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Multivariate Stochastic Volatility-Double Jump Model: an application for oil assets

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Abstract

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We propose a new multivariate model to capture the presence of jumps in mean and conditional variance in the returns of oil prices and companies in this sector. The model is based on the presence of common factors associated with jumps in mean and variance, as it performs a decomposition of the conditional variance of each asset as the sum of the common factor plus a specific transitory factor in a multivariate stochastic volatility structure. The estimation is made through Bayesian methods using Markov Chain Monte Carlo. The model allows recovering the changes in prices and volatility patterns observed in this sector, relating the jumps with the events observed in the period 2000-2015. We apply the model to estimate risk management measures, hedging and portfolio allocation and performing a comparison with other multivariate models of condi-tional volatility. Based on the results, we may conclude that the proposed model has a better performance when used to calculate portfolio VaR, since it does not reject the hypothesis of correct nominal coverage with certain specifications presented in this work. Furthermore, we conclude that the model can be used to hedge oil price risks, through the optimal hedge ratio for a portfolio containing an oil company as-set (stock) and the oil price contract. When compared to the standard methodology based on GARCH models, our model performs well in this application.

Keywords: Oil Prices, Jumps, Stochastic Volatility, Risk Management
JEL Classification: C58; G11; G17

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1 Introduction

A stylized fact in the evolution of oil prices is the presence of sudden changes in prices and volatility patterns. These events have a major impact on the cost and revenue structure of the companies in this sector, and thus the correct identification of these events is critical to the financial planning process and ultimately to the processes of risk management and allocation of portfolios.

Due to the importance of the jumps and the presence of a persistent volatility in oil prices, a comprehensive literature has been developed to analyze these phenomena. We can cite as recent examples of this literature Askari and Krichene (2008), Lee et al. (2010), Larsson and Nossman (2011), Ozdemir et al. (2013) and Gronwald (2012).

Our work presents a new contribution to this literature by introducing a new multivariate model that allows the modeling of common jumps in mean and conditional variance for log returns of oil prices and of companies in this sector. Our model allows us to identify which jumps are shared between oil and stock prices, and also allows a decomposition of permanent and transient effects on the conditional volatility structure. The model allows to estimate the common factor in the conditional volatility of oil sector, and the volatility for each asset is given by this common factor plus a specific mean reverting component. This analysis of common jumps is inserted into the literature of estimation of jumps in stock indexes (e.g. Kaeck and Alexander (2012), Kaeck and Alexander (2013)) and especially in the analysis of joint events in prices and asset returns, e.g. Asgharian and Bengtsson (2006), Bollerslev et al. (2008), Clements and Lia (2013), and Pukthuanthong and Roll (2014), and also has some connections with the methods used in jump detection in continuous time (Barndorff-Nielsen and Shephard (2006) and Jacod and Todorov (2009)).

This model generalizes different methodologies applied in the modeling of oil prices. It allows to capture jumps in the structure of the oil price returns, similar to the works of Larsson and Nossman (2011) and Baum and Zerilli (2015), but in our model these jumps are formulated as a common factor to all analyzed assets, and thus we can identify which jumps are common between oil and stock prices. This formulation can also be interpreted as regime switching model, where we have regimes with common shocks and regimes with only specific shocks for each asset. The model is also related to the analysis in Elyasiani et al. (2011), Narayan and Sharma (2011) and Sim and Zhou (2015), relating the impact of oil price fluctuations in the return dynamics of individual stocks and industries.

Our work is based on the use of a factor structure for the multivariate stochastic volality process. The use of factor structures in conditional volatility models has a long tradition in finance, for both models based on ARCH (Diebold and Nerlove (1989)) and stochastic volatility (Pitt and Shephard (1999)) specifications. Factorial designs and related principal components decompositions (e.g. Alexander (2002) and Hu and Tsay (2014)) allow parsimonious representation of volatility process, reducing the problems associated with the high number of parameters in multivariate models and also allowing a clearer interpretation of common volatility processes.

The conditional volatility structure of this model is based on the multivariate specification proposed in Laurini and Mauad (2015), where the conditional variance of each asset is decomposed into a common factor and a specific mean reverting factor. This common factor can be interpreted as the level of the conditional volatility, and is modelled using a structure of random level shifts proposed by Qu and Perron (2013). In this formulation the conditional variance is subject to the presence of jumps that represent changes in the level, and these jumps are modeled by a compound Binomial process. This formulation allows to capture in a simple and intuitive way the impact of jumps that alter the level of volatility for each asset, and can also be interpreted as a switching model for the conditional variance (e.g. Fong and See (2001) and Gias and Ramos (2014)), but where the number of regimes is not fixed, allowing great flexibility in capturing the risk patterns embedded in each asset, as discussed in Qu and Perron (2013) and Laurini and Mauad (2015).

We analyze both WTI (West Texas Intermediate) and Brent prices for oil, and the returns of four large oil companies: Petrobras, Exxon Mobil, Chevron and British Petroleum. The results show that there is a relation between these cited series, so that the model can in fact provide information for investors or oil traders in physical market and help with decision making. Companies such as Exxon and Chevron, besides Brent and WTI price series, show relatively low volatility persistence parameters when compared to Petrobras and British Petroleum. This means that their reversion to the average level of volatility is faster, indicating more autonomous volatility dynamics for Petrobras and BP in relation to shocks in oil prices.

To show the advantages of the specification proposed in this article, we performed comparative analyzes with other multivariate conditional volatility models used in the literature, and check the model performance in practical applications in risk management and portfolio allocation, calculating Value At Risk measures and building minimum global variance and hedge portfolios. The results indicate that the proposed model performs well in these applications, indicating the validity of the proposed formulation.

Calculating the dynamic Value at Risk (VaR) for the series we obtain better results when using the volatility estimated by our model when compared to the VaR calculation using the univariate calculation of volatility of a GARCH(1,1) model. Besides the VaR, we have also applied the model to the construction of hedge portfolios, minimizing the exposure of oil companies to the risk associated with changes in oil prices. We have built the hedge portfolios by combining individually Petrobras, Exxon, Chevron and British Petroleum with WTI oil, setting the optimal hedge portfolio by a given rule presented in this work. Comparing the results using the conditional variances and covariances estimated by the model proposed in this article and the DCC model, we can see that the portfolio that uses the optimal weight estimated using the double jumps model generates a smaller variance for most of the analyzed companies. More details on these applications are presented throughout the work.

This work has the following structure - in Section 2 we present the model and the Bayesian estimation methodology based on Markov Chain Monte Carlo. The database and chronology of the major events in the oil sector for the analysed period are described in Section 3, and the model estimation results and some adjustment measures are showed in Section 4. Applications in risk management and portfolio allocation are placed in Section 5, and the final conclusions are in Section 6.

2 Multivariate Double Jumps Model

The model is based on a generalization of the multivariate stochastic volatility with common jumps model proposed in Laurini and Mauad (2015), combining the structure of random jumps in volatility level originally proposed in Qu and Perron (2013), but introducing the possibility of common (joint) jumps on mean and conditional volatility. The model is based on the following specification:

$$y_{it} = exp\left(\frac{h_{it}}{2} + \frac{s_i^v \mu_t}{2}\right)(\varepsilon_{it}) \tag{1}$$

$$h_{it} = \phi_i h_{it-1} + \sigma_{\upsilon_i} \upsilon_{it} \tag{2}$$

$$\mu_t = \mu_{t-1} + \delta_t^v \sigma_\eta \eta_t \tag{3}$$

$$\gamma_{it} = s_i^m \delta_t^m \sigma_\nu \nu_t \tag{4}$$

$$h_{i,jt} = s_i^v s_j^v \mu_t \tag{5}$$

$$\delta_t^v \sim Bern(p_v),\, \delta_t^m \sim Bern(p_m) \tag{6}$$

$$\varepsilon_{it} \sim N(\gamma_{it}, 1),\, \upsilon_{it} \sim N(0,1),\, \nu_t \sim N(0,1) \tag{7}$$

The model is based on a multivariate stochastic volatility structure (Taylor (1986)), but modified to include the possibility of common jumps in the mean and conditional volatility. In this model y_{it} denote the set of observed series, log returns of financial assets. The conditional volatility process for each asset is obtained as the sum of a mean reverting component (specific) for each series, and a common factor, modeled using a compound Binomial process.

The common factor in the conditional variance is given by a compound Binomial process, represented by the latent component μ_t, with δ_t^v being a sequence of independent Bernoulli variables with common parameter p_v, representing the probability in jumps in variance. The transitory component in the volatility for the series i is represented by h_{it}, parameterized as first-order autoregressive processes, with persistence parameter ϕ_i and volatility σ_{υ_i}. The conditional volatility of each series is given by the sum of transitory component plus the common factor μ_t, multiplied by a scale factor s_i^v. We normalize s_1^v, the scale for WTI oil, to one to identify the volatility process, as usual in factor models.

In this representation the process μ_t depends on the realization of δ_t^v variable. If this variable is sampled with value zero (no jump), the process μ_t remains with the same value of prior period (μ_{t-1}). If the realization of the variable δ_t^v is one, indicating the occurrence of a jump, the process μ_t is given by the previous value μ_{t-1} plus an innovation from a white noise Gaussian process, with volatility given by the parameter σ_η, which represents the random intensity of jumps.

As we mentioned earlier, this structure can be interpreted as a regime changing model, but where the number of regimes is not fixed and determined by the observed data. The component μ_t represents the conditional mean of the process, subject to level changes. In this way the model can be thought of with a generalization with multiple long-run values for the variance process of the decomposition of transitory and permanent components of volatility used by Ahmed et al. (2012) to model oil prices, based on Component GARCH models.

Similarly the model can also be related to the decomposition of unconditional variance regimes based on Markov Regime Switching models, used to analyze the prices of Brent and Wti prices used in Zhang and Zhang (2015). This analysis is based on the presence of three regimes for unconditional variance, based on the formulation proposed by Hamilton (1989). Our model is more flexible for not assuming a fixed number of regimes, and allow the joint modeling of various assets. Our decomposition of transitory and permanent components based on common and specific volatility factors allows to analyze the dynamics of these series in a much more intuitive way. Thus, our model combines and generalizes other modeling methods applied to financial and oil prices.

The second component of the model is the structure of common jumps in the mean. This structure is given by the process γ_{it}, represented by a process of independent Bernoulli jumps δ_T^m multiplied by a factor of variance σ_V, which captures the size of jumps. Distinct from the process of jumps in conditional variance, the jump process in mean is not persistent and only impacts the returns in period t. Again we use scale parameters, denoted by s_i^m, to represent the intensity of jumps in each asset, where we also impose that the scale for WTI is equal to one. To incorporate the errors in mean for each asset, the component ε_{it} affecting each series is given by a Gaussian white noise process with mean γ_{it} and unit variance. When there is no jumps in the error process in mean reduces to a Gaussian white noise with unit variance, similar to a standard SV process (Eq. 1).

2.1 Estimation

To perform the inference procedure we use Bayesian estimation using Markov Chain Monte Carlo methods, generalizing the procedure proposed in Laurini and Mauad (2015). The methodology is based on a mixture of Gibbs sampling and Metropolis-Hastings algorithms to sample the parameters and latent components. In special, the model is based on a data augmentation process to sample the jump processes in mean and conditional variance.

We use a methodology based in the threshold exceedance methodology proposed by Albert and Chib (1993), using an auxiliary latent variable with uniform distribution to perform the sampling for the Bernoulli variables corresponding to the occurrence of jumps. The jump processes δ_t^v and δ_t^m assume value one if this auxiliary variables exceed specific threshold values, and assumes zero value if are sampled below this thresholds. The threshold values are calibrated in accordance with the probability of jumps p_v and p_m, whose prior probabilities are assumed as generated by Beta densities.

Our sampling procedure, similar to Laurini and Mauad (2015), is based on a mixture of Gibbs sampling algorithms for the conjugated densities and Metropolis-Hastings sampling for the non-conjugated processes. We avoid the linearization step in Qu and Perron (2013), using a Metropolis-Hastings procedure to sample the non-linear steps, and not needing the use of the mixture of normals sampler of Kim et al. (1998).

The data augmentation process determining the Bernoulli thresholding is given by a sequence of independent standard Uniform densities. We assume Beta priors for the probabilities of jumps in the model, independent Gaussian distributions for the persistence parameters ϕ_i and Gamma densities for all the volatility parameters. The hyperparameters used in the priors are disponible with the authors.

The sampling algorithm can be summarized by the following steps:

1. Initialize the latent variables and parameters h_{it}, μ_t, δ_t^v, δ_t^m, ϕ_i, σ_{υ_i}, p_v, σ_η, s_i^v, σ_V , s_i^m;

2. Sampling ϕ_i, s_i^v, σ_η, σ_{υ_i}, conditional on h_{it}, μ_t and the location of the jumps δ_t^v,

3. Sampling μ_t;

4. Sampling h_{it};

5. Sampling the process of jumps δ_t^v;
6. Sampling the probability of jumps p_v;
7. Sampling s_i^m, σ_V, σ_{υ_i}, conditional on the location of the jumps δ_t^m,
8. Sampling γ_{it};
9. Sampling the process of jumps δ_t^m;
10. Sampling the probability of jumps p_m;
11. Go back to step 2.

As commented above, the sampling procedure in steps 2-11 is based on Gibbs Sampling for the conjugated distributions and Metropolis-Hastings using the Slice Sampler of Neal (2003) for the non-conjugated posterior distributions, according to assumed prior distributions and the non-linear structure of the model. The inference procedure is based on a burn-in of 8,000 samples, and calculating the all posterior distributions using 24,000 additional samples[1].

3 Database

We apply the model in Section 2 for a set of six assets. In this group we include as oil price references the weekly log returns in US Dollars of WTI (West Texas Intermediate) and Brent oil prices, the two main references of price formation in oil market. The dataset also includes the dollar log-returns of Petrobras, Exxon, Chevron and British Petroleum, some of the largest oil companies in the world. The sample consists of weekly returns between 07/01/2000 and 01/09/2015, calculated using the closing price of every Friday. The total sample consists of 784 observations for each series. Although it's possible to use daily data, we use weekly data to avoid synchronization problems in the jumps between different series. As we are seeking common factors associated with jumps in the series, the weekly data provides enough time for a change in one of the series to impact the others and, therefore, the common jumps captured are more accurate using this frequency of data.

Figure 1 shows the returns for oil prices and the analyzed companies, and Table 1 presents descriptive statistics of the data used in this work. We can see in this table the presence of some stylized facts related to financial series, linked to the presence of conditional volatility structures. Returns have estimated means close to zero, and we

[1]Details on the Implementation details, the hyperparameters used in the priors, and convergence measures are not presented for space reasons but can be obtained from the authors.

can observe that the series are asymmetrical and have heavy tails, consistent with the presence of jumps and time varying volatility. The presence of jumps in the mean can also be observed by high maximum and minimum values observed in all series.

We can observe in Figure 1 that in 2014 the volatility is higher and the returns are lower. The oil prices have decreased since mid-2014, when WTI reached more than US$ 100,00 and fell to less than US$ 50,00 in about six months. Besides the high oil inventories around the world due to the application of new technologies in the sector, the demand was below expectations in 2014, leading to a sharp decline in prices. Moreover, China showed lower growth expectations in 2014, with lower demand for raw materials from other countries and less production, leading to higher volatility in the markets and declines in oil prices.

Another reason for this fall is the fact that America has became the largest oil producer in the world. Though it does not export crude oil, it now imports much less, creating a lot of spare supply. Besides, the Saudis and their Gulf allies have not tried to restore market prices, in order not to lose their market share. If they decided to curb their production, prices could rise, but the main benefits of such an action would go to countries like Iran and Russia. As Saudi Arabia has a large amount in reserves (US$ 900 bi), it can tolerate lower oil prices for a longer time. Besides, their oil extraction costs very little (aroud US$ 5 per barrel).

	WTI	Brent	Petrobras	Exxon	Chevron	British Petr.
mean	0.001	0.001	0.001	0.001	0.002	-0.001
sd	0.043	0.042	0.065	0.030	0.033	0.030
max	0.251	0.200	0.263	0.095	0.155	0.135
min	-0.192	-0.232	-0.364	-0.223	-0.317	-0.136
skewness	-0.455	-0.599	-0.640	-0.844	-1.231	-0.346
kurtosis	6.486	5.489	5.963	7.568	13.935	5.175

Table 1: *Descriptive Statistics*

	WTI	Brent	Petrobras	Exxon	Chevron	British Petr.
WTI	1.000	0.825	0.266	0.203	0.265	0.123
Brent	0.825	1.000	0.247	0.155	0.216	0.089
Petrobras	0.266	0.247	1.000	0.434	0.493	0.280
Exxon	0.203	0.155	0.434	1.000	0.817	0.437
Chevron	0.265	0.216	0.493	0.817	1.000	0.473
British Petr.	0.123	0.089	0.280	0.437	0.473	1.000

Table 2: *Correlation - Weekly log returns*

In Table 2 we present the linear correlation matrix estimated for the analyzed returns. As expected we noticed a strong correlation between WTI and Brent returns, and we can observe a significant correlation between Exxon and Chevron returns. This table reflects the patterns observed in Figure 1.

To set the context of the sector, we perform an annual chronology of major events in the period, and relate these events with major shocks identified by the model throughout the work. The main events related to the oil sector are presented below.

- 2000: Successive small falls in oil prices were derived from the increase in OPEC production quotas which sought to curb rising prices. This increase was the result of global economic recovery in late 90s (4 negative spikes at the beginning of the series).

- 2001: With the weaker US economy and increased production ceases recovery in prices. The terrorist attacks in Sept/2001 cause the first big negative spike in oil series. OPEC does not react in 2001.

- 2002: OPEC and non-OPEC countries (especially Russia) decide to cut production leading to the recovery in prices. At the end of the year, the low US inventories and internal problems in Venezuela (Chavez and strike at PDVSA) cause skyrocketing prices.

- 2003: OPEC decides to increase production, down prices; it is the second large negative spike at the beginning of the series. Thereafter (2003 and 2004) the recovery of the US economy and the strong demand coming from Asian growth drive prices sharply.

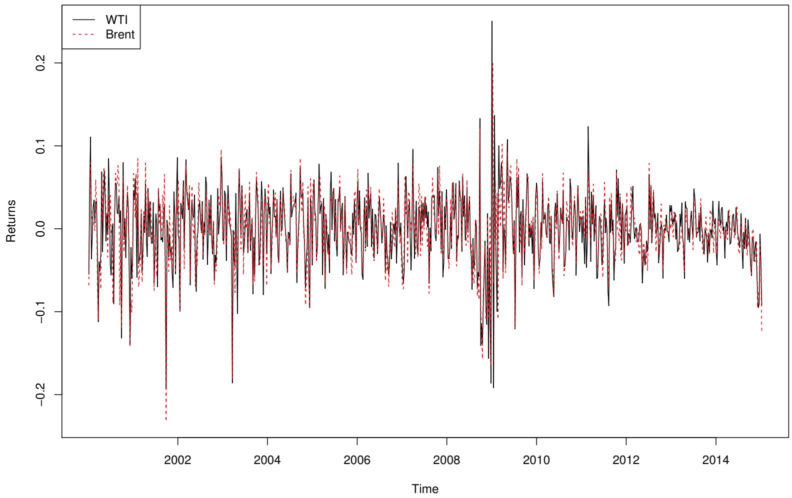
- 2005: Early in the second half damages from Katrina and Rita Hurricanes drive further prices.

- 2006: This year the inventories of OECD countries (Organization for Economic Co-operation and Development) grew a lot and in the second semester prices began a path of gradual decline (note that there is no large spikes in this movement). OPEC reacts with a cut in production and at the end of the year there is a positive spike.

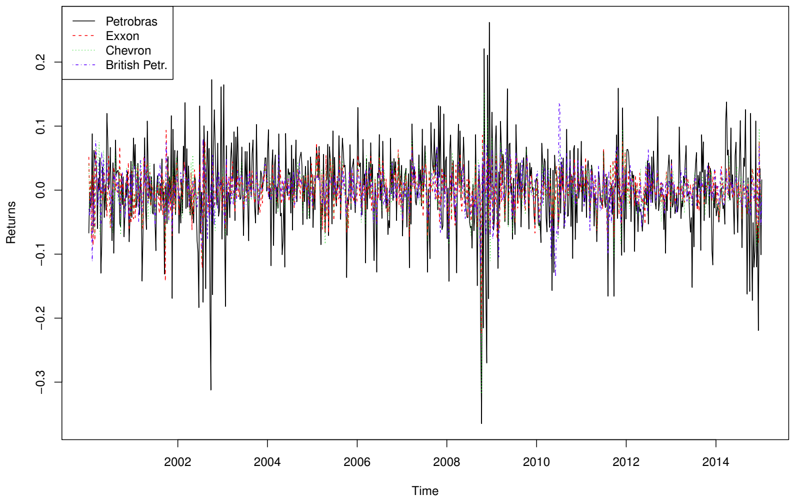
- 2007: OPEC keeps the production cut (positive prices spikes in the beginning of year). Begins a recessive process in the US economy.

- 2008: The recessive process weakens the stock prices and financial markets are positively affected by the good performance of commodities. Begins a speculative process with oil prices. The second semester is marked by the beginning of financial crisis, and the anticipation of global recession negatively affects prices. Amid the decline there is a period of strong volatility.

Figure 1: *Weekly log returns*



(a) Weekly log returns for WTI and Brent



(b) Weekly log returns for Petrobras, Exxon, Chevron and British Petroleum

- 2009: OPEC drastically cut production. Asia returns to increase demand and prices have strong recovery. Most positive spike in the series.
- 2011: At the beginning of the year prices shoot (large positive spike) as a result of lost production in Libya (Arab Spring) and the possible spread of the crisis by oil producing Arab countries. In the second semester the Libyan production is restored and prices recede (negative spike).
- 2012-2014: During this period the prices range between from US$ 80 to US$ 100 (from 2012 to mid-2014). With high prices there is a strong stimulus for new discoveries and application of new technologies (shale production). Oil inventories around the world increase and the demand grew below expectations. The recovery of the US economy was slow (and still ongoing). This results in a sharp decline in prices in the second half of 2014 (successive negative spikes).

Besides these aspects that explain the changes in oil prices, we can mention some specific events that affect the companies. The transitory volatility factors for British Petroleum and Petrobras show greater persistence compared to other companies. This pattern is also consistent with some relevant events with these companies.

The volatility of British Petroleum is pronounced in the first half of 2010, reflecting the accident in the Gulf of Mexico in April 2010, (known as BP oil spill, Macondo blowout). The platform leased to British Petroleum suffered an explosion and sank causing a leak that lasted for a long period, resulting in a substantial drop in its stock prices.

Petrobras was affected by a series of events from the beginning of our sample. In the second half of 2002 the uncertainty about the results in the presidential election affected stock market prices in Brazil, especially Petrobras. In late 2007 and early 2008 Petrobras prices were affected by the poor financial performance shown in the last quarter of 2007, when Petrobras presented a drop in profits of 17% caused by the appreciation of the Brazilian Real, reducing the revenue generated in US dollars, and the great capital injection made in the background the company's pension fund.

In the second half of 2010 Petrobras conducted a capitalization process, generating great speculation about what would be the offered price, and there was the unexpected second round in the presidential election, generating high volatility in prices. In late 2011 the return on equity (ROE) was 10%, in contrast to the value of 30% in 2004, showing to the stockholders the weakness of internal management and the increasing levels of leverage. In late 2014 there was a substantial drop in the oil prices, which coupled with research on internal corruption in the company led to consecutive declines in the company's prices and a sharp increase in volatility.

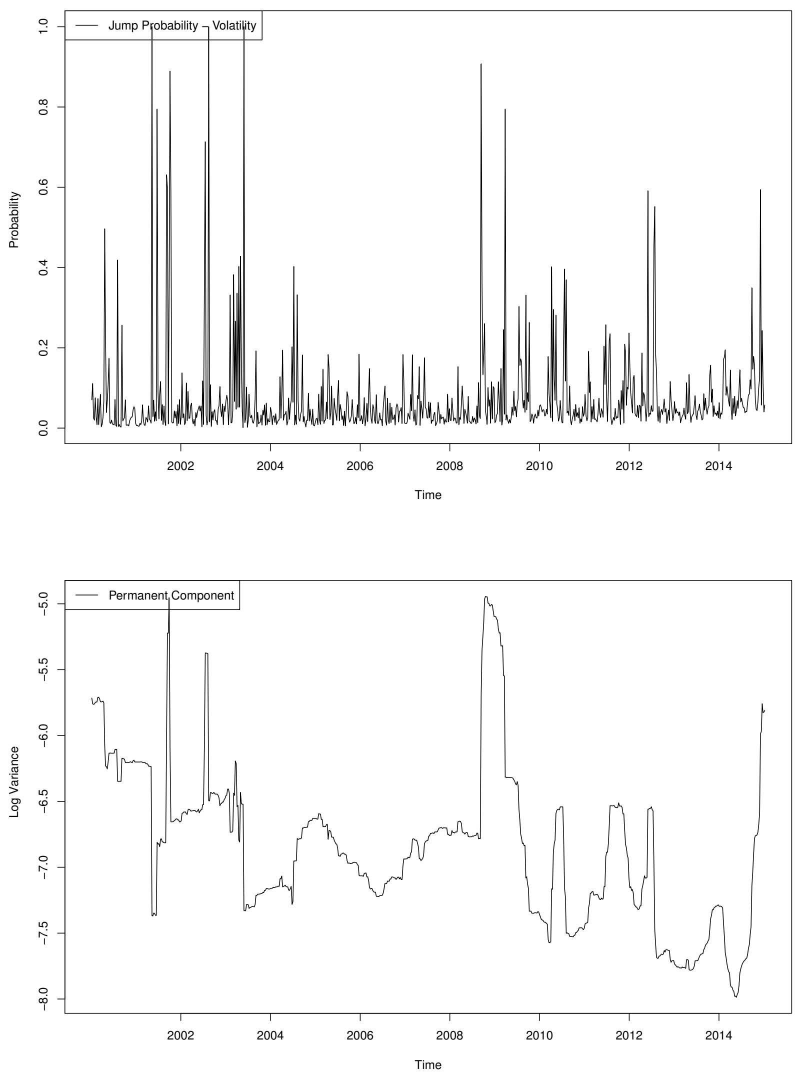
Our results are related to some other findings in the literature. Juvenal and Petrella (2015) obtains evidence that global demand shocks are responsible for most of price fluctuations, but speculative shocks also are important to explain the oil price increase between 2004 and 2008. A similar conclusion was find in Kaufmann (2011) and Morana (2013), and Cifarelli and Paladino (2010) relates the speculation process with the conditional variance observed in oil prices. Aastveit et al. (2014) geographically decompose the demand effects, and obtains evidence that the demand of emerging countries is more important than the demand of developed countries to explain the dynamics of prices and oil production, and that Europe and North America are more negatively affected for oil shocks than countries in Asia and South America, while Baumeister and Peersman (2013) indicate that time-varying short run oil demand and supply elasticities has an important role in the dynamics of the volatility of oil prices for the period 1986-2013.

4 Estimation Results

The results of the model estimation are presented in the next tables and figures. The posterior distribution of the estimated parameters are summarized in Table 3 in the Appendix, where we also show figures with the full estimated posterior distribution for the main parameters of the model. There are several interesting results in this estimation. The volatility persistence parameters ϕ_i can be separated into two groups, a first group having a lower persistence, for WTI, Brent, Exxon and Chevron, with the posterior means in the range of 0.25-0.32, and a second group with parameters higher than 0.91 for Petrobras and British Petroleum. A lower value indicates a rapid reversion to the average level of volatility, which is captured by the scaled common factor μ_t, while a higher persistence in the transitory factor indicates that the process of reversion to the common factor is slower. These results indicate that Petrobras and British Petroleum have more autonomous dynamics of conditional volatility in relation to the common factor.

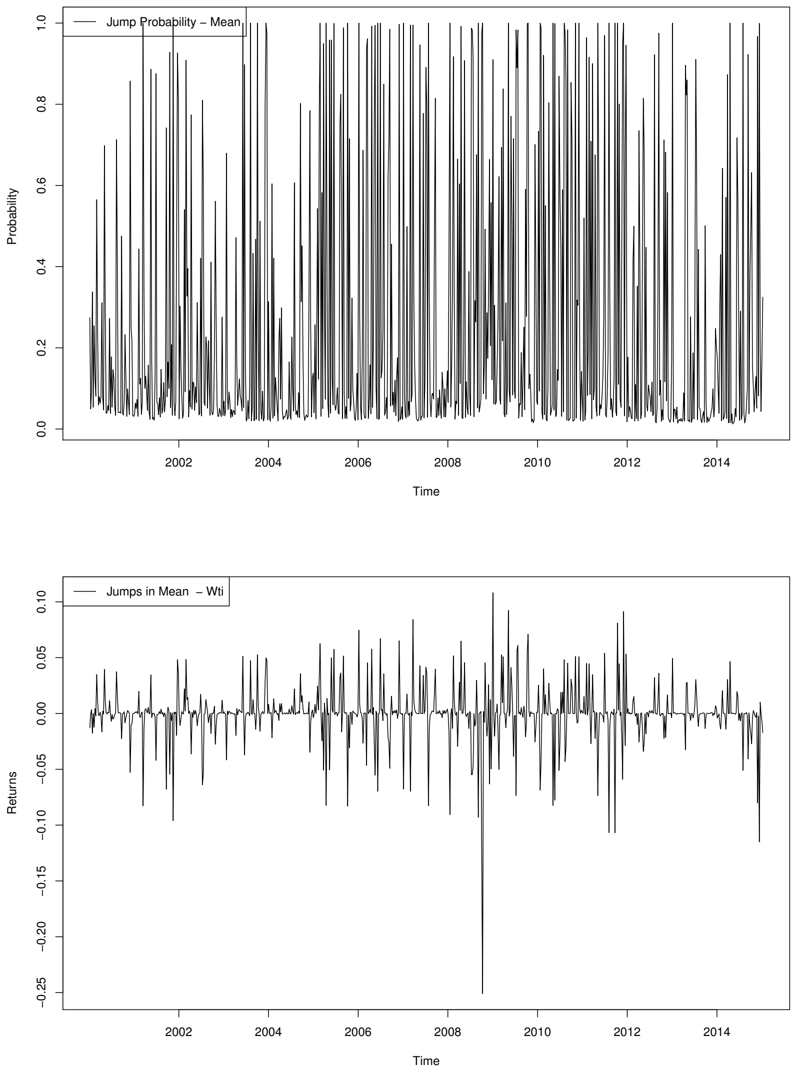
Regarding the common volatility factor, we can see that the posterior mean for the probability of jumps to this component is estimated as 0.07, indicating a significant presence of jumps in the conditional variance process. The parameter σ_η associated with the size of the shocks was estimated with posterior mean of 1.79. Recalling that this parameter is associated with the average log-variance, this value indicates that the jumps in volatility process can have a large magnitude. The estimated scale factors for the process of variance s_i^v show a pattern consistent with the dispersion of returns observed among these assets. Recalling that the common factor μ_t is the process of conditional log-variance, higher values for the scale factor s_i^v indicate a lower average variance for the series (a more negative log variance). The parameter associated with Petrobras was estimated with posterior mean of 0.89, indicating a higher exposure to the common variance factor, amplifying the effects of jumps. Conversely, the variance scale parameters for Exxon, Chevron and British Petroleum are estimated with values greater than one, leading to a lower exposure to variations in the common factor. Recalling that in this model the common factor μ_t process represents the level of the conditional variance, this result indicates that unconditional variances of these series will be relatively smaller, consistent with the observed series.

Figure 2: *Jump Process in Variance*



Regarding the parameters associated with jumps on the mean, we can also observe some significant effects. The parameter p_m, which measures the probability of common jumps on the mean, was estimated with posterior mean of 0.23. We can interpret this value as a form of regime switching model, indicating the presence of two distinct regimes. The first regime, associated with the presence of a common jump in mean, indicates the presence of a shock that is transmitted to all the series, while in the second regime without jumps the variation in returns is determined only by specific innovations in each series. In this case the jumps process is not connected only to the presence of extreme movements in returns, but indicates the presence of common shocks to all series.

Figure 3: *Jump Process in Mean*

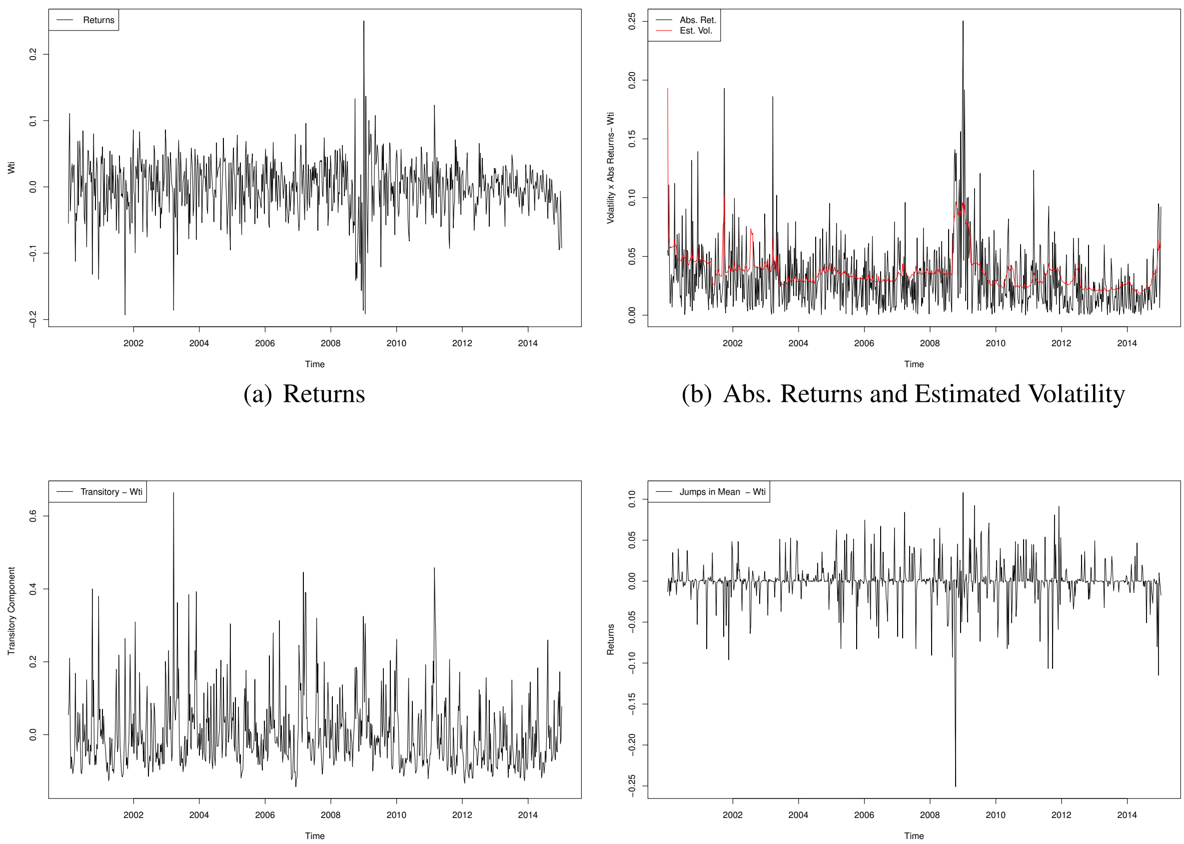


The parameter σ_V, which measures the intensity of the jumps in common factor for mean was estimated with posterior mean of 0.028, indicating that they explain a significant part of the total variance of all series. The scaling factors for the jumps on mean s_2^m are also consistent with these results. The parameter s_3^m, linked to Petrobras, was estimated with a posterior mean of 1.22, indicating that this company amplifies the observed jumps in the mean, while at the other end the parameter s_6^m connected to British Petroleum was estimated with a posterior mean of 0.47, indicating that this company is less exposed to common jumps in mean.

Figure 2 shows the estimated posterior probability of jumps in the variance, associated with the process δ_t^v, and the estimated posterior mean for the common factor of volatility μ_t. The pattern captured for the common factor is very close to the behavior observed for return series, in particular for the observed variation in the returns WTI oil. In particular, the common factor is consistent with the greater volatility observed in 2008-2009 and in late 2014, period with the largest jumps observed in the common factor μ_t.

In Figure 3 we present the posterior probability of jumps on mean, connected to the Bernoulli process δ_t^m, and the process of common jumps in mean γ_{it}. We can see that the posterior probability of jumps indicates a pattern consistent with the estimated parameter p_m, indicating the presence of many common shocks among the analyzed series. We can also note that the estimated γ_{it} process is also consistent with the price changes observed in the oil sector, as the large price variations in 2001, 2003, 2009 and the end of 2014.

Figure 4: *WTI*



(a) Returns

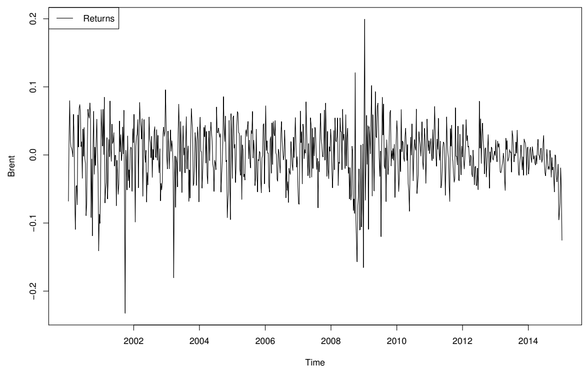
(b) Abs. Returns and Estimated Volatility

(c) Transitory - WTI

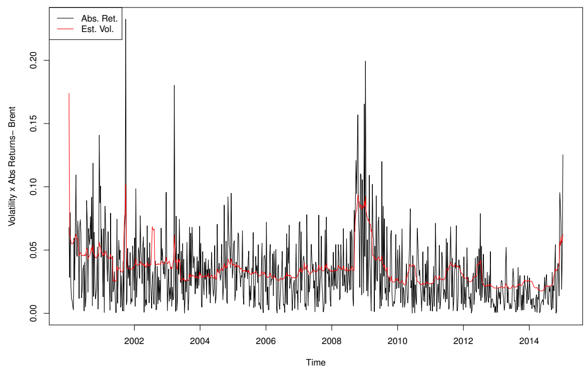
(d) Jumps in mean - WTI

Figures 4-9 show panels with the observed returns, a comparison between the absolute returns and the volatility adjusted by the model, the transitory factor and the common jump factor multiplied by the scaling factor for returns of WTI, Brent, Exxon, Chevron and British Petroleum. We can see that the adjusted volatility process is a proper fit for all assets when compared to absolute returns, the usual proxy for the true latent volatility process, capturing the observed changes in the level of these absolute returns. This can be observed on the Panel b of figures 4-9.

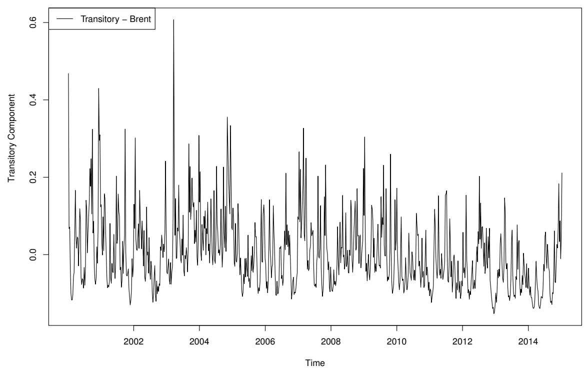
Figure 5: *Brent*



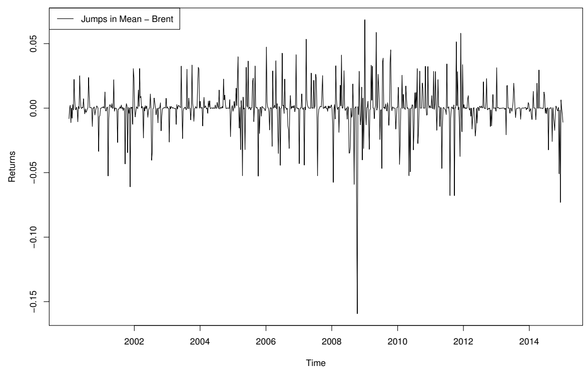
(a) Returns



(b) Abs. Returns and Estimated Volatility

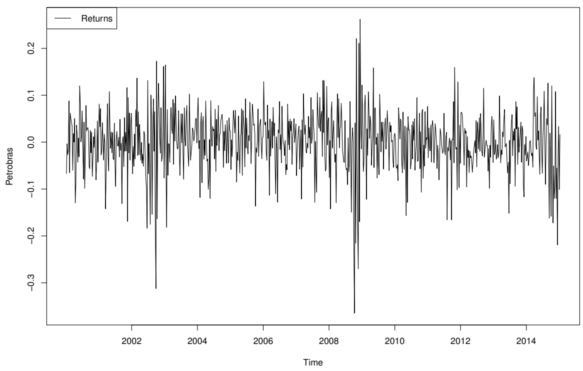


(c) Transitory - Brent

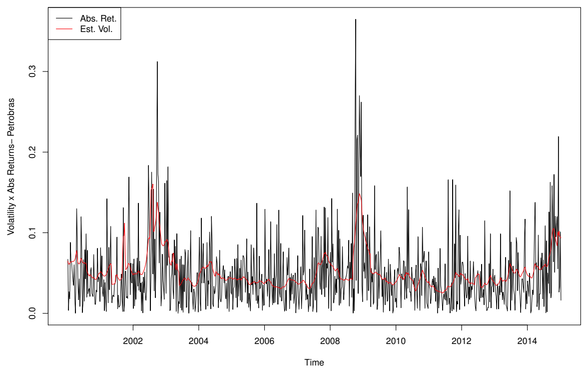


(d) Jumps in mean - Brent

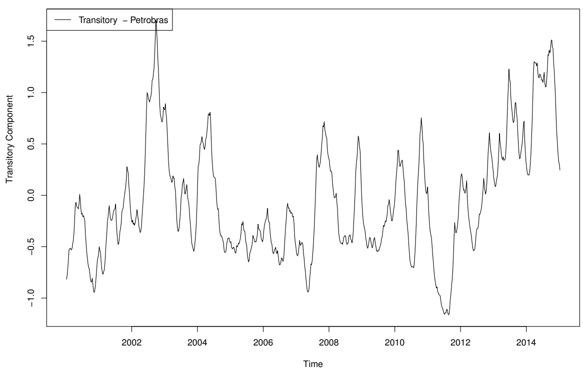
Figure 6: *Petrobras*



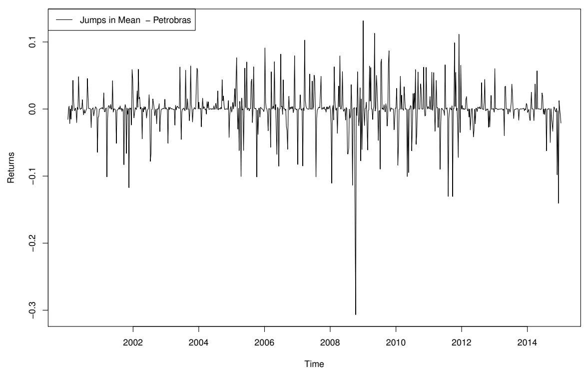
(a) Returns



(b) Abs. Returns and Estimated Volatility



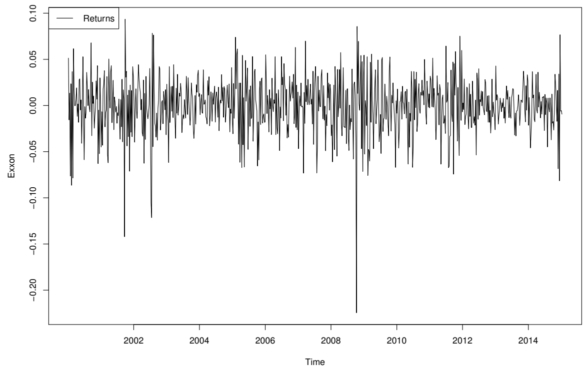
(c) Transitory - Petrobras



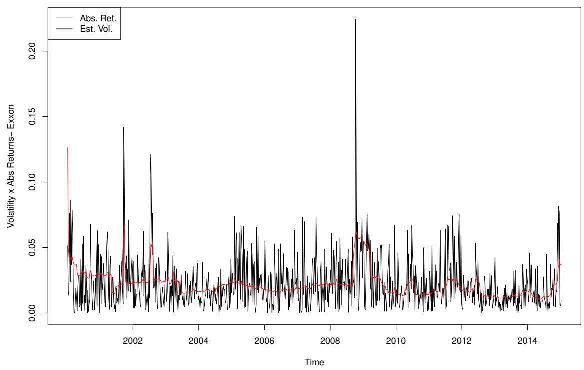
(d) Jumps in mean - Petrobras

The analysis for the WTI and Brent oil show that its returns seem close, and both are more volatile around 2009. Plotting the absolute returns with the estimated volatility for these series, as observed in Panel b of figures 4 and 5, it is possible to conclude that the estimation of the model for the volatility seem quite accurate, as it is for the other assets too. As for the comparison of the transitory component, we can observe that it seems more significant for the WTI prices than for the Brent ones. This fact can be verified in figure 14, where we can see that the density of the volatility persistence parameter for the WTI is concentrated on lower values when compared to the Brent prices. Therefore, we can conclude that jumps in the volatility of Brent returns tend to last longer than those in the WTI prices.

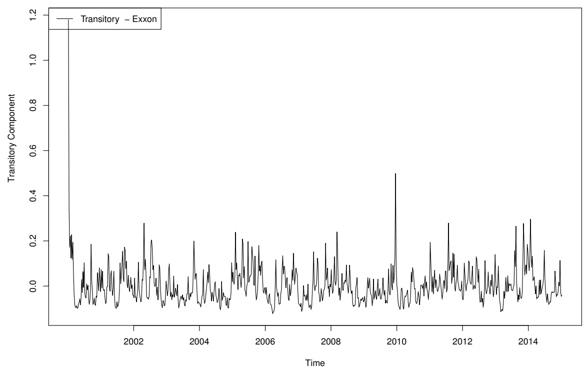
Figure 7: *Exxon*



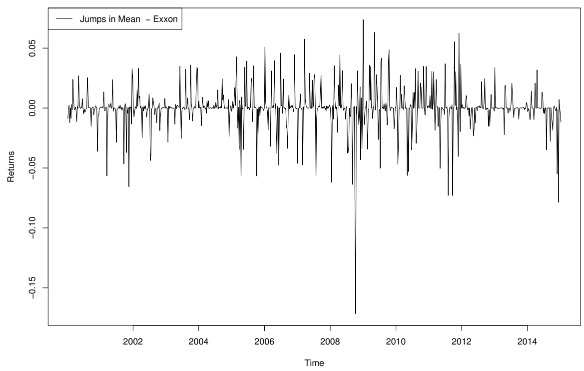
(a) Returns



(b) Abs. Returns and Estimated Volatility



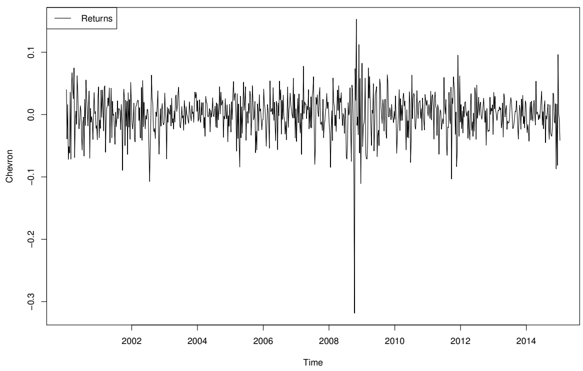
(c) Transitory - Exxon



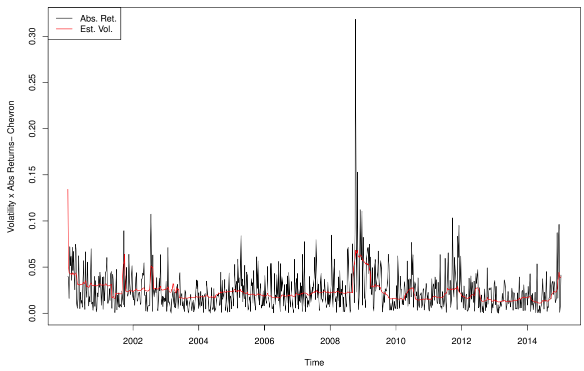
(d) Jumps in mean - Exxon

The transitory variance factors h_{it}, observed on Panel c of figures 4-9, are consistent with the results for the estimation of persistence parameters of ϕ_i. We can see that the transitory factors for WTI, Brent, Exxon and Chevron indicate a fast mean reversion to the common volatility factor, while for British Petroleum and Petrobras transitory factors are more persistent and has a greater range of values.

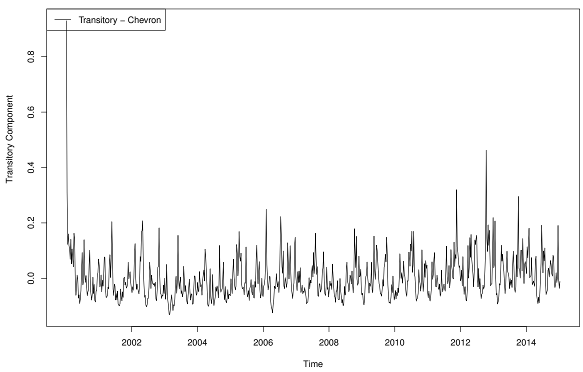
Figure 8: *Chevron*



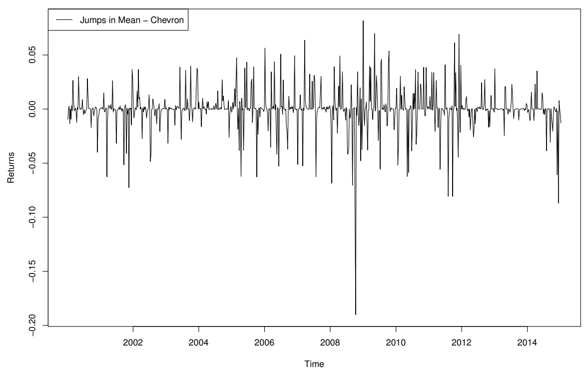
(a) Returns



(b) Abs. Returns and Estimated Volatility

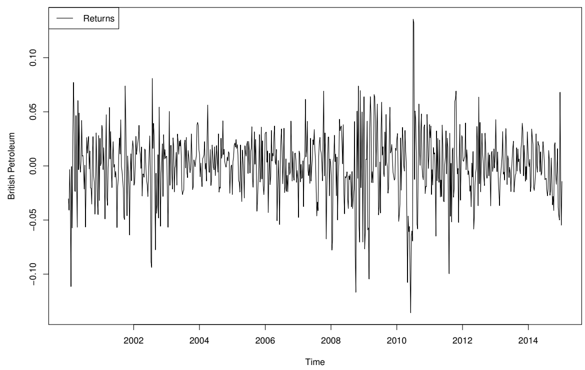


(c) Transitory - Chevron

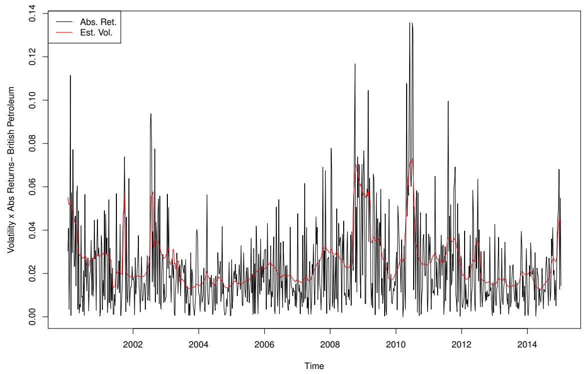


(d) Jumps in mean - Chevron

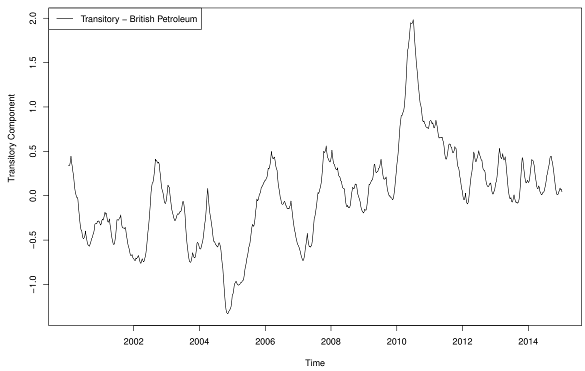
Figure 9: British Petroleum



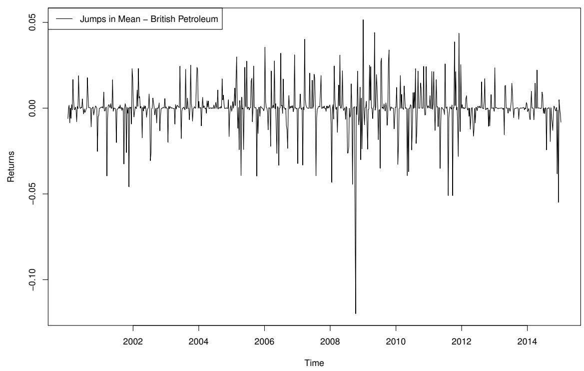
(a) Returns



(b) Abs. Returns and Estimated Volatility



(c) Transitory - British Petroleum



(d) Jumps in mean - British Petroleum

The slow mean reversion for the volatility of Petrobras can in part be explained by the high leverage ratio of the Brazilian company. Historically, the high levels of debt might have caused instability in the trading of Petrobras' stocks, leading to higher volatility.

4.1 Model Fit Measures

To quantify the quality of the adjustment of the proposed double jumps model, we performed a comparative analysis with some of the leading multivariate models of conditional volatility used in the analysis of financial time series. To perform this analysis, we use the common metric of comparison, using the square returns as a proxy of the true unobserved conditional variance.

Table 4 in the Appendix shows the mean error and root mean squared error of our model in comparison with four different models proposed in the literature. We compare our results with the DCC (Diagonal Conditional Correlation), DVEC (Diagonal Vec), CCC (Constant Conditional Correlation) and the Dynamic Copula-GARCH model. The dynamic correlation model (DCC) - is described in Engle (2002) as a class of multivariate models which have the flexibility of univariate GARCH models coupled with parsimonious parametric models for correlations. The diagonal vec model - DVEC, formulated by Bollerslev et al. (1988), is a multivariate generalization of univariate GARCH models, proposing GARCH-like structures for the conditional covariance. The CCC model, proposed by Bollerslev (1990) is based on the estimation of univariate GARCH models, with the conditional covariance being estimated assuming a constant correlation between series, and the Dynamic Copula-GARCH model joins the GARCH formulation for conditional variances with flexible dependence structures between series based on copula decomposition, e.g. Jondeau and Rockinger (2006). We do not present the details on these models; a complete survey about these specifications and general multivariate GARCH models can be found in Bauwens et al. (2006).

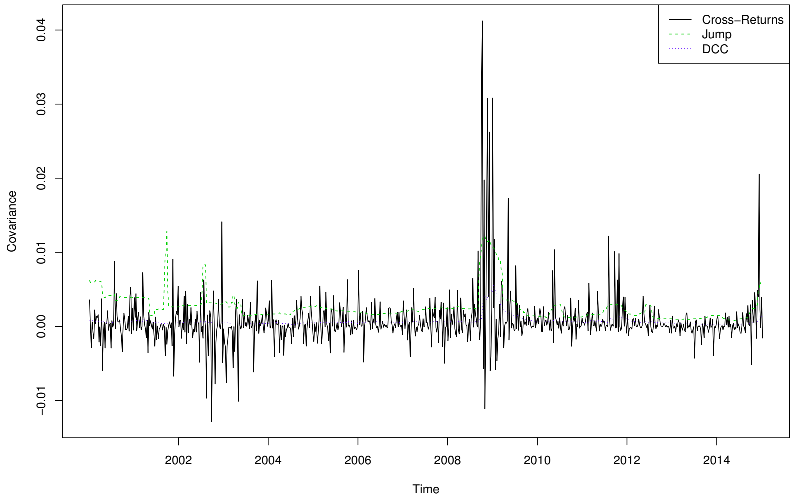
We use for the DCC model a specification based on GARCH(1,1) processes for the analyzed series and a DCC(1,1) order for the correlation structure. The DVEC model is also based on GARCH(1,1) formulations for conditional variances and covariances, and for CCC model we use an analogous specification. The Copula-GARCH model is based on a Student-t Copula to model the dependence between series, and GARCH(1,1) to each univariate process.

We can see in Table 4 that the multivariate SV-double jumps model proposed in this article has a overall performance superior the other models, when we look at the root mean squared error between the conditional variance adjusted by the models and the squared returns with smaller rmse for all analyzed series, except for Petrobras, where the DVEC, CCC and Copula-GARCH achieve a better fit.

The conditional covariance estimation in the proposed model is obtained through Equation (2.5), and indicates that the covariance between assets i and j is obtained as the product of scale parameters and the common factor μ_t, and thus all dependence is a function of exposure to the common variance factor. As an example, we show in Figures 10 and 11 the estimated covariances between WTI and Petrobras and Petrobras and Exxon estimated by our model, in conjunction with the covariance estimated by the DCC model and cross returns as proxy for the true latent covariance.

We present in Tables 5 and 6 (in the Appendix) the root mean squared error between the covariance estimated by the double jump and DCC models and cross returns as the proxy for true unobserved covariance. In general, the two models have a similar performance in the estimation of the conditional covariance, without a clear dominance of one model.

Figure 10: *Estimated Covariance - WTI and Petrobras*



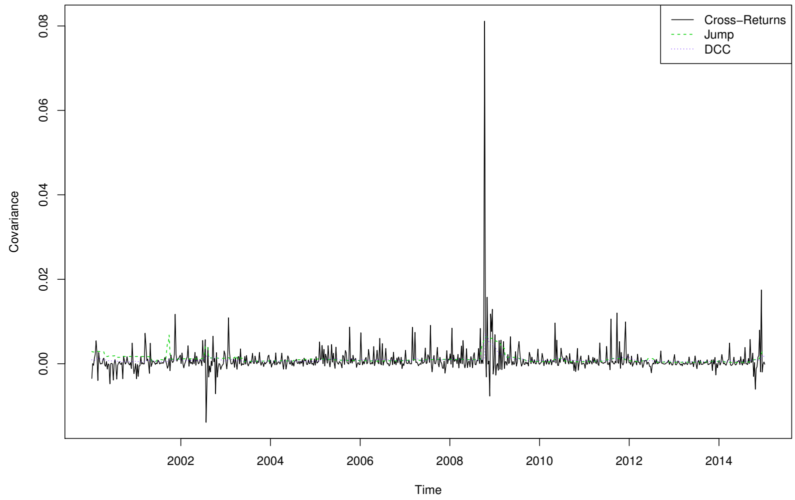
5 Applications in Risk Management and Portfolio Allocation

5.1 Value At Risk

To check the properties of the proposed model, we performed some practical tests using the model for the calculation of risk management measures and asset allocation procedures. The first application is the calculation of Value At Risk, a key measure in the measurement of extreme events and the impact of these events on assets and portfolios. See Hung et al. (2008) for VaR estimations applied to energy commodities. For this we realized dynamic Value At Risk estimations using a Gaussian approximation for the VaR. In this case the VaR of the portfolio in the period t is calculated as $VaR_t = \hat{y}_{it} - q_\alpha \mathscr{H}_{it}$, where $\hat{y}_{it}$ and $\mathscr{H}_{it}$ are the expected values estimated for the mean and conditional variance of the asset i in period t. We compare the dynamic calculation of VaR using estimates of volatility $\mathscr{H}_{it} = exp(h_{it}/2 + s_i^v \mu_t/2) + Var(\gamma_{it})$ obtained from our model and the univariate estimation of a GARCH (1,1) model, a common choice in the estimation of dynamic VaR measure.

In Tables 7 and 8 in the Appendix we present the proportions of violations using the VaR measures calculated using our model and the GARCH benchmark, for VaR levels of 5% and 1%. The Tables also show the p-value of the unconditional coverage test Christoffersen (1998), a usual specification procedure for this class of risk measures. The null hypothesis of this test is $E[I_t] = p$, while the alternative is $E[I_t] \neq p$, where $\{I_t\}_{t=1}^T$ is an indicator sequence constructed from a given interval forecast. In our case, the sequence is formed by the proportion of violations for each model and the parameter p is 5% and

Figure 11: *Estimated Covariance - Petrobras and Exxon*



1% (for tables 7 and 8 respectively). The significance level is 5% for both tables.

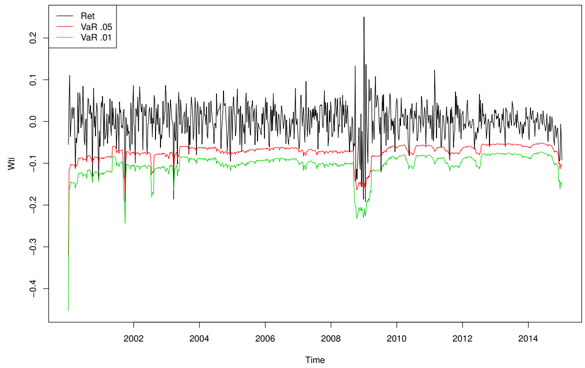
When we look at the results for the VaR with 5% level, we can see that the two methods of VaR calculation has similar performance, except for the WTI series where the Jump method has a lower proportion of violations than the 5% nominal coverage. In all other series the results are similar, indicating no rejection of the hypothesis of nominal coverage equal to 5%.

However, when we look at the 1% VaR level, which represent the risk of most extreme violations, we can see that the results based on the double jumps model are more adequate. The 1% nominal coverage is rejected for the series of Petrobras, Exxon, Chevron and British Petroleum when we use VaR-GARCH method, while in general we cannot reject the hypothesis of correct nominal coverage for the VaR estimation using the double jumps model, except for the WTI returns. However, this rejection is due to a lower than expected number of violations, indicating a more conservative VaR, which is less harmful in terms of exposure to extreme risks. Figure 12 shows the values of the dynamic VaR estimated using the SV-double jump model for the series, compared to the observed returns.

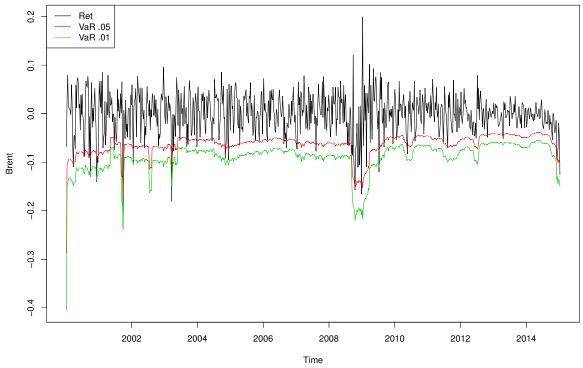
5.2 Hedging Oil Price Risk

An important application of multivariate conditional volatility model is the construction of hedge portfolios, minimizing the exposure of oil companies to the risk associated with changes in oil prices. For this, we can determine the optimal hedge ratio (Kroner and Ng (1998)) for a portfolio containing the asset a and oil o by the optimal weight given by:

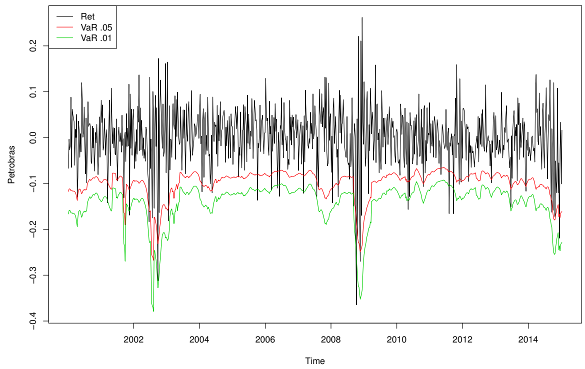
Figure 12: Value At Risk



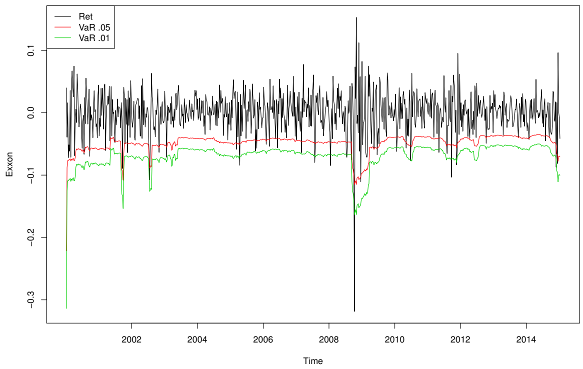
(a) WTI



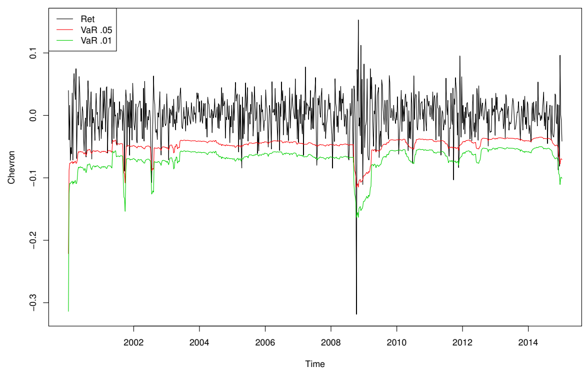
(b) Brent



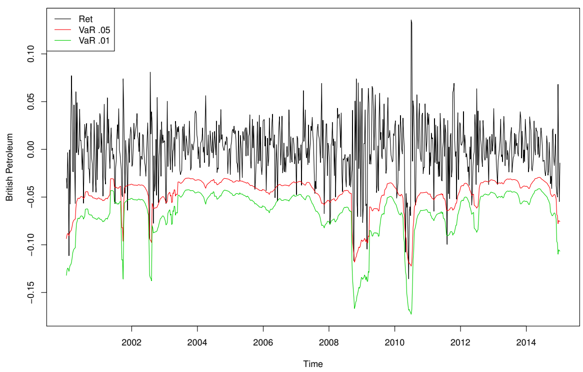
(c) Brent



(d) Exxon



(e) Chevron



(f) British Petroleum

$$w_{oa,t} = \frac{\mathscr{H}_{at} - \mathscr{H}_{o,at}}{\mathscr{H}_{ot} - 2\mathscr{H}_{o,at} - \mathscr{H}_{at}} \quad (8)$$

where:

$$w_{oa,t} = \begin{matrix} 0 & if\, w_{oa,t} < 0 \\ w_{oa,t} & if\, 0 \leq w_{oa,t} \leq 1 \\ 1 & if\, w_{oa,t} > 1 \end{matrix} \quad (9)$$

The $w_{oa,t}$ variable denotes the weight of the oil in a portfolio containing oil and assets, $\mathscr{H}_{at}$ the variance estimated for the asset a, $\mathscr{H}_t$ the conditional variance of oil and $\mathscr{H}_{o,at}$ the covariance between the two assets in the period t. The hedging strategy involves a short position in one asset and long position on the other. See Salisu and Oloko (2015) for an application of this methodology in risk management in the oil sector.

In this application we built the hedge portfolios by combining individually Petrobras, Exxon, Chevron and British Petroleum with WTI oil, setting the optimal hedge portfolio by the rule given by Equations 8 and 9. Table 9 (in the Appendix) shows the mean and the variance of the hedge portfolios.

We compare the results using the conditional variances and covariances estimated by the model proposed in this article and the DCC model. We can see in Table 9 that the hedge portfolio using the optimal weight estimated using the double jumps model generates a smaller variance for Petrobras, Exxon and Chevron, and has a worse performance for the portfolio of British Petroleum. This result indicates that the model performs well in this application, compared to the standard methodology based on GARCH models.

5.3 Portfolio Weights

An important application of multivariate volatility models is on dynamic portfolio management. In particular, multivariate volatility models are used in the construction of minimum variance portfolios, being a dynamic generalization of the Markowitz' portfolio theory. A common benchmark in these analyzes is to compare the performance of the global minimum variance portfolio constructed from the estimates of the conditional covariance matrix. The estimation of global minimum variance portfolios represent a simple way to compare the performance of variance estimation methods in allocation problems, and have some advantages relative to other procedures. See Amenc and Martellini (2002) and DeMiguel et al. (2009) for advantages and limitations of this methodology.

To check the performance of the model proposed in this work, we build global minimum variance portfolios using the returns of Exxon, Chevron and British Petroleum. A similar application was also performed including Petrobras, but in that case the weights of the global minimum variance portfolio for this company were generally near zero, due to the fact that the variance of this series is much higher than the others. Similarly to the previous section, we compared our results with an allocation based on the conditional covariance matrix estimated by the DCC model. In this experiment we compared the in-sample performance of these two portfolios, similar to the analyzes conducted in Engle (2009).

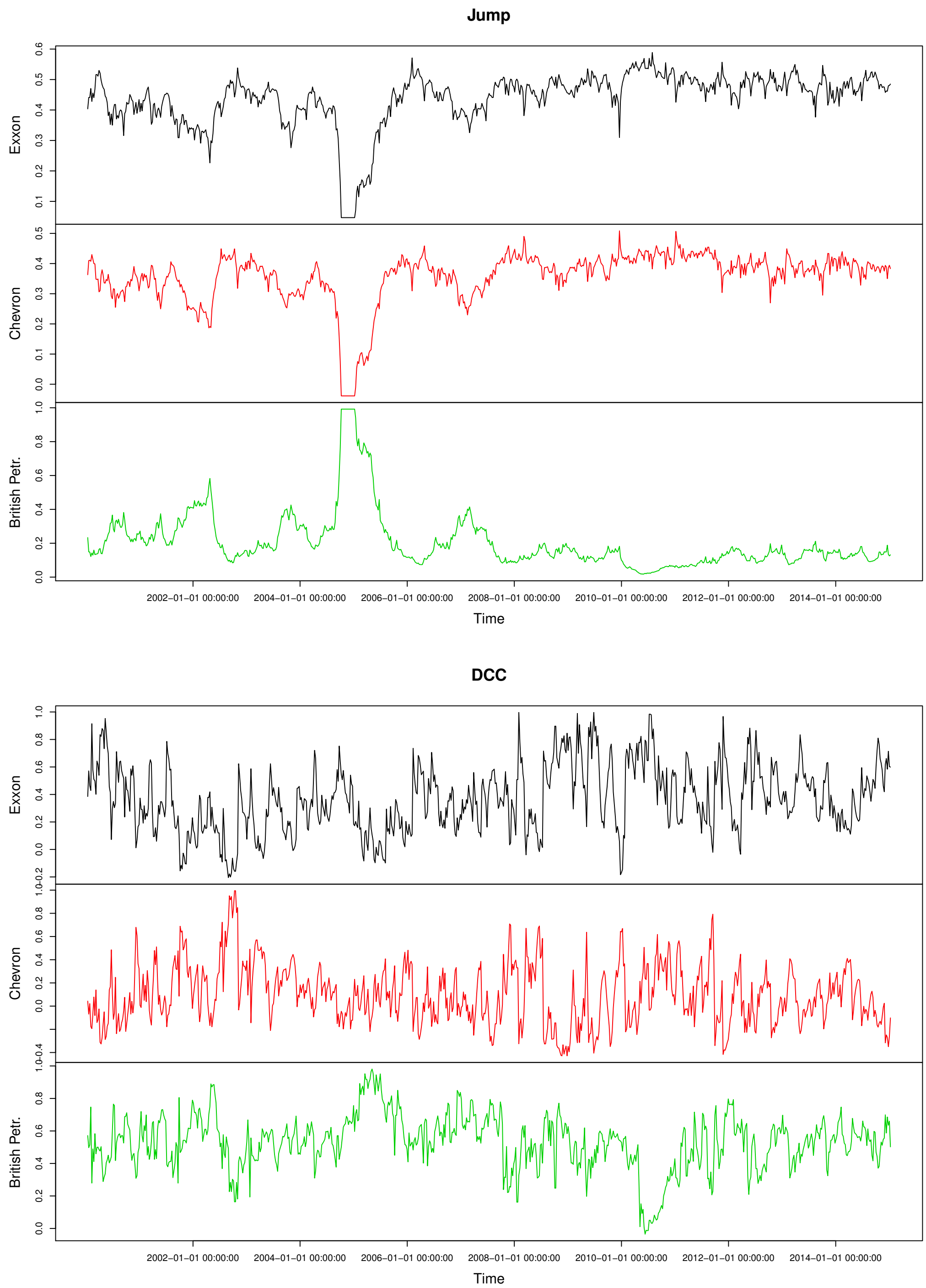
Table 10 in the Appendix presents some performance measures comparing the global minimum variance portfolios obtained by the two methods of estimating the co-variance matrix. We can see that the Sharpe Ratio of the portfolio of minimum variance obtained by double jumps model was 0.04864, compared to a Sharpe Ratio of 0.01147 obtained by the portfolio built using the DCC model. We test the statistical significante of difference between Sharpe Ratios using the robust procedures of Ledoit and Wolf (2008), finding HAC and block bootstrap p-values of 0.015 and 0.012, supporting the best performance of the double jump model in this application.

This result indicates a higher risk premium per unit of risk for the model proposed in this article, resulting in a superior allocation. The variance of the portfolio estimated by jumps model was estimated as 0.00072, slightly higher than the value of 0.00065 of the portfolio based on the DCC estimation. Although the variance of the portfolio is greater, the risk premium in the portfolio based on the jumps model is higher than what is generally considered a more appropriate allocation.

Another key point differentiating these two portfolios is in the rebalancing process, the process of buying and selling positions in each asset to maintain the optimal weights. In the last line of Table 10 we present the turnover of the portfolios, which is a measure of mean cumulative change in the portfolio weights. A portfolio with lower turnover indicates less rebalancing, and thus lower transaction costs associated with maintenance of the portfolio.

We can observe that the mean turnover of the portfolio based on the jumps model was 0.05167, compared to the mean turnover of 0.28090 of the portfolio based on the DCC estimation, indicating much lower transaction costs for the proposed model in this article. This result can be interpreted by viewing the evolution of the weights in the portfolio during the period. Figure 13 shows the optimal weights for the assets in the portfolios. We can observe that the variation between weeks in portfolio weights for the double jumps model is much inferior to the variations in DCC portfolios, which show a rather noisy allocation. The results indicate that a portfolio allocation based on proposed model gives a higher Sharpe ratio and much lower turnover, which are two desired characteristics in the asset allocation procedures, indicating that the method proposed in this work has practical advantages in the processes of risk management and portfolio allocation for the series under review.

Figure 13: *Portfolio Weights - Double Jump and DCC models*



6 Conclusion

In this paper we propose a new multivariate model that allows incorporating the presence of common factors related to joint jumps in the mean and conditional variance of oil prices returns and companies in this sector. The application of this model to the series of the stock returns of the some of main oil companies in the world (Exxon, Chevron, Petrobras and British Petroleum) and to the series of oil prices (Brent and WTI) shows that proposed model can adequately capture the dynamics of jumps and conditional volatility of the analyzed series.

The common factors estimated by the model can be associated with the main events in the oil sector, and thus the model allows an economic interpretation of shock transmission dynamics in this sector. Another important contribution of the proposed methodology is to allow different rates of mean reversion in the conditional volatility, through the decomposition of the total volatility as the sum of the common factor plus asset specific autoregressive processes. The results indicate that Exxon and Chevron have fast mean reversions in the conditional volatility, while Petrobras and British Petroleum have more autonomous volatility dynamics in relation to shocks in oil prices.

The model allows to incorporate some important features in the oil sector, especially the transmission of oil price shocks to returns and volatilities of the companies in this sector. This aspect is important for the financial planning of oil companies, and especially in risk management and portfolio allocation. Applying the model to the analysis of Value at Risk, hedging oil price risk and portfolio selection, the results show that the proposed model has generally outperformed the main methodology used in these procedures, based on GARCH models.

An important extension to the model proposed in this work is the introduction of a variable intensity in jumps processes , similar to Laszlo et al. (2015) and especially Xu and Perron (2014). This variable intensity structure may be important to characterize the changes in volatility observed in the oil industry in recent periods.

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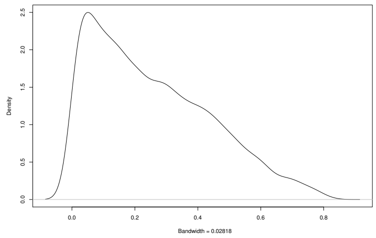
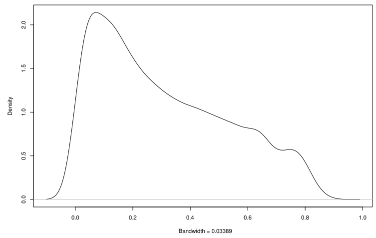
Appendix

Posterior Distribution for Selected Parameters

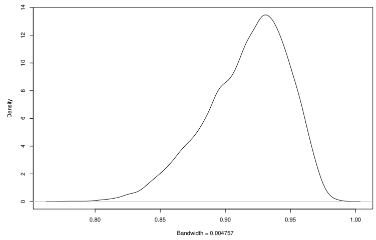
Figure 14: *Volatility Persistence -* ϕ_i



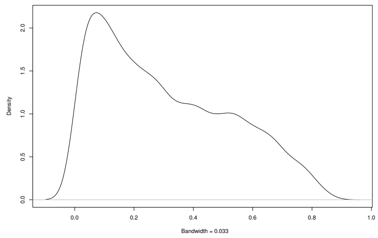
(a) WTI



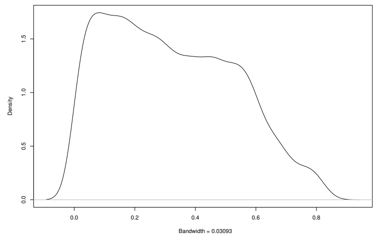
(b) Brent



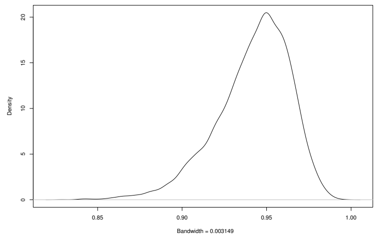
(c) Petrobras



(d) Exxon

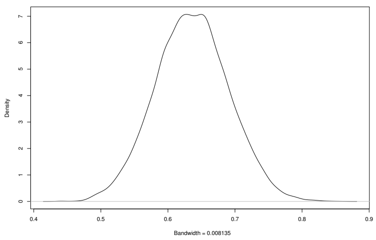


(e) Chevron

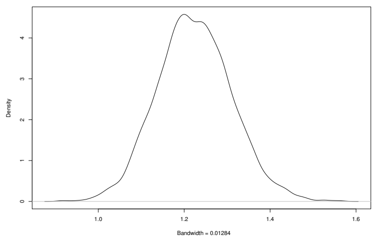


(f) British Petroleum

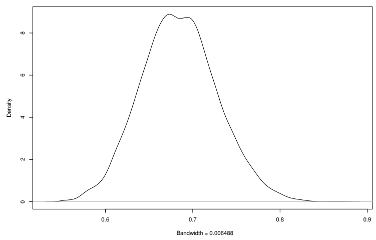
Figure 15: *Scale Parameters - Mean* s_i^m



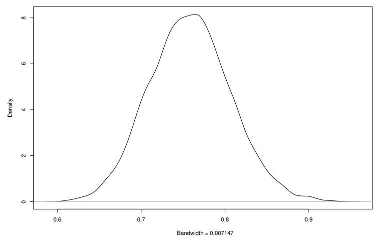
(a) Brent



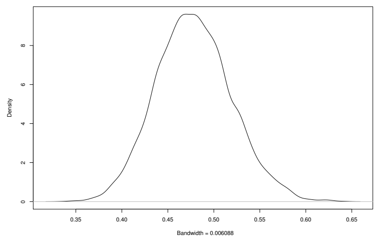
(b) Petrobras



(c) Exxon

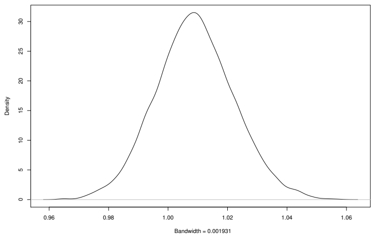


(d) Chevron

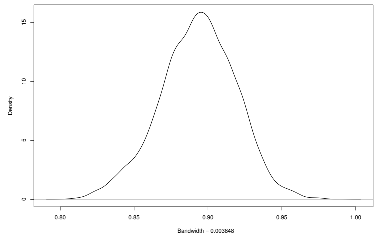


(e) British Petroleum

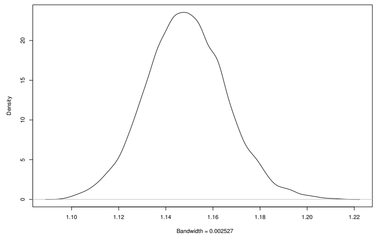
Figure 16: *Scale Parameters - Variance -* s_i^v



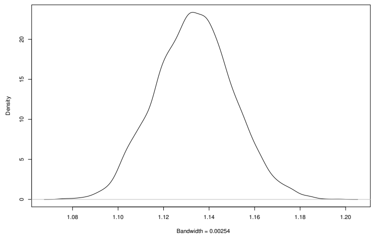
(a) Brent



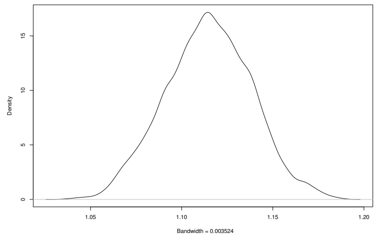
(b) Petrobras



(c) Exxon

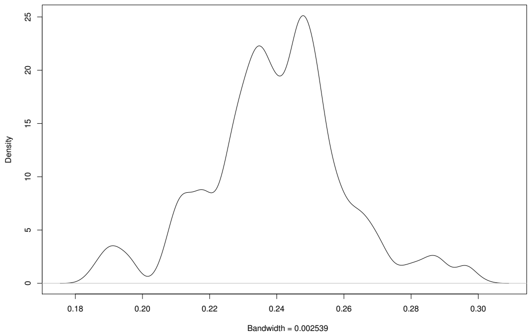


(d) Chevron

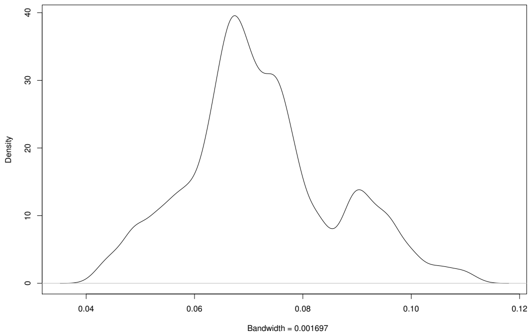


(e) British Petroleum

Figure 17: *Jump Probabilities*



(a) Mean



(b) Variance

(a) WTI parameters - s_1^v and s_1^m are normalized to one

	2.5%	Mean	97.5%	SD
ϕ_1	0.0084	0.2556	0.6818	0.1890
σ_{υ_1}	0.0558	0.1576	0.3775	0.0794

(b) Brent parameters

	2.5%	Mean	97.5%	SD
ϕ_2	0.0112	0.3086	0.7818	0.2272
σ_{υ_2}	0.0439	0.1273	0.2981	0.0660
s_2^v	0.9835	1.0090	1.0360	0.0132
s_2^m	0.5305	0.6360	0.7431	0.0545

(c) Petrobras parameters

	2.5%	Mean	97.5%	SD
ϕ_3	0.8452	0.9165	0.9662	0.0319
σ_{υ_3}	0.0420	0.0962	0.1954	0.0393
s_3^v	0.8391	0.8932	0.9412	0.0258
s_3^m	1.0608	1.2238	1.4007	0.0869

(d) Exxon parameters

	2.5%	Mean	97.5%	SD
ϕ_4	0.0116	0.3057	0.7601	0.2213
σ_{υ_4}	0.0424	0.1035	0.2039	0.0421
s_4^v	1.1156	1.1487	1.1833	0.0170
s_4^m	0.6026	0.6845	0.7724	0.0435

(e) Chevron parameters

	2.5%	Mean	97.5%	SD
ϕ_5	0.0131	0.3209	0.7460	0.2074
σ_{υ_5}	0.0461	0.1069	0.2296	0.0478
s_5^v	1.1012	1.1335	1.1675	0.0170
s_5^m	0.6662	0.7583	0.8564	0.0483

(f) British Petroleum parameters

	2.5%	Mean	97.5%	SD
ϕ_6	0.8912	0.9419	0.9762	0.0221
σ_{υ_6}	0.0288	0.0567	0.1054	0.0197
s_6^v	1.0681	1.1147	1.1609	0.0236
s_6^m	0.4017	0.4783	0.5638	0.0411

(g) Common Parameters

	2.5%	Mean	97.5%	SD
p_v	0.0475	0.0722	0.1014	0.0137
σ_η	1.0549	1.7945	2.9550	0.4884
p_m	0.1923	0.2398	0.2866	0.0206
σ_v	0.0227	0.0286	0.0364	0.0035

Notation: 1- WTI, 2- Brent, 3 - Petrobras, 4 - Exxon, 5 -Chevron, 6 - British Petroleum.

Table 3: *Posterior distribution of estimated parameters*

		Jump		DCC		DVEC		CCC		Copula
	me	rmse	me	rmse	me	rmse	me	rmse	me	rmse
WTI	-0.00028	0.00362	0.00001	0.00395	0.00003	0.00395	-0.00001	0.00393	0.00001	0.00395
Brent	-0.00031	0.00313	0.00009	0.00355	-0.00002	0.00399	-0.00007	0.00393	0.00002	0.00392
Petrobras	-0.00105	0.00807	-0.00015	0.00866	0.00239	0.00458	0.00229	0.00474	0.00225	0.00465
Exxon	-0.00032	0.00230	0.00002	0.00230	-0.00088	0.00432	-0.00091	0.00422	-0.00087	0.00418
Chevron	-0.00047	0.00381	0.00001	0.00395	-0.00073	0.00423	-0.00074	0.00406	-0.00069	0.00409
British Petr.	-0.00013	0.00148	-0.00002	0.00167	-0.00091	0.00426	-0.00093	0.00426	-0.00093	0.00426

Table 4: *Mean Squared Error*

	WTI	Petrobras	Exxon	Chevron	British Petr.	Brent
WTI	0.00362	0.00415	0.00196	0.00212	0.00197	0.00293
Petrobras	0.00415	0.00807	0.00361	0.00476	0.00287	0.00394
Exxon	0.00196	0.00361	0.00230	0.00286	0.00113	0.00208
Chevron	0.00212	0.00476	0.00286	0.00381	0.00128	0.00214
British Petr.	0.00197	0.00287	0.00113	0.00128	0.00148	0.00195
Brent	0.00293	0.00394	0.00208	0.00214	0.00195	0.00313

Table 5: *Root Mean Squared Error - Covariance Matrix - Double Jumps Model*

	WTI	Petrobras	Exxon	Chevron	British Petr.	Brent
WTI	0.00395	0.00362	0.00187	0.00213	0.00171	0.00322
Petrobras	0.00362	0.00866	0.00365	0.00486	0.00254	0.00336
Exxon	0.00187	0.00365	0.00230	0.00285	0.00117	0.00195
Chevron	0.00213	0.00486	0.00285	0.00395	0.00131	0.00211
British Petr.	0.00171	0.00254	0.00117	0.00131	0.00167	0.00158
Brent	0.00322	0.00336	0.00195	0.00211	0.00158	0.00355

Table 6: *Root Mean Squared Error- Covariance Matrix - DCC Model*

	Emp. Viol. Jump	Emp. Viol. GARCH	Coverage Test Jump	Coverage Test GARCH
WTI	0.026	0.065	0.001	0.059
Brent	0.046	0.055	0.682	0.512
Petrobras	0.045	0.057	0.566	0.325
Exxon	0.054	0.055	0.623	0.512
Chevron	0.046	0.055	0.682	0.512
British Petr.	0.040	0.052	0.190	0.743

Table 7: *Value At Risk 5%*

	Emp. Viol. Jump	Emp. Viol. GARCH	Coverage Test Jump	Coverage Test GARCH
WTI	0.003	0.013	0.029	0.371
Brent	0.004	0.013	0.102	0.371
Petrobras	0.014	0.026	0.275	0.000
Exxon	0.017	0.019	0.071	0.018
Chevron	0.014	0.022	0.275	0.003
British Petr.	0.005	0.018	0.208	0.044

Table 8: *Value At Risk 1%*

		Jump		DCC
	mean	var	mean	var
Petrobras	0.00008	0.00195	-0.00054	0.00198
Exxon	0.00017	0.00190	-0.00059	0.00191
Chevron	0.00155	0.00205	0.00141	0.00213
British Petr.	0.00136	0.00098	0.00071	0.00060

Table 9: *Hedge Effectivity*

	Jump	DCC
Sharpe ratio	0.04864	0.01147
Variance	0.00072	0.00065
Turnover	0.05167	0.28090

Results for Sharpe Ratio Difference Tests (Ledoit and Wolf (2008)) - p-value HAC - 0.015, p-value block bootstrap - 0.012.

Table 10: *Global Minimum Variance Portfolio Performance*