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Bargained Haircuts and Debt Policy Implications*

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Abstract

We extend the Cole and Kehoe model [7] by adding a Rubinstein bargaining game between creditors and debtor country to determine the share of debt repayment in a sovereign debt crisis. *Ex-post*, the possibility of partial repayment avoids the costly case of total default, as seen recently in Greece. *Ex-ante*, the effects are to increase the sovereign debt cap and delay the fiscal adjustment. In other words, expectations of a haircut in times of crisis relax leverage restrictions implied by financial markets and make government more lenient, suggesting caution with haircut adoption, especially when risk-free interest rates are low.

Keywords: partial default, speculative attacks, debt crisis
JEL Classification: F34, G01 and H63

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1 Introduction

Sovereign debt crises have often changed form but have not vanished from the global scene. The Cole and Kehoe [5] general equilibrium model clarified Mexico's 1994-95 debt crisis by showing that a self-fulfilling default could generate welfare losses for both parties involved: Mexico and its international creditors. Both could have been better off if international creditors had coordinated in rolling over the maturing debt.

International evidence has been increasing the appeal of a multiple equilibrium framework to appraise sovereign crises, typically noticeable by non-fundamental volatility[1]. Hence, multiplicity sustained by lack of coordination is recurrent in the literature. Based on a general setup encompassing ours, both Morris and Shin [15] and Angeletos and Werning [1] showed multiplicity may arise under general assumptions, including non-common knowledge.

Recently, Greek sovereign debt crisis, marked by bond price volatility, brought different topics to the economic debate. In particular, financial arrangements combining debtor rescue with creditor' losses divide opinions. On one hand, it helped to smooth the fiscal adjustment required for Greece to become solvent. On the other, economists have been highlighting an undesirable *ex-ante* effect: rescue packages reduce the incentives for fiscal discipline.

The anticipation of rescue in the case of crisis diminishes expected losses and makes creditors more willing to supply funds at lower rates, inducing debtors to keep the outstanding debt level, instead of decreasing it.

With this background in mind, we discuss the *ex-ante* market implications of this type of financial arrangement. We use the debt market structure proposed in the original Cole and Kehoe model, where government is financed only by external debt subject to speculative attacks. Looking at Greece, Italy, Ireland, Portugal and Spain (GIIPS), the biggest share of their outstanding government debt was mostly held by nonresidents (see Eurostat [9], Pisani-Ferry [17] and Merler and Pisani-Ferry [13]).

To include debt restructuration, we adapt the original model with a partial default technology, similar to the one discussed in Araujo, Leon and Santos [2]. We consider the possibility of debt renegotiation between international creditors and a debtor

[1] Sudden changes in outcomes, such as market prices of public bonds, without obvious comparable changes in the set of fundamentals, including public debt level, tax revenues and public expenditures.

country that is facing an imminent total default on its public debt. The renegotiation outcome determines a write-down of the face value of government bonds and therefore the actual return of sovereign debt in the aftermath of a crisis. The higher is the expected write-down, the lower the price of the bond is. Creditors understand they may recover significantly less than the principal invested and demand high nominal returns to bear the risk.

Accordingly, bond prices depend on the outcome of a Rubinstein bargaining game set to determine the optimal debt recovery rate on the defaulted debt. We discuss two possibilities for the Rubinstein bargaining game. In both, the central planner, which can be interpreted as the European Central Bank (ECB), plays the key role of enforcing the partial repayment[2].

As a representative of the monetary union, ECB should actively work to avoid total default of a member country, which is exemplified by Greece in our numerical exercise. We understand that a total and non negotiated default by a member country hurts more the monetary union strength than a partial and negotiated default. The debtor member country and the creditors also tend to prefer the partial default. The debtor prefers to keep the benefits of having a common currency and the foreign creditors understand the enormous difficulties of receiving claims on a non-negotiated defaulted sovereign debt.

In the first bargaining game considered, the central planner starts to talk and bargains with the group of international creditors on how to split the total cost of default, acting on behalf of the debtor country. Then, the central planner enforces the payment accordingly to the haircut agreement.

In the second, the central planner talks first again, but bargains with the debtor country, acting on behalf of the group of international creditors. We show in Figure 1 the quantitative results would be virtually the same independent of the case considered[3].

Another important assumption made is the size of the default penalty, denoted by the productivity loss, which encourages fiscal adjustment. In Greek case, as

[2]Here, the central planner is a player in a bargaining game, though in fact its importance to solve sovereign crises involves other aspects. Arellano and Bai [3] discussed some of them.

[3]For further discussions on debt recovery rates and equilibrium implications, see Mora [14] and Yue [20]. In the last, the debt recovery rate is determined in a Nash bargaining game which also affects the country's ex-ante incentives to default.

total default prevents access not only to the international bond market but also to the common currency benefits, one should expect higher productivity loss from total default than from partial default[4]. Therefore, partial default comes with lower penalty, and then smoothes the *ex-post* crisis costs when compared with total default. However, *ex-ante*, it also reduces the concerns about crisis. We highlight this moral hazard effect by showing, in a numerical exercise portraying Greek economy, that an economy remains in the crisis zone for a longer period of time when partial default is allowed, increasing debt crisis probability[5].

The qualitative policy prescriptions from our paper, with haircut possibility, are the same as the ones from the original Cole and Kehoe framework, without haircut possibility: lengthening the maturity of the external debt and reducing its level to avoid a sovereign crisis.

As a novelty, we present the side effect of a haircut possibility for debt policy: partial default increases the room for leverage. It causes an upward shift of the upper limit of the crisis zone (risky debt cap) and of the lower limit of the crisis zone (risk-free debt cap) for different maturities of the sovereign debt, as discussed in proposition. It also decelerates the optimal fiscal policy to exit the crisis zone. Kirsch and Rühmkorf [12] do not consider haircuts, but instead official financial assistance, and find a result similar to ours: financial assistance raises debt level.

2 Defaultable debt model with bargained haircut

Our framework closely resembles that of the original Cole and Kehoe dynamic stochastic general equilibrium, where high external debt can lead to credit constraints followed by strategic default. We modify their model and apply a partial default technology. The debt recovery rate is endogenous and depends on a Rubinstein bargaining game. We analyze the *ex-ante* implications of the recovery rate for the optimal debt policy and for the limits on leverage.

[4] Conesa and Kehoe [8] also consider bailout of government debt by official lenders in a debt crisis and highlight that the bailout cost is smaller than the cost of a total default.

[5] In each period, the crisis probability is defined exogenously, but the more periods the economy remains in the crisis zone, the higher is the cumulative chance of a crisis occurrence. One may argue that crisis probability per period itself should increase with partial default, a case to be considered in further extensions.

Accordingly, there is one good produced with capital, k, inelastic labor supply and price normalized at one unit of the currency. The economy consists of four types of agents: consumers, international creditors, the government of the debtor country and the central planner. External debt, B, is acquired only by international creditors. There is a probability π of no rollover if its level is in the crisis zone. The cost of (partial) default[6] on debt is the exclusion from the international lending market leading to a permanent productivity loss.

We discuss two possible bargaining cases. In the first, a central planner bargains with the international creditors on how to split the total cost of default. They agree to a recovery rate on the total debt. Then, the central planner forces the debtor country to partially repay. In a second case, we assume the central planner bargains with the debtor country, which wishes to partially repay. The second case may be viewed as central planner and international creditors accepting the partial repayment to avoid total default and the further costs associated with an exit from the common currency area.

2.1 Description of market participants

Consumers and international creditors compose continuums of agents, each one of them with mass equal to one. Government and central planner are single agents.

Consumers

Each consumer at any time t maximizes the expected utility

$$\max_{c_t, k_{t+1}} E \sum_{t=0}^{\infty} \beta^t \left[c_t + v\left(g_t\right)\right] \tag{1}$$

subject to the budget constraint, given by

$$c_t + k_{t+1} - k_t \leq (1-\theta)\left[a_t f\left(k_t\right) - \delta k_t\right]$$

with $k_0 > 0$. At time t, the consumer chooses how many goods to save for the next period, k_{t+1}, and how much to consume at present, c_t. The utility has two parts: a linear function of private consumption, c_t, and a function v of government spending,

[6] In order to avoid excessive wordiness, henceforth we omit the word partial.

g_t. The function $v(.)$ is continuous, differentiable, strictly concave and increasing. The right-hand side of the budget constraint corresponds to the consumer's income from production after taxes, $\theta \in (0,1)$, and depreciation, δ. The production function, $f(.)$, is continuous, concave, differentiable and strictly increasing[7]. If the government decides to default, the productivity, a_t, suffers a permanent fall:

$$a_t = 1, \text{ if } a_{t-1} = 1 \text{ and there is no default in } t$$
$$a_t = \alpha, \text{ otherwise, } \alpha \in (0,1)$$

International creditors

Each international creditor at time t may purchase a quantity of debt b_{t+1} at price q_t, in order to solve the following problem:

$$\max_{x_t, b_{t+1}} E \sum_{t=0}^{\infty} \beta^t x_t \tag{2}$$

$$s.t.\ x_t + q_t b_{t+1} \leq \overline{x} + z_t b_t$$

given an initial amount of external debt

$$b_0 > 0$$

The creditors choose how many goods to consume, x_t, and the amount of government bonds to buy, b_{t+1}, given an endowment $\overline{x}$ of goods (creditors have "deep pockets"). The left-hand side of the budget constraint shows the expenditure on new debt, where q_t is the price of one-period bonds that pay one unit of the good at maturity if the government does not default. The right-hand side includes the revenue received from the bonds purchased in the previous period, $z_t b_t$. The decision variable z_t indicates whether the government defaults ($z = \phi$) or not ($z = 1$). If it defaults, then the creditors receive ϕb_t.

Government

The government is assumed benevolent in the sense that it maximizes consumers' welfare, but with no commitment to honor its obligations. In t, its decision variables

[7] $f(0) = 0;\ f\prime(0) = \infty;\ f\prime(\infty) = 0$

are new debt, B_{t+1}, and government consumption, g_t. It also chooses whether to default on debt, z_t, or not, according to the budget constraint:

$$\begin{aligned} g_t + z_t B_t &\leq \theta \left[a_t f(K_t) - \delta K_t \right] + q_t B_{t+1} \\ z_t &\in \{\phi, 1\}, \ \phi \in (0,1) \text{ and } g_t > 0 \end{aligned} \tag{3}$$

The left-hand side of expression (3) refers to government current consumption and the payment of its debt. The right-hand side includes revenue from income taxes and from selling new debt. The government is also assumed to have a strategic behavior since it foresees the optimal decision of the participants in its economy, including its own: c_t, k_{t+1}, q_t, z_t, and g_t, given the initial aggregate state of the economy, S_t, and its debt choice of B_{t+1}. The parameter ϕ is between zero and one as an outcome from the Rubinstein bargaining game, as discussed next.

Central planner (CP)

The central planner acts only in the event of default. Its role is to bargain with either international creditors or the debtor country to achieve some debt recovery rate ϕ, which defines the haircut level $(1 - \phi)$.

To compute results, as our focus is on the ex-ante effects from the negotiated default, we borrow from the literature the simple and convenient Rubinstein model of bargaining [18], characterized by complete information and unique perfect equilibrium agreement, in our specific case, the debt recovery rate. In a one-to-one format, with alternating offers, game only ends when one player accepts the offer of other player. Delays are costly. Counter-offers occur from t to $t + 1$. The central planner's goal depends on the bargaining case. We discuss two possible bargaining cases and note that, in both, the haircut level, reached right after default, is very similar to each other.

(CP) case 1: central planner bargains with international creditors

Central planner bargains with creditors such that they accept some debt recovery rate. We assume the creditors have incentives to accept the offer. If they do not, their receivables will depend on judicial processes, that tend to be expensive, long and unpredictable. Once the bargain is concluded, we assume that the central planner can enforce partial repayment. The central planner acts on behalf of the debtor country and its goal is to minimize the recovery rate.

Formally, whenever q_t goes to zero and B_t is in the crisis zone, the debtor country announces a total default on its external debt claiming it is not possible to honor contracts. Then, the debtor country is called by the central planner. The debtor country, via the central planner, proposes the haircut rate $(1-\phi)$, so as $z_t = \phi$. The payoffs resulting from the bargaining process are $(1-z_t)\,B_t$ for debtor country and $z_t B_t$ for creditors.

We also assume that time is costly: the welfare gain associated with the agreement increases with the velocity with which the deal is concluded. Therefore, an agreement in the first round is much better than the same agreement reached at later rounds of negotiation. The intertemporal discount rate of the debtor country is β, while the discount rate of the creditors, only during crisis, may shift to $\beta^{\pi} = (\beta - \varepsilon)$, with $\varepsilon \geq 0$. After solving the bargaining game, we reach ϕ equal to $\frac{\beta^{\pi}(1-\beta)}{1-\beta^{\pi}\beta}$. When $\varepsilon = 0$, ϕ is equal to $\left(\frac{\beta}{1+\beta}\right)$. If β^{π} is zero, our model matches Cole and Kehoe with ϕ equal to zero. We consider the possibility of $\varepsilon \geq 0$ to allow creditors to have a different β from the debtor country since they are facing a default on their bond holdings and are involved in a collective bargaining game, where their claims become effective through a representative member. The possibility of $\varepsilon \geq 0$ also gives flexibility to model different bargaining outcomes, including the same β for creditors and the debtor country $(\varepsilon = 0)$[8].

(CP) case 2: central planner bargains with debtor country

The central planner is the first to talk and acts on behalf of the creditors. Its goal is to maximize the debt recovery rate by bargaining with the debtor country. The debtor country always prefers to accept some haircut level negotiated with the central planner to avoid significantly higher costs from total default that causes exit from the monetary union.

The rest of the bargaining process is the same, leading to ϕ equal to $\frac{(1-\beta)}{1-\beta^{\pi}\beta}$. When $\varepsilon = 0$, ϕ is equal to $\frac{1}{1+\beta}$.

Moreover, as presented in Figure 1, we compare case 1 with case 2. We set β^{π} equal to 0.91 and vary β from 0.91 to 1, or equivalently, we vary ϵ from 0 to 0.09. Note

[8]See Kaminsky *et al.* [11] for another disscussion of investors asking higher discount rates to reflect a crisis rate.

that the higher are the discount factors, the lower is the difference between haircuts. Results show that the haircut levels, $(1-\phi)$, when a central planner bargains with the international creditors, is similar to the one when central planner bargains with the debtor country. Hereafter, we consider the second case to solve partial default and to reach the bargaining outcome as a function of the impatience of the players (β and β^{π}). In Figure 2, we show the sequential game tree for case 2.

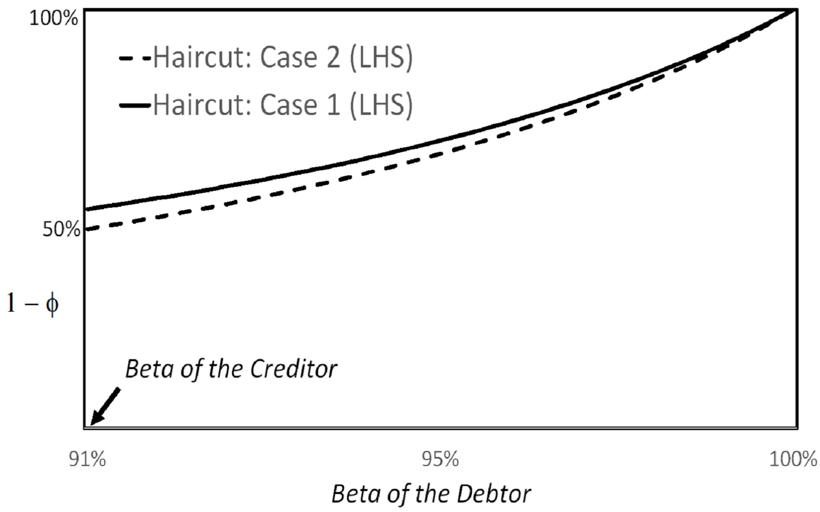

Figure 1: Equivalence of the two bargaining cases

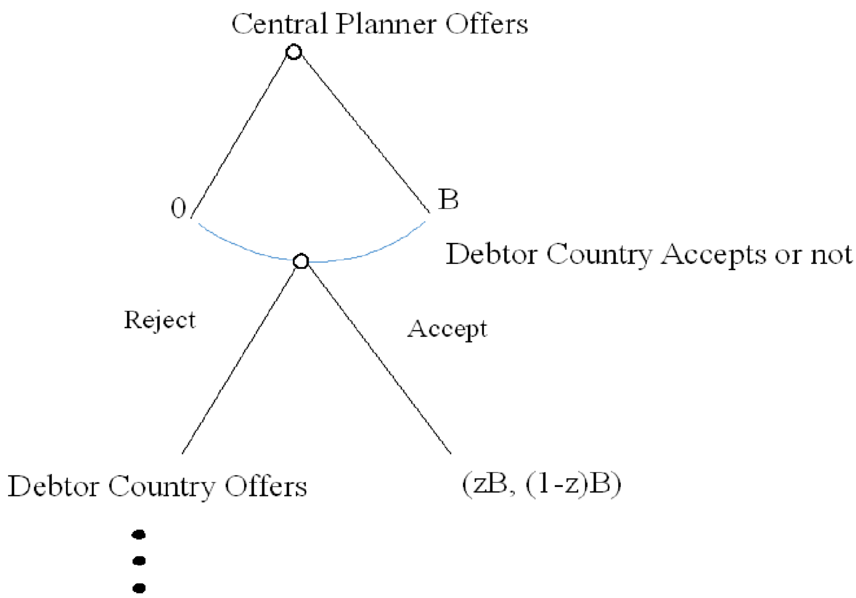

Figure 2: Sequential game tree

2.2 Uncertainty

Uncertainty about a speculative attack on debt is given by an exogenous probability π. The realization of a random variable ζ indicates the confidence that international creditors have that the government will not default on its debt. It is assumed independent over time and identically distributed according to an uniform[9]. When the debt is inside the crisis zone and $\zeta < \pi$, the price q goes to zero.

The variable ζ aims to capture the public perception of the refinancing risk faced by the indebted economy. When public perception deteriorates, each individual creditor is not willing to provide new funds to the debtor country. Since there is no debt rollover, the debtor country announces a total default to trigger the bargaining process. Once a renegotiation is in place, z moves from 0 to a positive value of z, equal to ϕ.

The price q_t of the outstanding debt B_t reflects rational expectations for any t, and is given either by $\beta(1 - \pi + \pi\phi)$, when debt is in the crisis zone, or by β, otherwise. However, to follow the same credit restriction explored in the original Cole and Kehoe model [5], we assume, during a confidence crisis, the price q going from $\beta\phi$ (rational price when $E[z] = \phi$) to 0, preventing the access to new funds within a crisis. One should interpret $q = 0$ as a shock on the price of new debt issuance during the crisis[10]. Such assumption rules out the following equilibrium result: new debt contracts being signed while maturing contracts are being discussed.

2.3 Timing of actions

In the initial period t, the aggregate state of the economy, S_t, is characterized by a positive amount of capital, K_t, of public debt, B_t, by the productivity level, a_{t-1}, equal to one, and by the realization of ζ_t. The hypothesis $a_{t-1} = 1$ denotes that no shock has hit the economy yet, so $z_{t-1} = 1$. The debt level, B_t, is assumed to be in the crisis zone, meaning that it is subject to a speculative attack with probability π.

After the realization of ζ_t, period uncertainty is solved. In period t, the government chooses at two different moments. First, it decides about new debt, B_{t+1}.

[9] Arellano [4] describes an income shock and assumes it as i.i.d. In our paper, the income is also affected, but via productivity, a_t.

[10] The international credit restriction resulted from speculative attack on sovereign debt may be a response to a change in economic fundamentals not explicitly described in the model, such as: a persistent change in prices of a key commodity exported by the country, fear of change in the government preferences after national elections, or a sudden reduction of international liquidity.

Next, given the creditors' action described in the price q_t, it decides whether or not to default. If the government decides not to default, then the choice of $z_t = 1$ determines government spending, g_t. If the government decides to default, then a bargaining game takes place to decide the optimal debt recovery rate ϕ. Given the debt recovery rate, ϕ, government consumption, g_t, is determined [11].

The timing of actions within period t, given that a default has not occurred yet, is:

- ζ_t is realized and the aggregate state is $S_t = (K_t, B_t, a_{t-1}, \zeta_t)$, with $a_{t-1} = 1$
- The government, taking $q_t(S_t, B_{t+1})$ as given, chooses B_{t+1}
- The international creditors, taking $q_t(S_t, B_{t+1})$ as given, choose whether or not to purchase b_{t+1}
- The government chooses whether or not to default, z_t.
  - No default: $z_t = 1$ and the level of spending, g_t, is determined
  - Default: $z_t = \phi$ and the level of spending, g_t, is determined (central planner intermediates a bargaining game, which results in a debt recovery rate of ϕ)
- Finally, consumers, taking a_t as given, choose c_t and k_{t+1}

2.4 Equilibrium

We define an equilibrium where market participants choose their actions sequentially, starting with consumers who choose last.

Consumers take as given the aggregate state, S, and the government's decisions, $G \equiv (z, g, B')$, to maximize their utility by choosing k' that solves the following equation:

$$\frac{1}{\beta} = (1-\theta)\left[f\prime(k')E_t\left(a'\right) - \delta\right] + 1$$

k' takes three values depending on $E(a')$: k^n, for $E(a') = 1$; k^π, for $E(a') = 1 - \pi + \alpha\pi$; and k^d, for $E(a') = \alpha$. In equilibrium, their choice of k' is equal to the aggregate capital

[11] Sunspot π is considered independent from ϕ.

level K'. International creditors act competitively and are risk neutral. They purchase new bonds whenever the price, q, makes the expected return at least equal to $1/\beta$:

$$1/\beta = E_t\left(z'\right)/q$$

At the same time, a competitive international credit market prevents an expected return higher than $1/\beta$. In equilibrium, q may take three values: β when $E(z') = 1$, $\beta(1 - \pi + \pi\phi)$ when $E(z') = (1 - \pi + \pi\phi)$, and 0 during a confidence crisis, when each individual creditor is not willing to provide new funds to debtor country. At this time, a bargaining process takes place to determine z_t.

Finally, the government anticipates optimal capital accumulation, k', and the price that makes international creditors indifferent to purchasing debt, q.

An equilibrium is defined as a list of value functions V_c for the representative consumer, V_b, for the representative creditors, and V_g, for the government; of policy functions $G \equiv (z, g, B')$ for the government, $C \equiv (c,\ k')$ for the consumer; of a price function, q; of a recovery rate function ϕ; and an expression for aggregate capital, K', such that:

(i) given $G = (\ z, g, B')$, $C \equiv (c,\ k')$ solves the consumer's problem and V_c is her value function;

(ii) given B', q, z and ϕ, B' chosen by the government solves the creditor's problem, when $b = B$ and V_b is the value function for the representative creditor;

(iii) given q, c, K', g, and z, B' solves the government's problem and V_g is the value function for the government;

(iv) q solves $q = \beta E(z')$[12]

(v) $\phi\left(\varepsilon\right)$ results from the triggered bargaining problem when $q = 0$;

(vi) given S, $B' = b'$;

(vii) given S, $K' = k'$.

[12] For new issuance during the crisis we consider $E(z') = 0$, to reflect the debt market closure.

3 The Crisis Zone

The crisis zone for one-period government bonds is defined as the debt interval, $\left[\underline{B}, \overline{B}\right]$, for which it is optimal for the government to honor contracts in the absence of a speculative attack and to respond with default to an attack. In our setup, when a speculative attack occurs, a debt recovery rate ϕ is negotiated among the central planner, the debtor country and its creditors. Formally, consider the government payoffs $V(s, B\prime, q, z)$, given $S = (B, K, a_{-1}, \zeta)$ and after new debt B' has been sold conditional upon decision z, price q, and risk π. Debt level B_0 is in the crisis zone if and only if $B_0 \in \left[\underline{B}, \overline{B}\right]$, where

$$\begin{aligned} \underline{B} &\equiv \max B \text{ such that } \{V(s,0,0,1) \geq V(s,0,0,\phi)\} \text{ and } S = (B, K^n, 1, \zeta \leq \pi) \\ \overline{B} &\equiv \max B \text{ such that } \{V(s,B,\beta,1) \geq V(s,B,\beta,\phi)\} \text{ and } S = (B, K^n, 1, \zeta > \pi) \end{aligned}$$

The crisis zone can be constructed for different debt maturities. To include maturity, one should consider a policy that converts an initial quantity of one-period bonds, B, into equal quantities B_N of bonds of maturity $1, ..., T$ (see Cole and Kehoe [5], p. 327). The policy prescription from their model is to lengthen the maturity of the debt. One may argue that another effect of lengthening the maturity is that the debtor country increases its room for leverage. The possibility of partial default has a similar effect. The debt limits for crisis zone, $\underline{B}$ and $\overline{B}$, increase with ϕ for all T. Increasing indebtedness results from creditors, during normal times, perceiving ϕB being repaid in case of a default, not a nil quantity of B, relaxing the limits for sovereign leverage.

> *Proposition: For any positive debt maturity $T > 0$ and for $\phi \in \left[0, \overline{\phi}(T)\right)$, the crisis zone limits ($\underline{B}$ and $\overline{B}$) become higher with partial default than the respective limits obtained in the original Cole and Kehoe model [7] (special case where $\phi = 0$). The parameter ϕ need to be upper-bounded to preserve the incentives for default and to avoid empty crisis zone.* (proof: see Appendix A).

The intuition for the upper limit for ϕ, which depends on T and is denoted by $\overline{\phi}(T)$, is as follows: since default has permanent adverse effects due to lower productivity and results from an optimal decision, it is necessary to ensure that default, at least, improves the fiscal position in the short run. Therefore, there is a limit for the partial repayment ϕ. For high values of ϕ, default would be so costly that it would never occur. To have partial repayment in equilibrium, it is sufficient to limit ϕ by $\overline{\phi}(T) = \min\left\{\beta^T; \left(1-\beta^T\right)\frac{B(1-\beta)-(g_n-g_d)}{B(1-\beta)}; \left(1-\beta^T\right)\frac{g_d}{g_n}\right\}$. The first limit assures the partial default is on the principal amount of the debt, *i.e.* partial default implies negative return on bonds. Note that after a payment of ϕ, the bond return rate is equal to $\left(\frac{\phi}{\beta^T}-1\right)$. The second assures that reduced flow of government revenue from both tax collection $(g_n - g_d)$ and partial repayment on maturing debt $\left(\phi\frac{B(1-\beta)}{1-\beta^T}\right)$ must be lower than the total interest payment over the total outstanding debt $B(1-\beta)$ in case of no default. The third limit rules out negative public expenditure. It assures that, even under the highest feasible debt, $\frac{g_n}{1-\beta}$, ϕ is low enough to guarantee a non negative public expenditure.

3.1 Computed Crisis Zone - Greece

We compute the crisis zone for Greece considering different maturities. The parameters used in this numerical example are an attempt to capture Greek economy before its debt crisis in 2010. The discount factor, β, is given by the yearly yield on German government bonds, r, whose value fluctuated between 0.03 and 0.045 between 2007 and 2009. We consider r equal to 0.035, which makes the discount factor β equal to 0.97, according to the equilibrium condition associated with the risk-neutral behavior of international creditors. The tax rate, θ, is set equal to 0.30 (OECD [16]) to match the average Greece tax rate for the same period. The chosen functional form, $v(g)$, is the same used by Cole and Kehoe [5], $v(g) = \ln(g)$. The results are very sensitive to this specification, which besides determining the coefficient of risk aversion, also defines the relative importance of public expenditure to private-sector consumption[13]. Net income, $af(k) - \delta k$, is parameterized with $f(k) = k^\lambda$. Capital share, λ, is set equal to 0.4 and the yearly depreciation rate, δ, to 0.061 (Trabandt and Uhlig [19], Table 4). The parameter α is 0.95, assuming that default causes a permanent drop in productivity of 0.05. We consider case 2 of the bargaining process

[13] We could represent governments more concerned about private goods by replacing $ln(g)$ with $\frac{ln(g)}{2}$, for example.

and set $\varepsilon = 0.1$, leading to ϕ equal to 0.20, and a β^{π} equal to 0.87.

Furthermore, the probability of partial default, π is based on the *ex-ante* risk premium observed in financial markets, attained from the following equilibrium condition:

$$\frac{1}{\beta} = \left(1 + r^{Greece}\right)(1 - \pi) + \pi\phi \tag{4}$$

where r^{Greece} is the yearly real interest rate on Greece's debt. r^{Greece} varied between 0.04 and 0.06 from 2007 to 2009. We set $r^{Greece} = 0.055$ and therefore π^d was computed at 0.02. Conesa and Kehoe ([8], Table 1) choose 0.03.

In Figure 3, the two upper curves portray what Cole and Kehoe call the *stationary participation constraint* and the two lower curves represent what they refer to as the *no-lending continuation condition.* The former constraint is the highest (risk) debt level for which it is better not to default if international creditors renew their loans. The latter constraint is the highest (risk-free) debt level for which it is better not to default if there is no new lending. The region between both constraints is the crisis zone. The solid lines indicate the limits for the crisis zone for the original case $(\phi = 0)$, and the dotted lines, for $\phi = 0.2$. Figure 3 shows that Greek government debt relative to GDP of 1.129 was in the crisis zone in 2009[14], when the debt crisis started. Official loans were replaced by private credit in 2010. At the end of that year, a bargaining game ensued as official members of the European Monetary Union requested a crisis resolution mechanism for the region with the participation of the private sector. Finally, in February 2012, an agreement among Greece, private creditors and the official sector contemplated a major debt restructuring that resulted in old bonds being exchanged for new ones. Haircut calculations by Zettelmeyer et al. ([21], Figure 5) show there are large differences according to the remaining duration of the bonds. They compute higher haircuts, of almost 0.8 (or $\phi = 0.2$), for bonds with remaining duration of one year and, 0.5 (or $\phi = 0.5$), for 10 years. A word of caution for haircut decisions: if the possibility of haircuts after 2012 had been taken into account by the participants of the international sovereign bond market three years earlier, then the crisis zone for Greece would have shifted upwards, ceteris paribus, opening room for

[14] For duration up to 3 years, see http://www.tradingeconomics.com/greece/government-debt-to-gdp for national statistics.

more leverage. In fact, in 2012, the debt-to-GDP ratio increased to 1.569 (Eurostat [10]). As Figure 3 shows, considering a fixed ten-year maturity, a small change in the default technology (from $\phi = 0$ to $\phi = 0.2$) opens the door for about 35 percent of GDP in the *ex-ante* feasible leverage.

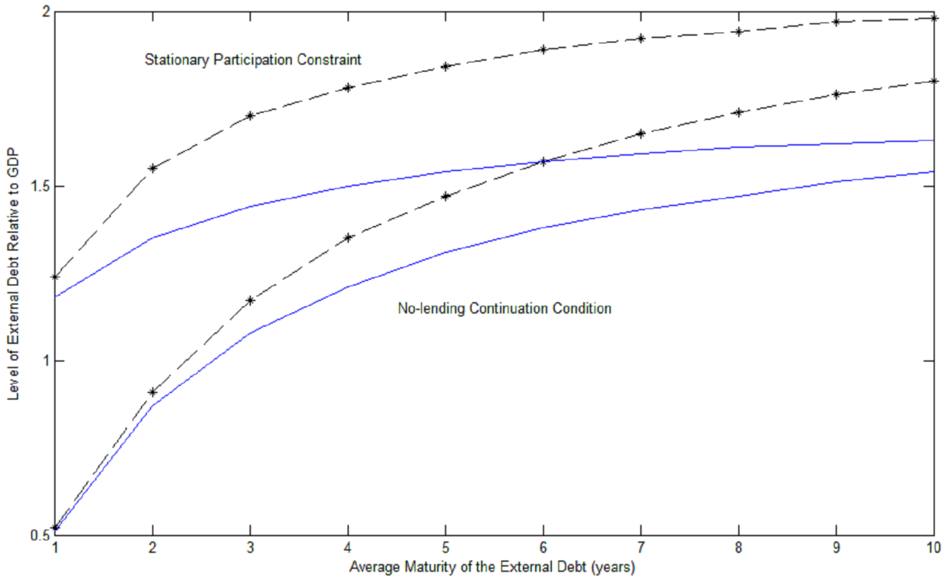

Figure 3: Haircut and the crisis zone (solid line, $\phi = 0$; dotted line, $\phi = 0.2$)

In Figure 4, all lines were plotted considering $\phi = 0$ and results show that the *ex-ante* effect of a lower risk-free interest rate, *i.e.* higher β, over the feasible leverage is very similar to the effect of haircut possibility. Lower interest rates on Greek bonds were observed in April 2014, after four years outside the international sovereign bond market. The country sold three billion euros of five-year maturity sovereign bonds with a 0.0495 yield (Financial Times, "Greek €3 bn bond sale snapped up", April 10, 2014). The ability to sell that amount is related to both (i) very low basic interest rates induced by central banks and (ii) low risk-spread of Greek bonds over German ones, the last resulting from the perception that the ECB would support Greek economy (and bonds).

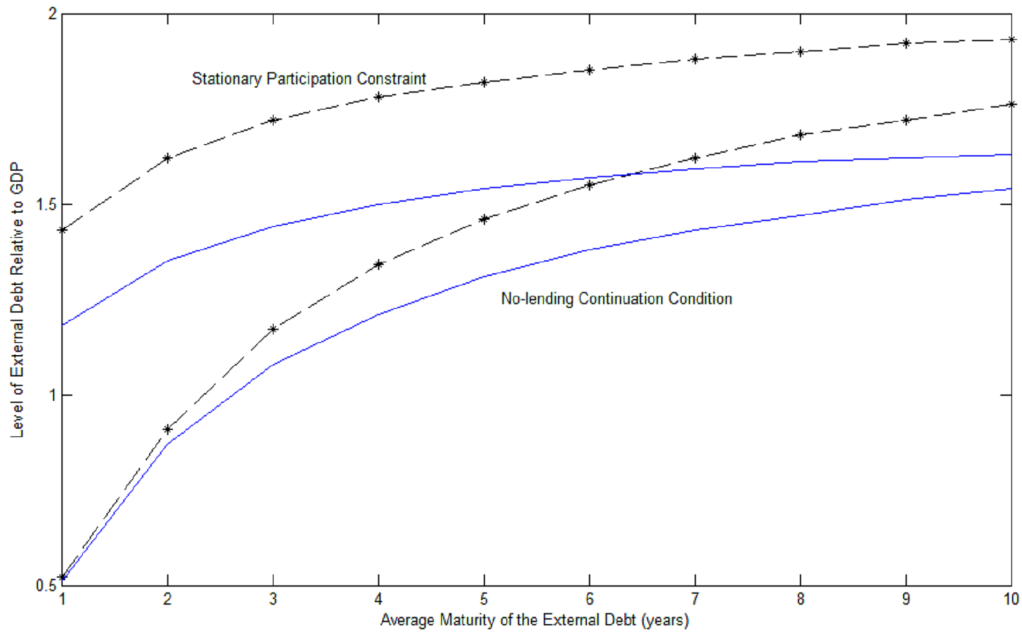

Figure 4: Eased monetary policy and the crisis zone (dotted: $r = 0.025$, solid: $r = 0.035$)

The results suggest that the combination of lower interest rates (higher β) and partial default possibility ($\phi > 0$) could lead to more leveraging of the public debt, instead of leading to the desired smoothed deleveraging. Nowadays, Greece is under the influence of both factors. Therefore, there should be extra caution regarding sovereign debt increase. Greek government should not be tempted by the attractively low interest rates that have been in effect in Europe with the cut in the ECB's benchmark interest.

3.2 Computed optimal fiscal policy

Next, we numerically compute the dynamic equilibrium and the effects of haircuts on the optimal debt policy. Such effects are intuitive. First, the partial repayment reduces the costs of default for creditors, making them more willing to lend while charging a lower premium. Second, as a partial default prevents productivity losses that are present under total default, therefore, in the aftermath of a crisis, the investment, output and tax collection tend to be higher than after an observed exit from the Euro area. From the government policy perspective, these benefits translate into a slower optimal adjustment when leaving the crisis zone, even when considering small deviations from the original total default case[15].

In Figure 5, the thinner line represents the exit from the crisis zone according to optimal debt policy by considering the original case ($\phi = 0$) and keeping all other parameters at the same values discussed before. The thicker line presents the debt policy function when ϕ jumps to 0.2. To capture not only the debt cost effect, but also the productivity effect, in Figure 5 we explicitly consider two productivity costs instead of one: a lower productivity loss associated with $\phi = 0.2$, namely $\alpha = 0.97$, and a higher productivity loss associated to $\phi = 0$, namely $\alpha = 0.95$. Previously, we kept α fixed and independent of ϕ to avoid losing generality. We do not need to assume different productivity levels to reach the qualitative results plotted in Figure 5, but it is convenient to see the debt cost effect on debt policy combined with the productivity effect.

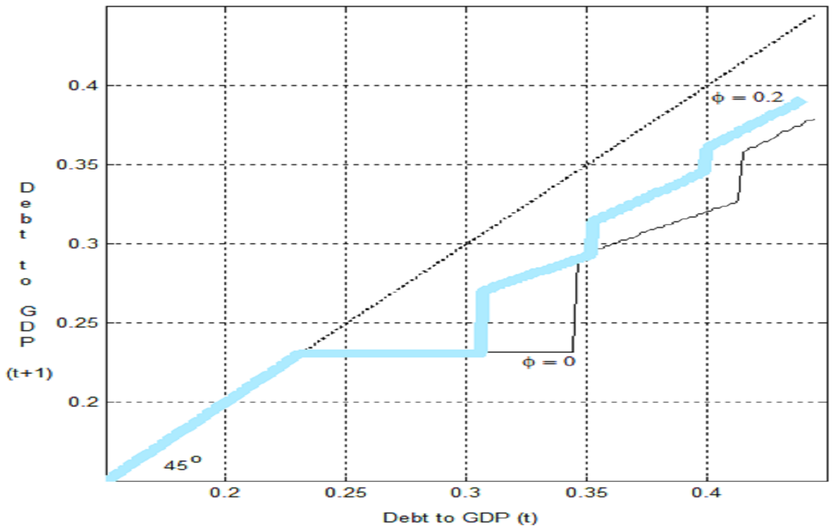

Figure 5: Haircuts and slower fiscal adjustments

4 Concluding Remarks

This paper discusses the effects of a now popular supportive policy in the aftermath of a sovereign crisis: haircuts resulting from debt renegotiation. We argue that haircuts on sovereign debt, despite reducing fiscal constraints, open the door for more leverage,

[15]Under low probability of crisis, the first effect (lower interest rates due to the expected haircut instead of no payment) is more than sufficient to decelerate the fiscal adjustment to exit the crisis zone.

delaying reductions in public expenditures. Combined with prolonged low interest rates, expectations of haircut in case of crisis contribute to an overleveraged economy, such as Greece, remaining in the crisis zone. An initial version of this paper[16] included productivity gains associated with increasing trade flows within a monetary union, following the productivity structure of Conesa and Kehoe [8]. For tractability, we replace the benefits of trade by a bargaining game setting an optimal haircut level due to a negotiated default. Implicitly, we assume prohibitive productivity costs in case of common currency exit (non-negotiated default). Extensions exploiting the international trade and trade benefits of common currency are challenging, but also promising.

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5 Appendix A

First, we highlight that, as already extensively discussed in the original Cole and Kehoe model ([5], [6], and [7]), a government that cares sufficiently more about private than public consumption or is sufficiently farsighted is guaranteed to have a non-empty crisis zone (i.e. $\overline{B} > \underline{B}$). Second, we highlight we are assuming the existence of a non-empty crisis zone for $\phi = 0$ and we show, in the end of this appendix, that the condition $\overline{B} > \underline{B}$ remains as long as ϕ has an upper limit. Next, we are going to detail the three conditions - (i), (ii), and (iii) - that ensure the result of our proposition, taking as given the existence of the crisis zone for $\phi = 0$. To derive such conditions note that the floor of the crisis zone, $\underline{B}$, is defined as the highest stationary debt level under which not defaulting ($z = 1$) is better than defaulting ($z = \phi < 1$), even when there is a speculative attack, *i.e.*, $q = 0$ and new debt is not available. Then, to compute the floor of the crisis zone, we need to compare $V(s, B\prime, 0, 1)$ with $V(s, B\prime, 0, \phi)$. Formally, $V(s, B\prime, 0, 1)$ is given by:

$$
\begin{aligned}
& V(s, B\prime, 0, 1) = \underline{C1} + \underline{G1} \\
& \underline{C1} \equiv \frac{(1-\theta)\left(f\left(K^n\right) - \delta K^n\right)}{1-\beta} \\
& \underline{G1} \equiv \frac{\left(1-\beta^T\right) v\left(g_n - \frac{B(1-\beta)}{1-\beta^T}\right) + \beta^T v\left(g_n\right)}{1-\beta} \\
& g_n \equiv \theta\left[f\left(K^n\right) - \delta K^n\right] \\
& K^n \text{ solves } \frac{1}{\beta} = (1-\theta)\left[f\prime(k') - \delta\right] + 1 \text{ and} \\
& \frac{\partial V(s, B\prime, 0, 1)}{\partial B} = \frac{\partial \underline{G1}}{\partial B} = -v\prime\left(g_n - \frac{B(1-\beta)}{1-\beta^T}\right) < 0
\end{aligned}
$$

where the payment of $\frac{B(1-\beta)}{1-\beta^T}$ must be made at maturities $1, 2, \ldots, T$, and the value function is decreasing in the current debt level as expected. $V(s, B\prime, 0, \phi)$ is given by:

$$
\begin{aligned}
&V(s, B\prime, 0, \phi) = \underline{C0} + \underline{G0} \\
&\underline{C0} \equiv (1-\theta)\left[\alpha f\left(K^n\right) - \delta K^n\right] + K^n - K^d + \frac{\beta(1-\theta)\left[\alpha f\left(K^d\right) - \delta K^d\right]}{1-\beta} \\
&\underline{G0} \equiv v\left(\theta\left(\alpha f\left(K^n\right) - \delta K^n\right) - \frac{\phi B(1-\beta)}{1-\beta^T}\right) + \\
&+\beta \frac{\left(1-\beta^{T-1}\right) v\left(g_d - \frac{\phi B(1-\beta)}{1-\beta^T}\right) + \beta^{T-1} v\left(g_d\right)}{1-\beta} \\
&g_d \equiv \theta\left[\alpha f\left(K^d\right) - \delta K^d\right] \\
&K^d \text{ solves } \frac{1}{\beta} = (1-\theta)\left[f\prime(k')\alpha - \delta\right] + 1 \\
&\frac{\partial V(s, B\prime, 0, \phi)}{\partial B} = \frac{\partial \underline{G0}}{\partial B} \text{ which is given by:} \\
&\left(\frac{-\phi}{1-\beta^T}\right)\left\{(1-\beta) v\prime\left(\theta\left(\alpha f\left(K^n\right) - \delta K^n\right) - \phi B_T\right) + \left(\beta - \beta^T\right) v\prime\left[g_d - \phi B_T\right]\right\} \\
&\text{where, } B_T \equiv \frac{B(1-\beta)}{1-\beta^T}
\end{aligned}
$$

and again, the debt payment of $\frac{\phi B(1-\beta)}{1-\beta^T}$ must be made during T periods, and the value function is decreasing in the current debt level as expected. For B sufficiently close to zero, it is trivial to conclude that $V(s(B \to 0), B\prime, 0, \phi) < V(s(B \to 0), B\prime, 0, 1)$, since after the default, the productivity becomes lower without significantly improving the public expenditure. Moreover, if B is high enough, higher than $g_n \frac{1-\beta^T}{(1-\beta)}$, then $z_t = 1$ is not an option as g cannot be negative. To ensure $z_t = \phi$ as the unique feasible option for this high debt level, ϕ must be lower than $\frac{g_d}{g_n}$[17]. Then, to assure the existence of the floor of the crisis zone, $\underline{B}$, it is sufficient to have $\phi < \frac{g_d}{g_n}$ and $\frac{\partial V(s,B\prime,0,1)}{\partial B} < \frac{\partial V(s,B\prime,0,\phi)}{\partial B}$, or, more than sufficient to have:

$$
\frac{v\prime\left(\theta\left(f\left(K^n\right) - \delta K^n\right) - \frac{B(1-\beta)}{1-\beta^T}\right)}{v\prime\left(g_d - \phi \frac{B(1-\beta)}{1-\beta^T}\right)} > 1 > \phi
$$

assured by

$$
\begin{aligned}
&\theta\left[f\left(K^n\right) - \delta K^n\right] - \theta\left[\alpha f\left(K^d\right) - \delta K^d\right] < \frac{B(1-\beta)}{1-\beta^T}(1-\phi) \\
&\phi < 1 - \frac{\theta\left(f\left(K^n\right) - \delta K^n\right) - \theta\left(\alpha f\left(K^d\right) - \delta K^d\right)}{\frac{B(1-\beta)}{1-\beta^T}}
\end{aligned}
$$

i.e., funds raised by the government due to the partial default must be higher than the reduction of the government revenue from tax collection.

[17] Redundant given condition (iii) - to be derived.

The cap of the crisis zone, $\overline{B}$, is defined as the highest stationary debt level under which not defaulting ($z = 1$) is better than defaulting ($z = \phi < 1$) when there is no speculative attack, *i.e.*, $q = \beta$ and new lending is available. Then, to compute the cap of the crisis zone, we need to compare $V(s, B\prime, \beta, 1)$ with $V(s, B\prime, \beta, \phi)$. Formally, $V(s, B\prime, \beta, 1)$ is given by:

$$
\begin{aligned}
&V(s, B\prime, \beta, 1) = \overline{C1} + \overline{G1} \\
&\overline{C1} \equiv \frac{(1-\theta)\left(f\left(K^n\right) - \delta K^n\right)}{1-\beta} \\
&\overline{G1} \equiv \frac{v\left(g_n - B\left(1-\beta\right)\right)}{1-\beta} \\
&\frac{\partial V(s, B\prime, \beta, 1)}{\partial B} = \frac{\partial \overline{G1}}{\partial B} = -v\prime\left(g_n - B\left(1-\beta\right)\right)
\end{aligned}
$$

where the payment of $B(1-\beta)$ is the interest rate charged on the total debt. The value function is decreasing in the current debt level as expected. $V(s, B\prime, \beta, \phi)$ is given by:

$$
\begin{aligned}
&V(s, B\prime, \beta, \phi) = \overline{C0} + \overline{G0} \\
&\overline{C0} \equiv (1-\theta)\left[\alpha f\left(K^n\right) - \delta K^n\right] + K^n - K^d + \frac{\beta}{1-\beta}\left[(1-\theta)\left(\alpha f\left(K^d\right) - \delta K^d\right)\right] \\
&\overline{G0} \equiv v\left(\theta\left(\alpha f\left(K^n\right) - \delta K^n\right) - B_T\left(\phi - \beta^T\right)\right) + \\
&+\frac{\beta\left[\left(1-\beta^{T-1}\right) v\left(g_d - \phi B_T\right) + \beta^{T-1} v\left(g_d\right)\right]}{1-\beta} \\
&B_T \equiv \frac{B\left(1-\beta\right)}{1-\beta^T} \text{ and } q_{_T} = \beta^T \\
&\left(-\frac{\partial V(s, B\prime, \beta, \phi)}{\partial B}\right) \text{ is given by:} \\
&\frac{(1-\beta)\left(\phi - \beta^T\right)}{1-\beta^T} v\prime\left(\theta\left(\alpha f\left(K^n\right) - \delta K^n\right) - B_T\left(\phi - \beta^T\right)\right) + \\
&+\phi \frac{\beta\left(1-\beta^{T-1}\right)}{1-\beta^T}\left[v\prime\left(g_d - \phi B_T\right)\right]
\end{aligned}
$$

For B sufficiently close to zero, it is trivial to conclude that $V(s\left(B \rightarrow 0\right), B\prime, \beta, \phi) < V(s\left(B \rightarrow 0\right), B\prime, \beta, 1)$, since after the default, the productivity becomes lower without significantly improving the public expenditure. Moreover, if B is high enough, higher than $\frac{g_n}{(1-\beta)}$, then $z_t = 1$ is not an option as g cannot be negative. To ensure $z_t = \phi$ as the unique feasible option for this high debt level, ϕ must be lower than $\left(1-\beta^T\right)\frac{g_d}{g_n}$. Given this condition, to ensure the existence of the cap of the crisis zone, $\overline{B}$, it is sufficient to have $\frac{dV(s,B\prime,\beta,1)}{dB} < \frac{dV(s,B\prime,\beta,\phi)}{dB}$, or, by considering $\left(\phi \leq \beta^T\right)$, it is sufficient to have:

$$\frac{v\prime\left(g_n - B\left(1-\beta\right)\right)}{v\prime\left(g_d - \phi B_T\right)} > \phi\beta$$

assured by

$$g_n - B\left(1-\beta\right) < g_d - \phi B_T$$
$$\phi B_T + \left(g_n - g_d\right) < B\left(1-\beta\right)$$

or

$$\phi < \left(1-\beta^T\right)\frac{B\left(1-\beta\right) - \left(g_n - g_d\right)}{B\left(1-\beta\right)}$$

Then, we have two conditions for the existence of $\overline{B}$. First, partial default must occur on the principal amount of the debt, *i.e.*, partial default must imply negative return on Bond $\left(\phi < \beta^T\right)$. Note that, after a payment of ϕ, the bond return-rate is equal to $\left(\frac{\phi}{\beta^T} - 1\right)$. Second, the flow reduction of the government revenue from both tax collection $\left(g_n - g_d\right)$ and partial repayment of maturing debt $\left(\phi B_T\right)$ must be lower than the total interest payment on the total debt $B\left(1-\beta\right)$. Therefore, the conditions for $\underline{B}$ and $\overline{B}$ to be well defined are the following:

$$\begin{aligned}
&\phi B_T + \left(g_n - g_d\right) < \frac{B\left(1-\beta\right)}{1-\beta^T} \\
&\phi B_T + \left(g_n - g_d\right) < B\left(1-\beta\right) \\
&\phi B_T + \left(g_n - g_d\right) < \beta^T B_T + \left(g_n - g_d\right) \\
&\phi < \left(1-\beta^T\right)\frac{g_d}{g_n}
\end{aligned}$$

and noting that the first condition is redundant, we can focus only in the following three conditions:

$$\begin{array}{lrcl}
(i) & \phi B_T + \left(g_n - g_d\right) & < & B\left(1-\beta\right) \\
(ii) & \phi & < & \beta^T \\
(iii) & \phi & < & \left(1-\beta^T\right)\dfrac{g_d}{g_n}
\end{array}$$

Again, as already discussed in the original paper ([5]), a government that cares sufficiently more about private than government consumption or is sufficiently farsighted is guaranteed to have a crisis zone ($\overline{B} > \underline{B}$). For a non-empty crisis zone, conditions (i), (ii), and (iii) are sufficient to ensure the result of proposition, *i.e.*, the higher the ϕ, the higher are the limits $\overline{B}$ and $\underline{B}$. Note that since having $\underline{B}$ increasing in ϕ, as $V(s, B\prime, 0, 1)$ does not depend on ϕ, it is sufficient to show that $\frac{\partial V(s,B\prime,0,\phi)}{\partial\phi} < 0$, which is true, since:

$$\begin{aligned}
&\Big[\left(1-\beta\right) v\prime\left(\theta\left(\alpha f\left(K^n\right) - \delta K^n\right) - \tfrac{\phi B(1-\beta)}{1-\beta^T}\right) + \\
&+\beta\left(1-\beta^{T-1}\right) v\prime\left(g_d - \tfrac{\phi B(1-\beta)}{1-\beta^T}\right)\Big] > 0
\end{aligned}$$

and to have $\overline{B}$ increasing in ϕ, it is sufficient to show that $\frac{\partial V(s,B\prime,\beta,\phi)}{\partial\phi} < 0$, which is true:

$$\begin{array}{c} -B_T \ \left\{ v\prime \left(\theta \left(\alpha f \left(K^n \right) - \delta K^n \right) - B_T \left(\phi - \beta^T \right) \right) + \right. \\ \left. + \frac{\beta\left[\left(1-\beta^{T-1}\right) v\prime\left(g_d - \phi B_T\right)\right]}{1-\beta} \right\} < 0 \end{array}$$

Finally, in order to assure that the crisis zone characterized under total default remains non-empty under partial default, i.e. to assure that $\underline{B}$ remains smaller than $\overline{B}$ when partial repayment increases, it is sufficient to show that:

$$\frac{\frac{\partial V(s,B\prime,\beta,\phi)}{\partial\phi}}{\frac{\partial V(s,B\prime,\beta,1)}{\partial B}} > \frac{\frac{\partial V(s,B\prime,0,\phi)}{\partial\phi}}{\frac{\partial V(s,B\prime,0,1)}{\partial B}} \Leftrightarrow$$

$$\frac{-B_T \left\{ v\prime \left(\theta \left(\alpha f \left(K^n \right) - \delta K^n \right) - B_T \left(\phi - \beta^T \right) \right) + \frac{\beta\left[\left(1-\beta^{T-1}\right) v\prime\left(g_d - \phi B_T\right)\right]}{1-\beta} \right\}}{-v\prime \left(g_n - B \left(1 - \beta \right) \right)}$$

$$>$$

$$\frac{-B_T \left[v\prime \left(\theta \left(\alpha f \left(K^n \right) - \delta K^n \right) - \phi B_T \right) + \frac{\beta\left(1-\beta^{T-1}\right)}{1-\beta} v\prime \left(g_d - \phi B_T \right) \right]}{-v\prime \left(g_n - \frac{B(1-\beta)}{1-\beta^T} \right)}$$

$$\Leftrightarrow$$

$$\frac{v\prime \left(g_n - \frac{B(1-\beta)}{1-\beta^T} \right)}{v\prime \left(g_n - B \left(1 - \beta \right) \right)}$$

$$>$$

$$\frac{\left[v\prime \left(\theta \left(\alpha f \left(K^n \right) - \delta K^n \right) - \phi B_T \right) + \frac{\beta\left(1-\beta^{T-1}\right)}{1-\beta} v\prime \left(g_d - \phi B_T \right) \right]}{\left\{ v\prime \left(\theta \left(\alpha f \left(K^n \right) - \delta K^n \right) - B_T \left(\phi - \beta^T \right) \right) + \frac{\beta\left[\left(1-\beta^{T-1}\right) v\prime\left(g_d - \phi B_T\right)\right]}{1-\beta} \right\}}$$

And as $\theta \left(\alpha f \left(K^n \right) - \delta K^n \right) - B_T < g_n - B_T < \theta \left(\alpha f \left(K^n \right) - \delta K^n \right) - \phi B_T$, then it is sufficient to have

$$\begin{aligned} B_T - B \left(1 - \beta \right) &> \phi B_T \\ \phi &< 1 - \left(1 - \beta^T \right) \\ \phi &< \beta^T \end{aligned}$$