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Monetary Policy Objectives and Money's Role in U.S. Business Cycles

Eurilton Araújo

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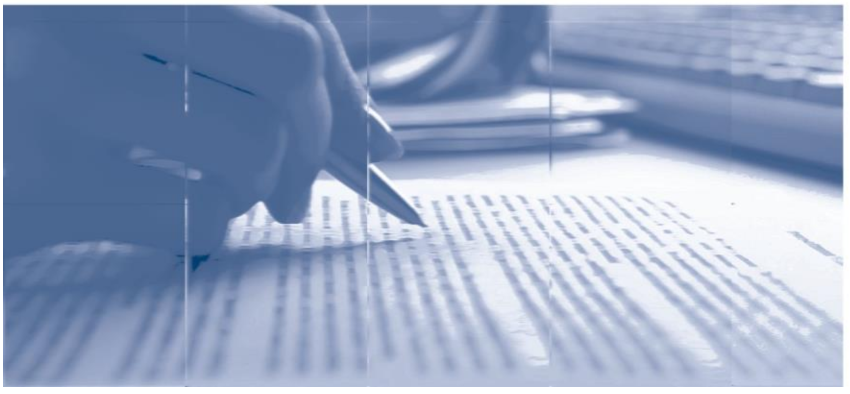

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Monetary Policy Objectives and Money's Role in U.S. Business Cycles *

Eurilton Araújo**

Abstract

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In a sticky-price model in which money can potentially play a key role in business cycles, I estimate monetary policy preference parameters under commitment in a timeless perspective. Empirical findings suggest that inflation stabilization and interest rate smoothing are the main objectives of monetary policy, with a very small role for output gap stabilization. Though the money growth rate is irrelevant as an argument in the Fed's objective function, its presence in structural equations improves model fit. Moreover, marginal likelihood comparisons show that the data favor Taylor rules over optimal policies. Finally, the way of describing monetary policy matters for macroeconomic dynamics.

Keywords: estimation, central bank preferences, optimal monetary policy
JEL Classification: E52, E58, E61

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1 Introduction

In the standard sticky-price new Keynesian model, as described in Galí (2008), monetary aggregates do not affect the equations describing inflation, interest rates and output dynamics. Furthermore, the central bank sets the interest rate and supplies any quantity of money demanded by economic agents at the given target rate. In sum, the canonical new Keynesian model, which is frequently employed to study monetary policy in academia and policy institutions, is block-recursive in money balances. Hence, the presence of a money demand equation imposes no restrictions on the dynamic behavior of key macroeconomic variables.

Evidence from estimated vector auto-regressions, such as Roush & Leeper (2003) and Favara & Giordani (2009), challenged this view for neglecting the role of money in business cycles. In addition, a burgeoning literature, based on estimated dynamic stochastic general equilibrium models, has started to find empirical evidence for the role of money in explaining macroeconomic fluctuations. Andrés et al. (2009), Poilly (2010), Canova & Menz (2011), Canova & Ferroni (2012), Benchimol & Fourçans (2012), Zanetti (2012) and Castelnuovo (2012) are examples of this literature. By supporting money as a relevant factor in business cycles, these papers contradict the early findings of Ireland (2004) and Andrés et al. (2006), who found no major role for money in cyclical fluctuations.

The literature above used Taylor rules to summarize monetary policy and documented a significant interest rate reaction to the growth rate of nominal money. To assess the potential role for the money growth rate as a monetary policy objective, i.e., an argument in the central bank's loss function, I depart from the specification of monetary policy shown in these papers, and replace the Taylor rule with optimal monetary policy under commitment in a timeless perspective.

As discussed in Svensson (2003), there are pitfalls in trying to infer the target variables that central banks may care about in their loss functions from the coefficients of simple monetary policy rules. In fact, a significant coefficient associated with the money growth rate may only signal that money is a useful indicator for forecasting inflation and the output gap, which are the only variables that the central bank cares about. Appendix A illustrates this point with an example based on the model studied in this paper. In short, a statistically significant variable in a Taylor rule is not necessarily a target variable in the central bank's loss function[1].

Following Dennis (2004, 2006), Ilbas (2010, 2012), Adolfson et al. (2011) and Givens (2012), I therefore specify the central bank's objective function as an intertemporal quadratic loss function to be minimized subject to the bank's information about the state of the economy and its view on the transmission mechanism. I then use quarterly data, ranging from 1984:Q1 to 2007:Q2, to estimate the model studied in Andrés et al. (2009) under optimal policy.

Empirical findings suggest that the Fed does not target the money growth rate and this variable is significant in estimated Taylor rules because it helps forecasting inflation and the output gap, which are themselves monetary policy objectives. The Fed's major concern is inflation stability and changes in interest rates are gradual, a typical conduct of central banks in normal times. There is evidence supporting the presence of money in the equations describing private agents' behavior. This presence indicates a more active role for money in explaining business cycles. Finally, optimal policies impose

[1] Kam et al. (2009) showed that, in small open economies under inflation-targeting, real exchange rates were significant macroeconomic variables in Taylor rules, but did not belong to the monetary authority's objective function.

additional restrictions on the equations characterizing the equilibrium, which are rejected by the data. Thus, compared to Taylor rules, optimal policies lead to alternative cyclical behavior of key macroeconomic variables but do not improve model fit.

The rest of this paper proceeds as follows. Section 2 sets out the model. Section 3 discusses the empirical methodology. Section 4 presents the main findings. Section 5 checks the robustness of some results. Finally, the last section concludes.

2 A sticky-price model with money

In this section, I present the log-linear approximation of the sticky price economy developed by Andrés et al. (2009), henceforth the ALSN model. This artificial economy, in contrast to the canonical new Keynesian model, features an explicit role for money.

In the ALSN model, money affects the description of the equilibrium through the specification of nonseparable preferences and portfolio adjustment costs.

First, the model assumes that household preferences are nonseparable in consumption and real money balances. This nonseparability assumption affects households' intertemporal rate of substitution in consumption. Consequently, the Euler equation characterizing output dynamics depends on real money balances.

In addition, nonseparable preferences alter intratemporal choices. In this context, real money balances affect labor supply and real marginal costs. Therefore, the new Keynesian Phillips curve, which describes inflation dynamics, depends on the evolution of real money balances over time.

Second, the presence of portfolio adjustment costs makes the demand for money a forward-looking equation. In the canonical new Keynesian model, the demand for money is a static equation and real money balances do not influence the dynamics of the remaining macroeconomic variables. For this reason, the analysis of the canonical new Keynesian model does not really need an explicit money demand equation.

Andrés et al. (2009) and Arestis et al. (2009) showed that a forward-looking money demand equation implies that movements in real money balances not accounted for by the static determinants of money demand (output and the nominal interest rate) reflect variations in expected natural rates of output. Since the natural rate of output is a function of the structural shocks, which ultimately drive macroeconomic dynamics, money therefore conveys information on the determinants of aggregate demand and supply beyond that contained in its static determinants. Because of this informational role, regardless of whether monetary aggregates appear or not explicitly in the Euler equation and in the new Keynesian Phillips curve, if central banks somehow incorporate them in their monetary policy strategy, money will have an active role in business cycles.

2.1 The log-linear equilibrium conditions

The following equations define a linear rational expectations model, approximately describing the equilibrium conditions of the ALSN model.

$$\begin{aligned}
\widehat{y}_t \quad = \quad & \frac{\phi_1}{\phi_1+\phi_2}\widehat{y}_{t-1}+\frac{\beta\phi_1+\phi_2}{\phi_1+\phi_2}E_t\widehat{y}_{t+1}-\frac{1}{\phi_1+\phi_2}\left(\widehat{r}_t-E_t\widehat{\pi}_{t+1}\right) \\
& -\frac{\beta\phi_1}{\phi_1+\phi_2}E_t\widehat{y}_{t+2}+\frac{\psi_2}{\psi_1}\left(\frac{1}{1-\beta h}\right)\left(\frac{1}{\phi_1+\phi_2}\right)\widehat{m}_t \\
& -\frac{\psi_2}{\psi_1}\left(\frac{1}{1-\beta h}\right)\left[\left(\frac{1+\beta h}{\phi_1+\phi_2}\right)E_t\widehat{m}_{t+1}-\left(\frac{\beta h}{\phi_1+\phi_2}\right)E_t\widehat{m}_{t+2}\right] \\
& -\frac{\psi_2}{\psi_1}\left(\frac{1-\beta h\rho_e}{1-\beta h}\right)\left(\frac{1-\rho_e}{\phi_1+\phi_2}\right)\widehat{e}_t+\left(\frac{1-\beta h\rho_a}{1-\beta h}\right)\left(\frac{1-\rho_a}{\phi_1+\phi_2}\right)\widehat{a}_t
\end{aligned} \tag{1}$$

$$\widehat{\pi}_t=\frac{\beta}{1+\beta\kappa}E_t\widehat{\pi}_{t+1}+\frac{\kappa}{1+\beta\kappa}\widehat{\pi}_{t-1}+\lambda\frac{\widehat{mc}_t}{1+\beta\kappa} \tag{2}$$

$$\begin{aligned}
\widehat{mc}_t \quad = \quad & (\chi+\phi_2)\widehat{y}_t-\phi_1\widehat{y}_{t-1}-\beta\phi_1E_t\widehat{y}_{t+1} \\
& -\frac{\psi_2}{\psi_1}\left(\frac{1}{1-\beta h}\right)\left[\widehat{m}_t-\beta hE_t\widehat{m}_{t+1}\right]+\frac{\psi_2}{\psi_1}\left(\frac{1-\beta h\rho_e}{1-\beta h}\right)\widehat{e}_t \\
& -\left(\frac{\beta h}{1-\beta h}\right)\left(1-\rho_a\right)\widehat{a}_t-(1+\chi)\widehat{z}_t
\end{aligned} \tag{3}$$

$$\begin{aligned}
\left[1+\delta_0(1+\beta)\right]\widehat{m}_t \quad = \quad & \gamma_1\widehat{y}_t-\gamma_2\widehat{r}_t+\left[\gamma_2(\overline{r}-1)(h\phi_2-\phi_1)-h\gamma_1\right]\widehat{y}_{t-1} \\
& -\left[\gamma_2(\overline{r}-1)\beta\phi_1\right]E_t\widehat{y}_{t+1}+\delta_0\widehat{m}_{t-1} \\
& +\left[\frac{\psi_2}{\psi_1}\left(\frac{\beta h\gamma_2(\overline{r}-1)}{1-\beta h}\right)+\delta_0\beta\right]E_t\widehat{m}_{t+1} \\
& -\left(\frac{\beta h\gamma_2(\overline{r}-1)}{1-\beta h}\right)\left(1-\rho_a\right)\widehat{a}_t \\
& +\left[1-\gamma_2(\overline{r}-1)\left(\frac{\psi_2}{\psi_1}\left(\frac{\beta h\rho_e}{1-\beta h}\right)+1\right)\right]\widehat{e}_t
\end{aligned} \tag{4}$$

$$\widehat{\mu}_t = \widehat{m}_t - \widehat{m}_{t-1} + \widehat{\pi}_t \tag{5}$$

$$\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \varepsilon_{at} \tag{6}$$

$$\widehat{e}_t = \rho_e \widehat{e}_{t-1} + \varepsilon_{et} \tag{7}$$

$$\widehat{z}_t = \rho_z \widehat{z}_{t-1} + \varepsilon_{zt} \tag{8}$$

The variables $\widehat{y}_t$, $\widehat{r}_t$, $\widehat{\pi}_t$, $\widehat{m}_t$, $\widehat{mc}_t$ and $\widehat{\mu}_t$ are output, the nominal interest rate, inflation, real money balances, real marginal costs and nominal money growth. The disturbances $\widehat{a}_t$, $\widehat{e}_t$ and $\widehat{z}_t$ are a preference shock, a money demand shock and a technology shock. I measure all variables in deviations from their steady-state values.

Equation (1) is the Euler equation that arises from the household choice problem and describes the aggregate demand in the artificial economy. Because preferences exhibit nonseparability between consumption and real money balances, terms involving real money balances and their expected values are part of the aggregate demand equation. The presence of habit persistence introduces a role for the lagged value of output as a factor explaining current output.

Equations (2) and (3) characterize the supply side of the model. Equation (2) is the new Keynesian Phillips curve that arises from firms' price-setting behavior. Equation (3) is an expression defining real marginal costs, which are an important driving force for inflation dynamics, according to equation (2).

The introduction of portfolio adjustment costs and the nonseparability across real money balances and consumption shape the form of the money demand relationship. In contrast to the traditional static money demand schedule, equation (4) shows that the real money balance is a forward-looking variable.

Equation (5) defines nominal money growth rate, and equations (6) to (8) specify the stochastic disturbances for the shocks, which follow AR(1) processes with normal innovations ε_{at}, ε_{et} and ε_{zt}, with zero mean and variance σ_j^2 for $j \in \{a, e, z\}$. The persistence parameters for the shocks are ρ_j for $j \in \{a, e, z\}$.

The compound parameters of the model are:
$\psi_1 = \left(\frac{-\Psi_1}{\overline{y}^{1-h}\Psi_{11}}\right)$, $\psi_2 = \left(\frac{-\Psi_{12}}{\overline{y}^{1-h}\Psi_{11}}\right)\left(\frac{\overline{m}}{\overline{e}}\right)$, $\phi_1 = \frac{\left(\frac{1}{\psi_1}-1\right)h}{1-\beta h}$, $\phi_2 = \frac{\frac{1}{\psi_1}+\left(\frac{1}{\psi_1}-1\right)\beta h^2-\beta h}{1-\beta h}$, $\chi = \frac{\varphi+\alpha}{1-\alpha}$, $\lambda = \left(\frac{(1-\theta)(1-\beta\theta)}{\theta}\right)\left(\frac{1-\alpha}{1+\alpha(\epsilon-1)}\right)$ and $\delta_0 = \frac{dc^2}{\overline{m}}$.

The variables $\overline{y}$, $\overline{m}$ and $\overline{e}$ are steady-state figures. In addition, $\overline{r}$ denotes the steady-state value for the gross nominal interest rate. The coefficients γ_1 and γ_2 are the long-run real income and interest rate response parameters.

The terms Ψ_1, Ψ_{11} and Ψ_{12} are the partial derivatives of the function Ψ, which summarizes how consumption and real money balances interact in the utility function of the representative household. I evaluate these derivatives at steady-state levels[2]. The parameter β is the household's discount factor, φ is the inverse of the Frisch labor supply elasticity, and h is a parameter controlling the degree of habit persistence in consumption. Finally, the coefficients c and d determine the shape of the portfolio adjustment cost function.

The technology parameter in the production function of intermediate goods is α, and the coefficient ϵ is the elasticity of substitution between

[2] To simplify notation, I omitted the arguments of Ψ, Ψ_1, Ψ_{11} and Ψ_{12} in the text. In fact, I evaluated these functions at the point $\left(\overline{y}^{1-h}, \frac{\overline{m}}{\overline{e}}\right)$.

the differentiated goods composing the production bundle. The Calvo parameter, which measures the degree of price stickiness, is θ. Additionally, the parameter κ measures the degree of price indexation.

The role of money in equations (1) to (3), which describe aggregate demand and aggregate supply, depends on the parameter ψ_2. If $\psi_2 = 0$, the terms involving real money balances and their expectations vanish in expressions (1) to (3). If $\psi_2 > 0$, real money and consumption are substitutes. In equation (4), the forward-looking nature of money demand depends on setting $\psi_2 \neq 0$ or on the presence of portfolio adjustment costs ($\delta_0 \neq 0$).

The output gap is a key variable for central banks when they set monetary policy. Following Smets & Wouters (2007) and Ilbas (2010, 2012), I define the output gap as the difference between actual output and the natural rate of output. The natural rate of output is the equilibrium output in a flexible-price version of the ALSN model. In this version of the model, $\widehat{mc}_t = 0$ since the price-markup is constant under flexible prices. In addition, there is no new Keynesian Phillips curve due to instantaneous price adjustments, implying $\widehat{\pi}_t = 0$ for all t. The variables $\widehat{y}_t^n$, $\widehat{r}_t^n$ and $\widehat{m}_t^n$, measured in deviations from their steady-state values, denote the natural rate of output, the interest rate and real money balances in the flexible-price equilibrium. Equations (1), (3) with $\widehat{mc}_t = 0$ and (4) to (8) characterize the flexible-price equilibrium and the vector $(\widehat{y}_t^n, \widehat{r}_t^n, \widehat{m}_t^n)$ solves this system of dynamic stochastic difference equations. Thus, the difference $\widehat{y}_t - \widehat{y}_t^n$ corresponds to the output gap, which is a model-based measure that indicates how efficiently resources are being employed.

Appendix B provides more details of the model. Next, I describe how the central bank conducts monetary policy. Specifically, I present a quadratic loss function that summarizes the Fed's policy preferences.

2.2 Monetary Policy

To close the model, I have to specify the behavior of the central bank. I treat the central bank as an optimizing agent in the same way I treat households and firms. In fact, the central bank chooses the best policy subject to the constraints imposed by private agents' behavior; it minimizes an intertemporal quadratic loss function under commitment.

I postulate an ad hoc functional form for the loss function, which is not microfounded and does not correspond to a second-order approximation of the representative agents' utility function. The approach of specifying an ad hoc loss function assumes that the central bank acts according to a specific mandate. As a consequence, the central bank is not a benevolent planner and the policy objective function is not welfare-based.

The formulation of a Ramsey policy problem, in which a benevolent planner maximizes the utility of the representative household, is theoretically the best approach from a public finance perspective. Nevertheless, households' preferences constrain the welfare-based objective function by imposing highly nonlinear structural restrictions, which are most likely misspecified with respect to the data-generating process. Therefore, from an empirical perspective, assuming that the monetary authority follows a mandate is a sensible strategy if the research goal is to infer the relative importance of targets that the central bank may care about. This strategy leads to free parameters in the loss function, which improves model fit.

To avoid these econometric drawbacks, the empirical papers on optimal policies in dynamic stochastic general equilibrium models, which I list in footnote 4, employed postulated ad hoc loss functions. Besides the econometric difficulties discussed before, I decide to use a postulated loss function because I am able to compare the estimated weights with the ones documented in this empirical literature. Moreover, in this paper, the weights reported in this previous research may inform the choice of priors for the parameters in the loss function.

In the context of the ALSN model, the analytical derivation of the loss function as an approximation of households' utility is a task beyond the scope of this paper. Woodford (2003, chapter 6) and Paustian & Stoltenberg (2008) obtained such loss function in a simple model with money. Since they considered a static money demand schedule, they were able to substitute out real money balances. Because of this substitution, monetary aggregates were absent from their utility-based measure. Hence, inflation, the output gap and the interest rate were the only arguments in their loss function.

In the case of the ALSN model, with the presence of equation (4) as a consequence of portfolio adjustment costs, the quadratic approximation will be an explicit function of $\widehat{y}_t - \widehat{y}_t^n$, $\widehat{m}_t$, $\widehat{\pi}_t$ and $\widehat{\mu}_t$, since one can write the expression for portfolio adjustment costs, denoted by G, as a function of inflation and the money growth rate. The weights on quadratic terms for $\widehat{m}_t$ and $\widehat{\mu}_t$ will hinge on the partial derivatives of the function Ψ, which summarizes how consumption and real money balances interact in the utility function of the representative household, as well as on the partial derivatives of G.

Söderström (2005) studied optimal monetary policy under discretion in a calibrated version of the canonical new Keynesian model with a loss function that included money as one of its arguments, but he did not estimate the central bank's preference parameters. To estimate the ALSN model under optimal policy, contrary to Söderström (2005), I follow Ilbas (2010, 2012) and Adolfson et al. (2011) and assume that the central bank optimizes under commitment in a timeless perspective[3].

The central bank minimizes $E_t \sum_{i=0}^{\infty} \beta^i Loss_{t+i}$ with $0 < \beta < 1$, subject to the equations describing the behavior of households and firms. The one-period ad hoc loss function includes inflation, the output gap, a smoothing component for the interest rate and money growth. The central bank targets these variables, which are the goals of monetary policy, according to the following objective function.

$$Loss_t = \widehat{\pi}_t^2 + q_y \left(\widehat{y}_t - \widehat{y}_t^n\right)^2 + q_r(\widehat{r}_t - \widehat{r}_{t-1})^2 + q_\mu \widehat{\mu}_t^2$$

The weights q_y, q_r and q_μ summarize the central bank's preferences concerning these goals. When estimating the ALSN model under optimal policy, I allow these parameters to be estimated freely, subject only to non-negativity constraints.

The term $q_r(\widehat{r}_t - \widehat{r}_{t-1})^2$ describes a preference for interest rate smoothing. Central banks typically set policy by changing incrementally the policy rate and many papers have included the change in the interest rate in the loss function[4]. These papers have also argued that adding this term in the loss function is relevant for capturing movements in interest rates observed in U.S. data.

According to its assigned mandate, the central bank pursues a nominal money growth target, which corresponds to the term $q_\mu \widehat{\mu}_t^2$. Transaction technologies[5] and portfolio adjustment costs are the usual ways of introducing

[3] I consider that the commitment occurred some time in the past; and, since the central bank does not disregard any previous commitment in a timeless perspective, the initial values of the Lagrange multipliers related to the optimal monetary policy problem are different from zero. In this case, the optimal policy is thus time consistent.

[4] For instance, Dennis (2004, 2006), Kam et al. (2009), Ilbas (2010, 2012), Adolfson et al. (2011) and Givens (2012).

[5] Croushore (1993) showed the equivalence between money in the utility function, as described in the ALSN model, and the specification of transaction technologies (shopping-

money in macroeconomic models. A more stable nominal money growth reduces fluctuations in transactions and in the costs of adjusting portfolios, leading to less volatile business cycles and more predictable costs for rebalancing portfolios in response to shocks. These effects of stabilizing nominal money growth on the economy are the rationale for the presence of $\widehat{\mu}_t$ in the loss function as a potential monetary policy objective.

In addition, since the ALSN model represents the central bank's view about the economy, the monetary authority takes into account the informational role of money, implicitly described by equation (4), as it minimizes the loss function subject to this model.

I compare the ALSN model estimates under optimal policy with the results obtained assuming that the Fed followed a simple Taylor rule. The optimal policy design introduces new state variables, which are the Lagrange multipliers associated with private agents' decisions. In this context, these new state variables, compared with the model with a Taylor rule, place more restrictions on the observable variables, which are the same across alternative monetary policy specifications. Because the Taylor rule is less restrictive than the optimal policy specification, big differences in model fit favoring the Taylor rule can suggest that the assumption of optimal policy under commitment is incompatible with the data employed in the estimation.

Furthermore, big differences between the parameter estimates under alternative specifications for the conduct of monetary policy can challenge the implicit assumption that the ALSN model is structural, i.e., its parameters are invariant to distinct formulations in modeling monetary policy. The estimation under different monetary policy specifications may indicate how plausible this assumption is.

time models).

I estimate the Taylor rule given by the following equation.

$$\widehat{r}_t = \rho_r \widehat{r}_{t-1} + (1 - \rho_r) \left[\rho_y \left(\widehat{y}_t - \widehat{y}_t^n \right) + \rho_\pi \widehat{\pi}_t + \rho_\mu \widehat{\mu}_t \right] + \varepsilon_{rt}$$

The parameters describing the rule are ρ_r, capturing interest rate inertia and the coefficients ρ_y, ρ_π and ρ_μ, capturing the response of the interest rate to the macroeconomic variables $\widehat{y}_t - \widehat{y}_t^n$, $\widehat{\pi}_t$ and $\widehat{\mu}_t$. The monetary policy shock is ε_{rt}. This rule is widely used in papers that investigate the role of money in sticky-price models, such as Andrés et al. (2009), Arestis et al. (2009), Poilly (2010), Canova & Menz (2011) and Castelnuovo (2012).

Since $\widehat{\mu}_t$ affects macroeconomic dynamics and summarizes additional information on the shocks hitting the economy, the central bank may react to this variable in order to stabilize the economy. Rather than specify a standard Taylor rule, I choose to work with a more general interest rate rule and let the data select the best fit specification.

Next, I summarize the findings of four related papers and compare their specifications with the benchmark model of this paper.

2.3 Related models

Ireland (2004) proposed a new Keynesian model that relaxed the typically employed assumption that households' preferences are separable in consumption and real money balances. Working with U.S. data, Ireland (2004) could not find empirical evidence to reject the assumption of separable preferences. Andrés et al. (2006) used Euro-area data and reached the same conclusions as Ireland (2004). By contrast, Andrés et al. (2009) found empirical support for money as a relevant factor in business cycles when they introduced portfolio adjustment costs in addition to the nonseparable preference channel. As an extension of Ireland (2004), Zanetti (2012) introduced a simple banking sector. Similar to considering the addition of portfolio adjustment costs, the introduction of the banking sector strengthened the role of money in the business cycle.

The difference between the first two papers and the model in Andrés et al. (2009) hinges on the specification of money demand. Ireland (2004) and Andrés et al. (2006) worked with a static money demand equation in contrast to Andrés et al. (2009), which specified a forward-looking money demand. Equation (4) represented, therefore, this money demand function. Expressions (1) to (3) and (5) to (8) are common to the three first papers. In Ireland (2004), however, equations (1) to (3) did not have backward-looking terms. Though structural parameters differ, the reduced forms for aggregate demand and supply schedules of Zanetti's model are similar to the log-linear equations (1) to (3) with $h = \kappa = 0$. In Zanetti (2012), money demand is static, equations (5) to (8) hold and an additional expression, which did not exist in the three previous papers, described the household's deposit constraint.

In this paper, equations (1) to (8) are identical to the expressions describing the equilibrium in Andrés et al. (2009), i.e., I specify private agents' behavior in the same way they did. The difference between the benchmark model of this paper and Andrés et al. (2009) lies on how the central bank sets monetary policy. This paper considers optimal policies, departing from Taylor rules, which characterized monetary policy in Ireland (2004), Andrés et al. (2006), Andrés et al. (2009) and Zanetti (2012).

3 Estimation

This section discusses the Bayesian approach to estimate dynamic stochastic general equilibrium models (DSGE) and presents the data set and the priors used in the estimation.

3.1 Econometric Strategy and Priors

I estimate the parameters using likelihood-based Bayesian methods as discussed in Dave & DeJong (2011) and An & Schorfheide (2007). In this paper, I use the Metropolis-Hastings algorithm to obtain draws from the posterior distribution, running separate chains composed of 950,000 draws, discarding the first 50% as initial burn-in. I assess the convergence of the estimations using diagnostic statistics described in Brooks & Gelman (1998). In addition, I use the Bayes factor to compare the fit of alternative models to the data.

The second columns in Tables 1 and 2 show the priors for the parameters and reports the mean and standard deviation of each prior distribution. I used beta distributions for the parameters restricted to the interval $[0, 1]$ and inverse gamma distributions for standard errors of the shocks. I centered the priors in values consistent with the estimated parameters reported in Andrés et al. (2009) and Castelnuovo (2012). I calibrated some of the parameters in the ALSN model. Specifically, I followed the calibrated values reported in Castelnuovo (2012), setting $\beta = 0.9925$, $\alpha = \frac{1}{3}$, $\epsilon = 6$ and $\overline{r} = 1.0158$.

The assumption of optimal monetary policy under commitment leads to a time-inconsistent policy. To interpret the results as the outcome of an optimal policy from the timeless perspective, which is time-consistent, I initialize the estimation according to a pre-sample period of 20 quarters. This method for dealing with time-inconsistency follows the econometric strategy in Ilbas

(2010, 2012). Next, I discuss the data used in estimating the ALSN model.

3.2 Data

I collected quarterly U.S. data from the FRED database, which is housed by the Federal Reserve Bank of St. Louis. The variables are real output, real money balances, inflation and the short-term interest rate. Real GDP is the measure of real output, real money balances equal nominal M2 money stock divided by the GDP deflator, inflation is the quarterly variation in GDP deflator and the Fed funds rate measures the nominal interest rate.

I worked with seasonally adjusted data, except for the nominal interest rate. I then expressed real output and real money balances in per-capita terms, employing the civilian non-institutional population. I used logarithmic scale for real output and real money balances. The observable series are: real GDP growth, inflation, the nominal interest rate and the growth rate of the real money balances. I removed the mean of all series prior to estimation.

In light of the evidence of parameter instability over time reported in Canova & Menz (2011), Canova & Ferroni (2012) and Castelnuovo (2012), I focus the analysis on the period after the Volcker disinflation and before the recent financial crisis. To be precise, the quarterly sample ranges from 1984:Q1 to 2007:Q2[6]. I chose this sample for two reasons. First, the model describes normal times and does not have features designed to explain financial crises. Second, I wanted to restrict the analysis to a period in which it would be reasonable to argue that the Fed followed a conventional monetary

[6]The sample agrees with the definition of the Great Moderation era in Smets & Wouters (2007). Indeed, the beginning of the sample is similar to the starting point defining the sample period in some other papers. For instance, in Ilbas (2010) and Givens (2012), which are papers that estimated policy preferences, the sample starts at some date in the first half of the 1980s. In addition, in Ireland (2004) and Zanetti (2012), the estimation starts at the first quarter of 1980.

policy, using the interest rate as its instrument to curb inflation[7].

4 Empirical Results

This section presents and discusses the main findings of this paper. I first present the results from the estimation under optimal policy. Next, I move to the analysis of the estimation results concerning the model in which the Taylor rule describes monetary policy. I then compare the fit of alternative models using marginal likelihoods and Bayes factors. To quantitatively assess the dynamics of macroeconomic variables in the models, I finally compute selected moments and impulse response functions.

4.1 Estimates with Optimal Policies

Table 1 shows the results from the estimation of the ALSN model under optimal monetary policy. There are three specifications under optimal monetary policy. The first specification, in the third column of this table, called *No Money*, is the ALSN model subject to the following restrictions: $q_\mu = \psi_2 = \delta_0 = 0$. The specification labeled *PA only* refers to the case in which money affects private agents' behavior, corresponding to the restriction $q_\mu = 0$. Finally, the specification with label *PA and CB* allows money to influence the behavior of private agents and the central bank's policy preference.

INSERT TABLE 1

According to Table 1, the main objectives of monetary policy, irrespective of the role of real money balances in the description of the equilibrium, are

[7] Walsh (2010) argues that the Fed's operating procedures to conduct monetary policy were fairly homogeneous in this period.

inflation stabilization and interest rate smoothing. Output gap stabilization is a far less important objective than these two objectives. Estimation results suggest that the relevance of the money growth rate as a monetary policy objective is virtually negligible. In fact, the interest rate smoothing parameter in the central bank's loss function is somewhat high, suggesting that this objective is more important than inflation. Though estimated policy preference parameters, in medium-scale models, suggest a prominent role for inflation, the range of values I found for the interest smoothing parameter is consistent with the findings reported in Dennis (2004, 2006) and Givens (2012) for the commitment case.

Introducing money substantially changes the estimation of the following parameters: ψ_1, γ_2, κ, θ, φ, q_r, ρ_a, ρ_e, ρ_z, σ_a, σ_e and σ_z. In models with money, κ and q_r tend to be smaller, γ_2 and θ tend to be higher and shocks are more persistent and volatile. Comparing the specifications *PA only* and *PA and CB*, the changes in estimated parameters are mild, the exception being φ and q_r.

The model with money opens up the channels through which this variable affects the equilibrium. Since money is present in the Euler equation and in the marginal cost equation, it affects demand and supply sides of the artificial economy. In addition, the dynamic money demand equation activates the informational role of money, which also modifies the equilibrium. All these changes introduce money as an additional state variable in the state space representation of the model, interacting with other state variables. The estimation of this new state space specification alters parameters' values related to demand and supply sides, as well as shocks. By contrast, in the model without monetary aggregates, money does not influence the equations describing inflation, interest rates and output dynamics.

The estimated values for ψ_2 is close to zero and δ_0 is far from zero. This characteristic indicates that portfolio adjustment costs are more important than nonseparability in giving money a distinctive role in explaining the business cycle. This result coincides with findings reported in Andrés et al. (2009), Arestis et al. (2009) and Castelnuovo (2012). The estimation yields a substantial degree of inflation and output inertia due to high values for the habit persistence parameter h and the price indexation parameter κ. Prices are very sticky due to high and possibly implausible values of θ[8]. All these features cast doubts on the plausibility of optimal policy under commitment as the best assumption to describe monetary policy. In fact, these features arise due to the specific cross equation restrictions induced by the optimizing behavior of the central bank. To fit the data and satisfy these restrictions, parameter estimates sometimes assume some implausible values.

Finally, I estimate the models assuming that the interest rate is subject to a measurement error under optimal policy. The variance of this measurement error is stable across specifications and smaller than the variance of the remaining stochastic disturbances.

Given that the aim of this paper is to provide some evidence on the role of money as a monetary policy objective, Figures 1 to 3 show priors and posteriors for the parameters of the model with no restrictions imposed on the weights in the loss function (the *PA and CB* specification). Since posterior distributions move away from priors, the data are informative about the parameters. Concerning the weights in the loss function, a comparison between posteriors and priors in Figure 2 shows that the data shift the prior distributions for q_y and q_μ to the left, suggesting negligible roles for the output gap and nominal money growth as monetary policy objectives.

[8] Kam et al. (2009), Ilbas (2010) and Canova & Ferroni (2012) also reported high and somewhat implausible values for the Calvo parameter θ.

INSERT FIGURES 1 TO 3

4.2 Estimates with Taylor Rules

Table 2 shows the results from the estimation of versions of the ALSN model in which Taylor rules describe monetary policy. For the three specifications shown in this table, I use the following labels: *No Money* ($\rho_\mu = \psi_2 = \delta_0 = 0$), *PA only (*$\rho_\mu = 0$) and *PA and CB* (unrestricted model).

The estimated structural parameters are relatively stable, with the exception of ψ_2, γ_2, ρ_e, σ_e and σ_z. Compared with the optimal policy models, there is more parameter stability across specifications. Inflation inertia, as measured by κ, is smaller. The high estimated degree of habit persistence h continues to induce output inertia. Though still high, the parameter θ lies in a more reasonable interval for the Calvo probability.

As in the estimations under optimal policies, portfolio adjustment costs seem to be more important than nonseparability in modeling the role of money for business cycles. The Taylor rules exhibit a moderate degree of interest rate inertia. Monetary policy shocks are not very volatile, with variance in line with the estimated variance of the measurement error in models with optimal policies.

INSERT TABLE 2

Comparing the best fit specification under Taylor rules with the best model under optimal policies, which are respectively *Taylor Rule-PA and CB* and *optimal policy-PA only*, one can see that most of the parameter estimates in the equations describing private agents' behavior are very similar. The exceptions are κ, θ, ρ_a, ρ_e, ρ_z and σ_z. Some of these parameters control the degree of persistence implied by the model and assume high values under optimal policies. Therefore, the specifications under optimal policies imply more inertial dynamics for macroeconomic variables compared with models using Taylor rules. As explained in Galí (2008), this feature is intrinsic to optimal policies under commitment since they introduce history dependence.

4.3 Model Comparison

I compare the fit of the models under the optimal policy assumption with specifications in which a Taylor rule describes monetary policy. I use marginal data densities or marginal likelihoods to compare the empirical performance of these models.

Table 3 reports marginal likelihoods and Bayes factors for each model. The Bayes factor is the ratio of marginal likelihoods associated with alternative models, i.e., $BF = \frac{p(Y_T|M_1)}{p(Y_T|M_2)}$, where $p(Y_T|M_j)$ is the marginal likelihood of model M_j. I report Bayes factors in $\log_{10}$ scale, that is, I compute the following expression $\log_{10}(BF)$. In this way, I can express orders of magnitude in a more compact scale since the ratio between marginal data densities may involve large ranges of numerical values.

I normalize the Bayes factor of the model under optimal policy without money ($q_\mu = \psi_2 = \delta_0 = 0$) to zero in $\log_{10}$ scale because this specification presents the worst empirical fit as measured by the marginal likelihood. To compare two alternative models, one just takes the differences between their $\log_{10}$-scaled Bayes factors. An improvement indicates some evidence in favor of the model with the highest marginal likelihood. The evidence is strong (decisive) if the improvement is greater than 1.5 (2). For example, in comparing the *optimal policy-PA only* model with the *optimal policy-PA and CB* model, the difference between the $\log_{10}$-scaled Bayes factors is 0.1368, favoring the *optimal policy-PA only* model, which has the highest marginal data density among the two specifications. In fact, in the *optimal policy-PA and CB* model, q_μ is close to zero. This fact justifies the small difference between the $\log_{10}$-scaled Bayes factors of the two models. For Taylor rules, since the difference between the $\log_{10}$-scaled Bayes factors is 1.7655 and the marginal likelihood of the *Taylor Rule-PA and CB* model is higher, a comparison between the *Taylor Rule-PA only* model and the *Taylor Rule-PA and CB* model indicates a strong evidence favoring the latter.

INSERT TABLE 3

Table 3 shows that the models with Taylor rules dominate the models with optimal policy under commitment in a timeless perspective. In fact, the data provide decisive evidence in favor of this specification. This finding suggests that he cross-equation restrictions associated with optimal policies are at odds with the Fed's behavior.

In addition, this result may indicate that the assumption of a central bank behaving according to the optimal monetary policy under commitment is not the best way to describe the data. This fact opens the door to alternative specifications for monetary policy, which could be conducted under discretion in an optimal way or could not be characterized by any simple optimization problem. Alternatively, this result also suggests the possibility of a misspecified central bank loss function. An investigation of these hypotheses is beyond the scope of this paper.

Model comparison shows that the presence of the money growth rate as a monetary policy objective does not improve model fit. The estimated Fed preference is consistent with a strategy that targets inflation and gradually adjusts interest rates.

In sum, incorporating money in the structural equations improves model fit under optimal policy. This evidence supports a relevant role for money in modeling private agents' behavior in small scale macroeconomic models. In contrast, there is no compelling evidence of a role for money also as a monetary policy objective. These findings suggest that money as an indicator variable, conveying information that improves the forecasts of inflation and economic activity, is the most plausible interpretation for the interest rate reaction to the growth rate of nominal money in Taylor rules.

4.4 Dynamic Properties

I assess the implications of introducing money for the dynamics of key macroeconomic variables.

Table 4 reports selected moments for the output gap, inflation and the interest rate. Under Taylor rules, the output gap and inflation are less volatile in the specification with money. In contrast, under optimal policies, the presence of money leads to more volatility in the output gap and inflation. Moreover, independent of the policy specification, interest rates are more volatile in models with money. Persistence patterns are somewhat similar in models with and without money, irrespective of how I describe monetary policy.

INSERT TABLE 4

The inspection of Table 4 reveals differences in key moments of macroeconomic variables associated with alternative descriptions of monetary policy. Here, I focus on the best fit models. Indeed, as a consequence of a low estimated q_y, the best optimal policy model increases the volatility and reduces the persistence of the output gap. Additionally, inflation is more volatile and persistent under this model. Regarding interest rates, they are much less volatile and extremely persistent under optimal policies. This last feature is due to an excessive degree of history dependence introduced by optimal policies.

The last column of Table 4 shows data-based moments[9]. The best specification for optimal policies generates moments that are less consistent with the data than the ones related to the best configuration for Taylor rules. This result agrees with Table 3, which suggests that Taylor rules are more in line with the data according to reported marginal likelihoods. Though optimal policies stand in contrast to the data, they capture well interest rate persistence. Taylor rules, on the other hand, introduce more volatility and less persistence in interest rates. Overall, the models in Table 4 cannot generate moments that match closely the ones from the data.

In Figures 4 to 7, I plot impulse response functions to further investigate the differences in macroeconomic dynamics implied by alternative monetary policy specifications. To save space, in these figures, for each way of modeling monetary policy, I consider only the best fit specification according to Table 3. The figures show impulse responses for the models with parameters calibrated at the posterior mean. I consider a 1% shock and I measure the impulse in percentage points. Moreover, since the size of the shock is the same across models, for a given shock, I report the impulse response of the models together in the same graph.

Figures 4 to 7 exhibit the responses to shocks of the following macroeconomic variables: inflation, the output gap, the nominal interest rate, real marginal costs, output and real money balances. Since monetary policy shocks are specific to Taylor rules, the responses under the best optimal policy configuration are absent in Figure 7. I first describe the effects of a shock on these variables in the Taylor rule model and then highlight the

[9] The date-based output gap is the HP-filtered output series.

differences between these effects and the responses under the optimal policy specification.

Figure 4 shows responses to a preference shock, which implies an increase in the intertemporal marginal rate of substitution. This shock increases current output, but this increment is less than the hike in its natural counterpart; hence, the output gap goes down. According to equation (3), preference shocks negatively affect real marginal costs; inflation therefore decreases on impact. Reacting to inflation and the output gap, the interest rate declines. According to equation (4), the shock leads to a decline in real money balances on impact. But the final response of this variable depends on the remaining terms in this equation. The strong output response more than cancels out this decline, and real money balances mildly increase in the Taylor rule model. For the optimal policy specification, interest rates barely move and real money balances decline due to the weak output response and the dominance of the initial effect of the shock.

INSERT FIGURE 4

Figure 5 presents responses to a money demand shock. Since consumption and real money balances are complementary goods ($\psi_2 < 0$), an increase in real money balances due to the shock triggers an increase in output. According to equation (3), since $\psi_2 < 0$, money demand shocks negatively affect real marginal costs; inflation therefore decreases on impact. Finally, interest rates increase responding to an increase in nominal money growth due to an increment in real money balances, which is strong in magnitude than the declines in inflation and the output gap. In the case of the Taylor rule model, this movement in interest rates also helps to bring inflation down. For the optimal policy specification, interest rates barely react. Excluding real marginal costs, output and the output gap, the responses of the remaining variables are weaker in comparison to the Taylor rule model.

INSERT FIGURE 5

Figure 6 displays responses to a technology shock. Output increases, though less than its natural counterpart, and inflation decreases. Further, interest rates decrease, reacting to a reduction in inflation and the output gap. Nominal money balances increase due to low interest rates and high output. In addition, under the best optimal policy, except for real marginal costs, the responses of macroeconomic variables are much weaker than those associated with the Taylor rule model. Finally, interest rates follow the same pattern shown in their reaction to preference and money demand shocks. Due to high estimated values for the Calvo probability θ, movements in economic activity do not have a great influence on inflation and the Phillips curve becomes less sensitive to developments in the output gap. Hence, the response of inflation to a positive technology shock is relatively muted.

INSERT FIGURE 6

Figure 7 reports responses to a monetary policy shock. Since the shock weakens aggregate demand, output, inflation and real marginal costs decrease on impact. Weak aggregate demand and high interest rates lead to a reduction in real money balances. Finally, the output gap goes down on impact because the decrease in actual output is bigger than the decrease in the natural rate of output.

INSERT FIGURE 7

Summing up, Figures 4 to 7 register more inertial responses of inflation and interest rates to shocks under the best fit model for optimal policies.

For most variables, the signs of impulse responses on impact are somewhat qualitatively similar irrespective of how I describe monetary policy. The exceptions are interest rates and real money balances. On the other hand, the magnitudes of impulse responses on impact differ a lot across models. For most responses of variables to shocks, the Taylor rule specification delivers bigger magnitudes on impact.

5 Sensitivity Analysis

In this section, I perform sensitivity analysis on some elements of the model to check the robustness of relevant results. First, I evaluate the sensitivity of the estimated weights in the loss function regarding alternative prior specifications. Second, I consider a model with a restricted version of the Taylor rule, without money and inertia. Third, I estimate the unrestricted version of the model under optimal policy (*PA and CB*) using artificial data from the *Taylor Rule-PA and CB* model, which is the best empirical model according to Table 3.

5.1 Estimation of the parameters in the loss function under alternative priors

This exercise checks the result concerning low estimated values for q_y and q_μ. Since the aim of this paper is to provide some evidence on the role of money as a monetary policy objective, estimation under alternative priors are useful to assess if the finding suggesting q_μ close to zero depends on how I set the priors for the parameters in the loss function.

I consider the following alternative priors:

- loose priors: Gamma distribution as the prior for the three weights with mean 0.5 and variance 0.6, which is three times larger than the one reported in Table 2.

- tight priors: Gamma distribution as the prior for the three weights with mean 0.5 and variance 0.2, which is half the variance specified in Table 2.

- weight-specific priors: the prior for q_y is Gamma with mean 0.25 and variance 0.4, the prior for q_r is Gamma with mean 0.75 and variance 0.5 and the prior for q_μ is Gamma with mean 0.5 and variance 0.6.

The set of priors labeled as *loose* are flatter than the priors in Table2 and, compared to the baseline case of this table, the posteriors may move around more relatively to the *loose* priors. Compared to Table 2, *tight* priors constrain the weights around 0.5 by imposing more restrictions on posteriors displacement relative to them. Since the priors do not need to be the same for the weights, I choose different mean and variance for each parameter in the loss function in the third set of priors. I set a loose prior for q_μ to try to restrict the prior influence on the estimated parameter. The mean values of the priors for q_y and q_r are based on Dennis (2004, 2006) and Givens (2012), which documented small magnitudes for q_y and large values for q_r.

Table 5 reports the posteriors for the parameters in the loss function under these alternative priors. Under *loose* and *weight-specific* priors, for q_y and q_μ, the posterior mean is small and, particularly for q_y, it is very close to zero. On the contrary, under these two sets of priors, the posterior mean for q_r assumes considerable magnitudes. The use of *tight* priors restricts shifts in the posterior mean relatively to the prior mean. In this case, though prior and posterior means are not so far from each other, for q_y and q_μ, the data still move the posterior mean towards zero. Again, for q_r, the posterior mean is far from zero than the prior mean. Irrespective of the set of priors, in terms of magnitudes, the posterior mean of q_r is always bigger than the posterior mean of q_μ, which is always bigger than the posterior mean of q_y.

INSERT TABLE 5

Figure 8 shows priors and posteriors for the parameters in the loss function for each set of priors. For q_y and q_μ, irrespective of the set of priors, the data shift the prior distribution to the left and posterior central tendency is smaller than prior central tendency. On the other hand, the data displace the prior distribution to the right for q_r. In other words, the data favor values closer to zero for q_y and q_μ. By contrast, the data locate the posterior for q_r around values far away from zero.

INSERT FIGURE 8

Finally, I also experimented with Normal-distributed priors, a priori relaxing the restriction confining the weights to be positive numbers. To save space, I do not present the results for this case since, for q_y and q_μ, the posterior mean is negative even with a less dispersed prior centered in a positive number. This situation illustrates the need to impose, a priori, a zero bound restriction on the parameters in the loss function, reinforcing the choice of a Gamma distribution as the prior for the weights.

5.2 A standard Taylor rule

An important finding of this paper is that Taylor rules dominate optimal policies in marginal likelihood comparisons, suggesting that models with Taylor rules describe better the data. Here, I consider a simple Taylor rule, without money and inertia to evaluate if the joint presence of these elements are responsible for the empirical preeminence of Taylor rules.

The following equation characterizes the estimated Taylor rules:

$$\widehat{r}_t = \rho_r \widehat{r}_{t-1} + (1 - \rho_r) \left[\rho_y \left(\widehat{y}_t - \widehat{y}_t^n \right) + \rho_\pi \widehat{\pi}_t + \rho_\mu \widehat{\mu}_t \right] + \varepsilon_{rt}$$

I compare the standard Taylor rule ($\rho_r = \rho_\mu = 0$) with the following specifications: a Taylor rule with interest rate smoothing ($\rho_r > 0$ and $\rho_\mu = 0$) and the general Taylor rule ($\rho_r > 0$ and $\rho_\mu > 0$). These two alternatives to the standard Taylor rule are the cases reported in Table 2. Table 6 shows marginal likelihoods and Bayes factors for each interest rate rule. I report the results related to the model in which money affects the structural equations describing private agents' behavior[10]. The general Taylor rule yields the best fit and the evidence favoring this rule is decisive. Therefore, disregarding the standard Taylor rule in Table 2 is just a way to focus on the most empirically plausible interest rate rules. Comparing Table 3 and Table 6, the standard Taylor rule fits the data better than any optimal policy specification. Hence, the superiority of Taylor rules in fitting the data continues to hold even for this particular case with less empirical support.

INSERT TABLE 6

5.3 The stability of parameter estimates across monetary policy specifications

Table 1 and 2 summarize the estimation of the ALSN model with Taylor rules and under optimal policies. By comparing these estimations, some parameters are unstable under distinct specifications for the conduct of monetary policy. This pattern may indicate that the ALSN model is not structural, i.e.,

[10] According to Table 3, introducing money in the equations associated with the private sector always improves model fit. The resuts are qualitatively the same for the standard new Keynesian model ($\psi_2 = \delta_0 = 0$). I did not report them for the sake of brevity.

its private-sector parameters are not invariant to alternative formulations in modeling monetary policy.

From an econometric point of view, estimated private-sector parameters may differ across policy specifications since different policies affect the mapping from the private-sector parameters to the parameters of the reduced-form linearized state-space representation of the model, which is the basis for likelihood computations.

Canova & Menz (2011) and Castelnuovo (2012) reported parameter instability over time in models with money and Taylor rules. Finally, when Canova & Ferroni (2012) restricted some parameters during the estimation, the unrestricted ones changed considerably.

One way to assess parameter instability across monetary policy specifications is to simulate data from the best ALSN model under Taylor rules and use these simulated data to re-estimate the model with optimal policy. I choose the best Taylor rule model as the data-generating process because it is the most empirically plausible model.

To carry out this exercise, I calibrate the parameters of the best Taylor rule model at their posterior means and generate artificial time series of length 5000 for the observable variables in the estimation. In this way, with long time series, small sample sizes do not influence the statistical properties induced by the Taylor rule model.

Table 7 presents the results. For some parameters, the true values are far away from the means of the estimated parameters using artificial data. Further, for most parameters, the true values are outside the 90% interval displayed in the fourth column. Overall, the results indicate that the estimation employing simulated data cannot recover the parameters of the data-generating model.

Interestingly, as in the estimation based on observed data, the likelihood shifts the prior distribution to the left for q_y and q_μ and displaces the prior distribution to the right for q_r, though these movements are less dramatic for the estimation that hinges on artificial data. Particularly, since interest rates based on the Taylor rule model are less persistent than actual interest rates, the magnitude of the posterior mean of q_r is not so big in the fourth column of Table 7 compared with the fifth column.

INSERT TABLE 7

6 Conclusion

Standard new Keynesian literature assigns a minimal role for monetary aggregates in explaining cyclical fluctuations. Alternative ways of introducing money in dynamic stochastic general equilibrium models have challenged this view and empirical research based on them suggested that monetary aggregates played an important role in explaining U.S. business cycles. This study also documented that the Fed reacted systematically to the growth rate of nominal money when a Taylor rule described monetary policy.

This response to money growth rates might be rationalized in two alternative ways. First, money growth could be a target variable in the Fed's loss function. Alternatively, money could be just an indicator variable, with no role as a monetary policy objective, being useful in forecasting inflation and economic activity. To gauge the plausibility of these alternative interpretations, I estimated the model studied in Andrés et al. (2009), in which money is a relevant factor, by replacing the Taylor rule with optimal monetary policy.

According to the empirical evidence, the presence of the money growth rate as a monetary policy objective does not improve model fit. Moreover, inflation variability and interest rate smoothing are the main objectives of monetary policy, irrespective of the role of money in the equations describing private agents' behavior. Additionally, the data favor models in which a Taylor rule describes monetary policy. These results suggest that considering money as an indicator variable is the most plausible rationale to account for the interest rate response to the money growth rate in Taylor rules.

Since macroeconomic variables behave differently across the two alternative ways of describing monetary policy, the choice of monetary policy specification matters for macroeconomic dynamics. Moreover, the introduction of money balances changes to some extent the model's transmission mechanism.

Future research can extend this paper in at least three directions. First, researchers can perform a cross country analysis on the role of money as a monetary policy objective. Second, an extension of this paper can evaluate the role of money in the central bank's objective function in the context of the model put forth by Canova & Ferroni (2012), which is a version of the medium-size structural macroeconometric model of Smets & Wouters (2007) with money. Finally, in the spirit of Givens (2012), an additional study can evaluate alternative ways to introduce optimal policies in models in which money plays a potential role in explaining business cycles.

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APPENDIX A

Optimal interest rate response to the money growth rate

My goal here is to stress the point made in the introduction on the pitfalls of interpreting Taylor rule coefficients as a barometer of the central bank's preference. I use the mean values of the priors in Table 1 to calibrate the model and consider the case in which $q_\mu = 0.75$ as well as the alternative specification with $q_\mu = 0$. I then compute optimal Taylor rules in a calibrated version of the ALSN model.

The computed Taylor rules are: $\widehat{r}_t = 0.684\widehat{r}_{t-1} + 0.052\widehat{y}_t + 1.471\widehat{\pi}_t + 0.443\widehat{\mu}_t$ for $q_\mu = 0.75$ and $\widehat{r}_t = 0.651\widehat{r}_{t-1} + 0.046\widehat{y}_t + 1.466\widehat{\pi}_t + 0.431\widehat{\mu}_t$ for $q_\mu = 0$.

In this calibrated version of the ALSN model, the optimal Taylor rules imply an interest rate that responds to money growth. This result remains, even in the case of no explicit concern for stabilizing the money growth rate in the central bank's loss function.

Svensson (2003) stressed the pitfalls in trying to infer what central banks may care about from the coefficients of simple monetary policy rules. Since the interest rate responds to money growth when the central bank's objectives do not include money growth stability, these optimal Taylor rules illustrate his point.

APPENDIX B
The ALSN Model

This appendix presents details of the model developed by Andrés et al. (2009). The economy consists of a representative household and a continuum of firms indexed by $j \in [0,1]$. The model abstracts from capital accumulation and features price stickiness.

- **Households**

The representative household maximizes the expected flow of utility given by the expression:

$$E_0 \sum_{t=0}^{\infty} \beta^t a_t \left[\Psi \left(\frac{C_t}{C_{t-1}^h}, \frac{M_t}{e_t P_t} \right) - \frac{N_t^{1+\varphi}}{1+\varphi} \right] - G(\cdot)$$

The variable C_t stands for aggregate consumption, $\frac{M_t}{P_t}$ represents real money balances and N_t denotes hours worked. The preference shock is a_t and the shock to the household's demand for real balances is e_t. The parameter β, restricted to be in the unity interval, is the discount factor. The parameter φ, a positive number, is the inverse of the Frisch labor supply elasticity. Finally, h is a parameter controlling the degree of habit persistence in consumption.

The preference specification allows for nonseparability between consumption and real money balances, as well as habit persistence in consumption. The function $\Psi(\cdot)$ summarizes all these features. Specifically, the intratemporal nonseparability between consumption and real money balances gives rise to an explicit real money balance term in the equations describing the supply and demand sides of the artificial economy.

In addition to the nonseparability channel, the presence of portfolio adjustment costs generates an alternative mechanism that gives money a role in the dynamic equations of the model. Moreover, the money demand equation becomes a dynamic forward-looking equation in which expectations of future interest rate matter.

Portfolio adjustment costs are probably small in magnitude. For this reason, I log-linearize the model around a steady state in which these costs are zero. In addition, the functional form for $G(\cdot)$ is compatible with small costs[11]. The portfolio adjustment cost function G follows the specification below.

$$G(\cdot) = \frac{d}{2}\left\{\exp\left[c\left(\frac{\frac{M_t}{P_t}}{\frac{M_{t-1}}{P_{t-1}}} - 1\right)\right] + \exp\left[-c\left(\frac{\frac{M_t}{P_t}}{\frac{M_{t-1}}{P_{t-1}}} - 1\right)\right] - 2\right\}$$

In each period the household faces the budget constraint given by the equation:

$$\frac{M_{t-1} + B_{t-1} + W_t N_t + T_t + D_t}{P_t} = C_t + \frac{\frac{B_t}{R_t} + M_t}{P_t}$$

The representative household enters the current period with money holdings M_{t-1} and bonds B_{t-1}, receiving lump-sum transfers T_t, dividends D_t and labor income $W_t N_t$, where W_t stands for nominal wages. The household purchases new bonds at nominal cost $\frac{B_t}{R_t}$, where R_t denotes the gross nominal interest rate between the current period t and the next $t+1$. Finally, the household will enter the next period with money holdings M_t.

[11] To quote Andrés et al. (2009): “An advantage of this portfolio adjustment cost specification is that for a wide range of c and d values, the portfolio adjustment costs incurred to carry out typical monetary transactions are trivial when converted into units of resources surrendered by the representative agent (...). Yet, at the same time, these costs imply substantial effects on money demand dynamics. The effects on dynamics, moreover, are supported by many existing empirical findings regarding money demand” (p. 761)

The choices variables for the household are consumption (C_t), hours (N_t), real money holdings ($\frac{M_t}{P_t}$) and bonds (B_t).

The representative household maximizes its expected utility subject to its budget constraint. The first-order conditions for this optimization problem are:

$$a_t N_t^{\varphi} = \lambda_t \frac{W_t}{P_t}$$

$$\lambda_t = \beta E_t \left(\frac{R_t \lambda_{t+1}}{\Pi_{t+1}} \right)$$

$$\lambda_t = E_t \left[a_t \frac{\Psi_1 \left(\frac{C_t}{C_{t-1}^h}, \frac{m_t}{e_t} \right)}{C_{t-1}^h} - h\beta \left(\frac{C_{t+1}}{C_t} \right) a_{t+1} \frac{\Psi_1 \left(\frac{C_{t+1}}{C_t^h}, \frac{m_{t+1}}{e_{t+1}} \right)}{C_t^h} \right]$$

$$\lambda_t = \left(\frac{a_t}{e_t} \right) \Psi_2 \left(\frac{C_t}{C_{t-1}^h}, \frac{m_t}{e_t} \right) - \frac{\partial G(m_t, m_{t-1})}{\partial m_t} - \beta E_t \left(\frac{\partial G(m_{t+1}, m_t)}{\partial m_t} - \frac{\lambda_{t+1}}{\Pi_{t+1}} \right)$$

I denote the Lagrange multiplier by λ_t. I also define two new variables: $m_t = \frac{M_t}{P_t}$ and $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ The terms Ψ_1 and Ψ_2 are the partial derivatives of the function Ψ with respect to $\frac{C_t}{C_{t-1}^h}$ and $\frac{m_t}{e_t}$. The symbols $\frac{\partial G(m_t, m_{t-1})}{\partial m_t}$ and $\frac{\partial G(m_{t+1}, m_t)}{\partial m_t}$ stand for the partial derivative of the portfolio adjustment cost function G with respect to m_t, when evaluated at t and $t+1$.

The first equation defines labor supply. The combination of the second and third expressions leads to the Euler equation. Finally, the blending of the second and fourth expressions yields the money demand equation.

The formulae for $\frac{\partial G(m_t,m_{t-1})}{\partial m_t}$ and $\frac{\partial G(m_t,m_{t-1})}{\partial m_t}$ are the following:

$$\frac{\partial G(m_t,m_{t-1})}{\partial m_t}=\frac{d}{2}\left\{\begin{array}{c}\exp\left[c\left(\frac{m_t}{m_{t-1}}-1\right)\right]\left(\frac{c}{m_{t-1}}\right)\\ +\exp\left[-c\left(\frac{m_t}{m_{t-1}}-1\right)\right]\left(-\frac{c}{m_{t-1}}\right)\end{array}\right\}$$

$$\frac{\partial G(m_{t+1},m_t)}{\partial m_t}=\frac{d}{2}\left\{\begin{array}{c}\exp\left[c\left(\frac{m_{t+1}}{m_t}-1\right)\right]\left(-\frac{cm_{t+1}}{m_t^2}\right)\\ +\exp\left[-c\left(\frac{m_{t+1}}{m_t}-1\right)\right]\left(\frac{cm_{t+1}}{m_t^2}\right)\end{array}\right\}$$

To derive equations (1) and (4), I log-linearize the second, third and fourth first-order conditions. Before this step, I need to find steady-state figures for output ($\overline{y}$), real money balances ($\overline{m}$) and the gross nominal interest rate ($\overline{r}$). For this end, I impose the following restrictions on the first-order conditions: $C_t = Y_t$, $a_t = a_{t+1} = \overline{a}$, $e_t = e_{t+1} = \overline{e}$, $z_t = \overline{z}$, $N_t = \overline{N}$, $\lambda_t = \lambda_{t+1} = \overline{\lambda}$, $C_{t-1} = C_t = C_{t+1} = \overline{y}$, $m_{t-1} = m_t = m_{t+1} = \overline{m}$ and $\Pi_{t+1} = \overline{\Pi} = 1$. The exogenous variables are $\overline{a}$, $\overline{e}$ and $\overline{z}$. In addition, according to the aggregate production function in steady-state[12], $\overline{N} = \left(\frac{\overline{y}}{\overline{z}}\right)^{\frac{1}{1-\alpha}}$ and $\overline{\frac{W}{P}} = (1-\alpha)\overline{z}\,\overline{N}^{-\alpha}$.

The equations characterizing the steady-state are:

$$\overline{a}\;\overline{y}^{\frac{\varphi+\alpha}{1-\alpha}} = (1-\alpha)\overline{z}^{\frac{(1+\varphi)}{1-\alpha}}\overline{\lambda}$$

$$\overline{r} = \frac{1}{\beta}$$

$$\overline{\lambda}\overline{y}^h = (1-\beta h)\overline{a}\Psi_1\left(\overline{y}^{1-h}, \frac{\overline{m}}{\overline{e}}\right)$$

[12] The production function $Y_t(j) = z_t N_t^{1-\alpha}(j)$ describes the technology for firm j. To aggregate this function across firms, I assume that there is no price dispersion in steady-state.

$$\overline{a}\Psi_2\left(\overline{y}^{1-h},\frac{\overline{m}}{\overline{e}}\right)=(1-\beta)\overline{e}\overline{\lambda}$$

The log-linearized versions of the second, third and fourth first-order conditions are:

$$\widehat{\lambda}_t=E_t\widehat{\lambda}_{t+1}+\widehat{r}_t-E_t\widehat{\pi}_{t+1}$$

$$\begin{aligned}\widehat{\lambda}_t &= \left(\frac{1-\beta h\rho_a}{1-\beta h}\right)\widehat{a}_t-\frac{\psi_2}{\psi_1}\left(\frac{1-\beta h\rho_e}{1-\beta h}\right)\widehat{e}_t+\beta\phi_1E_t\widehat{y}_{t+1}\\ &\quad -\phi_2\widehat{y}_t+\phi_1\widehat{y}_{t-1}+\frac{\psi_2}{\psi_1}\left(\frac{1}{1-\beta h}\right)\left[\widehat{m}_t-\beta hE_t\widehat{m}_{t+1}\right]\end{aligned}$$

$$\begin{aligned}&\Psi_{12}\overline{y}^{1-h}\left(\widehat{y}_t-h\widehat{y}_{t-1}\right)+\left(\Psi_{22}\frac{\overline{m}}{\overline{e}}-(1+\beta)\delta_0\right)\widehat{m}_t\\ &+\delta_0\widehat{m}_{t-1}+\beta\delta_0E_t\widehat{m}_{t+1}+\Psi_2\widehat{a}_t-\left(\Psi_{22}\frac{\overline{m}}{\overline{e}}+\Psi_2\right)\widehat{e}_t\\ =\ &\Psi_2\left(\widehat{\lambda}_t+\frac{\beta}{1-\beta}\widehat{r}_t\right)\end{aligned}$$

By combining the second and third log-linearized first-order conditions, one gets equation (1) of the main text, which is the Euler equation. The second and fourth first-order conditions lead to equation (4), which is the money demand schedule.

To simplify notation, I omitted the bivariate argument $\left(\overline{y}^{1-h},\frac{\overline{m}}{\overline{e}}\right)$ of Ψ_2, Ψ_{12} and Ψ_{22}, which are the partial derivatives of $\Psi\left(\overline{y}^{1-h},\frac{\overline{m}}{\overline{e}}\right)$. I discuss the coefficients ψ_1, ψ_2, ϕ_1, ϕ_2 and δ_0 in the main text. In addition, I define the coefficients γ_1 and γ_2 in equation (4) by the following expressions:

$$\gamma_2(\overline{r}-1) = \frac{\Psi_2}{\Psi_2 \frac{\psi_2}{\psi_1}\left(\frac{1}{1-\beta h}\right) - \Psi_{22}\frac{\overline{m}}{\overline{e}}}$$

$$\gamma_1 = \gamma_2\left[\overline{r}\frac{\overline{y}}{\overline{m}}\frac{\psi_2}{\psi_1}\left(\frac{1}{1-\beta h}\right) + (\overline{r}-1)\phi_2\right]$$

- **Firms and Price-Setting Behavior**

The production function $Y_t(j) = z_t N_t^{1-\alpha}(j)$ describes the technology for firm j. The variables $Y_t(j)$ and $N_t(j)$ represent output and work-hours hired from households. The technology shock is z_t and the parameter $(1-\alpha)$ measures the elasticity of output with respect to hours worked. The aggregate output is given by $Y_t = \left(\int_0^1 Y_t^{\frac{\epsilon-1}{\epsilon}}(j)dj\right)^{\frac{\epsilon}{\epsilon-1}}$, where ϵ is the elasticity of substitution. The price charged by firm j is $P_t(j)$ and the aggregate price level is P_t.

Real marginal costs for firm j are $MC_t^r(j) = \frac{z_t^{\frac{1}{\alpha-1}}}{1-\alpha}\frac{W_t}{P_t}Y_t^{\frac{\alpha}{1-\alpha}}(j)$ and aggregate real marginal costs are $MC_t = \frac{z_t^{\frac{1}{\alpha-1}}}{1-\alpha}\frac{W_t}{P_t}Y_t^{\frac{\alpha}{1-\alpha}}$.

Using the demand for $Y_t(j)$, given by $Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$, one gets the following expression involving $MC_t^r(j)$ and MC_t:

$$MC_t^r(j) = MC_t\left(\frac{P_t(j)}{P_t}\right)^{-\frac{\alpha\epsilon}{1-\alpha}}$$

Firms operate in a monopolistic competitive market and set prices in a staggered fashion using the scheme proposed by Calvo (1983). According to Calvo (1983), only a fraction of firms, given by $(1-\theta)$, is able to adjust prices. Therefore, each period, these firms reset their prices to maximize expected profits.

Following, Christiano, Eichenbaum & Evans (2005), I introduce an indexation mechanism in which firms that do not set prices optimally at time t will adjust their prices to lagged inflation, according to the equation $P_{t+\tau}(j) = P_{t+\tau-1}(j)(\pi_{t+\tau-1})^{\kappa}$, where the parameter κ indicates the degree of price indexation and π_t denotes inflation. This framework for price-setting behavior leads to a hybrid specification for inflation dynamics. Thus, inflation is a forward-looking variable, but some backward-looking component is necessary to describe inflation dynamics.

When the Calvo mechanism allows a firm to adjust its price, it chooses the new price P_t^* to maximize expected future profits. Hence, the price-setting problem is the following:

$$\underset{P_t^*}{Max}\ E_t \sum_{\tau=0}^{\infty} (\beta\theta)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left[\left(\frac{P_t^*}{P_{t+\tau}} \Pi_{t-1,t+\tau-1}^{\kappa} - MC_{t+\tau}^{r}(j) \right) Y_{t+\tau}(j) \right]$$

The variable $\beta^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t}$ is the stochastic discount factor and $\Pi_{t-1,t+\tau-1}$ is the accumulated inflation rate between $t-1$ and $t+\tau-1$.

Using the relationship between $MC_t^r(j)$ and MC_t, the price-setting problem becomes:

$$\underset{P_t^*}{Max}\ E_t \sum_{\tau=0}^{\infty} (\beta\theta)^{\tau} \frac{\lambda_{t+\tau}}{\lambda_t} \left\{ \begin{array}{c} \left[\frac{P_t^*}{P_{t+\tau}} \Pi_{t-1,t+\tau-1}^{\kappa} - MC_{t+\tau} \left(\frac{P_t^*}{P_{t+\tau}} \Pi_{t-1,t+\tau-1}^{\kappa} \right)^{-\frac{\alpha\epsilon}{1-\alpha}} \right] \\ \left(\frac{P_t^*}{P_{t+\tau}} \Pi_{t-1,t+\tau-1}^{\kappa} \right)^{-\epsilon} Y_{t+\tau} \end{array} \right\}$$

The first-order condition leads to the following equation:

$$\left(\frac{P_t^*}{P_t}\right)^{1+\frac{\alpha\epsilon}{1-\alpha}} = \frac{\epsilon}{\epsilon-1}\frac{E_t\sum_{\tau=0}^{\infty}(\beta\theta)^{\tau}\,\frac{\lambda_{t+\tau}}{\lambda_t}MC_{t+\tau}\left(\frac{\Pi^{\kappa}_{t-1,t+\tau-1}}{\Pi_{t,t+\tau}}\right)^{-\frac{\epsilon}{1-\alpha}}Y_{t+\tau}}{E_t\sum_{\tau=0}^{\infty}(\beta\theta)^{\tau}\,\frac{\lambda_{t+\tau}}{\lambda_t}\left(\frac{\Pi^{\kappa}_{t-1,t+\tau-1}}{\Pi_{t,t+\tau}}\right)^{1-\epsilon}Y_{t+\tau}}$$

Next, I define $p_t^* = \frac{P_t^*}{P_t}$ and use the auxiliary variables X_{1t} and X_{2t} to write the previous expression in its recursive formulation below.

$$(p_t^*)^{1+\frac{\alpha\epsilon}{1-\alpha}} = \frac{\epsilon}{\epsilon-1}\frac{X_{1t}}{X_{2t}}$$

$$X_{1t} = \lambda_t MC_t Y_t + \beta\theta\Pi_t^{-\frac{\kappa\epsilon}{1-\alpha}}E_t\Pi_{t+1}^{\frac{\epsilon}{1-\alpha}}X_{1t+1}$$

$$X_{2t} = \lambda_t Y_t + \beta\theta\Pi_t^{\kappa(1-\epsilon)}E_t\Pi_{t+1}^{\epsilon-1}X_{2t+1}$$

The aggregate price level P_t evolves as follows:

$$P_t = \left[\theta\left(P_{t-1}(\pi_{t-1})^{\kappa}\right)^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

The last four equations above characterize the non-linear Phillips curve.

The log-linear versions of the four previous equations are:

$$\left(1+\frac{\alpha\epsilon}{1-\alpha}\right)\widehat{p}_t^* = \widehat{x}_{1t} - \widehat{x}_{2t}$$

$$\widehat{x}_{1t} = (1-\beta\theta)(\widehat{\lambda}_t + \widehat{mc}_t + \widehat{y}_t) + \beta\theta E_t\left[\widehat{x}_{1t+1} + \frac{\epsilon}{1-\alpha}\left(\widehat{\pi}_{t+1} - \kappa\widehat{\pi}_t\right)\right]$$

$$\widehat{x}_{2t} = (1-\beta\theta)(\widehat{\lambda}_t + \widehat{y}_t) + \beta\theta E_t\left[\widehat{x}_{2t+1} + (\epsilon - 1)\left(\widehat{\pi}_{t+1} - \kappa\widehat{\pi}_t\right)\right]$$

$$\widehat{p}_t^* = \frac{\theta}{1-\theta}\left(\widehat{\pi}_t - \kappa\widehat{\pi}_{t-1}\right)$$

The combination of the four expressions above leads to equation (2), which is the new Keynesian Phillips curve.

To derive equation (3) of the main text, I log-linearize the expression defining aggregate real marginal costs, which is $MC_t = \frac{z_t^{\frac{1}{\alpha-1}}}{1-\alpha}\frac{W_t}{P_t}Y_t^{\frac{\alpha}{1-\alpha}}$.

In households' problem, the first expression in the set of first-order conditions is $a_t N_t^{\varphi} = \lambda_t \frac{W_t}{P_t}$ and, according to the production function, $N_t = \left(\frac{Y_t}{z_t}\right)^{\frac{1}{1-\alpha}}$. These two equations lead to the following expression for real wages:

$$\frac{W_t}{P_t} = a_t\left(\frac{1}{\lambda_t}\right)\left(\frac{Y_t}{z_t}\right)^{\frac{\varphi}{1-\alpha}}$$

Substituting the equation for $\frac{W_t}{P_t}$ in the formula for MC_t, the following alternative definition of real marginal costs obtains:

$$MC_t = a_t\frac{z_t^{\frac{1}{\alpha-1}}}{1-\alpha}\left(\frac{1}{\lambda_t}\right)\left(\frac{Y_t}{z_t}\right)^{\frac{\varphi}{1-\alpha}} Y_t^{\frac{\alpha}{1-\alpha}}$$

Log-linearizing the equation above yields the expression:

$$\widehat{mc}_t = \chi\widehat{y}_t - \widehat{\lambda}_t + \widehat{a}_t - (1+\chi)\widehat{z}_t$$

where $\chi = \frac{\varphi+\alpha}{1-\alpha}$.

To finally arrive at equation (3), I use the log-linearized version of the second first-order condition in households' problem to substitute out the variable $\widehat{\lambda}_t$.

- **The log-linear flexible-price equilibrium**

Since the price-markup is constant under flexible prices, $\widehat{mc}_t = 0$ in the flexible-price equilibrium. In addition, there is no new Keynesian Phillips curve due to instantaneous price adjustments, implying $\widehat{\pi}_t = 0$ for all t. The symbols $\widehat{y}_t^n$, $\widehat{r}_t^n$ and $\widehat{m}_t^n$ denote output, the interest rate and real money balances in the flexible-price equilibrium. The equations describing this equilibrium are:

$$\begin{aligned}(\chi + \phi_2)\widehat{y}_t^n &= \phi_1 \widehat{y}_{t-1}^n + \beta\phi_1 E_t \widehat{y}_{t+1}^n + \frac{\psi_2}{\psi_1}\left(\frac{1}{1-\beta h}\right)\left[\widehat{m}_t^n - \beta h E_t \widehat{m}_{t+1}^n\right] \\ &\quad -\frac{\psi_2}{\psi_1}\left(\frac{1-\beta h\rho_e}{1-\beta h}\right)\widehat{e}_t + \left(\frac{\beta h}{1-\beta h}\right)(1-\rho_a)\,\widehat{a}_t + (1+\chi)\widehat{z}_t\end{aligned}$$

$$\begin{aligned}\widehat{r}_t^n &= -(\phi_1+\phi_2)\widehat{y}_t^n + \phi_1\widehat{y}_{t-1}^n + (\beta\phi_1+\phi_2)\,E_t\widehat{y}_{t+1}^n - \beta\phi_1 E_t\widehat{y}_{t+2}^n \\ &\quad + \frac{\psi_2}{\psi_1}\left(\frac{1}{1-\beta h}\right)\widehat{m}_t^n - \frac{\psi_2}{\psi_1}\left(\frac{1}{1-\beta h}\right)\left[(1+\beta h)\,E_t\widehat{m}_{t+1}^n - \beta h E_t\widehat{m}_{t+2}^n\right] \\ &\quad -\frac{\psi_2}{\psi_1}\left(\frac{1-\beta h\rho_e}{1-\beta h}\right)(1-\rho_e)\,\widehat{e}_t + \left(\frac{1-\beta h\rho_a}{1-\beta h}\right)(1-\rho_a)\,\widehat{a}_t\end{aligned}$$

$$\begin{aligned}\left[1+\delta_0(1+\beta)\right]\widehat{m}_t^n &= \gamma_1\widehat{y}_t^n - \gamma_2\widehat{r}_t^n + \left[\gamma_2(\overline{r}-1)(h\phi_2-\phi_1) - h\gamma_1\right]\widehat{y}_{t-1}^n \\ &\quad - \left[\gamma_2(\overline{r}-1)\beta\phi_1\right]E_t\widehat{y}_{t+1}^n + \delta_0\widehat{m}_{t-1}^n \\ &\quad + \left[\frac{\psi_2}{\psi_1}\left(\frac{\beta h\gamma_2(\overline{r}-1)}{1-\beta h}\right) + \delta_0\beta\right]E_t\widehat{m}_{t+1}^n \\ &\quad - \left(\frac{\beta h\gamma_2(\overline{r}-1)}{1-\beta h}\right)(1-\rho_a)\,\widehat{a}_t \\ &\quad + \left[1 - \gamma_2(\overline{r}-1)\left(\frac{\psi_2}{\psi_1}\left(\frac{\beta h\rho_e}{1-\beta h}\right)+1\right)\right]\widehat{e}_t\end{aligned}$$

Tables and Figures

Table 1. Models with Optimal Policies

Parameters	Prior Shape (mean, std.dev.)	Posterior Distribution Mean [90% Interval] No Money	Money – PA only	Money – PA and CB
ψ_1	Gamma	0.3525	0.4379	0.4213
	(0.8, 0.1)	[0.3177,0.3867]	[0.3567, 0.5132]	[0.3432, 0.4937]
ψ_2	Normal	0	-0.0625	-0.0650
	(0.1, 0.1)	(calibrated)	[-0.0931, -0.0274]	[-0.0955, -0.0332]
h	Beta	0.9871	0.9885	0.9862
	(0.7,0.1)	[0.9802, 0.9945]	[0.9817, 0.9960]	[0.9783, 0.9948]
θ	Beta	0.8785	0.9622	0.9442
	(0.65, 0.1)	[0.8581, 0.8997]	[0.9431, 0.9796]	[0.9230, 0.9710]
κ	Beta	0.9386	0.6862	0.7378
	(0.5, 0.1)	[0.9237, 0.9529]	[0.5640, 0.8194]	[0.6235, 0.8540]
φ	Gamma	1.5291	0.9251	0.8367
	(1, 0.2)	[1.1397, 1.9220]	[0.6192, 1.2346]	[0.5402, 1.1140]
γ_1	Gamma	0.4940	0.4842	0.4543
	(0.5, 0.1)	[0.3310, 0.6486]	[0.3206, 0.6470]	[0.3032, 0.5963]
γ_2	Gamma	0.1591	0.3883	0.4380
	(0.2, 0.1)	[0.0396, 0.2707]	[0.2039, 0.5673]	[0.2185, 0.6392]
δ_0	Gamma	0	3.8140	3.8207
	(3.5, 0.2)	(calibrated)	[3.4838, 4.1600]	[3.4649, 4.1550]
q_y	Gamma	0.0071	0.0101	0.0212
	(0.5, 0.4)	[0.0001, 0.0157]	[0.0017, 0.0190]	[0.0027, 0.0410]
q_r	Gamma	5.5817	2.4070	1.2548
	(0.5, 0.4)	[4.1767, 7.0083]	[0.7411, 3.8239]	[0.2062, 2.5515]
q_μ	Gamma	0	0	0.0999
	(0.5, 0.4)	(calibrated)	(calibrated)	[0.0214, 0.1724]
ρ_a	Beta	0.3067	0.7445	0.6528
	(0.5, 0.2)	[0.1958, 0.4176]	[0.6222, 0.8792]	[0.4976, 0.8265]
ρ_e	Beta	0.3163	0.4224	0.4690
	(0.5, 0.2)	[0.1095, 0.5217]	[0.3496, 0.4987]	[0.3841, 0.5565]
ρ_z	Beta	0.5613	0.7829	0.7037
	(0.5, 0.2)	[0.2872, 0.8413]	[0.5283, 0.9839]	[0.4183, 0.9653]
σ_a	Inverse Gamma	0.0186	0.0298	0.0260
	(0.1, 2)	[0.0151, 0.0223]	[0.0189, 0.0400]	[0.0175, 0.0342]
σ_e	Inverse Gamma	0.0404	0.0837	0.0767
	(0.1, 2)	[0.0279, 0.0531]	[0.0698, 0.0977]	[0.0615, 0.0922]
σ_z	Inverse Gamma	0.0984	0.4004	0.3178
	(0.1, 2)	[0.0348, 0.1602]	[0.0373, 0.8373]	[0.0323, 0.6211]
σ_r	Inverse Gamma	0.0126	0.0121	0.0121
	(0.1, 2)	[0.0118, 0.0135]	[0.0118, 0.0125]	[0.0118, 0.0126]

Table 2. Models with Taylor Rules

Parameters	Prior Shape (mean, std.dev.)	Posterior Distribution Mean [90% Interval] No Money	Money – PA only	Money – PA and CB
ψ_1	Gamma (0.8, 0.1)	0.4117 [0.3390, 0.4796]	0.4201 [0.3457, 0.4908]	0.4050 [0.3330, 0.4705]
ψ_2	Normal (0.1, 0.1)	0 (calibrated)	0.0332 [0.0041, 0.0657]	-0.0312 [-0.0747, 0.0131]
h	Beta (0.7,0.1)	0.9603 [0.9393, 0.9824]	0.9570 [0.9345, 0.9797]	0.9542 [0.9303, 0.9794]
θ	Beta (0.65, 0.1)	0.8670 [0.8204, 0.9166]	0.8676 [0.8232, 0.9126]	0.8472 [0.7927, 0.9033]
κ	Beta (0.5, 0.1)	0.6607 [0.5520, 0.7710]	0.6167 [0.4985, 0.7328]	0.5828 [0.4637, 0.7037]
φ	Gamma (1, 0.2)	1.0149 [0.6874, 1.3322]	1.0108 [0.6868, 1.3250]	0.9715 [0.6485, 1.2825]
γ_1	Gamma (0.5, 0.1)	0.4895 [0.3273, 0.6429]	0.4966 [0.3354, 0.6578]	0.4975 [0.3329, 0.6551]
γ_2	Gamma (0.2, 0.1)	0.1899 [0.0444, 0.3280]	0.1691 [0.0392, 0.2925]	0.3160 [0.0711, 0.5604]
δ_0	Gamma (3.5, 0.2)	0 (calibrated)	3.8668 [3.5177, 4.2088]	3.8673 [3.5248, 4.2191]
ρ_r	Beta (0.7, 0.2)	0.5273 [0.3806, 0.6773]	0.5248 [0.3802, 0.6731]	0.5393 [0.3872, 0.6896]
ρ_y	Gamma (0.5, 0.2)	0.2222 [0.0580, 0.3810]	0.2113 [0.0607, 0.3644]	0.3334 [0.0965, 0.5593]
ρ_π	Gamma (1.5, 0.2)	1.4505 [1.1305, 1.7599]	1.4621 [1.1459, 1.7771]	1.4816 [1.1567, 1.8000]
ρ_μ	Gamma (0.5, 0.2)	0 (calibrated)	0 (calibrated)	0.6506 [0.2907, 1.0012]
ρ_a	Beta (0.5, 0.2)	0.4864 [0.3307, 0.6380]	0.5162 [0.3716, 0.6599]	0.4342 [0.2781, 0.5922]
ρ_e	Beta (0.5, 0.2)	0.3153 [0.1108, 0.5225]	0.5580 [0.4094, 0.7367]	0.5955 [0.4048, 0.8006]
ρ_z	Beta (0.5, 0.2)	0.4943 [0.2036, 0.7798]	0.4916 [0.2030, 0.7796]	0.4400 [0.1565, 0.7200]
σ_a	Inverse Gamma (0.1, 2)	0.0217 [0.0159, 0.0271]	0.0226 [0.0164, 0.0284]	0.0212 [0.0158, 0.0265]
σ_e	Inverse Gamma (0.1, 2)	0.0406 [0.0275, 0.0530]	0.0669 [0.0392, 0.0911]	0.0604 [0.0304, 0.0895]
σ_z	Inverse Gamma (0.1, 2)	0.0766 [0.0281, 0.1261]	0.0748 [0.0287, 0.1221]	0.0625 [0.0260, 0.1002]
σ_r	Inverse Gamma (0.1, 2)	0.0120 [0.0118, 0.0124]	0.0120 [0.0118, 0.0124]	0.0120 [0.0118, 0.0124]

Table 3. Model Comparison

Models	Marginal Likelihood	$log_{10}(Bayes\ Factor)$
Optimal Policy - No Money	1096.852355	0
Optimal Policy - PA Only	1252.842623	67.7457
Optimal Policy - PA and CB	1252.527630	67.6089
Taylor Rule - No Money	1192.358981	41.4780
Taylor Rule - PA Only	1276.607390	78.0666
Taylor Rule - PA and CB	1280,675023	79.8331

Note: PA stands for private agents and CB denotes Central Bank

Table 4. Volatility and Persistence

Volatility (%)	Taylor Rules		Optimal Policies		Data
and Persistence	No Money	Money(Best Fit)	No Money	Money(Best Fit)	
$\sigma(y-y^f)$	1.5500	1.0700	0.2500	4.8400	0.9001
$\sigma(\pi)$	0.2800	0.2500	0.1100	0.3100	0.2409
$\sigma(r)$	1.4500	1.5700	0.0600	0.1700	0.5688
$\rho(y-y^f)$	0.6472	0.5873	0.6033	0.4771	0.8667
$\rho(\pi)$	0.5381	0.4596	0.6194	0.6593	0.5561
$\rho(r)$	0.5365	0.4627	0.9881	0.9871	0.9588

Note: In each cell, posterior median. σ(.) denotes standard deviation and ρ(.) denotes the first autocorrelation

Table 5. Posteriors for the weights in the loss function under alternative priors

Parameters	Loose Prior	Tight Prior	Weight-specific Prior
q_y	0.0093 [0.0011, 0.0179]	0.1740 [0.0393, 0.3145]	0.0072 [0.0008, 0.0141]
q_r	3.6446 [1.0466, 6.2030]	0.7733 [0.4324, 1.1155]	2.3852 [0.7210, 3.8398]
q_μ	0.0415 [0.0001, 0.0932]	0.2837 [0.1444, 0.4218]	0.0427 [0.0001, 0.0960]

Table 6. Comparing Taylor Rules

Models	Marginal Likelihood	$log_{10}(Bayes\ Factor)$
Standard Taylor Rule	1270.388252	0
Taylor rule with smoothing	1276.607390	2.7009
General Taylor Rule	1280.675023	4.4674

Table 7. Optimal Policy: Estimation based on an artificial data set

Parameters	Prior Shape (mean, std.dev.)	True Parameters from the best Taylor Rule Model (posterior mean)	Posterior Distribution Mean [90% Interval]	
			Estimation using an artificial data set	Baseline Estimation in Table 1 (PA and CB)
ψ_1	Gamma (0.8, 0.1)	0.4050	0.3613 [0.3178, 0.3976]	0.4213 [0.3432, 0.4937]
ψ_2	Normal (0.1, 0.1)	-0.0312	-0.0348 [-0.0399, -0.0298]	-0.0650 [-0.0955, -0.0332]
h	Beta (0.7,0.1)	0.9542	0.9798 [0.9737, 0.9863]	0.9862 [0.9783, 0.9948]
θ	Beta (0.65, 0.1)	0.8472	0.8861 [0.8752, 0.8978]	0.9442 [0.9230, 0.9710]
κ	Beta (0.5, 0.1)	0.5828	0.5417 [0.5141, 0.5696]	0.7378 [0.6235, 0.8540]
φ	Gamma (1, 0.2)	0.9715	0.4653 [0.3300, 0.5998]	0.8367 [0.5402, 1.1140]
γ_1	Gamma (0.5, 0.1)	0.4975	0.3066 [0.2099, 0.3991]	0.4543 [0.3032, 0.5963]
γ_2	Gamma (0.2, 0.1)	0.3160	0.8694 [0.7256, 1.0172]	0.4380 [0.2185, 0.6392]
δ_0	Gamma (3.5, 0.2)	3.8673	3.2695 [3.0430, 3.5186]	3.8207 [3.4649, 4.1550]
q_y	Gamma (0.5, 0.4)	-	0.1098 [0.0666, 0.1506]	0.0212 [0.0027, 0.0410]
q_r	Gamma (0.5, 0.4)	-	0.9430 [0.7140, 1.1598]	1.2548 [0.2062, 2.5515]
q_μ	Gamma (0.5, 0.4)	-	0.2615 [0.2056, 0.3170]	0.0999 [0.0214, 0.1724]
ρ_a	Beta (0.5, 0.2)	0.4342	0.3121 [0.2684, 0.3551]	0.6528 [0.4976, 0.8265]
ρ_ε	Beta (0.5, 0.2)	0.5955	0.4130 [0.3707, 0.4563]	0.4690 [0.3841, 0.5565]
ρ_z	Beta (0.5, 0.2)	0.4400	0.5363 [0.2523, 0.8185]	0.7037 [0.4183, 0.9653]
σ_a	Inverse Gamma (0.1, 2)	0.0212	0.0211 [0.0173, 0.0250]	0.0260 [0.0175, 0.0342]
σ_ε	Inverse Gamma (0.1, 2)	0.0604	0.0769 [0.0709, 0.0832]	0.0767 [0.0615, 0.0922]
σ_z	Inverse Gamma (0.1, 2)	0.0625	0.1209 [0.0461, 0.1946]	0.3178 [0.0323, 0.6211]
σ_r	Inverse Gamma (0.1, 2)	0.0120	0.0151 [0.0149, 0.0154]	0.0121 [0.0118, 0.0126]

Figure 1: Estimated Parameters under Optimal Policy 1 - Prior and Posterior

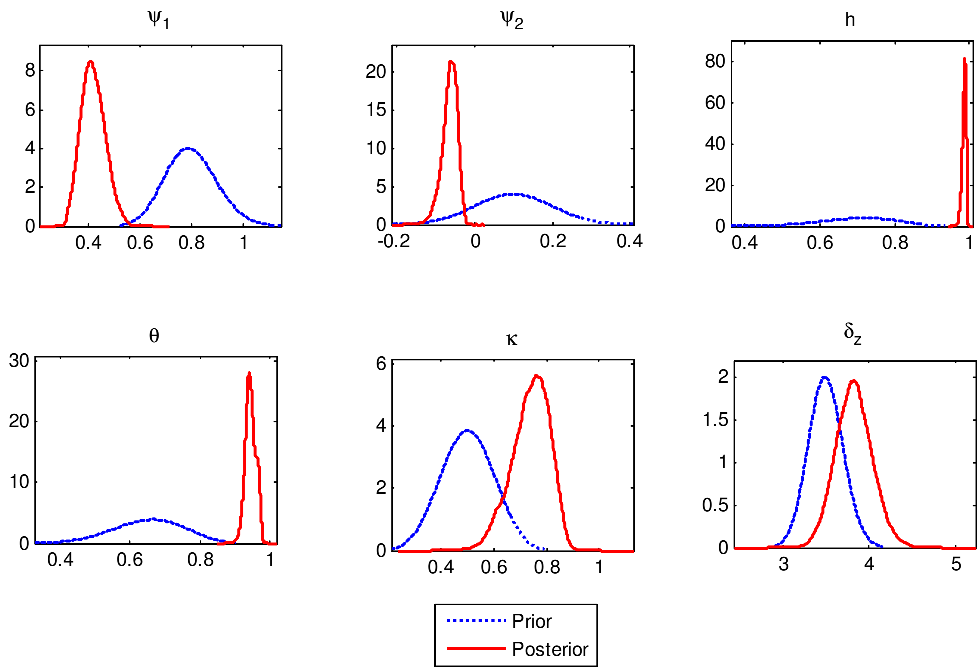

Figure 2: Estimated Parameters under Optimal Policy 2 - Prior and Posterior

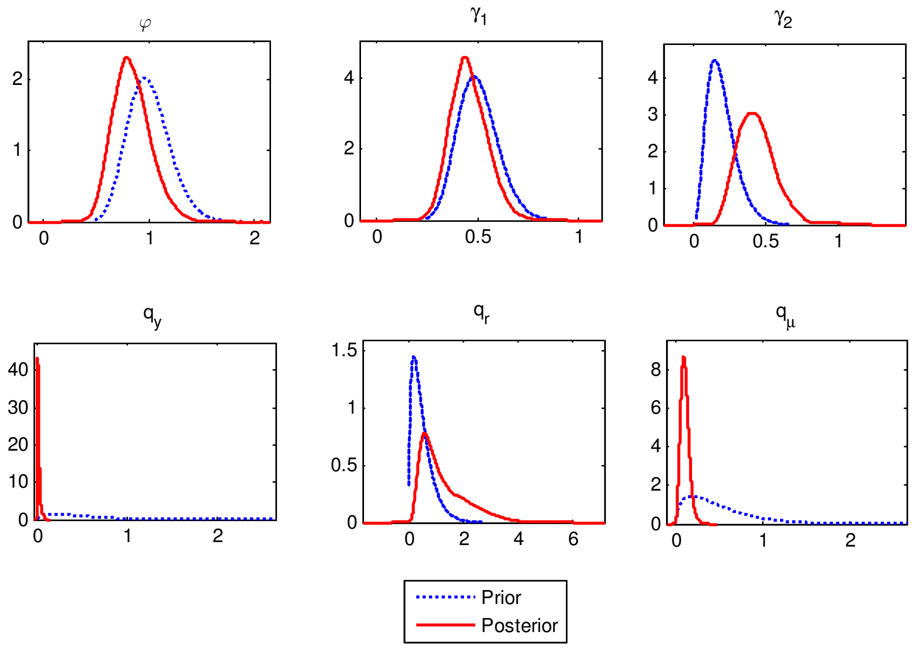

Figure 3: Estimated Parameters under Optimal Policy 3 - Prior and Posterior

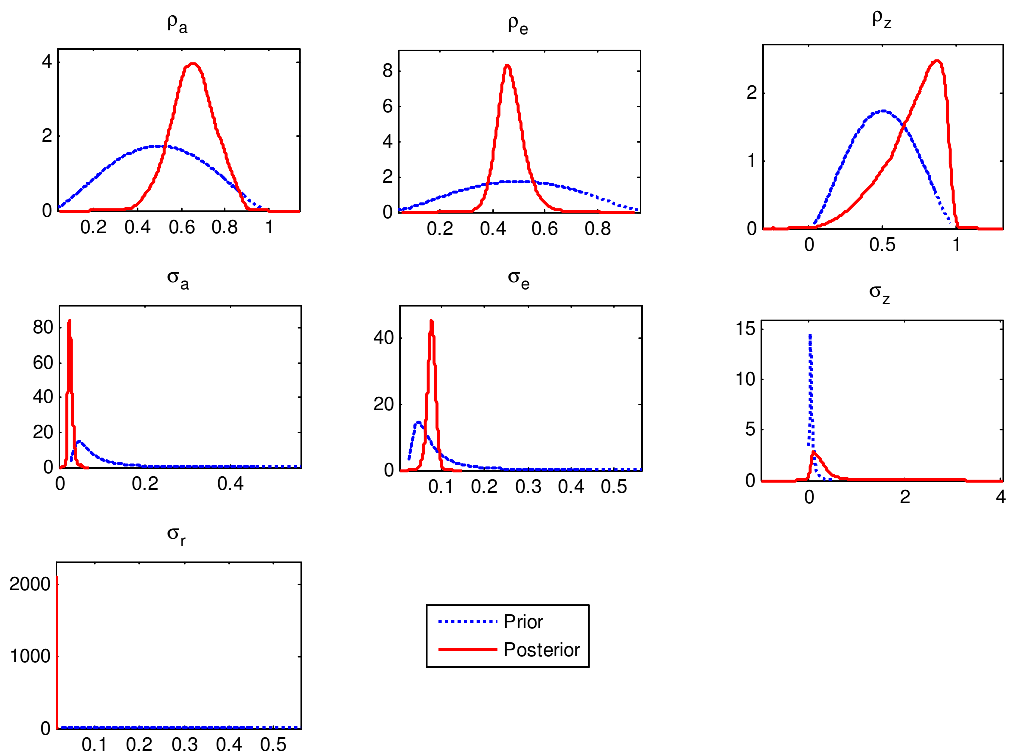

Figure 4: Impulse Responses to preference shocks e_a

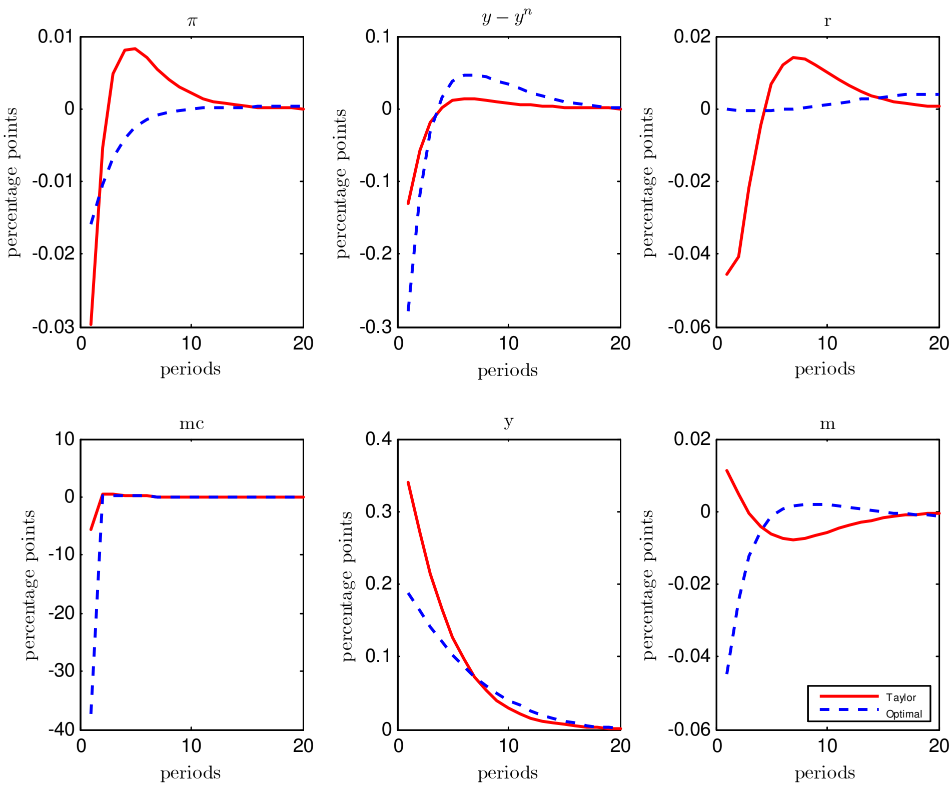

Figure 5: Impulse Responses to money demand shocks e_e

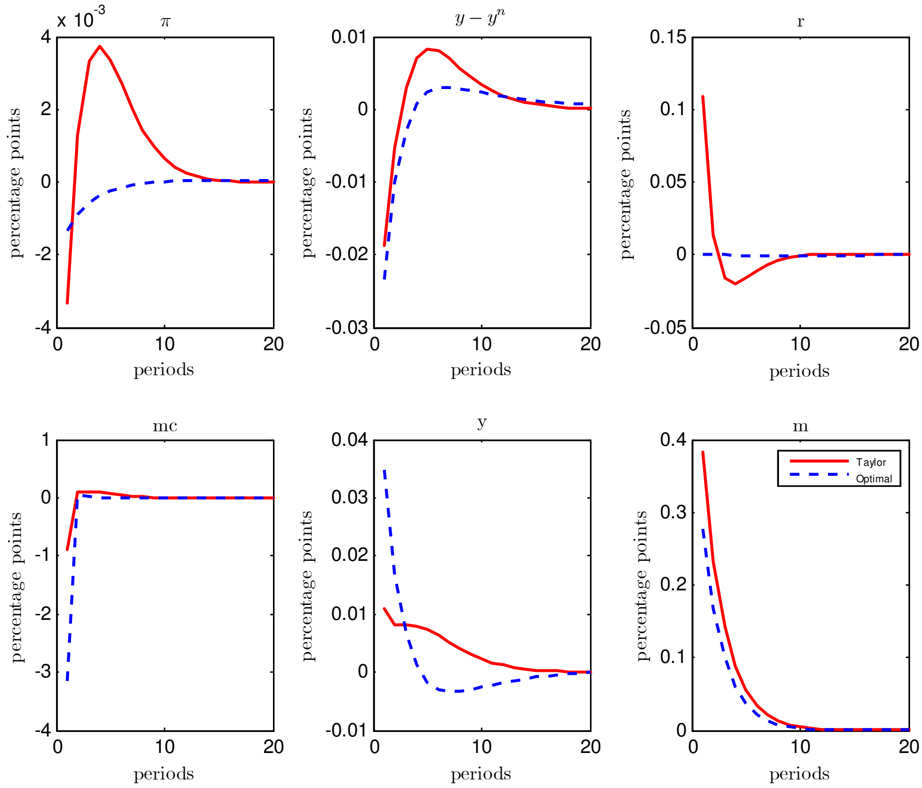

Figure 6: Impulse Responses to technology shocks e_z

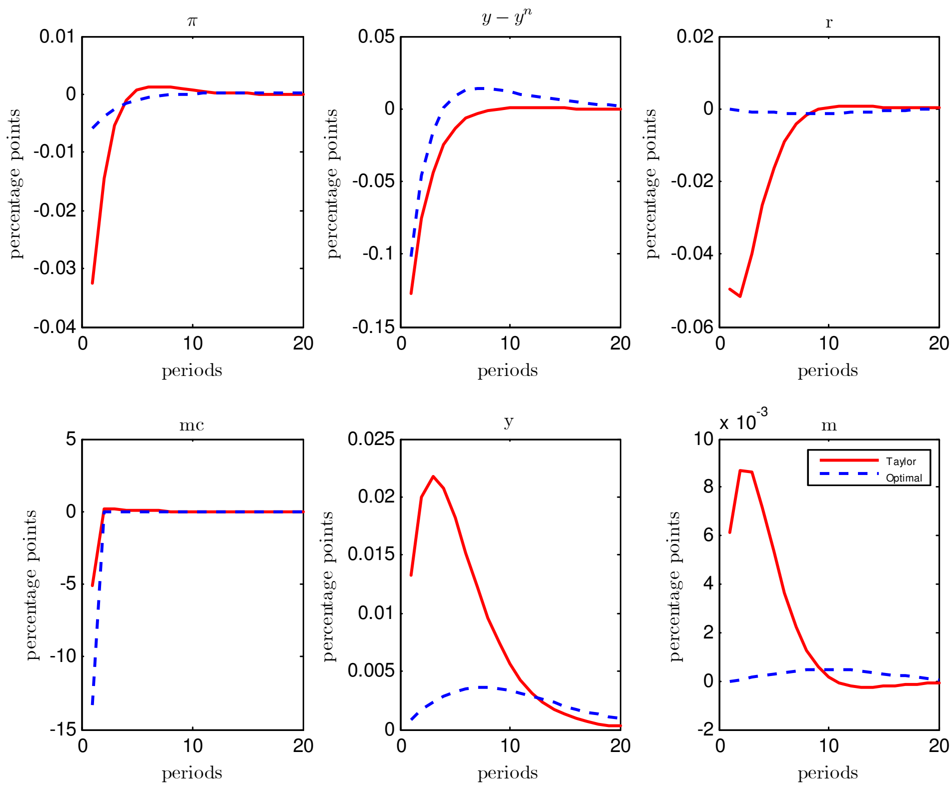

Figure 7: Impulse Responses to monetary policy shocks e_r

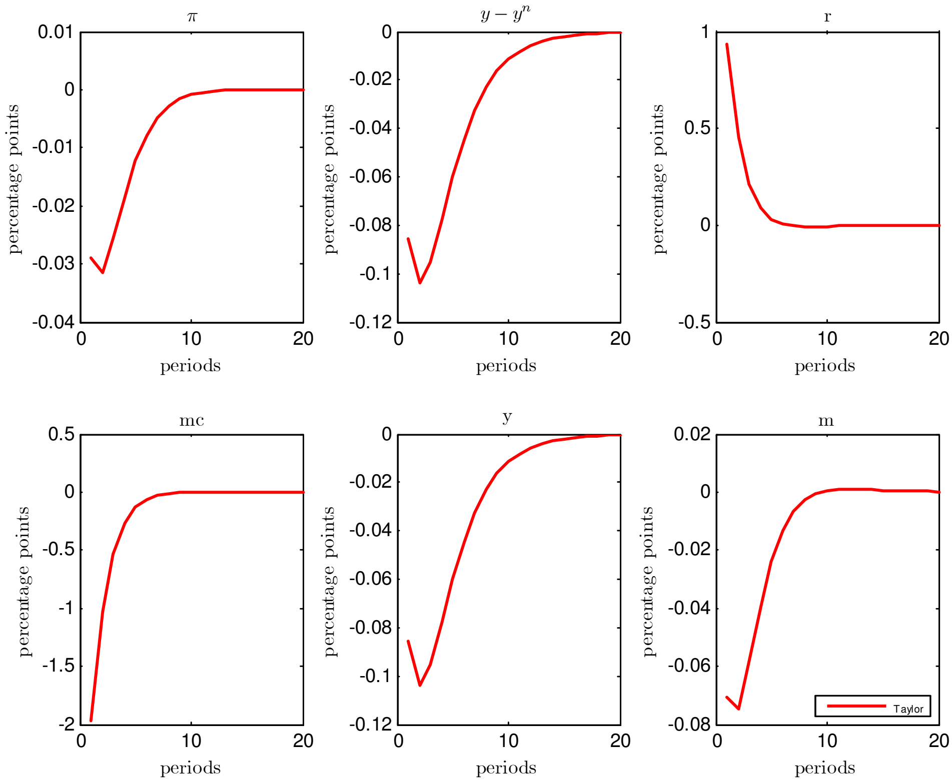

Figure 8: Parameters in the Loss Function under alternative priors