ISSN 1518-3548

BANCO CENTRAL DO BRASIL

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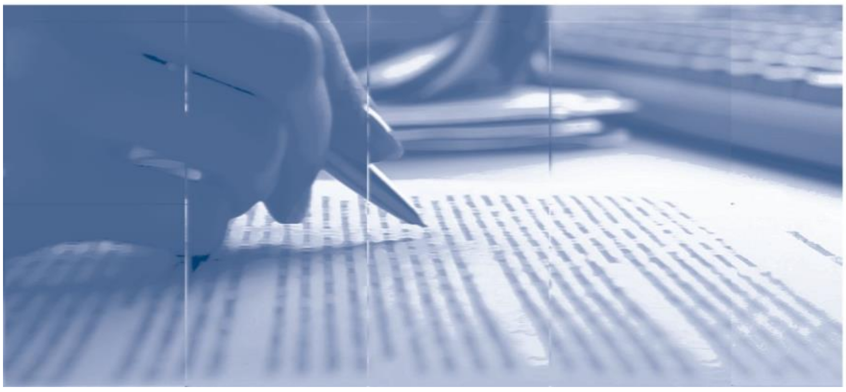
December, 2014

Working Papers



376

ISSN 1518-3548
CGC 00.038.166/0001-05

Working Paper Series	Brasília	n. 376	December	2014	p. 1-60

Working Paper Series

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Determinacy and Learnability of Equilibrium in a Small Open Economy with Sticky Wages and Prices*

Eurilton Araújo**

Abstract

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If the central bank attempts to minimize a welfare loss function in a small-open economy model with nominal wage and price rigidities, it has been argued that a monetary policy rule that responds to consumer price index (CPI) inflation performs better than rules that react to competing inflation measures. From the perspective of determinacy and learnability of rational expectations equilibrium (REE), this paper suggests that a rule that responds to CPI inflation does not significantly increase the central bank's ability to promote the convergence of an economy to a determinate and learnable REE, nor does it hasten the speed of this convergence when compared with rules that react to contending inflation measures.

Keywords: determinacy, inflation, learnability, monetary policy rules
JEL Classification: E13, E31, E52

* I acknowledge financial support from the Brazilian Council of Science and Technology (CNPq). The views expressed in this paper are my own and should not be interpreted as representing the positions of the Banco Central do Brasil or its board members.

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1 Introduction

Despite the behavior of many inflation-targeting central banks, most research on small-open economies with sticky prices, such as Galí and Monacelli (2005), suggests that the monetary authority should respond to domestic inflation rather than to consumer price index (CPI) inflation. In a small open economy with sticky wages and prices, Rhee and Turdaliev (2013) and Campolmi (2014) compared monetary policy rules that reacted to different inflation measures according to their welfare losses. In this context, they found that CPI inflation performed better than some contending measures, including domestic inflation.

Since researchers have also assessed the desirability of an interest rate rule by examining the determinacy and learnability properties that this rule induces in equilibrium, this paper reevaluates the results presented in Rhee and Turdaliev (2013) and Campolmi (2014) from the perspective of determinacy and learnability of rational expectations equilibrium (REE). In the context of an open-economy model featuring nominal wage and price rigidities, the main contribution of this paper is to revisit, through the lens of determinacy and learnability, the following questions: which inflation index should monetary policy rules react to and should these rules include responses to the exchange rate?

This paper also addresses the effects of introducing nominal wage rigidity and trade openness on determinacy and learnability conditions associated with competing interest rate rules. Moreover, following Ferrero (2007) and Christev and Slobodyan (2013), I investigate how the specification of interest rate rules matters for the speed of convergence of an economy to a determinate and E-stable REE through an adaptive learning process. Indeed, the speed of learning is an additional yardstick through which I evaluate the set of monetary policy rules studied in this paper.

The main finding of this paper suggests that, when compared with rules that react to competing inflation measures, the rule that responds to CPI inflation does not provide any noticeable improvement in the central bank's ability to promote convergence of an economy to a determinate and learnable REE. Moreover, for this rule, the learning algorithm converges slowly or with the same speed implied by alternative rules. Hence, in contrast to the evaluation based on welfare losses, the rule that responds to CPI inflation does not exhibit superior performance from the viewpoint of determinacy and E-stability.

Next, I introduce some basic ideas on equilibrium determinacy and learnability. Following this initial discussion, I then briefly review the related literature.

Equilibrium determinacy and learnability, also known as expectational stability (E-stability), have become important criteria for the design and evaluation of monetary policy rules in new Keynesian models. By definition, a determinate REE is unique, free from self-fulfilling fluctuations, and non-explosive. In addition, a REE is E-stable if agents who do not initially possess rational expectations coordinate upon it after using least-squares adaptive learning methods to acquire knowledge about the law of motion governing macroeconomic dynamics. A suitably designed interest rate rule promotes the convergence of an economy to a determinate and E-stable REE.

Evans and Honkapohja (2001) and Bullard and Mitra (2002, 2007) provided the foundations to analyze determinacy and E-stability of equilibria under the assumption of rational expectations. Since the publication of these papers, a burgeoning literature examining conditions that ensure determinacy and E-stability of REE in extensions of the new Keynesian model has emerged. For instance, Llosa and Tuesta (2009) studied a model featuring the cost channel of monetary policy and Duffy and Xiao (2011) investigated the effects of capital accumulation on learnability of REE.

The literature has also considered the role of labor market frictions for determinacy and E-stability of REE. In this respect, Kurozumi and Van Zandweghe (2012) introduced search and matching frictions and Best (2012) studied a closed economy model with nominal wage and price rigidities as described in Erceg et al. (2000). Furthermore, a branch of the literature has focused on introducing asset markets in the new Keynesian framework and gauging their effects on standard results concerning determinacy and learnability of REE. Machado (2013), Xiao (2013) and Pfajfar and Santoro (2014) are examples of this line of research.

The literature reviewed above has focused on closed economy models. Though extensions of the new Keynesian model incorporating open economy considerations abound, papers emphasizing issues concerning determinacy and learnability of REE in open-economy environments are relatively scarce. Linnemann and Schabert (2006), Llosa and Tuesta (2008), Bullard and Schaling (2009) and Zanna (2009) exemplify the research on determinacy and learnability of equilibria in open-economy models.

Llosa and Tuesta (2008) studied the small open economy model developed by Galí and Monacelli (2005) and showed that the degree of openness interacted with particular interest rate rules, changing the relevance of the Taylor principle as a condition ensuring determinacy and E-stability of REE. In a previous paper, Linnemann and Schabert (2006) found similar results by studying a more restricted class of interest rate rules that responded solely to inflation measures.

Bullard and Schaling (2009) analyzed a two country model and pointed out that rules that included responses to international economic conditions engendered feedback between the two economies even in the case of no explicit feedback caused by other factors. Finally, Zanna (2009) studied purchasing power parity (PPP) exchange rate rules in a small open economy model and concluded that these rules, adopted by some governments in developing economies, induced indeterminate and learnable sunspot equilibria.

This paper is closely related to the research mentioned in the preceding paragraphs. For instance, Llosa and Tuesta (2008) and Best (2012) are special cases of the small open economy model of this paper. In fact, the model in Llosa and Tuesta (2008) corresponds to setting the degree of wage rigidity to zero whereas the specification in Best (2012) is equivalent to the situation of zero degree of openness. In contrast to these papers, I emphasize determinacy and learnability properties as performance criteria to address the question of which inflation measure should a central bank respond to.

In this paper, I present numerical results on determinacy and E-stability of equilibria for a small open economy model with sticky nominal wages and prices in which interest rate rules respond to alternative inflation measures. As illustrated by Colciago (2011) for a closed economy with rule-of-thumb consumers and capital, the introduction of nominal wage rigidity substantially alters the determinacy properties of interest rate rules. Therefore, it is important to disentangle the role played by different forms of nominal rigidities for determinacy and E-stability of REE and gauge how the addition of nominal wage rigidity changes determinacy and learnability properties of monetary policy rules in a small open economy.

The paper proceeds as follows. Section 2 sets out the model. Section 3 presents the numerical findings on determinacy and E-stability of REE. Section 4 discusses the speed of convergence of determinate and E-stable equilibria. Finally, the last section concludes. An appendix provides details of the model.

2 The model

In this section, I present the log-linear approximation of the model discussed in Rhee and Turdaliev (2013) and Campolmi (2014). This model is the small-open economy studied in Galí and Monacelli (2005) with the labor market characteristics found in Erceg et al. (2000). There is monopolistic competition in the labor market and households set nominal wages following the scheme proposed by Calvo (1983).

Regarding the introduction of nominal wage rigidity, Christiano et al. (2005) and Smets and Wouters (2007) pointed out that this type of nominal rigidity improved the ability of closed-economy medium-scale models to fit U.S. data. This improvement has also been documented for open-economy medium-scale models, as stressed by Adolfson et al. (2008), Jääskelä and Nimark (2011) and Dong (2013).

As pointed out by Campolmi (2014), sticky nominal wages alter the dynamics of the Galí and Monacelli (2005) model since wage markups fluctuate in response to domestic and foreign shocks. In the presence of nominal wage rigidity, fluctuations in CPI inflation induce movements in real wages, which affect wage markups. Hence, changes in wage markups lead to fluctuations in wage inflation and in firms' marginal costs. These movements translate into more volatile wage and domestic inflation rates. In this context, the central bank improves welfare by reacting to CPI inflation since this reaction directly reduces the volatility of wage inflation and indirectly curtails the volatility of domestic inflation. By decreasing these volatilities, the central bank also promotes welfare-enhancing reductions in wage and price dispersions.

The change in model dynamics engendered by the introduction of sticky nominal wages has implications for determinacy and E-stability of REE. The investigation of these implications is therefore the goal of this paper.

After I present the basic equations of the model, I discuss alternative interest rate rules representing how the central bank conducts monetary policy and I then describe how to calibrate the parameters.

2.1 The environment

Following Galí and Monacelli (2005), I consider a small open economy as one among a continuum of infinitesimally small economies constituting the world economy. There is a continuum of households who populate this small open economy; each household derives utility from the consumption of both domestically produced and imported goods and disutility from labor. In the goods market, there is monopolistic competition and prices are sticky, following the price-setting scheme of Calvo (1983). Labor is immobile across countries and there is monopolistic competition in the labor market with workers setting their wages in a way similar to how firms set their prices. In addition, markets are complete at the international level and the law of one price always holds for individual goods.

After the log-linearization of the equilibrium conditions around the steady state, the following equations describe the small open economy:

$$\widetilde{y}_t = E_t\left(\widetilde{y}_{t+1}\right) - \frac{1}{\sigma_\alpha}\left[r_t - E_t\left(\pi_{H,t+1}\right)\right] - \frac{rr_t^n}{\sigma_\alpha} \tag{1}$$

$$\pi_{H,t} = \beta E_t(\pi_{H,t+1}) + \kappa_{ph}\widetilde{y}_t + \lambda_{ph}\widetilde{w}_t \tag{2}$$

$$\pi_{W,t} = \beta E_t(\pi_{W,t+1}) + \kappa_w \widetilde{y}_t - \lambda_w \widetilde{w}_t \tag{3}$$

$$\widetilde{w}_t = \widetilde{w}_{t-1} + \pi_{W,t} - \pi_{H,t} - \alpha\sigma_\alpha \left(\widetilde{y}_t - \widetilde{y}_{t-1}\right) + \alpha\Delta s_t^n - \Delta w_t^n \tag{4}$$

The variables $\widetilde{y}_t$, $\pi_{H,t}$, $\pi_{W,t}$, $\widetilde{w}_t$ and r_t represent the domestic output gap, domestic inflation, nominal wage inflation (the log difference in nominal wages), the real wage gap and the domestic interest rate. The output gap is the difference between domestic output and the natural level of output, both measured in log. The definition of the real wage gap is analogous. The natural level of output and the natural real wage are the output level and the real wage that would occur in the absence of nominal rigidities. Besides this, rr_t^n, Δs_t^n and Δw_t^n denote the natural level of the real interest rate, the variation in the natural level of the terms of trade and the variation in the natural real wage. These variables are conceptually similar to the natural level of output.

The expectation operator E_t represents rational expectations as well as alternative expectation formation mechanisms. The system (1) to (4) is thus valid both under rational expectations and under adaptive learning[1].

Equation (1) is the Euler equation that arises from the household choice problem and describes the aggregate demand in the artificial economy, also known as the dynamic IS equation. Equation (2) is the new Keynesian Phillips curve that arises from firms' price-setting behavior. Equation (3) is the wage inflation equation due to the introduction of monopolistic competition and nominal rigidity in the labor market. Finally, equation (4) combines an identity linking the variation in the real wage gap to the variation in the

[1] I am assuming that the law of iterated expectations holds in order to derive the approximate linear model.

natural real wage gap with a relationship between the terms of trade gap, denoted by $\widetilde{s}_t$, and the output gap $\widetilde{y}_t$. The identity is $\Delta\widetilde{w}_t = \Delta w_t - \Delta w_t^n = \pi_{W,t} - \pi_{CPI,t} - \Delta w_t^n$, where $\pi_{CPI,t}$ is CPI inflation. The relationship between $\widetilde{s}_t$ and $\widetilde{y}_t$ is given by the expression[2] $\widetilde{s}_t = \sigma_\alpha \, \widetilde{y}_t$.

The equations above involve several deep parameters. The parameter β stands for the discount factor, σ measures the degree of relative risk aversion implicit in the utility function, φ is the inverse of the labor supply elasticity, γ is the elasticity of substitution between imported goods, η is the elasticity of substitution between domestic and foreign goods and α is the inverse of the home bias in preferences, which is a measure of the degree of trade openness. The degrees of price and wage stickiness are θ_{ph} and θ_w, which correspond to the probabilities that prices and wages will not be adjusted under the scheme proposed by Calvo (1983). Finally, the parameter ε_w stands for the elasticity of substitution between distinct types of labor in the monopolistic competitive labor market. Since the production technology is linear in labor, the parameter ε, which denotes the elasticity of substitution between types of goods produced in the home country, does not influence any equation after the log-linearization of the equilibrium conditions around the steady state.

The composite parameters κ_{ph}, λ_{ph}, κ_w, λ_w, σ_α and Ω are convolutions of the deep parameters described in the preceding paragraph. The expressions defining these parameters are: $\kappa_{ph} = \alpha\sigma_\alpha\lambda_{ph}$, $\kappa_w = (\sigma - \alpha\sigma_\alpha\Omega + \varphi)\lambda_w$, $\lambda_{ph} = \frac{(1-\theta_{ph})(1-\beta\theta_{ph})}{\theta_{ph}}$, $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w\varphi)}$, $\sigma_\alpha = \frac{\sigma}{(1-\alpha)+\alpha\Omega}$ and $\Omega = \gamma\sigma + (1-\alpha)(\eta\sigma - 1)$.

The degree of openness (α) affects the dynamics of price and wage inflation since the slopes κ_{ph} and κ_w depend on it. The parameter α also influences the dynamic IS equation since σ_α governs the sensitivity of the output gap to the real interest rate. In sum, the interactions between the de-

[2] Galí and Monacelli (2005) obtain this expression by manipulating equations (29) and (34) of their paper.

gree of openness and nominal rigidities shape the dynamics of the small-open economy.

Next, I present some ancillary equations that are helpful in analyzing the model. I show these equations in their log-linearized forms.

The assumption of complete international financial markets leads to the uncovered interest rate parity condition $E_t(\Delta e_{t+1}) = r_t - r_t^*$, where e_t is the nominal exchange rate and r_t^* is the world interest rate.

The expression $s_t = p_{F,t} - p_{H,t}$ defines the terms of trade, where $p_{F,t}$ and $p_{H,t}$ are the log-level of foreign prices and domestic prices in domestic currency. The definition of the terms of trade leads to $\Delta s_t = \pi_{F,t} - \pi_{H,t}$, where $\pi_{F,t}$ and $\pi_{H,t}$ are foreign and domestic inflation, measured as the log-difference in price levels.

The definition of CPI inflation is $\pi_{CPI,t} = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t}$. Using the equation $\Delta s_t = \pi_{F,t} - \pi_{H,t}$, the expression[3] $\pi_{CPI,t} = \pi_{H,t} + \alpha\Delta s_t$ emerges.

The equation $p_{F,t} = e_t + p_t^*$ holds under the law of one price and implies $\pi_{F,t} = \Delta e_t + \pi_t^*$, where p_t^* stands for the log-level of the world price index and $\pi_t^* = p_t^* - p_{t-1}^*$ is the world inflation rate.

By combining $\Delta s_t = \pi_{F,t} - \pi_{H,t}$ and $\pi_{F,t} = \Delta e_t + \pi_t^*$, one gets $\Delta s_t = \Delta e_t + \pi_t^* - \pi_{H,t}$. The terms of trade gap is the log-deviation of the terms of trade from their natural levels, which are the terms of trade that would have prevailed in the absence of nominal rigidities. After using this definition, the expression $\Delta s_t = \Delta e_t + \pi_t^* - \pi_{H,t}$ becomes $\widetilde{s}_t = \widetilde{s}_{t-1} + \Delta e_t + \pi_t^* - \pi_{H,t} + \Delta s_t^n$.

Following Llosa and Tuesta (2008), I assume that the global variables r_t^* and π_t^* are kept constant at their steady state levels, which are both normalized to zero. The variables rr_t^n, Δs_t^n and Δw_t^n are driven by mutually independent first-order autoregressive processes. This is an assumption that

[3] To derive equation (4), I use this expression to substitute out CPI inflation.

facilitates determinacy and E-stability analysis based on the system (1) to (4) supplemented by an interest rate rule. In fact, the microfoundations of the model relate these variables to domestic technology and foreign output shocks.

2.2 Interest rate rules

A monetary policy rule for the domestic interest rate r_t complements the system (1) to (4). This complete dynamic system of stochastic difference equations describes an equilibrium.

I follow Llosa and Tuesta (2008) and consider two alternative specifications: contemporaneous data and forecast-based data.

The expression for the contemporaneous specification is:

$$r_t = \phi_r r_{t-1} + \phi_p \pi_{m,t} + \phi_y \widetilde{y}_t + \phi_e \Delta e_t \tag{5}$$

The forecast-based data interest rate rule is:

$$r_t = \phi_r r_{t-1} + \phi_p E_t \pi_{m,t+1} + \phi_y E_t \widetilde{y}_{t+1} + \phi_e E_t \Delta e_{t+1} \tag{6}$$

The variable $\pi_{m,t}$ stands for some particular inflation measure m. I consider the following inflation measures: domestic inflation ($\pi_{H,t}$), CPI inflation ($\pi_{CPI,t}$), nominal wage inflation ($\pi_{W,t}$) and a composite inflation, denoted by $\pi_{COM,t}$ and computed according to the formula: $\pi_{COM,t} = (1 - \frac{\lambda_{ph}}{\lambda_{ph}+\lambda_w})\, \pi_{H,t} + \frac{\lambda_{ph}}{\lambda_{ph}+\lambda_w} \pi_{W,t}$. Rhee and Turdaliev (2013) considered these inflation indices whereas Campolmi (2014) looked at the first three measures.

The composite measure deserves some explanation. In the model with $\alpha = 0$, which corresponds that proposed by Erceg et al. (2000), if the elasticity of substitution between types of goods is proportional to the elasticity of substitution between types of labor and $\kappa_{ph} = \kappa_w$, equations (2) and (3) collapse into a single new Keynesian Phillips curve written in terms of $\pi_{COM,t}$. Furthermore, in this situation, there is no trade-off between stabilizing $\widetilde{y}_t$ and $\pi_{COM,t}$ and the optimal monetary policy is to target and fully stabilize $\pi_{COM,t}$.

Even when the conditions described above are not satisfied, $\pi_{COM,t}$ is still an interesting inflation measure because the stickier component is weighted more heavily, emphasizing that monetary policy should attenuate market distortions due to nominal rigidities by stabilizing an inflation measure that underlines their role in macroeconomic dynamics. In fact, since the weight $\frac{\lambda_{ph}}{\lambda_{ph}+\lambda_w}$ is increasing (decreasing) with the degree of wage (price) rigidity, the relative weight of domestic price (wage) inflation in $\pi_{COM,t}$ is increasing with the degree of price (wage) stickiness.

The parameters describing the rules are ϕ_r, capturing interest rate inertia, and the coefficients ϕ_p, ϕ_y and ϕ_e, capturing the response of the interest rate to the macroeconomic variables $\pi_{m,t}$, $\widetilde{y}_t$ and Δe_t or to their expectations $E_t\pi_{m,t+1}$, $E_t\widetilde{y}_{t+1}$ and $E_t\Delta e_{t+1}$.

For each rule, I consider the following cases: benchmark ($\phi_r = \phi_e = 0$), managed exchange rate ($\phi_r = 0$), interest rate smoothing ($\phi_e = 0$) and managed exchange rate with interest rate smoothing, which implies no restrictions on the coefficients ϕ_r and ϕ_e.

To integrate the rules (5) and (6) in the system (1) to (4), I have to eliminate Δe_t and $E_t\Delta e_{t+1}$ from these interest rate rules. In equation (5), I combine expressions $\widetilde{s}_t = \widetilde{s}_{t-1} + \Delta e_t + \pi_t^* - \pi_{H,t} + \Delta s_t^n$ and $\widetilde{s}_t = \sigma_\alpha \, \widetilde{y}_t$ to substitute out Δe_t. In equation (6), I use the uncovered interest rate parity condition $E_t(\Delta e_{t+1}) = r_t - r_t^*$ to eliminate $E_t\Delta e_{t+1}$. After these manipulations and the normalization of r_t^* and π_t^*, the expressions for the interest rules are:

$$
\begin{aligned}
r_t &= \phi_r r_{t-1} + (\phi_p g_1 + \phi_p g_3 + \phi_e)\pi_{H,t} + (\phi_p g_2)\pi_{W,t} \\
&\quad + [\phi_y + \sigma_\alpha(\phi_p g_3 + \phi_e)]\widetilde{y}_t - \sigma_\alpha(\phi_p g_3 + \phi_e)\widetilde{y}_{t-1} \\
&\quad - (\phi_p g_3 + \phi_e)\Delta s_t^n
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
r_t &= \frac{\phi_r}{(1-\phi_p g_3 - \phi_e)} r_{t-1} + \frac{\phi_p g_1}{(1-\phi_p g_3 - \phi_e)} E_t \pi_{H,t+1} \\
&\quad + \frac{\phi_p g_2}{(1-\phi_p g_3 - \phi_e)} E_t \pi_{W,t+1} + \frac{\phi_y}{(1-\phi_p g_3 - \phi_e)} E_t \widetilde{y}_{t+1}
\end{aligned} \tag{8}
$$

In equations (7) and (8), I introduce the auxiliary parameters g_1, g_2 and g_3, which control the inflation measures to which the central bank responds. In this context, the possible configurations are:

- domestic inflation ($\pi_{H,t}$): $g_1 = 1$, $g_2 = 0$ and $g_3 = 0$
- CPI inflation ($\pi_{CPI,t}$): $g_1 = 1 - \alpha$, $g_2 = 0$ and $g_3 = \alpha$
- nominal wage inflation ($\pi_{W,t}$): $g_1 = 0$, $g_2 = 1$ and $g_3 = 0$
- composite inflation ($\pi_{COM,t}$): $g_1 = 1 - \frac{\lambda_{ph}}{\lambda_{ph}+\lambda_w}$, $g_2 = \frac{\lambda_{ph}}{\lambda_{ph}+\lambda_w}$ and $g_3 = 0$

The complete system characterizing the equilibrium comprises equations (1) to (4) and either (7) or (8) as a description of monetary policy.

2.3 Calibration

In the baseline calibration of the model, I choose the parameters as follows.

- *Preferences.* Following Rhee and Turdaliev (2013) and Campolmi (2014), I set the elasticity of labor supply to $\frac{1}{3}$ by specifying $\varphi = 3$, and the discount factor is $\beta = 0.99$. The coefficient of risk aversion is $\sigma = 5$, which is the value employed by Llosa and Tuesta (2008).

- *Goods and labor markets.* Again, I stick to the values found in Rhee and Turdaliev (2013) and Campolmi (2014). Therefore, $\varepsilon_w = 6$ and $\theta_{ph} = \theta_w = 0.75$.

- O*pen-economy parameters.* Following Galí and Monacelli (2005), Llosa and Tuesta (2008) and Campolmi (2014), I set $\alpha = 0.4$. This figure matches the import to GDP ratio for Canada. In accordance with Llosa and Tuesta (2008), I set $\gamma = 1$ and $\eta = 1.5$. According to Campolmi (2014), this parameterization, with $\Omega > 1$, corresponds to the case in which domestic and foreign goods are substitutes in utility.

To perform robustness checks of the benchmark rules, I focus on the role of introducing nominal wage rigidity and trade openness. To study the effect of each feature on the equilibrium, I change only one of the parameters in each alternative scenario, keeping the others constant at their baseline values. I consider the following additional scenarios: a situation with price rigidity only in which $\theta_w = 0$ and a closed economy scenario in which $\alpha = 0$.

3 Determinacy and E-stability

It would be nice to derive analytical results characterizing determinacy and learnability of REE under alternative interest rate rules. This derivation, however, is not always possible, except in some special cases, such as the small open economy model investigated in Llosa and Tuesta (2008). Indeed, with the addition of sticky nominal wages, the dimension of the system is now five and its reduction to a lower dimension, which would facilitate analytical results, is too complicated in the small open economy model with nominal wage and price stickiness. For this reason, I adopt a simulation approach and provide numerical findings.

This same approach is present in some other papers that have investigated complex models, such as Duffy and Xiao (2011), Kurozumi and Van Zandweghe (2012) and Xiao (2013). The simulation exercise goes as follows. For a given interest rate rule, I vary the coefficients of inflation and the output gap over a specified range of values. This range defines a fine grid of points that covers plausible and empirically relevant scenarios. For each pair of coefficients of inflation and the output gap, I check the conditions for determinacy and E-stability of REE. Next, I describe and discuss these conditions.

3.1 Methodology

I can write the system (1) to (4) supplemented by (7) or (8) as:

$$A_0 x_t = A_1 E_t x_{t+1} + A_2 x_{t-1} + A_3 v_t \tag{9}$$

$$v_t = R v_{t-1} + \zeta_t$$

where $x_t = [\widetilde{y}_t \ \pi_{H,t} \ \pi_{W,t} \ \widetilde{w}_t \ r_t]'$ and $v_t = [rr_t^n \ \Delta s_t^n \ \Delta w_t^n]'$.

The 3×1 random vector ζ_t is independent and identically distributed with mean zero and variance-covariance matrix Σ.

The matrices A_0, A_1 and A_2 are 5×5; A_3 is 5×3 and R is 3×3.

If the inverse matrix A_0^{-1} exists, the system (9) becomes:

$$x_t = BE_t x_{t+1} + Dx_{t-1} + Kv_t \tag{10}$$

where $B = A_0^{-1}A_1$, $D = A_0^{-1}A_2$ and $K = A_0^{-1}A_3$.

The matrices B and D are 5×5, whereas K is 5×3.

For determinacy analysis, I write the system (10) as:

$$E_t z_{t+1} = J_1 z_t + J_2 v_t \tag{11}$$

where $z_t = [x_t \; x_{t-1}]'$.

The matrices of system (11) are:

$$J_1 = \begin{bmatrix} B & 0_{5\times5} \\ 0_{5\times5} & I_{5\times5} \end{bmatrix}^{-1} \begin{bmatrix} I_{5\times5} & -D \\ I_{5\times5} & 0_{5\times5} \end{bmatrix}$$

$$J_2 = \begin{bmatrix} B & 0_{5\times5} \\ 0_{5\times5} & I_{5\times5} \end{bmatrix}^{-1} \begin{bmatrix} -K \\ 0_{5\times3} \end{bmatrix}$$

I represent null matrices by $0_{n\times m}$ and the symbol $I_{n\times n}$ denotes identity matrices.

According to Farmer (1999), the REE is determinate if the number of stable eigenvalues of J_1 is equal to the number of predetermined variables of system (11).

The method for E-stability analysis follows the standard approach of Evans and Honkapohja (2001), which I summarize below.

Under adaptive learning, economic agents use recursive least-squares updating to form expectations. They have a forecasting model known as the perceived law of motion (PLM), which is based on the MSV (minimum state variable) solution of the linear system of rational expectations.

Consider the system:

$$x_t = BE_t x_{t+1} + Dx_{t-1} + Kv_t$$
$$v_t = Rv_{t-1} + \zeta_t \tag{12}$$

The MSV solution has the form:

$$x_t = a + bx_{t-1} + cv_t \tag{13}$$

where a is 5×1, b is 5×5 and c is 5×3.

The assumptions about agents' information set at time t are important in deriving E-stability conditions. In this paper, the information set is the same used in Llosa and Tuesta (2008) and Best (2012) and corresponds to the vector $(1, x_{t-1}^{'}, v_t^{'})^{'}$. Under this time t information set, the expectations are:

$$E_t x_{t+1} = (I + b)a + b^2 x_{t-1} + (bc + cR)v_t.$$

where I denotes the 5×5 identity matrix.

The insertion of the expectations $E_t x_{t+1}$ above in system (12) gives the actual law of motion (ALM):

$$x_t = B(I + b)a + (Bb^2 + D)x_{t-1} + (Bbc + BcR + K)v_t$$

The perceived law of motion (PLM) used in the least-squares learning algorithm is equation (13) and the map from PLM to ALM is:

$$T(a, b, c) = (B(I + b)a,\ Bb^2 + D,\ Bbc + BcR + K)$$

The vector $(\overline{a}, \overline{b}, \overline{c})$ is the fixed point of the map from PLM to ALM, known as the T-map, and corresponds to a REE of system (12).

To define E-stability, researchers have studied the local stability of the following matrix differential equation at $\overline{\xi} = (\overline{a}, \overline{b}, \overline{c})$:

$$\frac{d\xi}{d\tau} = T(\xi) - \xi \tag{14}$$

where $\xi = (a, b, c)$ and τ denotes a "notional" time.

Evans and Honkapohja (2001) established the connection between E-stability and the convergence of real-time learning rules for a class of dynamic models that included the small-open economy model of this paper. In fact, the conditions for E-stability govern the convergence of the least-squares learning algorithm to a REE. Before I state these conditions, I have to compute the following derivative matrices:

$$DT_a(\overline{a}, \overline{b}) = B(I + \overline{b}) \tag{15}$$

$$DT_b(\overline{b}) = \overline{b} \otimes B + I \otimes B\overline{b} \tag{16}$$

$$DT_c(\overline{b}, \overline{c}) = R^{'} \otimes \overline{b} + I \otimes B\overline{b} \tag{17}$$

where the operator $\otimes$ denotes the Kronecker product.

These derivative matrices correspond to the Jacobian of the T-map $T(\xi)$.

Finally, the REE solution of system (12) is E-stable or learnable under the following conditions: all real parts of the eigenvalues of the derivative matrices above are lower than 1.

3.2 Results

Figures 1 to 12, displayed at the end of the paper, show the regions of determinacy and E-stability associated with alternative specifications for monetary policy rules described by equations (7) and (8). In all simulations, I vary the policy parameters ϕ_p and ϕ_y of the interest rate rules. The ranges allowed for ϕ_p and ϕ_y cover empirically relevant cases. In particular, these ranges for both parameters comprise a fine grid of values between 0 and 5, with an increment step size of 0.01. This setup yields 250,000 parameter configurations and I evaluate each pair $\left(\phi_p, \phi_y\right)$ in the grid, checking numerically whether a given configuration satisfies the conditions for determinacy and learnability.

I investigate a benchmark rule in which I fix $\phi_r = \phi_e = 0$. For the rule with managed exchange rate (MER), I set $\phi_r = 0$ and $\phi_e = 0.6$, following the calibration of Llosa and Tuesta (2008). Analogously, for the rule with interest rate smoothing (IRS), I use the specification in Best (2012) and set $\phi_r = 0.65$ and $\phi_e = 0$. I also experiment with alternative values for ϕ_r and ϕ_e. In the succeeding paragraphs, I will comment on the findings that emerged from these experiments. Finally, if the rule features managed exchange rate and interest rate smoothing, the calibration is $\phi_r = 0.65$ and $\phi_e = 0.6$.

3.2.1 Contemporaneous data rules

Figures 1 to 6 show that the regions of determinacy and E-stability are quite similar under different inflation measures. Indeed, there is no single inflation index that clearly generates, for all rules, regions of determinacy and E-stability that are significantly larger than the ones related to alternative inflation measures. Responding to CPI inflation therefore does not comparatively increase the central bank's ability to promote the convergence of an economy to a determinate and E-stable REE.

Furthermore, in the simulations I performed, determinate equilibria are always E-stable and indeterminate equilibria are always E-unstable, irrespective of the monetary policy rule considered. Finally, the introduction of nominal wage rigidity and a high degree of trade openness enlarges the regions of determinacy and E-stability. This enlargement also occurs if the monetary policy rule features MER, IRS or both.

Under positive inflation, the central bank's response has to be sufficiently aggressive to generate high real interest rates to reduce aggregate demand and bring inflation down. This aggressive response imposes restrictions on the parameters of the interest rate rule in order to guarantee determinacy and E-stability.

If the central bank's response is not sufficiently aggressive, under rational expectations, an exogenous increase in expected inflation or, under learning, an upward departure of inflation forecasts from the levels associated with rational expectations, would be consistent with lower real interest rates and a consequent increase in the output gap, which would feed back to increase current and expected inflation even more, creating an inflationary spiral. Under rational expectations, the initial exogenous rise in expected inflation is self-fulfilling whereas, under learning, there is no way to reverse the departure of inflation forecasts from the values consistent with rational expectations.

The similarity of the regions of determinacy and E-stability related to different inflation measures suggests that what is relevant for determinacy and learnability is the central bank's ability to respond strongly to movements in some nominal anchor. Exactly which nominal price it cares about is not so important. Since alternative inflation measures tend to co-move, the central bank's strong response to a specific inflation index tends to stabilize the remaining inflation measures.

The introduction of nominal wage rigidity relaxes some of the restrictions on the coefficients of monetary policy rules and enlarges the region of determinacy and E-stability. In fact, nominal wage stickiness dampens the movements in real wages associated with shocks that affect economic activity, such as a non-fundamental increase in inflation expectations. Thus, aggregate demand shifts driven by these shocks do not lead to extreme variations in wage markups and firms' marginal costs, requiring a less pronounced corrective action by the central bank. This less aggressive response increases the combination of parameters compatible with determinacy and E-stability.

As a consequence of the expenditure-switching effect between domestic and foreign goods due to changes in the terms of trade, the degree of openness increases the sensitivity of the Phillips curves to the output gap and the central bank does not need to engender a strong variation in aggregate demand to control inflation. A less aggressive response of the interest rate imposes lesser restrictions on the parameters of the monetary policy rule and thus enlarges the regions of determinacy and learnability.

In fact, a certain degree of exchange rate management helps to avoid indeterminacy and instability under learning. Equation (7) shows that in the MER specification of equation (5) there is an indirect response of interest rates to domestic inflation, with coefficient ϕ_e. In this case, the direct response to any contending inflation measure does not have to be very strong, implying determinate and E-stable equilibria for relative small values of ϕ_p. Thus, the direct reaction toward movements in the exchange rate relaxes determinacy and learnability conditions because it implies an extra response to domestic inflation.

If the monetary policy rule features IRS, the central bank's response lasts for some quarters in the future. This long-lasting response can be less aggressive than a short-lived response in a monetary policy rule without persistence and can achieve, through expectations, the same variation in the output gap needed to stabilize inflation. This effect enlarges the region of determinacy and E-stability for rules with persistent interest rate movements.

3.2.2 Forecast-based data rules

Figures 7 to 12 show that the regions of determinacy and E-stability are quite similar if the rule reacts to non-CPI measures. For the baseline calibration (open economy with sticky nominal wages and prices), if the rule responds to CPI inflation, the region of determinacy and E-stability is the smallest one compared with the specifications that react to alternative inflation measures. Therefore, responding to non-CPI inflation increases the central bank's ability to lead the economy to a determinate and E-stable REE compared with situations in which interest rates respond to CPI inflation.

Besides, for the baseline calibration, indeterminate equilibria can be E-stable irrespective of the monetary policy rule considered. In addition, determinate equilibria can be E-unstable if the rule responds to CPI inflation and IRS is present. The introduction of nominal wage rigidity enlarges the regions of determinacy and E-stability but openness has the opposite effect. This enlargement also occurs if the monetary policy rule features IRS. On the other hand, when compared with the benchmark, the rule with MER alone shrinks the regions of determinacy and E-stability.

A positive degree of openness α together with the interest rate response to expected CPI inflation in the policy rule narrows both determinate and E-stable areas. After normalizing r_t^* and π_t^*, I use the uncovered interest rate parity condition $E_t(\Delta e_{t+1}) = r_t$ and the definition of expected CPI inflation $E_t\pi_{CPI,t+1} = (1- \alpha)E_t\pi_{H,t+1} + \alpha E_t\Delta e_{t+1}$ to interpret the reduction of the determinate and E-stable area as a consequence of the interaction between monetary policy activism and openness.

In fact, an increase (decrease) in the interest rate due to expected CPI inflation (deflation) triggered by a non-fundamental shock induces an expected depreciation (appreciation) of the nominal exchange rate, according to the uncovered interest rate parity condition. These movements in the expected exchange rate reinforce the expectation of higher (lower) CPI inflation, according to the expression defining this variable. This reinforcement effect increases with the degree of openness α. In addition, the response to expected CPI inflation ϕ_p cannot be very strong in order to avoid substantial changes in the expected exchange rate.

Summing up, if either the degree of openness or the aggressiveness of monetary policy in responding to expected CPI inflation is sufficiently high, the REE tends to be indeterminate and private agents would not learn it if they use least-squares algorithms.

The mechanism described above becomes stronger under monetary policy rules with MER since the interest rate responds directly to the expected exchange rate. Consequently, the region of determinacy and E-stability shrinks even more in the presence of MER. The effects of introducing nominal wage rigidity and monetary policy rules with IRS are the same as in the case of contemporaneous data rules, leading to an enlargement of the determinate and E-stable area.

Sometimes the central bank's actions are not strong enough to prevent a REE driven by non-fundamental shocks to expected inflation, but are capable of reversing the departure of inflation forecasts from the values consistent with rational expectations. Under this circumstance, indeterminate equilibria can be E-stable as reported in Figures 7, 9, 10, 11 and 12. On the other hand, the degree of aggressiveness of an interest rate response consistent with a determinate REE may not lead to convergence if one starts away from this REE. Figures 11 and 12 show this case, displaying a situation in which forecast-based rules respond to CPI inflation and feature IRS.

In short, a monetary policy rule may not guarantee the convergence of the artificial economy to a determinate REE in which private agents would be able to learn it over time if their initial expectations were not rational. Indeed, the properties of the MSV solution are critical for E-stability because the dynamics of the system start from within a small neighborhood of a particular REE. In other words, dynamics along off-equilibrium paths are crucial for E-stability but irrelevant for determinacy conditions. Bullard and Mitra (2007) discussed the connection between the nature of the MSV solution and conditions that ensure determinacy and E-stability in a simple new Keynesian model with interest rate inertia.

Finally, the results presented in this section are very similar do the findings reported in Llosa and Tuesta (2008), as well as in Linnemann and Schabert (2006). These papers studied small-open economies with price rigidity as the only nominal friction. Therefore, from the perspective of determinacy and learnability, the introduction of nominal wage rigidity did not qualitatively alter the main results concerning the role of openness and the fact that forecast-based rules that respond to CPI inflation impose substantial restrictions on determinacy and E-stability conditions. Nevertheless, models with rigid nominal wages showed wider determinate and E-stable areas, mitigating to some extent the indeterminacy of equilibrium and E-instability associated with some monetary policy rules.

4 Speed of Convergence

The preceding analysis examined the asymptotic properties of the least-squares learning algorithm, by establishing and checking the conditions for E-stability of REE. Here, in the context of the artificial economy of this paper, I investigate how the design of monetary policy rules can affect the speed of convergence of the learning process to a determinate REE. In fact, since one needs to be sure that the learning process will eventually converge to a unique equilibrium, I restrict the analysis to the region of determinacy and E-stability for the chosen parameter configurations.

The speed of convergence is an important metric because it captures how the REE influences the system dynamics under learning. Fast convergence indicates that the dynamics of the artificial economy would stay very close to the dynamics of the REE, whereas slow convergence means that the transitional dynamics under learning would drive macroeconomic dynamics most of the time.

4.1 Methodology

One way to measure the speed of convergence is to simulate the learning process over time, as in Marcet and Sargent (1995). However, this approach is unfeasible due to the large number of parameter configurations and interest rate rules I consider. Indeed, I would have to simulate the learning process for each monetary policy rule and for each pair $\left(\phi_p, \phi_y\right)$ in the region of determinacy and E-stability. Christev and Slobodyan (2013) proposed an alternative way that explores the convergence properties of the matrix differential equation (14).

A measure of the speed of convergence, under this alternative approach, uses the real parts of the eigenvalues of the Jacobian of $T(\xi) - \xi$. The eigenvalues of this Jacobian correspond to the eigenvalues of the following derivative matrices $DT_a(\overline{a}, \overline{b}) - I$, $DT_b(\overline{b}) - I$ and $DT_c(\overline{b}, \overline{c}) - I$. Here, the E-stability conditions guarantee convergence if all real parts of the eigenvalues of these derivative matrices are negative. In fact, establishing that the real parts of the eigenvalues of the Jacobian of $T(\xi) - \xi$ are negative is equivalent to showing that all real parts of the eigenvalues of the Jacobian of the T-map are less than 1.

Even if the equilibrium satisfies E-stability conditions, there are situations in which negative eigenvalues lead to fast or slow convergence of the matrix differential equation (14) according to their magnitudes. This is the case because a linear combination of terms of the form $K_i e^{\lambda_i t}$ solves the linearized approximation of equation (14), where λ_i denotes an eigenvalue of the Jacobian with a negative real part and K_i is a constant to be determined.

If the Jacobian of $T(\xi) - \xi$ has p eigenvalues, denoted by λ_i, each one with a negative real part $\mathrm{Re}(\lambda_i)$, the following expression characterizes the speed of convergence S proposed by Christev and Slobodyan (2013):

$$S = Min\left\{ \left|\mathrm{Re}(\lambda_1)\right|, \left|\mathrm{Re}(\lambda_2)\right|, ..., \left|\mathrm{Re}(\lambda_p)\right| \right\}$$

where $|\mathrm{Re}(\lambda_i)|$ is the absolute value of $\mathrm{Re}(\lambda_i)$.

In fact, in the long run, the term $K_m e^{\lambda_m t}$ in the solution of the matrix differential equation (14) dominates the dynamics of ξ if λ_m is such that the speed of convergence S is $|\mathrm{Re}(\lambda_m)|$.

In sum, under adaptive learning, the speed of convergence governs how fast the linear approximation of equation (14) asymptotically approaches the REE.

4.2 Results

Following Christev and Slobodyan (2013), I report the smallest absolute value of the real part of the eigenvalues of the Jacobian as the measure for the speed of convergence. Since I focus the investigation on the regions of determinacy and E-stability, the eigenvalues λ_i are negative.

Tables 1 and 2 display the median and the standard deviation concerning the speed of convergence across simulations for each pair of monetary policy rule and inflation measure. I focus on the overall performance of each combination of rule and inflation measure over the range of values considered for the parameters describing monetary policy. For this reason, instead of highlighting specific calibrations yielding faster convergence, I choose to report these summary statistics.

Marcet and Sargent (1995) provided a sufficient condition to ensure that parameters in learning algorithms based on recursive least squares converge at root-t rate to a normal distribution centered at the REE. More precisely, the condition is that the real parts of all eigenvalues of the Jacobian of $T(\xi)-\xi$ are less than -0.5. In other words, if $|\mathrm{Re}(\lambda_i)| > 0.5$ for all eigenvalues, the requirement in Marcet and Sargent (1995) holds. If this restriction is not verified, the recursive algorithm is still able to converge but the effect of the initial conditions fails to wane at an exponential rate. Consequently, the rate of convergence is expected to be slower than root-t.

Tables 1 and 2 show that the condition in Marcet and Sargent (1995) is rarely satisfied on average, because the average of the largest real part of an eigenvalue of the Jacobian of $T(\xi)-\xi$ is bigger than -0.5, which corresponds to saying that the average value for S is less than 0.5 and, consequently, on average the condition $|\mathrm{Re}(\lambda_i)| > 0.5$ does not apply to the eigenvalues. Tables 1 and 2 display slow speeds of convergence in the sense that these eigenvalue-based measures reflect the fact that the rate of convergence of the parameters in learning algorithms based on recursive least squares is on average slower than root-t for the specifications I considered.

Marcet and Sargent (1995) suggested a numerical procedure based on Monte Carlo simulations for investigating the precise rate of convergence in situations in which their condition does not hold. This exercise is beyond the scope of this paper. The eigenvalue-based measure of Christev and Slobodyan (2013) suffices for the purpose of comparing the speed of convergence across different rules and across alternative inflation measures. Next, for each monetary policy rule, I discuss the results in detail.

4.2.1 Contemporaneous data rules

Table 1 reports median and the standard deviation (in parentheses) of the speed of convergence across simulations under contemporaneous data rules, as specified by equation (5).

The first column of Table 1 shows the inflation measures under investigation, which are domestic inflation ($\pi_{H,t}$), CPI inflation ($\pi_{CPI,t}$), nominal wage inflation ($\pi_{W,t}$) and a composite inflation ($\pi_{COM,t}$), as described in subsection 2.2.

I investigate a benchmark rule ($\phi_r = \phi_e = 0$) under three calibrations: the baseline calibration (BASE) described in subsection 2.3, a closed economy (CLOSED) in which $\alpha = 0$, and an economy with price rigidity only (PRICE), in which $\theta_w = 0$. These specifications correspond to columns 2 to 4 of Table 1. In addition, I study a managed exchange rate (MER) rule in which $\phi_r = 0$ and $\phi_e = 0.6$, an interest rate smoothing (IRS) rule in which $\phi_r = 0.65$ and $\phi_e = 0$. Finally, I also consider a rule with both MER and IRS ($\phi_r = 0.65$ and $\phi_e = 0.6$). I investigate the last three rules under the baseline calibration of subsection 2.3 and I report the results in columns 5 to 7 of Table 1.

In column 3 of Table 1, the results for $\pi_{H,t}$ and $\pi_{CPI,t}$ are the same, since in a closed economy there is no difference between domestic and CPI inflation.

In the specification with price rigidity only, the linearized equations describing the private sector's behavior do not depend on nominal wages. In fact, an Euler equation and a new Keynesian Phillips curve are the elements needed to characterize demand and supply. These equations supplemented by a monetary policy rule constitute a dynamic system in the variables $\widetilde{y}_t$, $\pi_{H,t}$ and r_t. Thus, in column 4 I report the results for the rule that responds to domestic or to CPI inflation, as studied by Llosa and Tuesta (2008).

Table 1. Speed of Convergence-contemporaneous data rules

index	*base*	*closed*	*price*	*mer*	*irs*	*mer/irs*
$\pi_{H,t}$	0.021205 (0.028822)	0.031819 (0.028702)	0.097657 (0.100534)	0.024896 (0.032612)	0.025221 (0.038582)	0.029270 (0.046057)
$\pi_{CPI,t}$	0.021181 (0.035691)	0.031819 (0.028702)	0.096632 (0.108593)	0.024819 (0.040998)	0.025137 (0.042885)	0.029123 (0.044360)
$\pi_{W,t}$	0.021171 (0.032119)	0.032142 (0.030559)	–	0.024837 (0.037843)	0.025161 (0.044992)	0.029178 (0.053774)
$\pi_{COM,t}$	0.021173 (0.031943)	0.032123 (0.030451)	–	0.024840 (0.037571)	0.025164 (0.044990)	0.029183 (0.053827)

Note: baseline model (base), closed economy (closed), price rigidity only (price), managed exchange rate (mer) and interest rate smoothing (irs). Index: domestic inflation, CPI inflation, nominal wage inflation and composite inflation.

For each contemporaneous data rule in Table 1, the results for the speed of convergence under alternative inflation measures are quite similar. If one uses the convergence speed as the metric to compare the performance of the contending inflation measures, there is no inflation index that clearly dominates the remaining measures.

Overall, the specifications converge slowly to the REE. According to the median values in columns 2 to 4, increasing openness and introducing nominal wage rigidity reduce the speed of convergence.

Comparing columns 5 to 7 to the benchmark (column 2), rules featuring MER or IRS lead to faster convergence. Indeed, using the baseline calibration (open economy with sticky nominal wages and prices), the combination of both features yields the highest speed of convergence for each inflation measure.

Table 1 shows that introducing nominal wage decreases the dispersion of the speed of convergence across parameter configurations for ϕ_p and ϕ_y. On the other hand, opening up the economy yields a mild increase in this dispersion.

Regarding the baseline calibration (open economy with sticky nominal wages and prices), Table 1 shows that rules featuring MER or IRS lead to more dispersed values for the speed of convergence compared with column 2. In fact, under the baseline calibration, the highest dispersion for the convergence speed across parameter configurations for ϕ_p and ϕ_y occurs under a rule displaying both MER and IRS.

The findings concerning contemporaneous data rules seem to reflect the size of the regions of determinacy and learnability. Since Table 1 reports averages, the results suggest that a larger region of determinacy and E-stability has more parameter configurations with high speed of convergence. Therefore, introducing MER and IRS, which leads to larger regions of determinacy and E-stability, also improves the average speed of convergence in contemporaneous data rules.

4.2.2 Forecast-based data rules

Table 2 shows the median and standard deviation (in parentheses) for the speed of convergence across simulations under forecast-based data rules, as specified by equation (6).

In Table 2, I consider the same inflation measures and the same configurations previously described for contemporaneous data rules. Table 2 is therefore similar to Table 1. Again, the results reported in column 3 for $\pi_{H,t}$ and $\pi_{CPI,t}$ are the same since in a closed economy there is no difference between domestic and CPI inflation.

Table 2. Speed of Convergence-forecast-based data rules

index	*base*	*closed*	*price*	*mer*	*irs*	*mer/irs*
$\pi_{H,t}$	0.034319 (0.038299)	0.031338 (0.030024)	0.439877 (0.256518)	0.084083 (0.061625)	0.029269 (0.042964)	0.047392 (0.074146)
$\pi_{CPI,t}$	0.010450 (0.042526)	0.031338 (0.030024)	0.230335 (0.27834)	0.010359 (0.042071)	0.018527 (0.039828)	0.022618 (0.026486)
$\pi_{W,t}$	0.034882 (0.043585)	0.031612 (0.032367)	—	0.087078 (0.074251)	0.029273 (0.049947)	0.047254 (0.065492)
$\pi_{COM,t}$	0.034809 (0.043578)	0.031600 (0.032226)	—	0.087007 (0.075684)	0.029273 (0.050419)	0.047269 (0.068584)

Note: baseline model (base), closed economy (closed), price rigidity only (price), managed exchange rate (mer) and interest rate smoothing (irs). Index: domestic inflation, CPI inflation, nominal wage inflation and composite inflation.

For each rule in Table 2, results for the speed of convergence regarding the specifications that respond to $\pi_{H,t}$, $\pi_{W,t}$ and $\pi_{COM,t}$ are quite similar. However, forecast-based rules that respond to CPI inflation ($\pi_{CPI,t}$) yield slower convergence to the REE than alternative specifications that respond to the remaining inflation measures. If one compares the performance of the contending inflation measures from the perspective of the speed of convergence, $\pi_{H,t}$, $\pi_{W,t}$ and $\pi_{COM,t}$ clearly dominate $\pi_{CPI,t}$. On the other hand, based on the speed of convergence, there is no clear ranking among the three non-CPI measures.

Except for the specification in which only prices are rigid, convergence to the REE is on average slow. Since rules that respond to $\pi_{CPI,t}$ behave differently, I discuss the results in Table 2 in two steps. First, I focus on rules based on non-CPI inflation measures and then on specifications that respond to CPI inflation.

For non-CPI inflation measures, according to the median values in columns 2 to 4, more open economies show faster convergence and introducing nominal wage rigidity reduces the speed of convergence. Compared with contemporaneous data rules, the role of openness is quite different, leading to a mild improvement in the speed of convergence for forecast-based rules.

For non-CPI inflation measures, comparing columns 5 to 7 to the benchmark (column 2) shows that the rule with MER leads to faster convergence whereas the rule with IRS slows down convergence relative to the benchmark. Here, compared with contemporaneous data rules, the effect of introducing IRS goes in the opposite direction. In the case of the rule with both MER and IRS, under the baseline calibration (open economy with sticky nominal wages and prices), these opposing effects balance each other, leading to a speed of convergence between the MER and IRS specifications.

Table 2 shows that for non-CPI inflation measures introducing a nominal wage decreases the dispersion of the speed of convergence across parameter configurations for ϕ_p and ϕ_y. On the other hand, opening up the economy increases this dispersion.

For the baseline calibration, the rule with MER shows the highest dispersion for the speed of convergence if it responds to $\pi_{W,t}$ and $\pi_{COM,t}$. With regard to rules that respond to $\pi_{H,t}$,the highest dispersion for the speed of convergence across parameter configurations for ϕ_p and ϕ_y occurs under the rule with both MER and IRS.

Considering rules that respond to CPI inflation, according to the median values in columns 2 to 4, increasing openness and introducing nominal wage rigidity reduce the speed of convergence.

Comparing columns 5 to 7 to the benchmark (column 2), the rule that responds to $\pi_{CPI,t}$ and features IRS leads to faster convergence whereas the rule that responds to $\pi_{CPI,t}$ and displays MER yields a small reduction in the convergence speed relative to the benchmark. In contrast to Table 1, the effect of introducing MER goes in the opposite direction compared with contemporaneous data rules. Further, under the baseline calibration (open economy with sticky nominal wages and prices), the combination of both features yields the highest speed of convergence for rules that respond to $\pi_{CPI,t}$. This finding stands in contrast to the results for forecast-based rules that respond to non-CPI inflation.

Table 2 shows that for CPI inflation, introducing a nominal wage decreases the dispersion of the speed of convergence across parameter configurations for ϕ_p and ϕ_y whereas this dispersion increases in a more open economy. The non-CPI inflation measures also follow this pattern.

For the baseline calibration, rules that respond to $\pi_{CPI,t}$ and feature MER, IRS or both reduce the dispersion of the speed of convergence compared with the benchmark rule, which also responds to $\pi_{CPI,t}$. Indeed, the benchmark rule, under the baseline calibration, shows the highest dispersion for the speed of convergence across parameter configurations for ϕ_p and ϕ_y, though its standard deviation is only slightly bigger than the one associated with the MER rule.

Concerning the speed of convergence of forecast-based data rules, the specifications that respond to $\pi_{CPI,t}$ behave differently from those that respond to non-CPI inflation measures. The effects of MER and IRS on the speed of convergence vary according to the inflation measure considered under forecast-based rules. Moreover, from the perspective of the speed of convergence, a desirable forecast-based monetary policy rule should respond only to non-CPI inflation measures in a small-open economy with nominal wage and price rigidities.

The low average speed of convergence associated with forecast-based rules that respond to CPI inflation seems to be a consequence of a smaller region of determinacy and E-stability in which too many parameter configurations have eigenvalues consistent with low speed of convergence.

5 Conclusion

In a small-open economy model featuring both price and nominal wage rigidities, I investigated the determinacy and learnability conditions of REE under a handful of possible monetary policy rules. Moreover, I analyzed the speed of learning under these rules, adding another metric through which the desirability of a given rule should be gauged.

I adopted a simulation approach and provided numerical results. First, for each specific interest rate rule, I checked the conditions for determinacy and E-stability of REE in a grid of values for the coefficients of inflation and the output gap, finding the region in which the equilibrium is determinate and learnable. Next, restricted to the region of determinacy and E-stability, I computed a measure for the speed of convergence of the learning process put forth by Christev and Slobodyan (2013).

Regarding contemporaneous data rules, the regions of determinacy and E-stability are similar under competing inflation measures. Hence, responding to CPI inflation does not comparatively increase the central bank's ability to promote the convergence of an economy to a determinate and E-stable REE. In addition, the introduction of nominal wage rigidity, a high degree of openness and monetary policy rules with managed exchange rate, interest rate smoothing or both enlarge the regions of determinacy and E-stability.

Concerning forecast-based data rules, if the rule responds to CPI inflation, the region of determinacy and E-stability is the smallest compared with specifications that react to alternative inflation measures. Therefore, the rules that react to non-CPI inflation dominate the rule that responds to CPI inflation and improve the central bank's ability to lead the economy to a determinate and E-stable REE. Further, the introduction of nominal wage rigidity and rules with interest rate smoothing enlarge the regions of determinacy and E-stability. On the other hand, a high degree of openness and rules with managed exchange rate shrink these regions.

Compared with models in which price rigidity is the only nominal friction, models that also feature rigid nominal wages mitigate to some extent the indeterminacy of equilibrium and E-instability associated with some monetary policy rules, but they do not qualitatively change the main results of sticky-price models of a small-open economy concerning the role of openness and the fact that forecast-based rules that respond to CPI inflation significantly narrow the shape of determinate and E-stable areas.

The speed of convergence is an additional yardstick through which I evaluated the set of monetary policy rules studied in this paper. In fact, contemporaneous data rules that respond to CPI inflation yield approximately the same speed of convergence as rules that react to contending inflation measures. Furthermore, forecast-based data rules that respond to CPI inflation lead to slow convergence compared with rules that react to non-CPI inflation measures. The rules with managed exchange rate and interest rate smoothing improve the speed of convergence under contemporaneous data rules whereas their effects on convergence under forecast-based data rules depend on the chosen inflation measure.

In sum, from the perspective of determinacy and learnability of rational expectations equilibrium (REE), this paper suggests that the rule that responds to CPI inflation does not provide any substantial improvement in the central bank's ability to promote the convergence of an economy to a determinate and learnable REE, nor does it lead to a faster convergence when compared with rules that react to contending inflation measures. Furthermore, the introduction of rigid nominal wages does not affect these findings.

The conclusions of the preceding paragraph put in perspective the results reported in Rhee and Turdaliev (2013) and Campolmi (2014) that support CPI inflation-targeting. In the context of the model presented in this paper, a rule that responds to CPI inflation may not be the best choice if the assessment does not hinge on welfare losses. This observation reinforces the need for more research on alternative performance criteria to evaluate different measures of inflation in monetary policy rules.

Future research can extend this paper in at least three directions. First, researchers can analyze additional interest rules with alternative measures for economic activity, such as the output growth, or study price-level-target rules. Second, an extension of this paper can search for a convex combination of the basic inflation measures with better detrminacy and learnability properties. Finally, it is worth conducting the same exercise of this paper in a medium-size model.

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APPENDIX

A small open economy model with sticky wages and prices

This appendix presents the problem facing households and firms in the model studied by Rhee and Turdaliev (2013) and Campolmi (2014). For details on the derivation of equations describing the equilibrium, I refer the reader to these papers.

The small open economy consists of a continuum of households and firms indexed by h and j, both belonging to the interval $[0, 1]$. The model abstracts from capital accumulation and features wage and price stickiness.

The world economy comprises a continuum of small open economies indexed by i in the interval $[0, 1]$. These economies share identical preferences, technology and market structure. Since each economy is of measure zero, its domestic policy decisions do not affect the remaining countries.

- **Households and wage-setting behavior**

The representative household h maximizes the expected flow of utility given by the expression:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(h)^{1+\varphi}}{1+\varphi} \right]$$

The parameter β, restricted to be in the unity interval, is the discount factor and σ measures the degree of relative risk aversion. Finally, the parameter φ, a positive number, is the inverse of the Frisch labor supply elasticity. The variable C_t stands for aggregate consumption and $N_t(h)$ denotes labor supply.

In fact, C_t is an index that combines bundles of domestic and imported goods according to the expression $C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]$, where

$C_{H,t}$ is an index of consumption of domestic goods and $C_{F,t}$ is an index of imported goods. The parameter α measures the share of domestic consumption allocated to imported goods and can be interpreted as an index of openness; the parameter η controls the substitutability between domestic and foreign goods.

The following functions define the indices of consumption $C_{H,t}$ and $C_{F,t}$:

$$C_{H,t} = \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$
$$C_{F,t} = \left(\int_0^1 C_{i,t}{}^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}}$$

The parameter ε denotes the elasticity of substitution between types of goods produced in the home country, which are indexed by j, while γ measures the substitutability between goods produced in different foreign economies.

Here, $C_{i,t}$ is the aggregate quantity of goods imported from country i and consumed domestically. By symmetry, since all economies share the same structure, the expression for $C_{i,t}$ is:

$$C_{i,t} = \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The variable $C_{i,t}(j)$ stands for types of goods produced in a given country i, which are also indexed by j.

In each period the household faces the budget constraint given by the equation:

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di$$
$$+ E_t \left[\Psi_{t,t+1} D_{t+1} \right] \leq D_t + W_t(h) N_t(h) + T_t$$

The symbol $P_{H,t}(j)$ represents the price of domestic good j and $P_{i,t}(j)$ is the price of variety j imported from country i in domestic currency.

The representative domestic household receives lump-sum net transfers T_t and labor income $W_t(h)N_t(h)$, where $W_t(h)$ stands for nominal wages. The variable D_t is the payoff in t of the portfolio held in $t-1$ and $\Psi_{t,t+1}$ denotes the stochastic discount factor for one-period ahead nominal payoffs.

From the optimal allocation of any given consumption expenditure within each category of goods, I get the price indices:

$$\begin{aligned}
P_t &= \left[(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \\
P_{H,t} &= \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \\
P_{F,t} &= \left(\int_0^1 P_{i,t}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}} \\
P_{i,t} &= \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}
\end{aligned}$$

One can show that the following equalities hold:

$$\begin{aligned}
\int_0^1 P_{H,t}(j)C_{H,t}(j)dj &= P_{H,t}C_{H,t} \\
\int_0^1 P_{i,t}(j)C_{i,t}(j)dj &= P_{i,t}C_{i,t} \\
\int_0^1 P_{i,t}C_{i,t}di &= P_{F,t}C_{F,t} \\
P_{H,t}C_{H,t} + P_{F,t}C_{F,t} &= P_tC_t
\end{aligned}$$

Based on the relations above, I can rewrite the budget constraint as follows.

$$P_tC_t + E_t\left[\Psi_{t,t+1}D_{t+1}\right] \leq D_t + W_t(h)N_t(h) + T_t$$

The household h chooses consumption C_t, labor $N_t(h)$ and bonds D_t in order to maximize the expected flow of utility subject to the budget constraint. According to Erceg et al. (2000), there is also a wage-setting decision stage in which each individual h may choose $W_t(h)$ optimally.

Next, I discuss the wage decision. The representative household h supplies differentiated labor inputs $N_t(j,h)$ for the production of each variety j. Indeed, total labor supplied and the aggregate wage index are given by $N_t(h) = \int_0^1 N_t(j,h)dj$ and $W_t = \left(\int_0^1 W_t(h)^{1-\varepsilon_w} dh\right)^{\frac{1}{1-\varepsilon_w}}$.

Following Erceg et al. (2000), in each period, only a fraction $(1-\theta_w)$ of households can reset wages optimally in order to maximize the expected flow of utility. The log-linear rule that approximates the optimal wage-setting strategy is the following:

$$\overline{w_t} = \frac{1-\beta\theta_w}{1+\varphi\varepsilon_w} \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[\mu^w + mrs_{t+k} + \varphi\varepsilon_w w_{t+k} + p_{t+k}\right]$$

The symbol $\overline{w_t}$ denotes the log of the newly set nominal wage $\overline{W_t}$, the log of the steady-state wage mark-up is $\mu^w = \log(\frac{\varepsilon_w}{\varepsilon_w - 1})$, mrs_t stands for the marginal rate of substitution between consumption and labor in log, w_t is the log of nominal wages and p_t represents the log of the consumer price index.

Under the assumed wage-setting scheme, the aggregate wage index evolves according to the equation:

$$W_t = \left[\theta_w W_{t-1}^{1-\varepsilon_w} + (1-\theta_w)(\overline{W_t})^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}}$$

The log-linearization around the steady state yields the following formula for the wage inflation $\pi_{W,t}$:

$$\pi_{W,t} = (1-\theta_w)(\ \overline{w_t} - w_{t-1})$$

After some algebra, the combination of the expressions describing the optimal wage-setting strategy and the evolution of the aggregate wage index results in equation (3) of the main text.

- **Firms and price-setting behavior**

The production function $Y_t(j) = A_t N_t(j)$ describes the technology for firm j. The variables $Y_t(j)$ and $N_t(j)$ represent output and an index of labor input used by firm j. The technology shock is A_t, with $a_t = \log(A_t)$ following a first order autoregressive process.

The expression $N_t(j) = \left(\int_0^1 N_t(j,h)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dh \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$ defines the index of labor input $N_t(j)$ and the aggregate output is given by $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$, where ε_w is the elasticity of substitution between labor varieties and ε is the elasticity of substitution across different good varieties.

Firms operate in a monopolistic competitive market and set prices in staggered fashion using the scheme proposed by Calvo (1983), in which only a fraction $(1 - \theta_{ph})$ of firms can adjust prices. In this context, in each period these firms reset their prices to maximize expected profits. The following log-linear rule approximates the optimal price-setting strategy:

$$\overline{p_{H,t}} = \mu^{ph} + (1 - \beta\theta_{ph}) \sum_{k=0}^{\infty} (\beta\theta_{ph})^k E_t \left[mc_{t+k} + p_{H,t+k} \right]$$

The variable $\overline{p_{H,t}}$ represents the newly set domestic prices $\overline{P_{H,t}}$ in log, $\mu^{ph} = \log(\frac{\varepsilon}{\varepsilon-1})$ is the log of the steady-state markup, mc_t stands for the log of real marginal costs and $p_{H,t}$ denotes the log of the domestic price index $P_{H,t}$.

Real marginal costs in log scale are given by $mc_t = -v + w_t - p_{H,t} - a_t$, where w_t is the log of nominal wages and $v = \log(1 - \tau)$, with τ representing an employment subsidy. This subsidy neutralizes the distortion due to firms' market power, leaving the economy with nominal stickiness as the only effective distortion. Hence, the flexible price equilibrium is efficient. This equilibrium defines the natural level of macroeconomic variables.

Under the assumed price-setting behavior, the equation below describes the dynamics of the domestic price index:

$$P_{H,t} = \left[\theta_{ph} P_{H,t-1}^{1-\varepsilon} + (1-\theta_{ph})(\overline{P_{H,t}})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

The log-linearization around the steady state yields the formula involving domestic inflation $\pi_{H,t}$:

$$\pi_{H,t} = (1-\theta_{ph})(\ \overline{p_{H,t}} - p_{H,t-1})$$

After some algebra, the expressions describing the optimal price-setting strategy and the dynamics of the domestic price index lead to the new Keynesian Phillips curve as stated by equation (2) of the main text.

- **Market clearing conditions and monetary policy**

The domestic good market clearing condition yields the following expression:

$$Y_{H,t}(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j) di$$

After some algebra involving the definitions of the demand functions for $C_{H,t}(j)$ and $C_{H,t}^i(j)$, I have:

$$Y_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\Xi_{it} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} C_t^i di\right]$$

The variable $C_{H,t}^i(j)$ denotes the demand from country i of good j produced in the home economy, Ξ_{it} is the nominal exchange rate and $P_{F,t}^i$ is the price index for goods imported by country i expressed in its own currency. Lastly, P_t^i is the consumer price index for households living in country i.

Using the definition of aggregate output $Y_t = \left(\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$, I get the following expression:

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha)+\alpha\right] \int_0^1 \left(S_t^i S_{i,t}\right)^{\gamma-\eta} Q_{i,t}^{\eta-\frac{1}{\sigma}} di$$

The effective terms of trade for country i is $S_t^i = \frac{\Xi_{it} P_{F,t}^i}{P_{H,t}}$ and $S_{i,t} = \frac{P_{i,t}}{P_{H,t}}$ denotes the bilateral terms of trade between country i and the domestic economy H. Finally, $Q_{i,t} = \frac{E_{it} P_t^i}{P_t}$ represents the bilateral real exchange rate between countries i and H.

The labor market clearing condition is:

$$N_t = \int_0^1 N_t(j)dj = \int_0^1 N_t(h)dh.$$

To close the model, I assume that the central bank follows the interest rate rules described in subsection 2.2 of the main text.

FIGURES

Figure 1: Contemporaneous Data Rule: Benchmark-baseline

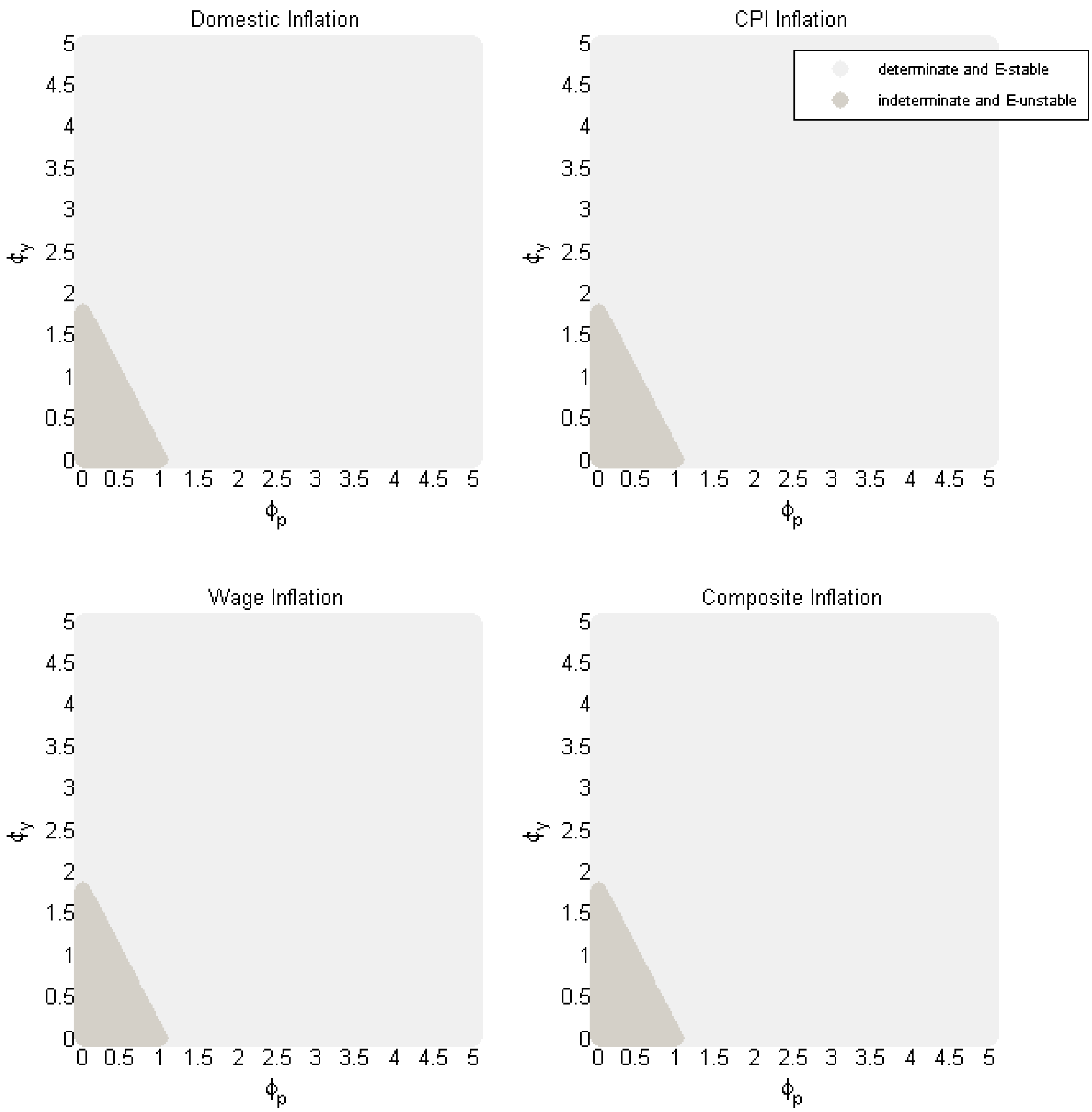

Figure 2: Contemporaneous Data Rule: Benchmark-closed economy

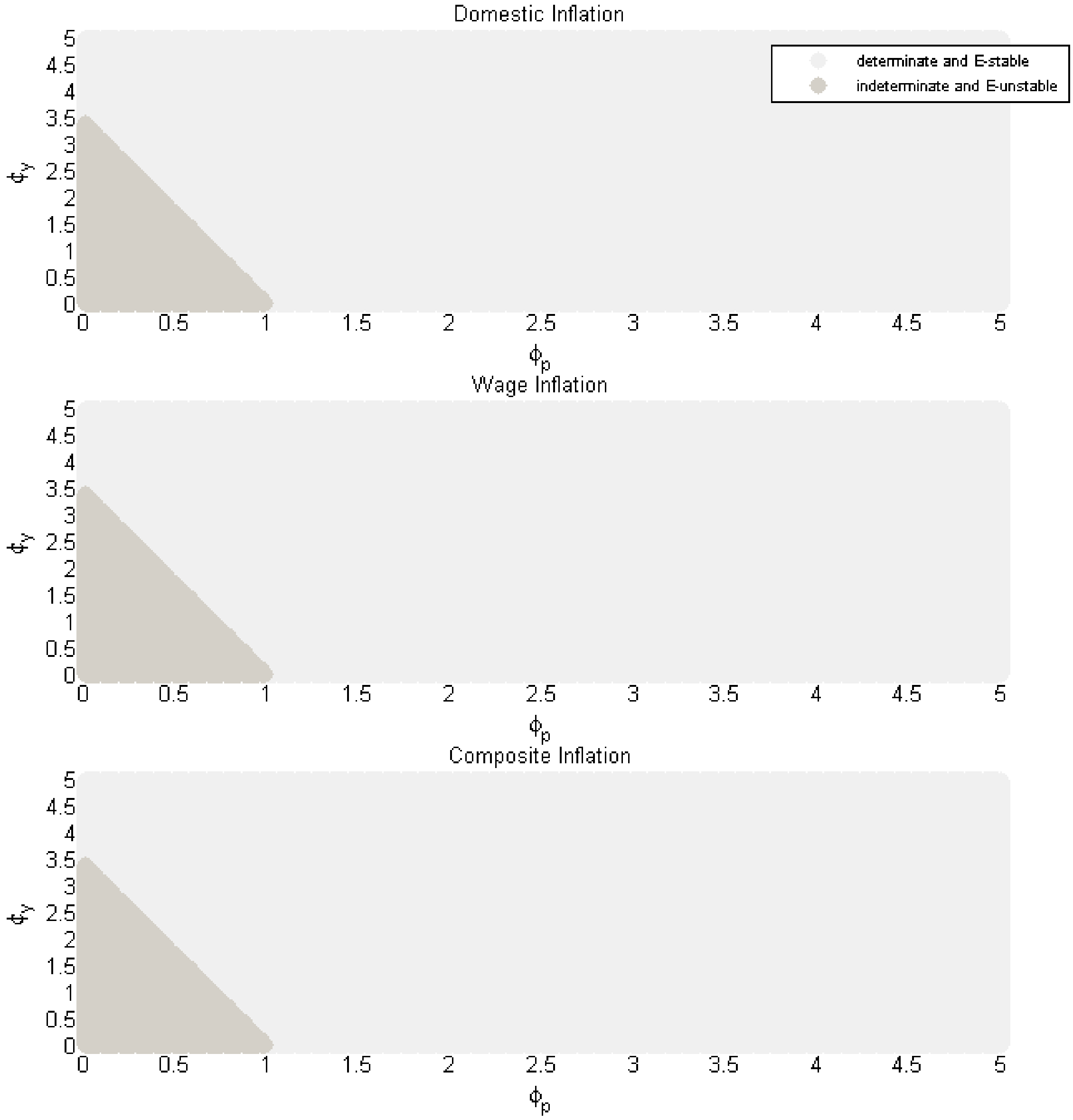

Figure 3: Contemporaneous Data Rule: Benchmark-price rigidity only

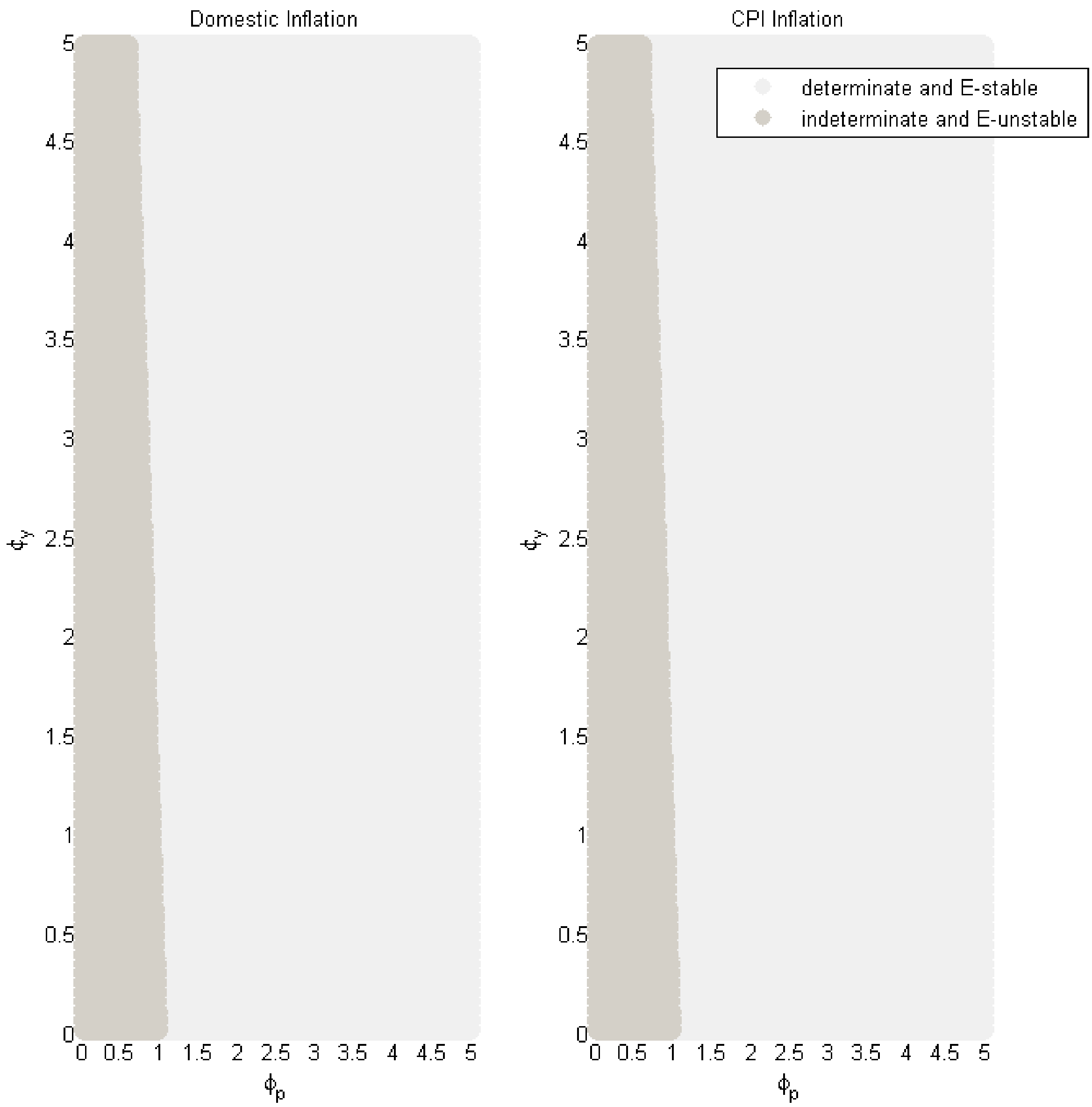

Figure 4: Contemporaneous Data Rule: Managed Exchange Rate

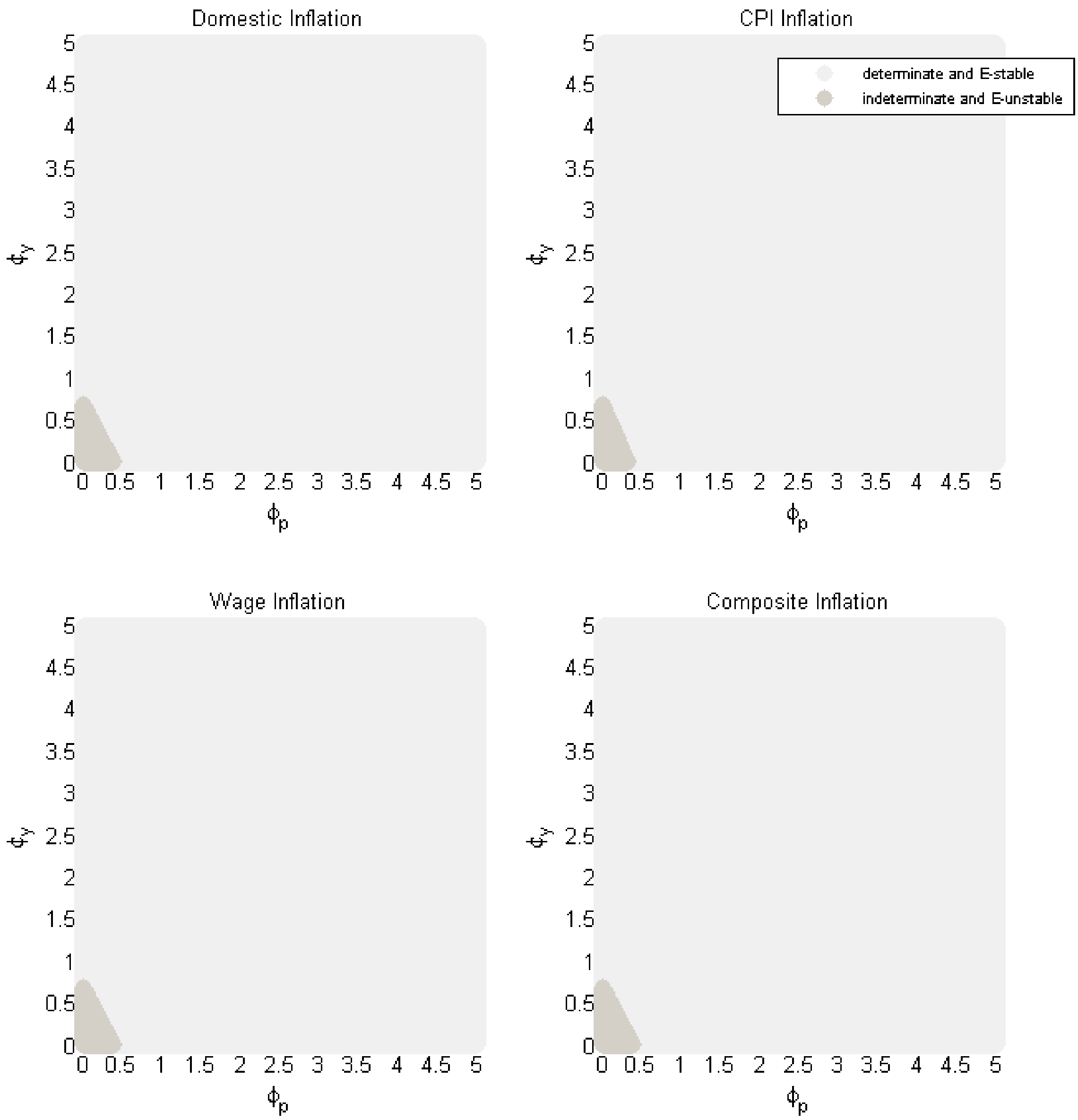

Figure 5: Contemporaneous Data Rule: Interest Rate Smoothing

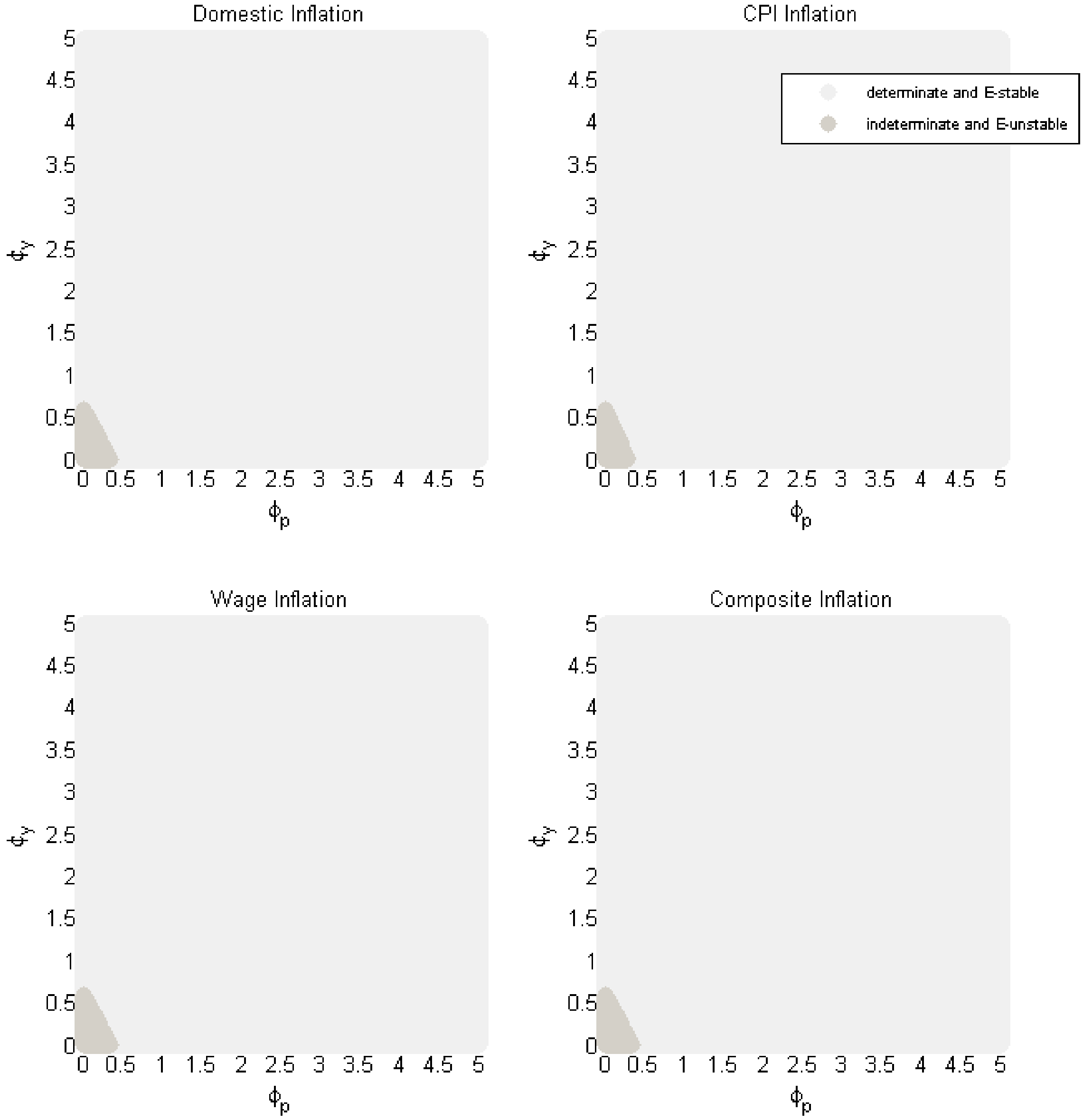

Figure 6: Contemporaneous Data Rule: Managed Exchange Rate and Interest Rate Smoothing

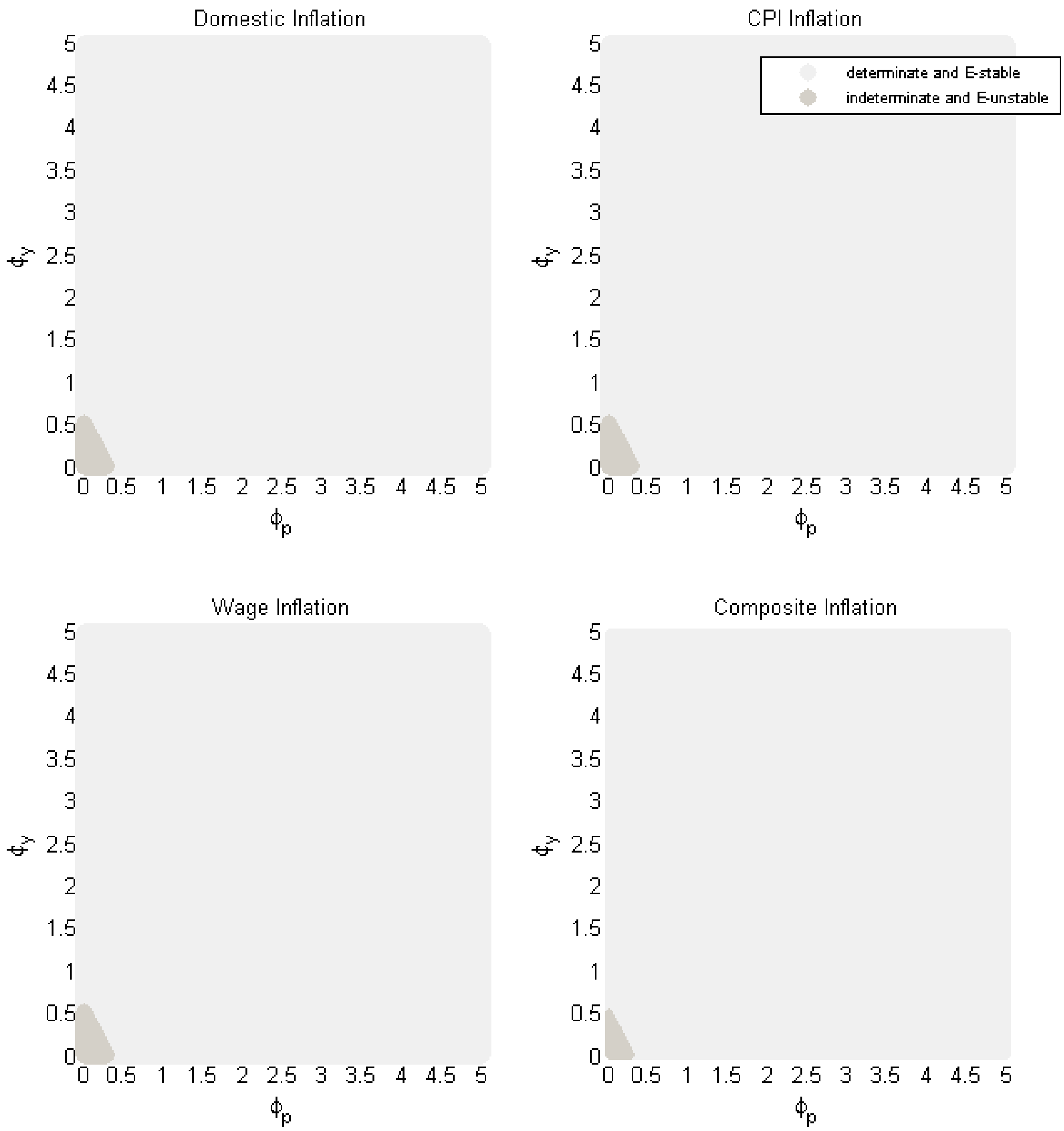

Figure 7: Forecast-based Data Rule: Benchmark-baseline

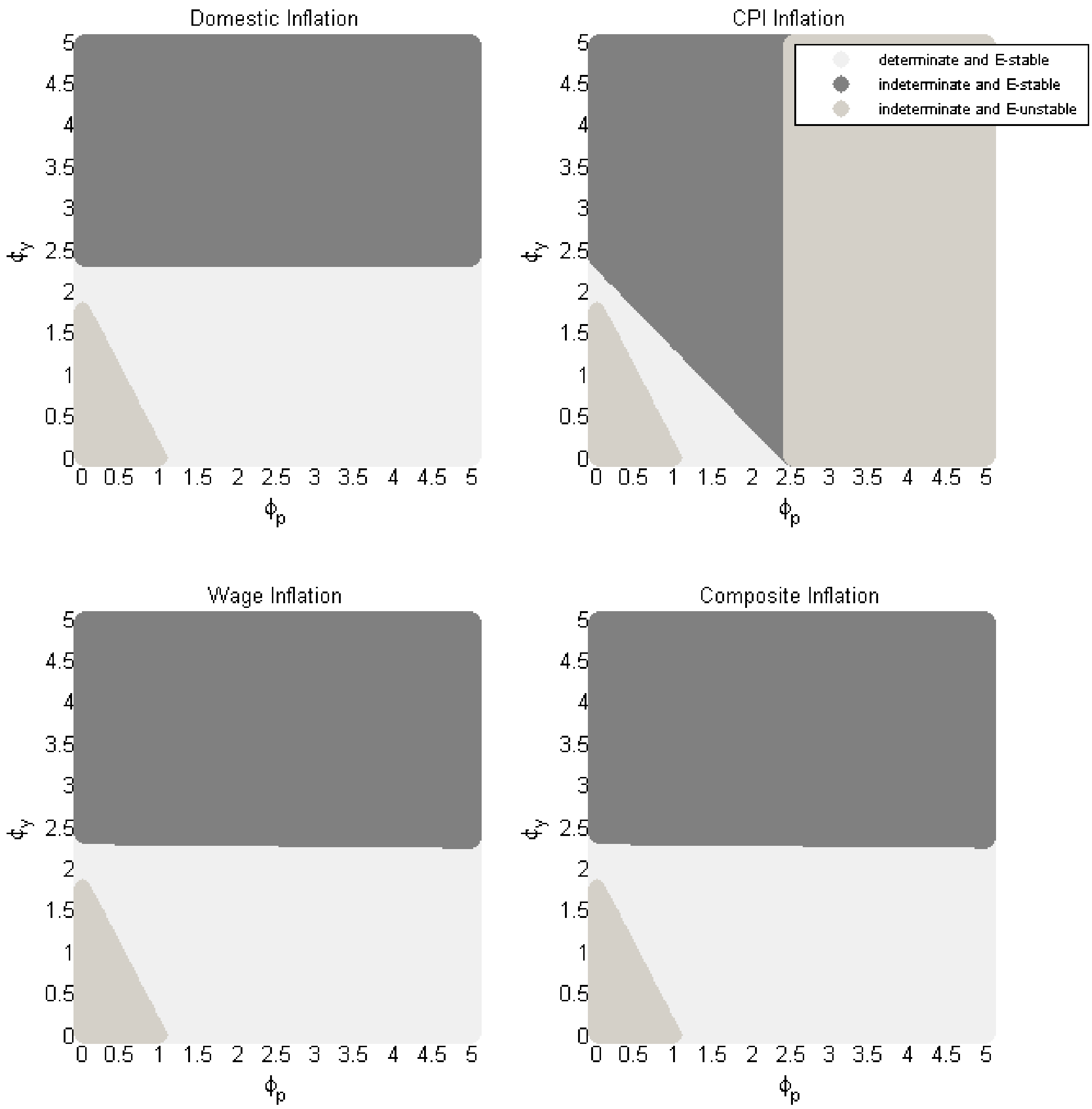

Figure 8: Forecast-based Data Rule: benchmark-closed economy

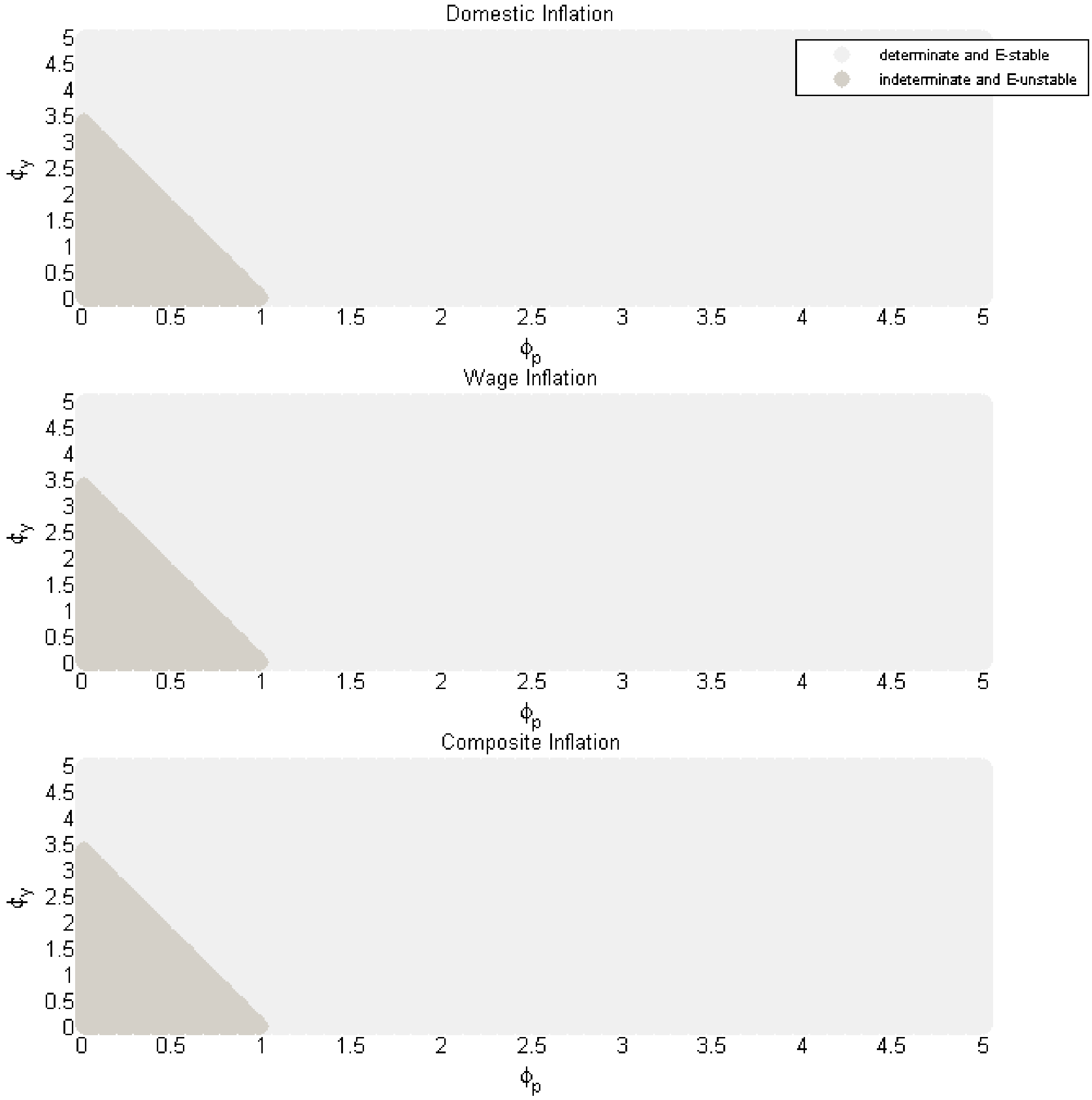

Figure 9: Forecast-based Data Rule: Benchmark-price rigidity only

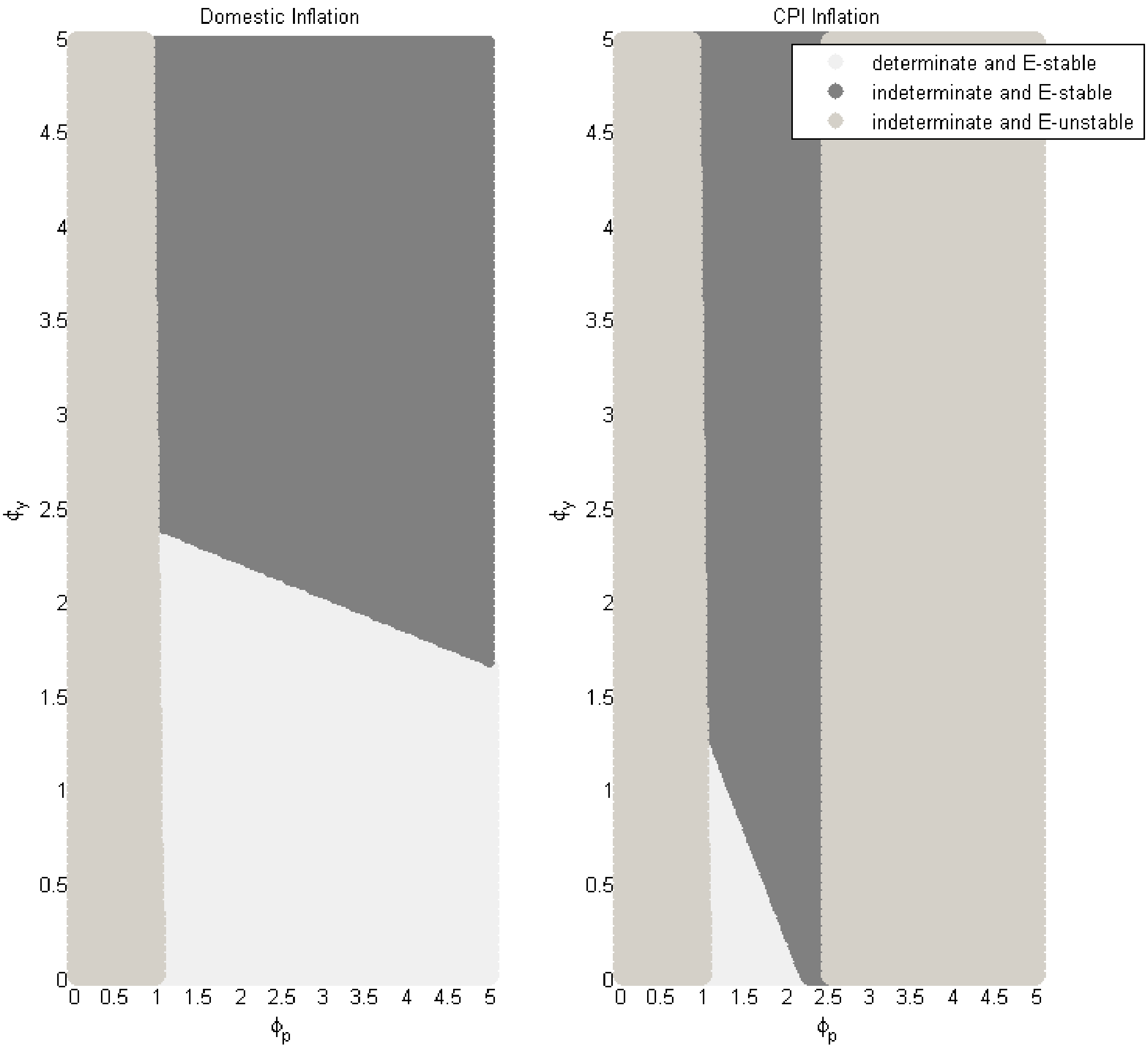

Figure 10: Forecast-based Data Rule: Managed Exchange Rate

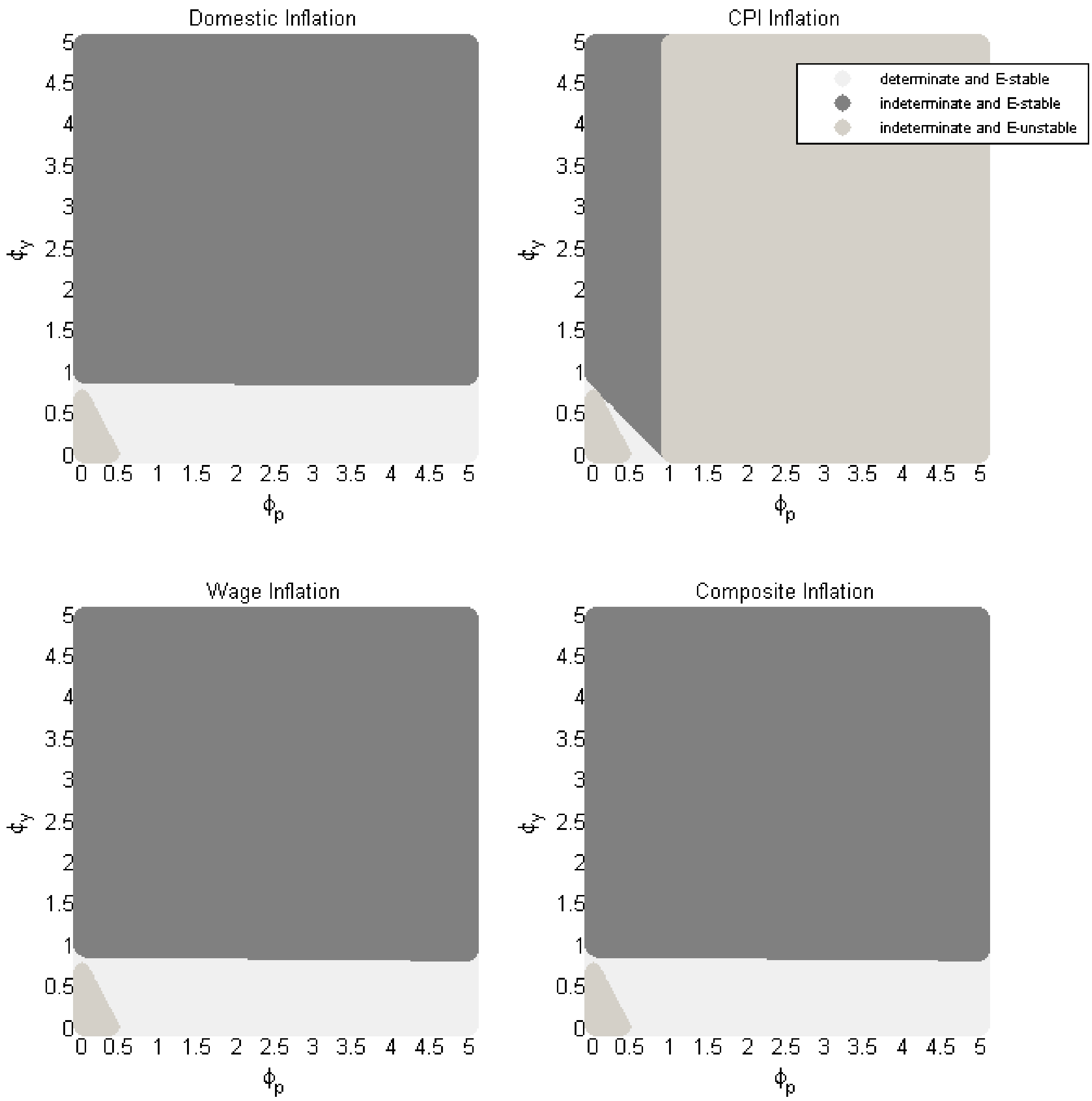

Figure 11: Forecast-based Data Rule: Interest Rate Smoothing

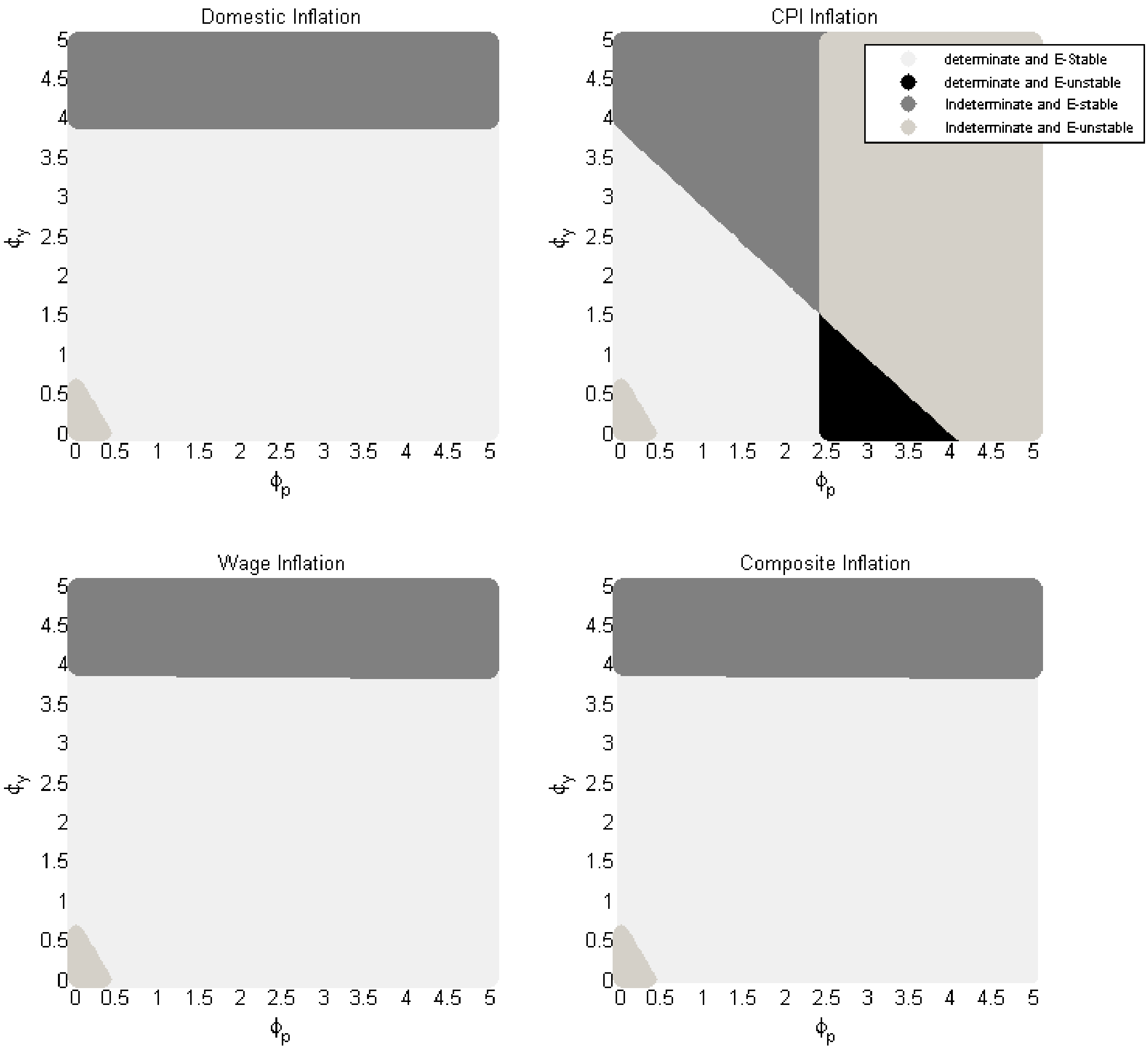

Figure 12: Forecast-based Data Rule: Managed Exchange Rate and Interest Rate Smoothing