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Wagner Piazza Gaglianone, Osmani Teixeira de Carvalho Guillén and
Francisco Marcos Rodrigues Figueiredo

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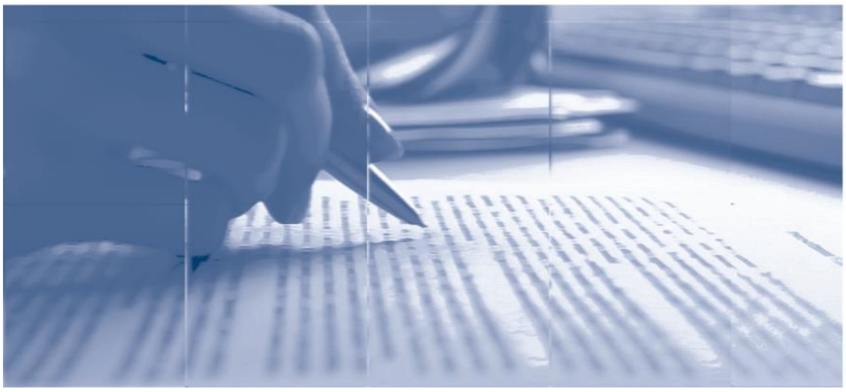

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Local Unit Root and Inflationary Inertia in Brazil*

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Abstract

The Working Papers should not be reported as representing the views of the Banco Central do Brasil. The views expressed in the papers are those of the author(s) and do not necessarily reflect those of the Banco Central do Brasil.

In this paper, we study the persistence of Brazilian inflation using quantile regression techniques. To characterize the inflation dynamics we employ the Quantile Autoregression model (QAR) of Koenker and Xiao (2004, 2006), where the autoregressive coefficient may assume different values in distinct quantiles, allowing testing the asymmetry hypothesis for the inflation dynamics. Furthermore, the model allows investigating the existence of a local unit root behavior, with episodes of mean reversion sufficient to ensure stationarity. In other words, the model enables one to identify locally unsustainable dynamics, but still compatible with global stationarity; and it can be reformulated in a more conventional random coefficient notation to reveal the periods of local non-stationarity. Another advantage of this technique is the estimation method, which does not require knowledge of the innovation process distribution, making the approach robust against poorly specified models. An empirical exercise with Brazilian inflation data and its components illustrates the methodology. As expected, the behavior of inflation dynamics is not uniform across different conditional quantiles. In particular, the results can be summarized as follows: (i) the dynamics is stationary for most quantiles; (ii) the process is non-stationary in the upper tail of the conditional distribution; (iii) the periods associated with local unsustainable dynamics can be related to those of increased risk aversion and higher inflation expectations; and (iv) out-of-sample forecasting exercises show that the QAR model at the median quantile level can exhibit, in some cases, lower mean squared error (MSE) compared to the random walk and AR forecasts.

Keywords: Inflation, Persistence, Quantile Autoregression.

JEL Classification: C14, C22, E31.

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1 Introduction

There are several methods available in the literature to estimate the inflationary inertia (or persistence). The simplest approach consists on regressing the inflation rate on its own lags and, then, computing the sum of autoregressive coefficients. Methods that are more sophisticated include, for instance, the estimation of reduced-form Phillips curves, or even building structural macroeconomic models representing the inflationary dynamics based on latent factors and Kalman filtering. See Rudd and Whelan (2007) or Pivetta and Reis (2007) for further details.

In developed economies, it is well known that inflation persistence has been diminishing over the past decades according to several studies in the literature (e.g., Mishkin, 2007; Stock and Watson, 2007). For instance, there has been a reduction of persistence in the U.S., especially after the 90s, due to the great moderation period. There is also evidence of declining persistence in the G7 countries according to Cecchetti et al. (2007).

In Brazil, such evidence is less clear-cut, since some studies indicate a diminishing persistence, whereas others point out the opposite results. For example, Machado and Portugal (2014) decompose inflationary inertia into three components (deviations of expectations from the actual monetary policy target; persistence of the factors driving inflation; and the usual intrinsic measure of persistence evaluated through lagged inflation terms) and conclude that intrinsic persistence declined between 1995 and 2011, while the other components remained relatively stable. On the contrary, Roache (2014) compares inflation targeting countries and concludes that inflationary persistence in Brazil augmented until 2013, in particular, for periods with inflationary shocks. In turn, Oliveira and Petrassi (2014), based on a sample of 40 countries since 1995, suggest that persistence remained relatively stable in Brazil.

In this paper, we tackle such issue from a different perspective. The objective here is to study the persistence of Brazilian inflation, and its main components, using quantile regression techniques.

Quantile regression is a statistical method for estimating models of conditional quantile function. Nowadays, it is applied in many fields since it allows for statistical inference on the entire conditional distribution. Based on a semiparametric approach, it provides a complete picture for analyzing statistical relationships, by showing how covariates influence the location, scale and shape of the entire response distribution.

In recent years, a great amount of empirical applications appeared in the time-series literature based on quantile regressions, such as: Koenker and Zhao (1996); Engle and Manganelli (2004); Koenker and Xiao (2006); Lima et al. (2008); Xiao (2009); Gaglianone and Lima (2012, 2014), Xiao (2014), among many others. Regarding the use of quantile autoregressions for analyzing inflation, Çiçek and Akar (2013) for Turkey; and Wolters and Tillman (2015) and Manzan and Zerom (2015) for United States are recent examples. In terms of Brazilian inflation, Maia and Cribari Neto (2006) analyzes the dynamics of IPCA and IGP-DI from August/1994 to April/2004. They found that the inflationary dynamics is not uniform across different quantiles.

To characterize the dynamics of inflation we used in this paper the Quantile Autoregression model (QAR), proposed by Koenker and Xiao (2002, 2004, 2006), in which the autoregressive coefficient may assume different values in distinct quantiles, allowing testing the asymmetry hypothesis for the inflation dynamics. Furthermore, the model allows investigating the existence of a local unit root behavior, with episodes of mean reversion sufficient to ensure stationarity. In other words, the model enables us to identify locally unsustainable dynamics, but still compatible with a global stationarity hypothesis of the investigated series. In addition, the model can be reformulated in a more conventional random coefficient notation, in order to reveal the periods of local non-stationarity. Another advantage of this technique is the estimation method, which does not require knowledge of the innovation process distribution, making this approach robust against poorly specified models.

It is used in this study data from the monthly Brazilian consumer price index (IPCA) and its components for the period from January 1995 until April 2014. To explore possible differences in disaggregated inflation data, we studied the main disaggregated components of the headline inflation: market prices and regulated prices. Additionally, we disaggregate market prices in two ways: i) tradables and non-tradables; and ii) services, food and beverages, and industrial goods. Therefore, our work differs from that one by Maia and Cribari-Neto (2006), by using a larger data sample that includes almost 15 years under inflation targeting and taking in account the components of IPCA inflation.

As expected, when we apply the usual unit root tests, the inflation series are stationary for post stabilization period. When we compare the choice of lags for the QAR model, the headline inflation, market prices and industrial goods are described by simple autoregressive model, whilst other disaggregated items are described by a more complex autoregressive dynamics. The choice of lags may indicate that disaggregated items have a higher temporal dependence caused by idiosyncratic movements, lost in the act of aggregation.

The "local" unit root analysis, based on the test of Koenker and Xiao (2004), was applied to investigate the non-stationary dynamics of the inflation rate in Brazil, in order to find the largest quantile level (denoted by $\tau_{crit.}$) in which the null hypothesis of a unit root can be rejected. The result shows that the critical quantile found using Brazilian inflation data belongs to a conditional quantile between the quantile level 0.7 and 0.8. If we use a finer grid we find that $\tau_{crit.} = 0.71$. Consequently, for approximately $H = 29\%$ of the periods the inflation rate exhibited a non-stationary behavior in the considered sample (1995-2014). To further understand the inflation inertia in the past two decades, we next reestimate the QAR(1) model by using a recursive estimation scheme and computing the respective H statistic. From this recursive exercise, we can infer that there was a gradual reduction of inflation inertia, which can be credited to the inflation targeting regime. For disaggregated data, regulated prices are in quantile 64%, while the market prices are located in the 77% quantile; whereas food and beverages inflation, for instance, belongs to the 90% quantile. This result indicates that disaggregated data may have a lower persistence. Moreover, out-of-sample forecasting exercises show that the QAR model might exhibit lower mean squared error (MSE) in comparison to standard autoregressive and random walk point forecasts.

The outline of the paper is as follows. In Section 2 we provide an overview of the methodology. Section 3 presents the empirical exercise and Section 4 concludes.

2 Methodology

2.1 The Quantile Autoregression (QAR) model

The techniques discussed in this paper are appropriate for investigating the dynamics of a weakly stationary and ergodic univariate process $\{y_t\}$. In a sequence of papers Koenker and Xiao (2002, 2004, 2006) introduced the so-called Quantile Autoregression (QAR) model. In this paper, we employ their approach to study inflationary inertia and separate non-stationary observations from stationary ones by using the QAR model. First, consider the following assumptions:

Assumption 1 Let $\{U_t\}$ be a sequence of i.i.d. standard uniform random variables.

Assumption 2 Let $\alpha_i\left(U_t\right)$, $i = 0, ..., p$ be comonotonic random variables.[1]

We define the p-th order autoregressive process as follows,

$$y_t = \alpha_0\left(U_t\right) + \alpha_1\left(U_t\right) y_{t-1} + ... + \alpha_p\left(U_t\right) y_{t-p}, \tag{1}$$

where α_j's are unknown functions $[0, 1] \to \mathbb{R}$ to be estimated. We refer the previous equation as the QAR(p) model. Given assumptions 1 and 2, the quantile of y_t conditional of the information set available at $t-1$, that is $\mathcal{F}_{t-1}$, is given by

$$Q_{y_t}(\tau \mid \mathcal{F}_{t-1}) = \alpha_0\left(\tau\right) + \alpha_1\left(\tau\right) y_{t-1} + ... + \alpha_p\left(\tau\right) y_{t-p},$$

where τ is the quantile of U_t and $\mathcal{F}_{t-1} = \left(y_{t-1}, ..., y_{t-p}\right)$ only includes here[2] the past values of the variable of interest y_t.

The QAR(p) model (1) can be reformulated in a more conventional random coefficient notation as follows:

$$y_t = \mu_0 + \beta_{1,t} y_{t-1} + ... + \beta_{p,t} y_{t-p} + u_t, \tag{2}$$

[1] Two random variables $X, Y : \Omega \to \mathbb{R}$ are said to be comonotonic if there exists a third random variable $Z : \Omega \to \mathbb{R}$ and increasing functions f and g such that $X = f(Z)$ and $Y = g(Z)$.

[2] See the Quantile Autoregressive Distributed-Lag model (QADL) of Galvão Jr et al. (2013), which also includes exogenous stationary covariates and generalizes the QAR model of Koenker and Xiao (2006).

where

$$\mu_0 = \mathbb{E}(\alpha_0(U_t)),$$
$$u_t = \alpha_0(U_t) - \mu_0,$$
$$\beta_{j,t} = \alpha_j(U_t), \quad j = 1, ..., p.$$

Thus, $\{u_t\}$ is an i.i.d. sequence of random variables with distribution $F(\cdot) = \alpha_0^{-1}(\cdot + \mu_0)$, and the $\beta_{j,t}$ coefficients are functions of this u_t innovation.

An alternative form of model (2) widely used in economic applications is the ADF (augmented Dickey-Fuller) representation (3), in which the first order autoregressive coefficient plays an important role in measuring persistence in economic and financial time series, which in our case will be crucial to determine the dynamics of the inflation rate:

$$y_t = \mu_0 + \alpha_{1,t} y_{t-1} + \sum_{j=1}^{p-1} \alpha_{j+1,t} \Delta y_{t-j} + u_t, \tag{3}$$

where, corresponding to (1),

$$\alpha_{1,t} = \sum_{i=1}^{p} \alpha_i(U_t),$$
$$\alpha_{j+1,t} = -\sum_{i=j}^{p} \alpha_i(U_t), \qquad j = 1, ..., p.$$

Under some regularity conditions, if $\alpha_{1,t} = 1$, then y_t contains a unit root and is persistent; and if $\mid \alpha_{1,t} \mid < 1$, then y_t is stationary. Notice that equations (1), (2) and (3) are all equivalent representations of the adopted econometric model. More on regularity conditions underlying model (1) are found in Koenker and Xiao (2004, 2006).[3]

[3] For instance, Koenker and Xiao (2006) show that under some mild conditions the QAR(p) process y_t is covariance stationary and satisfies a central limit theorem.

2.2 Model estimation

Provided that the right hand side of (1) is monotone increasing in U_t, it follows that the τ-th conditional quantile function of y_t can be written as

$$Q_{y_t}(\tau \mid y_{t-1}, ..., y_{t-p}) = \alpha_0(\tau) + \alpha_1(\tau) y_{t-1} + ... + \alpha_p(\tau) y_{t-p}, \tag{4}$$

or more compactly as

$$Q_{y_t}(\tau \mid y_{t-1}, ..., y_{t-p}) = x_t' \alpha(\tau),$$

where $x_t' = (1, y_{t-1}, ..., y_{t-p})'$. The transition from (1) to (4) is an immediate consequence of the fact that for any monotone increasing function g and a standard uniform random variable, U, we have:

$$Q_{g(U)}(\tau) = g(Q_U(\tau)) = g(\tau),$$

where $Q_U(\tau) = \tau$ is the quantile function of U_t. Analogous to quantile estimation, quantile autoregression estimation involves the solution to the problem

$$\min_{\{\alpha \in R^{p+1}\}} \sum_{t=1}^{n} \rho_\tau (y_t - x_t' \alpha), \tag{5}$$

where ρ_τ is defined as in Koenker and Basset (1978):

$$\rho_\tau(u) = \begin{cases} \tau u, u \geq 0 \\ (\tau - 1) u, u < 0 \end{cases}.$$

It is worth mentioning that the quantile regression method is robust in distributional assumptions, a property that is inherited from the robustness of the ordinary sample quantiles. In addition, it is not the magnitude of the dependent variable that matters in quantile regression, but its position relative to the estimated hyperplane. As a result, the estimated coefficients are less sensitive to outlier observations than, for example, the standard OLS estimator. This superiority over OLS estimator is, in fact, common to any M-estimator.

Autoregressive order choice

Equation (1) gives our p-th order quantile autoregression model. We now discuss how to choose the optimal lag length p. We follow Koenker and Machado (1999) in testing for the null hypothesis of exclusion for the p-th control variable $\alpha_p(\tau)$ as it follows:

$$H_0 : \alpha_p(\tau) = 0, \quad \text{for all } \tau \in \Gamma, \tag{6}$$

where Γ is some (discrete) index set $\Gamma \subset (0,1)$. Let $\widehat{\alpha}(\tau)$ denote the minimizer of

$$\widehat{V}(\tau) = \min_{\{\alpha \in \mathbb{R}^{p+1}\}} \sum \rho_\tau (y_t - x_t'\alpha),$$

where $x_t' = (1, y_{t-1}, y_{t-2}, ..., y_{t-p})'$and $\widetilde{\alpha}(\tau)$ denotes the minimizer for the corresponding constrained problem without the p-th autoregressive variable, with

$$\widetilde{V}(\tau) = \min_{\{\alpha \in \mathbb{R}^{p}\}} \sum \rho_\tau (y_t - x_{1t}'\alpha),$$

where $x_{1t}' = \left(1, y_{t-1}, y_{t-2}, ..., y_{t-(p-1)}\right)'$. Thus, $\widehat{\alpha}(\tau)$ and $\widetilde{\alpha}(\tau)$ denote the unrestricted and restricted quantile regression estimates. Koenker and Machado (1999) state that one can test the null hypothesis (6) using a related version of the likelihood process for a quantile regression with respect to several quantiles. Suppose that the $\{u_t\}$ are i.i.d. but drawn from some distribution F. The LR statistics at a fixed quantile τ is derived, under some regularity conditions, as it follows:

$$L_n(\tau) = \frac{2\left(\widetilde{V}(\tau) - \widehat{V}(\tau)\right)}{\tau(1-\tau)\, s(\tau)}, \tag{7}$$

where $s(\tau)$ is the sparsity function, defined by:

$$s(\tau) = \frac{1}{f\left(F^{-1}(\tau)\right)}.$$

The sparsity function, also known as the "quantile-density function", plays the role of a nuisance parameter. In order to carry out a joint test about the significance of the p-th autoregressive coefficient with respect to the set of quantiles Γ, Koenker and

Machado (1999) suggest using the Kolmogorov-Smirnov type statistics:

$$\sup_{\tau \in \Gamma} L_n(\tau).$$

The authors show that under the null hypothesis (6):

$$\sup_{\tau \in \Gamma} L_n(\tau) \rightsquigarrow \sup_{\tau \in \Gamma} Q_1^2(\tau),$$

where $Q_1(\cdot)$ is a Bessel process of order 1. Critical values for $sup\ Q_q^2(\cdot)$ are extensively tabled in Andrews (1993).

Global stationarity

Given the choice of the optimal lag length p, one must check for global stationarity of the y_t process, in order to verify whether y_t is covariance stationary in the sense of Koenker and Xiao (2006). An approach for testing the unit root property is to examine it over a range of quantiles $\tau \in \Gamma$, instead of focusing only on a selected quantile τ, by using a Kolmogorov-Smirnov (KS) type test based on the regression quantile process for $\tau \in \Gamma$. In this sense, Koenker and Xiao (2006) proposed the following quantile regression based statistics for testing the null of a unit root:[4]

$$QKS = \sup_{\tau \in \Gamma} | U_n(\tau) |, \tag{8}$$

where $U_n(\tau)$ is the coefficient based statistics given by:

$$U_n(\tau) = n(\widehat{\alpha}_1(\tau) - 1).$$

Koenker and Xiao (2004) suggest the approximation of the limiting distribution of (8) under the null hypothesis by using the autoregressive bootstrap (ARB). An alternative way is to approximate the distribution under the null using the residual based block bootstrap procedure (RBB). The advantages of the RBB over ARB are documented in Lima and Sampaio (2005). In this paper, we conduct usual unit root tests (e.g. ADF) to check for global stationarity.

[4] The presence of a unit root implies that a shock today has a long-lasting impact.

Local unit root test

In this section, we introduce the Koenker-Xiao test, which is used to test the null hypothesis $H_0 : \alpha_1(\tau) = 1$, for a fixed $\tau \in (0, 1)$. We express the null hypothesis in the ADF representation as:

$$H_0 : \alpha_1 (\tau) = 1, \text{ for selected quantiles } \tau \in (0, 1) .$$

In order to test such a hypothesis, Koenker and Xiao (2004) proposed a statistic similar to the conventional augmented Dick-Fuller (ADF) t-ratio statistic. The t_n statistic is the quantile autoregression counterpart of the ADF t-ratio test for a unit root and is given by:

$$t_n (\tau) = \frac{\widehat{f (F^{-1} (\tau))}}{\sqrt{\tau (1 - \tau)}} \left(Y'_{-1} P_X Y_{-1}\right)^{\frac{1}{2}} \left(\widehat{\alpha}_1 (\tau) - 1\right),$$

where $\widehat{f (F^{-1} (\tau))}$ is a consistent estimator of $f (F^{-1} (\tau))$; Y_{-1} is the vector of lagged dependent variables (y_{t-1}) and P_X is the projection matrix onto the space orthogonal to $X = (1, \Delta y_{t-1}, ..., \Delta y_{t-p+1})$. Koenker and Xiao (2004) show that the limiting distribution of $t_n (\tau)$ can be written as:

$$t_n (\tau) \Rightarrow \delta \left(\int_0^1 \underline{W}_1^2 \right)^{-\frac{1}{2}} \int_0^1 \underline{W}_1 dW_1 + \sqrt{1 - \delta^2} N (0, 1)$$

where $\underline{W}_1 (r) = W_1 (r) - \int_0^1 W_1 (s) \, ds$ and $W_1 (r)$ is a standard Brownian Motion. Thus, the limiting distribution of $t_n (\tau)$ is nonstandard and depends on parameter δ given by:

$$\delta = \delta (\tau) = \frac{\sigma_{\omega\psi} (\tau)}{\sigma_\omega \sqrt{\tau (1 - \tau)}},$$

which can be consistently estimated. The terms $\sigma_{\omega\psi} (\tau)$ and σ_ω come from the long run covariance matrix of a bivariate Brownian motion and the critical values for the statistic $t_n (\tau)$ are provided by Hansen (1995, p.1155) for values of δ^2 in steps of 0.1 (for intermediate values of δ^2, Hansen suggests obtaining critical values by interpolation).

2.3 Identifying non-stationary observations

The QAR model can play a useful role in expanding the territory between classical stationary linear time series and their unit root alternatives. To see this, consider the following example of a simple QAR(1) model discussed in Lima et al. (2008):

$$y_t = \alpha_0 (U_t) + \alpha_1 (U_t) y_{t-1}. \tag{9}$$

Suppose that $\alpha_1 (U_t) = U_t + 0.5$. In this case, note that if $0.5 \leq U_t < 1$ then the model generates y_t according to a non-stationary dynamics, however, for smaller realizations of U_t, there is a mean reversion tendency. This way, the model exhibits a form of asymmetric persistence in the sense that sequences of strongly positive innovations of the i.i.d. standard uniform random variable U_t tend to reinforce its non-stationary like behavior, while occasional smaller realizations induce mean reversion and thus undermine the persistence of the process. Therefore, it is possible to have locally non-stationary time series being globally stationary.[5]

How to separate periods of (local) stationarity from periods where y_t exhibits non-stationary behavior? Lima et al. (2008) tackle this issue by defining the critical quantile ($\tau_{crit.}$) as the largest quantile $\tau \in \Gamma = (0,1)$ such that $\alpha_{1,t} (\tau) = \sum_{i=1}^{p} \alpha_i(\tau) < 1$, where τ is the quantile of U_t. The critical quantile $\tau_{crit.}$ can easily be identified by using the Koenker and Xiao (2004) test for $H_0 : \alpha_{1,t} (\tau) = 1$ over a grid of selected quantiles $\tau \in \Gamma = (0,1)$. In turn, the critical conditional quantile $Q_{y_t}(\tau_{crit.} \mid \mathcal{F}_{t-1})$ is defined as the τ-th conditional quantile function evaluated at $\tau = \tau_{crit.}$.

Now, let $\Omega = (t_1, t_2, ...t_T)$ be the set of all observations T and assume that for the subset of time periods $\Upsilon \subset \Omega$, the time series y_t exhibits non-stationary behavior (unit root model). Lima et al. (2008) show that: $Q_{y_t}(\tau_{crit.} \mid \mathcal{F}_{t-1}) < y_t$ for all $t \in \Upsilon$, that is, the critical conditional quantile of y_t will always be lower than y_t for all periods in which y_t exhibits a unit-root behavior.

Moreover, by comparing both time series y_t and $\widehat{Q}_{y_t}(\tau_{crit.} \mid \mathcal{F}_{t-1})$, one can compute the statistic $H = \frac{1}{T}\sum_{t=1}^{T} I_{t\, \left\{y_t > \widehat{Q}_{y_t}(\tau_{crit.} \mid \mathcal{F}_{t-1})\right\}}$, which represents the percentage of pe-

[5] For instance, reproducing the example of Lima et al. (2008), if at a given period $t = t_A$, $U_{t_A} = 0.2$, then $\alpha_1 (U_{t_A}) = 0.7$ and the model will present a mean reversion tendency at $t = t_A$. However, if at $t = t_B$, $U_{t_B} = 0.5$, then $\alpha_1 (U_{t_B}) = 1$, and y_t will have a local unit-root behavior.

riods in which y_t exhibits (local) non-stationary behavior, where T is the sample size and I_t is an indicator function such that $I_t = \begin{cases} 1 \;\; ; \text{ if } \; y_t > \widehat{Q}_{y_t}(\tau_{crit.} \mid \mathcal{F}_{t-1}) \\ 0 \;\; ; \text{ otherwise} \end{cases}$. Under some mild conditions, one can further show, as expected, that $\underset{T\to\infty}{\text{plim}}(H) = 1 - \tau_{crit.}$ See Lima et al. (2008) for further details. In next section, we apply these results to a set of inflation rates in Brazil in order to reveal those periods in which the inflation rate y_t exhibited non-stationary dynamics.

Note that our definition of inflationary inertia (i.e. the statistic H) is the frequency of non-stationary periods observed in the sample, instead of the usual definition of inertia related to the sum of the autoregressive coefficients, for instance, in an OLS regression.

3 Empirical Exercise

3.1 Data

We focus our analysis on the dynamics of the monthly inflation rate in Brazil, as measured by IPCA, which is a consumer price index (CPI) used to compute the official inflation target. Our goal is to estimate the QAR(p) model for the inflation rate based on the IPCA and its main components. In this sense, we study the monthly headline inflation (IPCA) as well as its two main components: (1) the market prices; and (2) the regulated and monitored prices.[6]

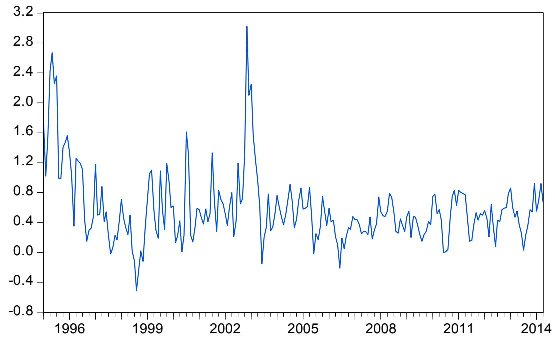
In addition, we investigate the dynamics of the market prices disaggregated in two ways: (i) tradables and (ii) non-tradables; or, alternatively, disaggregated as: (a) services, (b) food and beverages, and (c) industrial goods.[7] The sample period ranges from January 1995 to April 2014. The complete dataset is publicly available in the Time Series Management System (SGS) in the BCB website (www.bcb.gov.br).

Figure 1 shows the CPI inflation rate since 1995, as measured by the Brazilian Institute of Geography and Statistics (IBGE). The graph exhibits the more recent period, after the Brazilian (inflation) stabilization plan in mid 1994.

[6] The regulated prices are defined as those that are relatively insensitive to domestic demand and supply conditions or that are in someway regulated by a public agency.

[7] The "industrial goods" is an artifical series of inflation, defined here as the (log) difference of the market price inflation and the sum of services and food and beverages' monthly inflation rates.

Figure 1 - Inflation rate (IPCA) % per month



Descriptive statistics for the headline IPCA inflation as well as its components are presented in Table 1. The last column of Table 1 shows the weights of each component in the aggregate IPCA index.

Table 1 - Inflation and its components:

descriptive statistics for monthly rates and weights

Inflation	Mean	Median	Maximum	Minimum	Std. Dev.	Weight (%)
1) headline	0.59	0.49	3.02	-0.51	0.49	100.00
1.1) regulated prices	0.78	0.42	5.86	-1.11	1.09	22.70
1.2) market prices	0.54	0.49	2.99	-0.45	0.48	77.30
1.2.i) tradables	0.46	0.37	3.58	-0.66	0.56	35.70
1.2.ii) non-tradables	0.63	0.46	4.44	-0.40	0.68	41.60
1.2.a) services	0.71	0.48	6.91	-0.36	0.89	35.60
1.2.b) food and beverages	0.56	0.56	5.85	-1.28	0.86	24.80
1.2.c) industrial goods	0.36	0.33	2.23	-1.60	0.47	16.90

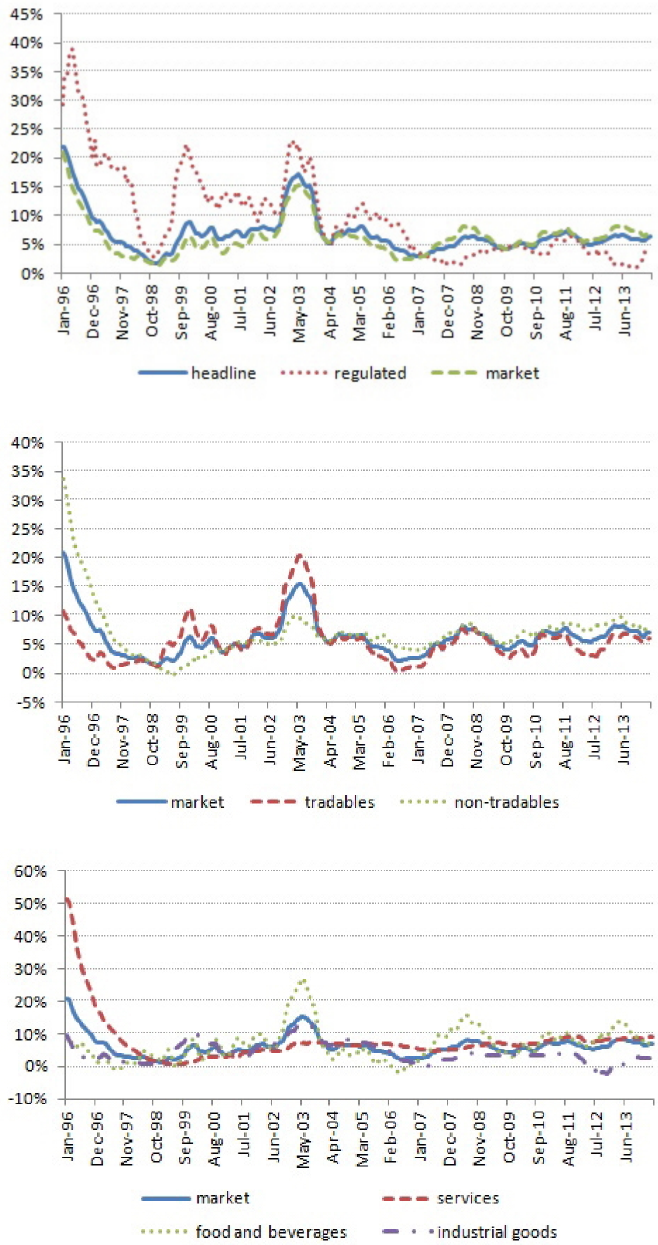
Notes: The sample period ranges from January 1995 to April 2014.

Weights of sub-indexes, in respect to the headline inflation, are as of April 2014.

After several unsuccessful economic plans that attempt to fight the inertial inflation in Brazil, the inflation stabilization was achieved with the implementation of the Real Plan in June 1994. The average monthly inflation measured by IPCA in the first half of 1994 was 43.1% whereas the average of the second half was just 2.9%. There has been a structural change in the dynamics of inflation and the behavior on this variable post 1994 has displayed more stable character.

Considering the twelve-month inflation rates shown in Figure 2, we observe the data for the headline inflation as well as its components are affected by the period of disinflation. We can also observe a spike of inflation in 2002. This inflationary episode was result of the capital flight and consequently devaluation of the Real that occurred before Lula's presidential election.

Figure 2 - Disaggregated inflation rates (% 12 months)



Among the components of market prices, services and food and beverages prices are the most volatiles and they present the greatest average monthly change. But the behavior of the components are not uniform through the sample period. Table 2 shows the average yearly inflation rates for 4 subsamples.

Table 2 - Yearly Average Inflation Rates

Inflation	1995-1999	2000-2004	2005-2009	2010-2014*
1) headline	9.7	8.6	4.7	6.1
1.1) regulated prices	17.1	12.5	4.6	3.7
1.2) market prices	8.2	7.2	4.8	6.9
1.2.i) tradables	5.8	8.2	3.7	5.6
1.2.ii) non-tradables	11.2	6.1	5.9	8.1
1.2.a) services	16.1	5.5	6.0	8.6
1.2.b) food and beverages	4.7	8.7	5.7	8.7
1.2.c) industrial goods	5.5	7.4	3.0	2.2

Note: *For 2014, data through April.

Remaining indexation and realignment of prices in the process of privatization occurred through the 1990s explains mostly of the behavior of services in the first subsample and the regulated prices[8] in the first two subsamples. The huge devaluation of the Real currency in 2002 affected the prices of tradable goods as expected. For the period of 2005-2009, we see a more homogeneous behavior with more inflation regarding services. Such feature is more salient in the last period along with a similar behavior of food and beverages. Since there is heterogeneous behavior among the components of aggregate inflation, it is advisable to take into account this feature in the empirical analysis.

Usual unit root tests for headline inflation and its components using Augmented Dickey-Fuller (ADF) and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) approaches are displayed in Table 3.

[8] For a description of the behavior of regulated prices in the 1990s and early 2000 see Figueiredo and Ferreira (2002).

Table 3 - Usual unit root tests

	monthly		12 months	
Inflation	ADF	KPSS	ADF	KPSS
1) headline	-7.11	0.17	-4.50	0.34
1.1) regulated prices[3]	-12.63	0.06	-5.00	0.09
1.2) market prices	-6.37	0.14	-4.49	0.08
1.2.i) tradables	-7.66	0.12	-3.25	0.12
1.2.ii) non-tradables	-5.70	0.30	-5.71	0.16
1.2.a) services	-6.17	0.22	-3.78	0.20
1.2.b) food and beverages	-7.82	0.24	-9.20	0.04
1.2.c) industrial goods	-8.10	0.06	-8.10	0.06

Notes: 1) Critical values (1%, 5%, 10%): ADF (intercept) (-3.46, -2.87, -2.57) and KPSS (intercept) (0.74, 0.46, 0.35).

2) ADF (intercept+trend) (-4.00, -3.43, -3.14) and KPSS (intercept+trend) (0.22, 0.15, 0.12).

3) Equation includes intercept and trend. Equations for other series only include intercept.

In terms of ADF tests, the null hypothesis of unit root is rejected in all cases for a significance level of 1% to both monthly and 12-month rates except for tradables where the null is rejected in a 5% significance level. Similar results are found for KPSS tests and the null hypothesis of stationarity is not rejected in all cases.

3.2 In-sample analysis

In this paper, we choose to use seasonal adjusted data (X12 filtering) to avoid a more complex dynamics in the quantile autoregressive process. Another reason to deal with filtered data is to properly discriminate the source of non-stationarity, that is, to focus on local unit roots and to avoid seasonal unit root processes (see Ghysels and Osborn, 2001). In addition, the QAR modelling and the respective testing procedures are not originally designed to deal with seasonal effects.[9]

[9] Although some inflation components in Brazil clearly exhibit some seasonal patterns (e.g. services inflation), a proper investigation considering the raw data (without seasonal adjustment) would require additional econometric tools outside the main focus of this paper. We leave this route as a

We start investigating the behavior of headline inflation over the past two decades (January1995-April2014). In appendix, we present the results for the other inflation components presented in Table 1. First, we determine the autoregressive order of the QAR(p) model (1) using the Kolmogorov-Smirnov test based on LR statistics, following Koenker and Machado (1999). We start estimating the quantile regression next presented with $p_{\max} = 6$, that is:

$$Q_{y_t}(\tau \mid y_{t-1}, ..., y_{t-p}) = \alpha_0(\tau) + \alpha_1(\tau) y_{t-1} + ... + \alpha_6(\tau) y_{t-6}.$$

The index set used for quantiles is $\tau \in \Gamma = [0.1, 0.9]$ with steps of 0.05. Next, we test if the sixth order covariate is relevant, based on the null hypothesis:

$$H_0 : \alpha_6(\tau) = 0, \quad for\ all\ \tau \in \Gamma.$$

The results are reported in Table 4. Using critical values obtained in Andrews (1993), we can infer that the autoregressive variable y_{t-6} can be excluded from our econometric model.

Table 4: Choice of the autoregressive order

excluded variable	$\sup_{\tau\in\Gamma} L_n(\tau)$ estimate	H_0	Result at 5%
y_{t-6}	0.71	$\alpha_6(\tau) = 0$	do not reject
y_{t-5}	1.06	$\alpha_5(\tau) = 0$	do not reject
y_{t-4}	2.65	$\alpha_4(\tau) = 0$	do not reject
y_{t-3}	4.81	$\alpha_3(\tau) = 0$	do not reject
y_{t-2}	3.01	$\alpha_2(\tau) = 0$	do not reject

The 5% and 10% critical values are 9.31 and 7.36, respectively.

Since the sixth order is not relevant, we proceed the analysis by checking whether the fifth order covariate is relevant (or not).[10] Thus, we considered the null hypothesis:

suggestion for future research.

[10] As usual, we performed the test for exclusion of y_{t-6} with same sample size used to test the exclusion of y_{t-5}.

$$H_0 : \alpha_5(\tau) = 0, \quad for\ all\ \tau \in \Gamma,$$

whose results are also presented in Table 4. Indeed, we verify that all variables from the sixth up to the second autoregressive covariates can be excluded. Thus, the optimal choice of lag length in our model is $p = 1$ and this order will be used in the subsequent estimation and hypothesis tests. In other words, the model of the conditional quantiles estimated for the headline inflation is the following: $Q_{y_t}(\tau \mid y_{t-1}) = \alpha_0(\tau) + \alpha_1(\tau)\, y_{t-1}$. The point estimates of the respective functions $\widehat{\alpha}_i(\tau), i = \{0; 1\}$, for a discrete grid of quantiles τ are presented in Table 5.[11]

Table 5 : Estimated QAR(1) model

for the headline inflation (IPCA)

τ	$\widehat{\alpha}_0(\tau)$	Std. Error	$\widehat{\alpha}_1(\tau)$	Std. Error
0.10	0.02	0.06	0.50	0.12
0.20	0.06	0.05	0.55	0.12
0.30	0.08	0.03	0.61	0.07
0.40	0.10	0.04	0.67	0.08
0.50	0.11	0.03	0.78	0.07
0.60	0.13	0.03	0.80	0.07
0.70	0.17	0.04	0.87	0.09
0.80	0.23	0.04	0.91	0.07
0.90	0.33	0.04	0.98	0.07

Note: Standard errors are computed from a Huber sandwich covariance matrix.

Next, we conduct two usual tests to further investigate the estimated quantile process. The first test is the so-called "Slope Equality Testing" due to Koenker and Bassett (1982). The idea is to check for slope equality across quantiles as a robust test of heteroskedasticity. The null hypothesis is given by $Ho : \alpha(\tau_1) = \alpha(\tau_2) = ... = \alpha(\tau_k)$ and these restrictions on the coefficients, along the set of quantiles $\tau \in$

[11] Compared to Maia and Cribari Neto (2006), our results, based on a larger data sample, indicate a higher autoregressive point estimate $\widehat{\alpha}_1(\tau)$ for the median quantile $\tau = 0.5$ (that is, 0.63 from Maia and Cribari Neto, whereas in this paper we find 0.78).

$[\tau_1, ..., \tau_k]$, can be tested through a Wald statistic, which is distributed as a chi-squared distribution.

The second test is called "Symmetry Testing" and is based on Newey and Powell (1987), which proposes a less restrictive hypothesis of symmetry for asymmetric least squares estimators, but the approach can also be used for quantile regressions. The idea of the test is that if the conditional distribution of y_t is symmetric, then it follows that $Ho : \alpha(0.5) = \frac{\alpha(\tau)+\alpha(1-\tau)}{2}$. This null hypothesis can be evaluated through a Wald test on the quantile process.[12]

The results for the headline inflation suggest that the inflationary dynamics in Brazil is not uniform across distinct quantile levels. The test results indicate a rejection of the null hypothesis (at 5% significance level) in the slope equality test; whereas the null for the symmetry test cannot be rejected at the same significance level (see Table 9 for further details).

Now, we conduct the "local" unit root analysis by using the Koenker and Xiao (2004) test. In order to investigate the non-stationary dynamics of the inflation rate in Brazil , we need to test the null hypothesis $H_0 : \alpha_1(\tau) = 1$ at various quantiles by using the t-ratio test $t_n(\tau)$ proposed by Koenker and Xiao (2004). Table 6 reports the results. The second column displays the estimate of the autoregressive term at each decile. Note that, in accordance with our theoretical model, $\widehat{\alpha}_1(\tau)$ is monotonic increasing in τ, and it is close to unity when we move towards upper quantiles. Table 6 also shows that the null hypothesis $H_0 : \alpha_1(\tau) = 1$ is rejected against the alternative hypothesis $H_1 : \alpha_1(\tau) < 1$ for all $\tau \in [0.1; 0.7]$. The critical values were obtained by interpolation of the critical values reported in Hansen (1995). The last column summarizes the local analysis.

Table 6 shows that the critical quantile found using Brazilian inflation data belongs to a conditional quantile between the quantile level 0.7 and 0.8. Indeed, using a finer grid we find that $\tau_{crit.} = 0.71$. Consequently, for approximately $H = 29\%$ of the periods the inflation rate exhibited a non-stationary behavior in the considered sample (1995-2014). Figure 3 displays the estimated coefficients of the QAR(1) model and its 95% confidence bands (and respective OLS estimates for comparison purposes)

[12] Newey and Powell point out that if it is known (*a priori*) that the errors are i.i.d., but possibly asymmetric, one can restrict the null hypothesis to only test the restriction for the intercept.

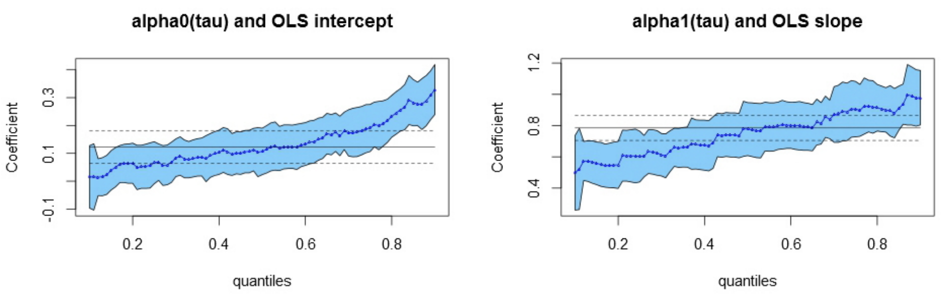
and Figure 4 presents the respective non-stationary periods (i.e. vertical gray bars), represented by the indicator variable $I_t = \begin{cases} 1 \ ; \ \text{if} \ \ y_t > \widehat{Q}_{y_t}\left(\tau_{crit.} = 0.71 \mid \mathcal{F}_{t-1}\right) \\ 0 \ ; \ \text{otherwise} \end{cases}$.

Table 6 : Koenker-Xiao test

τ	$\widehat{\alpha}_1(\tau)$	$t_n(\tau)$	δ^2	H_0: $\alpha_1(\tau) = 1$
0.10	0.50	-10.14	-2.37	reject
0.20	0.55	-18.43	-2.47	reject
0.30	0.61	-13.40	-2.57	reject
0.40	0.67	-9.85	-2.61	reject
0.50	0.78	-7.64	-2.69	reject
0.60	0.80	-5.66	-2.68	reject
0.70	0.87	-2.98	-2.73	reject
0.80	0.91	-1.69	-2.70	do not reject
0.90	0.98	-0.29	-2.60	do not reject

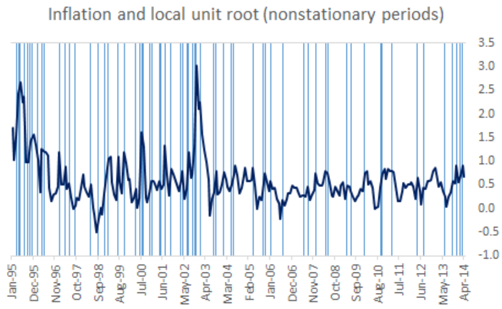
Note: Last column's results are related to a 5% level of significance.

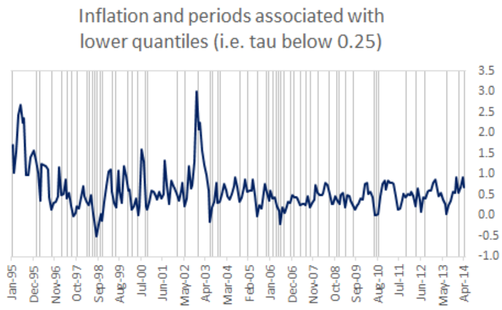
Figure 3 - Estimated coefficients (IPCA, headline inflation, 1995-2014)



Notice from Figure 4 that the non-stationary dynamics is observed for 29% of the observations, although the unit root behavior seems to be concentrated in the beginning of the sample. Due to the nonlinear dynamics of y_t, it is possible to identify different inflation regimes by estimating, for several historical periods, the respective statistic H previously described in section 2.3.

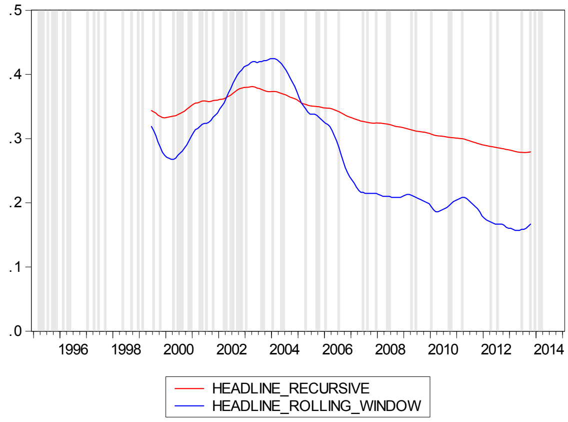
Figure 4 - Inflation (IPCA) and periods associated with local unit root (upper panel) or lower quantiles (bottom panel)





To further understand the inflation inertia in the past two decades, we next reestimate the QAR(1) model by using a recursive estimation scheme. To do so, we estimate the model coefficients for the sample Jan1995-Dec1998, with 48 observations, and compute the respective H statistic. Then, we add a new observation (Jan1999), reestimate the model, and compute again the H statistic. We continue this way until we reach the full sample with 232 observations (Jan1995-April2014). We also computed the H statistics based on a rolling window scheme. The result is presented in Figure 5. From this exercise, we can infer that there was a gradual reduction of inflation inertia in Brazil, which can be credited to the inflation targeting regime.

Figure 5 - Frequency of periods with non-stationary dynamics with recursive and rolling window estimation of the H statistic



Note: The vertical bars represent the non-stationary periods.

An interesting question one could ask is about the relationship between the non-stationary periods of inflation and other macroeconomic variables. Although this is not the main objective of the paper, we next provide a flavor of such analysis by comparing the QAR results for the headline inflation with a measure of country risk aversion (Embi+Br) and also a market expectations series for the twelve-month ahead cumulated inflation.[13]

Indeed, the periods associated with local unsustainable inflationary dynamics can be related to those of increased risk aversion and higher inflation expectations (see Appendix B and C for further details). In addition, we further apply the QAR model to those macro series (Embi+Br and inflation expectations) in order to construct its own series of non-stationary periods (that is, $I_t^{Embi+Br}$ and I_t^{Expect}).

The respective estimations are based on the samples: Jan1995-Apr2014 (Embi+Br) and Nov2001-Apr2014 (inflation expectations) and lead to a QAR(2) model for the Embi+Br time series (resulting in $H^{Embi} = 47\%$) and a QAR(4) model for the inflation expectations (with $H^{Expect} = 10\%$). The comparison of the non-stationary periods regarding headline inflation, Embi+Br and inflation expectations is presented in Tables 7 and 8. In particular, notice from the Granger causality tests that the non-stationary periods of the Embi+Br series seem to anticipate those regarding the inflationary

[13] We use the Focus survey of market expectations conducted by the Central Bank of Brazil (available at: https://www3.bcb.gov.br/expectativas/publico/en/serieestatisticas).

dynamics.

Table 7 - Sample correlations of non-stationary periods I_t for IPCA, Embi+Br and Inflation Expectations

number of lags (j)	$Corr(I_t^{IPCA}; I_{t-j}^{Embi+Br})$	$Corr(I_t^{IPCA}; I_{t-j}^{Expect.})$
0	-0.006	0.204
1	0.152	0.038
2	0.020	0.149
3	0.243	0.038
4	0.078	0.038
5	-0.046	-0.072
6	0.061	0.094

Table 8 - Granger causality test between non-stationary periods I_t for IPCA, Embi+Br and Inflation Expectations

Null hypothesis	p-value
$I_t^{Embi+Br}$ does not Granger Cause I_t^{IPCA}	0.006
I_t^{IPCA} does not Granger Cause $I_t^{Embi+Br}$	0.156
I_t^{Expect} does not Granger Cause I_t^{IPCA}	0.573
I_t^{IPCA} does not Granger Cause I_t^{Expect}	0.531
I_t^{Expect} does not Granger Cause $I_t^{Embi+Br}$	0.918
$I_t^{Embi+Br}$ does not Granger Cause I_t^{Expect}	0.307

Note: The pairwise Granger causality test is based on 4 lags.

Next, we investigate the disaggregated components of inflation. Table 9 summarizes the results (detailed in Appendix A), regarding such components of the headline inflation for the same investigated period (1995-2014).

Table 9 - Summary of results for inflation disaggregated components

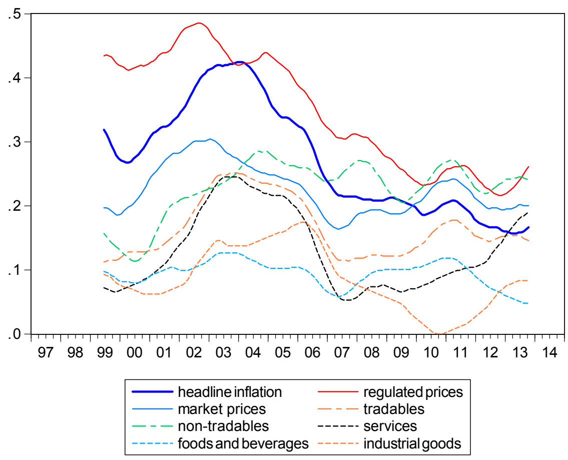
Inflation	optimal lag	$\tau_{crit.}$	$H_{1995-2014}$	slope test	symmetry test
1) headline	1	0.71	0.29	0.0059	0.2087
1.1) regulated prices	4	0.64	0.36	0.0000	0.0820
1.2) market prices	1	0.77	0.23	0.1226	0.0774
1.2.i) tradables	2	0.84	0.16	0.0331	0.0917
1.2.ii) non-tradables	2	0.77	0.23	0.0000	0.6858
1.2.a) services	3	0.86	0.14	0.0000	0.9735
1.2.b) food and beverages	2	0.90	0.10	0.0597	0.2877
1.2.c) industrial goods	1	0.90	0.10	0.6008	0.4518

Notes: Sample: Jan1995-Apr2014. P-values in the last two columns. Each test (slope equality and symmetry) includes intercept and all slopes' responses along the grid of quantiles $\tau = [0.1, 0.2, ..., 0.9]$.

$$Ho^{slope\ test}: \alpha(\tau_1) = \alpha(\tau_2) = ... = \alpha(\tau_k) \ ; Ho^{symmetry\ test}: \alpha(0.5) = \frac{\alpha(\tau) + \alpha(1-\tau)}{2}$$

Figure 6 - Inertia measure for inflation and its components

(rolling window estimation of H, which is the ratio of non-stationary periods)



When we compare the choice of lags, the headline inflation, market prices and industrial goods are described by simple autoregressive model, while services and regulated prices are described by a more complex autoregressive dynamics. The choice of lags may indicate that disaggregated items have a higher temporal dependence caused by idiosyncratic movements, lost in the act of aggregation.

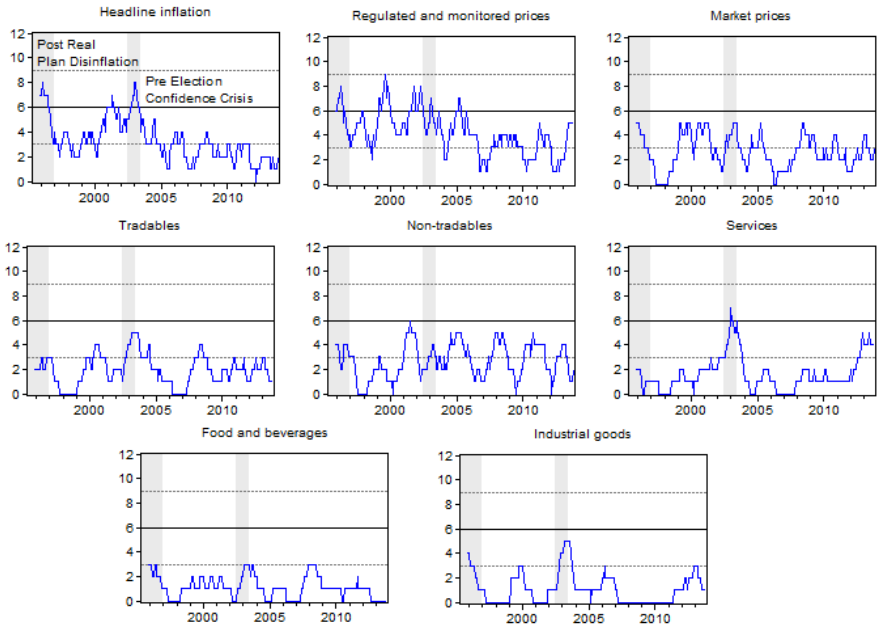
By analyzing the critical quantile, the headline inflation belongs to the quantile 71%, whereas its two main components (regulated and market prices) are located in quantiles 64% and 77%, respectively. In turn, the disaggregated components of market prices all belong to higher quantiles (e.g., services, foods and beverages, and industrial goods are located in quantiles 86%, 90% and 90%, respectively). This result indicates that disaggregated data may have a lower persistence (i.e., higher critical quantile). In other words, the quantile autoregression approach, applied to Brazilian inflation data, suggests that aggregation increases persistence; corroborating previous findings in the literature (e.g. Altissimo, Mojon and Zaffaroni, 2009).

The results of the statistical H for the whole sample (1995-2014) are in line with previous results.[14] For disaggregated data, we find a non-stationary behavior in a smaller percentage when compared to aggregate data. When we analyze the H statistic calculated recursively, there is a temporal decrease in this except in services inflation, which shows a significant increase pattern since mid 2009.

In order to identify the periods of non-stationary behavior for the aggregate IPCA and its components, Figure 7 displays the accumulated number of local non-stationary episodes in 12 months for headline inflation and its components. The two highlighted areas shows the disinflation post Real Plan period (May/1995 to November/1996) and the confidence crisis before Lula's election (July/2002 to May/2003). Additionally, to make the inspection of Figure 7 more straightforward, three lines were drawn in the charts: Two dotted lines for 3 and 9 episodes and one bold line representing 6 episodes of (local) non-stationary behavior.

[14] As a historical curiosity, we also include the analysis for the period of hyperinflation (1980-1994). In this case we impose the (unrealistic) assumption that the data are stationary for this period. The test of Koenker and Machado (1999) indicates that a first-order autoregressive process would describe well this dataset. The critical quantile is the first quantile ($\tau_{crit.} = 0.10$), which shows the existence of high persistence, since the H statistic indicates that in 90% of cases there was a non-stationary behavior for this period. We leave as a sugestion for future research the investigation of the 1980-1994 period using the second difference of price indexes, that is, the first difference of inflation rates, which are expected to show an overall stationary behavior.

Figure 7 - Twelve-month accumulated number of local non-stationary episodes



Regarding the headline inflation, the observed periods with more occurrences of non-stationary episodes coincides with the highlights one. Furthermore, the reduction of occurrences is patent in the last part of the sample. The reduction of non-stationarities is also evident for the regulated and monitored prices and it could explain at least partially the reduction observed in the headline inflation. As mentioned before, the process of the price realignment explains the behavior of monitored prices in the first part of the sample. It also noticeable a hike in the latest period covered by the sample used in this paper. Additionally, food and beverages displayed a stationary like behavior all over the entire sample.

3.3 Out-of-sample forecasting exercise

In this section, we compare the out-of-sample point forecast performance of the QAR model (at median quantile), with standard AR and random walk counterparts.[15] To do so, we use data over the period January 1995-December 2004 ($T = 120$ observations) for model estimation (training sample) and reserve the remaining data for out-of-sample forecasting. We construct point forecasts for horizons $h = 1, ..., 36$ months;

[15] For each price index, we estimate a QAR model using the same lag length indicated in the previous section.

and the evaluation sample for $h = 1$ ranges from January 2005-April 2014 (i.e. 112 point forecasts).

To evaluate the performance of the competing models, we estimate them by using both recursive estimation (increasing sample size) as well as rolling window estimation (with a fixed sample of $T = 120$ monthly observations) which is more suitable in the presence of structural breaks. In the later case, each model is initially estimated using the first 120 observations and the one-month-ahead (up to $h = 36$ months-ahead) point forecasts are generated. We, then, drop the first data point, add an additional observation at the end of the sample, re-estimate the models and generate again out-of-sample forecasts. This process is repeated along the remaining data. See Morales-Arias and Moura (2013) for a detailed discussion about rolling window and recursive forecasting.

We compute the Mean Squared Error (MSE) of each model for every forecast horizon and generate MSE ratios of each model in respect to the quantile regression (benchmark model). The results for the headline inflation are presented in Table 10; and for the remaining price indexes are shown in Appendix D. We also statistically test (pairwise) the difference between the MSE loss functions of the quantile regression forecast (at median quantile), in respect to the other approaches, based on the Diebold and Mariano (1995) and West (1996) test[16], in the recursive estimation scheme, and on the unconditional predictive ability test of Giacomini and White (2006), in the rolling window case. In both tests, the null hypothesis assumes equal predictive ability of two competing forecasts.

In respect to the MSE ratios for the headline inflation, it is worth noting the relatively good performance of the QAR model in comparison to the AR and random walk forecasts in several horizons and both sampling schemes. In respect to the other price indexes, excepting the results for non-tradables and food and beverages, the QAR model is again indicated as the best model (based on the MSE ratio) among the competing investigated models, in many cases of the two sampling schemes and considered forecast horizons. The Diebold-Mariano-West and Giacomini-White tests

[16] The variances entering the test statistics use the Newey and West (1987) estimator, with a bandwidth of 0 at the 1-month horizon and 1.5*horizon in the other cases, following Clark (2011, supplementary appendix) and Clark and McCracken (2012, p.61).

corroborate these results in many situations.[17]

Table 10 - MSE ratios for the headline inflation

Panel A - Recursive estimation

Model	Description	h=1	h=2	h=3	h=6	h=9	h=12	h=24	h=36
1	Random walk	1.171 ***	1.288 ***	1.379 ***	1.318 *	1.374	1.314 *	2.150 **	1.784 *
		[3.453]	[3.197]	[2.871]	[1.662]	[1.654]	[1.667]	[2.483]	[1.918]
		(0.001)	*(0.002)*	*(0.005)*	*(0.1)*	*(0.101)*	*(0.099)*	*(0.015)*	*(0.059)*
2	AR(1)	1.028 **	1.055 **	1.087 **	1.220 ***	1.283 ***	1.332 ***	1.371 ***	1.404 ***
		[2.414]	[2.231]	[2.428]	[2.836]	[3.11]	[3.346]	[3.774]	[4.026]
		(0.017)	*(0.028)*	*(0.017)*	*(0.006)*	*(0.002)*	*(0.001)*	*(0)*	*(0)*
3	AR(2)	1.017	1.049 **	1.077 **	1.190 ***	1.245 ***	1.297 ***	1.351 ***	1.383 ***
		[1.232]	[2.173]	[2.499]	[2.757]	[2.959]	[3.241]	[3.711]	[3.974]
		(0.22)	*(0.032)*	*(0.014)*	*(0.007)*	*(0.004)*	*(0.002)*	*(0)*	*(0)*
4	AR(3)	1.015	1.044 **	1.061 **	1.146 **	1.183 ***	1.215 ***	1.230 ***	1.254 ***
		[1.061]	[2.066]	[2.244]	[2.478]	[2.733]	[3.08]	[3.38]	[3.77]
		(0.291)	*(0.041)*	*(0.027)*	*(0.015)*	*(0.007)*	*(0.003)*	*(0.001)*	*(0)*
5	AR(4)	0.989	0.980	0.966	1.025	1.032	1.064 *	1.095 **	1.110 ***
		[-0.437]	[-1.086]	[-1.323]	[0.732]	[0.877]	[1.889]	[2.454]	[3.315]
		(0.663)	*(0.28)*	*(0.189)*	*(0.466)*	*(0.382)*	*(0.062)*	*(0.016)*	*(0.001)*
6	QAR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Best model		**5**	**5**	**5**	**6**	**6**	**6**	**6**	**6**

Panel B - Rolling window estimation

Model	Description	h=1	h=2	h=3	h=6	h=9	h=12	h=24	h=36
1	Random walk	1.246 **	1.340 ***	1.406 **	1.359 *	1.434 *	1.340 **	2.285 ***	2.024 ***
		[5.298]	[8.477]	[6.107]	[3.242]	[3.399]	[4.218]	[22.533]	[14.021]
		(0.021)	*(0.004)*	*(0.013)*	*(0.072)*	*(0.065)*	*(0.04)*	*(0)*	*(0)*
2	AR(1)	1.015	1.019	1.020	1.095	1.092	1.107	1.101	1.145
		[0.231]	[0.232]	[0.125]	[1.065]	[0.852]	[1.089]	[0.83]	[2.672]
		(0.631)	*(0.63)*	*(0.724)*	*(0.302)*	*(0.356)*	*(0.297)*	*(0.362)*	*(0.102)*
3	AR(2)	1.030	1.030	1.035	1.106	1.099	1.106	1.097	1.147
		[0.855]	[0.576]	[0.363]	[1.352]	[0.987]	[1.128]	[0.791]	[2.644]
		(0.355)	*(0.448)*	*(0.547)*	*(0.245)*	*(0.32)*	*(0.288)*	*(0.374)*	*(0.104)*
4	AR(3)	1.042	1.042	1.043	1.100	1.097	1.098	1.086	1.137
		[1.796]	[1.058]	[0.535]	[1.173]	[0.932]	[1.006]	[0.71]	[2.666]
		(0.18)	*(0.304)*	*(0.465)*	*(0.279)*	*(0.334)*	*(0.316)*	*(0.4)*	*(0.103)*
5	AR(4)	1.031	1.006	0.994	1.049	1.058	1.064	1.061	1.112
		[0.919]	[0.028]	[-0.012]	[0.397]	[0.443]	[0.597]	[0.495]	[2.324]
		(0.338)	*(0.868)*	*(0.912)*	*(0.529)*	*(0.505)*	*(0.44)*	*(0.482)*	*(0.127)*
6	QAR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Best model		**6**	**6**	**5**	**6**	**6**	**6**	**6**	**6**

Notes: Forecast horizons (h) in months. Sample estimation (h=1): Jan1995-Dec2004. Sample evaluation (h=1): Jan2005-Apr2014. Rolling window estimation is based on T=120 observations. The median quantile regression (model 6) is used as benchmark in both panels to compute the MSE ratios (MSE of model 6 in denominator). The Diebold-Mariano-West test is used in Panel A, and the Giacomini-White test is employed in Panel B. In both panels, positive test statistics indicate that the loss of model m is greater than the benchmark (QAR) loss. The null in both tests assumes equal predictive ability. Test statistics are presented [in brackets] and p-values are shown (in parenthesis). *, **, and *** indicate rejection of the null at 10%, 5% and 1% levels, respectively.

[17] This result in favor of the quantile-based point forecasts can partially be explained by the ability of the (median) quantile regression in dealing with outlier observations (in our case, extreme non-anticipated inflationary shocks), which might affect the performance of the remaining models, exclusively based on Ordinary Least Squares (OLS) estimation, which is only designed to account for average responses.

4 Conclusions

The purpose of this article is to study the persistence of Brazilian inflation using quantile regression techniques. To characterize the dynamics of inflation we used the Quantile Autoregression model (QAR) proposed by Koenker and Xiao (2002, 2004, 2006). Based on a QAR, we find evidence of important heterogeneity associated with the inflation dynamics in Brazil, that cannot be described by processes simply modelling the conditional mean and estimated with a standard OLS setup. Moreover, the upper quantiles of the inflation rate process in Brazil seem to exhibit a strong persistence that can be well described by processes of autoregressive type when the economy is in distress periods.

As expected, when we apply the usual unit root tests, the inflation series are stationary for the post stabilization period. When we compare the choice of lags for the QAR model, the headline inflation, market prices and industrial goods are described by simple autoregressive model, whilst other disaggregated items are described by a more complex autoregressive dynamics. The choice of lags may indicate that disaggregated items have a higher temporal dependence caused by idiosyncratic movements, which are potentially lost (or mutually canceled) in the act of price-component aggregation.

The "local" unit root analysis based on the test of Koenker and Xiao (2004) was applied to investigate the non-stationary dynamics of the inflation rate in Brazil. The result shows that the critical quantile using Brazilian inflation data belongs to the level $\tau_{crit.} = 0.71$. Consequently, for approximately $H = 29\%$ of the periods the inflation rate exhibited a non-stationary behavior in the considered sample (1995-2014). To further understand the inflation inertia in the past two decades, we next reestimate the QAR(1) model by using a recursive estimation scheme and compute the respective H statistic, which is our suggested proxy for inflationary inertia. From this recursive exercise, we can infer that there was a gradual reduction of inflation inertia, which can be credited to the inflation targeting regime. For disaggregated data, the results indicate lower persistence.

The relationship between the non-stationary periods of inflation were compared with a measure of risk aversion and a market expectation series. From the results of Granger causality tests, the non-stationary periods of the measure of risk aversion seems to anticipate those regarding the inflationary dynamics.

We also performed an out-of-sample forecast exercise comparing the QAR model against random walk and autoregressive models. For the headline inflation, based on MSE ratios, the QAR outperformed the other models for several forecast horizons. Regarding the components of the headline inflation, excepting for the results of non-tradables and food beverages, the QAR models displayed a good performance among the competing investigated models in many cases.

Regarding future research, possible extensions might include checking how seasonality affects the empirical results. Other route would be to investigate the inflation data before the Real plan. Preliminary results using a sample data starting in 1980 shows a strong non-stationary dynamics for the most part of the sample.

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Appendix A: Results for the headline inflation and its disaggregates

Table A.1: Choice of the autoregressive order

excluded variable	headline inflation	regulated prices	market prices	tradables	non-tradables	services	food and beverages	industrial goods
y_{t-6}	0.71	4.93	2.86	0.88	5.01	0.97	4.91	2.19
y_{t-5}	1.06	1.61	2.69	2.04	7.86	6.37	1.91	2.79
y_{t-4}	2.65	$12.48^{(*)}$	2.50	1.85	2.99	4.68	1.72	1.68
y_{t-3}	4.81	-	0.77	1.72	5.92	$16.46^{(*)}$	1.54	6.41
y_{t-2}	3.01	-	3.88	$9.45^{(*)}$	$23.79^{(*)}$	-	$25.09^{(*)}$	2.46

Notes: H_0: $\alpha_j(\tau) = 0$, where j is the lag related to the excluded variable y_{t-j}. Table entries are estimates of $\sup_{\tau \in \Gamma} L_n(\tau)$.

The 5% and 10% critical values are 9.31 and 7.36, respectively. (*) means rejection of Ho at 5% level of significance.

Table A.2: Koenker-Xiao test

	headline inflation			regulated prices			market prices			tradables			non-tradables		
τ	$\alpha_1(\tau)$	$t_n(\tau)$	δ^2	$\alpha_1(\tau)$	$t_n(\tau)$	δ^2	$\alpha_1(\tau)$	$t_n(\tau)$	δ^2	$\alpha_1(\tau)$	$t_n(\tau)$	δ^2	$\alpha_1(\tau)$	$t_n(\tau)$	δ^2
0.1	0.50	-10.14	-2.37	0.29	-11.66	-2.16	0.66	-7.88	-2.31	0.32	-11.22	-2.26	0.68	-5.77	-2.34
0.2	0.55	-18.43	-2.47	0.32	-14.94	-2.27	0.75	-7.43	-2.53	0.50	-10.35	-2.47	0.72	-8.01	-2.42
0.3	0.61	-13.40	-2.57	0.52	-10.54	-2.44	0.71	-9.70	-2.62	0.57	-11.36	-2.55	0.76	-8.34	-2.51
0.4	0.67	-9.85	-2.61	0.64	-8.73	-2.53	0.68	-11.38	-2.69	0.61	-8.97	-2.56	0.80	-7.35	-2.58
0.5	0.78	-7.64	-2.69	0.73	-8.24	-2.57	0.75	-8.61	-2.73	0.60	-8.36	-2.60	0.80	-7.27	-2.61
0.6	0.80	-5.66	-2.68	0.80	-4.70	-2.56	0.74	-7.67	-2.75	0.63	-7.26	-2.61	0.79	-8.81	-2.60
0.7	0.87	-2.98	-2.73	$0.96^{(*)}$	-0.76	-2.63	0.79	-4.89	-2.75	0.70	-5.76	-2.62	0.79	-5.37	-2.61
0.8	$0.91^{(*)}$	-1.69	-2.70	$1.07^{(*)}$	0.74	-2.69	$0.89^{(*)}$	-1.93	-2.72	0.69	-3.74	-2.57	$0.96^{(*)}$	-0.62	-2.65
0.9	$0.98^{(*)}$	-0.29	-2.60	$1.29^{(*)}$	1.94	-2.60	$0.88^{(*)}$	-1.55	-2.55	0.65	-3.34	-2.60	$1.09^{(*)}$	1.17	-2.62

Notes: $Ho: \alpha_1(\tau) = 1$. (*) means that Ho cannot be rejected at 5% level of significance.

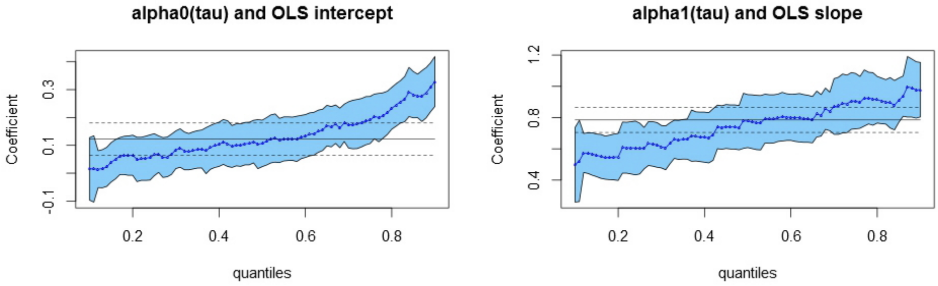
Table A.3: Koenker-Xiao test (cont.)

	services			food and beverages			industrial goods		
τ	$\alpha_1(\tau)$	$t_n(\tau)$	δ^2	$\alpha_1(\tau)$	$t_n(\tau)$	δ^2	$\alpha_1(\tau)$	$t_n(\tau)$	δ^2
0.1	0.80	-4.58	-2.22	0.39	-6.05	-2.34	0.54	-3.93	-2.47
0.2	0.80	-8.74	-2.40	0.43	-10.06	-2.47	0.58	-5.75	-2.53
0.3	0.82	-9.09	-2.33	0.49	-8.36	-2.56	0.61	-7.07	-2.56
0.4	0.81	-8.79	-2.33	0.52	-6.50	-2.58	0.66	-7.12	-2.57
0.5	0.85	-9.11	-2.38	0.49	-7.63	-2.60	0.64	-7.23	-2.58
0.6	0.87	-8.28	-2.34	0.49	-7.07	-2.60	0.64	-8.14	-2.59
0.7	0.87	-6.52	-2.34	0.67	-3.92	-2.58	0.68	-5.66	-2.57
0.8	0.89	-3.11	-2.33	0.65	-4.77	-2.57	0.64	-3.70	-2.60
0.9	$1.03^{(*)}$	0.70	-2.34	0.60	-2.84	-2.49	0.56	-3.11	-2.51

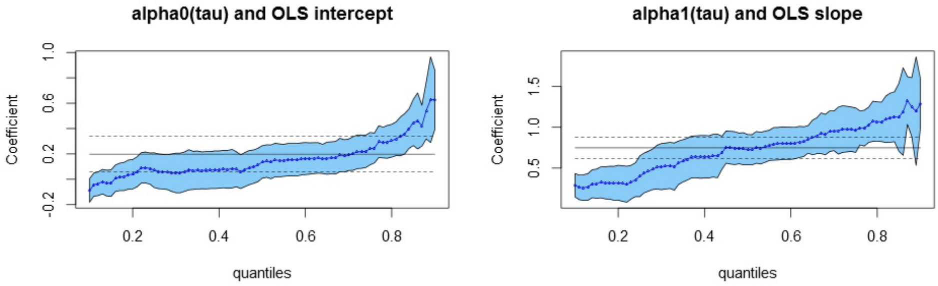
Notes: $Ho : \alpha_1(\tau) = 1$. (*) means that Ho cannot be rejected at 5% level of significance.

Figure A.1 - Estimated coefficients (1995-2014)

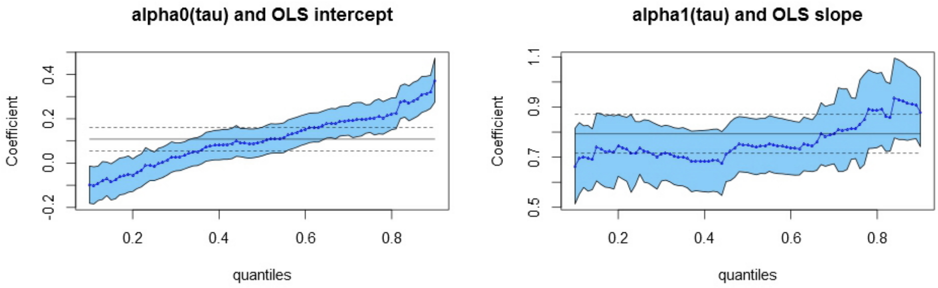
Headline inflation



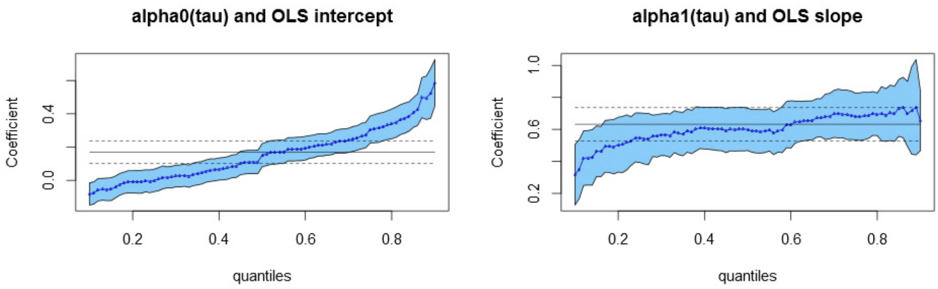
Regulated and monitored prices



Market prices



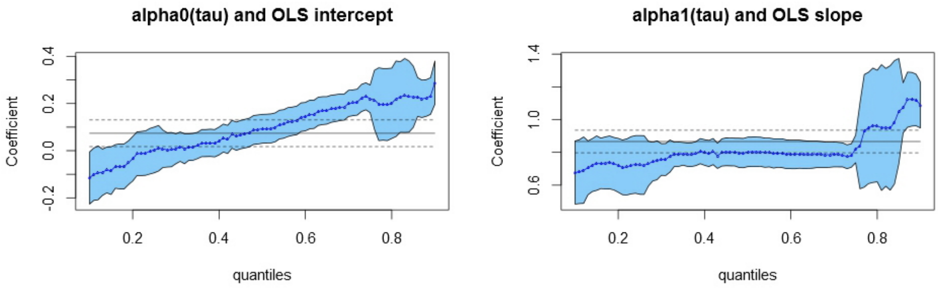
Tradables



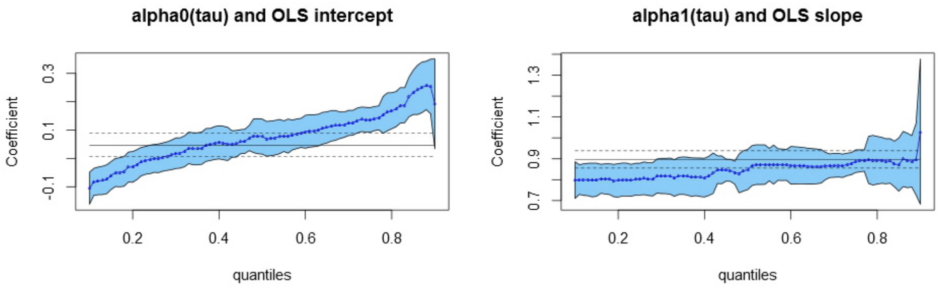
Note: We omitted coefficients of higher lags (if applicable) to save space.

Figure A.2 - Estimated coefficients (1995-2014)

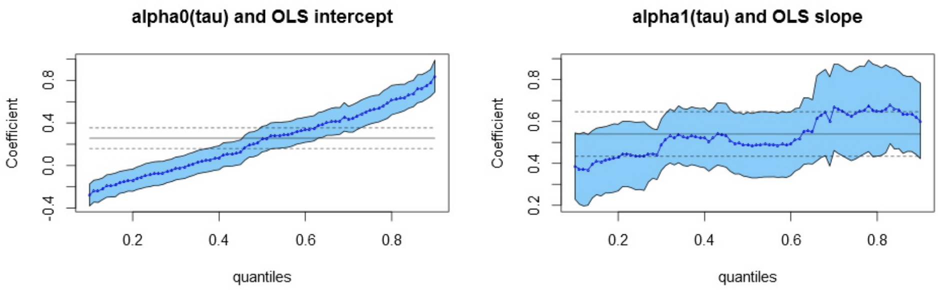
Non-tradables



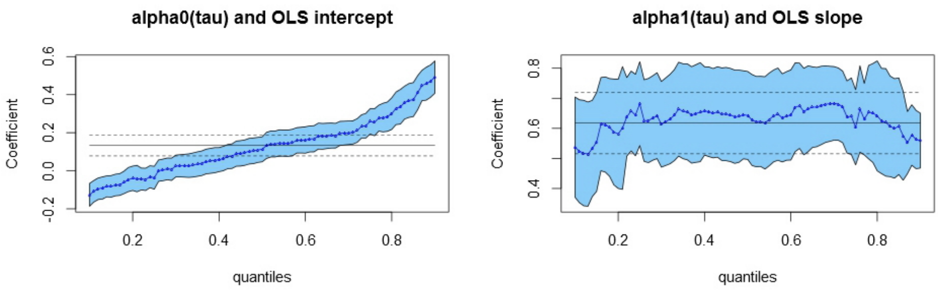
Services



Food and beverages



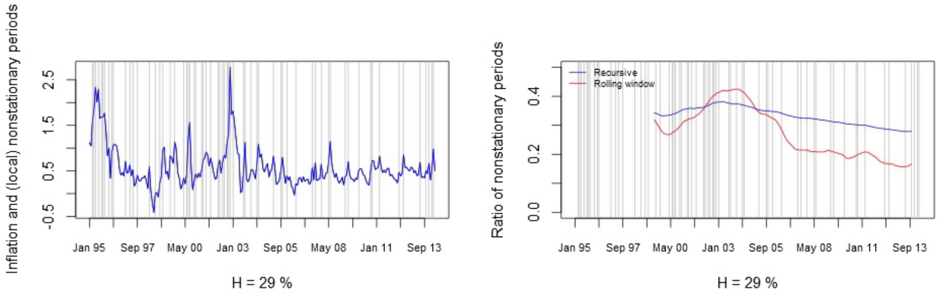
Industrial goods



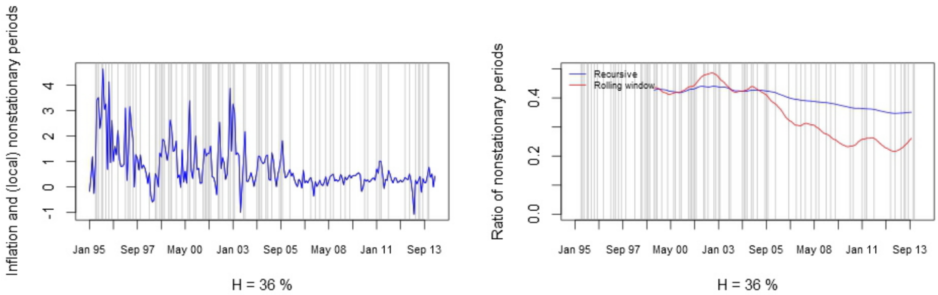
Note: We omitted coefficients of higher lags (if applicable) to save space.

Figure A.3 - Local unit root and ratio of periods with non-stationary inflation

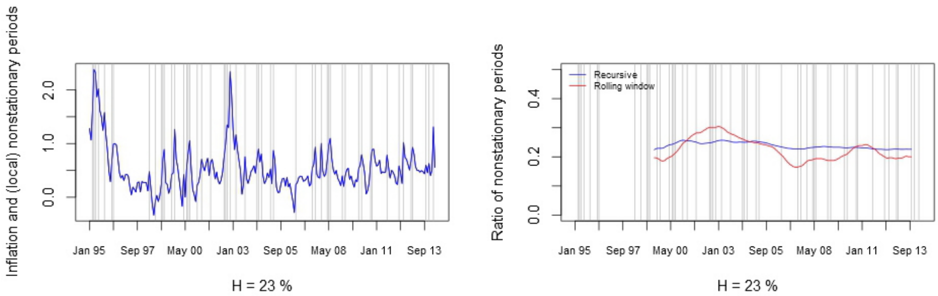
Headline inflation



Regulated and monitored prices



Market prices



Tradables

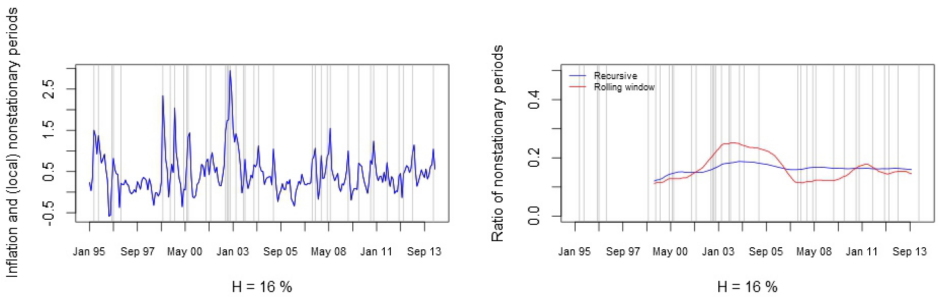
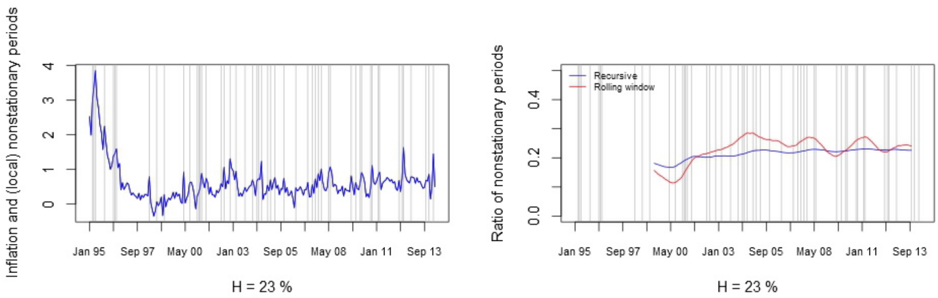
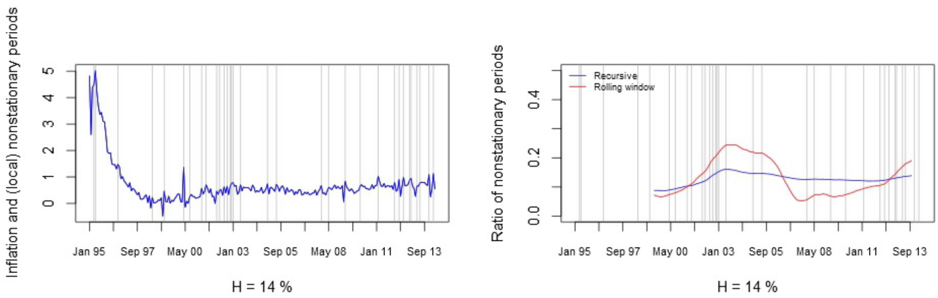


Figure A.4 - Local unit root and ratio of periods with non-stationary inflation

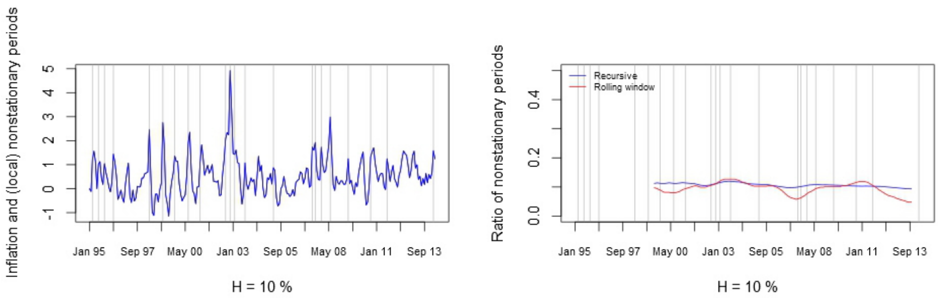
Non-tradables



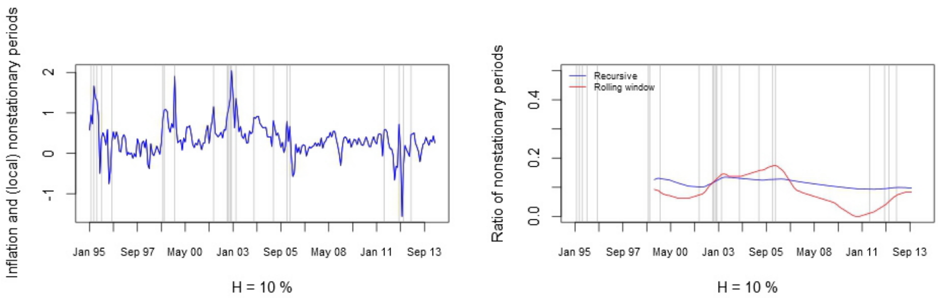
Services



Food and beverages

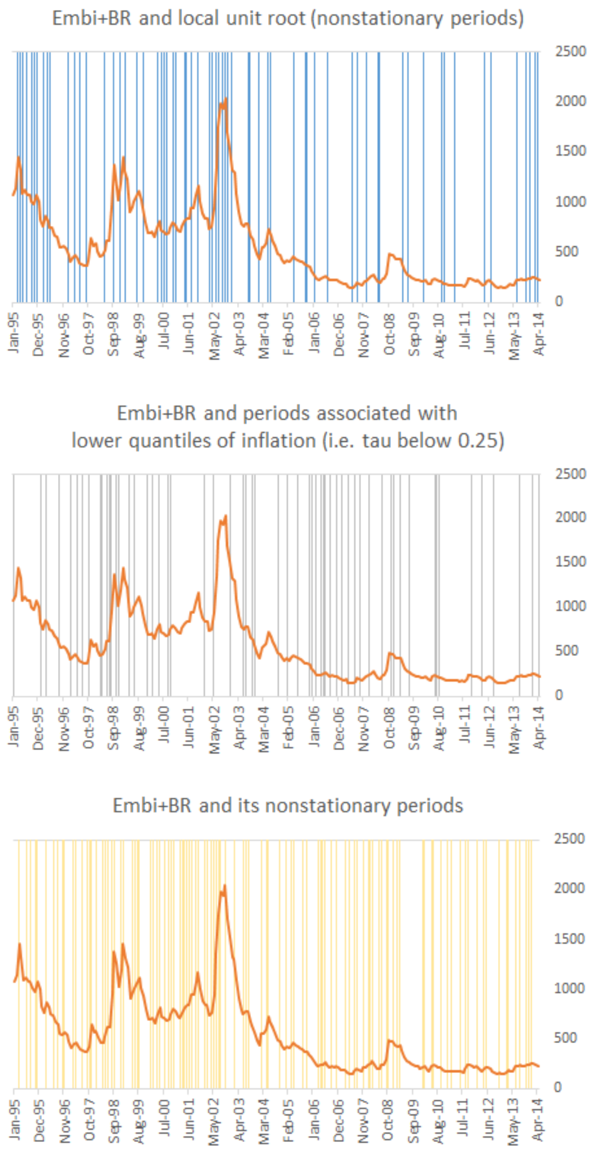


Industrial goods



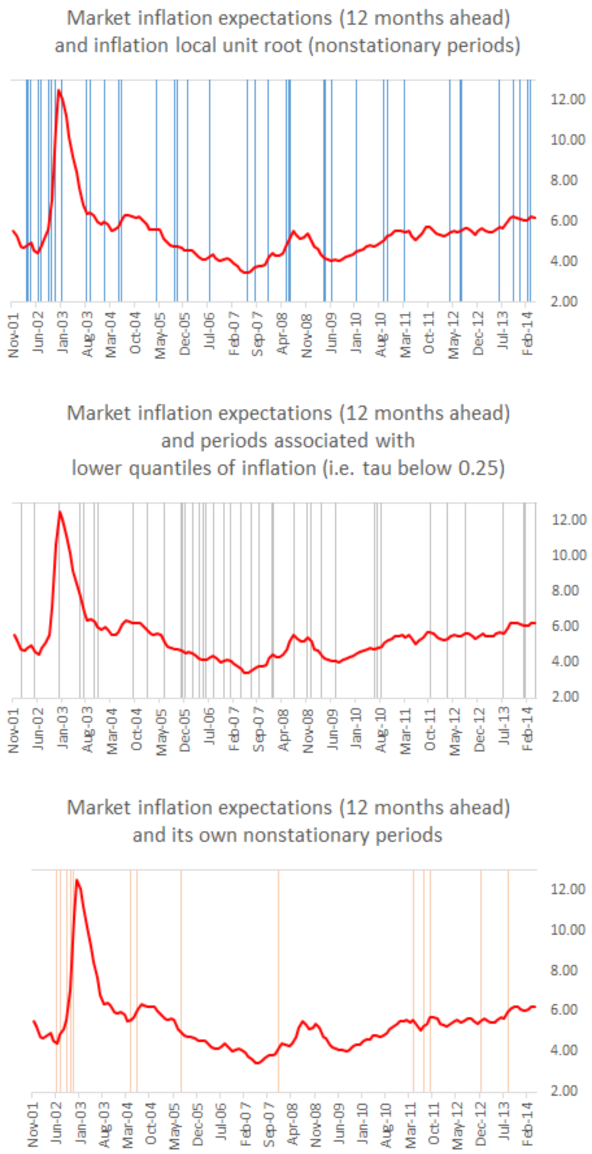
Appendix B

Figure B.1 - Risk aversion proxied by EMBI+BR and periods associated with inflation local unit root (first panel) or lower quantiles (second panel) or Embi and its own non-stationary periods (third panel)



Appendix C

Figure C.1 - Inflation expectations and periods associated with inflation local unit root (first panel) or lower quantiles (second panel) or expectations and its own non-stationary periods (third panel)



Appendix D

Table D.1 - MSE ratios for regulated prices

Panel A - Recursive estimation

Model	Description	h=1		h=2		h=3		h=6		h=9		h=12		h=24		h=36	
1	Random walk	1.224	*	1.486	**	1.540	**	1.112		1.253		1.127		1.014		1.036	
		[1.961]		[2.513]		[2.483]		[0.58]		[1.027]		[0.479]		[0.076]		[0.184]	
		(0.052)		*(0.013)*		*(0.015)*		*(0.563)*		*(0.307)*		*(0.633)*		*(0.94)*		*(0.854)*	
2	AR(1)	1.885	***	2.631	***	3.200	***	3.568	***	3.088	***	3.028	***	2.712	***	2.768	***
		[6.784]		[6.544]		[6.908]		[8.954]		[9.257]		[11.557]		[11.846]		[12.555]	
		(0)		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*	
3	AR(2)	1.618	***	1.955	***	2.467	***	3.178	***	2.980	***	3.002	***	2.723	***	2.781	***
		[5.309]		[6.198]		[6.643]		[8.139]		[8.795]		[11.178]		[11.623]		[12.457]	
		(0)		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*	
4	AR(3)	1.492	***	1.736	***	2.071	***	2.784	***	2.752	***	2.852	***	2.659	***	2.718	***
		[4.631]		[5.685]		[6.238]		[7.349]		[8.094]		[10.413]		[11.327]		[12.258]	
		(0)		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*	
5	AR(4)	1.340	***	1.488	***	1.695	***	2.260	***	2.404	***	2.653	***	2.753	***	2.860	***
		[4.463]		[5.041]		[5.39]		[6.534]		[7.109]		[8.665]		[10.57]		[12.198]	
		(0)		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*	
6	QAR	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
Best model		**6**		**6**		**6**		**6**		**6**		**6**		**6**		**6**	

Panel B - Rolling window estimation

Model	Description	h=1		h=2		h=3		h=6		h=9		h=12		h=24		h=36	
1	Random walk	1.204		1.473	**	1.626	**	1.225		1.479		1.385		1.282	*	1.312	**
		[0.906]		[5.096]		[6.284]		[1.104]		[2.091]		[1.429]		[3.652]		[4.821]	
		(0.341)		*(0.024)*		*(0.012)*		*(0.293)*		*(0.148)*		*(0.232)*		*(0.056)*		*(0.028)*	
2	AR(1)	1.348	***	1.664	***	2.016	***	2.259	***	2.120	***	2.252	***	2.201	***	2.313	***
		[8.401]		[17.557]		[22.949]		[29.861]		[22.12]		[18.799]		[11.414]		[18.906]	
		(0.004)		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0.001)*		*(0)*	
3	AR(2)	1.411	***	1.694	***	2.025	***	2.233	***	2.103	***	2.241	***	2.189	***	2.307	***
		[11.676]		[21.003]		[25.527]		[29.161]		[22.164]		[19.386]		[11.811]		[19.407]	
		(0.001)		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0.001)*		*(0)*	
4	AR(3)	1.343	***	1.543	***	1.760	***	2.087	***	2.043	***	2.204	***	2.169	***	2.285	***
		[10.076]		[17.514]		[21.227]		[26.265]		[20.705]		[18.845]		[11.686]		[19.161]	
		(0.002)		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0.001)*		*(0)*	
5	AR(4)	1.278	***	1.448	***	1.637	***	1.963	***	1.960	***	2.144	***	2.150	***	2.272	***
		[11.902]		[20.291]		[23.204]		[25.718]		[21.343]		[20.247]		[12.034]		[18.36]	
		(0.001)		*(0)*		*(0)*		*(0)*		*(0)*		*(0)*		*(0.001)*		*(0)*	
6	QAR	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
Best model		**6**		**6**		**6**		**6**		**6**		**6**		**6**		**6**	

Table D.2 - MSE ratios for market prices

Panel A - Recursive estimation

Model	Description	h=1		h=2		h=3		h=6		h=9		h=12		h=24		h=36	
1	Random walk	1.201	***	1.326	***	1.469	***	1.500	**	1.396	*	1.764	***	2.214	***	1.557	**
		[3.018]		[3.446]		[3.49]		[2.465]		[1.681]		[3.838]		[3.021]		[2.248]	
		(0.003)		*(0.001)*		*(0.001)*		*(0.015)*		*(0.096)*		*(0)*		*(0.003)*		*(0.028)*	
2	AR(1)	1.035		1.036		1.043		0.977		0.910		0.943		0.765	*	0.684	**
		[1.455]		[0.797]		[0.692]		[-0.192]		[-0.621]		[-0.365]		[-1.899]		[-2.396]	
		(0.149)		*(0.427)*		*(0.49)*		*(0.848)*		*(0.536)*		*(0.716)*		*(0.061)*		*(0.019)*	
3	AR(2)	1.051	*	1.037		1.043		0.983		0.929		0.949		0.763	*	0.685	**
		[1.707]		[0.764]		[0.652]		[-0.129]		[-0.454]		[-0.302]		[-1.824]		[-2.298]	
		(0.091)		*(0.447)*		*(0.516)*		*(0.898)*		*(0.651)*		*(0.763)*		*(0.072)*		*(0.024)*	
4	AR(3)	1.051		1.031		1.028		0.962		0.916		0.925		0.757	**	0.692	**
		[1.62]		[0.655]		[0.458]		[-0.321]		[-0.601]		[-0.515]		[-2.161]		[-2.591]	
		(0.108)		*(0.514)*		*(0.648)*		*(0.749)*		*(0.549)*		*(0.607)*		*(0.033)*		*(0.012)*	
5	AR(4)	1.021		0.990		0.967		0.941		0.907		0.917		0.778	***	0.727	***
		[0.637]		[-0.251]		[-0.682]		[-0.632]		[-0.877]		[-0.757]		[-2.647]		[-3.03]	
		(0.526)		*(0.802)*		*(0.497)*		*(0.529)*		*(0.382)*		*(0.451)*		*(0.01)*		*(0.003)*	
6	QAR	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
Best model		**6**		**5**		**5**		**5**		**5**		**5**		**4**		**2**	

Panel B - Rolling window estimation

Model	Description	h=1		h=2		h=3		h=6		h=9		h=12		h=24		h=36	
1	Random walk	1.270	**	1.408	***	1.572	***	1.541	**	1.447	*	1.828	***	2.216	***	1.528	***
		[4.369]		[9.609]		[11.47]		[6.109]		[3.178]		[29.93]		[13.669]		[9.258]	
		(0.037)		*(0.002)*		*(0.001)*		*(0.013)*		*(0.075)*		*(0)*		*(0)*		*(0.002)*	
2	AR(1)	1.028		1.028		1.028		0.960		0.937		0.964		0.766	***	0.671	***
		[0.53]		[0.287]		[0.201]		[-0.138]		[-0.233]		[-0.071]		[-14.067]		[-23.114]	
		(0.466)		*(0.592)*		*(0.654)*		*(0.71)*		*(0.629)*		*(0.79)*		*(0)*		*(0)*	
3	AR(2)	1.038		1.021		1.019		0.966		0.949		0.966		0.764	***	0.671	***
		[0.776]		[0.151]		[0.082]		[-0.093]		[-0.14]		[-0.061]		[-13.831]		[-22.649]	
		(0.378)		*(0.697)*		*(0.775)*		*(0.761)*		*(0.708)*		*(0.805)*		*(0)*		*(0)*	
4	AR(3)	1.042		1.035		1.027		0.969		0.958		0.959		0.755	***	0.672	***
		[0.862]		[0.363]		[0.17]		[-0.072]		[-0.093]		[-0.09]		[-15.438]		[-22.521]	
		(0.353)		*(0.547)*		*(0.68)*		*(0.788)*		*(0.76)*		*(0.765)*		*(0)*		*(0)*	
5	AR(4)	1.045		1.027		1.015		0.961		0.948		0.953		0.748	***	0.675	***
		[0.779]		[0.213]		[0.053]		[-0.125]		[-0.158]		[-0.13]		[-17.108]		[-23.829]	
		(0.377)		*(0.644)*		*(0.817)*		*(0.724)*		*(0.691)*		*(0.719)*		*(0)*		*(0)*	
6	QAR	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
Best model		**6**		**6**		**6**		**2**		**2**		**5**		**5**		**3**	

Table D.3 - MSE ratios for tradables

Panel A - Recursive estimation

Model	Description	h=1	h=2	h=3	h=6	h=9	h=12	h=24	h=36
1	Random walk	1.238 ***	1.509 ***	1.734 ***	1.833 ***	1.466 *	1.867 ***	2.359 ***	2.096 ***
		[2.843]	[2.922]	[3.643]	[3.295]	[1.74]	[4.111]	[3.588]	[3.237]
		(0.005)	*(0.004)*	*(0)*	*(0.001)*	*(0.085)*	*(0)*	*(0.001)*	*(0.002)*
2	AR(1)	1.010	1.035	1.047	1.058	1.010	0.983	0.866	0.851
		[0.243]	[0.592]	[0.606]	[0.431]	[0.068]	[-0.122]	[-0.959]	[-1.052]
		(0.809)	*(0.555)*	*(0.546)*	*(0.668)*	*(0.946)*	*(0.903)*	*(0.34)*	*(0.296)*
3	AR(2)	1.006	1.014	1.028	1.078	1.041	0.991	0.867	0.854
		[0.171]	[0.209]	[0.304]	[0.502]	[0.27]	[-0.058]	[-0.923]	[-0.998]
		(0.864)	*(0.835)*	*(0.762)*	*(0.617)*	*(0.788)*	*(0.954)*	*(0.359)*	*(0.321)*
4	AR(3)	1.008	1.014	1.027	1.070	1.035	0.986	0.864	0.851
		[0.229]	[0.216]	[0.298]	[0.467]	[0.232]	[-0.094]	[-0.96]	[-1.038]
		(0.819)	*(0.83)*	*(0.766)*	*(0.641)*	*(0.817)*	*(0.926)*	*(0.34)*	*(0.302)*
5	AR(4)	1.030	1.034	1.044	1.065	1.023	0.961	0.850	0.841
		[0.807]	[0.527]	[0.509]	[0.431]	[0.165]	[-0.308]	[-1.177]	[-1.255]
		(0.421)	*(0.599)*	*(0.612)*	*(0.667)*	*(0.87)*	*(0.759)*	*(0.242)*	*(0.214)*
6	QAR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Best model		**6**	**6**	**6**	**6**	**6**	**5**	**5**	**5**

Panel B - Rolling window estimation

Model	Description	h=1	h=2	h=3	h=6	h=9	h=12	h=24	h=36
1	Random walk	1.194 **	1.433 ***	1.689 ***	1.906 ***	1.582 **	1.997 ***	2.457 ***	2.157 ***
		[5.174]	[9.144]	[15.11]	[16.276]	[4.292]	[26.466]	[30.794]	[37.569]
		(0.023)	*(0.002)*	*(0)*	*(0)*	*(0.038)*	*(0)*	*(0)*	*(0)*
2	AR(1)	0.996	1.016	1.064	1.121	1.102	1.111	0.923	0.903
		[-0.015]	[0.112]	[0.908]	[1.013]	[0.489]	[0.46]	[-0.539]	[-0.753]
		(0.904)	*(0.738)*	*(0.341)*	*(0.314)*	*(0.484)*	*(0.498)*	*(0.463)*	*(0.385)*
3	AR(2)	0.991	0.994	1.037	1.130	1.131	1.104	0.920	0.899
		[-0.076]	[-0.015]	[0.251]	[0.936]	[0.731]	[0.387]	[-0.577]	[-0.809]
		(0.783)	*(0.903)*	*(0.616)*	*(0.333)*	*(0.393)*	*(0.534)*	*(0.448)*	*(0.369)*
4	AR(3)	1.001	1.003	1.051	1.138	1.131	1.108	0.918	0.899
		[0.002]	[0.003]	[0.491]	[1.055]	[0.734]	[0.417]	[-0.594]	[-0.799]
		(0.966)	*(0.958)*	*(0.484)*	*(0.304)*	*(0.392)*	*(0.518)*	*(0.441)*	*(0.371)*
5	AR(4)	1.025	1.022	1.078	1.157	1.142	1.102	0.912	0.899
		[0.542]	[0.145]	[0.928]	[1.178]	[0.836]	[0.384]	[-0.668]	[-0.798]
		(0.462)	*(0.703)*	*(0.335)*	*(0.278)*	*(0.361)*	*(0.536)*	*(0.414)*	*(0.372)*
6	QAR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Best model		**3**	**3**	**6**	**6**	**6**	**6**	**5**	**3**

Table D.4 - MSE ratios for non-tradables

Panel A - Recursive estimation

Model	Description	h=1		h=2		h=3		h=6		h=9		h=12		h=24		h=36	
1	Random walk	1.284	***	1.355	***	1.412	***	1.447	***	1.458	**	1.693	***	1.615	***	1.048	
		[3.019]		[3.252]		[3.076]		[2.735]		[2.469]		[3.173]		[3.268]		[0.317]	
		(0.003)		*(0.002)*		*(0.003)*		*(0.007)*		*(0.015)*		*(0.002)*		*(0.002)*		*(0.752)*	
2	AR(1)	1.119	**	1.072	*	1.040		0.958		0.914		0.908		0.760	***	0.716	***
		[2.075]		[1.822]		[0.881]		[-0.805]		[-1.334]		[-1.317]		[-3.564]		[-4.284]	
		(0.04)		*(0.071)*		*(0.38)*		*(0.423)*		*(0.185)*		*(0.191)*		*(0.001)*		*(0)*	
3	AR(2)	1.047	**	1.067	**	1.070	*	1.005		0.969		1.015		0.896	***	0.812	***
		[2.392]		[2.438]		[1.712]		[0.091]		[-0.508]		[0.244]		[-2.813]		[-4.569]	
		(0.018)		*(0.016)*		*(0.09)*		*(0.928)*		*(0.612)*		*(0.808)*		*(0.006)*		*(0)*	
4	AR(3)	1.034	**	1.052	**	1.048		0.990		0.963		0.990		0.895	***	0.862	***
		[2.56]		[2.323]		[1.539]		[-0.254]		[-0.919]		[-0.239]		[-3.976]		[-4.33]	
		(0.012)		*(0.022)*		*(0.127)*		*(0.8)*		*(0.36)*		*(0.812)*		*(0)*		*(0)*	
5	AR(4)	1.021		0.993		0.972		0.982		0.993		1.014		0.973	*	0.973	
		[0.905]		[-0.479]		[-1.6]		[-0.714]		[-0.29]		[0.507]		[-1.692]		[-1.164]	
		(0.367)		*(0.633)*		*(0.113)*		*(0.477)*		*(0.772)*		*(0.613)*		*(0.094)*		*(0.248)*	
6	QAR	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
Best model		**6**		**5**		**5**		**2**		**2**		**2**		**2**		**2**	

Panel B - Rolling window estimation

Model	Description	h=1		h=2		h=3		h=6		h=9		h=12		h=24		h=36	
1	Random walk	1.394	**	1.535	***	1.516	***	1.412	**	1.419	**	1.730	***	1.714	***	1.084	
		[4.37]		[10.009]		[8.009]		[5.641]		[5.172]		[13.08]		[63.424]		[0.711]	
		(0.037)		*(0.002)*		*(0.005)*		*(0.018)*		*(0.023)*		*(0)*		*(0)*		*(0.399)*	
2	AR(1)	0.962		0.933		0.944	*	0.966		0.953		0.967		0.920	**	0.918	
		[-0.542]		[-2.577]		[-2.796]		[-0.477]		[-0.913]		[-0.367]		[-3.891]		[-1.374]	
		(0.462)		*(0.108)*		*(0.094)*		*(0.49)*		*(0.339)*		*(0.545)*		*(0.049)*		*(0.241)*	
3	AR(2)	0.955		0.948	**	0.943	**	0.933	*	0.933		0.953		0.927	*	0.939	
		[-2.17]		[-4.153]		[-5.197]		[-3.631]		[-2.572]		[-0.983]		[-3.721]		[-1.66]	
		(0.141)		*(0.042)*		*(0.023)*		*(0.057)*		*(0.109)*		*(0.321)*		*(0.054)*		*(0.198)*	
4	AR(3)	0.971		0.958	*	0.946	**	0.933	**	0.933	*	0.952		0.928	*	0.945	
		[-1.286]		[-3.638]		[-4.781]		[-4.045]		[-2.839]		[-1.082]		[-3.802]		[-1.564]	
		(0.257)		*(0.056)*		*(0.029)*		*(0.044)*		*(0.092)*		*(0.298)*		*(0.051)*		*(0.211)*	
5	AR(4)	0.972		0.963	*	0.949	**	0.924	**	0.933	*	0.947		0.930	*	0.956	
		[-1.198]		[-2.908]		[-4.637]		[-5.296]		[-2.923]		[-1.39]		[-3.656]		[-1.442]	
		(0.274)		*(0.088)*		*(0.031)*		*(0.021)*		*(0.087)*		*(0.238)*		*(0.056)*		*(0.23)*	
6	QAR	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
Best model		**3**		**2**		**3**		**5**		**5**		**5**		**2**		**2**	

Table D.5 - MSE ratios for services

Panel A - Recursive estimation

Model	Description	h=1		h=2		h=3		h=6		h=9		h=12		h=24		h=36	
1	Random walk	1.558	**	1.560	**	1.268	**	1.201		1.109		0.995		0.845		0.616	***
		[2.574]		[2.489]		[2.064]		[1.525]		[0.768]		[-0.035]		[-1.184]		[-4.763]	
		(0.011)		*(0.014)*		*(0.041)*		*(0.13)*		*(0.444)*		*(0.972)*		*(0.24)*		*(0)*	
2	AR(1)	1.376	**	1.265	*	0.993		0.855	**	0.776	***	0.694	***	0.533	***	0.452	***
		[2.261]		[1.843]		[-0.12]		[-2.4]		[-3.426]		[-4.069]		[-6.375]		[-8.797]	
		(0.026)		*(0.068)*		*(0.904)*		*(0.018)*		*(0.001)*		*(0)*		*(0)*		*(0)*	
3	AR(2)	1.134		1.153	**	0.997		0.960		0.893	***	0.883	***	0.926	**	1.002	
		[1.547]		[2.005]		[-0.094]		[-1.128]		[-2.826]		[-2.647]		[-2.13]		[0.066]	
		(0.125)		*(0.047)*		*(0.926)*		*(0.262)*		*(0.006)*		*(0.009)*		*(0.036)*		*(0.948)*	
4	AR(3)	1.048	*	1.043		0.987		0.963	*	0.931	***	0.931	***	0.960	*	1.011	
		[1.667]		[1.417]		[-0.769]		[-1.866]		[-2.819]		[-2.655]		[-1.889]		[0.5]	
		(0.098)		*(0.159)*		*(0.444)*		*(0.065)*		*(0.006)*		*(0.009)*		*(0.062)*		*(0.618)*	
5	AR(4)	1.080		1.070		1.009		0.986		0.982		0.991		1.016		1.061	***
		[1.003]		[1.292]		[0.215]		[-1.141]		[-1.01]		[-0.617]		[0.975]		[3.422]	
		(0.318)		*(0.199)*		*(0.83)*		*(0.257)*		*(0.315)*		*(0.539)*		*(0.332)*		*(0.001)*	
6	QAR	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
Best model		**6**		**6**		**4**		**2**		**2**		**2**		**2**		**2**	

Panel B - Rolling window estimation

Model	Description	h=1		h=2		h=3		h=6		h=9		h=12		h=24		h=36	
1	Random walk	1.716	***	1.738	***	1.381	**	1.432	***	1.416	***	1.385	**	1.373	***	0.987	
		[7.612]		[6.74]		[5.342]		[8.85]		[7.574]		[5.092]		[13.75]		[-0.016]	
		(0.006)		*(0.009)*		*(0.021)*		*(0.003)*		*(0.006)*		*(0.024)*		*(0)*		*(0.901)*	
2	AR(1)	1.177	**	1.254	***	1.277	***	1.166	***	1.186	***	1.184	***	1.151	**	1.105	
		[5.978]		[7.239]		[6.747]		[8.397]		[8.142]		[11.301]		[4.5]		[0.548]	
		(0.014)		*(0.007)*		*(0.009)*		*(0.004)*		*(0.004)*		*(0.001)*		*(0.034)*		*(0.459)*	
3	AR(2)	1.100		1.148	*	1.134	*	1.122	**	1.172	***	1.161	***	1.130	*	1.155	
		[1.793]		[3.22]		[3.35]		[6.589]		[10.497]		[8.936]		[3.776]		[2.61]	
		(0.181)		*(0.073)*		*(0.067)*		*(0.01)*		*(0.001)*		*(0.003)*		*(0.052)*		*(0.106)*	
4	AR(3)	0.993		1.029		1.038	*	1.057	*	1.112	***	1.125	***	1.129	**	1.172	**
		[-0.054]		[2.278]		[2.845]		[3.365]		[7.014]		[6.917]		[5.609]		[5.634]	
		(0.816)		*(0.131)*		*(0.092)*		*(0.067)*		*(0.008)*		*(0.009)*		*(0.018)*		*(0.018)*	
5	AR(4)	0.982		1.023		1.030		1.018		1.053		1.082	*	1.113	**	1.171	***
		[-0.221]		[0.805]		[1.268]		[0.643]		[2.405]		[3.373]		[5.78]		[8.643]	
		(0.639)		*(0.37)*		*(0.26)*		*(0.423)*		*(0.121)*		*(0.066)*		*(0.016)*		*(0.003)*	
6	QAR	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
Best model		**5**		**6**		**6**		**6**		**6**		**6**		**6**		**1**	

Table D.6 - MSE ratios for food and beverages

Panel A - Recursive estimation

Model	Description	h=1	h=2	h=3	h=6	h=9	h=12	h=24	h=36
1	Random walk	1.234	1.506 **	1.738 **	1.440	1.466	1.715 ***	2.403 ***	2.123 ***
		[1.623]	[2.206]	[2.502]	[1.5]	[1.517]	[3.41]	[3.5]	[2.668]
		(0.107)	*(0.03)*	*(0.014)*	*(0.137)*	*(0.132)*	*(0.001)*	*(0.001)*	*(0.009)*
2	AR(1)	1.018	0.992	0.965	0.889	0.879 *	0.879 *	0.862 **	0.862 ***
		[0.302]	[-0.107]	[-0.405]	[-1.569]	[-1.847]	[-1.926]	[-2.455]	[-2.901]
		(0.763)	*(0.915)*	*(0.686)*	*(0.12)*	*(0.068)*	*(0.057)*	*(0.016)*	*(0.005)*
3	AR(2)	0.959	0.929	0.917	0.916	0.886 *	0.879 *	0.855 **	0.860 ***
		[-1.349]	[-1.49]	[-1.455]	[-1.317]	[-1.8]	[-1.906]	[-2.512]	[-2.909]
		(0.18)	*(0.139)*	*(0.149)*	*(0.191)*	*(0.075)*	*(0.06)*	*(0.014)*	*(0.005)*
4	AR(3)	0.971	0.928	0.913	0.907	0.892 *	0.886 **	0.871 **	0.874 ***
		[-0.952]	[-1.526]	[-1.515]	[-1.558]	[-1.902]	[-2.005]	[-2.522]	[-3.011]
		(0.343)	*(0.13)*	*(0.133)*	*(0.122)*	*(0.06)*	*(0.048)*	*(0.014)*	*(0.004)*
5	AR(4)	0.985	0.943	0.929	0.922	0.896 *	0.889 **	0.874 **	0.878 ***
		[-0.51]	[-1.283]	[-1.329]	[-1.354]	[-1.923]	[-2.027]	[-2.517]	[-3.032]
		(0.611)	*(0.202)*	*(0.187)*	*(0.179)*	*(0.057)*	*(0.045)*	*(0.014)*	*(0.003)*
6	QAR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Best model		**3**	**4**	**4**	**2**	**2**	**3**	**3**	**3**

Panel B - Rolling window estimation

Model	Description	h=1	h=2	h=3	h=6	h=9	h=12	h=24	h=36
1	Random walk	1.260 *	1.575 **	1.836 ***	1.522 *	1.509	1.737 ***	2.427 ***	2.183 ***
		[3.213]	[5.323]	[7.623]	[3.054]	[2.316]	[13.269]	[32.568]	[27.403]
		(0.073)	*(0.021)*	*(0.006)*	*(0.081)*	*(0.128)*	*(0)*	*(0)*	*(0)*
2	AR(1)	1.049	1.059	1.041	0.906	0.868	0.888	0.862 **	0.850 ***
		[0.691]	[0.434]	[0.189]	[-1.414]	[-2.585]	[-1.839]	[-4.81]	[-22.892]
		(0.406)	*(0.51)*	*(0.664)*	*(0.234)*	*(0.108)*	*(0.175)*	*(0.028)*	*(0)*
3	AR(2)	0.997	0.987	0.972	0.931	0.884	0.884	0.851 **	0.848 ***
		[-0.004]	[-0.043]	[-0.192]	[-1.021]	[-2.232]	[-2.078]	[-5.195]	[-24.978]
		(0.95)	*(0.837)*	*(0.662)*	*(0.312)*	*(0.135)*	*(0.149)*	*(0.023)*	*(0)*
4	AR(3)	1.002	0.989	0.975	0.918	0.881	0.883	0.853 **	0.851 ***
		[0.002]	[-0.034]	[-0.132]	[-1.36]	[-2.349]	[-2.115]	[-5.163]	[-25.34]
		(0.961)	*(0.854)*	*(0.717)*	*(0.244)*	*(0.125)*	*(0.146)*	*(0.023)*	*(0)*
5	AR(4)	1.017	1.010	0.998	0.938	0.882	0.884	0.851 **	0.851 ***
		[0.159]	[0.029]	[-0.001]	[-0.797]	[-2.338]	[-2.11]	[-5.153]	[-25.464]
		(0.69)	*(0.866)*	*(0.977)*	*(0.372)*	*(0.126)*	*(0.146)*	*(0.023)*	*(0)*
6	QAR	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Best model		**3**	**3**	**3**	**2**	**2**	**4**	**5**	**3**

Table D.7 - MSE ratios for industrial goods

Panel A - Recursive estimation

Model	Description	h=1		h=2		h=3		h=6		h=9		h=12		h=24		h=36	
1	Random walk	1.264	*	1.520	**	1.560	*	1.577		1.410		1.721	***	1.689	**	1.108	
		[1.849]		[2.133]		[1.935]		[1.532]		[1.45]		[3.125]		[2.467]		[0.459]	
		(0.067)		*(0.035)*		*(0.056)*		*(0.129)*		*(0.15)*		*(0.002)*		*(0.016)*		*(0.647)*	
2	AR(1)	1.016		1.039	**	1.064	***	1.110	***	1.120	***	1.135	***	1.143	***	1.131	***
		[1.525]		[2.388]		[2.738]		[3.523]		[3.338]		[3.611]		[4.556]		[4.388]	
		(0.13)		*(0.019)*		*(0.007)*		*(0.001)*		*(0.001)*		*(0.001)*		*(0)*		*(0)*	
3	AR(2)	1.024		1.025		1.031		1.067	***	1.081	***	1.104	***	1.112	***	1.101	***
		[1.285]		[1.55]		[1.318]		[2.846]		[2.781]		[3.264]		[4.324]		[4.26]	
		(0.202)		*(0.124)*		*(0.19)*		*(0.005)*		*(0.006)*		*(0.002)*		*(0)*		*(0)*	
4	AR(3)	1.001		0.996		1.007		1.041	*	1.052	*	1.089	***	1.108	***	1.096	***
		[0.041]		[-0.106]		[0.197]		[1.665]		[1.936]		[2.719]		[4.251]		[4.218]	
		(0.968)		*(0.916)*		*(0.844)*		*(0.099)*		*(0.056)*		*(0.008)*		*(0)*		*(0)*	
5	AR(4)	0.998		0.982		0.999		1.022		1.017		1.039	*	1.046	***	1.040	***
		[-0.077]		[-0.455]		[-0.019]		[0.963]		[0.735]		[1.718]		[2.857]		[3.33]	
		(0.939)		*(0.65)*		*(0.985)*		*(0.338)*		*(0.464)*		*(0.089)*		*(0.005)*		*(0.001)*	
6	QAR	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
Best model		**5**		**5**		**5**		**6**		**6**		**6**		**6**		**6**	

Panel B - Rolling window estimation

Model	Description	h=1		h=2		h=3		h=6		h=9		h=12		h=24		h=36	
1	Random walk	1.257	*	1.505	**	1.546	**	1.572		1.399		1.680	***	1.646	**	1.073	
		[2.73]		[4.696]		[4.198]		[2.174]		[1.874]		[6.678]		[5.539]		[0.046]	
		(0.098)		*(0.03)*		*(0.04)*		*(0.14)*		*(0.171)*		*(0.01)*		*(0.019)*		*(0.829)*	
2	AR(1)	1.037		1.070	**	1.054		1.102	***	1.126	***	1.132	***	1.162	***	1.176	***
		[1.84]		[4.85]		[1.875]		[7.252]		[10.642]		[10.345]		[47.546]		[17.024]	
		(0.175)		*(0.028)*		*(0.171)*		*(0.007)*		*(0.001)*		*(0.001)*		*(0)*		*(0)*	
3	AR(2)	1.067	*	1.063	*	1.027		1.073		1.102	***	1.124	***	1.159	***	1.171	***
		[2.977]		[2.764]		[0.259]		[2.568]		[7.149]		[9.104]		[50.339]		[17.349]	
		(0.084)		*(0.096)*		*(0.611)*		*(0.109)*		*(0.008)*		*(0.003)*		*(0)*		*(0)*	
4	AR(3)	1.037		1.019		1.001		1.059		1.063		1.099	**	1.155	***	1.167	***
		[0.874]		[0.166]		[0]		[0.742]		[2.601]		[4.213]		[55.499]		[17.97]	
		(0.35)		*(0.684)*		*(0.991)*		*(0.389)*		*(0.107)*		*(0.04)*		*(0)*		*(0)*	
5	AR(4)	1.069		1.052		1.040		1.085		1.079	**	1.105	**	1.152	***	1.166	***
		[2.215]		[1.373]		[0.4]		[1.654]		[3.886]		[4.192]		[54.247]		[15.6]	
		(0.137)		*(0.241)*		*(0.527)*		*(0.198)*		*(0.049)*		*(0.041)*		*(0)*		*(0)*	
6	QAR	1.000		1.000		1.000		1.000		1.000		1.000		1.000		1.000	
Best model		**6**		**6**		**6**		**6**		**6**		**6**		**6**		**6**	