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Labor Markets in Heterogenous Sectors*

Sergio A. Lago Alves†

Abstract

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I expand the standard model with labor frictions and matching function, to account to the endogenous decision to either leave the labor market or migrate to a different sector, after a stochastic training period. Sectors (manufacturing and services) are asymmetric, firms are subject to price stickiness, have specific labor force, post vacancies advertisement and explore both the intensive as the extensive margin of labor. After estimating the model with 13 quarterly data from the goods and labor market, from 2003:Q1 to 2014:Q4, I show that the estimated version of this model is able to account for the heterogeneous dynamics of the labor and goods market in Brazil.

Keywords: DSGE models, Monetary Policy, Sector Heterogeneity, Manufacturing and Services Sectors, Labor Markets, Intensive and Extensive Margins of Labor, Imperfect and Incomplete Labor Mobility, Bayesian Inference, Brazilian Economy.

JEL classification: C11, E24, E31, E52, J21, J22, J31, J62, J64.

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1 Introduction

During the last 15 years the goods and labor markets in Brazil have been showing what Alves and Correa (2013) called the Brazilian Labor Market Dichotomy. The authors conjecture and find some evidence that this phenomenon was driven from deep sectoral heterogeneity between the manufacturing and services sectors.

Section 2 broadens the stylized facts related to this labor market dichotomy and presents evidence that the effects of sector heterogeneity have been more evident after the 2008-2009 Great Recession crisis. For instance, the unemployment rate have kept a decreasing path, even though activity measures were also led to fall: (a) the GDP growth rate; (b) the participation rate; and (c) important measures from the manufacturing sector only, such as GDP gap, employed workers, hours per worker and inflation rate.

On the one hand, those puzzling facts suggests that any analysis on production, labor market and inflation using Brazilian data must consider the strong heterogeneity of the services and manufacturing sectors, and must consider both the intensive and the extensive margins of labor in both sectors. On the other hand, increasing the sophistication of a general equilibrium model to account to such a level of heterogeneity might make inference much harder.

In this context, I aim at answering two important questions: (i) Which modelling features does a Dynamic General Equilibrium Model need in order to imbed the strong heterogeneity of the goods and labor markets in the services and manufacturing sectors, and account for those stylized facts observed in the Brazilian economy? (ii) How do sectoral labor and goods markets quantities respond to monetary policy and labor market shocks?

For answering the first question, I expand the standard DMP model[1] (after Diamond (1982), Mortensen (1982) and Pissarides (1985)), with search and matching frictions to account for equilibrium unemployment, to account to the endogenous decision to either leave the labor market or reallocate to a different sector, after a stochastic training period. Sectors (manufacturing and services) are asymmetric, firms are subject to sector-specific price stickiness and labor productivity, have specific labor force, post vacancies adver-

[1]While searching for jobs, unemployed workers earn monetary transfers and leisure benefits. Firms search for workers and post job vacancies at a cost. Search frictions prevents all unemployed workers from getting a job and firm from filling all available vacancies. Instead, the probability that an unemployed worker is matched into a new job depends on the total unemployed labor force and on the total number of vacancies. After a match occurs, individual wages are set by a Nash bargaining between the newly hired worker and the firm.

tisement and explore both the intensive as the extensive margin of labor. For simplicity and better understanding the labor market interactions with the goods market, I consider a closed economy, with constant stock of capital and two sectors only: services and manufacturing.

In the labor market modelling part, I bring two important contributions. First, in order to account for an endogenous leave of the labor market, I assume that searching for a job is a burden, captured by a constant disutility per unemployed worker. This assumption is simple, but rich enough to capture the trade-off between searching for a job for an uncertain period of time, which brings unemployment compensations and the expectation of a salary in the future, and stop looking for a job for a while, which ends the frustration of unsuccessfully searching a job for a while, even though losing unemployment compensations. If was not for this burden, unemployed workers with consumption insurance, as the ones that come back to their parents home or share a big household in which some of them have a job, would voluntarily prefer to remain unemployed. Indeed, they will consume just as an employed worker and have lower disutility to work.

In the literature, Christiano et al. (2010) uses a similar, but not as simple, way to account for endogenous involuntary unemployment. They assume that the disutility is a convex function of the time spent to search for a job, which in turns increases the chances of obtaining one. The way I model the burden, even though simpler, allows me for similar results.

Second, I model an asymmetric cost of reallocation to a different sector. Unemployed workers should leave the labor market, for a stochastic period of time, to specialize on the necessary skills for working in the other sector. When searching for a job in a different firm of the same sector, no specialization cost is imposed.

As in Thomas (2011) and Alves (2012), I assume that firms simultaneously make decisions on pricing and both the intensive and extensive margins of labor, so that labor is firm-specific. This interaction between pricing and firm-specific labor induces richer dynamics in both the goods and labor market.

Addressing the second question, I estimate log-linearized version of the model and obtain empirical responses to a monetary policy shock.

Estimation of 39 deep parameters and 13 standard deviations of the heterogeneous model is done using Bayesian technique with a Metropolis-Hasting MCMC algorithm and flat priors,[2] using 13 observed quarterly variables, from 2003:Q1 to 2014:Q4: manufacturing (detrended) GDP, services (detrended) GDP, tradables inflation rate from Brazilian CPI (as a consumer-based proxy for the inflation rate of the manufacturing sector), non-tradables inflation rate from Brazilian CPI (as a consumer-based proxy for the inflation rate of the services sector), working-age population, participation rate, employed workers at the manufacturing sector, employed workers at the services sector, hours per worker at the manufacturing sector, aggregate hours per worker, separation rate at the manufacturing sector, total mass of hired workers, and nominal interest rate. After obtaining 6,000,000 draws from a MCMC sampler, I keep the last 1,000,000 draws for inference and Bayesian impulse response exercises.

In the labor makert, the major empirical findings are: (*i*) workers from the manufacturing sector who are out of the labor market take longer to return (about 6 months) than workers from the service sector (about 3 months); (*ii*) workers from the manufacturing sector reallocate much faster to the service sector (about 7 months) than workers from the services sector (about 10 years) - in this regard, the information content in the sample strongly suggest that reallocation from services to manufacturing were really rare; (*iii*) the combination of greater labor market tightness and smaller search frictions in the services sector is the major explanation why unemployed workers find it easier to get a job in the service sector than those of the manufacturing one; (*iv*) on the other hand, workers' bargaining power in the manufacturing sector is much larger than that of the service sector. As a result, the average salary in the service sector are more correlated with the unemployment compensation, which is also very correlated with the minimum wage in Brazil. The results also suggest that salary bargaining is much more efficient in the manufacturing sector.

The data also support the evidence that there is no labor supply puzzle in the Brazilian labor market, i.e. I find that labor is just weakly elastic to salaries in Brazil.

The results also suggest that prices are much stickier and much more persistent in the manufacturing sector than in the services sector. Since prices are more flexible in the services sector, they adjust faster to shocks. However, strategic complementarities induce sectoral inflation rates not to detach much from each other, so that sectors do not lose long-run demand attractiveness.

In case of aggregate shocks, the relative demand for both sectors will be different

[2] I only consider Uniform prior distributions, defined in supports large enough to contain about the whole region where the likelihood function is appreciable.

due to the fact of prices are more flexible in the services sector. This effect is combined with the strong heterogeneity characterizing both sectors to produce different responses in the goods and labor markets. Finally, after aggregate shocks, sectoral GDP and output responses in the services sector are weaker and take longer to start responding than in the manufacturing sector. This is dues to the fact that prices adjust faster in the services sector.

Responses of labor market quantities have two important features. The first one is that the dynamics of labor market quantities, both in the aggregate as in sectoral measurements, are much more persistent than those of the goods sector. The second one is that aggregate responses of labor market variables qualitatively follow those in the services sector. This is due to the fact that about 75% of employed workers are in this sector, and this share is large enough to dominate the aggregate dynamics.

As for the dynamics after a monetary policy shock, the results imply that it is the manufacturing sector which suffers more. The fall in employment, hours, real salaries, GDP and output is much stronger in the manufacturing than in the services sector. The model is also able to capture what is know as labor hoarding, for hours tend to fall much faster than employment after the shock.

The remainder of his paper is organized as follows. Section 2 describe stylized facts of the goods and labor market in Brazil. Section 3 describes the model. Section 4 estimates the model, while Section 5 shows some impulse responses from selected shocks. Section 6 concludes.

2 Stylized facts

All variables described in this section were released by the Brazilian Institute of Geography and Statistics (IBGE). In particular, the labor market variables are obtained from the IBGE's Employment Monthly Survey (PME). At first glance, the most impressing fact is the ever-decreasing path of the unemployment rate, which was is not accompanied by increasing GDP growth rates. GDP was strongly hit by the 2008-2009 crisis, whereas the unemployment rate was barely affected at all, as depicted in figure 1 (Panel A).

Alves and Correa (2013) state and find strongly evidence that this dichotomy is part of a big picture describing two different sectors in Brazil, i.e. manufacturing and services.

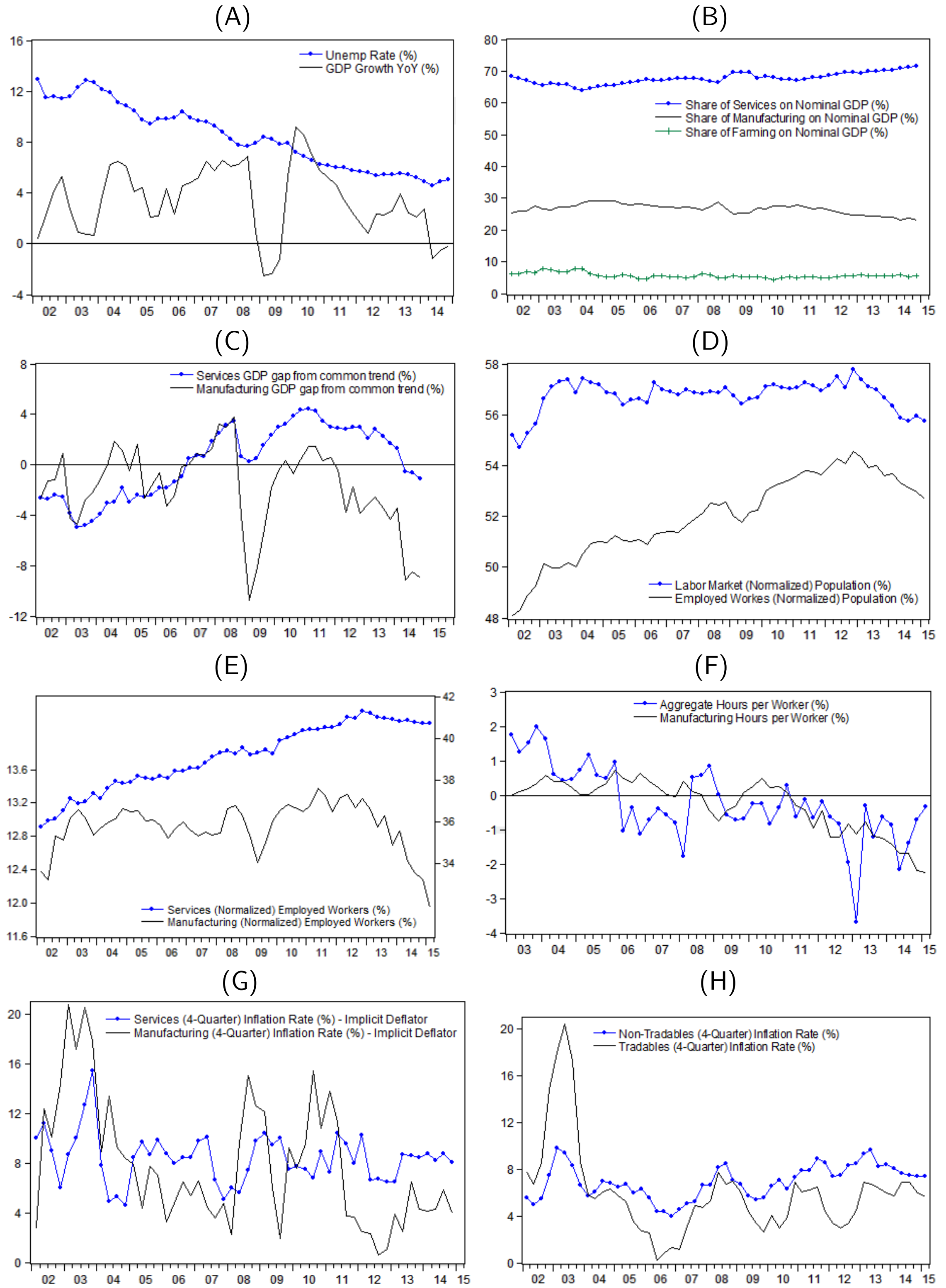

Figure 1: GDP, Labor Market and Inflation

For comparison, the services sector represents about 68% of Brazilian nominal GDP, as depicted in panel B of figure 1. Since the farming sector represents only 5.5% of Brazilian nominal GDP, and is strongly intensive in capital in Brazil, I embed it into the manufacturing GDP for the analysis I do in this paper.

Indeed, as we look closer at sectoral specific data, we find that those two sectors are very heterogeneous in many dimensions. For instance, Panel C of figure 1 shows the GDP gaps of both sectors from their common (long-run) log-linear trend.

Even obtained from different data, using a simpler method, this picture agrees with the findings of Alves and Correa (2013): the manufacturing sectoral GDP behave been struggling since 2011, after barely recovering from the (2008-2009) second Great Recession, while the services sectoral GDP behave been doing fine since 2007 and was barely affected by the crisis. Actually, only by mid-2014 this sector has been showing signs of struggle.

In the labor market, the annual growth rate of the active-age population has decreased from 1.7% in the early 2000's to about 1.2% by the 2010's, perhaps reflecting a demographic change towards an older population. Nevertheless, when normalizing by the active-age population, labor market stocks are more informative. Panel D of figure 1 depicts the (normalized) labor market population, i.e. the participation rate, and (normalized) employed workers.

Even though the labor market population remains stable for most of the sample, expect for falls at the beginning (2002) and the end of the sample (2013-2015), the mass of employed workers has been steadily increasing until the end of 2012, when both the participation rate and (normalized) employed workers started to decrease. Since the fall in the participation rate was larger than the fall of the latter, the unemployment continued to fall after 2012.

Panel E of figure 1 depicts the (normalized) masses of employed workers in the services and manufacturing sectors. Many features of the labor market suggest a strong heterogeneity. The first one is the fact that the services sector employs about 75% of the Brazilian working population. The remaining features come from their dynamics over time. Note that while the (normalized) employed population at the manufacturing sector remains stable for most of the sample, it has three periods of remarkable falls: (i) the beginning of the sample (2002); (ii) the second Great Recession (2008-2009); and (iii) the end of the sample (2013-2015). As for the services sector, its (normalized) employed population has been steadily increasing until the end of 2012, when its growth rate came to a halt. Note also that, differently of what happened in the manufacturing sector, the second Great Recession had almost no effect on the employed population of the services sector. Panel F of figure 1 depicts actual hours per worker, both in the aggregate (PME) as in the manufacturing sector (PIMES), described in terms of percentage deviations from their sample averages. Note that hours per worker have important variability over the cycle and have different sectoral dynamics.

As for sectoral inflation rates, panel G of figure 1 shows the 4-Quarter inflation rates of the implicit deflators of the services and manufacturing GDP's. Note that, even though the inflation rate of the manufacturing sector is more volatile, its level is much lower that of the services sector. And the gap between them seemed have become even larger more after 2011. In order to compare with inflation rates observed by consumers, panel H shows the 4-Quarter inflation rates, from the Brazilian Broad Consumer Price Index (IPCA) of non-tradable and tradable goods, as also considered in Alves and Correa (2013). Note that the main message is the same, including the gap opening from 2012 to 2015, since most of non-tradable goods comes from the services sector and most of tradable goods comes from the manufacturing sector. As expected for consumption goods, the volatilities are smaller than that from the implicit deflators.

3 The model

The model is depicted in Figure 2, which makes it easier to understand the whole structure in the analytical part. The representative household consumes consumption goods and have a continuum of workers, which can be hired or lose their jobs.

The labor market is subject to two sources of inefficiency: (i) workers can only work in their home economy; and (ii) there are search and match frictions. Finally, wages and hours are decided in a flexible Nash bargaining framework.

In each economy, consumption goods $z \in (0,1)$ can be either *manufactured* $(\mathfrak{m})$ or *services* $(\mathfrak{s})$ and are produced in two broad sectors $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$, i.e. there is a $\mathfrak{w}_{\mathfrak{m}}$ mass of goods $z_{\mathfrak{m}} \in \mathcal{Z}_{\mathfrak{m}} \equiv (0, \bar{z}_{\mathfrak{m}}]$ from sector $\mathfrak{m}$, and a $\mathfrak{w}_{\mathfrak{s}} = 1 - \mathfrak{w}_{\mathfrak{m}}$ mass of goods $z_{\mathfrak{s}} \in \mathcal{Z}_{\mathfrak{s}} \equiv (\bar{z}_{\mathfrak{m}}, 1]$ from sector $\mathfrak{s}$. Whenever convenient, I use the notation z when the results are independent of the firm type.

In the producing industry, differentiated firms produce all sort of consumption goods. Firms use labor in both the extensive and the intensive margins, post job vacancies at a cost and make price decisions.

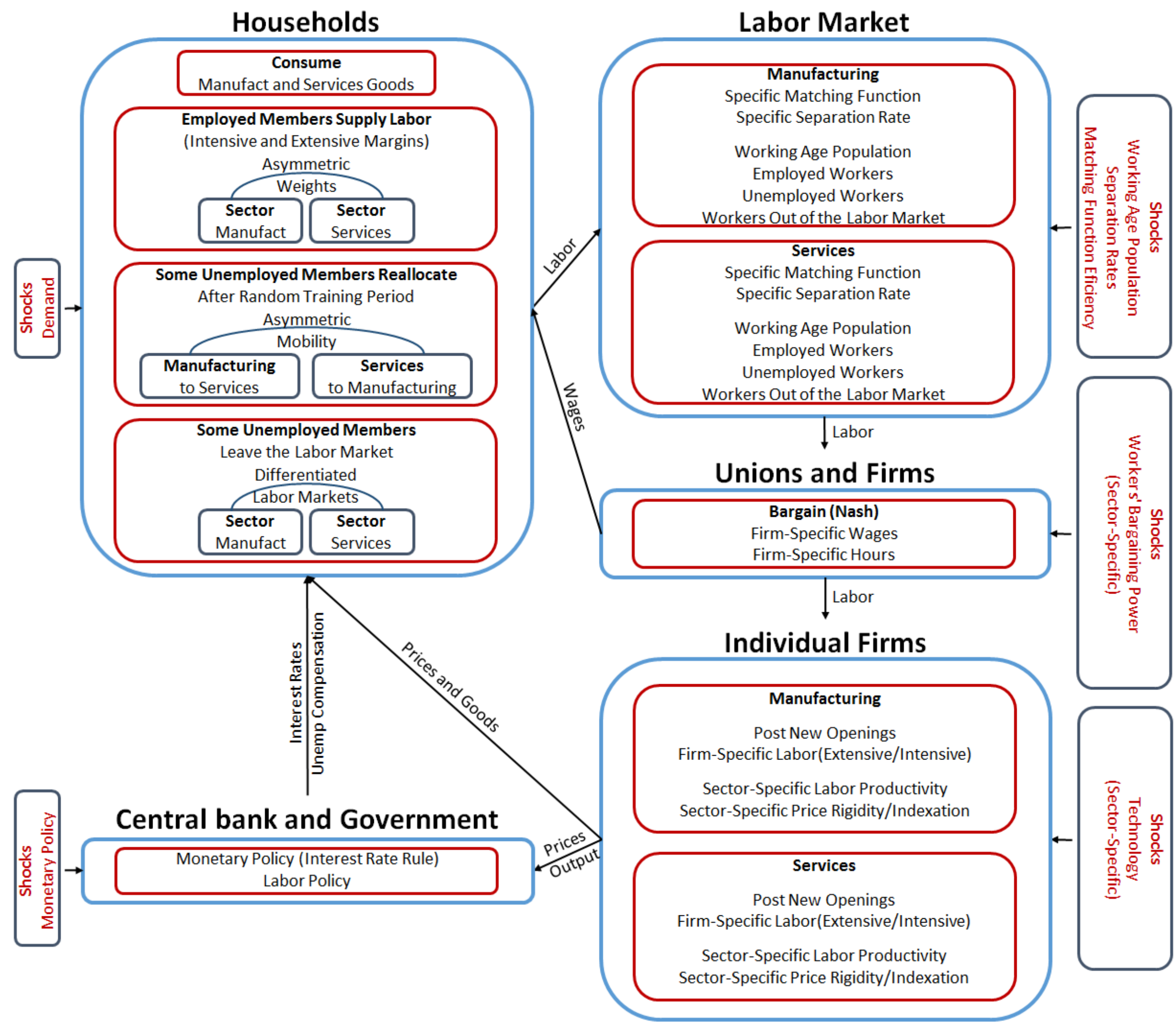

Figure 2: Model Structure

3.1 Labor flows

At the end of period t, the representative household has $\ell_t^{\mathfrak{p}}$ members at working age who care about all future generations. The size $\ell_t^{\mathfrak{p}}$ of the representative family is exogenous, stochastic, stationary, and its unconditional mean is normalized to unity, i.e. $E\ell_t^{\mathfrak{p}} = 1$.

Out of the $\ell_t^{\mathfrak{p}}$ members in the representative household, ℓ_t members are in the labor market (employed or unemployed) and $\ell_t^{\mathfrak{o}}$ members are out of the labor market. The quantities satisfy $\ell_t^{\mathfrak{p}} \equiv (\ell_t + \ell_t^{\mathfrak{o}})$. Even though the family size is an exogenous variable, the flows in and out the labor market are endogenously decided.

Within the household, $\mathsf{n}_t(z_{\mathfrak{c}}) \in (0, \ell_t)$ members are employed in firm $z_{\mathfrak{c}}$, from sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$. Labor is firm-specific and, due to labor market frictions, not all members are employed. In this context, $\mathsf{n}_t \equiv \int_0^1 \mathsf{n}_t(z)\, dz$ and $\mathsf{n}_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{m}_{\mathfrak{c}}} \int_{\mathfrak{c}} \mathsf{n}_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$ are the end-of-period employment aggregates in the economy as a whole and at sector $\mathfrak{c}$. During each period, $\mathsf{m}_t(z_\mathfrak{c})$ workers are matched into firm $z_\mathfrak{c}$. In this context, $\mathsf{m}_t \equiv \int_0^1 \mathsf{m}_t(z)\,dz$ and $\mathsf{m}_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_\mathfrak{c}} \int_\mathfrak{c} \mathsf{m}_t(z_\mathfrak{c})\,dz_\mathfrak{c}$ are the aggregate new matches in the economy as a whole and at sector $\mathfrak{c}$. The definitions imply:

$$\mathsf{n}_t = \mathfrak{w}_\mathfrak{m}\mathsf{n}_{\mathfrak{m},t} + \mathfrak{w}_\mathfrak{s}\mathsf{n}_{\mathfrak{s},t} \quad ; \; \mathsf{m}_t = \mathfrak{w}_\mathfrak{m}\mathsf{m}_{\mathfrak{m},t} + \mathfrak{w}_\mathfrak{s}\mathsf{m}_{\mathfrak{s},t} \tag{1}$$

While unemployed, workers might get a job within their own sectors according to a matching technology, described in the end of this section, without bearing any extra cost.

After not being matched during each period in sector $\mathfrak{c}$, however, a mass $\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}$ of unemployed workers decide it is better not to search for a job for a while, and possibly decide it is time to reallocate to the other sector. In any case, those workers leave the labor force of sector $\mathfrak{c}$ and enroll at the specialization school of sector $\mathfrak{c} \in \mathcal{F}_\mathfrak{c}$, where she catches up with frontier skills needed for either returning the original sector or working in the other sector. Training is not easy, though. With probability $\delta^{\mathfrak{c}}_{\mathfrak{c}}$, each worker returns to the labor force of sector $\mathfrak{c}$ in the beginning of next period. With probability $\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}$, she become fully specialized for working at sector $\bar{\mathfrak{c}} \neq \mathfrak{c}$ and decide it is better to reallocate to this sector in the beginning of next period.[3] In any case, specialized workers become part of the masses of beginning-of-period unemployed workers.

By the end of each period, $\mathsf{m}^{\mathfrak{o}}_t$ individuals have left the labor force, while $\ell^{\mathfrak{o}}_t$ aggregates all individuals out of the labor force:

$$\begin{aligned} \mathsf{m}^{\mathfrak{o}}_t &\equiv \mathfrak{w}_\mathfrak{m}\mathsf{m}^{\mathfrak{o}}_{\mathfrak{m},t} + \mathfrak{w}_\mathfrak{s}\mathsf{m}^{\mathfrak{o}}_{\mathfrak{s},t} && (2) \\ \ell^{\mathfrak{o}}_t &\equiv \mathfrak{w}_\mathfrak{m}\ell^{\mathfrak{o}}_{\mathfrak{m},t} + \mathfrak{w}_\mathfrak{s}\ell^{\mathfrak{o}}_{\mathfrak{s},t} && (3) \end{aligned}$$

where $\ell^{\mathfrak{o}}_{\mathfrak{c},t}$ is the mass of individuals out of the labor force of each sector.

At the beginning of each period, employed members separate from their jobs at an exogenous time-varying rate $\rho_\mathfrak{c} \in (0,1)$, which I assume to evolve according to the following stationary process about its steady state level $\bar{\rho}_\mathfrak{c}$:

$$\frac{\rho_{\mathfrak{c},t}}{\bar{\rho}_\mathfrak{c}} = \epsilon^{\rho}_{\mathfrak{c},t} \left(\frac{\rho_{\mathfrak{c},t-1}}{\bar{\rho}_\mathfrak{c}} \right)^{\phi^{\rho}_{\mathfrak{c}}} \tag{4}$$

where $\epsilon^{\rho}_{\mathfrak{c},t}$ is the sector-$\mathfrak{c}$ specific shock on the separation rate and $\phi^{\rho}_{\mathfrak{c}} \in (0,1)$.

Simultaneously, some individuals die and others come to working-age. I capture this

[3] Note that whenever $\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}} > \delta^{\mathfrak{c}}_{\bar{\mathfrak{c}}}$, it is easier to migrate from sector $\mathfrak{c}$ to sector $\bar{\mathfrak{c}}$ than from sector $\bar{\mathfrak{c}}$ to sector $\mathfrak{c}$.

fluctuation by assuming a constant exogenous death rate $\rho_{\mathfrak{d}} \in (0,1)$, affecting the masses of individuals in and out the labor market, and an exogenous net flow of $\mathsf{m}_{\ell,t}$ extra individuals evenly enrolling at specialization schools:

$$\left(\mathsf{m}_{\ell,t} - \bar{\mathsf{m}}_{\ell}\right) = \phi_{\ell}\left(\mathsf{m}_{\ell,t-1} - \bar{\mathsf{m}}_{\ell}\right) + \epsilon_{\ell,t} \tag{5}$$

where $\bar{\mathsf{m}}_{\ell}$ is the steady state level of extra individuals coming to working-age, $\epsilon_{\ell,t}$ is a shock to the mass of individuals coming to working-age and $\phi_{\ell} \in (0,1)$. and $\phi_{\ell} \in (0,1)$.

Based on those features, the laws of motion of employed members are described by

$$\begin{aligned} \mathsf{n}_t\left(z_{\mathfrak{c}}\right) &= \left(1-\rho_{\mathfrak{d}}\right)\left(1-\rho_{\mathfrak{c},t-1}\right)\mathsf{n}_{t-1}\left(z_{\mathfrak{c}}\right) + \mathsf{m}_t\left(z_{\mathfrak{c}}\right) \\ \mathsf{n}_{\mathfrak{c},t} &= \left(1-\rho_{\mathfrak{d}}\right)\left(1-\rho_{\mathfrak{c},t-1}\right)\mathsf{n}_{\mathfrak{c},t-1} + \mathsf{m}_{\mathfrak{c},t} \end{aligned} \tag{6}$$

The sectoral masses of individuals out of the labor market evolve as follows:

$$\ell^{\mathfrak{o}}_{\mathfrak{c},t} = \left(1-\rho_{\mathfrak{d}}\right)\ell^{\mathfrak{o}}_{\mathfrak{c},t-1} - \mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t} - \mathsf{m}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t} + \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t} + \mathsf{m}_{\ell,t} \tag{7}$$

where $\mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t}$ and $\mathsf{m}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t}$ denotes the flow of workers out of the labor force of sector $\mathfrak{c}$ who either returns to sector $\mathfrak{c}$ to search for a job or reallocates to sector $\bar{\mathfrak{c}}$, $\mathsf{m}^{\mathfrak{c}}_{\mathfrak{o},t}$ is the flow of workers coming from out of the labor market into sector $\mathfrak{c}$, and $\mathsf{m}_{\mathfrak{o},t}$ is the total flow of workers coming from out of the labor market. Those masses are defined as follows:

$$\mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t} = \delta^{\mathfrak{c}}_{\mathfrak{c}}\left(1-\rho_{\mathfrak{d}}\right)\ell^{\mathfrak{o}}_{\mathfrak{c},t-1} \quad ; \quad \mathsf{m}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t} = \delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\left(1-\rho_{\mathfrak{d}}\right)\ell^{\mathfrak{o}}_{\mathfrak{c},t-1} \tag{8}$$

$$\mathsf{m}^{\mathfrak{c}}_{\mathfrak{o},t} \equiv \mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t} + \tfrac{\mathfrak{w}_{\bar{\mathfrak{c}}}}{\mathfrak{w}_{\mathfrak{c}}}\mathsf{m}^{\mathfrak{c}}_{\bar{\mathfrak{c}},t} \quad ; \quad \mathsf{m}_{\mathfrak{o},t} \equiv \mathfrak{w}_{\mathfrak{m}}\mathsf{m}^{\mathfrak{m}}_{\mathfrak{o},t} + \mathfrak{w}_{\mathfrak{s}}\mathsf{m}^{\mathfrak{s}}_{\mathfrak{o},t} \tag{9}$$

The beginning-of-period unemployment aggregates u_t and $\mathsf{u}_{\mathfrak{c},t}$ account for unemployed members at the end of last period u^e_{t-1} and $\mathsf{u}^e_{\mathfrak{c},t-1}$ (defined further on), added to recently separated workers and workers returning from out of the labor market. Because I use a quarterly frequency calibration, I follow Ravenna and Walsh (2010) in distinguishing beginning-of-period from end-of-period unemployment aggregates. This strategy accounts for time-aggregation issues. The laws of motion are the following:

$$\begin{aligned} \mathsf{u}_{\mathfrak{c},t} &= \left(1-\rho_{\mathfrak{d}}\right)\left(\mathsf{u}^e_{\mathfrak{c},t-1} + \rho_{\mathfrak{c},t-1}\mathsf{n}_{\mathfrak{c},t-1}\right) + \mathsf{m}^{\mathfrak{c}}_{\mathfrak{o},t} \\ \mathsf{u}_t &= \mathfrak{w}_{\mathfrak{m}}\mathsf{u}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}}\mathsf{u}_{\mathfrak{s},t} \end{aligned} \tag{10}$$

Considering the masses $\mathsf{m}_{\mathfrak{c},t}$ and $\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}$ of unemployed workers either matched into a new job in sector $\mathfrak{c}$ or leaving the labor force, the end-of-period unemployment aggregates u^e_t and $\mathsf{u}^e_{\mathfrak{c},t}$ and labor forces ℓ_t and $\ell_{\mathfrak{c},t}$ and are defined as follows:

$$\mathsf{u}^e_{\mathfrak{c},t} = \mathsf{u}_{\mathfrak{c},t} - \mathsf{m}_{\mathfrak{c},t} - \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t} \quad ; \; \mathsf{u}^e_t = \mathfrak{w}_{\mathfrak{m}} \mathsf{u}^e_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \mathsf{u}^e_{\mathfrak{s},t} \tag{11}$$

$$\ell_{\mathfrak{c},t} = \mathsf{u}^e_{\mathfrak{c},t} + \mathsf{n}_{\mathfrak{c},t} \quad ; \; \ell_t = \mathfrak{w}_{\mathfrak{m}} \ell_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \ell_{\mathfrak{s},t} \tag{12}$$

Total and sectoral working-age populations are defined as follows:

$$\ell^{\mathfrak{p}}_{\mathfrak{c},t} \equiv \ell_{\mathfrak{c},t} + \ell^{\mathfrak{o}}_{\mathfrak{c},t} \quad ; \; \ell^{\mathfrak{p}}_t \equiv \mathfrak{w}_{\mathfrak{m}} \ell^{\mathfrak{p}}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \ell^{\mathfrak{p}}_{\mathfrak{s},t} \tag{13}$$

From (12),(11), (10), (6) and (9), we obtain alternative laws of motion for $\ell_{\mathfrak{c},t}$ and ℓ_t:

$$\ell_{\mathfrak{c},t} = (1 - \rho_{\mathfrak{d}}) \, \ell_{\mathfrak{c},t-1} + \mathsf{m}^{\mathfrak{c}}_{\mathfrak{o},t} - \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t} \quad \ell_t = (1 - \rho_{\mathfrak{d}}) \, \ell_{t-1} + \mathsf{m}_{\mathfrak{o},t} - \mathsf{m}^{\mathfrak{o}}_t \tag{14}$$

From (13), (7), (14), and (9), we obtain alternative laws of motion for $\ell^{\mathfrak{p}}_{\mathfrak{c},t}$ and $\ell^{\mathfrak{p}}_t$:

$$\ell^{\mathfrak{p}}_{\mathfrak{c},t} = (1 - \rho_{\mathfrak{d}}) \, \ell^{\mathfrak{p}}_{\mathfrak{c},t-1} + \tfrac{\mathfrak{w}_{\bar{\mathfrak{c}}}}{\mathfrak{w}_{\mathfrak{c}}} \mathsf{m}^{\mathfrak{c}}_{\bar{\mathfrak{c}},t} - \mathsf{m}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t} + \mathsf{m}_{\ell,t} \quad ; \; \ell^{\mathfrak{p}}_t = (1 - \rho_{\mathfrak{d}}) \, \ell^{\mathfrak{p}}_{t-1} + \mathsf{m}_{\ell,t} \tag{15}$$

which implies that the working-age population $\ell^{\mathfrak{p}}_t$ evolves according to a completely exogenous AR(1) process. Note that the condition $E\ell^{\mathfrak{p}}_t = 1$ implies that $E\mathsf{m}_{\ell,t} = \bar{\mathsf{m}}_\ell = \rho_{\mathfrak{d}}$.

Standard end-of-period unemployment rates $\mathfrak{u}^e_t$ and $\mathfrak{u}^e_{\mathfrak{c},t}$ are defined as

$$\mathfrak{u}^e_t \equiv \tfrac{\mathsf{u}^e_t}{\ell_t} \quad ; \; \mathfrak{u}^e_{\mathfrak{c},t} \equiv \tfrac{\mathsf{u}^e_{\mathfrak{c},t}}{\ell_{\mathfrak{c},t}} \tag{16}$$

while participation rates $\mathfrak{r}_t$ and $\mathfrak{r}_{\mathfrak{c},t}$ are defined according to:

$$\mathfrak{r}_t \equiv \tfrac{\ell_t}{\ell^{\mathfrak{p}}_t} \quad ; \; \mathfrak{r}_{\mathfrak{c},t} \equiv \tfrac{\ell_{\mathfrak{c},t}}{\ell^{\mathfrak{p}}_{\mathfrak{c},t}} \tag{17}$$

Firm z posts $\mathsf{v}^e_t(z)$ job vacancies at the end of each period, and hence $\mathsf{v}_t(z) \equiv \mathsf{v}^e_{t-1}(z)$ is the mass of job openings at firm z available at the beginning of period t. Therefore, I define $\mathsf{v}^e_t \equiv \int_0^1 \mathsf{v}^e_t(z)\, dz$ and $\mathsf{v}^e_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \mathsf{v}^e_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$ as the total end-of-period number of vacancy postings in the economy as a whole and in sector $\mathfrak{c}$. Similarly, I define v_t and $\mathsf{v}_{\mathfrak{c},t}$

as the corresponding beginning-of-period job openings. Those quantities satisfy:

$$\begin{aligned} \mathsf{v}_t = \mathsf{v}^e_{t-1} \quad &; \; \mathsf{v}_{\mathfrak{c},t} = \mathsf{v}^e_{\mathfrak{m},t-1} \\ \mathsf{v}_t = \mathfrak{w}_{\mathfrak{m}} \mathsf{v}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \mathsf{v}_{\mathfrak{s},t} \quad &; \; \mathsf{v}^e_t = \mathfrak{w}_{\mathfrak{m}} \mathsf{v}^e_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \mathsf{v}^e_{\mathfrak{s},t} \end{aligned} \tag{18}$$

In this context, p_t, q_t and θ_t are the economy wide job-finding rate, matching rate and labor market tightness rate within the period. Those rates, and their corresponding sectoral peers, are defined as follows:

$$\begin{aligned} \mathsf{p}_t \equiv \tfrac{\mathsf{m}_t}{\mathsf{u}_t} \;\; ; \; \mathsf{p}_{\mathfrak{c},t} \equiv \tfrac{\mathsf{m}_{\mathfrak{c},t}}{\mathsf{u}_{\mathfrak{c},t}} \quad \mathsf{q}_t \equiv \tfrac{\mathsf{m}_t}{\mathsf{v}_t} \;\; ; \; \mathsf{q}_{\mathfrak{c},t} \equiv \tfrac{\mathsf{m}_{\mathfrak{c},t}}{\mathsf{v}_{\mathfrak{c},t}} \quad \theta_t \equiv \tfrac{\mathsf{v}_t}{\mathsf{u}_t} \;\; ; \; \theta_{\mathfrak{c},t} \equiv \tfrac{\mathsf{v}_{\mathfrak{c},t}}{\mathsf{u}_{\mathfrak{c},t}} \\ \mathsf{p}^e_t \equiv \tfrac{\mathsf{m}_t}{\mathsf{u}^e_t} \;\; ; \; \mathsf{p}^e_{\mathfrak{c},t} \equiv \tfrac{\mathsf{m}_{\mathfrak{c},t}}{\mathsf{u}^e_{\mathfrak{c},t}} \quad \mathsf{q}^e_t \equiv \tfrac{\mathsf{m}_t}{\mathsf{v}^e_t} \;\; ; \; \mathsf{q}^e_{\mathfrak{c},t} \equiv \tfrac{\mathsf{m}_{\mathfrak{c},t}}{\mathsf{v}^e_{\mathfrak{c},t}} \quad \theta^e_t \equiv \tfrac{\mathsf{v}^e_t}{\mathsf{u}^e_t} \;\; ; \; \theta^e_{\mathfrak{c},t} \equiv \tfrac{\mathsf{v}^e_{\mathfrak{c},t}}{\mathsf{u}^e_{\mathfrak{c},t}} \end{aligned} \tag{19}$$

The sectoral matching functions have standard Cobb-Douglas forms:[4]

$$\mathsf{m}_{\mathfrak{c},t} \equiv \eta_{\mathfrak{c},t} \mathsf{v}^{1-a_{\mathfrak{c}}}_{\mathfrak{c},t} \mathsf{u}^{a_{\mathfrak{c}}}_{\mathfrak{c},t} \tag{20}$$

where $a_{\mathfrak{c}} \in (0,1)$ and $\eta_{\mathfrak{c},t}$ measures the efficiency of the matching technology of sector $\mathfrak{c}$, which evolves according to the following stationary process about its steady state level:

$$\frac{\eta_{\mathfrak{c},t}}{\bar{\eta}_{\mathfrak{c}}} = \epsilon^{\eta}_{\mathfrak{c},t} \left(\frac{\eta_{\mathfrak{c},t-1}}{\bar{\eta}_{\mathfrak{c}}} \right)^{\phi^{\eta}_{\mathfrak{c}}} \tag{21}$$

where $\epsilon^{\eta}_{\mathfrak{c},t}$ is the sector-$\mathfrak{c}$ specific shock on the efficiency of the matching technology and $\phi^{\rho}_{\mathfrak{c}} \in (0,1)$.

All previous relations imply the following identity:

$$\mathsf{p}_t = \mathfrak{w}_{\mathfrak{m}} \frac{\mathsf{u}_{\mathfrak{m},t}}{\mathsf{u}_t} \mathsf{p}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \frac{\mathsf{u}_{\mathfrak{s},t}}{\mathsf{u}_t} \mathsf{p}_{\mathfrak{s},t}$$

The intuition for this result is that the economy wide job-finding rate p_t can be computed using conditional probabilities. The conditional probability that an unemployed worker, at the beginning of period t, finds a job during during the period, given that she was in sector $\mathfrak{p}$ is $\mathsf{p}_{\mathfrak{c},t}$. Recall now that $\mathsf{u}_t = \sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} \mathsf{u}_{\mathfrak{c},t}$. It implies that the probabilities that an unemployed worker is either from sector $\mathfrak{m}$ or $\mathfrak{s}$, at the beginning of period t, are just the masses ratios:

$$\mathsf{p}^{\mathsf{u}}_{\mathfrak{c},t} \equiv \frac{\mathfrak{w}_{\mathfrak{c}} \mathsf{u}_{\mathfrak{c},t}}{\mathsf{u}_t} \tag{22}$$

[4] In the literature of search frictions in the labor market, the standard form is Cobb-Douglas (e.g. Shimer (2005) and Pissarides (2000)).

As long as matching functions depend only on sectoral rates, such as unemployment and vacancies masses, firms are unable to influence the sectoral matching rate $\mathsf{q}_{\mathfrak{c},t}$. Firms and unions know this result, but do not internalize the specific form of the aggregate matching function. Therefore, the individual matching functions satisfy $\mathsf{m}_t(z_{\mathfrak{c}}) = \mathsf{q}_{\mathfrak{c},t}\mathsf{v}_t(z_{\mathfrak{c}})$.

In the context of asymmetric sectors, I define the aggregate and sectoral rates p_t^o and $\mathsf{p}_{\mathfrak{c},t}^o$ according to which unemployed workers decide to leave the labor market:

$$\mathsf{p}_t^o \equiv \frac{\mathsf{m}_t^o}{(1-\mathsf{p}_t)\mathsf{u}_t} \quad ; \; \mathsf{p}_{\mathfrak{c},t}^o \equiv \frac{\mathsf{m}_{\mathfrak{c},t}^o}{(1-\mathsf{p}_{\mathfrak{c},t})\mathsf{u}_{\mathfrak{c},t}} \tag{23}$$

where $(1-\mathsf{p}_t)\,\mathsf{u}_t$ and $(1-\mathsf{p}_{\mathfrak{c},t})\,\mathsf{u}_{\mathfrak{c},t}$ are the masses of aggregate and sectoral unemployed workers who are not matched into new jobs during the period.

All previous relations imply the following alternative definitions for u_t^e and $\mathsf{u}_{\mathfrak{c},t}^e$:

$$\mathsf{u}_t^e = (1-\mathsf{p}_t^o)(1-\mathsf{p}_t)\,\mathsf{u}_t \quad ; \; \mathsf{u}_{\mathfrak{c},t}^e = \left(1-\mathsf{p}_{\mathfrak{c},t}^o\right)(1-\mathsf{p}_{\mathfrak{c},t})\,\mathsf{u}_{\mathfrak{c},t}$$

For an unemployed worker at the beginning of period t in sector $\mathfrak{c}$, the expected spell $\mathsf{T}_{\mathfrak{c},t}^u$ until being matched into a job (in any sector) evolves according to:[5]

$$\begin{aligned} \mathsf{T}_{\mathfrak{c},t}^u = \;& \mathsf{p}_{\mathfrak{c},t}\bar{\mathsf{t}} \\ +\;& (1-\mathsf{p}_{\mathfrak{c},t})\left[1+\left(1-\mathsf{p}_{\mathfrak{c},t}^o\right)E_t\mathsf{T}_{\mathfrak{c},t+1}^u\right] \\ +\;& (1-\mathsf{p}_{\mathfrak{c},t})\left[\mathsf{p}_{\mathfrak{c},t}^o\left(\delta_{\mathfrak{s}}^{\bar{\mathfrak{c}}}E_t\mathsf{T}_{\bar{\mathfrak{c}},t+1}^u+\delta_{\mathfrak{c}}^{\mathfrak{c}}E_t\mathsf{T}_{\mathfrak{c},t+1}^u+\frac{1-\delta_{\mathfrak{c}}^{\bar{\mathfrak{c}}}-\delta_{\mathfrak{c}}^{\mathfrak{c}}}{\delta_{\mathfrak{c}}^{\bar{\mathfrak{c}}}+\delta_{\mathfrak{c}}^{\mathfrak{c}}}\right)\right] \end{aligned} \tag{24}$$

where $\bar{\mathsf{t}} \in (0,1)$ is the average time within a period in which a recently laid-off worker remains unemployed when he is matched to a new job by the end of the same period.

Therefore, the expected spell T_t^u until being matched into a job, independently of the sector status, evolves according to:

$$\mathsf{T}_t^u = \mathfrak{p}_{\mathfrak{m},t}^{\mathsf{u}}\mathsf{T}_{\mathfrak{m},t}^u + \mathfrak{p}_{\mathfrak{s},t}^{\mathsf{u}}\mathsf{T}_{\mathfrak{s},t}^u \tag{25}$$

3.2 Domestic households

Besides making optimal consumption allocation, as described further on, the representative household is specialized in producing sectoral consumption bundles for own consumption and to be sold to firms as intermediate goods for posting vacancies. This

[5] The proof is shown in the Appendix.

market is competitive and hence the household makes zero profit out of it.

3.2.1 Consumption bundles

Consumption bundles are defined in terms of the economy wide consumption of goods from sectors $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$. Households are in charge to produce $\mathfrak{C}_{\mathfrak{c},t}$ units of *sectoral consumption bundles* to be consumed by different agents in the economy wide.

For that, the household needs to buy goods from domestic firms, i.e. $\boldsymbol{c}_t(z_{\mathfrak{c}})$ units of manufactured good $z_{\mathfrak{c}}$, and use the following Dixit and Stiglitz (1977) CES technologies:

$$(\mathfrak{C}_{\mathfrak{c},t})^{\frac{\phi-1}{\phi}} = \left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\right)^{\frac{1}{\phi}} \int_{\mathfrak{c}} \boldsymbol{c}_t(z_{\mathfrak{c}})^{\frac{\phi-1}{\phi}} dz_{\mathfrak{c}}$$

at total cost $P_{\mathfrak{c},t}\mathfrak{C}_{\mathfrak{c},t} \equiv \int_{\mathfrak{c}} p_t(z_{\mathfrak{c}})\, \boldsymbol{c}_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$, where $P_{\mathfrak{c},t}$ is the aggregate price of the sectoral bundle $\mathfrak{C}_{\mathfrak{c},t}$, and $\phi > 1$ is the elasticity of substitution between goods in the same sector.

The representative household consumes $C_{\mathfrak{c},t}$ units of the sectoral consumption bundle $\mathfrak{C}_{\mathfrak{c},t}$ and has utility over the *aggregate consumption* C_t, defined according to the following CES technology:

$$(C_t)^{\frac{\phi-1}{\phi}} = \sum_{\mathfrak{c}} (\mathfrak{w}_{\mathfrak{c}})^{\frac{1}{\phi}} (C_{\mathfrak{c},t})^{\frac{\phi-1}{\phi}}$$

at total cost $P_t C_t \equiv \sum_{\mathfrak{c}} P_{\mathfrak{c},t} C_{\mathfrak{c},t}$, where P_t is the aggregate consumer price index.

Generalizing Ravenna and Walsh (2010), I assume that each firm $z_{\mathfrak{c}}$ needs to buy $\mathsf{c}_t^{\mathsf{v}\mathfrak{m}}(z_{\mathfrak{c}})$ and $\mathsf{c}_t^{\mathsf{v}\mathfrak{s}}(z_{\mathfrak{c}})$ units of sectoral consumption bundles from sectors $\mathfrak{m}$ and $\mathfrak{s}$ in order to post $\mathsf{v}_t^e(z_{\mathfrak{c}})$ units of end-of-period job vacancies, according to the following CES technology:

$$(\mathsf{c}_t^{\mathsf{v}}(z_{\mathfrak{c}}))^{\frac{\phi-1}{\phi}} = (\mathfrak{w}_{\mathfrak{m}})^{\frac{1}{\phi}} (\mathsf{c}_t^{\mathsf{v}\mathfrak{m}}(z_{\mathfrak{c}}))^{\frac{\phi-1}{\phi}} + (\mathfrak{w}_{\mathfrak{s}})^{\frac{1}{\phi}} (\mathsf{c}_t^{\mathsf{v}\mathfrak{s}}(z_{\mathfrak{c}}))^{\frac{\phi-1}{\phi}}$$

at total cost $P_t\mathsf{c}_t^{\mathsf{v}}(z_{\mathfrak{c}}) \equiv P_{\mathfrak{m},t}\mathsf{c}_t^{\mathsf{v}\mathfrak{m}}(z_{\mathfrak{m}}) + P_{\mathfrak{s},t}\mathsf{c}_t^{\mathsf{v}\mathfrak{s}}(z_{\mathfrak{s}})$, where $\mathsf{c}_t^{\mathsf{v}}(z_{\mathfrak{c}})$ is proportional to the firm's end-of-period posted vacancies,

$$\mathsf{c}_t^{\mathsf{v}}(z_{\mathfrak{c}}) \equiv \varsigma_{\mathsf{v}\mathfrak{c}} \mathsf{v}_t^e(z_{\mathfrak{c}}) \qquad (26)$$

and $\varsigma_{\mathsf{v}\mathfrak{c}}$ is a sector-$\mathfrak{c}$ specific proportionality parameter.

In order to simplify the notation, let $\wp_{\mathrm{j},t}$ denote the relative price of the sectoral consumption bundle with price $P_{\mathrm{j},t}$ with respect to the consumption aggregate price P_t:

$$\wp_{\mathrm{j},t} \equiv \frac{P_{\mathrm{j},t}}{P_t} \quad ; \; \wp_{\mathrm{j},t} = \wp_{\mathrm{j},t-1}\frac{\Pi_{\mathrm{j},t}}{\Pi_t} \qquad (27)$$

Expenditure minimization implies the following relations:

$$
\begin{aligned}
&\left(\mathfrak{C}_{\mathfrak{c},t}\right)^{\frac{\phi-1}{\phi}} = \left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\right)^{\frac{1}{\phi}} \int_{\mathfrak{c}} \boldsymbol{c}_t\left(z_{\mathfrak{c}}\right)^{\frac{\phi-1}{\phi}} dz_{\mathfrak{c}} \quad &&; \; P_{\mathfrak{c},t}^{1-\phi} = \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} p_t\left(zc\right)^{1-\phi} dz_{\mathfrak{c}} \\
&\boldsymbol{c}_t\left(z_{\mathfrak{c}}\right) = \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \mathfrak{C}_{\mathfrak{c},t} \left(\frac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi} &&; \; P_{\mathfrak{c},t}\mathfrak{C}_{\mathfrak{c},t} = \int_{\mathfrak{c}} p_t\left(z_{\mathfrak{c}}\right) \boldsymbol{c}_t\left(z_{\mathfrak{c}}\right) dz_{\mathfrak{c}}
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
&\left(C_t\right)^{\frac{\phi-1}{\phi}} = \sum_{\mathfrak{c}} \left(\mathfrak{w}_{\mathfrak{c}}\right)^{\frac{1}{\phi}} \left(C_{\mathfrak{c},t}\right)^{\frac{\phi-1}{\phi}} \quad &&; \; 1 = \sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} \left(\wp_{\mathfrak{c},t}\right)^{1-\phi} \\
&C_{\mathfrak{c},t} = \mathfrak{w}_{\mathfrak{c}} C_t \left(\wp_{\mathfrak{c},t}\right)^{-\phi} &&; \; C_t = \sum_{\mathfrak{c}} \wp_{\mathfrak{c},t} C_{\mathfrak{c},t}
\end{aligned}
\tag{29}
$$

$$
\begin{gathered}
\mathsf{c}_t^{\mathsf{vm}}\left(z_{\mathfrak{c}}\right) = \mathfrak{w}_{\mathfrak{m}} \mathsf{c}_t^{\mathsf{v}}\left(z_{\mathfrak{c}}\right)\left(\wp_{\mathfrak{m},t}\right)^{-\phi} \quad ; \; \mathsf{c}_t^{\mathsf{vs}}\left(z_{\mathfrak{c}}\right) = \mathfrak{w}_{\mathfrak{s}} \mathsf{c}_t^{\mathsf{v}}\left(z_{\mathfrak{c}}\right)\left(\wp_{\mathfrak{s},t}\right)^{-\phi} \\
\mathsf{c}_t^{\mathsf{v}}\left(z_{\mathfrak{c}}\right) = \wp_{\mathfrak{m},t} \mathsf{c}_t^{\mathsf{vm}}\left(z_{\mathfrak{c}}\right) + \wp_{\mathfrak{s},t} \mathsf{c}_t^{\mathsf{vs}}\left(z_{\mathfrak{c}}\right)
\end{gathered}
\tag{30}
$$

Let $\mathsf{c}_{\mathfrak{c},t}^{\mathsf{vm}} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \mathsf{c}_t^{\mathsf{vm}}\left(z_{\mathfrak{c}}\right) dz_{\mathfrak{c}}$ and $\mathsf{c}_{\mathfrak{c},t}^{\mathsf{vs}} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \mathsf{c}_t^{\mathsf{vs}}\left(z_{\mathfrak{c}}\right) dz_{\mathfrak{c}}$ denote aggregate consumptions of goods from sectors $\mathfrak{m}$ and $\mathfrak{s}$ used as intermediates for posting vacancies by firms in production sector $\mathfrak{c}$. Those aggregates imply the following relations:

$$
\mathsf{c}_{\mathfrak{m},t}^{\mathsf{vc}} \equiv \mathfrak{w}_{\mathfrak{c}} \varsigma_{\mathsf{vm}} \mathsf{v}_{\mathfrak{m},t}^{e} \left(\wp_{\mathfrak{c},t}\right)^{-\phi} \quad ; \; \mathsf{c}_{\mathfrak{s},t}^{\mathsf{vc}} \equiv \mathfrak{w}_{\mathfrak{c}} \varsigma_{\mathsf{vs}} \mathsf{v}_{\mathfrak{s},t}^{e} \left(\wp_{\mathfrak{c},t}\right)^{-\phi}
\tag{31}
$$

Since the household supplies consumption goods at zero profit, the household itself and firms buy $C_{\mathfrak{c},t}$, $\mathsf{c}_{\mathfrak{m},t}^{\mathsf{vc}}$ and $\mathsf{c}_{\mathfrak{s},t}^{\mathsf{vc}}$ units of the economy wide consumption bundle type $\mathfrak{c}$ at aggregate price $P_{\mathfrak{c},t}$. Equilibrium requires:

$$
\mathfrak{C}_{\mathfrak{c},t} = C_{\mathfrak{c},t} + \mathfrak{w}_{\mathfrak{m}} \mathsf{c}_{\mathfrak{m},t}^{\mathsf{vc}} + \mathfrak{w}_{\mathfrak{s}} \mathsf{c}_{\mathfrak{s},t}^{\mathsf{vc}} = \mathfrak{w}_{\mathfrak{c}} \mathfrak{C}_t \left(\wp_{\mathfrak{c},t}\right)^{-\phi}
\tag{32}
$$

where $\mathfrak{C}_t = \sum_{\mathfrak{c}} \wp_{\mathfrak{c},t} \mathfrak{C}_{\mathfrak{c},t}$ is the aggregate expenditure over all consumption sectors $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}}$. Using the previous results, I obtain

$$
\mathfrak{C}_t = C_t + \mathfrak{w}_{\mathfrak{m}} \varsigma_{\mathsf{vm}} \mathsf{v}_{\mathfrak{m},t}^{e} + \mathfrak{w}_{\mathfrak{s}} \varsigma_{\mathsf{vs}} \mathsf{v}_{\mathfrak{s},t}^{e}
\tag{33}
$$

Note that $\mathsf{c}_{\mathfrak{m},t}^{\mathsf{vc}}$ and $\mathsf{c}_{\mathfrak{s},t}^{\mathsf{vc}}$ are to be interpreted as firms' intermediate consumption, which should be netted out when computing the model's GDP. See Section 3.3 for more details.

3.2.2 Optimal consumption allocation

As in Merz (1995), I assume full risk sharing of consumption among household members, employed, unemployed and out of the labor market.[6] All $\ell_t^{\mathfrak{p}}$ household members pool

[6] Some authors have been making efforts to model imperfect consumption insurance and fully capture the distortions caused by unemployment. See e.g. Christiano et al. (2010).

their income, and hence the household consumes $C_{\mathfrak{c},t}$ units of each type-$\mathfrak{c}$ consumption bundle. Unemployed workers earn monetary transfers from the government until they are matched into a firm. That generates $\mathfrak{w}_{\mathfrak{m}} P_t \varpi^c_{\mathfrak{m},t} \mathsf{u}^e_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} P_t \varpi^c_{\mathfrak{s},t} \mathsf{u}^e_{\mathfrak{s},t}$ in nominal income for the household, where $\varpi^c_{\mathfrak{m},t}$ and $\varpi^c_{\mathfrak{s},t}$ are sectoral aggregate real unemployment compensations, which evolve according to the following exogenous processes:

$$\begin{array}{ll} \varpi^c_{\mathfrak{m},t} = \epsilon_{\varpi,t} \left(\varpi^c_{\mathfrak{m},t-1}\right)^{\phi_\varpi} \left(\gamma^c_{\mathfrak{m}} \tilde{\varpi}_{\mathfrak{m},t-1}\right)^{1-\phi_\varpi} & ; \; \varpi^c_{\mathfrak{s},t} = \epsilon_{\varpi,t} \left(\varpi^c_{\mathfrak{s},t-1}\right)^{\phi_\varpi} \left(\gamma^c_{\mathfrak{s}} \tilde{\varpi}_{\mathfrak{s},t-1}\right)^{1-\phi_\varpi} \\ \tilde{\varpi}_{\mathfrak{m},t} \equiv \left(\bar{\varpi}_{\mathfrak{m}}\right)^{\phi^{ss}_\varpi} \left(\varpi_{\mathfrak{m},t}\right)^{1-\phi^{ss}_\varpi} & ; \; \tilde{\varpi}_{\mathfrak{s},t} \equiv \left(\bar{\varpi}_{\mathfrak{s}}\right)^{\phi^{ss}_\varpi} \left(\varpi_{\mathfrak{s},t}\right)^{1-\phi^{ss}_\varpi} \end{array} \tag{34}$$

where $\epsilon_{\varpi,t}$ is an aggregate shock to unemployment compensation, $\varpi_{\mathfrak{c},t}$ is the aggregate salary at sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}}$, $\bar{\varpi}_{\mathfrak{c}}$ is the steady state level of the aggregate salary, $\gamma^c_{\mathfrak{c}}$ is the steady state fraction of the aggregate salary given as unemployment compensation, and $\phi_\varpi \in (0, 1)$.

The economy wide aggregate real unemployment compensation ϖ^c_t is defined as follows:

$$\varpi^c_t = \frac{1}{\mathsf{u}^e_t} \sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} \varpi^c_{\mathfrak{c},t} \mathsf{u}^e_{\mathfrak{c},t} \tag{35}$$

Consumption over consumption bundle C_t provides an external habit formation utility[7] $u_t \equiv \mathfrak{u}_{\mathfrak{u},t} \frac{\left(C_t - \iota_{\mathfrak{u}} \tilde{C}_{t-1}\right)^{1-\sigma}}{(1-\sigma)}$ for each household member, where $\tilde{C}_t$ is the average consumption level which equals C_t in equilibrium, σ is the reciprocal of the intertemporal rate of substitution, $\iota_{\mathfrak{u}} \in (0, 1)$ is the habit formation parameter, and $\mathfrak{u}_{\mathfrak{u},t}$ is a preference shock, which evolves according to:

$$\frac{\mathfrak{u}_{\mathfrak{u},t}}{\bar{\mathfrak{u}}_{\mathfrak{u}}} = \epsilon_{\mathfrak{u},t} \left(\frac{\mathfrak{u}_{\mathfrak{u},t-1}}{\bar{\mathfrak{u}}_{\mathfrak{u}}}\right)^{\phi_{\mathfrak{u}}}$$

where $\epsilon_{\mathfrak{u},t}$ is the preference innovation and $\phi_{\mathfrak{u}} \in (0, 1)$.

As in Alves (2012), the representative household has unions specialized in negotiating wage and hours with firms. Union $z_{\mathfrak{s}}$ represents all $\mathsf{n}_t(z_{\mathfrak{c}})$ workers when bargaining with firm $z_{\mathfrak{s}}$ on hours per worker $h_t(z_{\mathfrak{c}})$ and nominal hourly wages $W_t(z_{\mathfrak{c}}) = P_t w_t(z_{\mathfrak{c}})$, where $w_t(z_{\mathfrak{c}})$ is the real wage. Whenever convenient, I consider instead the total nominal and real salaries over the period $\mathcal{W}_t(z_{\mathfrak{c}}) = W_t(z_{\mathfrak{c}}) h_t(z_{\mathfrak{c}})$ and $\varpi_t(z_{\mathfrak{c}}) = w_t(z_{\mathfrak{c}}) h_t(z_{\mathfrak{c}})$. Total hours worked at firm $z_{\mathfrak{s}}$ is defined as $H_t(z_{\mathfrak{c}}) \equiv \mathsf{n}_t(z_{\mathfrak{c}}) h_t(z_{\mathfrak{c}})$.

Representing the workers, the union's disutility to $H_t(z_{\mathfrak{c}})$ is $\upsilon_t(z_{\mathfrak{c}}) \equiv \chi \frac{H_t(z_{\mathfrak{c}})^{1+\nu}}{(1+\nu)}$, where ν is the reciprocal of Frisch labor elasticity. Since the unions belong to the representative household, the average disutility function per family is $\upsilon_t \equiv \int_0^1 \upsilon_t(z)\, dz$.[8]

[7] See e.g. Abel (1990) and Gali (1994).

[8] Using a unions-based aggregate disutility function instead of a workers-based one allows me to derive

Even though members out of the labor market consume $C_{\mathfrak{c},t}$ units of each type-$\mathfrak{c}$ consumption bundle, they make no monetary contribution to the household budget. However, being out of the labor market might be an optimal decision if being unemployed is a burden. Indeed, searching for a job is time consuming and annoying. This rationale justifies involuntary unemployment, and may be one of the causes for leaving the labor market.

A simple way to capture this phenomenon is to assume that the burden of being unemployed generates extra disutility $\upsilon_t^{\mathsf{u}}\mathsf{u}_t^e$ to the household:

$$\upsilon_t^{\mathsf{u}}\mathsf{u}_t^e \equiv \mathfrak{w}_{\mathfrak{m}}\bar{\upsilon}_{\mathfrak{m}}^{\mathsf{u}}\mathsf{u}_{\mathfrak{m},t}^e + \mathfrak{w}_{\mathfrak{s}}\bar{\upsilon}_{\mathfrak{s}}^{\mathsf{u}}\mathsf{u}_{\mathfrak{s},t}^e \tag{36}$$

where $\bar{\upsilon}_{\mathfrak{m}}^{\mathsf{u}}$ and $\bar{\upsilon}_{\mathfrak{s}}^{\mathsf{u}}$ are fixed sector-specific homogeneous disutility parameters faced by unemployed workers. In this case, members out of the labor market contribute for the household by avoiding extra disutilities. In the end of the day, a trade-off arises because leaving the labor market also reduces the number of job matches and, as a consequence, reduces the expected household income.

The representative household maximizes its welfare $\mathcal{U}_t = \max \left(\ell_t^{\mathfrak{p}} u_t - \upsilon_t - \upsilon_t^{\mathsf{u}}\mathsf{u}_t^e\right) + E_t\beta\mathcal{U}_{t+1}$, subject to the budget constraint and the equations related to the labor market (not shown for being irrelevant for now). Let λ_t denote the Lagrange multiplier on the nominal budget constraint. For simplification, I aggregate unemployment compensations with the unemployment disutilities into what I call net unemployment compensations $\varpi_{\mathfrak{m},t}^u$ and $\varpi_{\mathfrak{s},t}^u$, defined as follows:

$$\varpi_{\mathfrak{m},t}^u \equiv \varpi_{\mathfrak{m},t}^c - \tfrac{\bar{\upsilon}_{\mathfrak{m}}^{\mathsf{u}}}{\lambda_t P_t} \quad ; \; \varpi_{\mathfrak{s},t}^u \equiv \varpi_{\mathfrak{s},t}^c - \tfrac{\bar{\upsilon}_{\mathfrak{s}}^{\mathsf{u}}}{\lambda_t P_t} \tag{37}$$

The aggregate net unemployment compensation ϖ_t^u is defined as follows:

$$\varpi_t^u \equiv \frac{1}{\mathsf{u}_t^e}\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}}\varpi_{\mathfrak{c},t}^u\mathsf{u}_{\mathfrak{c},t}^e \tag{38}$$

Therefore, the representative household chooses C_t, A_{t+1}, and $\mathcal{B}_{t+1}$ to solve:

---

closed form equations describing the dynamics of the aggregate disutility to work in Section 3.4, which is an important variable for understanding the amplified volatilities under trend inflation. The dynamics implied by the labor flows and by the Calvo price setting convolute in such a way that the derivation is not possible otherwise. The unions-based disutility also allows me to obtain the firms' supply equations with no need to guess the loglinearized function forms to deal with the issue on firms' specific labor, as done in Thomas (2008).

$$\mathcal{U}_t = \max\ \ell_t^{\mathfrak{p}} u_t - \upsilon_t + \lambda_t \left[A_t + \mathcal{B}_t I_{t-1} + P_t \sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} \varpi^u_{\mathfrak{c},t} \mathsf{u}^e_{\mathfrak{c},t} - \Xi_t + P_t d_t \right.$$
$$\left. + \sum_{\mathfrak{c}} \int_{\mathfrak{c}} \mathsf{n}_t (z_{\mathfrak{c}}) \mathcal{W}_t (z_{\mathfrak{c}}) \, dz_{\mathfrak{c}} - \ell_t^{\mathfrak{p}} P_t C_t - E_t Q_{t+1} A_{t+1} - \mathcal{B}_{t+1} \right]$$
$$+ E_t \beta \mathcal{U}_{t+1}$$

where d_t denotes real profits from all firms, Ξ_t denotes lump-sum taxes net of transfers from the government, $\mathcal{B}_t$ is the value of one-period non-contingent domestic bonds at the end of period t, $I_t \equiv 1 + i_t$ is the gross interest rate on the domestic bond, A_t is the aggregate state-contingent value of the portfolio of financial securities held at the beginning of period t, E_t is the time-t expectations operator, and Q_{t+1} is the stochastic discount factor from $t+1$ to t.

The first-order conditions are the non-arbitrage condition $E_t Q_{t+1} = 1/I_t$, and the Euler equations

$$1 = \beta E_t \left(\frac{u'_{t+1}}{u'_t} \frac{I_t}{\Pi_{t+1}} \right) \quad ; \ Q_t = \beta \frac{u'_t}{u'_{t-1}} \frac{1}{\Pi_t} \tag{39}$$

where $u'_t \equiv \mathfrak{u}_{\mathrm{u},t} (C_t - \iota_{\mathrm{u}} C_{t-1})^{-\sigma}$ is the marginal utility to consumption. in equilibrium, $\Pi_t \equiv 1 + \pi_t$ is the gross inflation rate, and $\lambda_t = u'_t / P_t$ is the Lagrange multiplier on the budget constraint.

In equilibrium, demand for financial securities matches their supply by individuals, so that the aggregate state-contingent value of the portfolio held at the beginning of period t is $A_t = 0$, $\forall t$.

3.2.3 Leaving the labor force

Before deriving the optimal rules for sectoral migration, I present some comments and definitions. Individuals take the predetermined variables n_t, u_t, v_t, p_t and q_t, and their sectoral peers, as given. In this context, $\theta_t^f \equiv \theta_{t+1}$, $\mathsf{p}_t^f \equiv \mathsf{p}_{t+1}$, $\mathsf{q}_t^f \equiv \mathsf{q}_{t+1}$, and their sectoral peers, are key in deriving the optimal masses out of the labor forces in this section and the wage and the aggregate job market curves in Section 3.3.1.[9]

The job-finding rate for being matched at firm $z_{\mathfrak{c}}$, at sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$, is $\mathsf{p}_t (z_{\mathfrak{c}}) \equiv \mathsf{m}_t (z_{\mathfrak{c}}) / (\mathfrak{w}_{\mathfrak{c}} \mathsf{u}_{\mathfrak{c},t})$. The rate satisfies $\mathsf{p}_{\mathfrak{c},t} = \int_{\mathfrak{c}} \mathsf{p}_t (z_{\mathfrak{c}}) \, dz_{\mathfrak{c}}$. Likewise, the firm's vacancy share in the sector is $\mathsf{s}_t (z_{\mathfrak{c}}) \equiv \mathsf{v}_t (z_{\mathfrak{c}}) / (\mathfrak{w}_{\mathfrak{c}} \mathsf{v}_{\mathfrak{c},t})$. Note that $\mathsf{s}_t (z_{\mathfrak{c}})$ also equals $\mathsf{p}_t (z_{\mathfrak{c}}) / \mathsf{p}_{\mathfrak{c},t}$, the probability that the worker is matched into firm $z_{\mathfrak{c}}$, conditioned on obtaining a new job

[9] Note that end-of-period variables θ_t^e, p_t^e and q_t^e are not the same as lead variables θ_t^f, p_t^f and q_t^f.

in the sector. It implies that $\int_{\mathfrak{c}} \mathsf{s}_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}} = 1$.

Finally, the mass of workers matched into firm $z_{\mathfrak{c}}$ at period $t+1$ can be computed as follows:

$$\mathsf{m}_{t+1}(z_{\mathfrak{c}}) = \mathsf{q}_{\mathfrak{c},t+1}\mathsf{v}_{t+1}(z_{\mathfrak{c}}) = \frac{\mathsf{m}_{\mathfrak{c},t+1}}{\mathsf{v}_{\mathfrak{c},t+1}}\mathsf{v}_{t+1}(z_{\mathfrak{c}}) = \frac{\mathsf{p}_{\mathfrak{c},t+1}\mathsf{u}_{\mathfrak{c},t+1}}{\mathsf{v}_{\mathfrak{c},t+1}}\mathsf{v}_{t+1}(z_{\mathfrak{c}}) = \mathfrak{w}_{\mathfrak{c}}\mathsf{s}_{t+1}(z_{\mathfrak{c}})\,\mathsf{p}_{\mathfrak{c},t+1}\mathsf{u}_{\mathfrak{c},t+1}$$

For notation purposes, let $\upsilon_t'(z_{\mathfrak{c}}) \equiv \partial\upsilon_t(z_{\mathfrak{c}})/\partial H_t(z_{\mathfrak{c}}) = (1+\nu)\,\upsilon_t(z_{\mathfrak{c}})/H_t(z_{\mathfrak{c}})$.

Individuals may lack full information when deciding on whether leaving the labor market or reallocating to the other sector, considering myopic expectations on future flows $\tilde{E}_t\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}$ as given, which match the aggregate expectation in equilibrium $E_t\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}$. In order to capture this phenomenon, I assume that the household faces additional real adjustment costs $\frac{\varsigma_{\mathsf{m}\mathfrak{c}}}{2}\left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}}{\tilde{E}_t\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}}-1\right)^2$ on changes of the masses of unemployed workers leaving the labor market. I also assume that myopic expectations clears in equilibrium, i.e. $\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}/\tilde{E}_t\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1} = E_t\left(\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}/\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}\right)$.

Let me now rewrite the representative family's problem, including extra restrictions from labor flows and using the notation of net unemployment compensations $\varpi^u_{\mathfrak{c},t}$. They do not bind previously derived first order conditions, and hence are not included for computing the optimal consumption allocation. The additional restrictions are the ones described by equations (8), (9), (10), (11), (12), (5).

Therefore, when deciding the optimal mass of unemployed workers to leave the labor market or reallocate to a different sector, the representative household chooses $\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}$, $\ell_{\mathfrak{c},t}$, $\ell^{\mathfrak{o}}_{\mathfrak{c},t}$, $\mathsf{u}_{\mathfrak{c},t+1}$ and $\mathsf{n}_{t+1}(z_{\mathfrak{c}})$ to solve:[10]

$$\begin{aligned}
\mathcal{U}_t \; = \; \max \; & \ell^{\mathfrak{p}}_t u_t \; - \; \textstyle\sum_{\mathfrak{c}}\int_{\mathfrak{c}} \upsilon_t(\mathsf{n}_t(z_{\mathfrak{c}})h_t(z_{\mathfrak{c}}))dz_{\mathfrak{c}} \\
& + \; \lambda_t\left[A_t+\mathcal{B}_tI_{t-1}+\Xi_t+P_td_t-P_t\textstyle\sum_{\mathfrak{c}}\frac{\varsigma_{\mathsf{m}\mathfrak{c}}}{2}\left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}}{\tilde{E}_t\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}}-1\right)^2-\ell^{\mathfrak{p}}_tP_tC_t-E_tQ_{t+1}A_{t+1}-\mathcal{B}_{t+1}\right] \\
& + \; \lambda_t\left[\textstyle\sum_{\mathfrak{c}}\int_{\mathfrak{c}}\mathsf{n}_t(z_{\mathfrak{c}})\mathcal{W}_t(z_{\mathfrak{c}})dz_{\mathfrak{c}}+\sum_{\mathfrak{c}}P_t\mathfrak{w}_{\mathfrak{c}}\varpi^u_{\mathfrak{c},t}\left(\ell_{\mathfrak{c},t}-\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathsf{n}_t(z_{\mathfrak{c}})dz_{\mathfrak{c}}\right)\right] \\
& + \; E_t\textstyle\sum_{\mathfrak{c}}\int_{\mathfrak{c}}\lambda^{cn}_t(z_{\mathfrak{c}})\left[(1-\rho_{\mathfrak{d}})\left(1-\rho_{\mathfrak{c},t}\right)\mathsf{n}_t(z_{\mathfrak{c}})+\mathfrak{w}_{\mathfrak{c}}\eta_{\mathfrak{c},t}\mathsf{v}^{1-a_{\mathfrak{c}}}_{\mathfrak{c},t+1}\mathsf{u}^{a_{\mathfrak{c}}}_{\mathfrak{c},t+1}\mathsf{s}_{t+1}(z_{\mathfrak{c}})-\mathsf{n}_{t+1}(z_{\mathfrak{c}})\right]dz_{\mathfrak{c}} \\
& + \; \textstyle\sum_{\mathfrak{c}}\lambda^{\ell}_{\mathfrak{c},t}\left[\mathsf{u}_{\mathfrak{c},t}-\eta_{\mathfrak{c},t}\mathsf{v}^{1-a_{\mathfrak{c}}}_{\mathfrak{c},t}\mathsf{u}^{a_{\mathfrak{c}}}_{\mathfrak{c},t}-\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}+\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathsf{n}_t(z_{\mathfrak{c}})dz_{\mathfrak{c}}-\ell_{\mathfrak{c},t}\right] \; + \; \sum_{\mathfrak{c}}\lambda^{\mathfrak{o}}_{\mathfrak{c},t}\left[(1-\rho_{\mathfrak{d}})\left(1-\delta^{\mathfrak{c}}_{\mathfrak{c}}-\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\right)\ell^{\mathfrak{o}}_{\mathfrak{c},t-1}+\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}+\mathsf{m}_{\ell,t}-\ell^{\mathfrak{o}}_{\mathfrak{c},t}\right] \\
& + \; \textstyle\sum_{\mathfrak{c}}\lambda^{\mathsf{u}}_{\mathfrak{c},t}\left[(1-\rho_{\mathfrak{d}})\left(\ell_{\mathfrak{c},t}-\frac{(1-\rho_{\mathfrak{c},t})}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathsf{n}_t(z_{\mathfrak{c}})dz_{\mathfrak{c}}+\delta^{\mathfrak{c}}_{\mathfrak{c}}\ell^{\mathfrak{o}}_{\mathfrak{c},t}+\frac{\mathfrak{w}_{\bar{\mathfrak{c}}}}{\mathfrak{w}_{\mathfrak{c}}}\delta^{\mathfrak{c}}_{\bar{\mathfrak{c}}}\ell^{\mathfrak{o}}_{\bar{\mathfrak{c}},t}\right)-\mathsf{u}_{\mathfrak{c},t+1}\right] \; + \; \beta E_t\mathcal{U}_{t+1}
\end{aligned}$$

Let Q^{π}_t, defined below, denote the real stochastic discount factor. Recall also that $\lambda_tP_t = u_t'$, where u_t' is the marginal utility to consumption. Therefore, the first order

[10] The proof is shown in the Appendix.

conditions to pin down $\mathsf{m}^{\mathsf{o}}_{\mathfrak{c},t}$ can be simplified to:[11]

$$\begin{aligned} 0 &= \mathfrak{w}_{\mathfrak{c}}\varpi^{\mathsf{o}}_{\mathfrak{c},t} - \mathfrak{w}_{\mathfrak{c}}\varpi^{\ell}_{\mathfrak{c},t} - \varsigma_{\mathsf{m}\mathfrak{c}}\left(\frac{\mathsf{m}^{\mathsf{o}}_{\mathfrak{c},t}}{\mathsf{m}^{\mathsf{o}}_{\mathfrak{c},t-1}} - 1\right)\frac{1}{\mathsf{m}^{\mathsf{o}}_{\mathfrak{c},t-1}} \\ &\quad + \varsigma_{\mathsf{m}\mathfrak{c}} E_t Q^{\pi}_{t+1}\left(\frac{\mathsf{m}^{\mathsf{o}}_{\mathfrak{c},t+1}}{\mathsf{m}^{\mathsf{o}}_{\mathfrak{c},t}} - 1\right)\left(\frac{\mathsf{m}^{\mathsf{o}}_{\mathfrak{c},t+1}}{\mathsf{m}^{\mathsf{o}}_{\mathfrak{c},t}}\right)\frac{1}{\mathsf{m}^{\mathsf{o}}_{\mathfrak{c},t}} \end{aligned} \tag{40}$$

$$\varpi^{\ell}_{\mathfrak{c},t} = \varpi^{u}_{\mathfrak{c},t} + (1-\rho_{\mathfrak{d}})\,\Lambda^{\mathsf{u}}_{\mathfrak{c},t} \tag{41}$$

$$\varpi^{\mathsf{o}}_{\mathfrak{c},t} = (1-\rho_{\mathfrak{d}})\left(\delta^{\mathfrak{c}}_{\mathfrak{c}}\Lambda^{\mathsf{u}}_{\mathfrak{c},t} + \delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\Lambda^{\mathsf{u}}_{\bar{\mathfrak{c}},t}\right) + (1-\rho_{\mathfrak{d}})\left(1-\delta^{\mathfrak{c}}_{\mathfrak{c}} - \delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\right) E_t Q^{\pi}_{t+1}\varpi^{\mathsf{o}}_{\mathfrak{c},t+1} \tag{42}$$

$$\Lambda^{\mathsf{u}}_{\mathfrak{c},t} = a_{\mathfrak{c}}\mathsf{p}_{\mathfrak{c},t+1}E_t\int_{\mathfrak{c}}\Lambda^{cn}_t(z_{\mathfrak{c}})\,\mathsf{s}_{t+1}(z_{\mathfrak{c}})\,dz_{\mathfrak{c}} + E_tQ^{\pi}_{t+1}(1-a_{\mathfrak{c}}\mathsf{p}_{\mathfrak{c},t+1})\,\varpi^{\ell}_{\mathfrak{c},t+1} \tag{43}$$

$$\begin{aligned} \Lambda^{cn}_t(z_{\mathfrak{c}}) &= E_tQ^{\pi}_{t+1}\left[-\frac{\upsilon'_{t+1}(z_{\mathfrak{c}})\,h_{t+1}(z_{\mathfrak{c}})}{u'_{t+1}} + \varpi_{t+1}(z_{\mathfrak{c}}) - \varpi^{u}_{\mathfrak{c},t+1}\right] \\ &\quad + E_tQ^{\pi}_{t+1}\left[\varpi^{\ell}_{\mathfrak{c},t+1} + (1-\rho_{\mathfrak{d}})\left(1-\rho_{\mathfrak{c},t+1}\right)\left(\Lambda^{cn}_{t+1}(z_{\mathfrak{c}}) - \Lambda^{\mathsf{u}}_{\mathfrak{c},t+1}\right)\right] \end{aligned} \tag{44}$$

where

$$\varpi^{\mathsf{o}}_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}}\frac{\lambda^{\mathsf{o}}_{\mathfrak{c},t}}{u'_t} \quad \varpi^{\ell}_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}}\frac{\lambda^{\ell}_{\mathfrak{c},t}}{u'_t} \quad ; \; \Lambda^{\mathsf{u}}_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}}\frac{\lambda^{\mathsf{u}}_{\mathfrak{c},t}}{u'_t} \quad ; \; \Lambda^{cn}_t(z_{\mathfrak{c}}) \equiv \frac{\lambda^{cn}_t(z_{\mathfrak{c}})}{u'_t} \quad ; \; Q^{\pi}_t \equiv Q_t\Pi_t \tag{45}$$

The main message behind those results are: (*i*) once contolling for reallocation costs, unemployed workers must be indifferent between staying at the sectoral labor market, leaving it or trying to reallocate to the other sector; and (*ii*) decisions on reallocating must consider the relative net benefits from being unemployed on both sectors.

3.3 Firms

Goods are produced in sectors $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$. Each firm produces with technology

$$y_t(z_{\mathfrak{c}}) = \mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t H_t(z_{\mathfrak{c}})^{\varepsilon_{\mathfrak{c}}} \tag{46}$$

where $\varepsilon_{\mathfrak{c}} \in (0,1)$, $H_t(z_{\mathfrak{c}}) \equiv h_t(z_{\mathfrak{c}})\,\mathsf{n}_t(z_{\mathfrak{c}})$ is the *total hours worked*, A_t is the aggregate technology shock, and $\mathsf{a}_{\mathfrak{c},t}$ is the sector-$\mathfrak{c}$ idiosyncratic technology shock.

The technology shocks A_t and $\mathsf{a}_{\mathfrak{c},t}$ are stationary exogenous processes, described by

$$\frac{\mathsf{A}_t}{\bar{\mathsf{A}}} = \epsilon_{\mathsf{A},t}\left(\frac{\mathsf{A}_{t-1}}{\bar{\mathsf{A}}}\right)^{\phi_{\mathsf{A}}} \quad ; \quad \frac{\mathsf{a}_{\mathfrak{c},t}}{\bar{\mathsf{a}}_{\mathfrak{c}}} = \epsilon^{\mathsf{a}}_{\mathfrak{c},t}\left(\frac{\mathsf{a}_{\mathfrak{c},t-1}}{\bar{\mathsf{a}}_{\mathfrak{c}}}\right)^{\phi^{\mathsf{a}}_{\mathfrak{c}}}$$

[11] The proof is shown in the Appendix.

where $\epsilon_{\mathsf{A},t}$ and $\epsilon^{\mathsf{a}}_{\mathfrak{c},t}$ are the aggregate and sector-$\mathfrak{c}$ idiosyncratic technology innovations, $\phi_{\mathsf{A}} \in (0,1)$ and $\phi^{\mathsf{a}}_{\mathfrak{c}} \in (0,1)$.

Let $P_{\mathfrak{c},t}\mathcal{Y}_{\mathfrak{c},t} \equiv \int_{\mathfrak{c}} p_t(z_{\mathfrak{c}})\, y_t(z_{\mathfrak{c}})\, dz_{\mathfrak{c}}$ denote the aggregate revenue from sales, where $\mathcal{Y}_{\mathfrak{c},t}$ is the sector-$\mathfrak{c}$ *gross output.* In this context, $\mathcal{Y}_t$ denote the economy *gross output*:

$$\mathcal{Y}_t \equiv \sum_{\mathfrak{c}} \wp_{\mathfrak{c},t}\mathcal{Y}_{\mathfrak{c},t} \tag{47}$$

Firms' market clearing conditions are

$$y_t(z_{\mathfrak{c}}) = \boldsymbol{c}_t(z_{\mathfrak{c}}) \quad ; \ \mathcal{Y}_{\mathfrak{c},t} = \mathfrak{C}_{\mathfrak{c},t} \quad ; \ \mathcal{Y}_t = \mathfrak{C}_t \tag{48}$$

where $\mathfrak{C}_{\mathfrak{c},t}$ and $\mathfrak{C}_t$ are again the sectoral and the economy wide consumption bundles, as defined in Section 3.2.1. Using the market clearing conditions and the demand functions, I obtain the demand functions:

$$y_t(z_{\mathfrak{c}}) = \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}}\mathcal{Y}_{\mathfrak{c},t}\left(\tfrac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi} \quad ; \ \mathcal{Y}_{\mathfrak{c},t} = \mathfrak{w}_{\mathfrak{c}}\mathcal{Y}_t\left(\wp_{\mathfrak{c},t}\right)^{-\phi} \tag{49}$$

As mentioned in Section 3.2.1, $\mathsf{c}^{\mathsf{vc}}_{\mathfrak{m},t}$ and $\mathsf{c}^{\mathsf{vc}}_{\mathfrak{s},t}$ represent firms' intermediate consumption and must be netted out when computing sector-$\mathfrak{c}$ and the economy-wide GDP's, defined below:

$$Y_{\mathfrak{c},t} \equiv C_{\mathfrak{c},t} \quad ; \ Y_t \equiv C_t \tag{50}$$

3.3.1 Wage bargaining

Recall that each firm $z_{\mathfrak{c}}$ at sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$ uses labor in both the intensive $h_t(z_{\mathfrak{c}})$ and extensive $\mathsf{n}_t(z_{\mathfrak{c}})$ margins according to the technology $y_t(z_{\mathfrak{c}}) = \mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t H_t(z_{\mathfrak{c}})^{\varepsilon_{\mathfrak{c}}}$, where $H_t(z_{\mathfrak{c}}) = h_t(z_{\mathfrak{c}})\,\mathsf{n}_t(z_{\mathfrak{c}})$.

The total real salary per period $\varpi_t(z_{\mathfrak{c}}) = w_t(z_{\mathfrak{c}})\, h_t(z_{\mathfrak{c}})$ and hours per worker $h_t(z_{\mathfrak{c}})$ are decided by Nash bargaining and maximize $b_{\mathfrak{c},t}\log(\mathfrak{U}_t(z_{\mathfrak{c}})) + (1 - b_{\mathfrak{c},t})\log(\mathfrak{J}_t(z_{\mathfrak{c}}))$, where $\mathfrak{U}_t(z_{\mathfrak{c}})$ and $\mathfrak{J}_t(z_{\mathfrak{c}})$ are the worker's and firm's real match surpluses when the marginal worker is matched into firm $z_{\mathfrak{c}}$. As in Ravenna and Walsh (2011), I assume that the workers' bargaining power $b_{\mathfrak{c},t}$ is time-varying and evolves according to a stationary process about its steady state level $\bar{b}_{\mathfrak{c}}$:

$$\frac{b_{\mathfrak{c},t}}{\bar{b}_{\mathfrak{c}}} = \epsilon^{b}_{\mathfrak{c},t}\left(\frac{b_{\mathfrak{c},t-1}}{\bar{b}_{\mathfrak{c}}}\right)^{\phi^{b}_{\mathfrak{c}}} \tag{51}$$

where $\epsilon^b_{\mathfrak{c},t}$ is the sector-$\mathfrak{c}$ specific shock on the bargaining power and $\phi^b_{\mathfrak{c}} \in (0,1)$.

I derive the aggregate wage curve and the aggregate job creation curve, shown below. My analysis departs from Thomas (2011) by assuming that hours must be set to maximize the total surplus, as will be optimal both from the firm's as the union's perspectives, and not assuming that firms internalize the existence of a wage schedule, as a function of only hours, prior to optimization. Thomas (2011), on the other hand, assume that firms individually set hours to maximize their discounted flow of profits.

For notation purposes, let $\upsilon'_t(z_{\mathfrak{c}}) \equiv \partial \upsilon_t(z_{\mathfrak{c}}) / \partial H_t(z_{\mathfrak{c}})$ denote the marginal disutility to work, $\varpi'_t(z_{\mathfrak{c}}) \equiv \partial \varpi(h_t(z_{\mathfrak{c}})) / \partial h_t(z_{\mathfrak{c}})$ denote the marginal real salary, $w'_t(z_{\mathfrak{c}}) \equiv \partial w(h_t(z_{\mathfrak{c}})) / \partial h_t(z_{\mathfrak{c}})$ denote the marginal real wage. Recall also that $\lambda_t P_t = u'_t$, where u'_t is the marginal utility to consumption, and $Q^\pi_t \equiv Q_t \Pi_t$ is the real stochastic discount factor.

Bargaining takes place taking the extensive margin $\mathsf{n}_t(z_{\mathfrak{c}})$ as given, as soon as new hired workers arrive, in the beginning of period t, slightly after prices $p_t(z_{\mathfrak{c}})$ are set. Therefore, due to the demand function, total current revenue $\bar{\mathcal{R}}_t(z_{\mathfrak{c}})$ is also given. Therefore, subject to the law of motion of its employment stock, its production and demand functions, the firm chooses $\mathsf{v}^e_t(z_{\mathfrak{c}})$ and $\mathsf{n}_{t+1}(z_{\mathfrak{c}})$ to maximize its expected present discounted sum of nominal profits $\mathcal{J}_t(z_{\mathfrak{c}})$:

$$\begin{aligned}
\mathcal{J}_t(z_{\mathfrak{c}}) = \max & \left[\mathcal{R}_t(z_{\mathfrak{c}}) - P_t w_t(z_{\mathfrak{c}}) H_t(z_{\mathfrak{c}}) - P_t \varsigma_{\mathsf{v}\mathfrak{c}} \mathsf{v}^e_t(z_{\mathfrak{c}})\right] \\
& + P_t \lambda^n_t(z_{\mathfrak{c}}) E_t \left[(1-\rho_{\mathfrak{d}})\left(1-\rho_{\mathfrak{c},t}\right) \mathsf{n}_t(z_{\mathfrak{c}}) + \mathsf{q}^f_{\mathfrak{c},t} \mathsf{v}^e_t(z_{\mathfrak{c}}) - \mathsf{n}_{t+1}(z_{\mathfrak{c}})\right] \\
& + E_t Q_{t+1} \mathcal{J}_{t+1}(z_{\mathfrak{c}})
\end{aligned}$$

where $\mathcal{R}_t(z_{\mathfrak{c}}) \equiv p_t(z_{\mathfrak{c}}) y_t(z_{\mathfrak{c}})$ is the revenue function, which is written as follows, once we consider the production and demand functions:

$$\mathcal{R}_t(z_{\mathfrak{c}}) = P_{\mathfrak{c},t} \left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}} \mathcal{Y}_{\mathfrak{c},t}\right)^{\frac{1}{\phi}} (y_t(z_{\mathfrak{c}}))^{1-\frac{1}{\phi}} = P_{\mathfrak{c},t} \left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}} \mathcal{Y}_{\mathfrak{c},t}\right)^{\frac{1}{\phi}} \left[\mathsf{a}_{\mathfrak{c},t} \mathsf{A}_t \left(h_t(z_{\mathfrak{c}}) \mathsf{n}_t(z_{\mathfrak{c}})\right)^{\varepsilon_{\mathfrak{c}}}\right]^{1-\frac{1}{\phi}}$$

The first order conditions are:

$$\begin{aligned}
\mathsf{v}^e_t(z_{\mathfrak{c}}) : \quad & \lambda^n_{\mathfrak{c},t} \equiv \lambda^n_t(z_{\mathfrak{c}}) = \tfrac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^f_{\mathfrak{c},t}} \\
\mathsf{n}_{t+1}(z_{\mathfrak{c}}) : \quad & \lambda^n_t(z_{\mathfrak{c}}) = E_t Q^\pi_{t+1} \mathfrak{J}_{t+1}(z_{\mathfrak{c}})
\end{aligned} \tag{52}$$

where $\mathfrak{J}_t(z_{\mathfrak{c}}) \equiv \frac{1}{P_t} \frac{\partial \mathcal{J}_t(z_{\mathfrak{c}})}{\partial \mathsf{n}_t(z_{\mathfrak{c}})}$ is the real value of the marginal worker to the firm, i.e. the firm's real match surplus, which is computed by means of the Envelope Theorem. The real value of the marginal worker to the firm can be written as follows:[12]

$$\mathfrak{J}_t(z_\mathfrak{c}) = \frac{\varepsilon_\mathfrak{c}}{\mu}\frac{P_{\mathfrak{c},t}}{P_t}\left(\frac{1}{\mathfrak{w}_\mathfrak{c}}\frac{\mathcal{Y}_{\mathfrak{c},t}}{y_t(z_\mathfrak{c})}\right)^{\frac{1}{\phi}}\frac{y_t(z_\mathfrak{c})}{\mathsf{n}_t(z_\mathfrak{c})} - \varpi_t(z_\mathfrak{c}) + \left(1-\rho_{\mathfrak{dc},t}\right)\frac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^f_{\mathfrak{c},t}} \tag{53}$$

where

$$\rho_{\mathfrak{dc},t} \equiv 1 - (1-\rho_\mathfrak{d})\left(1-\rho_{\mathfrak{c},t}\right)$$

Let $\mathfrak{U}_t(z_\mathfrak{c}) \equiv \frac{1}{\lambda_t P_t}\frac{\partial \mathcal{U}_t}{\partial \mathsf{n}_t(z_\mathfrak{c})}$ denote the real match surplus enjoyed by the marginal worker matched into firm $z_\mathfrak{c}$, in monetary units:

$$\mathfrak{U}_t(z_\mathfrak{c}) = \varpi_t(z_\mathfrak{c}) - \varpi^u_{\mathfrak{c},t} + \varpi^\ell_{\mathfrak{c},t} - \frac{\upsilon'_t(z_\mathfrak{c})\, h_t(z_\mathfrak{c})}{u'_t} + \left(1-\rho_{\mathfrak{dc},t}\right)\left(\Lambda^{cn}_t(z_\mathfrak{c}) - \Lambda^{\mathsf{u}}_{\mathfrak{c},t}\right)$$

The solution to the Nash bargaining is $\frac{\mathfrak{U}_t(z_\mathfrak{c})}{\mathfrak{J}_t(z_\mathfrak{c})} = \frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}$,[13] which implies that the total surplus $\mathfrak{T}_t(z_\mathfrak{c}) \equiv \mathfrak{U}_t(z_\mathfrak{c}) + \mathfrak{J}_t(z_\mathfrak{c})$ is proportional to $\mathfrak{U}_t(z_\mathfrak{c})$ and $\mathfrak{J}_t(z_\mathfrak{c})$. This result implies that the household's and firms' Lagrange multipliers $\lambda^{cn}_t(z_\mathfrak{c}) = u'_t\Lambda^{cn}_t(z_\mathfrak{c})$ and $\lambda^n_t(z_\mathfrak{c})$ on the laws of motion of employment must satisfy:

$$\Lambda^{cn}_t(z_\mathfrak{c}) = \frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}\lambda^n_t(z_\mathfrak{c}) = \frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}\frac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^f_{\mathfrak{c},t}} \tag{54}$$

Plugging (54) into (43), I obtain:

$$\Lambda^{\mathsf{u}}_{\mathfrak{c},t} = \frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}a_\mathfrak{c}\varsigma_{\mathsf{v}\mathfrak{c}}\theta^f_{\mathfrak{c},t} + (1-a_\mathfrak{c}\mathsf{p}_{\mathfrak{c},t+1})\,E_t Q^\pi_{t+1}\varpi^\ell_{\mathfrak{c},t+1} \tag{55}$$

Therefore, I rewrite $\mathfrak{U}_t(z_\mathfrak{c})$ as

$$\begin{aligned}\mathfrak{U}_t(z_\mathfrak{c}) \;&=\; \varpi_t(z_\mathfrak{c}) - \varpi^u_{\mathfrak{c},t} - \left[\left(1-\rho_{\mathfrak{dc},t}\right)(1-a_\mathfrak{c}\mathsf{p}_{\mathfrak{c},t+1})\,E_t Q^\pi_{t+1}\varpi^\ell_{\mathfrak{c},t+1} - \varpi^\ell_{\mathfrak{c},t}\right] \\ &\quad + \left(1-\rho_{\mathfrak{dc},t}\right)\frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}\frac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^f_{\mathfrak{c},t}}(1-a_\mathfrak{c}\mathsf{p}_{\mathfrak{c},t+1}) - (1+\nu)\frac{1}{\mathsf{n}_t(z_\mathfrak{c})}\frac{\upsilon_t(z_\mathfrak{c})}{u'_t}\end{aligned}$$

Since $\mathfrak{U}_t(z_\mathfrak{c}) = \frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}\mathfrak{J}_t(z_\mathfrak{c})$, I obtain the equations describing the evolution dynamics

[12] The proof is shown in the appendix.

[13] The general solution of the Nash bargaining is $\frac{\mathfrak{U}_t(z_\mathfrak{c})}{\mathfrak{J}_t(z_\mathfrak{c})} = -\frac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}\frac{\partial\mathfrak{U}_t(z_\mathfrak{c})/\partial\varpi_t(z_\mathfrak{c})}{\partial\mathfrak{J}_t(z_\mathfrak{c})/\partial\varpi_t(z_\mathfrak{c})}$.

of the firm's salary as a function of $\lambda_t^r(z_\mathfrak{c})$:

$$\begin{aligned}\varpi_t(z_\mathfrak{c}) &= (1-b_{\mathfrak{c},t})\varpi^u_{\mathfrak{c},t} + (1-b_{\mathfrak{c},t})(1+\nu)\frac{1}{\mathsf{n}_t(z_\mathfrak{c})}\frac{\upsilon_t(z_\mathfrak{c})}{u'_t} + b_{\mathfrak{c},t}\frac{\varepsilon_\mathfrak{c}}{\mu}\frac{P_{\mathfrak{c},t}}{P_t}\left(\frac{1}{\mathfrak{w}_\mathfrak{c}}\frac{\mathcal{Y}_{\mathfrak{c},t}}{y_t(z_\mathfrak{c})}\right)^{\frac{1}{\phi}}\frac{y_t(z_\mathfrak{c})}{\mathsf{n}_t(z_\mathfrak{c})} \\ &+b_{\mathfrak{c},t}\left(1-\rho_{\mathfrak{d}\mathfrak{c},t}\right)a_\mathfrak{c}\varsigma_{\mathsf{v}\mathfrak{c}}\theta^f_{\mathfrak{c},t} \\ &+(1-b_{\mathfrak{c},t})\left[\left(1-\rho_{\mathfrak{d}\mathfrak{c},t}\right)(1-a_\mathfrak{c}\mathsf{p}_{\mathfrak{c},t+1})E_tQ^\pi_{t+1}\varpi^\ell_{\mathfrak{c},t+1}-\varpi^\ell_{\mathfrak{c},t}\right]\end{aligned}$$

Since $\mathfrak{J}_t(z_\mathfrak{c})$ and $\mathfrak{U}_t(z_\mathfrak{c})$ are proportional to $\mathfrak{T}_t(z_\mathfrak{c})$, unions and firms agree agree to set hours $h_t(z_\mathfrak{c})$ in order to maximize the total surplus, which implies:

$$\frac{\varepsilon_\mathfrak{c}}{\mu}\frac{P_{\mathfrak{c},t}}{P_t}\left(\frac{1}{\mathfrak{w}_\mathfrak{c}}\frac{\mathcal{Y}_{\mathfrak{c},t}}{y_t(z_\mathfrak{c})}\right)^{\frac{1}{\phi}}\frac{y_t(z_\mathfrak{c})}{\mathsf{n}_t(z_\mathfrak{c})} = \frac{\mu}{\varepsilon_\mathfrak{c}}(1+\nu)^2\frac{1}{\mathsf{n}_t(z_\mathfrak{c})}\frac{\upsilon_t(z_\mathfrak{c})}{u'_t}$$

Plugging this result in the salary equation, I obtain the firm's salary curve:

$$\begin{aligned}\varpi_t(z_\mathfrak{c}) &= (1-b_{\mathfrak{c},t})\varpi^u_{\mathfrak{c},t} + (1-b_{\mathfrak{c},t})\mathfrak{z}_{1\mathfrak{c},t}\frac{1}{\mathsf{n}_t(z_\mathfrak{c})}\frac{\upsilon_t(z_\mathfrak{c})}{u'_t} + b_{\mathfrak{c},t}\left(1-\rho_{\mathfrak{d}\mathfrak{c},t}\right)a_\mathfrak{c}\varsigma_{\mathsf{v}\mathfrak{c}}\theta^f_{\mathfrak{c},t} \\ &+(1-b_{\mathfrak{c},t})\left[\left(1-\rho_{\mathfrak{d}\mathfrak{c},t}\right)(1-a_\mathfrak{c}\mathsf{p}_{\mathfrak{c},t+1})E_tQ^\pi_{t+1}\varpi^\ell_{\mathfrak{c},t+1}-\varpi^\ell_{\mathfrak{c},t}\right]\end{aligned}$$

where

$$\mathfrak{z}_{1\mathfrak{c},t} \equiv (1+\nu)\left[1+\mu(1+\omega_\mathfrak{c})\tilde{b}_{\mathfrak{c},t}\right] \quad ; \ \omega_\mathfrak{c} \equiv \tfrac{1+\nu}{\varepsilon_\mathfrak{c}} - 1 \quad ; \ \tilde{b}_{\mathfrak{c},t} \equiv \tfrac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})} \tag{56}$$

It implies that the marginal salary is

$$\varpi'_t(z_\mathfrak{c}) = \mathfrak{z}_{2\mathfrak{c},t}\tfrac{\upsilon'_t(z_\mathfrak{c})}{u'_t} \quad ; \ \mathfrak{z}_{2\mathfrak{c},t} \equiv (1-b_{\mathfrak{c},t})\mathfrak{z}_{1\mathfrak{c},t}$$

The firm's job creation curve is then obtained by plugging those results into the firm's first order condition $\lambda^n_{\mathfrak{c},t} = E_tQ^\pi_{t+1}\mathfrak{J}_{t+1}(z_\mathfrak{c})$:

$$\frac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^f_{\mathfrak{c},t}} = E_tQ^\pi_{t+1}\left[\mathfrak{z}_{3\mathfrak{c}}\frac{\upsilon_{t+1}(z_\mathfrak{c})/\mathsf{n}_{t+1}(z_\mathfrak{c})}{u'_{t+1}} - \varpi_{t+1}(z_\mathfrak{c}) + \left(1-\rho_{\mathfrak{d}\mathfrak{c},t+1}\right)\frac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^f_{\mathfrak{c},t+1}}\right]$$

where

$$\mathfrak{z}_{3\mathfrak{c}} \equiv \mu(1+\nu)(1+\omega_\mathfrak{c})$$

Interestingly, note that (44) becomes a redundant result once we consider the system described by the firm's salary and job creation curves.

Let $\upsilon_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_\mathfrak{c}}\int_\mathfrak{c}\upsilon_t(z_\mathfrak{c})\,dz_\mathfrak{c}$ and $\varpi_{\mathfrak{c},t} \equiv \frac{1}{\mathsf{n}_{\mathfrak{c},t}}\frac{1}{\mathfrak{w}_\mathfrak{c}}\int_\mathfrak{c}\varpi_t(z_\mathfrak{c})\,\mathsf{n}_t(z_\mathfrak{c})\,dz_\mathfrak{c}$ denote the aggregate disutility and the the aggregate salary at sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}}$. Integrating over all firms in this sector, I obtain the following sectoral aggregate salary curve and aggregate job creation curve:

$$\begin{aligned} \varpi_{\mathfrak{c},t} = & \ (1-b_{\mathfrak{c},t})\,\varpi^{u}_{\mathfrak{c},t} + (1-b_{\mathfrak{c},t})\,\mathfrak{z}_{1\mathfrak{c},t}\tfrac{\upsilon_{\mathfrak{c},t}/\mathsf{n}_{\mathfrak{c},t}}{u'_t} + b_{\mathfrak{c},t}\left(1-\rho_{\mathfrak{d}\mathfrak{c},t}\right) a_{\mathfrak{c}}\varsigma_{\mathsf{v}\mathfrak{c}}\theta^{f}_{\mathfrak{c},t} \\ + & \ (1-b_{\mathfrak{c},t})\left[\left(1-\rho_{\mathfrak{d}\mathfrak{c},t}\right)\left(1-a_{\mathfrak{c}}\mathsf{p}_{\mathfrak{c},t+1}\right) E_t Q^{\pi}_{t+1}\varpi^{\ell}_{\mathfrak{c},t+1} - \varpi^{\ell}_{\mathfrak{c},t}\right] \end{aligned} \tag{57}$$

$$\tfrac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^{f}_{\mathfrak{c},t}} = \ E_t Q^{\pi}_{t+1}\left[\mathfrak{z}_{3\mathfrak{c}}\tfrac{\upsilon_{\mathfrak{c},t+1}/\mathsf{n}_{\mathfrak{c},t+1}}{u'_{t+1}} - \varpi_{\mathfrak{c},t+1} + \left(1-\rho_{\mathfrak{d}\mathfrak{c},t+1}\right)\tfrac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^{f}_{\mathfrak{c},t+1}}\right] \tag{58}$$

Note that sectoral salary curves are extended to account for the relative benefits from remaining at the labor market.

In this context, the economy wide aggregate salary ϖ_t is defined as follows:

$$\varpi_t = \frac{1}{\mathsf{n}_t}\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}}\varpi_{\mathfrak{c},t}\mathsf{n}_{\mathfrak{c},t} \tag{59}$$

Note now that:

$$\varpi_t(z_{\mathfrak{c}}) = \varpi_{\mathfrak{c},t} + \mathfrak{z}_{2\mathfrak{c},t}\left(\frac{\upsilon_t(z_{\mathfrak{c}})/\mathsf{n}_t(z_{\mathfrak{c}})}{u'_t} - \frac{\upsilon_{\mathfrak{c},t}/\mathsf{n}_{\mathfrak{c},t}}{u'_t}\right)$$

$$E_t Q^{\pi}_{t+1}\left(\mathfrak{z}_{3\mathfrak{c}} - \mathfrak{z}_{2\mathfrak{c},t+1}\right)\frac{\upsilon_{\mathfrak{c},t+1}/\mathsf{n}_{\mathfrak{c},t+1}}{u'_{t+1}} = E_t Q^{\pi}_{t+1}\left(\mathfrak{z}_{3\mathfrak{c}} - \mathfrak{z}_{2\mathfrak{c},t+1}\right)\frac{\upsilon_{t+1}(z_{\mathfrak{c}})/\mathsf{n}_{t+1}(z_{\mathfrak{c}})}{u'_{t+1}}$$

Since $\mathfrak{z}_{2\mathfrak{c},t}$ an exogenous autoregressive process, we can reasonably approximate the last result as

$$E_t Q^{\pi}_{t+1}\frac{\upsilon_{\mathfrak{c},t+1}/\mathsf{n}_{\mathfrak{c},t+1}}{u'_{t+1}} = E_t Q^{\pi}_{t+1}\frac{\upsilon_{t+1}(z_{\mathfrak{c}})/\mathsf{n}_{t+1}(z_{\mathfrak{c}})}{u'_{t+1}}$$

which implies that $E_t Q^{\pi}_{t+1}\varpi_t(z_{\mathfrak{c}}) = E_t Q^{\pi}_{t+1}\varpi_{\mathfrak{c},t}$, $\forall z_{\mathfrak{c}}$, i.e. there is ex-ante expected salary equalization between the firms in the same sector.

Let $\mathfrak{J}_{\mathfrak{c},t} \equiv \frac{1}{\mathsf{n}_{\mathfrak{c},t}}\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathfrak{J}_t(z_{\mathfrak{c}})\,\mathsf{n}_t(z_{\mathfrak{c}})\,dz_{\mathfrak{c}}$, $\mathfrak{U}_{\mathfrak{c},t} \equiv \frac{1}{\mathsf{n}_{\mathfrak{c},t}}\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathfrak{U}_t(z_{\mathfrak{c}})\,\mathsf{n}_t(z_{\mathfrak{c}})\,dz_{\mathfrak{c}}$ and $\mathfrak{T}_{\mathfrak{c},t} \equiv \mathfrak{U}_{\mathfrak{c},t} + \mathfrak{J}_{\mathfrak{c},t}$ denote the sectoral aggregate firms', workers and total real match surpluses at sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}}$. Integrating over all firms in this sector, I obtain:

$$\mathfrak{J}_{\mathfrak{c},t} = \mathfrak{z}_{3\mathfrak{c}}\frac{\upsilon_{\mathfrak{c},t}/\mathsf{n}_{\mathfrak{c},t}}{u'_t} - \varpi_{\mathfrak{c},t} + \left(1-\rho_{\mathfrak{d}\mathfrak{c},t}\right)\frac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^{f}_{\mathfrak{c},t}} \tag{60}$$

$$\mathfrak{U}_{\mathfrak{c},t} = \tfrac{b_{\mathfrak{c},t}}{(1-b_{\mathfrak{c},t})}\mathfrak{J}_{\mathfrak{c},t} \quad ; \ \mathfrak{T}_{\mathfrak{c},t} = \tfrac{1}{(1-b_{\mathfrak{c},t})}\mathfrak{J}_{\mathfrak{c},t} \quad ; \ \tfrac{\varsigma_{\mathsf{v}\mathfrak{c}}}{\mathsf{q}^{f}_{\mathfrak{c},t}} = E_t Q^{\pi}_{t+1}\mathfrak{J}_{\mathfrak{c},t+1} \tag{61}$$

3.3.2 Production firms - Marginal costs

The firm internalizes the fact that the marginal real salary is a function of its own production level $y_t(z_\mathfrak{c})$. Therefore, the real cost of a domestic firm $z_\mathfrak{c}$ at sector $\mathfrak{c} \in \mathcal{F}_\mathfrak{c} \equiv \{\mathfrak{m}, \mathfrak{s}\}$ is

$$\mathfrak{Cost}_t(z_\mathfrak{c}) = \mathfrak{s}_{\mathfrak{c},t}\varpi_t(z_\mathfrak{c})\,\mathsf{n}_t(z_\mathfrak{c}) + \varsigma_{\mathsf{v}\mathfrak{c}}\mathsf{v}_t^e(z_\mathfrak{c})$$

where $\mathfrak{s}_{\mathfrak{c},t}$ is an additional sector-$\mathfrak{c}$ specific cost shock over payroll, with unity average, which is not internalized during the bargaining process as well as when individuals optimally decide to leave the labor market or to reallocate to another sector. Of course, the sector-$\mathfrak{c}$ specific cost shocks will influence all those decisions in general equilibrium.

The shock evolves according to a stationary process about its steady state level $\bar{\mathfrak{s}}_\mathfrak{c} = 1$:

$$\frac{\mathfrak{s}_{\mathfrak{c},t}}{\bar{\mathfrak{s}}_\mathfrak{c}} = \epsilon_{\mathfrak{c},t}^{\mathfrak{s}}\left(\frac{\mathfrak{s}_{\mathfrak{c},t-1}}{\bar{\mathfrak{s}}_\mathfrak{c}}\right)^{\phi_\mathfrak{c}^\mathfrak{s}} \tag{62}$$

where $\epsilon_{\mathfrak{c},t}^{\mathfrak{s}}$ is the sector-$\mathfrak{c}$ specific innovation on the cost shock and $\phi_\mathfrak{c}^\mathfrak{s} \in (0,1)$.

The firm is free to adjust the intensive margin $h_t(z_\mathfrak{c})$. The extensive margin $\mathsf{n}_t(z_\mathfrak{c})$, however, depends only on previous decisions and hence is fixed during the period. Therefore, the real marginal cost is computed as follows:[14]

$$mc_t(z_\mathfrak{c}) = \varrho_\mathfrak{c}\epsilon_{\mathfrak{c},t}^{\mathfrak{mc}}\left(C_t - \iota_\mathrm{u}C_{t-1}\right)^\sigma\left(y_t(z_\mathfrak{c})\right)^{\omega_\mathfrak{c}} \tag{63}$$

where $\varrho_\mathfrak{c}$ is a composite parameter and $\epsilon_{\mathfrak{c},t}^{\mathfrak{mc}}$ is the marginal cost composite shock term, which aggregates all relevant shocks affecting the marginal cost:

$$\varrho_\mathfrak{c} \equiv \tfrac{\chi}{\varepsilon_\mathfrak{c}} \quad ; \; \epsilon_{\mathfrak{c},t}^{\mathfrak{mc}} \equiv \mathfrak{s}_{\mathfrak{c},t}\mathfrak{z}_{2\mathfrak{c},t}\left(\mathfrak{u}_{\mathrm{u},t}\right)^{-1}\left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-(1+\omega_\mathfrak{c})} \tag{64}$$

Since $\mathfrak{z}_{2\mathfrak{c},t}$ increases with the workers bargaining power $b_{\mathfrak{c},t}$ and the sector-$\mathfrak{c}$ specific cost $\mathfrak{s}_{\mathfrak{c},t}$ over payroll, the marginal cost also positively varies with shocks to $b_{\mathfrak{c},t}$ and $\mathfrak{s}_{\mathfrak{c},t}$.[15]

3.3.3 Price setting

With probability $(1-\alpha_\mathfrak{c})$, firm $z_\mathfrak{c}$ optimally readjusts its selling price to $p_t(z_\mathfrak{c}) = \bar{p}_{\mathfrak{c},t}$. With probability $\alpha_\mathfrak{c}$, its price is adjusted to $p_t(z_\mathfrak{c}) = p_{t-1}(z_\mathfrak{c})\,\Pi_{\mathfrak{c},t}^{ind}$, where $\Pi_{\mathfrak{c},t}^{ind} = \left(\Pi_{\mathfrak{c},t-1}\right)^{\iota_\mathfrak{c}}\left(\bar{\Pi}\right)^{\bar{\iota}}$, $\iota_\mathfrak{c} \in (0,1)$ is the indexation degree and $\bar{\iota} \in (0,1)$ is the indexation degree

[14] The proof is shown in the Appendix.
[15] The proof is shown in the Appendix.

over the long-run inflation target $\bar{\Pi}$. When optimally readjusting, firm $z_{\mathfrak{c}}$ sets its price to maximize its expected present discounted sum of profits, subject to the demand and marginal functions

$$y_{t+j}\left(z_{\mathfrak{c}}\right)=\left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\mathcal{Y}_{\mathfrak{c},t}\right)\mathcal{G}_{\mathfrak{c},t,t+j}\left(\frac{\Pi_{\mathfrak{c},t,t+j}}{\Pi_{\mathfrak{c},t,t+j}^{ind}}\right)^{\phi}\left(\frac{p_{t}\left(z_{\mathfrak{c}}\right)}{P_{\mathfrak{c},t}}\right)^{-\phi}$$

$$\begin{aligned}
mc_{t+j}\left(z_{\mathfrak{c}}\right) &= \varrho_{\mathfrak{c}}\epsilon_{\mathfrak{c},t+j}^{\mathfrak{mc}}\left(C_{t+j}-\iota_{\mathfrak{u}}C_{t+j-1}\right)^{\sigma}\left(y_{t+j}\left(z_{\mathfrak{c}}\right)\right)^{\omega_{\mathfrak{c}}} \\
&= \left(\frac{\Pi_{\mathfrak{c},t,t+j}}{\Pi_{\mathfrak{c},t,t+j}^{ind}}\right)^{\phi\omega_{\mathfrak{c}}}\varrho_{\mathfrak{c}}\epsilon_{\mathfrak{c},t+j}^{\mathfrak{mc}}\left(C_{t+j}-\iota_{\mathfrak{u}}C_{t+j-1}\right)^{\sigma}\left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\mathcal{Y}_{\mathfrak{c},t+j}\right)^{\omega_{\mathfrak{c}}}\left(\frac{p_{t}\left(z_{\mathfrak{c}}\right)}{P_{\mathfrak{c},t}}\right)^{-\phi\omega_{\mathfrak{c}}}
\end{aligned}$$

where

$$\mathcal{G}_{\mathfrak{c},t,t+j}\equiv\frac{\mathcal{Y}_{\mathfrak{c},t+j}}{\mathcal{Y}_{\mathfrak{c},t}}$$

Therefore, the problem is

$$\max_{\{p_t(z_{\mathfrak{c}})\}}E_t\sum_{j\geq 0}Q_{t,t+j}\alpha_{\mathfrak{s}}^{j}\left[p_{t}\left(z_{\mathfrak{c}}\right)\Pi_{\mathfrak{c},t,t+j}^{ind}y_{t+j}\left(z_{\mathfrak{c}}\right)-P_{t+j}\mathfrak{Cost}_{t+j}\left(z_{\mathfrak{c}}\right)\right]$$

The first order condition implies

$$\mu\varrho_{\mathfrak{c}}\epsilon_{\mathfrak{c},t}^{\mathfrak{mc}}\left(\wp_{\mathfrak{c},t}\right)^{-1}\left(C_{t}-\iota_{\mathfrak{u}}C_{t-1}\right)^{\sigma}\left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\mathcal{Y}_{\mathfrak{c},t}\right)^{\omega_{\mathfrak{c}}}=1 \quad , \quad \text{when } \alpha_{\mathfrak{s}}=0$$

and

$$\left(\frac{\bar{p}_{\mathfrak{c},t}}{P_{\mathfrak{c},t}}\right)^{(1+\phi\omega_{\mathfrak{c}})}=\frac{\mathcal{N}_{\mathfrak{c},t}}{\mathcal{D}_{\mathfrak{c},t}} \tag{65}$$

where

$$\mu=\tfrac{\phi}{(\phi-1)} \quad ; \ \mathcal{G}_{\mathfrak{c},t}\equiv\tfrac{\mathcal{Y}_{\mathfrak{c},t}}{\mathcal{Y}_{\mathfrak{c},t-1}}$$

and

$$\begin{aligned}
\mathcal{N}_{\mathfrak{c},t} &= \mu\varrho_{\mathfrak{c}}\epsilon_{\mathfrak{c},t}^{\mathfrak{mc}}\left(\wp_{\mathfrak{c},t}\right)^{-1}\left(C_{t}-\iota_{\mathfrak{u}}C_{t-1}\right)^{\sigma}\left(\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\mathcal{Y}_{\mathfrak{c},t}\right)^{\omega_{\mathfrak{c}}}+\alpha_{\mathfrak{c}}E_{t}\mathfrak{n}_{\mathfrak{c},t+1} \\
\mathcal{D}_{\mathfrak{c},t} &= 1+\alpha_{\mathfrak{c}}E_{t}\mathfrak{d}_{\mathfrak{c},t+1}
\end{aligned}$$

$$\begin{aligned}
\mathfrak{n}_{\mathfrak{c},t} &= Q_{t}\mathcal{G}_{\mathfrak{c},t}\Pi_{\mathfrak{c},t}\left(\frac{\Pi_{\mathfrak{c},t}}{\Pi_{\mathfrak{c},t}^{ind}}\right)^{\phi(1+\omega_{\mathfrak{c}})}\mathcal{N}_{\mathfrak{c},t} \\
\mathfrak{d}_{\mathfrak{c},t} &= Q_{t}\mathcal{G}_{\mathfrak{c},t}\Pi_{\mathfrak{c},t}\left(\frac{\Pi_{\mathfrak{c},t}}{\Pi_{\mathfrak{c},t}^{ind}}\right)^{(\phi-1)}\mathcal{D}_{\mathfrak{c},t}
\end{aligned}$$

The Calvo price setting structure implies:

$$1 = (1-\alpha_{\mathfrak{c}}) \left(\frac{\bar{p}_{\mathfrak{c},t}}{P_{\mathfrak{c},t}}\right)^{-(\phi-1)} + \alpha_{\mathfrak{c}} \left(\frac{\Pi_{\mathfrak{c},t}}{\Pi^{ind}_{\mathfrak{c},t}}\right)^{(\phi-1)}$$

3.3.4 Sectoral Phillips curves

In order to clarify the role of intersectoral interactions on price setting, I describe below the sectoral Phillips curves implied by log-linearized versions[16] of those first order conditions about equilibria in which the steady state level of inflation is $\bar{\Pi}$ and $\iota_{\mathfrak{c}} + \bar{\iota} = 1$.

$$\left(\hat{\pi}_{\mathfrak{c},t} - \hat{\pi}^{ind}_{\mathfrak{c},t}\right) = \beta E_t \left(\hat{\pi}_{\mathfrak{c},t+1} - \hat{\pi}^{ind}_{\mathfrak{c},t+1}\right) + \kappa_{\mathfrak{c}} \hat{x}_{\mathfrak{c},t} \quad ; \; \hat{\pi}^{ind}_{\mathfrak{c},t} = \iota_{\mathfrak{c}} \hat{\pi}_{\mathfrak{c},t-1}$$
$$\hat{x}_{\mathfrak{c},t} = \tfrac{1}{(\omega_{\mathfrak{c}}+\sigma)} \left(\hat{\epsilon}^{\mathfrak{mc}}_{\mathfrak{c},t} + \omega_{\mathfrak{c}} \widehat{\mathcal{Y}}_{\mathfrak{c},t} - \widehat{\wp}_{\mathfrak{c},t} + \tfrac{\sigma}{(1-\iota_{\mathfrak{c}})} \left(\hat{Y}_t - \iota_{\mathfrak{c}} \hat{Y}_{t-1}\right)\right)$$

where $\kappa_{\mathfrak{c}} \equiv \frac{(1-\alpha_{\mathfrak{c}})(1-\alpha_{\mathfrak{c}}\beta)}{\alpha_{\mathfrak{c}}} \frac{(\omega_{\mathfrak{c}}+\sigma)}{(1+\phi\omega_{\mathfrak{c}})}$, and $\hat{x}_{\mathfrak{c},t}$ represent the relevant activity gap related in the Phillips curve of sector $\mathfrak{c} \in \mathcal{F}_{\mathfrak{c}} \equiv \{\mathfrak{m}, \mathfrak{s}\}$.

Note that, as expected, $\hat{x}_{\mathfrak{c},t}$ depends on the sectoral marginal cost composite shock $\hat{\epsilon}^{\mathfrak{mc}}_{\mathfrak{c},t}$ and sectoral aggregate output $\widehat{\mathcal{Y}}_{\mathfrak{c},t}$. However, it also depends on its sectoral relative price $\widehat{\wp}_{\mathfrak{c},t}$ and aggregate GDP $\hat{Y}_t$. While $\widehat{\wp}_{\mathfrak{c},t}$ captures sectoral strategic complementarities in price setting, $\widehat{\mathcal{Y}}_{\mathfrak{c},t}$ also captures the costs of intermediate consumption needed for post vacancy openings, i.e. $\mathcal{Y}_{\mathfrak{c},t} = Y_{\mathfrak{c},t} + \mathfrak{w}_{\mathfrak{m}} \mathsf{c}^{\mathsf{vc}}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}} \mathsf{c}^{\mathsf{vc}}_{\mathfrak{s},t}$.

In this sense, the dynamics of the labor market directly affects price setting by means of $\widehat{\mathcal{Y}}_{\mathfrak{c},t}$ and $\hat{\epsilon}^{\mathfrak{mc}}_{\mathfrak{c},t}$. The role of the former was just described. As for $\hat{\epsilon}^{\mathfrak{mc}}_{\mathfrak{c},t}$, it varies with $\hat{\mathfrak{z}}_{2\mathfrak{c},t}$, which is strongly commoves with the workers bargaining power $\hat{b}_{\mathfrak{c},t}$.

3.4 Relative prices, aggregates and productivity

Modelling aggregate disutility functions $\upsilon_t \equiv \int \upsilon_t(z)\,dz$ and $\upsilon_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \upsilon_t(z_{\mathfrak{c}})\,dz_{\mathfrak{c}}$, and aggregate hours worked $H_t \equiv \int H_t(z)\,dz$ and $H_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} H_t(z_{\mathfrak{c}})\,dz_{\mathfrak{c}}$ requires more elaboration. Those variables can be rewritten as follows:[17]

$$\upsilon_{\mathfrak{c},t} \equiv \tfrac{\chi}{(1+\nu)} \left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-(1+\omega_{\mathfrak{c}})} \left(\tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} \mathcal{Y}_{\mathfrak{c},t}\right)^{(1+\omega_{\mathfrak{c}})} \left(\mathcal{P}_{\upsilon\mathfrak{c},t}\right)^{-\phi(1+\omega_{\mathfrak{c}})}$$
$$H_{\mathfrak{c},t} \equiv \left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-(1+\tilde{\omega}_{\mathfrak{s}})} \left(\tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} \mathcal{Y}_{\mathfrak{c},t}\right)^{(1+\tilde{\omega}_{\mathfrak{c}})} \left(\mathcal{P}_{H\mathfrak{c},t}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})}$$

$$\upsilon_t = \textstyle\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} \upsilon_{\mathfrak{c},t} \quad ; \; H_t = \textstyle\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} H_{\mathfrak{c},t}$$

[16] For any variable $\mathcal{X}_t$, the hatted representation $\widehat{\mathcal{x}}_t$ is its log-deviation from its steady state level $\bar{\mathcal{X}}$.
[17] The proof is shown in the Appendix.

where

$$\tilde{\omega}_{\mathfrak{c}} \equiv \frac{1}{\varepsilon_{\mathfrak{c}}} - 1$$

and $\mathcal{P}_{v\mathfrak{c},t}$ and $\mathcal{P}_{H\mathfrak{c},t}$ denote aggregate relative prices:

$$(\mathcal{P}_{v\mathfrak{c},t})^{-\phi(1+\omega_{\mathfrak{c}})} \equiv \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \left(\tfrac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi(1+\omega_{\mathfrak{c}})} dz_{\mathfrak{c}} \quad ; \quad (\mathcal{P}_{H\mathfrak{c},t})^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} \equiv \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \left(\tfrac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} dz_{\mathfrak{c}}$$

which evolve according to the following dynamics:

$$(\mathcal{P}_{v\mathfrak{c},t})^{-\phi(1+\omega_{\mathfrak{c}})} = (1-\alpha_{\mathfrak{c}}) \left(\frac{\bar{p}_{\mathfrak{c},t}}{P_{\mathfrak{c},t}}\right)^{-\phi(1+\omega_{\mathfrak{c}})} + \alpha_{\mathfrak{c}} \left(\frac{\Pi_{\mathfrak{c},t}}{\Pi^{ind}_{\mathfrak{c},t}}\right)^{\phi(1+\omega_{\mathfrak{c}})} (\mathcal{P}_{v\mathfrak{c},t-1})^{-\phi(1+\omega_{\mathfrak{c}})}$$

$$(\mathcal{P}_{H\mathfrak{c},t})^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} = (1-\alpha_{\mathfrak{c}}) \left(\frac{\bar{p}_{\mathfrak{c},t}}{P_{\mathfrak{c},t}}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} + \alpha_{\mathfrak{c}} \left(\frac{\Pi_{\mathfrak{c},t}}{\Pi^{ind}_{\mathfrak{c},t}}\right)^{\phi(1+\tilde{\omega}_{\mathfrak{c}})} (\mathcal{P}_{H\mathfrak{c},t-1})^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})}$$

Important indicators are the aggregate hours per worker h_t and $h_{\mathfrak{c},t}$, aggregate wages w_t and $w_{\mathfrak{c},t}$, and GDP per total hours ratios $\mathcal{A}_t$ and $\mathcal{A}_{\mathfrak{c},t}$. Those variables are defined as follows:

$$h_t \equiv \tfrac{H_t}{\mathsf{n}_t} \quad ; \; h_{\mathfrak{c},t} \equiv \tfrac{H_{\mathfrak{c},t}}{\mathsf{n}_{\mathfrak{c},t}} \quad ; \; w_t \equiv \tfrac{\varpi_t}{h_t} \quad ; \; w_{\mathfrak{c},t} \equiv \tfrac{\varpi_{\mathfrak{c},t}}{h_{\mathfrak{c},t}} \quad ; \; \mathcal{A}_t \equiv \tfrac{Y_t}{H_t} \quad ; \; \mathcal{A}_{\mathfrak{c},t} \equiv \tfrac{Y_{\mathfrak{c},t}}{H_{\mathfrak{c},t}}$$

3.5 Monetary and fiscal policies

The monetary authority is assigned a inflation target $\bar{\pi} \geq 0$ to pursuit and implements monetary policy according to the generalized Taylor rule:

$$\left(\frac{I_t}{\bar{I}}\right) = \mathfrak{u}_{\mathrm{i},t} \left(\frac{I_{t-1}}{\bar{I}}\right)^{\varphi_i} \left[\left(E_t \frac{\bar{\Pi}_{t+3}}{\bar{\Pi}}\right)^{\varphi_\pi} \left(\frac{Y_{t-1}}{\bar{Y}}\right)^{\varphi_y}\right]^{1-\varphi_i} \tag{66}$$

in which $\bar{\Pi}_t \equiv \left(\prod_{j=0}^{3} \Pi_{t-j}\right)^{1/4}$ is the 4-quarter geometric average of the (gross) inflation rate, the response parameters φ_i, φ_π, $\varphi_{\pi 1}$ and φ_y are consistent with stability and determinacy in equilibria with rational expectations, and $\mathfrak{u}_{\mathrm{i},t}$ is the monetary policy shock, which evolves according to:

$$\frac{\mathfrak{u}_{\mathrm{i},t}}{\bar{\mathfrak{u}}_{\mathrm{i}}} = \epsilon_{\mathrm{i},t} \left(\frac{\mathfrak{u}_{\mathrm{i},t-1}}{\bar{\mathfrak{u}}_{\mathrm{i}}}\right)^{\phi_{\mathrm{i}}}$$

where $\phi_{\mathrm{i}} \in (0,1)$ and $\epsilon_{\mathrm{i},t}$ is the monetary policy shock.

The government implements fiscal policy by setting lump-sum taxes net of transfers

Ξ_t to finance the nominal domestic debt $\mathcal{B}_t$ and unemployment compensation:

$$\mathcal{B}_{t+1} = \mathcal{B}_t \left(I_{t-1} \mathcal{R}_{t-1} \right) + P_t \sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} \varpi^{c}_{\mathfrak{c},t} \mathsf{u}^{e}_{\mathfrak{c},t} - \Xi_t$$

3.6 Aggregate real profits

Let d_t denote the aggregate real profit from all firms in the economy:

$$P_t d_t = \sum_{\mathfrak{c}} P_{\mathfrak{c},t} Y_{\mathfrak{c},t} - \sum_{\mathfrak{c}} \int_{\mathfrak{c}} \left(P_t \varpi_t \left(z_{\mathfrak{c}} \right) \mathsf{n}_t \left(z_{\mathfrak{c}} \right) + P_t \mathsf{c}^{\mathsf{v}}_t \left(z_{\mathfrak{c}} \right) \right) dz_{\mathfrak{c}}$$

Using previous results, I simplify this expression as follows:

$$d_t = Y_t - \left(\varpi_t \mathsf{n}_t + \sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} \varsigma_{\mathsf{v}\mathfrak{c}} \mathsf{v}^{e}_{\mathfrak{c},t} \right) \tag{67}$$

4 Inference

I estimate the log-linearized version of this model with a Bayesian technique and a Metropolis-Hastings (after Metropolis et al. (1953) and Hastings (1970)) MCMC algorithm. I obtain 6,000,000 draws from the MCMC sampler and keep the last 1,000,000 draws for inference and Bayesian impulse response exercises.[18] I consider 13 observed quarterly variables, from 2003:Q1 to 2014:Q4: manufacturing (detrended) GDP, services (detrended) GDP,[19] tradables inflation rate from Brazilian CPI (as a consumer-based proxy for the inflation rate of the manufacturing sector), non-tradables inflation rate from Brazilian CPI (as a consumer-based proxy for the inflation rate of the services sector), working-age population, participation rate, employed workers at the manufacturing sector, employed workers at the services sector, hours per worker at the manufacturing sector, aggregate hours per worker, separation rate at the manufacturing sector, total mass of hired workers (adjusted for formality), and nominal interest rate.

Hours per worker at the manufacturing sector and the separation rate at the manufacturing sector are from the IBGE's Monthly Industrial Survey (PIMES). The total mass of hired workers are from the Brazilian General List of Employed and Unemployed (CAGED) workers, released by the Brazilian Ministry of Labor. Since it only refers to

[18] Even though convergence occurred after about 1,500,000 draws, I kept on sampling to guarranty a good stationary distribution.

[19] I detrend the (log) manufacturing and services GDP measures using their common linear growth rate.

workers in formal sector, I adjust this measure using the formality rate from the IBGE's Employment Monthly Survey (PME). The nominal interest rate is the Brazilian Central Bank rate (Selic). All remaining variables are released by the Brazilian Institute of Geography and Statistics (IBGE). In particular, the labor market variables are obtained from the IBGE's Employment Monthly Survey (PME).

As for the Brazilian CPI measures, I extract them from the Broad Consumer Price Index (IPCA). Moreover, in order to deal with their observed seasonal pattern and avoid artificial dynamics created by usual seasonal adjustments algorithms (e.g. X12), I consider additional endogenous variables $\bar{\Pi}_{\mathfrak{c},t} \equiv \left(\prod_{j=0}^{3} \Pi_{\mathfrak{c},t-j}\right)^{1/4}$, defined as 4-quarter geometric average of the (gross) inflation rate at sector $\mathfrak{c} \in \{\mathfrak{m}, \mathfrak{s}\}$. In this context, I consider sectoral 12-month average inflation rates as observed variables for inference purposes.

I considered 13 shocks: 2 sector-c specific shocks on the separation rate $\left(\epsilon^{\rho}_{\mathfrak{m},t} \text{ and } \epsilon^{\rho}_{\mathfrak{s},t}\right)$, the shock to the mass of individuals coming to working-age $(\epsilon_{\ell,t})$, 2 sector-c specific shocks on the efficiency of the matching technology $\left(\epsilon^{\eta}_{\mathfrak{m},t} \text{ and } \epsilon^{\eta}_{\mathfrak{s},t}\right)$, the preference innovation $(\epsilon_{\mathfrak{u},t})$, 2 sector-$c$ specific innovations on the idiosyncratic technology shocks $\left(\epsilon^{\mathfrak{a}}_{\mathfrak{m},t} \text{ and } \epsilon^{\mathfrak{a}}_{\mathfrak{s},t}\right)$, 2 sector-$c$ specific shocks on the bargaining power $\left(\epsilon^{b}_{\mathfrak{m},t} \text{ and } \epsilon^{b}_{\mathfrak{s},t}\right)$, 2 sector-$c$ specific specific innovations on the marginal cost shocks $\left(\epsilon^{\mathfrak{s}}_{\mathfrak{m},t} \text{ and } \epsilon^{\mathfrak{s}}_{\mathfrak{s},t}\right)$, and the monetary policy shock $(\epsilon_{\mathrm{i},t})$.

I calibrate a few of the parameters. As for the death rate, I have detrended the working-age population by considering two log-linear constant growth rates (before and after 2009). The resulting residual implied an average death rate of $\rho_{\mathfrak{d}} = 0.495$. The mass of manufacturing firms were calibrated using the average elasticity of manufacturing prices and GDP's on the aggregate deflator and real level of GDP, from 1996 on. As a consequence, I obtained $w_{\mathfrak{m}} = 0.460$ (note that the weight of tradable goods on the IPCA inflation rate of market prices during the same period is 0.446). I set the elasticity of substitution at $\phi = 7$, which implies a price markup of $\mu = 1.17$.[20] As for the level of trend inflation, I considered the long-run Brazilian inflation target of $\bar{\pi} = 4.5$, and assumed full indexation between past inflation and inflation target, i.e. $\iota_{\mathfrak{c}}+\bar{\iota} = 1$. All steady state levels of exogenous shocks were set at 1, i.e. I have calibrated $\bar{\mathfrak{u}}_{\mathfrak{c}} = \bar{s}_{\mathfrak{m}} = \bar{s}_{\mathfrak{s}} = \bar{A} = \bar{a}_{\mathfrak{m}} = \bar{a}_{\mathfrak{s}} = 1$. I normalize and set the steady state level of hours per worker in the services sector at $\bar{h}_{\mathfrak{s}} = 1$. The steady state level of the participation rate is set at is sample average: $\bar{r} = 0.567$. The steady state level of the employment ratio of the manufacturing sector

[20] This value is consistent with the range used in the literature. For instance, Ravenna and Walsh (2008) assumes $\mu = 1.20$, while Thomas (2011) uses $\mu = 1.15$.

is set at is sample average: $\widetilde{\mathfrak{n}}^{e}_{\mathfrak{m}} \equiv \frac{\mathfrak{w}_{\mathfrak{m}}\bar{\mathfrak{n}}_{\mathfrak{m}}}{\bar{\mathfrak{n}}} = 0.249$. I also assume that the monetary policy shock $u_{\mathfrak{i},t}$ is a white noise, i.e. I impose $\phi_{\mathfrak{i}} = 0$. Finally, the subjective discount parameter was set at $\beta = 0.982$ in order to match the average ex-post real interest rate.

The calibration strategy, coupled with estimated parameters, define the distribution of the following parameters: (*i*) sector-specific proportionality parameters ς_{vm} and ς_{vs} in firms' end-of-period posted vacancies; (*ii*) sector-specific homogeneous disutility parameters $\bar{\upsilon}^{\mathsf{u}}_{\mathfrak{m}}$ and $\bar{\upsilon}^{\mathsf{u}}_{\mathfrak{s}}$ faced by unemployed workers; (*iii*) steady state levels of sector-specific efficiency parameters $\bar{\eta}_{\mathfrak{m}}$ and $\bar{\eta}_{\mathfrak{s}}$ in matching functions; and (*iv*) union's disutility ancillary parameter χ.

My standard approach is considering flat marginal prior distributions for all 39 estimated deep parameters and 13 standard deviations, i.e. all priors are set to be uniform distributions on very large support sets, so that inference is not biased at all by ill-designed prior distributions. Figure 3 shows marginal priors and posterior densities for the heterogeneous model. Note that, despite the use of flat priors, all parameters are well identified with sufficiently narrow marginal posterior distributions. Table 1 shows the posterior estimation of the deep parameters, some key steady state levels such as the share of unemployed workers coming from the manufacturing sector $\bar{\mathfrak{p}}^{\mathsf{ue}}_{\mathfrak{m}} \equiv \frac{\mathfrak{w}_{\mathfrak{m}}\bar{\mathfrak{u}}^{e}_{\mathfrak{m}}}{\bar{\mathfrak{u}}^{e}}$, and shocks standard deviations for the heterogeneous model.

In order to ensure that $0 \leq \delta^{\mathfrak{c}}_{\mathfrak{c}} + \delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}} < 1$ and assuming that average transition durations are no longer than 12 years, I use restricted prior densities $\delta^{\mathfrak{c}}_{\mathfrak{c}} \sim U(\delta^{*}, 1-\delta^{*})$, where $\delta^{*} = 1/(12 * 4 \text{ quarters}) = 0.02083$, and consider the normalizing transformation $\bar{\delta}^{\bar{\mathfrak{c}}}_{\mathfrak{c}} \equiv \frac{(\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}} - \delta^{*})}{(1-\delta^{*}-\delta^{\mathfrak{c}}_{\mathfrak{c}})}$, which I estimate with standard Uniform prior density $\bar{\delta}^{\bar{\mathfrak{c}}}_{\mathfrak{c}} \sim U(0,1)$.[21]

As for the estimated parameters for the labor market, I find that while workers from the manufacturing sector who are out of the labor market take longer to return ($Mode\left(\frac{1}{\delta^{\mathfrak{m}}_{\mathfrak{m}}}\right) \approx 2.1$ quarters) than workers from the service sector ($Mode\left(\frac{1}{\delta^{\mathfrak{s}}_{\mathfrak{s}}}\right) \approx 1.1$ quarters), workers from the manufacturing sector reallocate much faster to the service sector ($Mode\left(\frac{1}{\delta^{*}+\bar{\delta}^{\mathfrak{s}}_{\mathfrak{m}}(1-\delta^{*}-\delta^{\mathfrak{m}}_{\mathfrak{m}})}\right) \approx 2.4$ quarters) than workers from the services sector ($Mode\left(\frac{1}{\delta^{*}+\bar{\delta}^{\mathfrak{m}}_{\mathfrak{s}}(1-\delta^{*}-\delta^{\mathfrak{s}}_{\mathfrak{s}})}\right) \approx 10.3$ years) - in this regard, the information content in the sample strongly suggest that reallocation from services to manufacturing were really rare.[22]

[21] Note that this transformation ensures that $0 \leq \delta^{\mathfrak{c}}_{\mathfrak{c}} + \delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}} < 1$.

[22] The mean, median and 95% HPD credible interval durations are: 2.1, 2.1 and $(2.0, 2.2)$ quarters for $1/\delta^{\mathfrak{m}}_{\mathfrak{m}}$; 1.1, 1.1 and $(1.1, 1.2)$ quarters for $1/\delta^{\mathfrak{s}}_{\mathfrak{s}}$; 2.4, 2.4 and $(2.2, 2.6)$ quarters for $1/\left(\delta^{*} + \bar{\delta}^{\mathfrak{s}}_{\mathfrak{m}}(1-\delta^{*}-\delta^{\mathfrak{m}}_{\mathfrak{m}})\right)$; and 9.5, 9.8 and $(5.4, 11.7)$ years for $1/\left(\delta^{*} + \bar{\delta}^{\mathfrak{m}}_{\mathfrak{s}}(1-\delta^{*}-\delta^{\mathfrak{s}}_{\mathfrak{s}})\right)$.

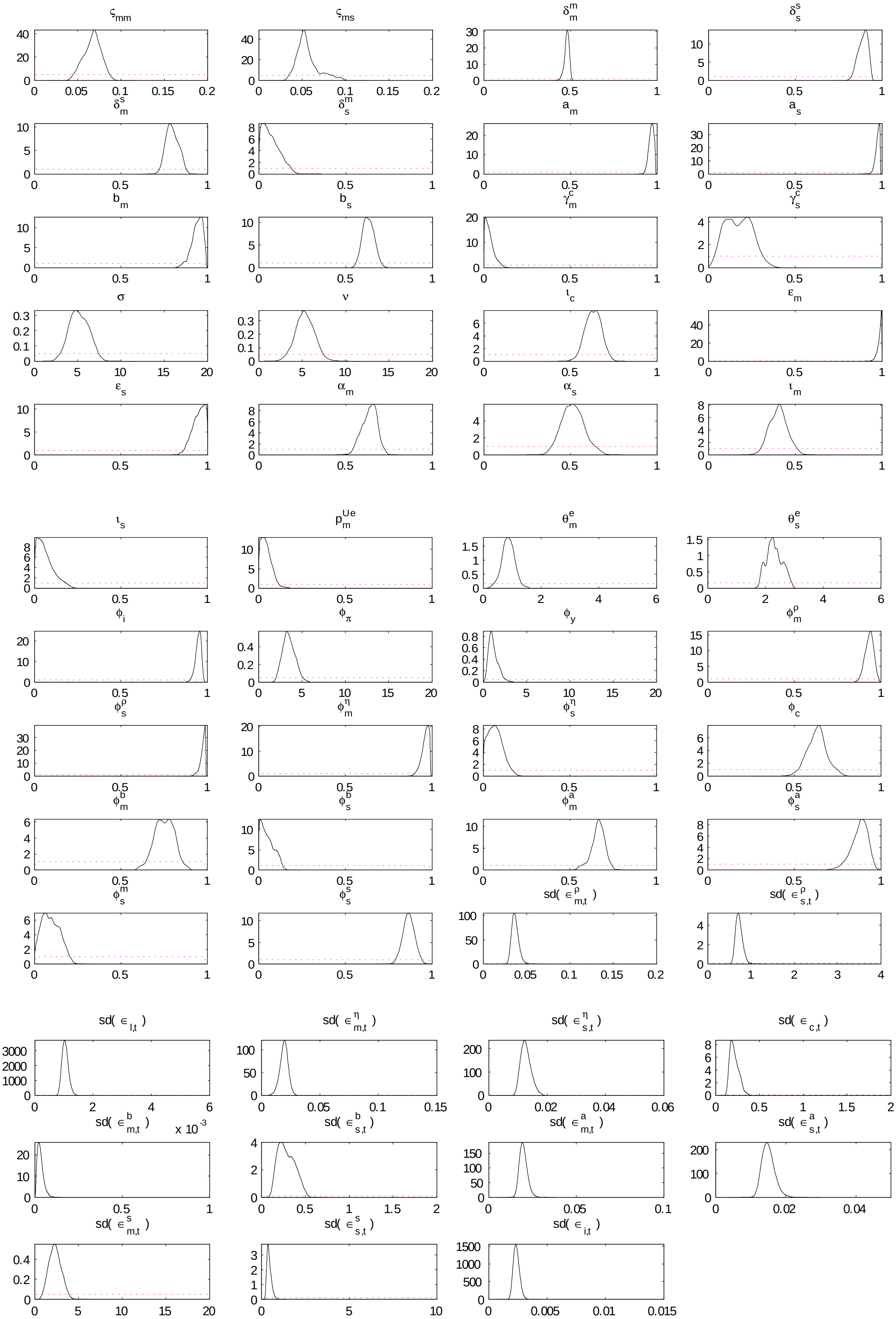

Figure 3: Posterior (black) and Prior (dotted) Marginal Densities - Heterogeneous Model

Table 1: Estimated Parameters and Standard Deviations

Parameter		Parameter		Parameter	
$\varsigma_{\mathfrak{mm}}$	0.067 (0.050,0.083)	$\alpha_{\mathfrak{m}}$	0.637 (0.561,0.702)	$\phi^{\mathfrak{a}}_{\mathfrak{m}}$	0.662 (0.600,0.729)
$\varsigma_{\mathfrak{ms}}$	0.056 (0.036,0.078)	$\alpha_{\mathfrak{s}}$	0.513 (0.402,0.618)	$\phi^{\mathfrak{a}}_{\mathfrak{s}}$	0.875 (0.798,0.949)
$\delta^{\mathfrak{m}}_{\mathfrak{m}}$	0.479 (0.457,0.502)	$\iota_{\mathfrak{m}}$	0.402 (0.316,0.487)	$\phi^{\mathfrak{m}}_{\mathfrak{s}}$	0.095 (0.007,0.172)
$\delta^{\mathfrak{s}}_{\mathfrak{s}}$	0.890 (0.846,0.934)	$\iota_{\mathfrak{s}}$	0.065 (0.000,0.136)	$\phi^{\mathfrak{s}}_{\mathfrak{s}}$	0.865 (0.809,0.921)
$\bar{\delta}^{\mathfrak{s}}_{\mathfrak{m}}$	0.796 (0.740,0.857)	$\bar{\mathfrak{p}}^{ue}_{\mathfrak{m}}$	0.045 (0.000,0.087)	$\epsilon^{\rho}_{\mathfrak{m},t}$	0.036 (0.030,0.042)
$\bar{\delta}^{\mathfrak{m}}_{\mathfrak{s}}$	0.070 (0.000,0.141)	$\bar{\theta}^{e}_{\mathfrak{m}}$	0.861 (0.500,1.230)	$\epsilon^{\rho}_{\mathfrak{s},t}$	0.719 (0.591,0.843)
$a_{\mathfrak{m}}$	0.966 (0.946,1.000)	$\bar{\theta}^{e}_{\mathfrak{s}}$	2.307 (1.848,2.741)	$\epsilon_{\ell,t}$	0.001 (0.001,0.001)
$a_{\mathfrak{s}}$	0.974 (0.957,1.000)	φ_i	0.947 (0.921,0.974)	$\epsilon^{\eta}_{\mathfrak{m},t}$	0.019 (0.013,0.024)
$\bar{b}_{\mathfrak{m}}$	0.939 (0.895,0.989)	φ_π	3.476 (2.315,4.640)	$\epsilon^{\eta}_{\mathfrak{s},t}$	0.012 (0.009,0.015)
$\bar{b}_{\mathfrak{s}}$	0.631 (0.577,0.685)	φ_y	1.154 (0.331,2.004)	$\epsilon_{\mathfrak{c},t}$	0.211 (0.137,0.291)
$\gamma^{c}_{\mathfrak{m}}$	0.033 (0.000,0.069)	$\phi^{\rho}_{\mathfrak{m}}$	0.932 (0.891,0.972)	$\epsilon^{b}_{\mathfrak{m},t}$	0.031 (0.005,0.055)
$\gamma^{c}_{\mathfrak{s}}$	0.173 (0.049,0.290)	$\phi^{\rho}_{\mathfrak{s}}$	0.972 (0.951,0.993)	$\epsilon^{b}_{\mathfrak{s},t}$	0.285 (0.139,0.438)
σ	5.166 (3.423,7.041)	$\phi^{\eta}_{\mathfrak{m}}$	0.960 (0.930,0.991)	$\epsilon^{\mathfrak{a}}_{\mathfrak{m},t}$	0.019 (0.016,0.023)
ν	5.287 (3.502,7.074)	$\phi^{\eta}_{\mathfrak{s}}$	0.070 (0.000,0.127)	$\epsilon^{\mathfrak{a}}_{\mathfrak{s},t}$	0.015 (0.012,0.018)
$\iota_{\mathfrak{c}}$	0.631 (0.557,0.705)	$\phi_{\mathfrak{c}}$	0.628 (0.535,0.724)	$\epsilon^{\mathfrak{s}}_{\mathfrak{m},t}$	2.325 (1.163,3.477)
$\varepsilon_{\mathfrak{m}}$	0.985 (0.968,1.000)	$\phi^{b}_{\mathfrak{m}}$	0.750 (0.659,0.839)	$\epsilon^{\mathfrak{s}}_{\mathfrak{s},t}$	0.433 (0.239,0.626)
$\varepsilon_{\mathfrak{s}}$	0.946 (0.895,1.000)	$\phi^{b}_{\mathfrak{s}}$	0.051 (0.000,0.108)	$\epsilon_{\mathfrak{i},t}$	0.002 (0.002,0.003)

T=48, N of Series: 13, point estimate: posterior mean, parentheses: 95% *HPD credible intervals*
 # Kept Draws: 1000000, diagnostics: model log marginal likelihood (lml): 1338.16

Central estimates suggest that although the matching elasticity to unemployed workers in the service sector is only slightly larger ($a_{\mathfrak{s}} \approx 0.974 > a_{\mathfrak{m}} \approx 0.966$) than in the manufacturing sector, the workers' bargaining power in the manufacturing sector is much larger than the bargaining power in the service sector $\left(\bar{b}_{\mathfrak{m}} \approx 0.94 > \bar{b}_{\mathfrak{s}} \approx 0.63\right)$. As a consequence, the average salary in the service sector are much more correlated with the unemployment compensation, which is also very correlated with the minimum wage in Brazil. The estimates also suggest that salary bargaining is much more efficient in the manufacturing sector, as the marginal posterior distribution of $\bar{b}_{\mathfrak{m}}$ almost matches that of $a_{\mathfrak{m}}$,[23] whereas it is not the case in the services sector. Note also that the estimated values for those relevant labor market parameters $\left(a_{\mathfrak{c}} \text{ and } \bar{b}_{\mathfrak{c}}\right)$ are much larger in Brazil than what is found in developed countries, whose estimates range in $[0.2\ ,\ 0.7]$.[24]

[23] See Hosios (1990) for the intuition behind this result.

[24] See e.g. Andolfatto (1996), Blanchard and Diamond (1989), Flinn (2006), Hagedorn and Manovskii (2008), Hall (2005), Merz (1995), Mortensen and Nagypal (2007), and Shimer (2005).

I also find that the labor market is much tighter in the services sector, on average, than in the manufacturing sector $\left(\bar{\theta}^{e}_{\mathfrak{s}} \approx 2.31 > \bar{\theta}^{e}_{\mathfrak{m}} \approx 0.86\right)$, which means that firms in the services sector have much more vacancy openings, relative to unemployed workers, than those from the manufacturing sector. That result explains why unemployed workers find it easier to get a job in the services sector than in the manufacturing one.

The estimates also suggest that unemployed workers from the manufacturing sector tend to be much more sluggish when leaving the labor market than those from the services sector ($\varsigma_{\mathfrak{mm}} \approx 0.07 > \varsigma_{\mathfrak{ms}} \approx 0.06$). Also, the manufacturing sector's share of unemployed workers is about $\bar{\mathfrak{p}}^{\mathsf{ue}}_{\mathfrak{m}} \approx 0.05$ of the total mass of unemployed workers. This result is a consequence of two facts: (*i*) the sample average share of the working population in the manufacturing sector is about 25% of the total working population; and (*ii*) the average a great deal of unemployed workers from the manufacturing sector tends to reallocate to the services sector.

As for the sectoral steady state ratios of the unemployment compensations over aggregate salaries, $\gamma^{c}_{\mathfrak{m}} \approx 0.03$ and $\gamma^{c}_{\mathfrak{s}} \approx 0.17$, their central estimates suggest that manufacturing workers have much larger earnings losses when are unemployed than those from the services sector. Notwithstanding, those losses are large in both sectors. Since I do not use any wage;salary observed series during the estimations, I carry out a simple robustness check for those ratios by computing the sample average ratio of the expenses from unemployment compensation and wage bonus, released by the Brazilian Ministry of Finances, over the salaries/wages components from the nominal income, released by IBGE, from 2002:Q1 on. I find an empirical ratio of 0.20. Even though this value is slightly larger than what implied by the central values of $\gamma^{c}_{\mathfrak{m}}$ and $\gamma^{c}_{\mathfrak{s}}$, they all agree that earning loses when unemployed are a big deal in Brazil.

The data also suggests that the reciprocal of the marginal rate of intertemporal substitution is larger to what is found in the US ($\sigma \approx 5.16$),[25] but well identified. As for the reciprocal ν of the Frisch elasticity, its marginal posterior distribution suggest that there is no labor supply puzzle in the Brazilian labor market, i.e. its central estimate implies that labor is just weakly elastic $\left(\frac{1}{\nu} \approx \frac{1}{5.28} = 0.19\right)$ to salaries in Brazil. This result is in line with the international micro evidence. Indeed, Chetty et al. (2011) shows that macro evidence tends to mimic micro evidence when labor is split into its extensive and intensive margins in macroeconomic models.

As for the goods market, workers are slightly more productive, on average, in the

[25]See e.g. Smets and Wouters (2007).

manufacturing than those from the services sector ($\varepsilon_{\mathfrak{m}} \approx 0.99 > \varepsilon_{\mathfrak{s}} \approx 0.95$). And prices are much stickier ($\alpha_{\mathfrak{m}} \approx 0.64 > \alpha_{\mathfrak{s}} \approx 0.51$) and much more indexed ($\iota_{\mathfrak{m}} \approx 0.40 > \iota_{\mathfrak{s}} \approx 0.06$) in the manufacturing than in the services sector.

As for shocks' standard deviations, it is worthy discussing the ones related to the sector-specific cost shocks over payroll. Note that the st. deviation of manufacturing innovations is about 5 times as large as that of the services sector $\left(\sigma_{\epsilon^{\mathfrak{s}}_{\mathfrak{m},t}} \approx 2.33 > \sigma_{\epsilon^{\mathfrak{s}}_{\mathfrak{s},t}} \approx 0.43\right)$. On the other hand, manufacturing cost shocks are not as persistent as those of the services sector. The shock inertia in the services sector that is about 9 times as large as that of the services sector ($\phi^{\mathfrak{s}}_{\mathfrak{s}} \approx 0.87 > \phi^{\mathfrak{m}}_{\mathfrak{s}} \approx 0.10$).

This result is in line with the fact that those cost shocks capture possible modelling mispecification issues when puting the model to the data. The most important one is that I have considered a closed economy model, as a simplification assumption. In inference exercises using Brazilian data, whose manufacturing sector is highly sensible to international shocks, those sector-specific cost shocks over payroll absorb variations coming from the rest of the world, and help me get around mispecification issues.

5 Impulse responses

Since prices are more flexible in the services sector, they adjust faster to shocks. However, strategic complementarities induce sectoral inflation rates not to detach much from each other, so that sectors do not lose long-run demand attractiveness.

In case of aggregate shocks, the relative demand for both sectors will be different due to the fact of prices are more flexible in the services sector. This effect is combined with the strong heterogeneity characterizing both sectors to produce different responses in the goods and labor markets. This channel explains most of the sectoral responses presented in this section.

Responses of labor market quantities have two important features. The first one is that the dynamics of labor market quantities, both in the aggregate as in sectoral measurements, are much more persistent than those of the goods sector. The second one is that aggregate responses of labor market variables qualitatively follow those in the services sector. This is due to the fact that about 75% of employed workers are in this sector, and this share is large enough to dominate the aggregate dynamics.

I also plot GDP per total hours ratios $\mathcal{A}_t$ and $\mathcal{A}_{\mathfrak{c},t}$ in order to show that the signal induced by such commonly used indicators can be very misleading whenever there are no technology, productivity or cost shocks hitting the economy. As shown in Section 5.1, for instance, a pure contractionary monetary shock leads $\mathcal{A}_t$ and $\mathcal{A}_{\mathfrak{c},t}$ to rise.

5.1 Impulse responses to monetary policy shock

Figure 4 shows the impulse responses to a monetary policy shock (1 p.p. annualized). Note that, as expected, aggregate inflation, GDP, output, real salary, real wages, employment and hours fall. The responses are also consistent with labor hoarding, hours (intensive margin) fall on impact, whereas employment (extensive margin) takes longer to reduce.

Note that the reduction of aggregate real salaries and wages makes the whole household poorer. Recall that, due to risk sharing within the household, all members have the same consumption level regardless of their employment or labor market participation status. Therefore, the consumption level of all members evenly fall after the reduction real salaries and wages. Since welfare falls, workers that were previously out of the labor market are induced to return as unemployed workers, increasing the participation rate. This fact leads the unemployment rate to rise, despite of the fall in employment.

As for the sectoral responses, it is in the services sector tat we ovserve a rise in the sectoral participation rate. This is due to the fact that the increase in the unemployment duration is about twice as large in the manufacturing sector as it is in the services sector. Moreover, real wages and salaries are not as much affected in the services sector.

As more workers become unemployed in the manufacturing sector and they find it easier to reallocate to the services sector, they tend to reallocate after a small training period out of the labor market. Therefore, the participation rate in the manufacturing sector actually falls.

The responses of the services output have mixed dynamics. The monetary shock induces aggregate consumption, which equals aggregate output, to fall. Since prices are much more flexible in this sector, it adjusts faster, increasing the relative demand for services goods vis-a-vis manufacturing goods. Therefore, services real output is much less affected by monetary policy changes.

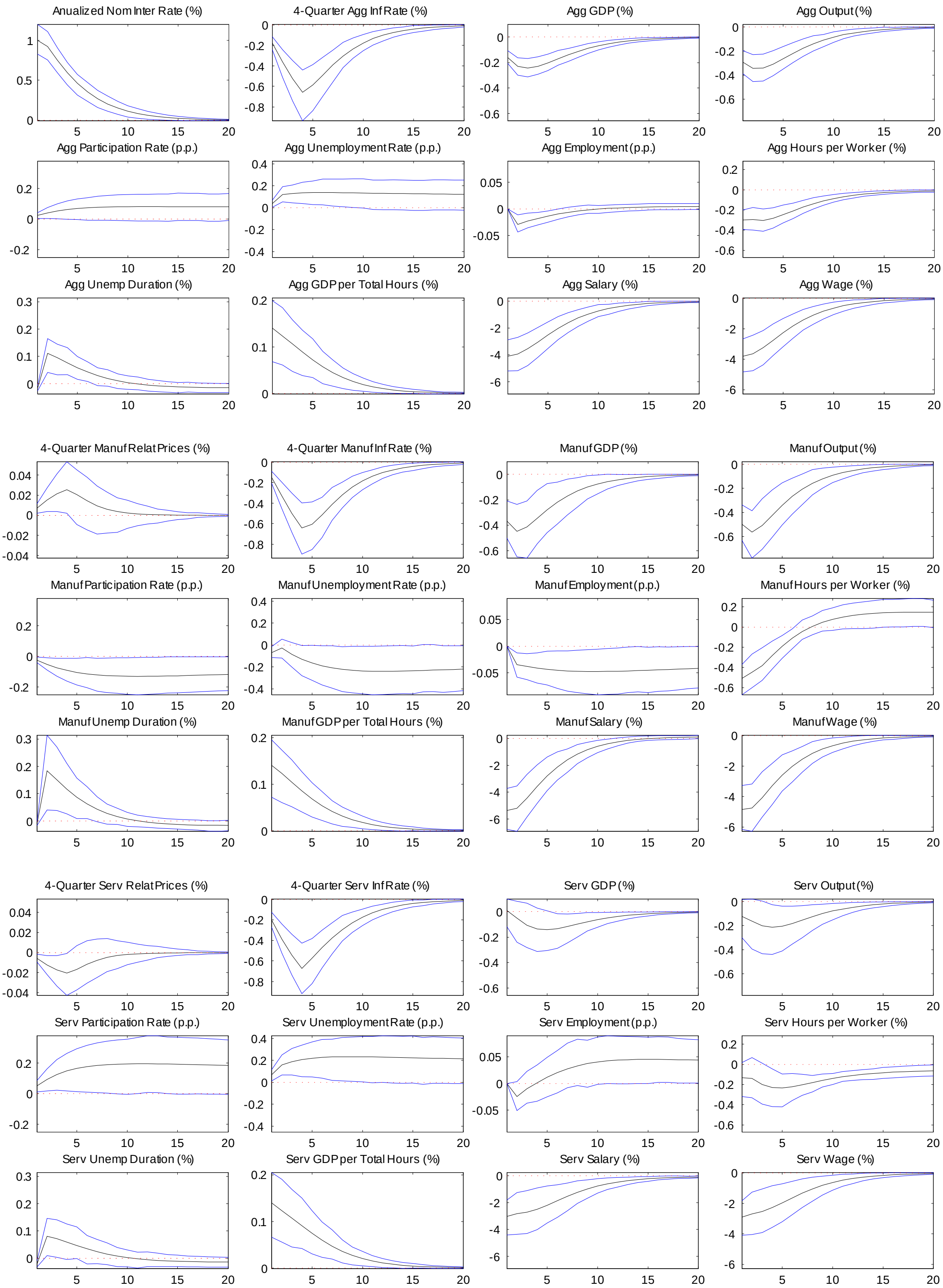

Figure 4: IRFs - 1 p.p. Shock to Annualized Nom Interest Rate - Heterog Model

In a nutshell, while manufacturing output falls on impact and take about one quarter to reach their maximum contraction, services output takes much longer to react, reaching its maximum contraction after four to five quarters. Moreover, contraction in the manufacturing sector is about three to four times as large as that of the services sector.

In the short run, the activity level in the services sector tends not to fall. This is caused mostly by the fact that most of the initial shock is absorbed by prices in this sector, as they are relatively more flexible than those of the manufacturing sector. In parallel, wages and salaries are falling and the availability of unemployed workers is increasing in this sector. Therefore, producing services goods becomes cheaper,[26] and so the sector is more resilient in the short run. In the medium run (5 to 20 quarters), the fall in aggregate demand finally induces the services sector to reduce production.

As for sectoral GDP, we must discount intermediate consumption from output in order to obtain value added. In this model, the only intermediate consumption is what firms consumes in order to post vacancies. After the monetary shock, both sectors reduce their expenses in vacancy postings. Therefore, the fall in sectoral GDP is not as strong as that of sectoral output.

Due to the sectoral heterogeneity, the services sector faces an increasing supply of unemployed workers. Since wages and salaries are falling everywhere, services firms find it optimal to reduce even further their expenses in vacancy postings when compared to what is done in the manufacturing sector. Therefore, the difference between the fall of sectoral GDP and output is more pronounced in the services sector.

In the manufacturing sector, the fall in employment is strong and long-lasting, i.e. it does not recover as fast as output and GDP. Therefore, the sector is characterized by a jobless recovery after the monetary shock. As for hours per worker, it initially fall during the first 5 quarters after the shock hits. However, since the recovering of manufacturing employment is very sluggish, workers that manage to keep their jobs tend to work more hours by 5 quarters after the shock.

In the services sector, the fall in employment is short-lived.[27] However, it is in the intensive margin of labor that adjustments are mostly done. Hours per worker fall and tend not to recover as fast as services output and GDP.

[26] Note that this results may not arise in models in which firms need to borrow to finance its inputs allocations.

[27] This result may be due to the fact that I have not modelled government policies adopted to offset the effects of the 2008-2009 crisis. It is a Brazilian anecdotal result that those policies benefitted the services labor market the most.

5.2 Impulse responses to sectoral technology shocks

Figures 5 (manufacturing) and 6 (services) show impulse responses to 1% sectoral specific technology shocks for the heterogeneous model.

Aggregate responses are similar no matter whether the technology shock is hitting the manufacturing or the services sector. However, amplitudes are larger when the technology shock hits the services sector. Note that, as expected, aggregate inflation falls while aggregate GDP and output rise.

Due to price rigidity, firms tend not to adjust prices as much and aggregate employment and hours per worker tend to fall. Even though that result may be at odds with the prediction of RBC models with search frictions, this happens whenever the price stickiness is sufficiently large.

Since aggregate prices do not adjust instantaneously, it is costly for firms (on average) to increase hours and post more vacancies to accommodate the increase in production. As a consequence, aggregate employment falls. Empirical evidence for this puzzling response has been found in many postwar countries, including the US, as the findings of Gali (1999, 2010) strongly suggest.

Therefore, the increase of unemployed workers tend to reduce real wages and salaries during the bargaining process. This reduction, in turn, induces a fraction of aggregate unemployed workers to leave the labor market, and so the aggregate participation rate falls. The net effect on the aggregate unemployment rate is a slightly increase in the short run, followed by a long-lasting fall.

After a technology shock in the manufacturing sector, manufacturing GDP and output rise, whereas those of the services sector fall. The effect is the opposite when a technology shock hits the services sector, instead. Moreover, the fall in real wages and salaries is stronger in the sector hit by the technology shock. And in the first quarters after the shock, sectors with stronger reductions in real wages and salaries experience a short-run increase on the mass of workers that leave the labor market.

As for the remaining quantities of the labor market, symmetry no longer holds dues to the huge difference in the reallocation rates. Manufacturing unemployed workers find it much easier to reallocate to the services sector than the other way around.

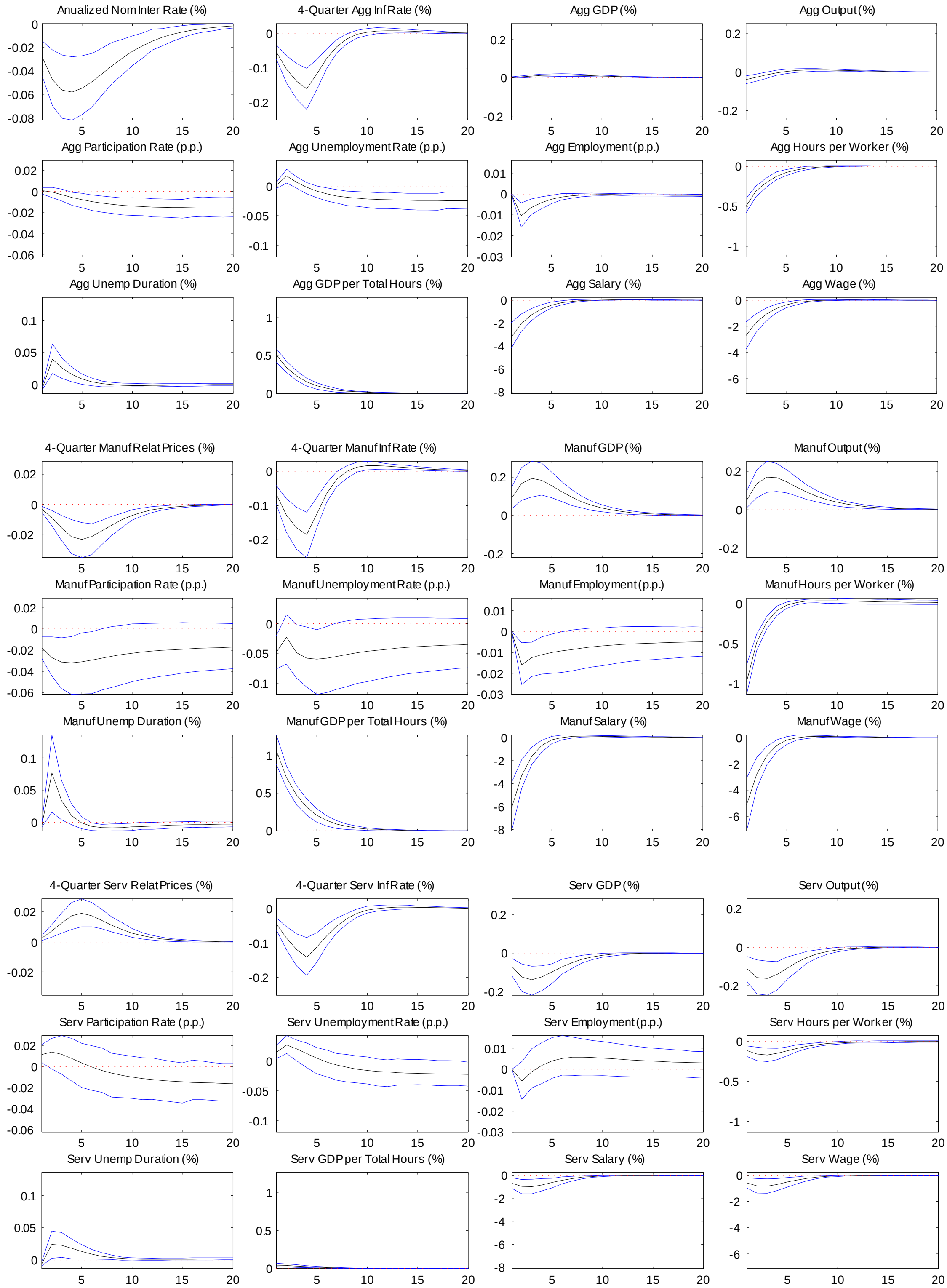

Figure 5: IRFs - 1% Shock to Manuf Technology - Heterog Model

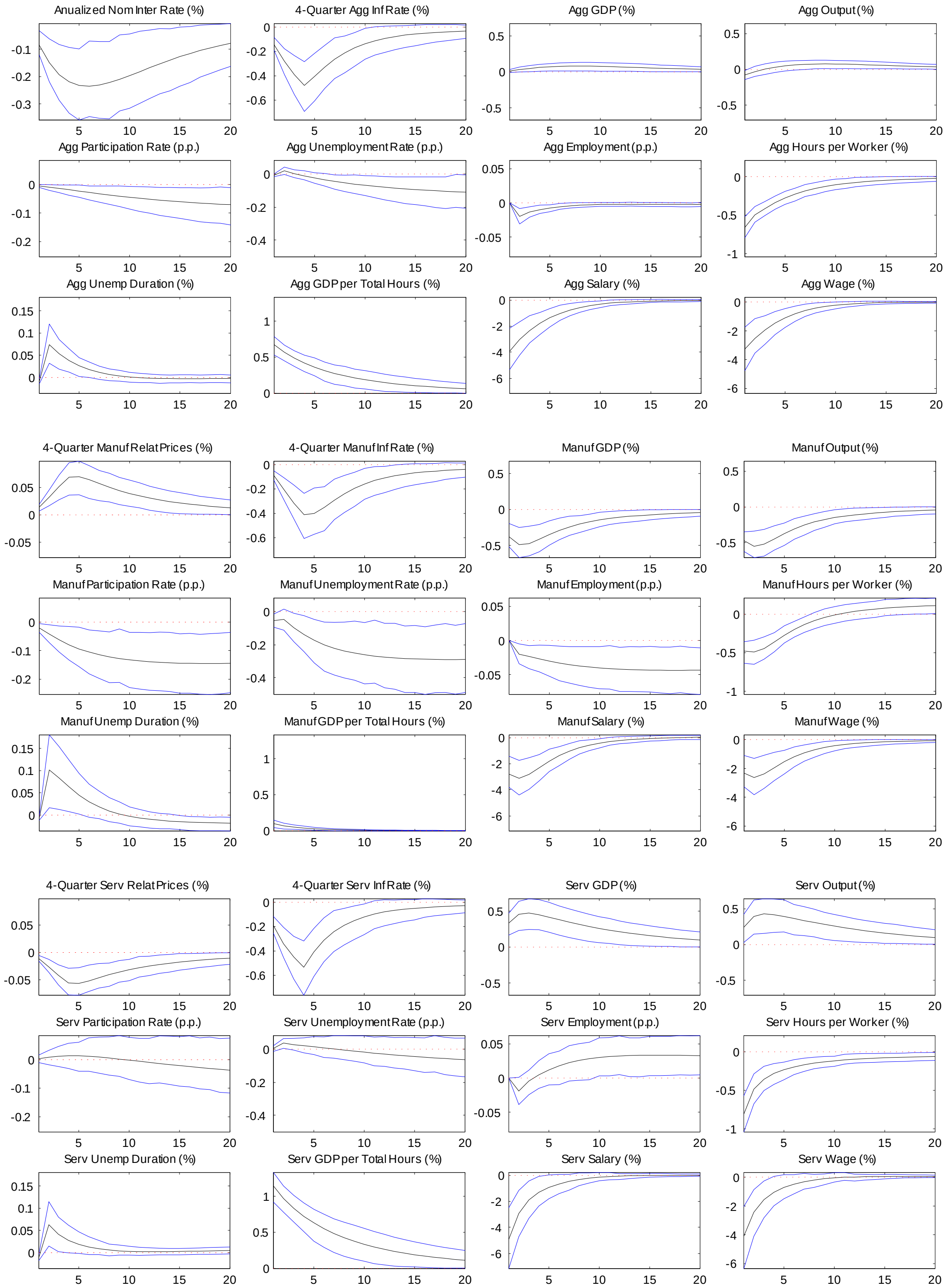

Figure 6: IRFs - 1% Shock to Serv Technology - Heterog Model

If the technology shock hits the manufacturing sector, the reduction of manufacturing employment, coupled with smaller reduction in wages in the services sector, leads to an increase in the reallocation of unemployment workers from the manufacturing to the services sector. That phenomena leads to a even larger reduction in the participation rate of the manufacturing sector and a increase in that of the services sector. After watching a strong increase in the sectoral labor supply, services firms find it easier to hire more workers without expending much on vacancy postings. As a result, medium-run employment tends to resiliently rise in the services sector, even though output and GDP have fallen.

If the technology shock hits the services sector, the strong time-cost imposed on unemployed works that want to reallocate to the manufacturing sector prevents a symmetric reallocation channel to hold. Therefore, the participation rate in the services sector tends not to respond. In addition, the strong fall in real wages and salaries induces services firms to post more vacancies and increase employment in the medium-run. Manufacturing firms, on the other hand, face strong output reduction and small reduction in real wages and salaries. This result induces firms not to hire as much and so manufacturing employment falls. This reduction is also long-lived. The fact that real wages and salaries are strongly falling in the services sector discourages unemployed workers from the manufacturing sector to reallocate. They tend to leave the labor market, instead.

5.3 Impulse responses to sectoral cost shocks

Figures 7 (manufacturing) and 8 (services) show impulse responses to 100% sectoral specific cost shocks over payroll for the heterogeneous model.

Similarly to the case of technology shocks, aggregate responses are similar no matter whether the cost shock is hitting the manufacturing or the services sector. And again, amplitudes are larger when the cost shock hits the services sector.

But now, responses to services technology shock are about one order of magnitude larger than those generated after the cost shock hits the manufacturing sector. By construction, those shocks are not internalized during the bargaining process or when individuals optimally decide either to leave the labor market or reallocate. In general equilibrium, though, those decisions are indirectly affected .

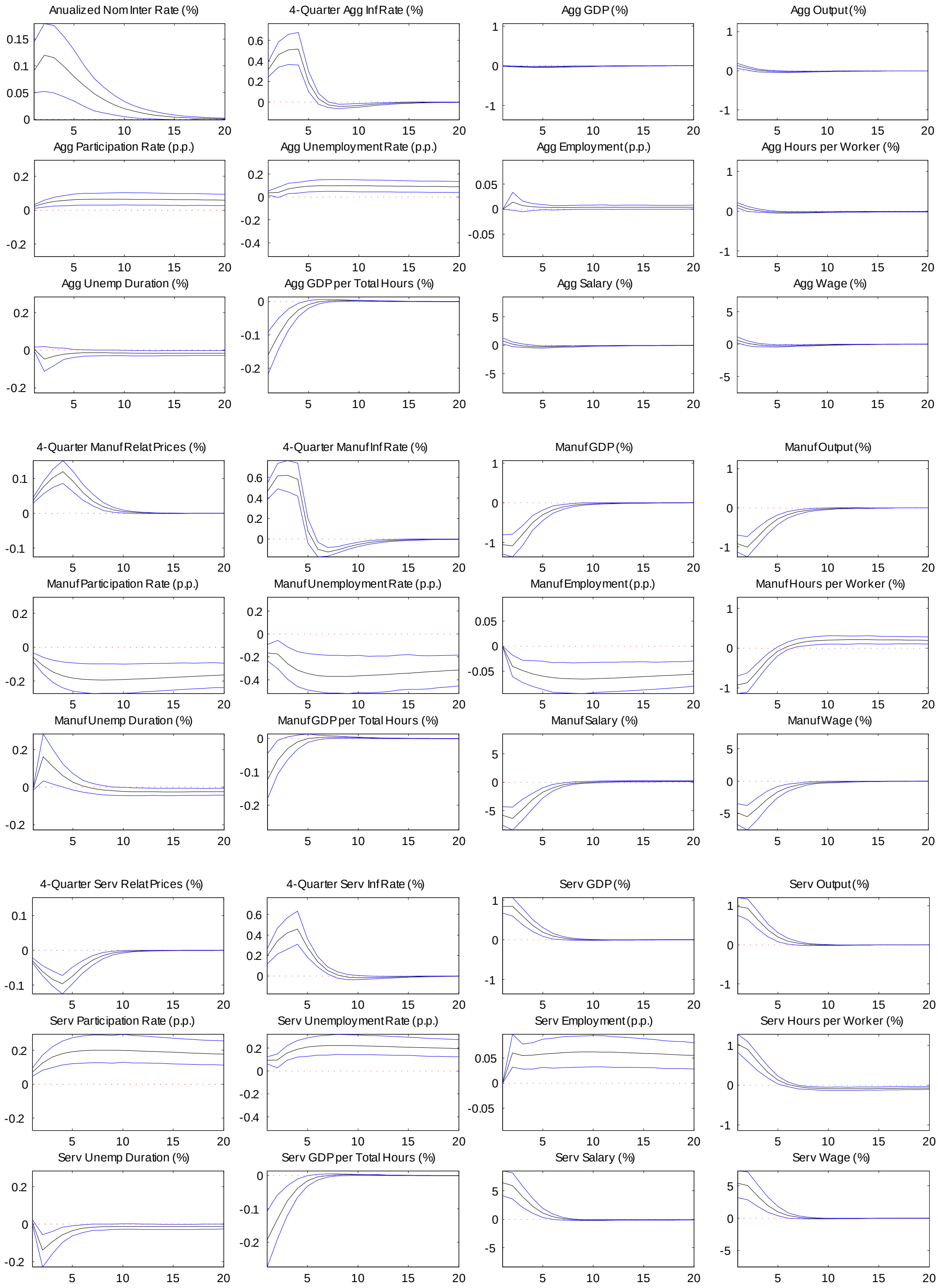

Figure 7: IRFs - 100% Shock to Manuf Marg Cost - Heterog Model

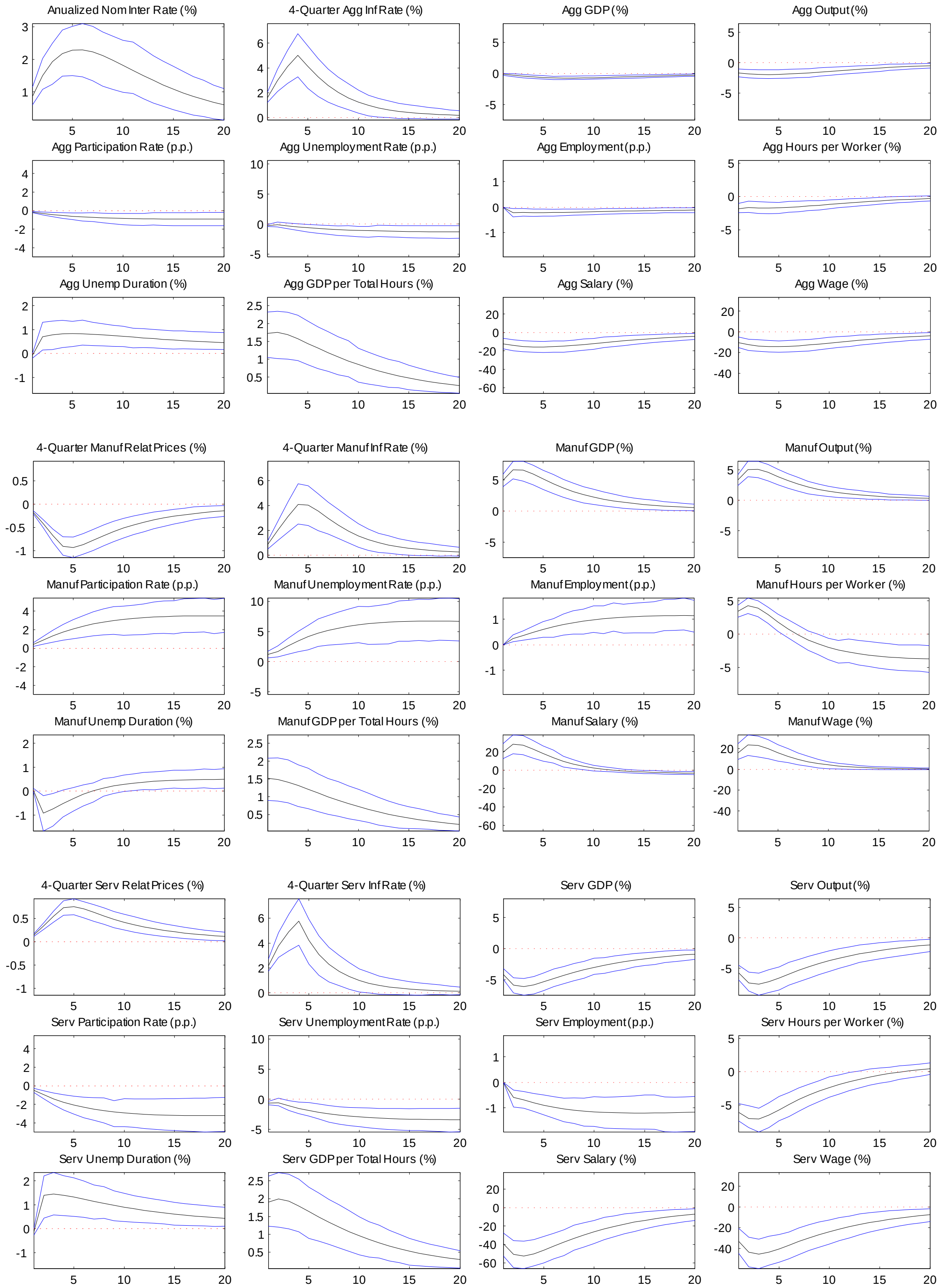

Figure 8: IRFs - 100% Shock to Serv Marg Cost - Heterog Model

In this context, the huge sectoral asymmetry created by the difficulty in reallocating from the services to the manufacturing sector does not seem to make a big difference after the economy is hit by each of the sectoral cost shocks. Qualitatively, the responses present an almost symmetric pattern, differing of course on amplitudes and dynamics.

After hit by a sector-$\mathfrak{c}$ specific cost shock, inflation strongly rises in all sectors, whereas aggregate GDP and output experience only a modest fall. The reason for that is is described as follows. As producing becomes more costly in sector $\mathfrak{c}$, firms in this sector find it optimal to reduce production, vacancy postings and new hirings, while decreasing the number of working hours per employee. As a consequence, bargained real wages and salaries fall.

Prices rise first in sector $\mathfrak{c}$, increasing its relative prices with respect to those from sector $\bar{\mathfrak{c}}$. Strategic complementarities induce prices to also rise in sector $\bar{\mathfrak{c}}$. Since firms in sector $\bar{\mathfrak{c}}$ are not hit by sector-$\mathfrak{c}$ specific cost shock, they optimally decide not to rise their prices as much as in sector $\mathfrak{c}$ in order to gain relative demand preferences from the household. Therefore, the demand for sector-$\mathfrak{c}$ specific goods rise, inducing firms in this sector to increase production, vacancy postings and new hirings. As a consequence, bargained real wages and salaries rise.

In the household's perspective, the fall in real salaries and wages in sector $\mathfrak{c}$ is almost completely offset by the rise of those quantities are in sector $\bar{\mathfrak{c}}$. Indeed, aggregate real salaries and wages are almost immune to sectoral cost shocks. Since the household does not get poorer, or richer, member out of the labor market have no incentive to return.

On the other hand, unemployed workers from sector $\mathfrak{c}$ realize that unemployed workers in sector $\bar{\mathfrak{c}}$ have better perspectives both in terms of unemployment duration as of real salaries and wages. Therefore, reallocation from sector $\mathfrak{c}$ to $\bar{\mathfrak{c}}$ increases. As a consequence, the participation rate at sector $\mathfrak{c}$ falls, while that of sector $\bar{\mathfrak{c}}$ rises. Those effects on the participation rates are so strong that surpass the chances on sectoral employment. And so the unemployment rate in sector $\mathfrak{c}$ falls, while that of sector $\bar{\mathfrak{c}}$ rises.

5.4 Impulse responses to bargaining power shocks

Figures 9 (manufacturing) and 10 (services) show impulse responses to 10% sectoral specific bargaining power shocks for the heterogeneous model.

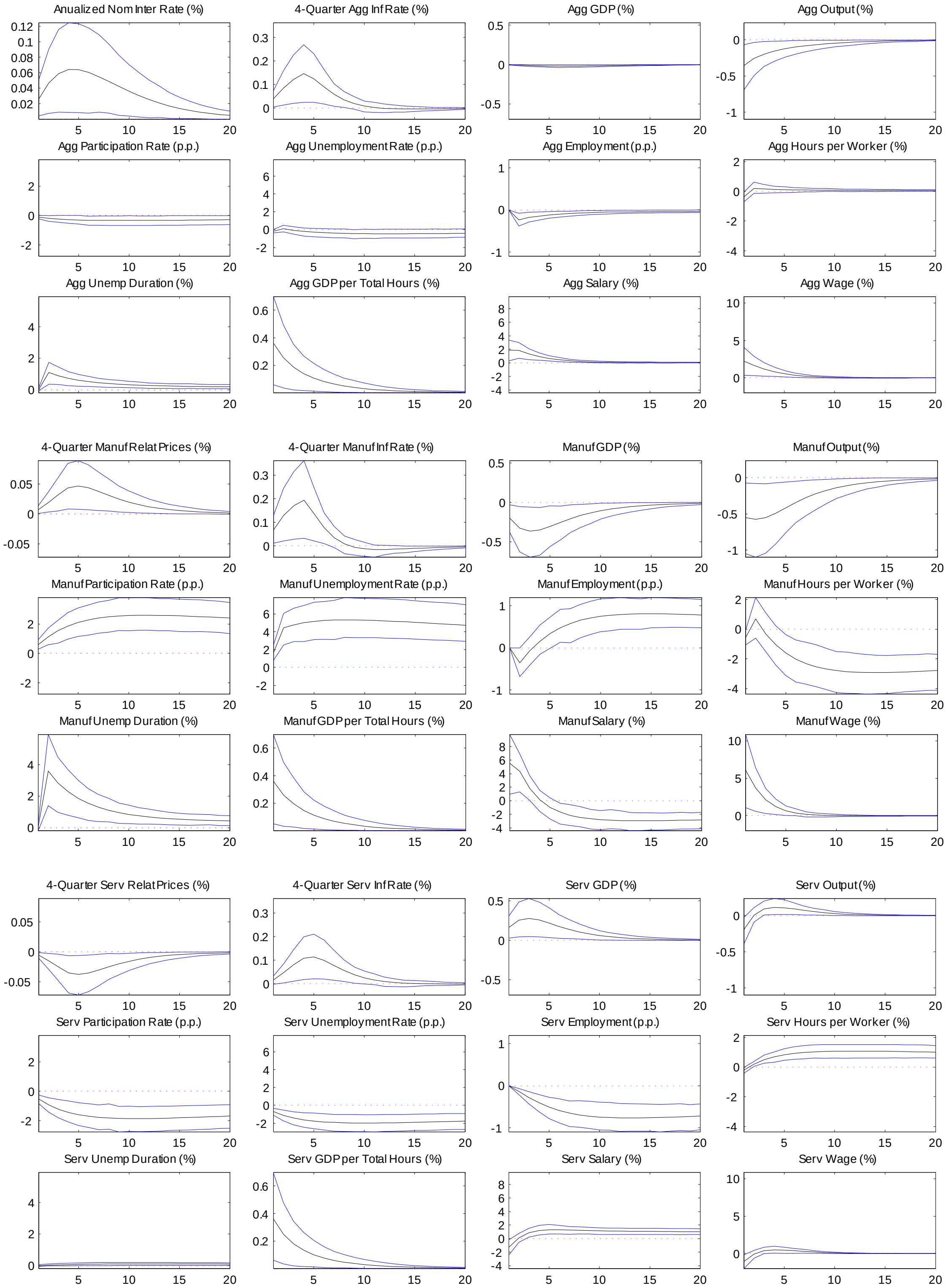

Figure 9: IRFs - 10% Shock to Manuf Bargain Power - Heterog Model

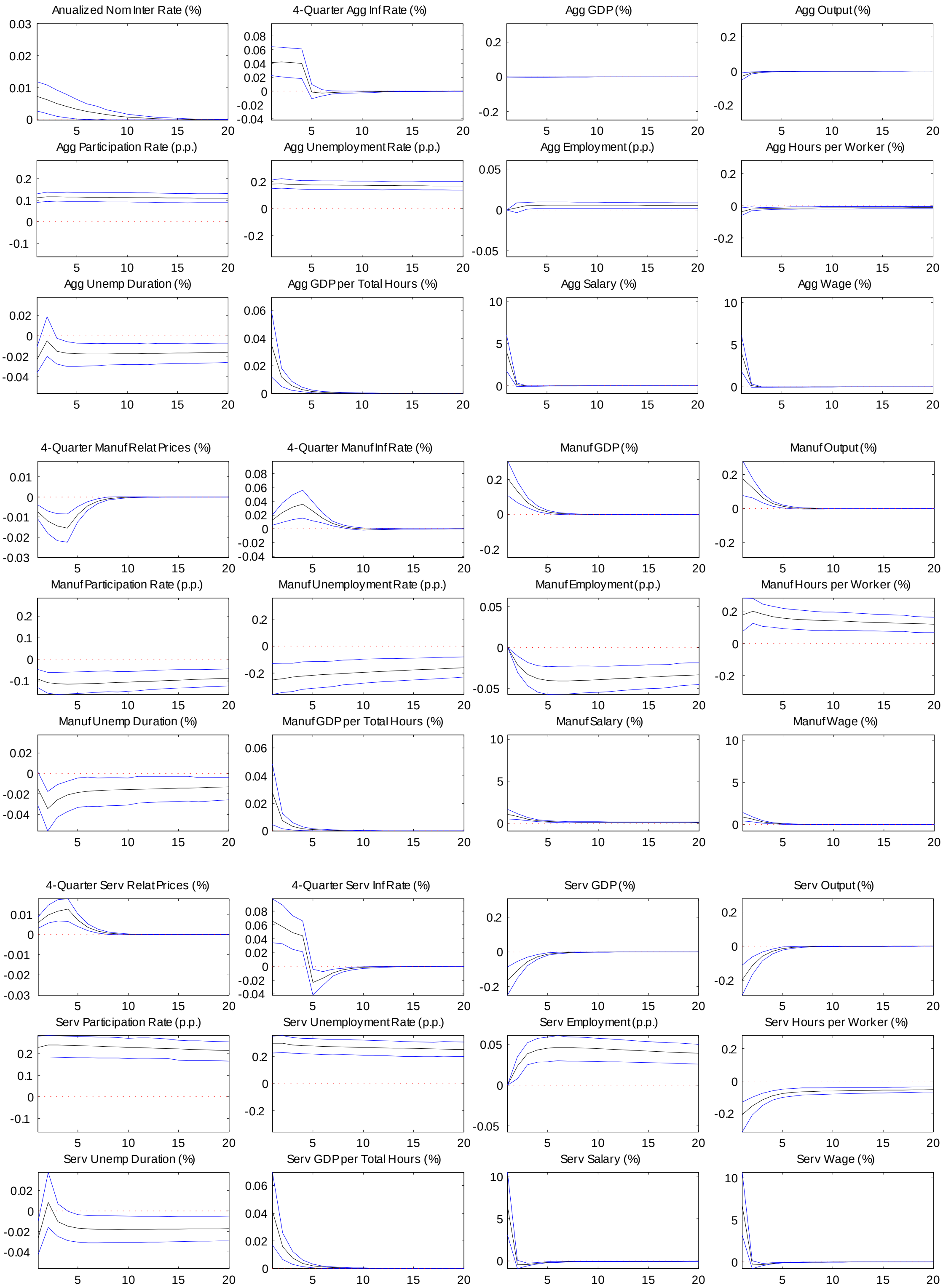

Figure 10: IRFs - 10% Shock to Serv Bargain Power - Heterog Model

In this case, a couple of responses have symmetrical counterparts when the shock hits the other sector instead. If sector $\mathfrak{c}$ is hit by the bargaining power shock, real wages and salaries rise because unions get a larger share of the total net surplus generated by each marginal worker hired by firm. Unemployed workers from sector $\bar{\mathfrak{c}}$ realize that unemployed workers in sector $\mathfrak{c}$ have better perspectives in terms of real salaries and wages and decide to reallocate from sector $\bar{\mathfrak{c}}$ to $\mathfrak{c}$.

As a consequence, the participation rate at sector $\bar{\mathfrak{c}}$ falls, while that of sector $\mathfrak{c}$ rises. The reduction in the size of the labor market in sector $\bar{\mathfrak{c}}$ also causes real salaries and wages to slightly rise in this sector. However, this is not enough to mitigate the flow towards sector $\mathfrak{c}$.

The increase in the stock of unemployed workers in sector $\mathfrak{c}$ naturally leads, by means of the matching function, to increases in new hirings and employment in this sector. Due to matching frictions, the increase in employment is not enough to absorb the rise in the stock of unemployed workers, which cause the unemployment rate to increase. The opposite phenomena happens in sector $\bar{\mathfrak{c}}$, where the participation rate, employment and the unemployment rate fall.

The increase in salaries and wages in sector $\mathfrak{c}$ causes the firms' marginal costs to rise, which in turn is passed into a price hikes. Due to strategic complementarities, prices in sector $\bar{\mathfrak{c}}$ also increase, but not as much. As a consequence, the relative price of sector $\mathfrak{c}$ increases and the household increase the relative demand for goods from sector $\bar{\mathfrak{c}}$. After watching their relative demand being reduced and having their profit flows decreased, due to a smaller share of the total surplus, firms from sector $\mathfrak{c}$ optimally decide to reduce production. Since employment is increasing, firms get to impose a working hours reduction in sector $\mathfrak{c}$ during the bargaining process.

5.5 Impulse responses to separation rate shocks

Figures 11 (manufacturing) and 12 (services) show impulse responses to 100% sectoral specific separation rates shocks for the heterogeneous model.

After a pure separation rate shock hits sector $\mathfrak{c}$, wage bargaining immediately converge to a fall in real salaries and wages and in hours per worker in this sector. Since labor costs are falling, production gets cheaper in sector and rises, which leads to a parallel increase in vacancy postings.

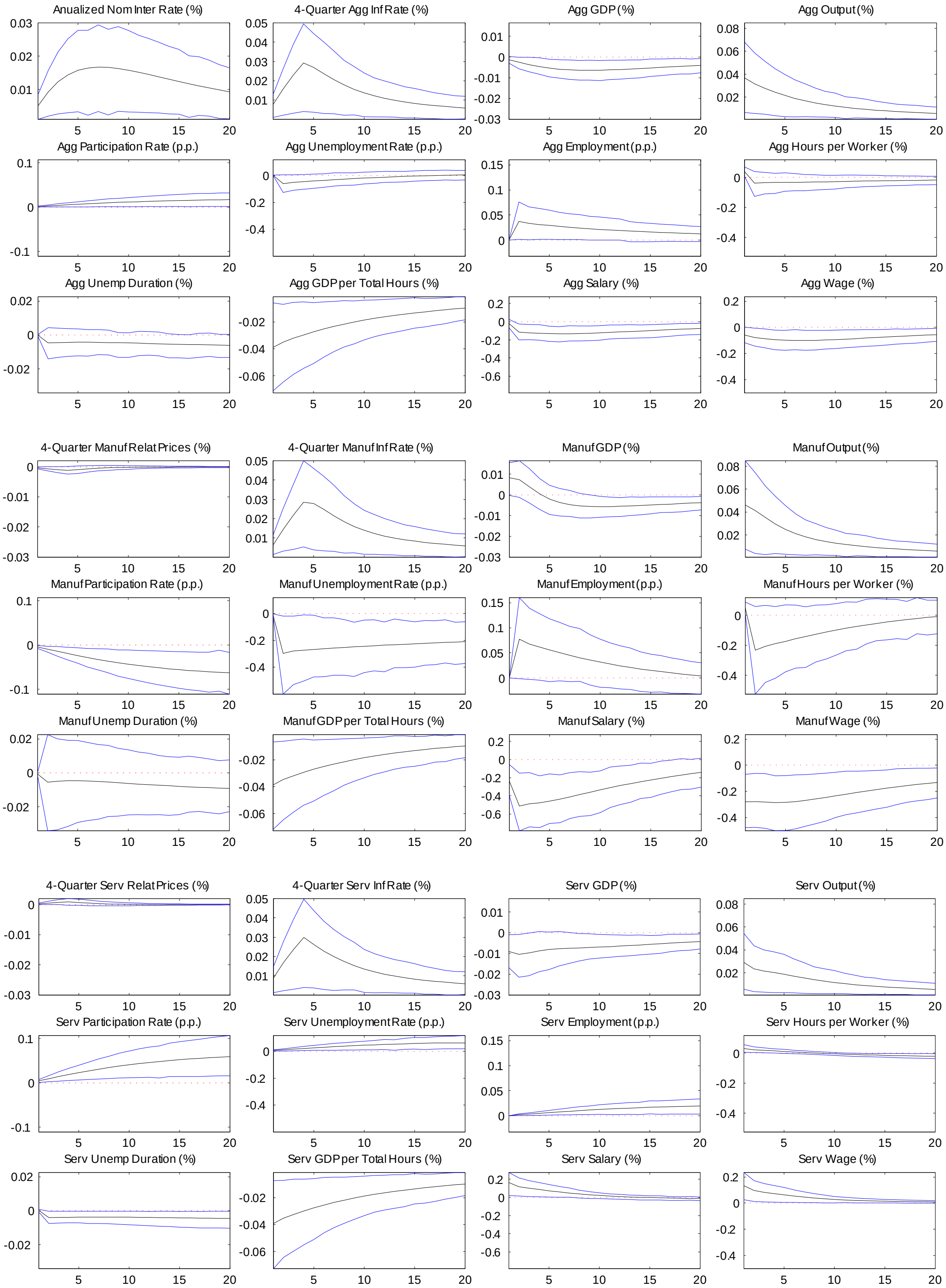

Figure 11: IRFs - 100% Shock (Doubling) to Manuf Separ Rate - Heterog Model

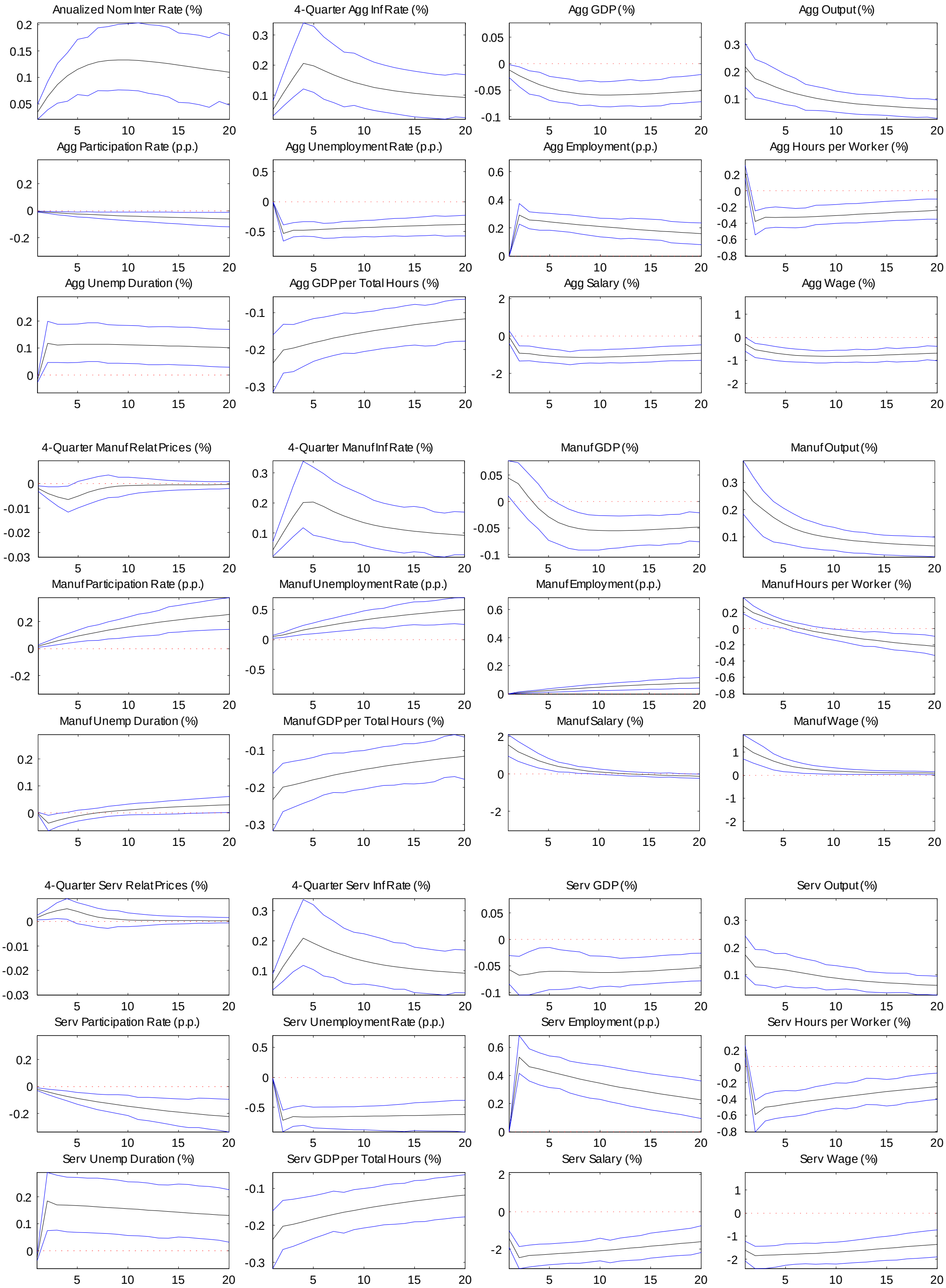

Figure 12: IRFs - 100% Shock (Doubling) to Serv Separ Rate - Heterog Model

The rise in intermediate consumption needed to increase vacancy postings in sector $\mathfrak{c}$ makes GDP to be smaller than output in this sector. In a nutshell, the rise in the separation rate shock hits sector $\mathfrak{c}$ without a reduction in production leads to a greater labor churning in this sector, followed by a slightly reduction in unemployment duration (even though not statistically different from zero).

Since firms need to buy differentiated goods from both sectors in order to post vacancies (e.g. advertising, etc.), output in sector $\bar{\mathfrak{c}}$ also rises as vacancy postings in sector $\mathfrak{c}$ increases. This leads to an additional increase in vacancy postings in sector $\bar{\mathfrak{c}}$ and so employment in this sector rises. The bargaining process, in turn, leads to larger real wages and salaries in sector sector $\bar{\mathfrak{c}}$. The rise in intermediate consumption needed to increase vacancy postings in sector $\bar{\mathfrak{c}}$ is large enough to offset the increase in output, and so GDP falls in sector $\bar{\mathfrak{c}}$.

Finally, due to the spike in the separation rate accompanied by a reduction in real salaries and wages, unemployed worker from sector $\mathfrak{c}$ realize that unemployed workers in sector $\bar{\mathfrak{c}}$ are better off in terms of real salaries and wages and decide to reallocate from sector $\mathfrak{c}$ to $\bar{\mathfrak{c}}$. This flow leads to a fall in the participation rate of sector $\mathfrak{c}$ and a rise in that of sector $\bar{\mathfrak{c}}$. When accounting for employment changes, the net effect is a fall in the unemployment rate of sector $\mathfrak{c}$ and a rise in that of sector $\bar{\mathfrak{c}}$.

5.6 Impulse responses to aggregate utility shock

Figure 13 shows impulse responses to a 10% aggregate utility shock for the heterogeneous model.

As the aggregate shock hits, aggregate consumption and output rise. In order to account for that, hours rise instantaneously and are latter followed by employment. The bargaining process leads to larger real salaries and wages, which lead to increased production costs and prices.

Since prices are not as rigid in the services sector as it is in the manufacturing sector, prices readjust faster in the services sector, which leads to a relative demand preference over goods produced in the manufacturing sector.

After prices has increased, intermediated consumption needed to post vacancies decreases, which leads to a reduction in unemployment. Hours per worker is used instead to compensate the fall in employment.

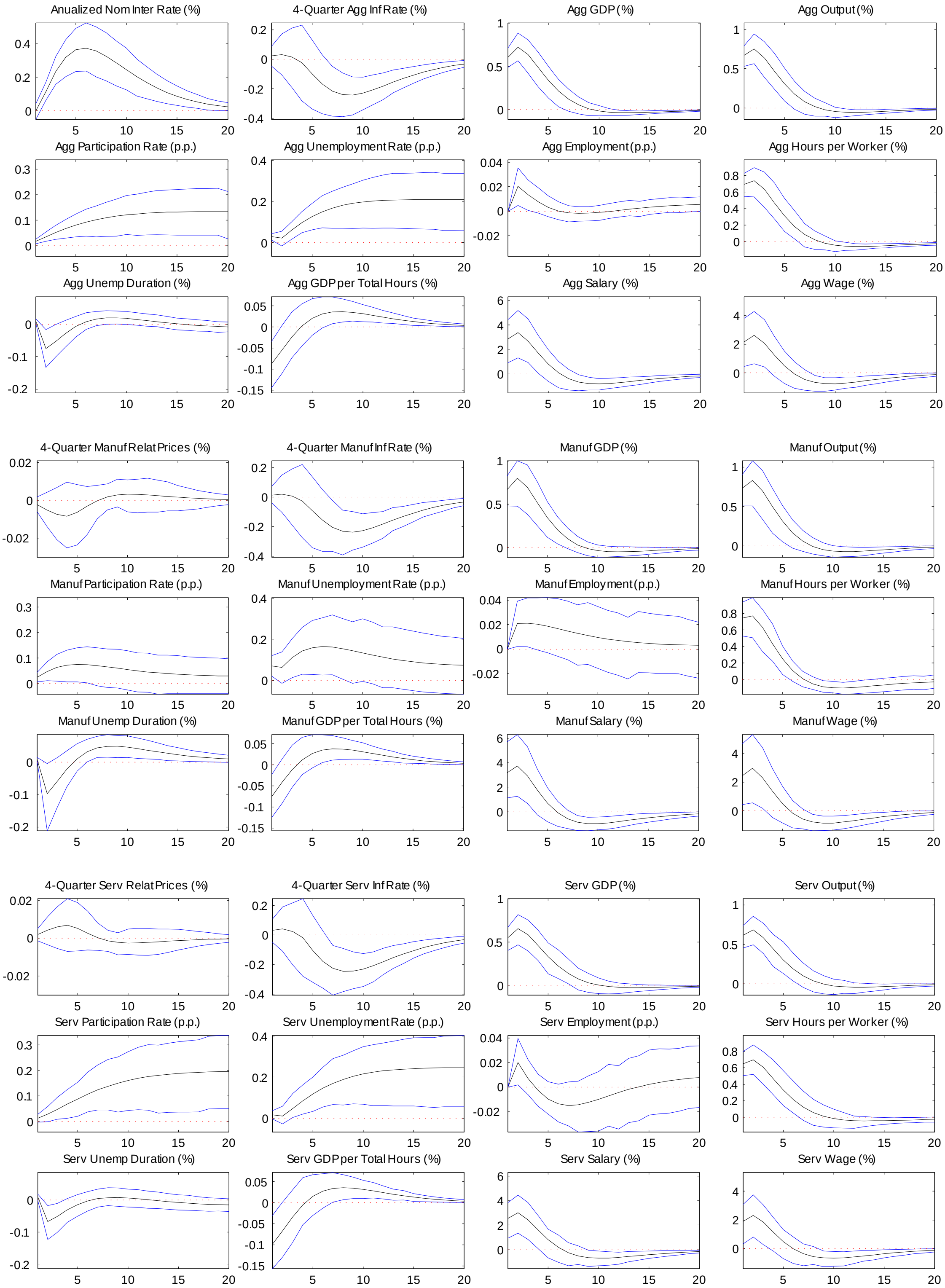

Figure 13: IRFs - 10% Demand Shock (Consumption Utility) - Heterog Model

Responding to the rise in the activity level and expected inflation, monetary policy rises the nominal interest rate. Recall that the nominal interest rate has a direct impact in the aggregate salary curves (57), by means of the discounted expectation term $E_t Q^{\pi}_{t+1} \varpi^{\ell}_{\mathfrak{c},t+1}$, where $E_t Q_{t+1} = 1/I_t$.

Coupled with the aggregate consumption channel, a tighten monetary policy has a strong effect on real salaries, which fall below theirs steady state levels after a while. That ends up leading to similar falls in the sectoral inflation rates.

In the labor market, the fact that vacancy openings are not heating up prevents labor reallocation to be significant. On the other, the strong rise on real salaries, wages and hours ends up encouraging workers to return to the labor market, and so participation rates increase. That phenomena offsets the rise on employment, and hence unemployment rates increase in both sectors.

5.7 Impulse responses to working-age population shock

Figure 13 shows impulse responses to a negative 1% working-age population shock for the heterogeneous model. Note that, qualitatively, both sectors respond the same way.

The response in goods market quantities, i.e. inflation and output, is negligible. on the other hand, as expected, labor markets quantities respond more intensively. We also observe that some manufacturing variables repond more agressively than those from the services sector. As the working-age population falls, the sectoral masses of unemployed workers fall by the same ratio. This is due to the fact that the shock is defined as evenly affecting both quantities. As a direct consequence, by of sectoral matching functions, the amount of unemployed workers being matched in each sector decreases.

The fall in labor supply and employment induces, during bargaining, a rise in real salaries. Hours per worker also increases in both sectors. However, the magnitudes obtained in the manufacturing sector is about 50% larger than those of the services sector. The effect on real wages is negligible.

As the household members gets richer, as real salaries increases and the size of the household (by which divide consumption) decreases, some unemployed workers leave the labor market. As a consequence, sectoral participation rates decrease. As those members leave the labor market and stop earning unemployment compensations, the household gets poorer. The latter effect offsets gains obtained by larger real salaries, and so the net effect on consumption (and output) is negligible.

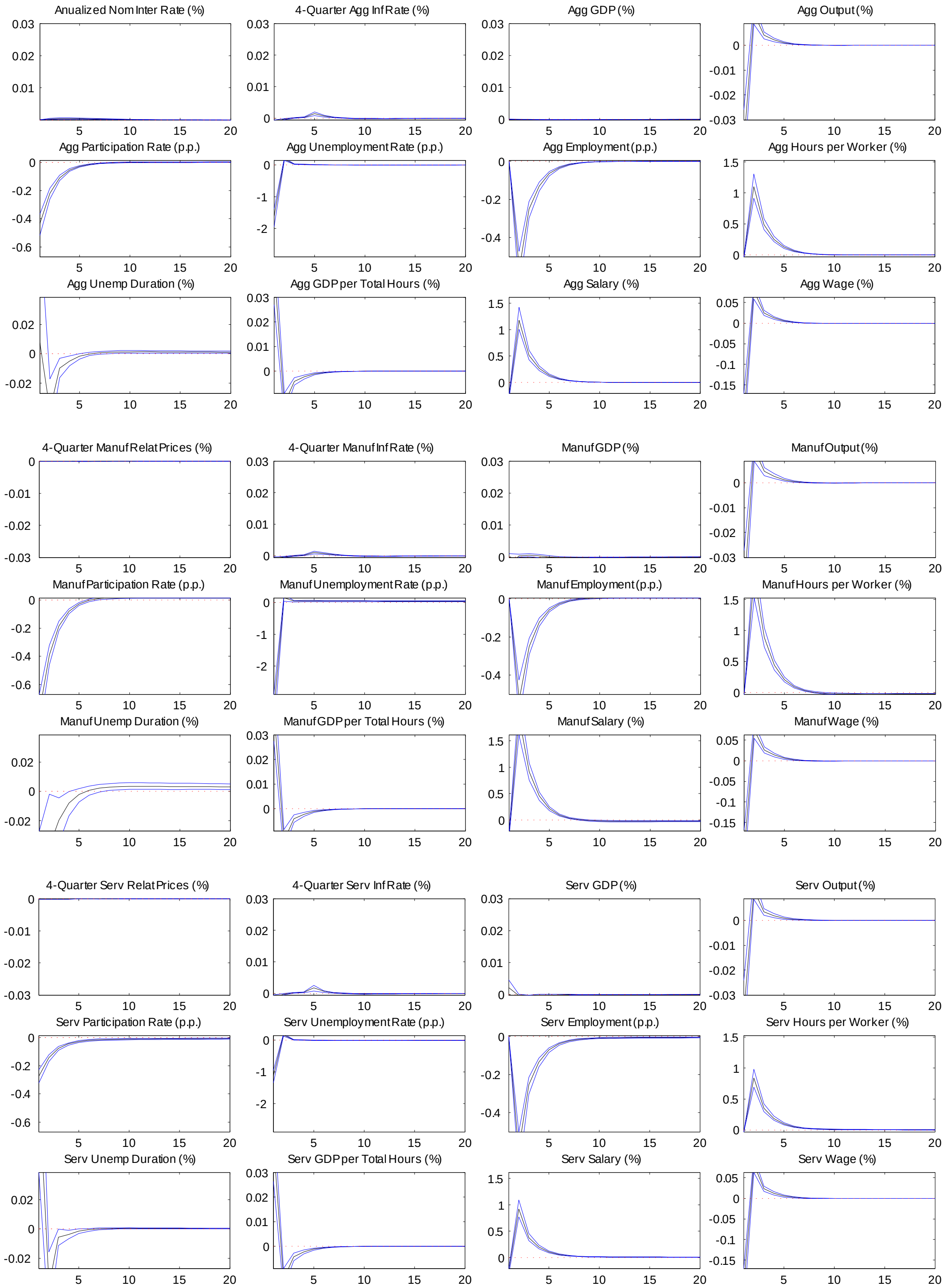

Figure 14: IRFs - 1% Working-Age Population Shock - Heterog Model

6 Conclusions

This paper presents and estimate a novel way to model the labor and goods markets with heterogeneous sectors in Brazil, endogenizing the optimal decision to reallocate to another sector or leave the labor market.

In the labor makert, the major empirical findings are: (*i*) workers from the manufacturing sector who are out of the labor market take longer to return (about 6 months) than workers from the service sector (about 3 months); (*ii*) workers from the manufacturing sector reallocate much faster to the service sector (about 7 months) than workers from the services sector (about 10 years) - in this regard, the information content in the sample strongly suggest that reallocation from services to manufacturing were really rare; (*iii*) the combination of greater labor market tightness and smaller search frictions in the services sector is the major explanation why unemployed workers find it easier to get a job in the service sector than those of the manufacturing one; (*iv*) on the other hand, workers' bargaining power in the manufacturing sector is much larger than that of the service sector. As a result, the average salary in the service sector are more correlated with the unemployment compensation, which is also very correlated with the minimum wage in Brazil. The results also suggest that salary bargaining is much more efficient in the manufacturing sector.

The data also support the evidence that there is no labor supply puzzle in the Brazilian labor market, i.e. I find that labor is just weakly elastic to salaries in Brazil.

The results also suggest that prices are much stickier and much more persistent in the manufacturing sector than in the services sector. Since prices are more flexible in the services sector, they adjust faster to shocks. However, strategic complementarities induce sectoral inflation rates not to detach much from each other, so that sectors do not lose long-run demand attractiveness.

In case of aggregate shocks, the relative demand for both sectors will be different due to the fact of prices are more flexible in the services sector. This effect is combined with the strong heterogeneity characterizing both sectors to produce different responses in the goods and labor markets. Finally, after aggregate shocks, sectoral GDP and output responses in the services sector are weaker and take longer to start responding than in the manufacturing sector. This is dues to the fact that prices adjust faster in the services sector.

Responses of labor market quantities have two important features. The first one is that the dynamics of labor market quantities, both in the aggregate as in sectoral measurements, are much more persistent than those of the goods sector. The second one is that aggregate responses of labor market variables qualitatively follow those in the services sector. This is due to the fact that about 75% of employed workers are in this sector, and this share is large enough to dominate the aggregate dynamics.

As for the dynamics after a monetary policy shock, the results imply that it is the manufacturing sector which suffers more. The fall in employment, hours, real salaries, GDP and output is much stronger in the manufacturing than in the services sector. The model is also able to capture what is know as labor hoarding, for hours tend to fall much faster than employment after the shock.

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A Some derivations

Proposition 1 *For an unemployed worker at the beginning of period t in sector $\mathfrak{c}$, the expected spell $\mathsf{T}^u_{\mathfrak{c},t}$ until being matched into a job (in any sector) evolves according to:*

$$\begin{aligned} \mathsf{T}^u_{\mathfrak{c},t} = {} & \mathsf{p}_{\mathfrak{c},t}\bar{\mathsf{t}} \\ + {} & (1-\mathsf{p}_{\mathfrak{c},t})\left[1+\left(1-\mathsf{p}^{\mathfrak{o}}_{\mathfrak{c},t}\right)E_t\mathsf{T}^u_{\mathfrak{c},t+1}\right] \\ + {} & (1-\mathsf{p}_{\mathfrak{c},t})\left[\mathsf{p}^{\mathfrak{o}}_{\mathfrak{c},t}\left(\delta^{\bar{\mathfrak{c}}}_{\mathfrak{s}}E_t\mathsf{T}^u_{\bar{\mathfrak{c}},t+1}+\delta^{\mathfrak{c}}_{\mathfrak{c}}E_t\mathsf{T}^u_{\mathfrak{c},t+1}+\frac{1-\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}-\delta^{\mathfrak{c}}_{\mathfrak{c}}}{\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}+\delta^{\mathfrak{c}}_{\mathfrak{c}}}\right)\right] \end{aligned}$$

where $\bar{\mathsf{t}}\in(0,1)$ is the average time within a period in which a recently laid-off worker remains unemployed when he is matched to a new job by the end of the same period.

Proof. Note that the expected time spell in a specialization school of sector $\mathfrak{p}\neq\bar{\mathfrak{p}}$ is

$$\mathsf{T}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t}=\left(1-\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}-\delta^{\mathfrak{c}}_{\mathfrak{c}}\right)\left(1+E_t\mathsf{T}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t+1}\right)=\frac{1-\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}-\delta^{\mathfrak{c}}_{\mathfrak{c}}}{\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}+\delta^{\mathfrak{c}}_{\mathfrak{c}}}$$

, while $\mathsf{T}^u_{\mathfrak{c},t}$ is computed as follows:

$$\mathsf{T}^u_{\mathfrak{c},t}=\mathsf{p}_{\mathfrak{c},t}\bar{\mathsf{t}}+(1-\mathsf{p}_{\mathfrak{c},t})\Big[1+\left(1-\mathsf{p}^{\mathfrak{o}}_{\mathfrak{c},t}\right)E_t\mathsf{T}^u_{\mathfrak{c},t+1}+\mathsf{p}^{\mathfrak{o}}_{\mathfrak{c},t}\Big(\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}E_t\mathsf{T}^u_{\bar{\mathfrak{c}},t+1}+\delta^{\mathfrak{c}}_{\mathfrak{c}}E_t\mathsf{T}^u_{\mathfrak{c},t+1}+\left(1-\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}-\delta^{\mathfrak{c}}_{\mathfrak{c}}\right)\left(1+E_t\mathsf{T}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t+2}\right)\Big)\Big]$$

■

Proposition 2 *The representative family's problem, including extra restrictions from labor flows and using the notation of net unemployment compensations $\varpi^u_{\mathfrak{c},t}$, can be rewritten as follows:*

$$\begin{aligned} \mathcal{U}_t = {} & \max\ \ell^{\mathfrak{p}}_t u_t - \textstyle\sum_{\mathfrak{c}}\int_{\mathfrak{c}}\upsilon_t(\mathsf{n}_t(z_{\mathfrak{c}})h_t(z_{\mathfrak{c}}))dz_{\mathfrak{c}}+\beta E_t\mathcal{U}_{t+1} \\ & +\lambda_t\left[A_t+\mathcal{B}_tI_{t-1}+\Xi_t+P_td_t-P_t\textstyle\sum_{\mathfrak{c}}\frac{\varsigma_{\mathsf{m}\mathfrak{c}}}{2}\left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}}{\tilde{E}_t\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}}-1\right)^2-\ell^{\mathfrak{p}}_tP_tC_t-E_tQ_{t+1}A_{t+1}-\mathcal{B}_{t+1}\right] \\ & +\lambda_t\left\{\textstyle\sum_{\mathfrak{c}}\int_{\mathfrak{c}}\mathsf{n}_t(z_{\mathfrak{c}})\mathcal{W}_t(z_{\mathfrak{c}})dz_{\mathfrak{c}}+\sum_{\mathfrak{c}}P_t\mathfrak{w}_{\mathfrak{c}}\varpi^u_{\mathfrak{c},t}\left(\ell_{\mathfrak{c},t}-\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathsf{n}_t(z_{\mathfrak{c}})dz_{\mathfrak{c}}\right)\right\} \\ & +E_t\textstyle\sum_{\mathfrak{c}}\int_{\mathfrak{c}}\lambda^{cn}_t(z_{\mathfrak{c}})\left[(1-\rho_{\mathfrak{d}})\left(1-\rho_{\mathfrak{c},t}\right)\mathsf{n}_t(z_{\mathfrak{c}})+\mathfrak{w}_{\mathfrak{c}}\eta_{\mathfrak{c},t}\mathsf{v}^{1-a_{\mathfrak{c}}}_{\mathfrak{c},t+1}\mathsf{u}^{a_{\mathfrak{c}}}_{\mathfrak{c},t+1}\mathsf{s}_{t+1}(z_{\mathfrak{c}})-\mathsf{n}_{t+1}(z_{\mathfrak{c}})\right]dz_{\mathfrak{c}} \\ & +\textstyle\sum_{\mathfrak{c}}\lambda^{\ell}_{\mathfrak{c},t}\left[\mathsf{u}_{\mathfrak{c},t}-\eta_{\mathfrak{c},t}\mathsf{v}^{1-a_{\mathfrak{c}}}_{\mathfrak{c},t}\mathsf{u}^{a_{\mathfrak{c}}}_{\mathfrak{c},t}-\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}+\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathsf{n}_t(z_{\mathfrak{c}})dz_{\mathfrak{c}}-\ell_{\mathfrak{c},t}\right] \\ & +\textstyle\sum_{\mathfrak{c}}\lambda^{\mathfrak{o}}_{\mathfrak{c},t}\left[(1-\rho_{\mathfrak{d}})\left(1-\delta^{\mathfrak{c}}_{\mathfrak{c}}-\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\right)\ell^{\mathfrak{o}}_{\mathfrak{c},t-1}+\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}+\mathsf{m}_{\ell,t}-\ell^{\mathfrak{o}}_{\mathfrak{c},t}\right] \\ & +\textstyle\sum_{\mathfrak{c}}\lambda^{\mathsf{u}}_{\mathfrak{c},t}\left[(1-\rho_{\mathfrak{d}})\left(\ell_{\mathfrak{c},t}-\frac{(1-\rho_{\mathfrak{c},t})}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathsf{n}_t(z_{\mathfrak{c}})dz_{\mathfrak{c}}+\delta^{\mathfrak{c}}_{\mathfrak{c}}\ell^{\mathfrak{o}}_{\mathfrak{c},t}+\frac{\mathfrak{w}_{\bar{\mathfrak{c}}}}{\mathfrak{w}_{\mathfrak{c}}}\delta^{\mathfrak{c}}_{\bar{\mathfrak{c}}}\ell^{\mathfrak{o}}_{\bar{\mathfrak{c}},t}\right)-\mathsf{u}_{\mathfrak{c},t+1}\right] \end{aligned}$$

Proof. The additional restrictions are the ones described by equations (8), (9), (10), (11), (12), (5):

$$\begin{aligned} \mathsf{n}_{\mathfrak{c},t}\equiv\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathsf{n}_t\left(z_{\mathfrak{c}}\right)dz_{\mathfrak{c}} \quad &;\quad \mathsf{m}_{\mathfrak{c},t}\equiv\eta_{\mathfrak{c},t}\mathsf{v}^{1-a_{\mathfrak{c}}}_{\mathfrak{c},t}\mathsf{u}^{a_{\mathfrak{c}}}_{\mathfrak{c},t} \\ \mathsf{u}^e_{\mathfrak{c},t}=\mathsf{u}_{\mathfrak{c},t}-\mathsf{m}_{\mathfrak{c},t}-\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t} \quad &;\quad \ell_{\mathfrak{c},t}=\mathsf{u}^e_{\mathfrak{c},t}+\mathsf{n}_{\mathfrak{c},t} \\ \ell_{\mathfrak{c},t}=\mathsf{u}_{\mathfrak{c},t}-\eta_{\mathfrak{c},t}\mathsf{v}^{1-a_{\mathfrak{c}}}_{\mathfrak{c},t}\mathsf{u}^{a_{\mathfrak{c}}}_{\mathfrak{c},t}-&\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}+\frac{1}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathsf{n}_t\left(z_{\mathfrak{c}}\right)dz_{\mathfrak{c}} \end{aligned}$$

$$
\begin{gathered}
\mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t} = \delta^{\mathfrak{c}}_{\mathfrak{c}}\left(1-\rho_{\mathfrak{d}}\right)\ell^{\mathfrak{o}}_{\mathfrak{c},t-1} \quad ; \ \mathsf{m}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t} = \delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\left(1-\rho_{\mathfrak{d}}\right)\ell^{\mathfrak{o}}_{\mathfrak{c},t-1} \\
\ell^{\mathfrak{o}}_{\mathfrak{c},t} = \left(1-\rho_{\mathfrak{d}}\right)\ell^{\mathfrak{o}}_{\mathfrak{c},t-1} - \mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t} - \mathsf{m}^{\bar{\mathfrak{c}}}_{\mathfrak{c},t} + \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t} + \mathsf{m}_{\ell,t} \\
\ell^{\mathfrak{o}}_{\mathfrak{c},t} = \left(1-\rho_{\mathfrak{d}}\right)\left(1-\delta^{\mathfrak{c}}_{\mathfrak{c}}-\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\right)\ell^{\mathfrak{o}}_{\mathfrak{c},t-1} + \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t} + \mathsf{m}_{\ell,t}
\end{gathered}
$$

$$
\begin{gathered}
\mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t+1} = \delta^{\mathfrak{c}}_{\mathfrak{c}}\left(1-\rho_{\mathfrak{d}}\right)\ell^{\mathfrak{o}}_{\mathfrak{c},t} \quad ; \ \mathsf{m}^{\mathfrak{c}}_{\bar{\mathfrak{c}},t+1} = \delta^{\mathfrak{c}}_{\bar{\mathfrak{c}}}\left(1-\rho_{\mathfrak{d}}\right)\ell^{\mathfrak{o}}_{\bar{\mathfrak{c}},t} \\
\mathsf{n}_{\mathfrak{c},t} \equiv \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathsf{n}_t\left(z_{\mathfrak{c}}\right)dz_{\mathfrak{c}} \qquad ; \ \mathsf{m}^{\mathfrak{c}}_{\mathfrak{o},t+1} \equiv \mathsf{m}^{\mathfrak{c}}_{\mathfrak{c},t+1} + \tfrac{\mathfrak{w}_{\bar{\mathfrak{c}}}}{\mathfrak{w}_{\mathfrak{c}}}\mathsf{m}^{\mathfrak{c}}_{\bar{\mathfrak{c}},t+1} \\
\mathsf{u}^{e}_{\mathfrak{c},t} = \ell_{\mathfrak{c},t} - \mathsf{n}_{\mathfrak{c},t} \quad ; \ \mathsf{u}_{\mathfrak{c},t+1} = \left(1-\rho_{\mathfrak{d}}\right)\left(\mathsf{u}^{e}_{\mathfrak{c},t} + \rho_{\mathfrak{c},t}\mathsf{n}_{\mathfrak{c},t}\right) + \mathsf{m}^{\mathfrak{c}}_{\mathfrak{o},t+1} \\
\mathsf{u}_{\mathfrak{c},t+1} = \left(1-\rho_{\mathfrak{d}}\right)\left(\ell_{\mathfrak{c},t} - \tfrac{\left(1-\rho_{\mathfrak{c},t}\right)}{\mathfrak{w}_{\mathfrak{c}}}\int_{\mathfrak{c}}\mathsf{n}_t\left(z_{\mathfrak{c}}\right)dz_{\mathfrak{c}} + \delta^{\mathfrak{c}}_{\mathfrak{c}}\ell^{\mathfrak{o}}_{\mathfrak{c},t} + \tfrac{\mathfrak{w}_{\bar{\mathfrak{c}}}}{\mathfrak{w}_{\mathfrak{c}}}\delta^{\mathfrak{c}}_{\bar{\mathfrak{c}}}\ell^{\mathfrak{o}}_{\bar{\mathfrak{c}},t}\right)
\end{gathered}
$$

$$
\begin{gathered}
\ell^{\mathfrak{o}}_t \equiv \mathfrak{w}_{\mathfrak{m}}\ell^{\mathfrak{o}}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}}\ell^{\mathfrak{o}}_{\mathfrak{s},t} \quad ; \ \ell_t = \mathfrak{w}_{\mathfrak{m}}\ell_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}}\ell_{\mathfrak{s},t} \\
\ell^{\mathfrak{p}}_t = \left(\ell_t + \ell^{\mathfrak{o}}_t\right) \quad ; \ \ell^{\mathfrak{p}}_t = \mathfrak{w}_{\mathfrak{m}}\ell_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}}\ell_{\mathfrak{s},t} + \mathfrak{w}_{\mathfrak{m}}\ell^{\mathfrak{o}}_{\mathfrak{m},t} + \mathfrak{w}_{\mathfrak{s}}\ell^{\mathfrak{o}}_{\mathfrak{s},t}
\end{gathered}
$$

Therefore, the first order conditions are:

$$
\begin{array}{ll}
\text{For } \mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}\text{:} & \lambda^{\mathfrak{o}}_{\mathfrak{c},t} - \lambda^{\ell}_{\mathfrak{c},t} - \varsigma_{\mathsf{m}\mathfrak{c}}E_t\left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}} - 1\right)\frac{1}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}}P_t\lambda_t = 0 \\
\text{For } \ell_{\mathfrak{c},t}\text{:} & \lambda^{\ell}_{\mathfrak{c},t} = \mathfrak{w}_{\mathfrak{c}}\varpi^{u}_{\mathfrak{c},t}\lambda_t P_t + \left(1-\rho_{\mathfrak{d}}\right)\lambda^{\mathsf{u}}_{\mathfrak{c},t} + \beta E_t\frac{\partial\mathcal{U}_{t+1}}{\partial\ell_{\mathfrak{c},t}} \\
\text{For } \ell^{\mathfrak{o}}_{\mathfrak{c},t}\text{:} & \lambda^{\mathfrak{o}}_{\mathfrak{c},t} = \left(1-\rho_{\mathfrak{d}}\right)\left(\delta^{\mathfrak{c}}_{\mathfrak{c}}\lambda^{\mathsf{u}}_{\mathfrak{c},t} + \frac{\mathfrak{w}_{\mathfrak{c}}}{\mathfrak{w}_{\bar{\mathfrak{c}}}}\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\lambda^{\mathsf{u}}_{\bar{\mathfrak{c}},t}\right) + \beta E_t\frac{\partial\mathcal{U}_{t+1}}{\partial\ell^{\mathfrak{o}}_{\mathfrak{c},t}} \\
\text{For } \mathsf{u}_{\mathfrak{c},t+1}\text{:} & \lambda^{\mathsf{u}}_{\mathfrak{c},t} = a_{\mathfrak{c}}\mathsf{p}_{\mathfrak{c},t+1}\mathfrak{w}_{\mathfrak{c}}E_t\int_{\mathfrak{c}}\lambda^{cn}_t\left(z_{\mathfrak{c}}\right)\mathsf{s}_{t+1}\left(z_{\mathfrak{c}}\right)dz_{\mathfrak{c}} + \beta E_t\frac{\partial\mathcal{U}_{t+1}}{\partial\mathsf{u}_{\mathfrak{c},t+1}} \\
\text{For } \mathsf{n}_{t+1}\left(z_{\mathfrak{c}}\right)\text{:} & \lambda^{cn}_t\left(z_{\mathfrak{c}}\right) = \beta E_t\frac{\partial\mathcal{U}_{t+1}}{\partial\mathsf{n}_{t+1}\left(z_{\mathfrak{c}}\right)}
\end{array}
$$

Using the Envelope Theorem, the evolution dynamics of $\partial\mathcal{U}_t/\partial\mathsf{n}_t\left(z_{\mathfrak{c}}\right)$, $\partial\mathcal{U}_t/\partial\mathsf{u}_{\mathfrak{c},t}$, $\partial\mathcal{U}_t/\partial\ell^{\mathfrak{o}}_{\mathfrak{c},t-1}$, $\partial\mathcal{U}_t/\partial\ell_{\mathfrak{c},t-1}$ and $\partial\mathcal{U}_t/\partial\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t-1}$ are described by

$$
\begin{aligned}
\frac{\partial\mathcal{U}_t}{\partial\mathsf{n}_t\left(z_{\mathfrak{c}}\right)} &= -\upsilon'_t\left(z_{\mathfrak{c}}\right)h_t\left(z_{\mathfrak{c}}\right) + \left[\varpi_t\left(z_{\mathfrak{c}}\right) - \varpi^{u}_{\mathfrak{c},t}\right]P_t\lambda_t \\
&\quad + \frac{1}{\mathfrak{w}_{\mathfrak{c}}}\lambda^{\ell}_{\mathfrak{c},t} + \left(1-\rho_{\mathfrak{d}}\right)\left(1-\rho_{\mathfrak{c},t}\right)\left(\lambda^{cn}_t\left(z_{\mathfrak{c}}\right) - \frac{1}{\mathfrak{w}_{\mathfrak{c}}}\lambda^{\mathsf{u}}_{\mathfrak{c},t}\right)
\end{aligned}
$$

$$
\frac{\partial\mathcal{U}_t}{\partial\mathsf{u}_{\mathfrak{c},t}} = \left(1 - a_{\mathfrak{c}}\mathsf{p}_{\mathfrak{c},t}\right)\lambda^{\ell}_{\mathfrak{c},t}
$$

$$
\frac{\partial\mathcal{U}_t}{\partial\ell^{\mathfrak{o}}_{\mathfrak{c},t-1}} = \left(1-\rho_{\mathfrak{d}}\right)\left(1-\delta^{\mathfrak{c}}_{\mathfrak{c}}-\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\right)\lambda^{\mathfrak{o}}_{\mathfrak{c},t}
$$

$$
\frac{\partial\mathcal{U}_t}{\partial\ell_{\mathfrak{c},t-1}} = 0
$$

$$
\frac{\partial\mathcal{U}_t}{\partial\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t-1}} = P_t\lambda_t\varsigma_{\mathsf{m}\mathfrak{c}}\left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t-1}} - 1\right)\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t-1}}\frac{1}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t-1}}
$$

Let Q^{π}_t, defined below, denote the real stochastic discount factor. Recall also that $\lambda_t P_t = u'_t$, where u'_t is the marginal utility to consumption. Therefore, the first order conditions to pin down $\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}$ can be simplified as follows:

$$
\mathfrak{w}_{\mathfrak{c}}\varpi^{\mathfrak{o}}_{\mathfrak{c},t} - \mathfrak{w}_{\mathfrak{c}}\varpi^{\ell}_{\mathfrak{c},t} - \varsigma_{\mathsf{m}\mathfrak{c}}\left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t-1}} - 1\right)\frac{1}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t-1}} + \varsigma_{\mathsf{m}\mathfrak{c}}E_t Q^{\pi}_{t+1}\left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}} - 1\right)\left(\frac{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t+1}}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}}\right)\frac{1}{\mathsf{m}^{\mathfrak{o}}_{\mathfrak{c},t}} = 0
$$

$$
\varpi^{\ell}_{\mathfrak{c},t} = \varpi^{u}_{\mathfrak{c},t} + \left(1-\rho_{\mathfrak{d}}\right)\Lambda^{\mathsf{u}}_{\mathfrak{c},t}
$$

$$
\varpi^{\mathfrak{o}}_{\mathfrak{c},t} = \left(1-\rho_{\mathfrak{d}}\right)\left(\delta^{\mathfrak{c}}_{\mathfrak{c}}\Lambda^{\mathsf{u}}_{\mathfrak{c},t} + \delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\Lambda^{\mathsf{u}}_{\bar{\mathfrak{c}},t}\right) + \left(1-\rho_{\mathfrak{d}}\right)\left(1-\delta^{\mathfrak{c}}_{\mathfrak{c}}-\delta^{\bar{\mathfrak{c}}}_{\mathfrak{c}}\right)E_t Q^{\pi}_{t+1}\varpi^{\mathfrak{o}}_{\mathfrak{c},t+1}
$$

$$
\Lambda^{\mathsf{u}}_{\mathfrak{c},t} = a_{\mathfrak{c}}\mathsf{p}_{\mathfrak{c},t+1}E_t\int_{\mathfrak{c}}\Lambda^{cn}_t\left(z_{\mathfrak{c}}\right)\mathsf{s}_{t+1}\left(z_{\mathfrak{c}}\right)dz_{\mathfrak{c}} + E_t Q^{\pi}_{t+1}\left(1 - a_{\mathfrak{c}}\mathsf{p}_{\mathfrak{c},t+1}\right)\varpi^{\ell}_{\mathfrak{c},t+1}
$$

$$\Lambda_t^{cn}(z_\mathfrak{c}) = E_t Q_{t+1}^{\pi}\left[-\frac{\upsilon'_{t+1}(z_\mathfrak{c})\,h_{t+1}(z_\mathfrak{c})}{u'_{t+1}} + \varpi_{t+1}(z_\mathfrak{c}) - \varpi_{\mathfrak{c},t+1}^{u}\right]$$
$$+E_t Q_{t+1}^{\pi}\left[\varpi_{\mathfrak{c},t+1}^{\ell} + (1-\rho_{\mathfrak{d}})(1-\rho_{\mathfrak{c},t+1})\left(\Lambda_{t+1}^{cn}(z_\mathfrak{c}) - \Lambda_{\mathfrak{c},t+1}^{\mathsf{u}}\right)\right]$$

where

$$\varpi_{\mathfrak{c},t}^{\mathsf{o}} \equiv \tfrac{1}{\mathfrak{w}_\mathfrak{c}}\tfrac{\lambda_{\mathfrak{c},t}^{\mathsf{o}}}{u'_t} \quad \varpi_{\mathfrak{c},t}^{\ell} \equiv \tfrac{1}{\mathfrak{w}_\mathfrak{c}}\tfrac{\lambda_{\mathfrak{c},t}^{\ell}}{u'_t} \quad ; \Lambda_{\mathfrak{c},t}^{\mathsf{u}} \equiv \tfrac{1}{\mathfrak{w}_\mathfrak{c}}\tfrac{\lambda_{\mathfrak{c},t}^{\mathsf{u}}}{u'_t} \quad ; \Lambda_t^{cn}(z_\mathfrak{c}) \equiv \tfrac{\lambda_t^{cn}(z_\mathfrak{c})}{u'_t} \quad ; Q_t^{\pi} \equiv Q_t \Pi_t$$

■

Proposition 3 *The firm's real marginal cost is computed as follows:*

$$mc_t(z_\mathfrak{c}) = \varrho_\mathfrak{c}\epsilon_{\mathfrak{c},t}^{\mathsf{mc}}(C_t - \iota_\mathrm{u} C_{t-1})^{\sigma}(y_t(z_\mathfrak{c}))^{\omega_\mathfrak{c}}$$

where

$$\varrho_\mathfrak{c} \equiv \tfrac{\chi}{\varepsilon_\mathfrak{c}} \quad ; \epsilon_{\mathfrak{c},t}^{\mathsf{mc}} \equiv \mathfrak{s}_{\mathfrak{c},t}\mathfrak{z}_{2\mathfrak{c},t}(\mathfrak{u}_{\mathrm{u},t})^{-1}(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t)^{-(1+\omega_\mathfrak{c})}$$

Since $\mathfrak{z}_{2\mathfrak{c},t}^{\mathfrak{s}}$ increases with the workers bargaining power $b_{\mathfrak{c},t}$ and the sector-$\mathfrak{c}$ specific cost $\mathfrak{s}_{\mathfrak{c},t}$ over payroll, the marginal cost also varies with shocks to $b_{\mathfrak{c},t}$ and $\mathfrak{s}_{\mathfrak{c},t}$.

Proof. Note that

$$mc_t(z_\mathfrak{c}) = \mathfrak{s}_{\mathfrak{c},t}\varpi'_t(z_\mathfrak{c})\frac{\partial h_t(z_\mathfrak{c})}{\partial y_t(z_\mathfrak{c})}\mathsf{n}_t(z_\mathfrak{c})$$

$$\varpi'_t(z_\mathfrak{c}) = \mathfrak{z}_{2\mathfrak{c},t}^{\mathfrak{s}}\frac{\upsilon'_t(z_\mathfrak{c})}{u'_t}$$

$$h_t(z_\mathfrak{c}) = \frac{1}{\mathsf{n}_t(z_\mathfrak{c})}(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t)^{-\frac{1}{\varepsilon_\mathfrak{c}}}(y_t(z_\mathfrak{c}))^{\frac{1}{\varepsilon_\mathfrak{c}}}$$

$$\begin{aligned}
\upsilon_t(z_\mathfrak{c}) &= \chi\frac{H_t(z_\mathfrak{c})^{1+\nu}}{(1+\nu)}\\
\upsilon'_t(z_\mathfrak{c}) &= \chi(H_t(z_\mathfrak{c}))^{\nu}\\
&= \chi\left((\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t)^{-\frac{1}{\varepsilon_\mathfrak{c}}}(y_t(z_\mathfrak{c}))^{\frac{1}{\varepsilon_\mathfrak{c}}}\right)^{\nu}\\
&= \chi(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t)^{-\frac{\nu}{\varepsilon_\mathfrak{c}}}(y_t(z_\mathfrak{c}))^{\frac{\nu}{\varepsilon_\mathfrak{c}}}
\end{aligned}$$

$$u'_t \equiv \mathfrak{u}_{\mathfrak{c},t}(C_t - \iota_\mathfrak{c} C_{t-1})^{-\sigma}$$

Therefore

$$mc_t(z_\mathfrak{c}) = \frac{\chi}{\varepsilon_\mathfrak{c}}\mathfrak{s}_{\mathfrak{c},t}\mathfrak{z}_{2\mathfrak{c},t}^{\mathfrak{s}}(\mathfrak{u}_{\mathfrak{c},t})^{-1}(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t)^{-(1+\omega_\mathfrak{c})}(C_t - \iota_\mathfrak{c} C_{t-1})^{\sigma}(y_t(z_\mathfrak{c}))^{\omega_\mathfrak{p}}$$

Moreover, as for the effects of $b_{\mathfrak{c},t}$ and $\mathfrak{s}_{\mathfrak{c},t}$ on the sector-$\mathfrak{c}$ specific cost $\mathfrak{s}_{\mathfrak{c},t}$:

$$\frac{\partial\left(\mathfrak{s}_{\mathfrak{c},t}\mathfrak{z}_{2\mathfrak{c},t}^{\mathfrak{s}}\right)}{\partial b_{\mathfrak{c},t}} = \mathfrak{s}_{\mathfrak{c},t}\mathfrak{z}_{2\mathfrak{c},t}^{\mathfrak{s}}\left[\frac{\mathfrak{s}_{\mathfrak{c},t}\omega_\mathfrak{c}}{[(1-b_{\mathfrak{c},t}) + b_{\mathfrak{c},t}\mathfrak{s}_{\mathfrak{c},t}(1+\omega_\mathfrak{c})](1-b_{\mathfrak{c},t}+b_{\mathfrak{c},t}\mathfrak{s}_{\mathfrak{c},t})}\right] > 0$$

$$\frac{\partial\left(\mathfrak{s}_{\mathfrak{c},t}\mathfrak{z}_{2\mathfrak{c},t}^{\mathfrak{s}}\right)}{\partial \mathfrak{s}_{\mathfrak{c},t}} = \mathfrak{s}_{\mathfrak{c},t}\mathfrak{z}_{2\mathfrak{c},t}^{\mathfrak{s}}\left[\frac{1}{\mathfrak{s}_{\mathfrak{c},t}} + \frac{\omega_\mathfrak{c} b_{\mathfrak{c},t}(1-b_{\mathfrak{c},t})}{[(1-b_{\mathfrak{c},t}) + b_{\mathfrak{c},t}\mathfrak{s}_{\mathfrak{c},t}(1+\omega_\mathfrak{c})](1-b_{\mathfrak{c},t}+b_{\mathfrak{c},t}\mathfrak{s}_{\mathfrak{c},t})}\right] > 0$$

■

Proposition 4 *Aggregate disutility functions $\upsilon_t \equiv \int \upsilon_t(z)\,dz$ and $\upsilon_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_\mathfrak{c}}\int_\mathfrak{c}\upsilon_t(z_\mathfrak{c})\,dz_\mathfrak{c}$ and aggregate hours worked $H_t \equiv \int H_t(z)\,dz$ and $H_{\mathfrak{c},t} \equiv \frac{1}{\mathfrak{w}_\mathfrak{c}}\int_\mathfrak{c} H_t(z_\mathfrak{c})\,dz_\mathfrak{c}$ can be rewritten* *as follows:*

$$\upsilon_{\mathfrak{c},t} \equiv \tfrac{\chi}{(1+\nu)} \left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-(1+\omega_{\mathfrak{c}})} \left(\tfrac{1}{\mathfrak{w}_{\mathfrak{c}}}\mathcal{Y}_{\mathfrak{c},t}\right)^{(1+\omega_{\mathfrak{c}})} \left(\mathcal{P}_{\upsilon\mathfrak{c},t}\right)^{-\phi(1+\omega_{\mathfrak{c}})}$$
$$H_{\mathfrak{c},t} \equiv \left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-(1+\tilde{\omega}_{\mathfrak{s}})} \left(\tfrac{1}{\mathfrak{w}_{\mathfrak{c}}}\mathcal{Y}_{\mathfrak{c},t}\right)^{(1+\tilde{\omega}_{\mathfrak{c}})} \left(\mathcal{P}_{H\mathfrak{c},t}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})}$$

$$\upsilon_t = \textstyle\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}}\upsilon_{\mathfrak{c},t} \quad ; \; H_t = \textstyle\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} H_{\mathfrak{c},t}$$

where $\tilde{\omega}_{\mathfrak{c}} \equiv \frac{1}{\varepsilon_{\mathfrak{c}}} - 1$*, and* $\mathcal{P}_{\upsilon\mathfrak{c},t}$ *and* $\mathcal{P}_{H\mathfrak{c},t}$ *denote aggregate relative prices:*

$$\left(\mathcal{P}_{\upsilon\mathfrak{c},t}\right)^{-\phi(1+\omega_{\mathfrak{c}})} \equiv \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \left(\tfrac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi(1+\omega_{\mathfrak{c}})} dz_{\mathfrak{c}} \quad ; \; \left(\mathcal{P}_{H\mathfrak{c},t}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} \equiv \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \left(\tfrac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} dz_{\mathfrak{c}}$$

Proof. Note that

$$\begin{aligned} y_t\left(z_{\mathfrak{c}}\right) &= \mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t H_t\left(z_{\mathfrak{c}}\right)^{\varepsilon_{\mathfrak{c}}} \\ H_t\left(z_{\mathfrak{c}}\right) &= \left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-\frac{1}{\varepsilon_{\mathfrak{c}}}} \left(y_t\left(z_{\mathfrak{c}}\right)\right)^{\frac{1}{\varepsilon_{\mathfrak{c}}}} = \left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-(1+\tilde{\omega}_{\mathfrak{c}})} \left(y_t\left(z_{\mathfrak{c}}\right)\right)^{(1+\tilde{\omega}_{\mathfrak{c}})} \\ \upsilon_t\left(z_{\mathfrak{c}}\right) &= \chi\frac{H_t\left(z_{\mathfrak{c}}\right)^{1+\nu}}{(1+\nu)} = \frac{\chi}{(1+\nu)} \left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-(1+\omega_{\mathfrak{c}})} \left(y_t\left(z_{\mathfrak{c}}\right)\right)^{(1+\omega_{\mathfrak{c}})} \end{aligned}$$

$$y_t\left(z_{\mathfrak{c}}\right) = \frac{1}{\mathfrak{w}_{\mathfrak{c}}} Y_{\mathfrak{c},t} \left(\frac{p_t\left(z_{\mathfrak{c}}\right)}{P_{\mathfrak{c},t}}\right)^{-\phi}$$

$$\upsilon_{\mathfrak{c},t} \equiv \tfrac{\chi}{(1+\nu)} \left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-(1+\omega_{\mathfrak{p}})} \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \left(y_t\left(z_{\mathfrak{c}}\right)\right)^{(1+\omega_{\mathfrak{c}})} dz_{\mathfrak{c}} \quad ; \; \upsilon_t = \textstyle\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}}\upsilon_{\mathfrak{c},t}$$
$$H_{\mathfrak{c},t} \equiv \left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-(1+\tilde{\omega}_{\mathfrak{p}})} \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \left(y_t\left(z_{\mathfrak{c}}\right)\right)^{(1+\tilde{\omega}_{\mathfrak{c}})} dz_{\mathfrak{c}} \quad ; \; H_t = \textstyle\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} H_{\mathfrak{c},t}$$

Therefore:

$$\upsilon_{\mathfrak{c},t} \equiv \tfrac{\chi}{(1+\nu)} \left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-(1+\omega_{\mathfrak{c}})} \left(\tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} Y_{\mathfrak{c},t}\right)^{(1+\omega_{\mathfrak{c}})} \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \left(\tfrac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi(1+\omega_{\mathfrak{c}})} dz_{\mathfrak{c}}$$
$$H_{\mathfrak{c},t} \equiv \left(\mathsf{a}_{\mathfrak{c},t}\mathsf{A}_t\right)^{-(1+\tilde{\omega}_{\mathfrak{c}})} \left(\tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} Y_{\mathfrak{c},t}\right)^{(1+\tilde{\omega}_{\mathfrak{c}})} \tfrac{1}{\mathfrak{w}_{\mathfrak{c}}} \int_{\mathfrak{c}} \left(\tfrac{p_t(z_{\mathfrak{c}})}{P_{\mathfrak{c},t}}\right)^{-\phi(1+\tilde{\omega}_{\mathfrak{c}})} dz_{\mathfrak{c}}$$

$$\upsilon_t = \textstyle\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}}\upsilon_{\mathfrak{c},t} \quad ; \; H_t = \textstyle\sum_{\mathfrak{c}} \mathfrak{w}_{\mathfrak{c}} H_{\mathfrak{c},t}$$

■