External Shocks, Financial Volatility and Reserve Requirements in an Open Economy

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Abstract

The performance of a countercyclical reserve requirement rule is studied in a dynamic stochastic model of a small open economy with financial frictions, imperfect capital mobility, a managed float regime, and sterilized foreign exchange market intervention. Bank funding sources, domestic and foreign, are imperfect substitutes. The model is calibrated and used to study the effects of a temporary drop in the world risk-free interest rate. Consistent with stylized facts, the shock triggers an expansion in domestic credit and activity, asset price pressures, and a real appreciation. A credit-based reserve requirement rule helps to mitigate both macroeconomic and financial volatility, with the latter defined both in terms of a narrow measure based on the credit-to-output ratio, the ratio of capital flows to output, and interest rate spreads, and a broader measure that includes real asset prices as well. An optimal rule, based on minimizing a composite loss function, is also derived. Sensitivity tests, related to the intensity of sterilization, the degree of exchange rate smoothing, and the rule used by the central bank to set the cost of bank borrowing, are also performed, both in terms of the transmission process and the optimal rule.

JEL Classification Numbers: F41, H41, H54.

*University of Manchester, United Kingdom, and Centre for Growth and Business Cycle Research; **Central Bank of Turkey; ***Central Bank of Brazil. We are grateful to Alessandro Flamini, André Minella, Peter Montiel, and Fabia de Carvalho for helpful discussions and comments, and to Pengfei Jia for excellent research assistance. Appendices A to D are available upon request. The views expressed in this paper are our own.
1 Introduction

A key lesson of the global financial crisis is the importance, for containing systemic risks and preserve financial and economic stability, of going beyond a microprudential approach, focused solely on the regulation of individual institutions. The greater focus on systemic risk has stirred up a broad debate in academic and policy circles on how macroprudential regulation can prevent the build-up of asset price bubbles and unsustainable credit booms. Even though no consensus has yet emerged on what instruments are most appropriate under which circumstances, some of them (such as dynamic provisions, countercyclical capital requirements, leverage and liquidity ratios) have already been made part of the new Basel III regime for banking regulation (see Basel Committee on Banking Supervision (2011, 2013)). Indeed, a key instrument in the global framework for liquidity regulation introduced by Basel III is a minimum standard for managing liquidity risk: the liquidity coverage ratio (LCR), which requires each bank to hold a sufficient quantity of highly-liquid assets to survive a 30-day period of market stress. Its implementation began in January 2015.\footnote{Basel III also introduced another minimum standard for managing liquidity risk, the net stable funding ratio (NSFR), which is viewed as complementary to the LCR. The NSFR focuses on a one-year time horizon and establishes a minimum amount of stable funding each bank must obtain based on the liquidity characteristics of its assets and activities. Its implementation is scheduled to begin in January 2018. See Dietrich et al. (2014) for a discussion.}

Among these instruments figures a tool that has been used extensively, reserve requirements, which can be thought of a Basel III liquidity requirement. Among middle-income countries, Brazil and Turkey have been actively using reserve requirements. Both countries lowered rates during the Lehman crisis and increased them again in the period of large capital inflows that emerged in 2010 to mid-2011. The Central Bank of Brazil also used average reserve requirements as a mechanism to prevent disruptions in the interbank market following the Lehman Brothers’ episode. Specifically, average reserve requirements imposed on large and liquid banks were lowered if they extended credit to small and illiquid banks.\footnote{As discussed by Robitaille (2011), in Brazil, despite the country’s high reserve ratios, reserve requirements have been seen as an important tool for managing liquidity risk. See also Mimir et al. (2012) for Turkey.} Other Latin American countries, such as Colombia and Peru, have also used this instrument aggressively over the past years. Prior to the global crisis the central banks of both countries managed reserve requirements as a prudential tool to contain pressures on credit growth emanating from large capital inflows. In this context both central banks raised marginal reserve requirements during this period. In the case of Peru, marginal reserve requirements were also imposed on foreign-currency deposits. More recently, as capital inflows to Latin America surged, the Central Bank of Peru actively raised marginal reserve requirements again. More generally, there is evidence suggesting that central banks in middle-income countries (MICs) have often raised reserve requirements in response to capital inflows (Hoffmann and Löffler (2014)).
This paper contributes to the debate on the role of reserve requirements in several ways. It extends the model in Agénor et al. (2014) to account for several important financial and policy features of MICs: a managed float; sterilized foreign exchange market intervention; imperfect substitutability between deposits and central bank borrowing as sources of funding for commercial banks. The first extension is consistent with the evidence that suggests that many MICs operate a managed float regime, in which the central bank decides whether to use the foreign exchange associated with capital inflows—transacted or not through the central bank—to accumulate foreign reserves, and if so whether such accumulation should be sterilized.

Accordingly, our analysis accounts for the fact, as the central bank intervenes, it partially sterilizes the impact of its intervention on money supply through offsetting changes in its holdings of government bonds. This emphasis is consistent with the evidence that suggests that sterilized interventions are the main instrument used by many emerging market and developing country central banks to affect the exchange rate. While some central banks may have explicit exchange rate objectives in mind when setting interest rates, that is not their main instrument for influencing the exchange rate. As discussed by Chang (2008), Aizenman and Glick (2009), Devereux and Yetman (2014), sterilization activity has played an important role in central bank policy responses to surges in capital inflows. In addition, since the global financial crisis the use of foreign exchange intervention as a way to mitigate excessive volatility has intensified further in many of these countries. Even though the feasibility and effectiveness of sterilization remain in general a matter of debate (see for instance Daude and Levy Yeyati (2014)), a greater weight on mitigating exchange rate volatility may well have been one of the factors inducing central banks to sterilize capital flows more aggressively in recent years. At the same time, however, we also account for the fact that changes in official reserves may be driven by other considerations, namely, concerns about real appreciation, as well as self-insurance motives.

The model also accounts for imperfect substitutability between deposits and central bank liquidity as sources of bank funding. This is captured by assuming that the rate at which banks can borrow from the central bank incorporates a premium (above and beyond a base policy rate, determined through a Taylor rule), which depends on the ratio of existing borrowing to deposits, as a result of a “stigma” effect. Thus, because higher reserve requirements hamper the ability to attract deposits, they also lead (all else equal) to an increase in the cost of central bank liquidity, which in turn affects the cost at which private agents can borrow. As it turns out, this is the key channel through which changes in reserve requirements may prove countercyclical.

The remainder of the paper is organized as follows. Section 2 provides a brief background on the use of reserve requirements in MICs. Section 3 describes the model. As in Agénor et al. (2014), the model features imperfect capital mobility and a two-

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3 More generally, in a study of 136 countries over the period 1974-2003 for instance, Bastourre et al. (2009) found that countries with flexible exchange rate regimes tend to have higher ratios of reserves to output.
level financial intermediation system, which accounts for bank borrowing abroad—an important feature of cross-border capital flows in recent years. In addition, as noted earlier, several novel elements are introduced: exchange rate smoothing, self insurance, sterilized foreign exchange market intervention, and imperfect substitutability between bank borrowing from the central bank and deposits. The equilibrium and some key features of the steady state are discussed in Section 4, and an illustrative calibration is presented in Section 5. The results of our main experiment, a temporary drop in the world safe interest rate, are described in Section 6. As documented in a number of studies, shocks to world interest rates are a key source of domestic macroeconomic fluctuations in middle-income countries (see for instance Edwards (2010)). Sensitivity tests, involving alternative assumptions about the rule whereby the central bank sets the cost of bank borrowing, the degree of exchange rate smoothing, the intensity of sterilization, and endogenous reserve requirements (linked to credit growth and the credit-to-output ratio) are reported in Section 7. Optimal countercyclical reserve requirements are discussed in Section 8. The concluding section discusses the implications of the analysis for the design of macroprudential regimes in middle-income countries and identifies some potentially fruitful directions for future research.

2 Background

As noted by Gray (2011) reserve requirements usually serve three purposes, depending on circumstances: a microprudential function, a monetary control role (to affect market interest rates and monetary aggregates), and a liquidity management function (especially to sterilize excess reserves). The microprudential function relates to protection against liquidity and solvency risks; from that perspective, reserve requirements help to ensure adequate liquidity insurance in the event of funding outflows. This role is particularly important in countries where the lack of development of financial markets leaves an undersupply of effectively liquid assets.

Regarding the monetary control and liquidity management functions, the evidence suggests that in middle-income countries reserve requirements have proved to be particularly useful during episodes of strong capital inflows associated with changes in world interest rates and risk perceptions. As documented in a number of studies, these episodes have often been accompanied by an expansion in credit, an increase in aggregate demand, and high inflationary pressures. In such conditions, although interest rate hikes could restrain inflation, they may also attract more capital, which

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4See Hoggarth et al. (2010), Committee on International Economic Policy and Reform (2012), Herrmann and Mihaljek (2013), and Reinhardt and Riddiough (2014) for a discussion of the importance of cross-border bank flows in international capital movements (especially changes in the external liabilities of resident banks) during the run up to, and the immediate aftermath of, the global financial crisis.

5Ahmed and Zlate (2014) provide evidence that interest rate differentials and global risk aversion are important determinants of net private capital inflows to middle-income countries, especially (for portfolio flows) in the aftermath of the global financial crisis.
in turn can fuel further credit expansion. By contrast, an increase in reserve requirements would induce banks to lower deposit rates. The mechanics are as follows. In response to large capital inflows, central banks often intervene to buy foreign exchange and prevent an appreciation of the exchange rate. Concurrently, to avoid an expansion in the money supply and maintain price stability, they engage not only in open-market operations (sales of government bonds) but also in increases in reserve requirements. In an open economy the incentive to do so is particularly strong if the use of open-market operations to sterilize capital flows is costly, due to large differentials between the interest rate on assets used for these operations and the interest earned on foreign reserves. Because reserve requirements represent a tax on bank intermediation, they drive a wedge between the rate that a bank pays its depositors and its cost of funding. If bank deposits offer special transaction and liquidity services to households, the cost of higher reserve requirements would normally be passed on in full to depositors in the form of lower deposit rates. A similar outcome, albeit with a less than complete pass through, would occur if banks can only partly substitute reservable liabilities with other funding sources as a result for instance of information frictions or a less than perfectly elastic supply of liquidity from the central bank. In either case, the policy may lead to an increase in bank intermediation spreads through lower deposit rates, higher lending rates, or both.

In an early contribution, Reinhart and Reinhart (1999) found indeed that increases in reserve requirements in developing countries tend to raise lending rates and reduce deposit rates, whereas Gelos (2009) found that high reserve requirements are a key determinant of the comparatively high intermediation spreads observed in Latin America. In subsequent studies, Montoro and Moreno (2011), Tovar et al. (2012), Izquierdo et al. (2013), Armas et al. (2014), Cordella et al. (2014), Federico et al. (2014), and Glocker and Towbin (2015), all found that increases in reserve requirements tend to mitigate the expansion of credit in Latin America. In effect, during the recent global financial crisis, reserve requirements were used as a substitute to monetary policy, not only to curb lending growth but also to dampen inflationary pressures.

More recent thinking on bank reserve requirements has focused on their role as a systematic—as opposed to sporadic—countercyclical macroprudential instrument, aimed at mitigating systemic risks to the financial system. By requiring banking institutions to hold a fraction of their deposits (in the form of either cash or deposits at the monetary authority remunerated at below-market rates), mandatory reserves act as an implicit tax on financial intermediation; and by altering the cost of funding, they may be useful to reduce the volatility of credit. Increasing reserve requirements can restrain credit growth during expansions, while reducing them during downturns can

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6 Higher reserve requirements on bank deposits have been used to sterilize the effects of capital inflows not just on the monetary base but also on the broader money supply, as occurred for instance in China, India, and Morocco, among others, in recent years. See for instance Ma et al. (2013) for a discussion of China’s experience.

7 Mora (2014) also found evidence that increases in reserve requirements were contractionary for Lebanon.
provide additional resources to limit credit contractions. Thus, reserve requirements may have a significant impact on the credit channel and the business cycle.\textsuperscript{8}

Recent analytical contributions on the macroprudential role of reserve requirements, in both closed and open economies, include Prada (2008), Bianchi (2011), Montoro (2011), Glocker and Towbin (2012), Kashyap and Stein (2012), Mimir et al. (2012), Alper et al. (2014), Escudero et al. (2014), and Medina and Roldós (2014). An increase in required reserves turns deposits into an expensive source of funding, so that the interest rate on deposits falls. In Prada’s model, this fall leads not only to a drop in the demand for deposits but also in a reduction in credit as well, because deposits and credit are complements. Thus, the policy is countercyclical. Bianchi (2011) showed that, for a generic bank balance sheet, capital and reserve requirements have similar effects and may therefore be thought of \textit{ex ante} (although not \textit{ex post}) as substitutes from a macroprudential perspective. Glocker and Towbin (2012) considered required reserves as an additional policy instrument and variations in loans as an additional target into an open-economy model with nominal rigidities and financial frictions. Their results imply that reserve requirements favor the price stability objective only if financial frictions are nontrivial and are more effective if there is a financial stability objective and debt is denominated in foreign currency. In their model, due to the endogeneity of monetary base, an increase in the reserve requirement rate increases loan-deposit spreads only if the remuneration of reserves is below the market rate. However, because they obtain opposite impact effects on consumption and investment, the overall effect on aggregate demand and inflation is ambiguous. Kashyap and Stein (2012) showed that the central bank can exploit a nonzero and time-varying scarcity value of reserves to tax the negative systemic externality from credit booms.

As discussed in more detail later, our analysis differs in significant ways from existing contributions. In contrast to the contributions mentioned earlier, it accounts for imperfect capital mobility, a two-level financial intermediation system, exchange rate smoothing, and sterilized intervention—all of them important features to understand the transmission of external shocks and policy responses to them. Moreover, as far as we know the rationale that it provides for the effectiveness (or lack thereof) of reserve requirements as a countercyclical instrument has not been highlighted before. In models where monetary policy targets a short-term interest rate, bank funding sources (deposits and central bank liquidity) are perfect substitutes, and there are no stigma effects associated with accessing central bank facilities, an increase in reserve requirements has no impact on lending rates despite making deposits more expensive; banks simply borrow more from the central bank at the prevailing rate. Without a change in the loan rate, it is difficult to generate a countercyclical role for reserve requirements; if anything, the opposite occurs—if intertemporal substitution effects are strong, the drop in the deposit rate tends to reduce savings and to increase cur-

\footnote{In dollarized economies, differentiated reserve requirement rates are also used as instruments to mitigate the risks associated with financial dollarization, most notably by building foreign exchange liquidity buffers. See for instance Armas et al. (2014) for a discussion of the Peruvian case.}
rent consumption—therefore generating a procyclical movement in private spending and thus possibly aggregate demand (see for instance Evandro and Takeda (2013), Agénor and Pereira da Silva (2014), and Escudero et al. (2014)). By contrast, a key feature of our analysis is that deposits and borrowing from the central bank are imperfect substitutes; this is captured by making a distinction between a “base” policy rate (determined through a standard Taylor rule) and the actual cost of borrowing for commercial banks. In turn, the latter incorporates a penalty rate that depends on the ratio of central bank borrowing to deposits, due to a “stigma” effect. Put differently, even though the central bank operates a standing facility, its supply of liquidity is not perfectly elastic at the base policy rate.9 As discussed later on, this assumption is critical to understand the countercyclical role of reserve requirements.

3 The Model

Consider a small open economy populated by six categories of agents: a representative household, intermediate goods-producing (IG) firms, a final good (FG) producer, a capital good (CG) producer, a commercial bank, the government, and the central bank. The country produces a continuum of intermediate goods, which are imperfect substitutes to a continuum of imported intermediate goods. Both categories of goods are aggregated to produce a homogeneous final good. In turn, the final good is consumed by the household and the government, used for investment (subject to additional costs) by the CG producer, or exported. Monopolistic competition prevails in the market for domestic intermediate goods and each intermediate good is produced or imported by a single firm.

3.1 Households

The household consumes the final good, demands housing services, supplies labor, and holds imperfectly substitutable domestic and foreign financial assets. It deposits funds in the bank at the beginning of the period and collects them (with interest) at the end of the period. It owns all domestic firms.

The objective of the representative household is to maximize

\[ U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{C_{t+s}^{1-\varsigma^{-1}}}{1-\varsigma^{-1}} + \eta_N \ln(1 - N_{t+s}) + \ln x_{t+s}^{\eta_2} H_{t+s}^{\eta_H} \right\}, \]

where \( C_t \) is consumption, \( N_t = \int_0^1 N_j^2 dj \), the share of total time endowment (normalized to unity) spent working, with \( N_j^2 \) denoting the number of hours of labor provided to IG producer \( j \), \( x_t \) a composite index of real monetary assets, \( H_t \) the stock of housing, \( \beta \in \)

9 A conceptually similar idea is developed in Alper et al. (2014), although in a very different setting. They also provide evidence for Turkey which suggests that central bank liquidity and household deposits are imperfect substitutes.
(0, 1) the subjective discount factor, $\zeta > 0$ the intertemporal elasticity of substitution in consumption, $E_t$ the expectation operator conditional on the information available at the beginning of period $t$, and $\eta_N, \eta_e, \eta_H > 0$. Housing services are proportional to their stock.

The composite monetary asset is a geometric average of real cash balances, $m^P_t$, and real bank deposits, $d_t$:

$$x_t = (m^P_t)^\nu d_t^{1-\nu}, \quad (2)$$

where $\nu \in (0, 1)$. Both $m^P_t$ and $d_t$ are measured in terms of the price of goods sold on the domestic market, $P^S_t$.

The household’s flow budget constraint is

$$m^P_t + d_t + b^P_t + z_t B^F_t + z^H_t \Delta H_t = w_t N_t - T_t - C_t + \frac{m^P_{t-1}}{1+\pi^t_t} + \frac{(1+i^D_{t-1})}{1+\pi^S_{t-1}} d_{t-1} + \frac{(1+i^B_{t-1})}{1+\pi^F_{t-1}} b^F_{t-1} \quad (3)$$

$$+ (1+i^F_{t-1}) z_t B^F_{t-1} + J^D_t + J^K_t + J^B,$$

where $z_t = E_t/P^S_t$ is the real exchange rate (with $E_t$ the nominal exchange rate), $z^H_t = P^H_t/P^S_t$ the real price of housing (with $P^H_t$ the nominal price), $1+\pi^S_t = P^S_t/P^S_{t-1}$, $H_t$ the stock of housing, $b^P_t$ ($B^F_t$) real (foreign-currency) holdings of one-period, non-contingent domestic (foreign) government bonds, $i^D_t$ the interest rate on bank deposits, $i^B_t$ and $i^F_t$ interest rates on domestic and foreign government bonds, respectively, $w_t$ the economy-wide real wage (measured in terms of the price of final goods sold domestically), $T_t$ real lump-sum taxes, $J^D_t = \int_0^1 (P^D_{jt}/P^S_t) dj$, $J^K_t$, and $J^B_t$, end-of-period profits of the IG producer, the CG producer, and the commercial bank.10 For simplicity, housing does not depreciate and domestic government bonds are held only at home.

The rate of return on foreign bonds is defined as

$$1+i^F_t = (1+i^W_t)(1-\theta^F_t), \quad (4)$$

where $i^W_t$ is the risk-free world interest rate and $\theta^F_t$ an endogenous spread, defined as

$$\theta^F_t = \frac{\theta^F_t}{2} B^F_t, \quad (5)$$

with $\theta^F_t > 0$.

The household maximizes (1) with respect to $C_t$, $N_t$, $m^P_{t+1}$, $d_{t+1}$, $b^F_{t+1}$, $B^F_{t+1}$, and $H_{t+1}$, subject to (3), (4), and (5) taking as given period-$t-1$ variables as well as $w_t$, $T_t$, and real profits. Thus, in contrast to models where the country’s borrowing premium depends on total net foreign indebtedness (as for instance in Gertler et al. (2007)),

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10 The definition of the real exchange rate assumes that the foreign-currency price of goods sold on markets abroad is normalized to unity.
the household internalizes the fact that its holdings of foreign bonds affect the risk premium that it faces on world capital markets. The first-order conditions are

\[ \mathbb{E}_t \left( \frac{C_{t+1}}{C_t} \right) = \beta \mathbb{E}_t \left( \frac{1 + i_t^B}{1 + \pi_{t+1}^B} \right), \]  

\[ N_t = 1 - \frac{\eta_N C_t^{1/\kappa}}{w_t}, \]  

\[ m_t^p = \frac{\eta_p \nu C_t^{1/\kappa} (1 + i_t^B)}{i_t^B}, \]  

\[ d_t = \frac{\eta_d (1 - \nu) C_t^{1/\kappa} (1 + i_t^B)}{i_t^B - i_t^B}, \]  

\[ z_t^H H_t^d = \left( 1 - \mathbb{E}_t \left( \frac{1 + \pi_{t+1}^H}{1 + i_t^B} \right) \right)^{-1} \eta_H C_t^{1/\kappa}, \]  

\[ 1 + i_t^B = (1 - \theta_0^{F,P} B_t^{F,P}) (1 + i_t^W) \mathbb{E}_t \left( \frac{E_{t+1}}{E_t} \right), \]

where \( 1 + \pi_{t+1}^H = P_{t+1}^H / P_t^H \) is the gross inflation rate in terms of nominal house prices between periods \( t \) and \( t + 1 \).

Equation (10) defines the demand for housing services, whereas equation (11) equates the expected marginal rates of return on domestic and foreign assets under the assumption of imperfect world capital markets; it can be rearranged to give the demand for foreign bonds as

\[ B_t^{F,P} = \frac{(1 + i_t^W) \mathbb{E}_t \left( E_{t+1} / E_t \right) - (1 + i_t^B)}{\theta_0^{F,P} (1 + i_t^W) \mathbb{E}_t \left( E_{t+1} / E_t \right)}. \]

which shows that the optimal level of household holdings of foreign bonds is a function of the difference between the expected, depreciation-adjusted world safe interest rate and the domestic bond rate.

The risk-free world interest rate follows a first-order autoregressive process:

\[ \frac{1 + i_t^W}{1 + i_t^W} = (1 + i_{t-1}^W)^{\rho_W} \exp(\xi_t^W), \]

where \( \rho_W \in (0, 1) \) and \( \xi_t^W \sim N(0, \sigma_{\xi^W}) \).

\[ ^{11} \text{Perfect capital mobility prevails when } \theta_0^{F,P} \rightarrow 0, \text{ in which case } 1 + i_t^B = (1 + i_t^W) \mathbb{E}_t \left( E_{t+1} / E_t \right), \]

which corresponds to the standard uncovered interest parity condition under risk neutrality. The specification used here follows Agénor (1997); see for instance Lartey (2012), Gabaix and Maggiori (2014), and Liu and Spiegel (2014) for alternative ways of modeling imperfect asset substitutability in an open economy.
3.2 Domestic Final Good

The FG producer imports a continuum of differentiated intermediate goods from the rest of the world and combines them with a similar continuum of domestically-produced intermediate goods to generate a domestic final good, in quantity \( Y_t \), which is sold both domestically (for consumption and investment) and abroad:

\[
Y_t = [\Lambda_D(Y^D_t)^{(\eta-1)/\eta} + (1 - \Lambda_D)(Y^F_t)^{(\eta-1)/\eta}]^{\eta/(\eta-1)},
\]

(13)

where \( \Lambda_D \in (0, 1) \), \( Y^D_t \) (\( Y^F_t \)) a quantity index of domestic (imported) intermediate goods, and \( \eta > 0 \) is the elasticity of substitution between baskets of domestic and imported composite intermediate goods. These baskets are defined as

\[
Y^i_t = \left\{ \int_0^1 [Y^i_{jt}]^{(\theta_i-1)/\theta_i} dj \right\}^{\theta_i/(\theta_i-1)}, \quad i = D, F
\]

(14)

where \( \theta_i > 1 \) is the elasticity of substitution between intermediate domestic goods among themselves (\( i = D \)), and imported goods among themselves (\( i = F \)), and \( Y^i_{jt} \) is the quantity of type-\( i \) intermediate good of category \( i \) (domestic or imported), with \( j \in (0, 1) \).

12 For simplicity, the number of both domestic and imported intermediate goods is normalized to unity.

The FG producer sells its output at a perfectly competitive price. Let \( P^D_{jt} \) denote the price of domestic intermediate good \( j \) set by firm \( j \), and \( P^F_{jt} \) the price of imported intermediate good \( j \), in domestic currency. Cost minimization yields the demand functions for each variety of intermediate goods:

\[
Y^i_{jt} = \left( \frac{P^i_{jt}}{P^t_i} \right)^{\theta_i} Y^i_t, \quad i = D, F
\]

(15)

where \( P^D_t \) and \( P^F_t \) are price indices for domestic and imported intermediate goods, respectively, which are given from the zero-profit condition as

\[
P^i_t = \left\{ \int_0^1 (P^i_{jt})^{1-\theta_i} dj \right\}^{1/(1-\theta_i)}, \quad i = D, F
\]

(16)

so that \( P^i_t Y^i_t = \int_0^1 P^i_{jt} Y^i_{jt} dj \).

Aggregating across firms yields the allocation of total demand between domestic and foreign goods:

\[
Y^D_t = \Lambda_D \eta \left( \frac{P^D_t}{P_t} \right)^{-\eta} Y^D_t, \quad Y^F_t = (1 - \Lambda_D) \eta \left( \frac{P^F_t}{P_t} \right)^{-\eta} Y^F_t,
\]

(17)

where \( P_t \) is the price of final output, given by

\[
P_t = [\Lambda_D \eta \left( \frac{P^D_t}{P_t} \right)^{-\eta} + (1 - \Lambda_D) \eta \left( \frac{P^F_t}{P_t} \right)^{-\eta}]^{1/(1-\eta)}.
\]

(18)
Given the focus of this paper, imperfect exchange rate pass-through is accounted for in a simple manner; the domestic-currency price of imports of intermediate good \( j \) is given by

\[
P_{jt}^F = E_t^{uF} E_{t-1}^{1-uF} WP_{jt}^F, \tag{19}
\]

where \( WP_{jt}^F \) is the foreign-currency price of imported good \( j \) and \( u^F \in (0, 1) \) measures the degree of exchange rate pass-through. Thus, \( u^F = 1 \) corresponds to complete pass-through (that is, pure producer currency pricing) and \( u^F = 0 \) no pass-through at all (that is, pure local currency pricing).\(^{13}\) Regardless of the value of \( u^F \), complete pass-through occurs in the long term.

The volume of goods sold abroad, \( Y_t^X \), depends on the domestic-currency price of exports of the final good, \( P_t^X \), relative to the price of goods sold on the domestic market:

\[
Y_t^X = \left( \frac{P_t^X}{P_t} \right)^\kappa, \tag{20}
\]

\[
P_t^X = E_t WP_t^X, \tag{21}
\]

where \( \kappa > 0 \) and \( WP_t^X \) is the foreign-currency price of exports.

With \( Y_t^S \) denoting the volume of goods sold on the domestic market, the output identity in volume terms is thus also given by

\[
Y_t = Y_t^S + Y_t^X. \tag{22}
\]

### 3.3 Domestic Intermediate Goods

Domestic IG producers, indexed by \( j \in (0, 1) \), produce intermediate goods by combining labor, \( N_{jt} \), and capital, \( K_{jt} \), and sell them on a monopolistically competitive market:

\[
Y_{jt}^D = N_{jt}^{1-\alpha} K_{jt}^\alpha, \tag{23}
\]

where \( \alpha \in (0, 1) \).\(^{14}\)

IG producers rent capital from the CG producer, at the rate \( r_t^K \), and pay for it after the sale of output. However, a fraction \( \kappa^W \in (0, 1) \) of wages must be paid in advance. To do so firm \( j \) borrows the real amount \( l_{jt}^W \) from the bank. The amount borrowed is therefore such that

\[
l_{jt}^W = \kappa^W w N_{jt}. \tag{24}
\]

Loans contracted for the purpose of financing working capital do not carry any risk, and are made at a rate that reflects only the marginal cost of borrowing from the

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\(^{13}\)See for instance Shi and Xu (2010) and Adolffson et al. (2014) for a full treatment with a monopolistically competitive import goods sector.

\(^{14}\)The analysis could be extended to account for the fact that the production of domestic intermediates also requires the use of imported intermediate goods (say, oil imports, \( Y_t^O \)), by using a Leontief technology of the form \( Y_{jt}^D = \min [N_{jt}^{1-\alpha} K_{jt}^\alpha / (1 - \alpha_O), Y_t^O / \alpha_O] \), where \( \alpha_O \in (0, 1) \), or a generalized Cobb-Douglas function, as in Liu and Spiegel (2014) for instance.
central bank, $i_t^C$. Total costs of firm $j$ in period $t$, $TC_{jt}$, are thus given by

$$TC_{jt} = (1 + i_t^C)K_j w_t N_{jt} + r^K_t K_{jt}.$$  

In standard fashion, cost minimization yields the capital-labor ratio and the unit real marginal cost, $mc_t$, as

$$\frac{K_{jt}}{N_{jt}} = \left( \frac{\alpha}{1 - \alpha} \right) \left[ \frac{(1 + K_j^c i_t^C) w_t}{r^K_t} \right], \quad (25)$$

$$mc_t = \left( \frac{r^K_t}{\alpha} \right)^{\alpha} \left[ \frac{(1 + K_j^c i_t^C) w_t}{1 - \alpha} \right]. \quad (26)$$

As in Rotemberg (1982), domestic IG producers incur a real cost in adjusting prices, of the form

$$\left( \frac{\Delta d}{2} \right) \left( \frac{\Delta j^D_t}{\Delta j^D_{t-1}} - 1 \right)^2 Y_t^D,$$

where $\phi_D \geq 0$. Each firm $j$ chooses a sequence of prices so as to maximize the discounted value of its current and future profits:

$$\{ P^D_{jt+s} \}_{s=0}^\infty = \arg \max_{P_t} \sum_{s=0}^\infty \beta^s \lambda_{t+s} J^D_{jt+s}, \quad (27)$$

where $J^D_{jt+s}$ denotes real profits at $t$, defined as

$$J^D_{jt} = \left( \frac{P^D_{jt}}{P_t} \right) Y^D_{jt} - mc_t Y^D_{jt} - \frac{\phi_D}{2} \left( \frac{P^D_{jt}}{P^D_{jt-1}} - 1 \right)^2 Y^D_t. \quad (28)$$

Taking $\{ mc_{t+s}, P^D_{t+s}, Y^D_{t+s} \}_{s=0}^\infty$ as given, and using (15) with $i = D$, the first-order condition for this maximization problem is:

$$\left( 1 - \theta_D \right) \left( \frac{P^D_{jt}}{P_t} \right)^{-\theta_D} \frac{1}{P_t} + \theta_D \left( \frac{P^D_{jt}}{P_t} \right)^{-\theta_D - 1} \frac{mc_t}{P^D_{jt}}, \quad (29)$$

$$-\phi_D \left\{ \left( \frac{P^D_{jt}}{P^D_{jt-1}} - 1 \right) \frac{1}{P^D_{jt-1}} \right\} + \beta \phi_D \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{P^D_{jt+1}}{P^D_{jt}} - 1 \right) \frac{P^D_{jt+1}}{P^D_{jt}} Y^D_{jt+1} \right\} = 0,$$

which determines the adjustment process of the nominal price $P^D_{jt}$.

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15 In this expression, the steady-state inflation rate in the price of goods sold domestically is assumed to be zero.

16 In standard fashion, IG firms (which are owned by households) are assumed to value future profits according to the household’s intertemporal marginal rate of substitution in consumption.
### 3.4 Capital Good

At the beginning of the period, the CG producer buys an amount \( I_t \) of the final good from the FG producer and combines it with the existing capital stock to produce new capital goods, \( K_{t+1} \), which is rented in the next period to IG producers at the rate \( r_{t+1}^K \). Aggregate capital accumulates as follows:

\[
K_{t+1} = \left\{ \frac{I_t}{K_t} - \frac{\Theta_K}{2} \left( \frac{K_{t+1} - K_t}{K_t} \right)^2 \right\} K_t + (1 - \delta) K_t, \tag{30}
\]

where \( K_t = \int_0^1 K_{jt} \, dj \), \( \delta \in (0, 1) \) is a constant rate of depreciation, and \( \Theta_K > 0 \) is a parameter that measures the magnitude of adjustment costs.

Investment goods must be paid in advance; the CG producer must therefore borrow from the bank:

\[
l_t = I_t. \tag{31}
\]

The household makes its exogenous housing stock, \( \bar{H} \), available without any direct charge to the CG producer, who uses it as collateral against which it borrows from the bank. Repayment is uncertain and occurs with probability \( q_t \in (0, 1) \). Expected repayment is thus \( q_t (1 + i_t^L) I_t + (1 - q_t) \kappa z_t^H \bar{H} \), where \( \kappa \in (0, 1) \) is the share of the value of the housing stock that can be effectively pledged as collateral.

Subject to (30) and (31), the CG producer chooses the level of capital \( K_{t+1} \) (taking the rental rate, the lending rate, and the existing capital stock, as given) so as to maximize the value of the discounted stream of dividend payments to the household:

\[
\{K_{t+s+1}\}_{s=0}^{\infty} = \arg\max_{K_{t+s+1}} \sum_{s=0}^{\infty} \mathbb{E}_t [\beta^s \lambda_{t+s} (1 + \pi_{t+s+1}^S) J_{t+s+1}^K], \tag{32}
\]

where \( \lambda_{t+s} = C_{t+s}^{-1/k} \) is the marginal utility value (in terms of consumption) of an additional currency unit of real profits at \( t+s \) and \( \mathbb{E}_t [\beta^s \lambda_{t+s} (1 + \pi_{t+s+1}^S) J_{t+s+1}^K] \) denotes expected real profits at the end of period \( t+s \), defined as

\[
\mathbb{E}_t [\beta^s \lambda_{t+s} (1 + \pi_{t+s+1}^S) J_{t+s+1}^K] = \mathbb{E}_t \left\{ \beta^s \lambda_{t+s} \left[ r_{t+s}^K K_{t+s} - [q_{t+s} (1 + i_{t+s}^L) I_{t+s} + (1 - q_{t+s}) \kappa z_{t+s}^H \bar{H}] \right] \right\}.
\]

Using (6), the first-order condition for maximization can be written as

\[
\mathbb{E}_t r_{t+1}^K = q_t (1 + i_t^L) \mathbb{E}_t \left\{ \left[ 1 + \Theta_K \left( \frac{K_{t+1}}{K_t} - 1 \right) \right] \left( \frac{1 + i_{t+1}^B}{1 + \Theta_{K+1}} \right) \right\}, \tag{33}
\]

\[
-\mathbb{E}_t \left\{ q_{t+1} (1 + i_{t+1}^L) \left[ 1 - \delta + \frac{\Theta_K}{2} \left( \frac{K_{t+2}}{K_{t+1}} \right)^2 - 1 \right] \right\},
\]

which relates the expected rate of return to capital to the (current and expected) repayment probability and the loan rate.
3.5 Commercial Bank

The bank supplies credit to IG producers as well, who use it to finance their labor costs prior to the sale of output. Its supply of loans is perfectly elastic at the prevailing lending rate. It also borrows on world capital markets and from the central bank. At the end of each period, it repays with interest household deposits and the liquidity borrowed from the central bank, and redeems in full its foreign debt. All profits are then distributed, the bank is liquidated, and a new bank opens at the beginning of the next period.

Real lending to IG producers and the CG producer, \( l_t \), is equal to, using (13) and (31),

\[
l_t = \int_0^1 l^W_j d\gamma + I^1_t = \kappa^W w_t N_t + I_t.
\]

(34)

The bank’s balance sheet in real terms is

\[
l_t + RR_t = d_t + z_t L_t^{F,B} + I_t^{C,B},
\]

(35)

where \( L_t^{F,B} \) is foreign borrowing (in foreign-currency terms), \( I_t^{C,B} \) borrowing from the central bank, and \( RR_t \) required reserves, which do not pay interest—a common practice, as documented by Gray (2011)—and are set as a fraction \( \mu_t^R \in (0, 1) \) of deposit liabilities:

\[
RR_t = \mu_t^R d_t.
\]

(36)

The bank’s cost of borrowing on world capital markets, \( i_t^{F,B} \), measured in foreign-currency terms, is defined as

\[
1 + i_t^{F,B} = (1 + i_t^W)(1 + \theta_t^{F,B}),
\]

(37)

where \( \theta_t^{F,B} \) is a risk premium that increases with the amount borrowed:

\[
\theta_t^{F,B} = \frac{\theta_0^{F,B}}{2} L_t^{F,B},
\]

(38)

where \( \theta_0^{F,B} > 0 \).

Assuming that \( \kappa z_t H_t \leq (1 + i_t^D)l_t^D \) to avoid a corner solution, the bank’s expected real profits at the end of period \( t \) (or beginning of \( t+1 \)) are defined as

\[
\mathbb{E}_t[(1 + \pi_{t+1}^S)J_t^{B}] = (1 + i_t^{C})l_t^W + q_t(1 + i_t^L)l_t^L + (1 - q_t)\kappa z_t^H H_t + \mu_t^R d_t
\]

\[-(1 + i_t^D)d_t - (1 + i_t^{C})l_t^{C,B} - (1 + i_t^{F,B})\mathbb{E}_t(\frac{E_{t+1}}{E_t})z_t L_t^{F,B},
\]

(39)

\[\footnote{A marginal reserve requirement regime could be modeled as \( RR_t = \mu_t^R d_t + \mu_t^{RR}(d_t - d_{t-1}) \), with \( \mu_t^R \in (0, 1) \) and \( \mu_t^{RR} > 0 \). This regime would be equivalent to (36) in the steady state. Note also that the bank holds no domestic bonds; as discussed in the next section, in equilibrium it has no incentive to do so.}\]
where \( q_t(1 + \nu^w_t)I_t + (1 - q_t)\kappa z^H_t \bar{H} \) is expected repayment (as defined earlier), and \( \mu^R_t d_t \) the reserve requirements held at the central bank and returned to the bank at the end of the period. The other terms are self explanatory.

The bank sets the deposit and lending rates and determines foreign borrowing so as to maximize expected profits:18

\[
i_t^D, i_t^L, L_t^{F,B} = \arg \max \mathbb{E}_t[(1 + \pi^S_{t+1})J^B_{t+1}]. \tag{40}
\]

Solving (40) subject to (34)-(39) yields

\[
i_t^D = (1 + \frac{1}{\eta_D})^{-1}(1 - \mu^R_t)i_t^C, \tag{41}
\]

\[
i_t^L = \frac{1 + i_t^C}{(1 + \eta_t^{-1})\kappa_t} - 1, \tag{42}
\]

\[
L_t^{F,B} = (1 + i_t^C) - (1 + i_t^W)\mathbb{E}_t(E_{t+1}/E_t), \tag{43}
\]

where \( \eta_D \) and \( \eta_I \) are interest elasticities of the supply of deposits and the CG demand for loans, respectively.

Equation (41) shows that the equilibrium deposit rate is a markup over the refinance rate, adjusted (downward) for the implicit cost of holding reserve requirements. Equation (42) indicates that the lending rate depends negatively on the repayment probability and positively on the marginal cost of borrowing from the central bank. Equation (43) states that foreign borrowing is decreasing in the cost of borrowing abroad and increasing in the cost of borrowing domestically from the central bank.

As in Agénor et al. (2013, 2014), the repayment probability depends positively on the effective collateral-CG loan ratio and the cyclical position of the economy:

\[
q_t = \left( \frac{\kappa z^H_t \bar{H}}{\kappa_t^z} \right)^{\varphi_1} \left( \frac{Y^S_t}{\bar{Y}^S} \right)^{\varphi_2}, \tag{44}
\]

with \( \varphi_1, \varphi_2 > 0 \) and \( \bar{Y}^S \) is the steady-state level of output sold domestically.19

Borrowing from the central bank is determined residually from the balance sheet constraint (35) together with (36):

\[
l_t^{C,B} = l_t - z_t L_t^{F,B} - (1 - \mu^R_t)d_t. \tag{45}
\]

---

18 Because deposits, loans, and borrowing (domestic and foreign) all mature at the end of each period, the maximization problem is static in nature.

19 A relationship similar to (44) can be formally derived by relating bank monitoring efforts and borrowers’ incentives to repay their loans (as in Agénor and Pereira da Silva (2014)) and by assuming that monitoring costs are countercyclical.
3.6 Central Bank

The central bank supplies liquidity to the commercial bank through a standing facility, at a price that reflects both a base policy rate (which is adjusted in response to inflation deviations from target and output deviations from its steady-state level) and a premium. It also engages in partial sterilization and reserve accumulation. Its balance sheet is given by

$$z_t R_t^F + b_t^C + l_{t^C}^C - nw_t = m_t + RR_t,$$

where $z_t R_t^F$ denotes international reserves, $b_t^C$ holdings of government bonds, $m_t$ the real supply of cash, and $nw_t$ the central bank’s real net worth.

We consider a managed float regime, in which the central bank (as a result of a self-insurance motive against real and financial external shocks) also has a target level of reserves, which is a multiple $\phi_1^R > 1$ of the value of imports, $WP_t^F Y_t^F$, and a fraction $\phi_2^R \in (0, 1)$ of the net foreign-currency liabilities of the private sector, $L_t^{F,B} - B_t^{F,P}$. The reserve accumulation rule is thus

$$R_t^F = \left( \frac{E_t}{E_{t-1}} \right)^{-\varphi_1^R} (R_{t-1})^{\varphi_2^R} \left\{ (\phi_1^R WP_t^F Y_t^F)^{\varphi_2^R} [\phi_2^R (L_t^{F,B} - B_t^{F,P})]^{1-\varphi_2^R} \right\}^{1-\varphi_1^R},$$

where $\varphi_1^R \geq 0$ measures the degree of smoothing (or “leaning against the wind”) of short-term movements in the nominal exchange rate, $\varphi_2^R \in (0, 1)$ the degree of persistence, and $\varphi_2^R \in (0, 1)$ the relative importance of the “trade” motive versus the “financial” motive. Thus, as long as $\varphi_1^R > 0$, the central bank buys (sells) reserves to stabilize the exchange rate when it appreciates (depreciates). In the particular case where $\varphi_1^R = 0$ and $\varphi_2^R = 1$, the stock of reserves remains constant over time and the exchange rate is fully flexible.

The central bank also adjusts its holdings of government bonds in order to sterilize the effects of its buying and selling of international reserves on the supply of cash.

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20 Note that in (47) self-insurance against trade shocks is captured only through an import coverage ratio, as is often the case in practice. A more general measure would be the trade (or current account) balance, which would imply a positive relationship between trade deficits and the level of reserves. At the same time, however, with a mercantilist motive instead of a self-insurance motive, the relationship could go in the opposite direction—there would be a positive relationship between trade or current account surpluses and reserves.

21 See for instance Palma and Portugal (2014) for evidence on exchange rate smoothing for Brazil, and Vujanovic (2011) for cross-country evidence. As documented in the literature, the reasons for the central bank wanting to smooth exchange rate movements may be related to fear of floating, concerns with competitiveness, and possibly financial stability, given that the bank does not (fully) hedge its foreign-currency risk in our setting. At the same time, there may be a fear of losing reserves, which would militate in favor of a relatively low value of $\varphi_1^R$, or induce the use of capital controls.

22 We also experimented with the expected depreciation rate, $E_t E_{t+1}/E_t$, in (47). The central bank therefore would now sell (buy) foreign reserves if it expects the exchange rate to depreciate (appreciate) one-period ahead. The results were essentially the same, except that, in contrast to what is shown later, the magnitude of the jump in reserves is smaller on impact.
Thus, its stock of bonds evolves according to

$$b_t^C - \frac{b_{t-1}^C}{1 + \pi_t^S} = -\kappa^F z_t \Delta R_t^F,$$  \hspace{1cm} (48)

where $\kappa^F \in (0, 1)$ measures the degree of sterilization.

Because all income received by the central bank is transferred to the government (as discussed later), changes in the nominal value of the central bank’s net worth are given by capital gains from exchange rate depreciation only ($\Delta NW_t = R_t^F \Delta E_t$). Using this result, taking first differences of (46) and substituting (48) in the resulting expression yields

$$m_t = \frac{m_{t-1}}{1 + \pi_t^S} + (1 - \kappa^F) z_t \Delta R_t^F + (i_t^{C,B} - \frac{i_{t-1}^{C,B}}{1 + \pi_t^S}) - (RR_t - \frac{RR_{t-1}}{1 + \pi_t^S}),$$  \hspace{1cm} (49)

which shows that, with full sterilization ($\kappa^F = 1$), changes in the domestic-currency value of foreign-exchange reserves would not affect directly the supply of cash. This is complete sterilization in a broad sense, as opposed to complete sterilization in a narrow sense, where the supply of cash remains constant. Indeed, in the present case, changes in foreign reserves could affect the supply of cash—and therefore interest rates and aggregate demand—indirectly, through changes in $i_t^{C,B}$ and $RR_t$.

The central bank sets its base policy rate, $i_t^R$, on the basis of a Taylor-type policy rule:

$$1 + i_t^R = \frac{(1 + i_{t-1}^R)}{1 + \hat{i}^R} \left( 1 + \frac{\pi_t^S}{1 + \hat{\pi}^S} \right)^{\epsilon_1} \left( \frac{\hat{Y}_t^S}{\hat{Y}_t^S} \right)^{\epsilon_2} \exp(\epsilon_t),$$  \hspace{1cm} (50)

where $\hat{i}^R$ and $\hat{Y}^S$ are the steady-state values of the policy rate and domestic sales of the final good, $\pi_t^S \geq 0$ the central bank’s headline inflation target, $\chi \in (0, 1)$ a coefficient measuring the degree of interest rate smoothing, $\epsilon_1, \epsilon_2 > 0$, and $\epsilon_t \sim N(0, \sigma_e)$.

The actual cost of borrowing for the commercial bank incorporates, in addition to the base policy rate, a penalty rate, $\theta_t^{C,B}$:

$$1 + i_t^C = (1 + i_t^R)(1 + \theta_t^{C,B}).$$  \hspace{1cm} (51)

In turn, the penalty rate is positively related to the ratio of central bank borrowing to deposits:

$$\theta_t^{C,B} = \theta_0^{C,B} \left( \frac{i_t^{C,B}}{d_t} \right),$$  \hspace{1cm} (52)

where $\theta_0^{C,B} > 0$. Thus, the central bank charges a penalty that increases with the amount borrowed. In addition, this amount is scaled by the bank’s core liabilities, that is, deposits. This helps to capture the fact that, from the perspective of the central bank, the composition of bank liabilities matters when setting borrowing terms. Indeed, in normal times central banks prefer commercial banks to raise deposits to fund their operations rather than borrow from them, so when the ratio $i_t^{C,B} / d_t$ is too high they
raise the cost of refinancing to discourage further borrowing and induce commercial banks to raise deposit rates and thereby improve incentives for households to raise their deposit holdings. In a sense, an \( t_t^{C,B}/d_t \) ratio that is too high creates a “stigma” effect, which raises funding costs either directly on borrowing from the central bank, as is modeled here, or indirectly through borrowing on the interbank market, as may be the case in practice (see Armentier et al. (2013) and Ennis and Weinberg (2013)).

### 3.7 Government

The government purchases the final good and issues nominal riskless one-period bonds to finance its deficit; it does not borrow abroad. In addition to lump-sum taxes, it also receives the interest income collected by the central bank on its foreign reserves and its loans to the commercial bank. It pays interest on the share of government debt held by the private sector. Its budget constraint is given by

\[
b_t - \frac{b_{t-1}}{1+\pi_t^s} = G_t - T_t + \frac{i_{t-1}^B b_{t-1}^P}{1+\pi_t^s} - \left( \frac{i_{t-1}^C t_{t-1}^{C,B}}{1+\pi_t^s} + z_{t+1}^W R_{t-1}^F \right),
\]

where \( b_t = b_t^C + b_t^P \) is the real stock of government bonds, and \( G_t \), real government spending. In what follows the government is assumed to keep its real stock of debt constant (\( b_t = b \), for all \( t \)) and to balance its budget by adjusting lump-sum taxes. The composition of the public debt therefore varies as a result of the open-market operations associated with the sterilized interventions (as described by (48)) conducted by the central bank.

Government purchases represent a fraction \( \psi \in (0,1) \) of domestic sales of the final good:

\[
G_t = \psi Y_t^S.
\]

The production structure and main real and financial flows between agents are summarized in Figure 1.

### 4 Equilibrium and Steady State

In a symmetric equilibrium, \( K_{jt} = K_t, N_{jt} = N_t, Y_{jt} = Y_t, P_{jt}^d = P_t^d \), for all \( j \in (0,1) \) and \( i = D,F \). All IG firms produce the same output and prices are the same across firms.

Equilibrium in the goods market requires that sales on the domestic market be equal to aggregate demand, inclusive of capital adjustment costs:

\[
Y_t^S = C_t + G_t + I_t + \frac{\phi D}{2} \left( \frac{P_t^D}{P_{t-1}^D} - 1 \right)^2 \left( \frac{P_t^D}{P_t^S} \right) Y_t^D,
\]

with the price of sales on the domestic market determined through the identity

\[
P_t Y_t = P^S_t Y_t^S + P^X_t Y_t^X.
\]
Bank loans to IG firms and the capital producer are made in the form of cash. The equilibrium condition of the market for cash is thus given by

\[ m_t = m_t^P + l_t, \]

where \( m_t \) (the supply of cash) is defined in (49).

Finally, the external budget constraint is given by

\[ WP_t X_t - WP_t X_t^F + i_{t-1}^W F_{t-1} + \theta_{t-1}^F B_{t-1}^F - \theta_{t-1}^F L_{t-1} - \Delta F_t = 0, \]

where \( F_t = R_t^F + B_{t}^F - L_{t}^F \) is the economy’s net foreign asset position.

The steady-state solution of the model is derived in Appendix B. Several of its key features are similar to those described in Agénor et al. (2013, 2014), so we refer to those papers for a more detailed discussion. In brief, with a headline inflation target \( \tilde{\sigma} = 0 \), the steady-state inflation rate \( \tilde{\sigma} \) is also zero. In addition to standard results (the steady-state value of the marginal cost, for instance, is given by \( (\tilde{\sigma}^D - 1)/\theta_D \)), the steady-state value of the repayment probability is

\[ \tilde{\sigma}^D = (1 + \bar{\sigma}^D)^{-1}(1 - \tilde{\sigma}^R) \tilde{\sigma}^C, \]

and \( \tilde{\sigma}^L = \beta^{-1}/(1 + \eta_I^{-1}) \tilde{\sigma} - 1 \). The cost of borrowing from the central bank is thus \( \tilde{\sigma}^C = \tilde{\sigma}^R + \tilde{\sigma}^C \). From these equations it can be shown that \( \tilde{\sigma}^D < \tilde{\sigma}^C \) and \( \tilde{\sigma}^L \leq \tilde{\sigma}^C \). In order to have \( \tilde{\sigma}^L > \tilde{\sigma}^C \) (to ensure that the bank has an incentive to borrow from the central bank), we impose \( 1/(1 + \eta_I^{-1}) \tilde{\sigma} > 1 \). This condition also implies that \( \beta^{-1}/(1 + \eta_I^{-1}) \tilde{\sigma} > 1 \), so that \( \tilde{\sigma}^L > 0 \). Because these conditions also imply that \( \tilde{\sigma}^L > \tilde{\sigma}^B \), in equilibrium the bank has no incentives to hold government bonds. Thus, \( \tilde{\sigma}^L > \tilde{\sigma}^C > \tilde{\sigma}^R = \tilde{\sigma}^B > \tilde{\sigma}^D \).

From (12), the steady-state value of the stock of foreign bonds held by the representative household is

\[ \tilde{B}_{t}^F = \tilde{i}_t^W - (\beta^{-1} - 1) \frac{1}{\theta_D^F(1 + \tilde{i}_t^W)}, \]

which is positive as long as the world risk-free interest rate exceeds the domestic real interest rate. The greater the degree of imperfections on the world capital markets (the higher \( \theta_D^F \)) is, the lower household holdings of foreign bonds are. From (47), the stock of reserves is equal to its target level:

\[ \tilde{R}_t^F = \frac{\phi_R^F WP_t Y_t^F\tilde{\sigma}_R}{[\phi_R^B (L_{t}^F B_{t}^F - B_{t}^F F_{t}^R)]^{e_{R-1}}}. \]

To analyze the response of the economy to shocks, we log-linearize the model around a nonstochastic, zero-inflation steady state. The log-linearized equations are summarized in Appendix C.

5 Baseline Calibration

To calibrate the model we dwell largely on Agénor et al. (2013, 2014). We therefore refer to those studies for a detailed discussion of some of our choices. In addition,
for some of the parameters that are deemed critical for this study, sensitivity analysis is reported in the next section. This is the case, in particular, for the nature of the monetary policy rule, the degree of exchange rate smoothing, and the intensity of sterilization.

Parameter values are summarized in Table 1. The discount factor $\beta$ is set at 0.985, which corresponds to an annual real interest rate of 6 percent. The intertemporal elasticity of substitution, $\zeta$, is 0.6, in line with estimates for middle-income countries (see Agénor and Montiel (2015)). The preference parameter for leisure, $\eta_{L}$, is set at 10, to capture a fairly inelastic supply of labor. The preference parameters for composite monetary assets, $\eta_{x}$, and housing, $\eta_{H}$, are both set at the same low value, 0.02. The share parameter in the index of money holdings, $\nu$, which corresponds to the relative share of cash in narrow money, is set at 0.35. This value is consistent with available data for MICs. The sensitivity of the spread to household foreign bond holdings, $\theta_{0}^{E:P}$, is set at 0.5.

The distribution parameter between domestic and imported intermediated goods in the production of the final good, $\Lambda_{D}$, is set at 0.7, to capture the case of an economy where the share of nontraded goods in total output remains high. The elasticity of substitution between baskets of domestic and imported composite intermediate goods, $\eta$, is set at 2.0. The elasticities of substitution between intermediate domestic goods among themselves, $\theta_{D}$, and imported goods among themselves, $\theta_{F}$, are set equal to the same value, 10. The pass-through coefficient is set at $\mu^{F} = 0.3$; this is line with the evidence suggesting a decline in the strength of the pass-through effect in recent years in both industrial and developing countries (see Bussière et al. (2013) and Devereux and Yetman (2014)). The price elasticity of exports, $\kappa$, is set equal to 0.9, a value consistent with the lower range of estimates for developing countries reported by Imbs and Méjean (2010).

The share of capital in domestic output of intermediate goods, $\alpha$, is set at 0.35. With $\theta_{D} = 10$, the steady-state value of the markup rate, $\theta_{D}/(\theta_{D} - 1)$, is equal to 11.1 percent. The adjustment cost parameter for prices of domestic intermediate goods, $\phi_{D}$, is set at 74.5 to capture a high degree of nominal price stickiness. The rate of depreciation of private capital, $\delta$, is set equal to 0.02. The adjustment cost incurred by the CG producer for transforming the final good into investment, $\Theta_{K}$, is set at 14. The share of labor costs financed in advance, $\kappa^{W}$, is set at a relatively high value, 0.8, in line with the evidence for middle-income countries.

Regarding the commercial bank, the effective collateral-loan ratio, $\kappa$, is set at 0.2. The elasticity of the repayment probability is set at $\varphi_{1} = 0.1$ with respect to the effective collateral-loan ratio and $\varphi_{2} = 0.3$ with respect to deviations in output from its steady state. Parameter $\theta_{0}^{E:B}$, which determines how the bank’s foreign borrowing responds to the differential in the cost of domestic and foreign borrowing, is set at 0.16; this value implies that bank foreign liabilities represent initially about 10 percent of their total liabilities.

Regarding the central bank, the reserve requirement rate $\mu^{R}$ is set at 0.1. The degree of persistence in the central bank’s policy response, $\chi$, is set at 0.8 whereas,
consistent with estimates of Taylor-type rules for MICs, responses of the base policy rate to inflation and output deviations, \( \varepsilon_1 \) and \( \varepsilon_2 \), are set at 2.0 and 0.5, respectively (see for instance Moura and Carvalho (2010)). The sensitivity of the penalty rate to the bank borrowing-required reserve ratio, \( \phi_{0}^{C,B} \), to 0.1 initially; sensitivity analysis is reported later on. The parameter characterizing the degree of exchange rate smoothing in the foreign reserves targeting rule, \( \varphi_{1}^{R} \), is set at 0.5 initially, to reflect a relatively low degree of intervention. The relative weight in the trade motive for self insurance is assumed to be predominant (compared to the capital account motive) and accordingly the parameter \( \varphi_{1}^{R} \) is set at 0.8, whereas the degree of persistence in the rule, \( \varphi_{1}^{R} \), is set at 0.8. Given that the model is log-linearized and solved in terms of deviations from the steady state, parameters \( \delta_{1}^{R} \) and \( \delta_{2}^{R} \) (which relate the targeted stock of foreign reserves to imports and net private foreign-currency liabilities, respectively) are both normalized to unity. The degree of sterilization, \( \kappa^{F} \), is set initially at a relatively low value, 0.2 and sensitivity analysis is conducted later on. Finally, the degree of persistence of the shock to the world risk-free rate, \( \rho_{W} \), is set at 0.8, which implies a fairly high degree of inertia.

6 Drop in the World Risk-Free Rate

To illustrate the impact of external shocks, consider a temporary drop in the world risk-free interest rate by 35 basis points at a quarterly rate, or about 141 basis points at an annual rate.

The results of this experiment are summarized in Figure 2. On impact, the shock lowers the return on foreign assets and the cost of borrowing abroad for the domestic bank. Thus, households’ holdings of foreign bonds decline, whereas the bank’s foreign liabilities increase; both combine to generate an inflow of capital, which leads to an appreciation of the nominal exchange rate. In turn, the nominal appreciation lowers the domestic price of imported intermediate goods and stimulates their demand as well as final good production. At the same time, it lowers inflation (measured in terms of the price of domestic sales) initially, which induces the central bank to reduce the base policy rate. The net effect, however, is an increase in that rate, given the boom in domestic sales. The inflow of foreign borrowing and the drop in the deposit rate induced by the initial fall in the base policy rate combine to reduce the central bank borrowing-deposit ratio, and thus the penalty rate, which in turn magnifies the reduction in the refinance rate. As a result the loan rate also falls, which further dampens inflation through a (reverse) cost channel.

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23 As documented by Aizenman and Glick (2009) and Agénor and Pereira da Silva (2013), even though the degree of sterilization (as measured by offset coefficients) has increased in recent years in many middle-income countries, it remains imperfect—especially in Latin America.

24 In analyzing the effects of this shock, we do not account for the fact that changes in foreign interest rates could affect foreign output, and thus domestic exports. Doing so allows us to isolate the pure financial effects of the shock.
The increase in imports raises the central bank’s desired and (to a smaller extent, given partial adjustment) actual stock of foreign reserves. With partial sterilization, the accumulation of foreign reserves tends to increase the monetary base. At the same time, the increase in foreign borrowing by the commercial bank reduces (at the initial level of loans) its domestic borrowing from the central bank, which tends to reduce the monetary base. The former effect dominates, implying an increase in the supply of cash. At the initial level of consumption, the nominal bond rate must therefore fall to raise the demand for cash and restore market equilibrium. Because expected future inflation increases, the real bond rate also falls, thereby inducing households to increase consumption today.

In addition to an intertemporal effect on consumption, the fall in the real bond rate leads to an increase in the demand for housing services, which tends to raise real estate prices. In turn, this raises the value of the collateral that firms can pledge. But because the real loan rate falls initially, borrowing for investment outlays increases—so much so that the collateral-loan ratio falls, which tends to reduce the repayment probability. But because of the expansion of output, the net effect on the probability of repayment is positive. The nominal loan rate therefore falls. Thus, aggregate demand (spending on goods sold domestically) unambiguously increases on impact. In addition to the level effect on final output, there is also a composition effect: the appreciation of the nominal and real exchange rates translates into a drop in the share of final output allocated to exports, and an increase in the share sold domestically.

Over time, the increase in investment raises the capital stock, which tends to lower the rental rate of capital and to raise the marginal product of labor and therefore gross wages. The increase in current consumption raises the marginal utility of leisure and induces households to reduce their supply of labor, thereby magnifying the initial upward pressure on wages associated with higher output and the increased demand for labor. The downward movement in the refinance rate (the rate at which intermediate goods producers borrow to finance their working capital needs) mitigates this initial pressure but nevertheless the effective wage rate increases on impact. Because the rental rate of capital does not change on impact (due to the one-period lag in capital accumulation), marginal costs rise as well in the first period. Over time, the reduction in the rental rate of capital induced by the boom in investment leads in a first phase to lower marginal costs, but the increase in the effective wage leads to higher inflation.

The results of this experiment show that, consistent with the evidence, external shocks that lead to large inflows of capital (a “sudden flood,” in the terminology of Agénor et al. (2014)) generate a domestic boom characterized by increases in asset prices and aggregate demand, an expansion in output, and—over time only, given the initial appreciation—inflationary pressures. Although the boom in domestic activity tends to raise the base policy rate, the drop in inflation (associated with the appreciation of the real exchange rate) tends to reduce it. Given the relative weights of inflation and output deviations in the Taylor rule, the net effect is a drop in the base policy rate. Nevertheless, the cost of bank borrowing falls by more than the policy rate because the repayment probability increases (thereby reducing the risk premium.
imposed on domestic borrowers) and because the penalty rate imposed by the central bank falls. Essentially, because the bank borrows more abroad, it borrows less from the central bank, which in turns reduces the penalty component of the refinance rate.

7 Sensitivity Analysis

To assess the sensitivity of the previous results, we conduct several experiments: a stronger response of the penalty rate to the central bank borrowing-deposit ratio, stronger exchange rate smoothing, and full sterilization of foreign exchange market interventions. In addition, we also illustrate the performance of the model with a countercyclical reserve requirement rule, whose optimality is discussed in the next section.

7.1 Sensitivity of Penalty Rate

Consider the case where the parameter that characterizes the setting of the penalty rate, $\vartheta_0^{C,B}$ in (52), takes a value of zero compared to the baseline value of 0.1. In that case, deposits and central bank liquidity are perfect substitutes as sources of liquidity for the bank and $i^R_t = i^C_t$.

The results are illustrated by the dotted lines in Figure 2. The behavior of most variables (except for the marginal cost) are qualitatively the same compared to the benchmark experiment, but there is a critical difference. The reduction in the policy rate (induced by the downward effect on inflation of the initial appreciation, as noted earlier) lowers the return on deposits, which leads to a drop in the demand for that category of assets. All else equal, this induces the bank to borrow more from the central bank. However, the bank also borrows abroad, which tends to reduce the amount that it borrows domestically. Given our calibration, the latter effect dominates and the penalty rate falls on impact. With $\vartheta_0^{C,B}$ positive, the drop in the penalty rate means that the cost of borrowing from the central bank (the refinance rate) falls by more than the drop in the base policy rate, whose behavior is determined by the Taylor rule. By contrast, with $\vartheta_0^{C,B} = 0$, the refinance rate and the policy rate fall by the same amount. Except for inflation, the imperfect substitutability between deposits and central bank borrowing mitigates the procyclical response of the financial system.

7.2 Exchange Rate Smoothing

Consider the case where the central bank engages in more aggressive exchange rate smoothing. This is illustrated by increasing the parameter $\varphi_1^R$ in the reserve target rule (47) from an initial value of 0.5 to 15. The results are displayed in Figure 3. The key difference is that, to mitigate the appreciation, the central bank intervenes more heavily, and therefore accumulates more reserves; with partial sterilization, the money supply expands by more than in the benchmark case, implying that the bond rate
must drop by more than before to restore equilibrium in the money market. However, the additional impact on consumption and the demand for land (and thus real house prices) remain rather subdued.

Because intervention smooths the path of the exchange rate (both nominal and real), it also leads to a smoother path of inflation, and thus of the base policy rate, which further mitigates inflationary pressures over time through a reverse cost channel. In addition, the smoother (future) path of the exchange rate this triggers higher foreign bank borrowing today, implying a larger drop in the central bank borrowing-deposit ratio. In turn, the reduction in the penalty rate translates into a more substantial drop in the refinance rate. Consequently, the fall in the loan rate is also larger, and investment expands by more than in the benchmark case. Thus, more aggressive exchange rate smoothing may actually be expansionary despite stabilizing the behavior of the currency.\textsuperscript{25} As discussed later, this has implications for the degree of aggressiveness of a countercyclical reserve requirement rule.

### 7.3 Full Sterilization

Consider the case now where the central bank engages in full sterilization of foreign exchange market intervention, which corresponds to $\kappa^F = 1$ in (49). Changes in the domestic-currency value of foreign-exchange reserves therefore have no direct effect on the supply of cash.

The results of this experiment are shown in Figure 4. Full sterilization mitigates volatility, first and foremost by smoothing the path of the nominal exchange rate. As a result, inflation and the base policy rate, and thus the refinance and loan rates, are less volatile, with all of these variables remaining below (above) their values in the benchmark case during the initial (later) phase of the adjustment process. However, the impact on the real side of the economy is negligible. These results therefore do not provide much support to some claims in the recent literature, according to which sterilized foreign exchange purchases under inflation targeting in an economy with an active credit channel may have expansionary consequences on aggregate demand through their negative impact on lending rates. Indeed, an argument often made is that even if sterilization succeeds in limiting domestic monetary expansion, it may not completely insulate an economy from the effects of capital inflows. If domestic interest-bearing assets are imperfect substitutes, then a capital inflow may be associated with a shift in the composition of demand for domestic interest-bearing assets, as well as with an increase in the total demand for such assets. In this case, unless the composition of domestic assets issued in sterilization operations matches that demanded by creditors, the structure of domestic asset returns would be altered. In turn, this could trigger a portfolio reallocation which, in the presence of wealth effects, may affect aggregate demand and prices (see Garcia (2012)). This is not the case in our simulations, in part

\textsuperscript{25}Although we do not report them here, similar results hold if the smoothing term in (47) is specified in terms of the real exchange rate, rather than the nominal value.
because the degree of persistence in the reserve accumulation rule, as measured by \( \varphi_2^R \), is quite large.\(^{26}\)

### 7.4 Endogenous Reserve Requirements

In the foregoing discussion it was assumed that the reserve requirement rate, \( \mu_i^R \), is kept constant. As noted earlier, in practice policymakers in MICs have often used reserve requirements as part of a countercyclical toolkit to mitigate credit fluctuations caused by the capital inflows associated with these shocks. Accordingly, we consider now the case where the central bank implements a countercyclical reserve requirement rule that relates changes in \( \mu_i^R \) to deviations in the ratio of investment loans to domestic output sales:

\[
\frac{\mu_i^R}{\mu_i^R} = (\frac{\mu_i^R}{\mu_i^R})\chi_1^R \left\{ \left( \frac{\mu_i^P}{\mu_i^P} \right)^{1-\chi_2^R} \right\},
\]  

where \( \chi_1^R \in (0, 1) \) and \( \chi_2^R > 0.\(^{27}\)

At the outset, it is worth noting that there is a key difference in the model between an increase in the base policy rate and an increase in the reserve requirement rate. On impact, a higher \( \mu_i^R \) lowers the deposit rate (and thus deposits); all else equal (that is, for a given level of foreign borrowing), the drop in deposits induces the bank to borrow more from the central bank. Both effects combine to raise the \( \delta_i^C/B/d_i \) ratio. This leads therefore to an increase in the penalty rate and in the cost of borrowing, \( i_i^C \). Even though in principle \((1 - \mu_i^R)i_i^C\), could either increase or fall, should it increase it will be by less than an increase in \( i_i^C \) induced by a rise in the base policy rate, \( i_i^R \). In fact, if \((1 - \mu_i^R)i_i^C\) falls, then \( i_i^D \) would also fall (see (41)), so an increase in the reserve requirement rate would not exacerbate private capital inflows, in contrast to an increase in the base policy rate. However, in the model capital flows depend directly on the bond rate, and not the deposit rate. Thus, much depends also on the indirect effects of these two policies.

The properties of the model with and without the endogenous rule (59) are illustrated in Figure 5, in the base case where \( \chi_1^R = 0.1 \) and \( \chi_2^R = 8 \); thus, the case considered is that of a fairly aggressive policy with little inertia. On impact, the response of the reserve requirement rate to the initial expansion in credit leads indeed to a drop in bank deposits and to an increase in the central bank borrowing-deposit ratio; as a result, the penalty rate increases, thereby mitigating the initial drop in the refinance rate and thus the lending rate. This dampens also the initial expansion in credit and investment. The supply of cash expands by less, requiring thereby a smaller drop in the bond rate—which in turn weakens incentives to consume today. As a result, aggregate demand expands by less than in the benchmark case; the endogenous response

\(^{26}\)Note that our analysis abstracts from the fact that sterilized intervention entails quasi-fiscal costs.

\(^{27}\)A forward-looking credit-growth rule is discussed in Mimir et al. (2012) and Escudero et al. (2014), whereas Montoro (2011) uses a contemporaneous rule in terms of the level of credit. We report later on an experiment with a credit growth rule.
of the reserve requirement rate is unambiguously countercyclical, in the sense that it mitigates the initial expansion in output, credit, and asset price pressures, even though the policy has more limited effects on inflation and the exchange rate. If anything, the exchange rate appreciates slightly more, because with the bond rate increasing, there is a slight reduction in household holdings of foreign bonds. As result, domestic inflation falls slightly more on impact—and so does the base policy rate.

To further illustrate how this policy operates, it is worth considering two scenarios: the case where (as discussed earlier) the parameter that characterizes the setting of the penalty rate, $\theta_{0}^{CB} = 0$ in (52), and the case where $\theta_{0}^{CB} = 0.15$, compared to the baseline value of 0.1. The first case corresponds to a situation where deposits and central bank borrowing are perfect substitutes. In that case, the base policy rate and the refinance rate are one and the same ($i_{d}^{R} = i_{d}^{F}$); a change in the central bank borrowing-deposit ratio has no effect on the loan rate, and thus no effect on credit and investment. If foreign borrowing does not change, all that happens when deposits fall is that the bank fully offsets the drop in market funding by borrowing more from the central bank. Put differently, for the countercyclical reserve requirement policy to be effective, deposits and central bank borrowing must be imperfect substitutes.\[28\]

The second case is illustrated in Figure 6. The interest rate and aggregate demand effects described in Figure 5 are now magnified. In fact, and in contrast to what obtains in the benchmark case, with the higher value of $\theta_{0}^{CB}$ the refinance rate actually increases on impact, whereas the loan rate barely falls. The smaller the degree of substitutability between deposits and central bank borrowing, the stronger the countercyclical effect of reserve requirements. This result is well illustrated in Figure 7, which shows how the impact effect of the shock on domestic absorption falls with values of $\chi_{2}^{R}$ ranging from 0 to 10.

Finally, we also considered the case where the countercyclical reserve requirement rule is specified not in terms of the credit-to-domestic sales ratio, as in (59), but in terms of deviations in nominal credit growth, $(1 + \pi_{t}^{S})(L_{t}^{R}/L_{t-1}^{R})$.\[29\] Figure 8 illustrates the results with the two rules, together with the benchmark case, for the same values of $\theta_{0}^{CB}$, $\chi_{1}^{R}$, and $\chi_{2}^{R}$ as in Figure 5. The figure shows that the results are qualitatively the same; the credit growth-based rule is also countercyclical, despite generating more volatility initially for some variables.

\[28\]If the shortfall in deposits is absorbed by increased bank borrowing abroad, the associated capital inflow would magnify the exchange rate appreciation, the reduction in inflation, and the expansion in economic activity. If the net effect is a higher base policy rate, the reserve requirement rule could be countercyclical—even if deposits and central bank liquidity are perfect substitutes.

\[29\]We also experimented with real credit growth, by using $L_{t}^{R}/L_{t-1}^{R}$ instead of $(1 + \pi_{t}^{S})L_{t}^{R}/L_{t-1}^{R}$ in (59); the results did not differ much from those reported here.
8 Optimal Reserve Requirement Rule

In the foregoing analysis we have considered arbitrary values of the parameters characterizing the reserve requirement rule (59). We now consider the case where the central bank is concerned with two objectives, macroeconomic stability and financial stability. Extending the methodology described in Agénor et al. (2013, 2014), we define macroeconomic (in)stability, $V_{t}^{M}$, in terms of a weighted average of the volatility of inflation deviations and output deviations, and financial (in)stability in terms of two composite measures: 

- a) a narrow index, $V_{t}^{F,N}$, defined as a weighted average of the volatilities of the ratio of investment loans to domestic sales of the final good, $i_{t}^{l}/Y_{t}^{S}$, the ratio of net capital inflows to domestic sales, $(L_{t}^{F,B} - B_{t}^{F,P})/Y_{t}^{S}$, and the loan-refinance rate spread, $i_{t}^{L} - i_{t}^{C}$; and
- b) a broad index, $V_{t}^{F,B}$, that adds to the variables included in the narrow index the volatility of real asset prices, measured by the volatility of real house prices and the volatility of the real exchange rate. For macroeconomic stability, we use weights of 0.7 in inflation deviations and 0.3 for output deviations. These weights reflect relatively greater concern with price stability. For the narrow financial stability index, the weights are 1/3 on each measure whereas for the broad index the weights are 0.3 each for the three variables included in the narrow index, and a weight of 0.05 on each asset price. In the latter case, the weighting ensures that financial variables continue to dominate in the definition of financial stability. This also reflects the fact that the evidence on the behavior of asset prices prior to financial crises appears to be less robust than other variables, especially credit (see Agénor and Montiel (2015)).

In addition, we also define a composite index of economic stability, $V_{t}$, defined with two sets of weights: first with equal weight 0.5 to each stability objective, and second with a weight of 0.8 for macroeconomic stability and 0.2 for financial stability. Thus, the central bank shows equal concerns with the two objectives in the first case, whereas in the second macroeconomic stability dominates. Formally, the central bank’s instantaneous policy loss function can be written as

$$V_{t} = [V_{t}^{M}(\sigma_{\pi s}^{2}, \sigma_{Ys}^{2})]^\zeta [V_{t}^{F}]^{1-\zeta},$$ (60)

with $\zeta = 0.5, 0.8$ and $^{31}$

$$V_{t}^{F,N} = V_{t}^{F,N}[\sigma_{\pi s/y}^{2}, \sigma_{(L_{F,B}-B_{F,P})/Ys}^{2}, \sigma_{(L_{-1})/Ys}^{2}], \quad V_{t}^{F,B} = V_{t}^{F,B}[V_{t}^{F,N}, \sigma_{\pi}^{2}, \sigma_{2}^{2}].$$

The goal of the central bank is now, for a given value of the persistence parameter $\chi_{1}^{R} = 0.1$ in the countercyclical rule (59), to determine the optimal value of $\chi_{2}^{R}$ so as to minimize the loss function (60).

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$^{30}$Changes in these weights have a relatively limited impact on the results. Individual volatility measures are based on the asymptotic (unconditional) variances of the relevant variable.

$^{31}$The loss function (60) is consistent with studies that take a second-order approximation of the utility of individuals and find that it differs from the standard case by including financial variables. See for instance Andrés et al. (2013) for an analytical derivation.
A numerical solution to this problem is illustrated in Figures 9 and 10, in the first case for $\zeta = 0.5$ and the second for $\zeta = 0.8$, and for both measures of financial stability. The figures show clearly that the relationship between the degree of aggressiveness of the countercyclical reserve rule and economic volatility follows a U-shape pattern. Intuitively, as the policy becomes more aggressive, volatility falls at first, because (as can be inferred from Figures 6, 7 and 8) the policy stabilizes credit, investment and domestic absorption. However, the more aggressive the policy becomes, the more volatile interest rates and deposits become; the volatility in domestic interest rates induces more volatility in capital flows, and therefore tends to increase financial volatility—so much so that it eventually dominates the gains in terms of reduced volatility in credit and aggregate demand. Thus, there exists an optimal value for $\chi_R^2$, which is about 6 in Figure 9 for both measures of financial stability, when the central bank attaches equal weights to macroeconomic stability and financial stability. By contrast, when the central bank attaches a higher weight to macroeconomic stability ($\zeta = 0.8$), Figure 10 shows that the optimal value of $\chi_2^R$ is much higher, at about 11. The reason is that the countercyclical reserve rule is particularly effective at mitigating fluctuations in inflation and output. Interestingly enough, this result can be seen as providing support for central banks that have used reserve requirements as a substitute for monetary policy—a situation where, presumably, preferences for macroeconomic stability are high, as captured by a high value of $\zeta$.

To further study the sensitivity of the results, Figure 11 considers an additional experiment: the case where foreign exchange market intervention aimed at smoothing the exchange rate, as captured by $\varphi_1^R$, is stronger. Only the narrow measure of financial stability is used, but this is sufficient to illustrate the main points of the analysis. The figure shows the results for $\varphi_1^R$ increasing from 0.5 to 1.5. They indicate that the stronger the exchange rate smoothing intervention, the more aggressive should also be the optimal response of the reserve requirement rate to credit. Intuitively, more aggressive smoothing means a smaller real appreciation over the entire adjustment path. Although (as can be inferred from Figure 3) inflation may drop by more initially, given its forward-looking nature, it also follows a smoother path subsequently. As a result, the increase in the base policy rate in the Taylor rule is smaller, implying a larger drop in the loan rate and magnifying the increase in investment and output (see again Figure 3). To counter these effects, the reserve requirement rule must therefore be more aggressive. What this simulation suggests, in effect, is that in principle an optimal combination of the degree of exchange rate smoothing and the degree of aggressiveness of the countercyclical reserve requirement rule—possibly combined with the intensity of sterilization—may exist. Interestingly enough, these combinations of policies have been widely used in middle-income countries, including Brazil and Turkey, in recent years.

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32 This conjecture is supported by an examination of the individual volatility measures that are used to calculate the composite indices of macroeconomic and financial (in)stability.
Concluding Remarks

Central banks in middle-income countries are often confronted with the dilemma of achieving several competing objectives with limited policy instruments. For instance, in response to large capital inflows, they may want to target a stable exchange rate and inflation rate as well as curb credit growth. However, although by raising the policy interest rate to tighten monetary policy the price stability objective may be met, the interest rate increase may attract additional capital inflows and magnify the appreciation of the domestic currency. As a result, central banks have often used reserve requirements as a substitute to monetary policy. Before the global financial crisis, the use of reserve requirements was indeed often motivated by monetary policy or microprudential objectives. Only recently has there been a formal recognition that reserve requirements can help address concerns arising from the procyclicality of the financial system. Accordingly, there has been renewed thinking about the role of reserve requirements as a macroprudential instrument.

In this paper, the performance of a countercyclical reserve requirement rule was studied in a dynamic stochastic model of a small open economy with financial frictions, imperfect capital mobility, a managed float regime, and sterilized foreign exchange market intervention. Deposits and central bank liquidity were also assumed to be imperfect substitutes as sources of bank funding. This was captured by assuming that the rate at which banks can borrow from the monetary authority incorporates a premium (above and beyond a base policy rate), which depends on the ratio of central bank borrowing to deposits, as a result of a stigma effect. The model was calibrated and used to study the effects of a temporary drop in the world risk-free interest rate. Consistent with the stylized facts, the simulations showed that this shock triggers an expansion in domestic credit and activity, asset price pressures, and a real appreciation. We also showed that a credit-based reserve requirement rule helps to mitigate both macroeconomic and financial volatility, with the latter defined both in terms of a narrow measure based on the credit-to-output ratio, the ratio of capital flows to output, and interest rate spreads, and a broader measure that includes also real asset prices. An optimal rule, based on minimizing a composite measure of economic stability (a loss function combining measures of both macroeconomic and financial stability), was also derived. The relationship between the degree of aggressiveness of the countercyclical reserve rule and economic volatility was shown to be nonmonotonic. At first, as the policy becomes more aggressive, volatility falls because it stabilizes credit, investment and domestic absorption. As the policy becomes more aggressive, it magnifies volatility in interest rates and bank deposits; in turn, higher volatility in domestic interest rates induces more volatility in capital flows, and therefore tends to increase financial volatility—so much so that it eventually dominates the gains in terms of reduced volatility in credit growth and aggregate demand.

Sensitivity tests, related to the intensity of sterilization, the degree of exchange rate smoothing, and the rule used by the central bank to set the cost of bank borrowing, were also performed, both in terms of the transmission process and the optimal rule. Among
other results, it was shown that the stronger the exchange rate smoothing policy is, the stronger should be the optimal response to credit growth in the countercyclical reserve rule. These results imply not only that an optimal countercyclical reserve requirement policy does exist, but also that in principle an optimal instrument combination (the degree of exchange rate smoothing, and the aggressiveness of the countercyclical reserve requirement rule, and possibly the intensity of sterilization) can be derived. Interestingly enough, these combinations of policies—supplemented by temporary capital controls in some cases—have been widely used in middle-income countries in recent years.

The foregoing analysis can be extended in a number of directions. First, the use of central bank bonds (held by commercial banks) for sterilization purposes could be added, to capture a common practice in some middle-income countries (see for instance Vargas et al. (2010, 2013)). Second, the model could be extended to account for the fact that reserve requirements represent a tax on banking activity, which may have an adverse effect on the financial condition and credit of depository institutions relative to that of other financial institutions. The bank’s optimization problem could thus be generalized to account for the fact that banks have an incentive to reduce the tax-like impact of (unremunerated) reserve requirements by evading them. Moreover, if changes in reserve requirements lead to disintermediation away from the banking sector and toward less-regulated channels, the consequence may be to distort markets and weaken financial stability. This may entail long-run costs, which may exceed the short-run stabilization benefits that the policy may generate. Finally, it may be useful to address the issue of the joint determination of the degree of exchange rate smoothing, the strength of sterilization, and the optimal response to credit growth in setting the reserve requirement rate.

Notwithstanding these various extensions, as it stands our analysis provides important lessons for policymakers in middle-income countries confronted with the dilemmas associated with external financial shocks. Our results support the view that to dampen the potentially destabilizing effects of large capital flows on asset prices and credit markets, countercyclical reserve requirements can be highly effective. Raising interest rates to contain aggregate demand pressures during episodes to sudden floods can be self-defeating, as this may induce more capital inflows; under such circumstances raising reserve requirements may help to not only contain aggregate demand, but also credit growth. This is an important message in the context of the ongoing debate about the choice and implementation of macroprudential instruments.

33Robitaille (2011) discusses how reserve requirement policy in Brazil taxes large banks to subsidize small banks that are exempt, but that over time banks have shifted from demand deposits with a high reserve requirement to other funding sources such as certificates of deposits.

34If banks hold excess reserves, reserve requirements are generally non-binding and the incentive to mitigate their impact would of course reduced.

35In Appendix D we discuss another potential channel through which reserve requirements can be countercyclical, through their impact on production costs. Preliminary results, however, suggest that this result is difficult to establish for a range of plausible values. We intend to pursue this line of research as well in the future.
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Table 1
Benchmark Calibration: Key Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Household</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.985</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
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<td>Elasticity of intertemporal substitution</td>
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<tr>
<td>$\eta_N$</td>
<td>10.0</td>
<td>Preference parameter for leisure</td>
</tr>
<tr>
<td>$\eta_x$</td>
<td>0.02</td>
<td>Preference parameter for money holdings</td>
</tr>
<tr>
<td>$\eta_H$</td>
<td>0.02</td>
<td>Preference parameter for housing</td>
</tr>
<tr>
<td>$\nu$</td>
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<td>Share parameter in index of money holdings</td>
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<tr>
<td>$\theta_{F,P}$</td>
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<td>Sensitivity of risk premium, household foreign bonds</td>
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<tr>
<td><strong>Production</strong></td>
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<tr>
<td>$\Lambda_D$</td>
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<td>distribution parameter, final good</td>
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<tr>
<td>$\eta$</td>
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<td>Elasticity of substitution, baskets of intermediate goods</td>
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<td>$\mu^F$</td>
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<td>Exchange rate pass-through, imported goods</td>
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<tr>
<td>$\kappa$</td>
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<td>Price elasticity of exports</td>
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<tr>
<td>$\theta_{D,F}$</td>
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<td>Share of capital, domestic intermediate goods</td>
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<td>$\phi_D$</td>
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<td>Share of labor costs financed in advance</td>
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<td><strong>Commercial Bank</strong></td>
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<tr>
<td>$\kappa$</td>
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<td>Effective collateral-loan ratio</td>
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<tr>
<td>$\varphi_1$</td>
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<td>Elasticity of repayment prob, collateral</td>
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<tr>
<td>$\varphi_2$</td>
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<td>Elasticity of repayment prob, cyclical output</td>
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<td>$\mu^R$</td>
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<td>Degree of interest rate smoothing</td>
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<td>Response of base policy rate to inflation deviations</td>
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<td>Response of base policy rate to output deviations</td>
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<td>Exchange rate smoothing parameter, foreign reserves rule</td>
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<td>Persistence parameter, foreign reserves rule</td>
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<td>Sterilization coefficient</td>
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<td>Persistence coefficient, reserve requirement rule</td>
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<td>$\chi_2^R$</td>
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<td>Sensitivity to credit growth, reserve requirement rule</td>
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<td><strong>World interest rate</strong></td>
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<td>$\rho_W$</td>
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<td>Persistence, shock to world risk-free rate</td>
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Figure 1
Agents, Production Structure and Main Real and Financial Flows in a Small Open Economy
Note: Interest rates, inflation rate and the repayment probability are measured in absolute deviations, that is, in the relevant graphs a value of 0.05 for these variables corresponds to a 5 percentage point deviation in absolute terms. The real exchange rate is defined as the ratio of the nominal exchange rate divided by the price of domestically-produced final goods sold on the domestic market.
Figure 3
Experiment: Transitory Drop in the World Risk-Free interest Rate
Low and High Intensity of Nominal Exchange Rate Smoothing
(Deviations from steady state)

Note: Interest rates, inflation rate and the repayment probability are measured in absolute deviations, that is, in the relevant graphs a value of 0.05 for these variables corresponds to a 5 percentage point deviation in absolute terms. The real exchange rate is defined as the ratio of the nominal exchange rate divided by the price of domestically-produced final goods sold on the domestic market.
Figure 4
Experiment: Transitory Drop in the World Risk-Free interest Rate
Partial and Full Sterilization of Foreign Exchange Intervention
(Deviations from steady state)

Note: Interest rates, inflation rate and the repayment probability are measured in absolute deviations, that is, in the relevant graphs a value of 0.05 for these variables corresponds to a 5 percentage point deviation in absolute terms. The real exchange rate is defined as the ratio of the nominal exchange rate divided by the price of domestically-produced final goods sold on the domestic market.
Figure 5
Experiment: Transitory Drop in the World Risk-Free interest Rate
Exogenous and Endogenous Countercyclical Reserve Requirement Rule,
\( \chi^R_1 = 0.1, \chi^R_2 = 8, \theta^C_B = 0.1 \)
(Deviations from steady state)

Note: Interest rates, inflation rate and the repayment probability are measured in absolute deviations, that is, in the relevant graphs a value of 0.05 for these variables corresponds to a 5 percentage point deviation in absolute terms. The real exchange rate is defined as the ratio of the nominal exchange rate divided by the price of domestically-produced final goods sold on the domestic market.
Figure 6
Experiment: Transitory Drop in the World Risk-Free interest Rate
Exogenous and Endogenous Countercyclical Reserve Requirement Rule,
\[ \chi^R_1 = 0.1, \chi^R_2 = 8, \theta^{CB}_0 = 0.15 \]
(Deviations from steady state)

Note: Interest rates, inflation rate and the repayment probability are measured in absolute deviations, that is, in the relevant graphs a value of 0.05 for these variables corresponds to a 5 percentage point deviation in absolute terms. The real exchange rate is defined as the ratio of the nominal exchange rate divided by the price of domestically-produced final goods sold on the domestic market.
Figure 7
Experiment: Transitory Drop in the World Risk-Free Interest Rate Impact Response of Domestic Absorption for Different Values of the Sterilization Coefficient and Aggressiveness of the Countercyclical Reserve Requirement Rule

Note: $\kappa^F$ is the sterilization coefficient, which varies between 0 and 1, and $\chi^F$ is a positive parameter that measures the response to credit growth in the countercyclical reserve requirement rule. The vertical axis measures percentage deviations of the first period response of domestic absorption (defined as the sum of private consumption, investment, and government spending) from its steady-state value.
Figure 8
Experiment: Transitory Drop in the World Risk-Free interest Rate
Alternative Endogenous Countercyclical Reserve Requirement Rules, 
\( \chi_1^R = 0.1, \chi_2^R = 8, \theta_{CB} = 0.1 \)
(Deviations from steady state)

Note: Interest rates, inflation rate and the repayment probability are measured in absolute deviations, that is, in the relevant graphs a value of 0.05 for these variables corresponds to a 5 percentage point deviation in absolute terms. The real exchange rate is defined as the ratio of the nominal exchange rate divided by the price of domestically-produced final goods sold on the domestic market.
Note: $\chi^R_2$ is a positive parameter that measures the response to credit growth in the countercyclical reserve requirement rule. The vertical axis measures economic volatility, in terms of a composite index of macroeconomic volatility and a composite index of financial volatility, both narrow and broad, as defined in the text.
Figure 10
Index of Economic Volatility and Aggressiveness of the Countercyclical of the Reserve Requirement Rule Weight of 0.8 to Macro, 0.2 to Financial Volatility

Note: $\chi^2_2$ is a positive parameter that measures the response to credit growth in the countercyclical reserve requirement rule. The vertical axis measures economic volatility, in terms of a composite index of macroeconomic volatility and a composite index of financial volatility, both narrow and broad, as defined in the text.
Figure 11
Index of Economic Stability (Equal Weights)
with Narrow index of Financial Stability
and for Alternative Values of Exchange Rate Smoothing

\( \phi^R_1 = 1.5 \)
(right scale)

\( \phi^R_1 = 0.5 \)
(left scale)