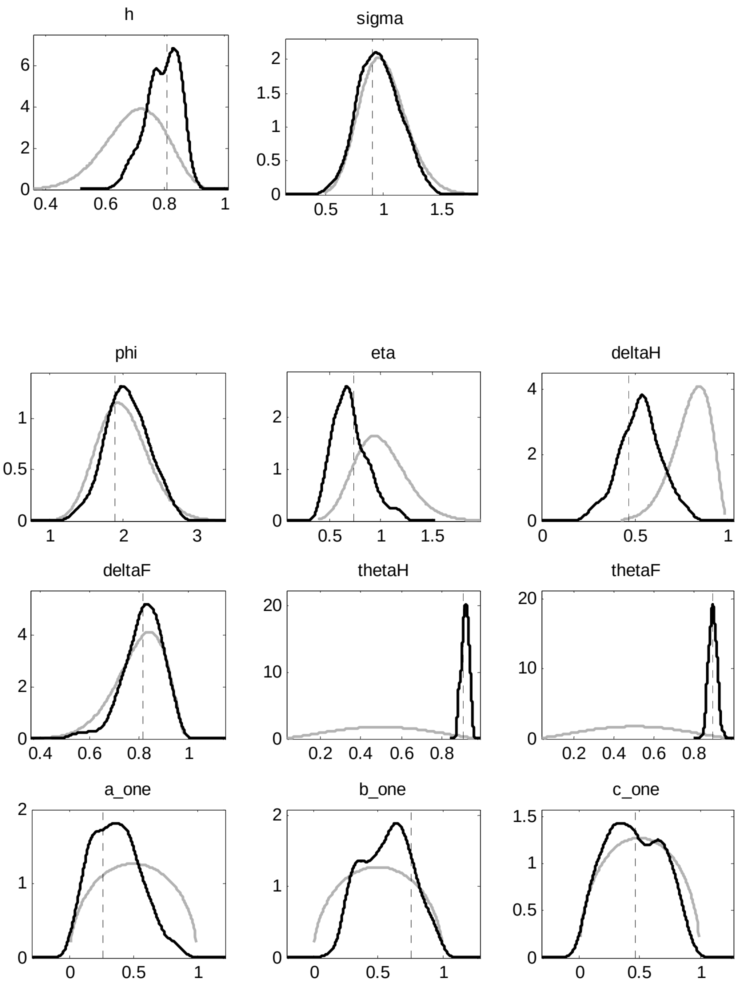
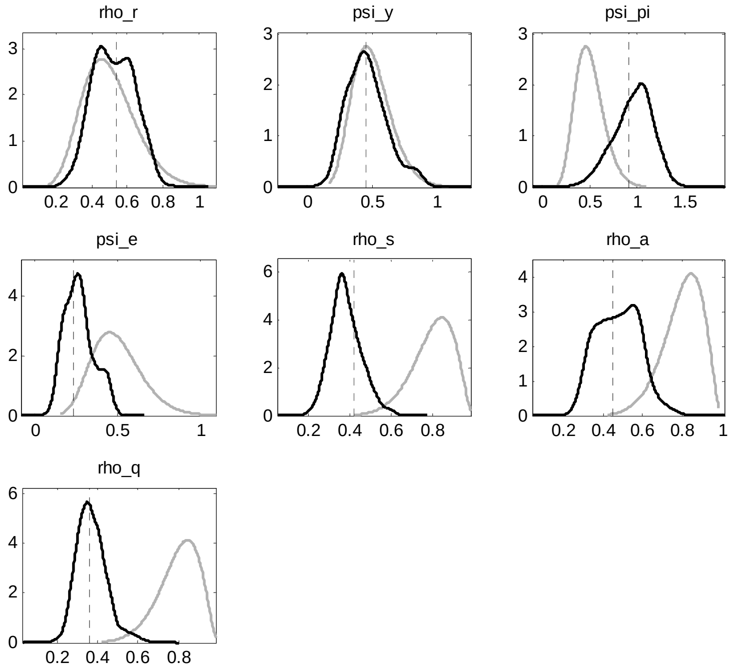


Figure 3: Posterior Distribution of Key Parameters. Prior (gray) and Posterior (black). (continued)



6.2. Fit of the models

We use the posterior odds ratio to examine the fit of three different model specifications relative to the benchmark model. In addition, the second order moments of key observables are compared to those implied by the estimated model.

Table 4 summarizes the model comparison based on the marginal likelihood of the models and the Bayes factor as the ratio of the marginal likelihoods of each model to the *benchmark* model. The marginal likelihoods were computed using the modified harmonic mean estimator proposed by Geweke (1999).

The marginal data density associated with the *no indexation* model is -614.21 and the Bayes factor is approximately 1.4×10^{30} in favor of this model versus the *benchmark* model. This apparently odd result was also found by Matheson (2010) who show that the best fitting model for three different countries is the one without habit formation and without price indexation. Del Negro et al. (2007) found that the two alternative model specifications (no indexation and no habit) are reject in favor of the fully specified model, although the rejection for the no indexation is not as strong as that for the no habit model. Silveira (2008) using data from Brazil found that both price indexation and habit formation are important to improve model´s fit though the evidence is less robust for price indexation.

The marginal data density of the *no habit* model is -711.04. The Bayes factor of the model versus the *benchmark* suggests that the better fit is achieved by the fully specified model. The same is also true for the *no exchange rate* model. The marginal data density is -714.24 and the Bayes factor suggests that the better fit is achieved by the fully specified model. Hence, we could not conclude that the central bank does not target exchange rate volatility via its interest rate decisions as in Kam et al. (2009) and Palma and Portugal (2014).

Table 4: Log marginal likelihoods and Bayes factors.

Specification	$\ln p(Y\|\mathcal{M})$	Bayes Factor
Benchmark model	-683.62	0
No habit	-711.04	-27.41
No indexation	-614.21	69.41
No exchange rate	-714.24	-30.62

Table 5 presents the variance and first order autocorrelations of four observables used to estimate the model: output (y), interest rate (r), real exchange rate (q) and inflation (π). The comparison exercise provides a measure of absolute fit rather than a measure of relative fit based on posterior odds ratios. It is possible to see that model does not match the second order properties of the data so well even though for some variables the implied variance and autocorrelations are close to their empirical counterparts. The variance implied by the estimated model for output and real exchange rate are lower than the variance of the data. For the first order autocorrelations the model provides a better characterization of the data. The implied autocorrelations are close to autocorrelations in the data.

Table 5: Data and implied variances and first order autocorrelations of output, interest rate, real exchange rate and inflation.

	Variance ($\times 10^{-3}$)		First order autocorrelation	
	Data	Model	Data	Model
y	0.8592	0.5192	0.5475	0.5703
r	0.1235	0.2127	0.9373	0.5110
q	6.3832	2.0491	0.2467	0.3400
π	0.0928	0.2473	0.4423	0.5561

6.3. Impulse responses

In this section we analyze the dynamic properties of the models when the economy is exposed to a technology and a monetary policy shock. We present the responses of domestic output growth, real exchange rate, inflation, interest rates and terms of trade to the two shocks that have received the most attention in the literature. Additionally, we compare whether the responses for *no indexation* model which is the best fitting model differ significantly from the *benchmark* model.

Figures 3 and 4 show mean responses to one-standard-deviation shocks computed based on the posterior draws for the benchmark and no indexation models. The figures show the impulse response functions with respect to technology and monetary shocks for the benchmark model (red solid lines), the no indexation model (blue solid lines) and 90% posterior bands (dotted lines).

A positive technology shock raises output. It has a substantial impact on inflation and a prolonged effect on the exchange rate and terms of trade. There is a reduction on the marginal cost due to the increase of productivity and as a result there is a reduction on the domestic prices. The terms of trade is also affect by the differential of domestic and foreign inflation showing an increase. Figures 3 shows that the responses of the variables to a technology shock appear to be similar for the no indexation and the benchmark models. However, the technology shock seems to have a greater effect in the benchmark model.

A monetary policy shock has an impact on output. Due to the real interest parity condition exchange rate depreciates. These movements reduce the prices of domestic goods and thereby have an impact on the terms of trade. Afterwards the interest rate is adjusted following the monetary policy rule. Figure 4 shows that except for the terms of trade the impulse responses are not very persistent since the models do not generate much internal propagation. As for the previous case, monetary shocks seem to have a greater effect in the benchmark model.

Figure 4: Impulse responses to a technology shock for the benchmark model (red solid lines), the no indexation model (blue solid lines) and 90% posterior bands (dotted lines).

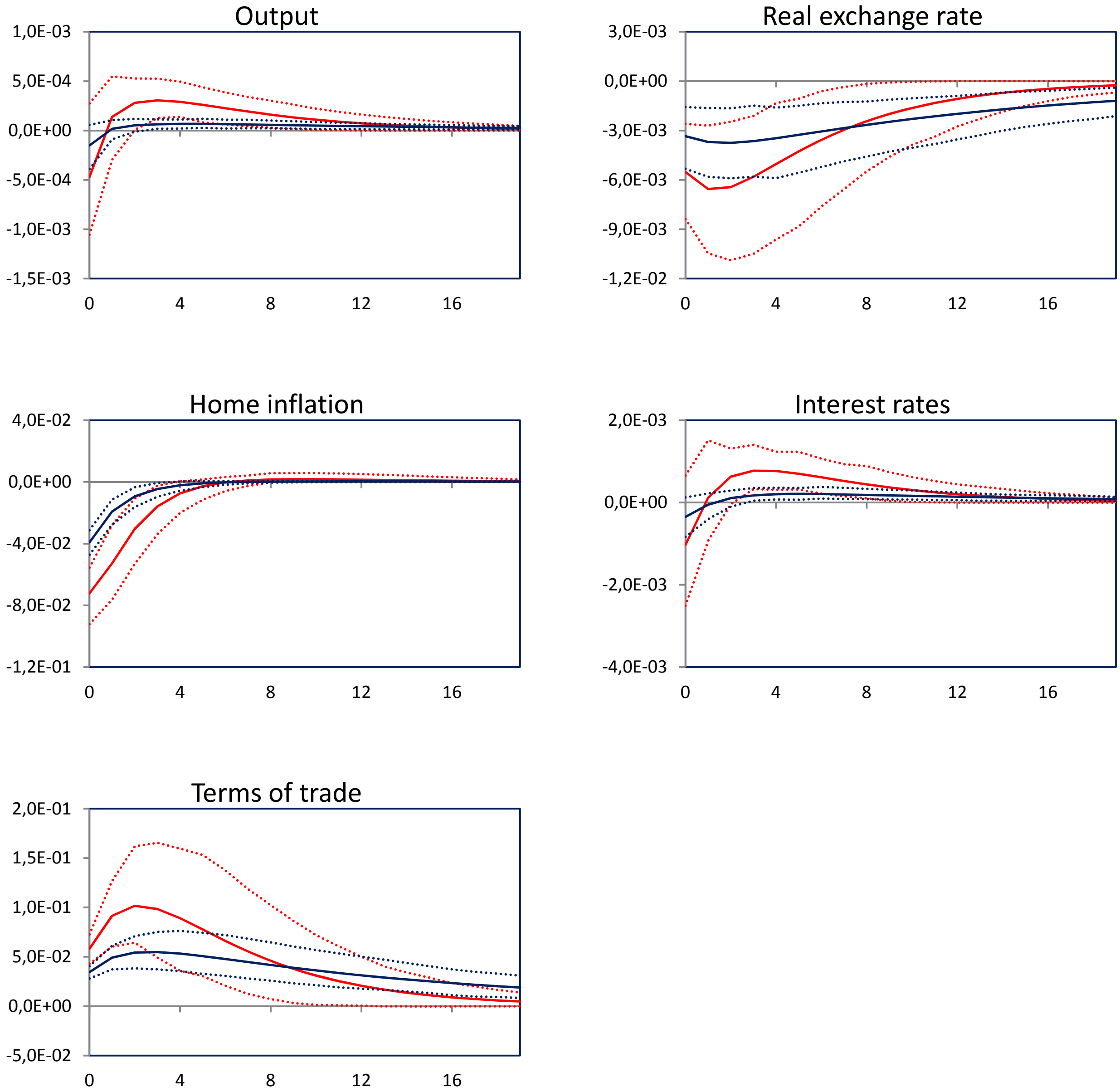
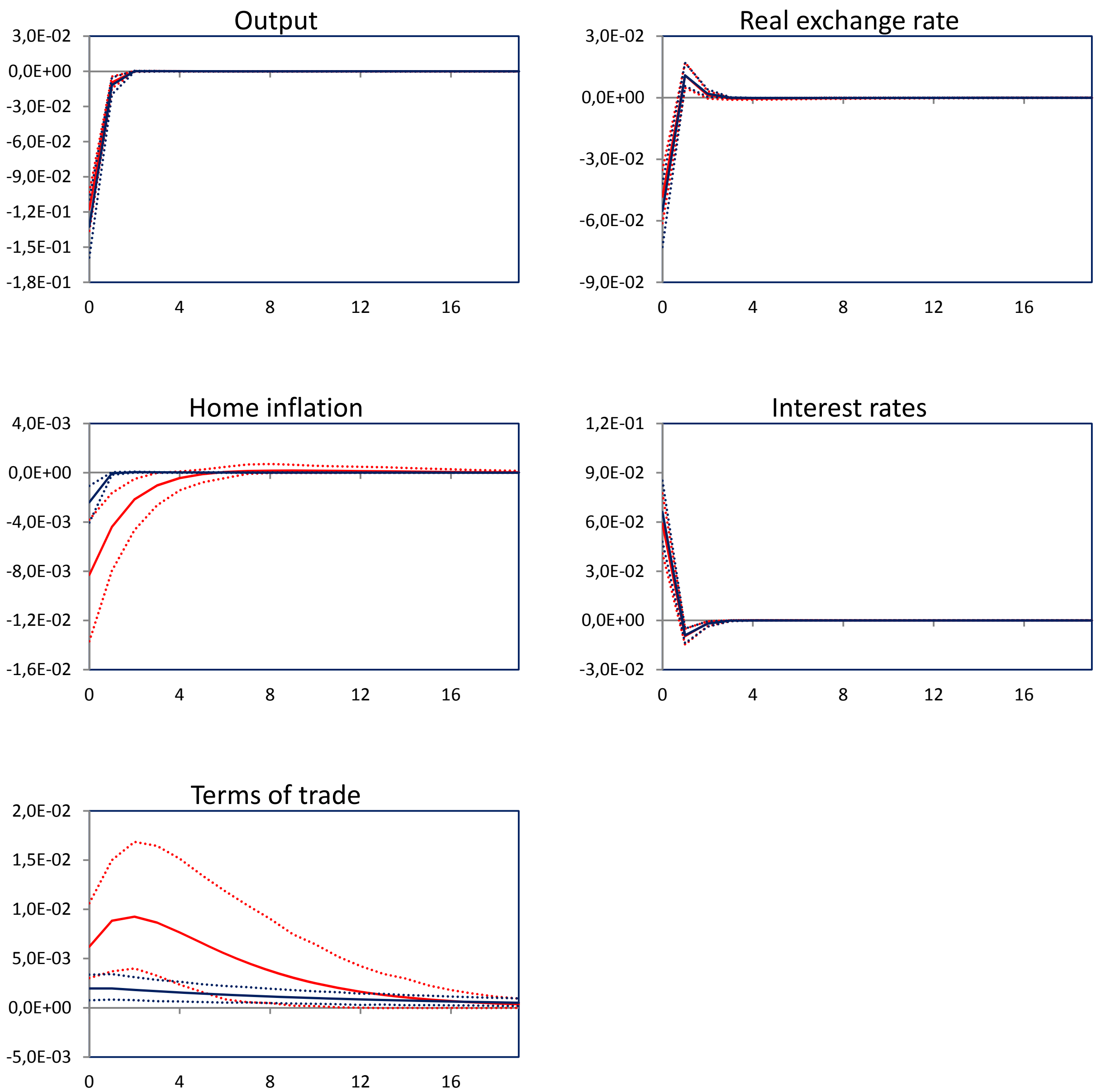


Figure 5: Impulse responses to a monetary policy shock for the benchmark model (red solid lines), the no indexation model (blue solid lines) and 90% posterior bands (dotted lines).



6.4. Comparisons to VARs

We compare the DSGE model to a VAR using the DSGE-VAR approach of Del Negro and Schorfheide (2004). To construct a DSGE-VAR we consider a vector autoregressive specification of the form:

$$y_t = \Phi_c + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} - u_t, \quad (41)$$

where y_t is a n×1 random vector and u_t is assumed to be normally distributed with zero mean $u_t \sim$ iid $N(0, \Sigma)$. Eq. (41) is a reduced form representation of a p-lag VAR. The VAR can be written as a multivariate linear regression model:

$$Y = X\Phi + U$$

where Y and U are $T \times n$ matrices composed of rows y_t' and u_t'. X is a $T \times k$ matrix with rows $x_t' = [y_{t-1}', \dots, y_{t-p}', 1]$ and $\Phi = \left[\Phi_0, \Phi_1, \dots, \Phi_p\right]'$. It is assumed that that data was transformed such that y_t is stationary. Let $\mathbb{E}_\theta^D[.]$ be the expectation under the DSGE model conditional on parameterization θ and define the autocovariance matrices:

$$\Gamma_{XX}(\theta) = \mathbb{E}_\theta^D[x_t x_t'], \quad \Gamma_{XY}(\theta) = \mathbb{E}_\theta^D[x_t y_t'].$$

A VAR approximation of the DSGE model can be obtained from the following restriction functions that relate the DSGE model parameters to the VAR parameters:

$$\Phi^*(\theta) = \Gamma_{XX}^{-1}(\theta)\Gamma_{XY}(\theta), \quad \Sigma^*(\theta) = \Gamma_{YY}(\theta) - \Gamma_{YX}(\theta)\Gamma_{XX}^{-1}(\theta)\Gamma_{XY}(\theta).$$

Del Negro et al. (2007) argue that if the VAR representation y_t deviates from the restrictions functions $\Phi^*(\theta)$ and $\Sigma^*(\theta)$, then the DSGE model is misspecified. In order to account for this potential misspecification it is assumed that there is a vector θ and matrices Φ^Δ and Σ^Δ such that the data are generated from the VAR with the following coefficient matrices:

$$\Phi = \Phi^*(\theta) + \Phi^\Delta, \quad \Sigma = \Sigma^*(\theta) + \Sigma^\Delta.$$

The matrices Φ^Δ and Σ^Δ capture deviations from the restriction functions $\Phi^*(\theta)$ and $\Sigma^*(\theta)$. For the specification of the prior distribution of Φ and Σ given the DSGE model parameters θ it is assumed that:

$$\Sigma|\theta \sim \mathcal{IW}(\lambda T \Sigma^*(\theta), \lambda T - k, n)$$

$$\Phi|\Sigma, \theta \sim \mathcal{N}\left(\Phi^*(\theta), \frac{1}{\lambda T}[\Sigma^{-1} \otimes \Gamma_{XX}(\theta)]^{-1}\right),$$

where $\mathcal{IW}$ is the inverted Wishart distribution. As An and Schorfheide (2007) explain the prior distribution can be interpreted as a posterior calculated from a sample of λT observations generated from the DSGE model with parameters θ. λ is a hyperparameter that scales the prior covariance matrix. The prior is diffuse for small values of λ. The larger λ, the more the estimates of Φ and Σ will approximate to the restrictions implied by the DSGE model. The prior distribution is proper provided that $\lambda T \geq k + n$.

The joint posterior density of VAR and DSGE model parameters can be factorized as follows:

$$p_\lambda(\Phi, \Sigma, \theta|Y) = p_\lambda(\Phi, \Sigma|Y, \theta)p_\lambda(\theta|Y).$$

with the λ subscript indicating the dependence of the posterior on the hyperparameter. The posterior distribution of Φ and Σ is also of the Inverted Wishart-Normal form (Del Negro and Schorfheide, 2011):

$$\Sigma|Y, \theta \sim \mathcal{IW}\left((1+\lambda)T\hat{\Sigma}_{\mathrm{b}}(\theta), (1+\lambda)T-k, n\right)$$

$$\Phi|Y, \Sigma, \theta \sim \mathcal{N}\left(\hat{\Phi}_{\mathrm{b}}(\theta), \Sigma \otimes (\lambda T\Gamma_{XX}(\theta) + X'X)^{-1}\right).$$

Therefore, the larger the weight λ of the prior, the closer the posterior mean of the VAR parameters is to $\Phi^*(\theta)$ and $\Sigma^*(\theta)$, the values that respect the cross-equation restrictions of the DSGE model. On the other hand, if $\lambda = (n + k)/T$ then the posterior mean is close to the OLS estimate $(X'X)^{-1}X'Y$ (An and Schorfheide, 2007).

The prior density $p(\theta)$ for the DSGE model parameters is determined by the specification of the prior distributions of the DSGE parameters. The marginal posterior density of θ can be obtained through the marginal likelihood of $p_\lambda(Y|\theta)$. A derivation is provided in Del Negro and Schorfheide (2004).

The empirical performance of the DSGE-VAR approach depends on the weight placed on the DSGE model restrictions. Del Negro et al. (2007) use the marginal data density as a criterion for the choice of λ:

$$p_\lambda(Y) = \int p_\lambda(Y|\theta)p(\theta)d\theta.$$

For computational reasons the hyperparameter λ is restrict to a finite grid Λ. The normalized $p_\lambda(Y)$ can be interpreted as posterior probabilities for λ if it is assigned equal prior probability to each grid point. Del Negro et al. (2007) define:

$$\hat{\lambda} = \underset{\lambda \in \Lambda}{argmax}\; p_\lambda(Y|\lambda).$$

If $p_\lambda(Y)$ peaks at an intermediate value of λ (between 0.5 and 2, for instance) the model is not well specified and a comparison between DSGE-VAR($\hat{\lambda}$) and DSGE-VAR(∞) impulse responses can potentially yield insights about the source of misspecification (Del Negro et al., 2007).

6.5. DSGE-VAR results

We assess the fit of the *no indexation*, *no habit*, *no exchange* and *benchmark* models using DSGE-VAR approach. We report the results based on a DSGE-VAR with four lags. The lag length of the DSGE-VAR is chosen to give a good approximation of the model[3]. We show in Table 6 that the VAR approximation error is small. In addition, it is assumed that λ lies on the finite grid $\Lambda \in \{0.25, 0.5, 0.75, 1, 5, 100\}$. The log marginal likelihoods of DSGE-VARs as a function of λ for each model are reported in Table 6.

The maximum values of the likelihood functions are attained for λ between 0.5 and 0.75. For $\lambda \geq 1$ the values of the marginal likelihoods decrease significantly. The functions have an inverted U-shape for all models, similar to those found in empirical applications by Del Negro and Schorfheide (2006). The substantial drop in marginal likelihood as λ increases is an evidence of the DSGE model misspecification. It reflects the fact that the fit of the DSGE model is worse than the DSGE-VAR($\hat{\lambda}$) model.

The last row reports the log marginal likelihood of the DSGE model. Comparing it to the DSGE-VAR(∞), which in our case it is assumed to be λ = 100, it is possible to check if the VAR can adequately approximate the state-space representation of the DSGE model. If the VAR approximation of the DSGE model were exact, then the values of the last two rows would be the same. In our case, they are quite close suggesting that the VAR approximation error is small.

Table 6: Log marginal likelihoods of DSGE–VAR and DSGE models

DSGE-VAR(λ)	*Benchmark*	*No indexation*	*No habit*	*No exchange*
λ = 0.25	-670.26	-607.92	-696.44	-706.18
λ = 0.5	-668.43	-605.37	-694.93	-702.16
λ = 0.75	-669.02	-605.43	-695.85	-708.63
λ = 1	-669.97	-605.99	-697.10	-710.22
λ = 5	-678.09	-611.84	-707.03	-715.93
λ = 100	-682.22	-615.36	-711.92	-717.93
DSGE model	-683.62	-614.21	-711.04	-714.24

[3] A VAR approximation of the DSGE model is typically not exact because the state-space representation of the linearized DSGE model generates moving average terms. The accuracy of approximation depends on the number of lags p (An and Schorfheide, 2007).

7 Conclusion

In this paper we estimated a DSGE model for a small open economy based on Kam et al. (2009) using Brazilian economy data for the period of inflation targeting followed by the Central Bank of Brazil since 1999.

We assessed in a Bayesian framework the empirical fit of four different specifications of the small open economy model. In particular, we tested the importance of two particular features of DSGE models: price indexation and habit formation. We also tested the case where the central bank is restricted to put no weight on exchange rate variability in its monetary policy function. Finally, the model´s ability to fit particular second-order characteristics was assessed.

Using posterior odds ratios we found that the best fitting model is the one without price indexation. Contrary to Palma and Portugal (2014) we could not conclude that the central bank does not target exchange rate volatility via its interest rate decisions. The comparison of second order moments showed that as for persistence the model provides a reasonable characterization of the data. The implied volatilities for output, interest rate, real exchange rate and inflation are not so close to their empirical counterparts. The analysis of the impulse response functions have showed that the dynamics of the variables are quite similar although the shocks have a greater effect in the benchmark model when compared to the model without price indexation.

Then, we investigated the potential model misspecification implied by invalid cross-coefficient restrictions on the time series representation generated by the DSGE model. The model misspecification was tested using the DSGE-VAR approach developed by Del Negro and Schorfheide (2004) which provides a reliable benchmark to evaluate the model´s misspecification. The maximum values of the likelihood functions were attained for λ between 0.5 and 0.75 considering the four competing model specifications. Therefore, there is an indication of misspecification in this highly stylized small open economy model.

The reasons for the model misspecification should be investigated in a future research. The model assumes that shocks across the home and foreign economy are independent. However, there is ample evidence of comovement in economy activity across countries. Alternative specifications that assume correlated cross-country shocks could partially resolve the problem of misspecification (Justiniano and Preston, 2010a).

The inclusion of financial frictions into the small open economy setting as in Christiano, Trabandt and Walentin (2011) could also improve the model´s fit.

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