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Sergio R. Stancato de Souza

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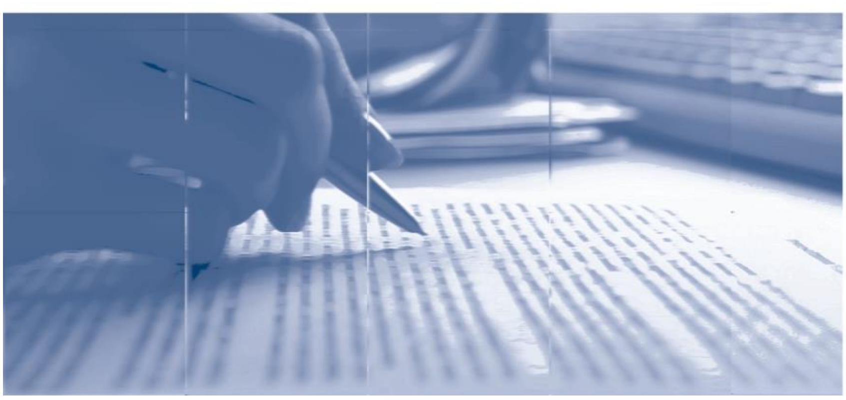

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# Capital Requirements, Liquidity and Financial Stability: the case of Brazil*

Sergio R. Stancato de Souza†

**Abstract**

*The Working Papers should not be reported as representing the views of the Banco Central do Brasil. The views expressed in the papers are those of the author(s) and do not necessarily reflect those of the Banco Central do Brasil.*

This paper simulates the effects of credit risk, changes in capital requirements and price shocks on the Brazilian banking system. We perform the analysis within the context of a model that integrates data on bilateral exposures in the interbank market with information about the liquidity profile of each financial institution. Asset prices in the model are determined endogenously as a function of the total volume of fire sales, thus creating the possibility that marking to market may trigger new rounds of fire sales and downward asset price spirals. The simulation results show that the Brazilian banking system is robust, as relatively large increases in the delinquency rate lead to only modest losses in the system. We also compute the contribution of each financial institution to systemic losses under severe shocks and find that contributions from medium-sized banks can be significant. However, if shocks become more severe, only large banks will contribute significantly to systemic losses.

Keywords: financial stability; capital requirements; risk attribution; fire sales.
JEL Classification: C63, G21, G28, G32.

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# 1 Introduction

In this paper, we study the reactions of a banking system subject to regulatory constraints under a variety of shocks. Our model represents a network of banks mutually exposed which also have claims and obligations towards agents that are external to the network. Each bank updates the market value of its assets and realizes losses due to both the delinquency of its debtors and to the fire sales of its assets, if they occur. When there are losses, they reflect directly on the bank's capital, that may fall below the regulatory minimum. In this case, the bank must restore compliance with capital requirements. One course of action it can adopt is to sell risky assets, as these constitute the basis for capital requirement computation. These risky assets are not perfectly liquid so that the sales the bank needs to perform to comply with legal requirements induce a price fall in these assets, increasing the magnitude of the original loss, which then feeds back into the minimum capital requirement. The assets are sold outside the network.

Our model has the framework proposed by Cifuentes et al. (2005), in which risky asset prices follow an exponential function decay with an exponent that is proportional to the share of the assets that are being sold. We divide risky assets into two categories: liquid and illiquid assets. Liquid assets are marked to market and have lower price decay than illiquid assets. The latter are not marked to market so that only banks that sell them suffer losses. When deciding which asset to sell, we assume a pecking order: liquid assets are sold first due to their greater liquidity.

The way banks react to adverse conditions may cause externalities to other banks within the network, be it through default propagation, be it through the price fall of marked-to-market assets caused by fire sales. The loss feedback mechanisms and the externalities we describe above amplify the impact from shocks on banks, creating the possibility that a modest shock produces important effects. In addition to this, the shock amplification due to fire sales caused by the need of continuous compliance with legal capital requirements provides evidence of the procyclicality of this prudential tool. In other words, in times in which banks are weakened by losses, the need to comply with capital requirements may well amplify vulnerabilities instead of mitigating them.

Our purpose in building this model is to provide a framework for systemic risk assessment or stress testing with three channels of transmission for shocks. The first is a direct contagion channel, operating through direct exposures between banks and the other two are indirect contagion channels, operating through asset prices. One of them refers to asset prices for assets subject to marking to market. In this case, contagion occurs both when a bank sells assets at a price that is lower than its original value and when there is a price fall and the unsold assets have to be marked to market. The last channel refers to asset prices for assets that are not marked to market. This channel transmits contagion when the corresponding assets are sold. The assets in this category are illiquid and mostly held to maturity while the assets that are marked to market are liquid and constantly traded.

The model does not take into account banks' behavioral reactions to shocks. Rather, the behavior of banks follows rules or constraints imposed by the regulator. This is one of the reasons given by Borio and Drehmann (2009) and Borio et al. (2014) for not using a given model as an early warning indicator. However, this type of model can be useful because economic agents make decisions constrained by rules defined by regulation or even by the agents themselves. It is common that banks themselves define rules to follow. One example for this is the maintenance of leverage by financial intermediaries, as studied in Adrian and Shin (2010). Concerned with regulator-proposed rules, we study the compliance with capital requirements. Both works find that these rules induce procyclicality. This means, specifically in the case of capital requirements, that a rule intended to keep individual banks safe, collectively may produce harmful effects to the system due to externalities. Procyclicality means amplification: the response to a shock feeds back in the same direction of the shock, thus, it may destabilize the system, i.e., it is possible that there is a shock that, applied to a given system, is such that a small increase produces a large additional loss. This is what is usually referred to as the knife-edge property to which the literature refers (for instance, see Gai and Kapadia (2010) and Hałaj and Kok (2013)).

We analyze the Brazilian banking system in December 2013 using this model. We apply four types of shocks: a parallel increase in bad default ratios, an increase in the capital requirement ratio, a drop in asset prices, and individual banks' idiosyncratic defaults. Given that the interbank exposures are relatively small, we find that for this banking system, the price shock transmission channels are much more relevant than the direct exposures channel. We also find that, for lower intensity shocks, the liquid assets price channel is the strongest, and for higher intensity shocks, the illiquid assets price channel is the strongest. This finding is conditional to the definition that, in our model, external illiquid tradable assets comprises lending operations and funding providing operations of banks. Analyzing individual banks contribution to losses in a scenario of a simultaneous increase in bad default ratios of 10 p.p., we find that the 4 banks that most contribute to 5 variables (total systemic losses, sales of liquid assets, sales of illiquid assets, liquid assets price fall and illiquid assets price fall) are relevant due to a combination of a lower capital ratio and the size of its stock of risky liquid assets compared to the aggregate stock of these assets. We finally analyze the robustness of the model with respect to price decay ratios and find that higher price decays lead this banking system into a region in which the knife-edge property is remarkable. We note that this increased sensitivity of prices with respect to the quantities sold is characteristic of crises turmoils. Finally, according to the rules we model, what accelerates the losses increase with respect to the shock's increase is the presence of big banks among these affected by the shock, as they need to sell comparatively high amounts of assets to become compliant.

This paper contributes with the literature in the following ways: 1) it includes a shock transmission channel for prices not related to mark to market processes, 2) it performs analyses for different plausible scenarios of shocks in credit, regulation, market conditions and idiosyncratic shocks, and 3) it investigates vulnerability sources for this banking system according to the process modeled.

This model is closely related to a model presented in Gauthier et al. (2012), following David and Lehar (2011), differing from these models by the addition of a shock transmission channel and by a modification in the debt seniority structure. Their computation of risk weights is also different from ours. We follow the ad-hoc price decay computation in Cifuentes et al. (2005).

The literature to which this paper belongs investigates the stability of a banking system organized in a network. Among the earlier papers in this literature, we cite Freixas et al. (2000) and Allen and Gale (2000). The first models systemic risk in an interbank market in which consumers are unsure about where they want to consume. This causes liquidity needs for banks, which prefer interbank credit lines to the cost of maintaining reserves. However, if a shock hits the system, banks face a positive probability of failure. Allen and Gale (2000), by their turn, study how the network structure of a stylized banking system affects its final equilibrium after a liquidity preference shock. Eisenberg and Noe (2001) propose a clearing algorithm for a generic financial network that always has a solution and, under a mild condition, has a unique solution. This provides a foundation for processing real payments systems and for use in simulations and are followed by a series of empirical works investigating financial stability in national payments systems and interbank networks. For instance, Upper and Worms (2004) analyze the effects of the bankruptcy of a bank in the German banking system, Cocco et al. (2005), analyze the Portuguese interbank market, Van Lelyveld and Liedorp (2006), the Dutch banking sector and Degryse and Nguyen (2007), the Belgian banking sector. Elsinger et al. (2006) extend the investigation of a chain of failure from an idiosyncratic break of a bank by including risk factors arising from a macroeconomic model and affecting banks assets classes exposed to them. The analysis is performed for the Austrian banking system. Cifuentes et al. (2005) propose a model integrating possible insolvencies in a banking system with fire sales of assets by banks pursuing adequacy to capital requirements. Decreased prices in markets in which fire sales have occurred affect simultaneously all banks, given that these assets are marked to market. After the $2007-09$ global crisis, Brunnermeier (2009)'s analysis of the mechanisms that amplified losses along the process highlights the need to include common exposures and loss spirals into financial stability simulations.

Related theoretical works include Caballero and Simsek (2009), who propose a model of fire sales and market breakdowns in which endogenous complexity plays a central role. The model illustrates how fire sales and complexity due to uncertainty and partial information can amplify losses, creating a downward spiral. Gai and Kapadia (2010) model two channels of contagion: direct counterparty exposures and indirect contagion due to asset prices, that arise when banks under distress have to sell long-term assets prematurely, causing loss of their value, along the lines of Cifuentes et al. (2005) and Shin (2008). Their findings suggest that financial systems exhibit a robust-yet-fragile tendency: while the probability of contagion may be low, effects can be large when there are problems. Besides, a priori indistinguishable shocks can have very different consequences for financial systems, depending on how the shocks hit the system. Adrian and Shin (2010) study a financial system in which balance sheets are continuously marked to market, requiring continuous adjustment by banks that strive to keep their leverage. It presents evidence of procyclicality of these adjustments. Battiston et al. (2012) present a model in which banks that suffered a default from a debtor are subject to a run by their short term liabilities creditors, as they are concerned about the bank's solvency and do not rollover their short term investments. This run may lead the bank that suffered a loss into illiquidity even if it is solvent, due to imperfect information. Lee (2013) studies systemic liquidity shortages under different network structures in a banking system in which banks suffer external liquidity withdrawals and have to call their clams against other banks and sell their external liquid assets. If necessary, a bank puts its illiquid assets to sale. He finds that a core-periphery network with a deficit money center produces the highest level of systemic risk.

Among the empirical works, we cite Alessandri et al. (2009), which describes a prototype of a quantitative framework for assessing systemic risk and applies it to make projections for system-wide banking assets in the United Kingdom. The framework comprises the banks' network and a macroeconomic model and considers not only loss management, as most models do, but also profit reinvestment by banks. Barnhill and Schumacher (2011) propose a model that integrates correlated systemic liquidity and solvency risks, in which risk factors are applied to their corresponding asset classes in the balance sheet. The authors demonstrate this methodology for a set of banks in the U.S.. van den End and Tabbae (2012) use firm-specific balance sheet data to build aggregate indicators of systemic risk from the Dutch supervisory liquidity report. They find that the balance sheet adjustments have been procyclical in crises, while the reactions during them were static instead of following the usual pecking order adopted in tranquil times.

Finally, Borio et al. (2014) review the state of the art of stress testing models, assessing their strengths and weaknesses. They argue that these models are not well-suited as early warning devices, which means that stress tests are not trustworthy for identifying vulnerabilities in seeming tranquil times. On the other hand, if properly designed, they can be quite effective as crises management tools. For improving the performance of macro stress tests, from a technical perspective, they suggest the improvement of the generation of non-linearities and feedback effects; from a broader perspective, they suggest that processes and governance are critical: it is important to test the system hard and distrust rosy results.

The remainder of the paper is structured as follows: the next section presents the model we employ, section 3, presents the data we use, in section 4, we test the Brazilian banking system, under different types of shocks, in section 5, we assess the individual banks contributions to losses, in section 6, we perform a robustness check with respect to different prices decay rates, and in section 7, we make final remarks.

# 2 Model

## 2.1 The banking system

We model the banking system as a set of $N$ banks, $\mathcal{N} = \{1, ..., N\}$, in which a bank $i$ is described by a simplified balance sheet as shown in Table 1.

Table 1: Bank $i$'s simplified balance sheet.

| Assets | Liabilities |
|---|---|
| $C_i$ | $D_i$ |
| $\sum_j x_{ji}$ | $\sum_j x_{ij}$ |
| $A_i^L$ | $L_i^0$ |
| $A_i^I$ | |
| $F_i^A$ | $V_i$ |

In Table 1, $C_i$ is the amount of bank $i$'s cash equivalent assets, $x_{ij}$ is the bank $i$'s total interbank liability to bank $j$, $A_i^L$, its external liquid assets, $A_i^I$, its external illiquid assets, $F_i^A$, its fixed assets, $D_i$, its total deposits, which are considered the highest seniority debts, and $L_i^0$, its total external debts.

We compute bank $i$'s liquid assets $\lambda_i$, its total assets $T_i^A$ and its total liabilities $T_i^L$ as follows:

$$\lambda_i = C_i + \sum\nolimits_j x^L_{ji} + A^L_i \tag{1}$$

$$x_{ji} = x^L_{ji} + x^I_{ji} \tag{2}$$

$$T^A_i = \lambda_i + A^I_i + F^A_i \tag{3}$$

$$T^L_i = D_i + \sum\nolimits_j x_{ij} + L^0_i + V_i \tag{4}$$

In Equation (2), $x^L_{ji}$ and $x^I_{ji}$ are the bank $j$'s liquid and illiquid interbank liabilities to bank $i$, while in Equation (4), $V_i$ is bank $i$'s net worth. For simplicity, we assume that $V_i$ is the bank's regulatory capital, i.e., it has to be at least equal to the bank's capital requirement, and that $L^0_i$ is the difference between the total debt $T^L_i$ and the other terms in Equation (4). Additionally, we consider that the interbank debts and the debts to counterparties external to the network have the same seniority and that, following Eisenberg and Noe (2001), the debt to the shareholders have the least seniority.

Following Cifuentes et al. (2005) and Gauthier et al. (2012), we assume that the interbank assets and the external liquid and illiquid assets are risky regarding capital adequacy computations. We also consider that external liquid assets $A^L_i$ (fixed income private securities, stocks and mutual funds quotes) are marked to market, while the external illiquid assets $A^I_i$ are not. Finally, we consider that only part of external illiquid assets (i.e. the credit portfolio) can be sold. The remaining illiquid assets are considered completely illiquid, thus they are not sold for capital adequacy adjustment purposes. External liquid assets can always be put to sale.

## 2.2 The capital adequacy process

### 2.2.1 Model basics

1. Timing

   We simulate the reactions of the banking system to shocks from its position on a given date. However, the simulations consider that the banking system's reaction is a sequence of cycles of solvency and capital adequacy assessments, and reactions through fire sales, performed until equilibrium is reached. Each cycle can be considered as lasting about a day.

2. Interbank liabilities network

   Let a system with $N$ banks mutually exposed have its liabilities network represented by the matrix $\boldsymbol{X}_{[N\times N]}$, whose elements $x_{ij}$ represent bank $i$'s liabilities towards bank $j$. Each bank $i$ has an endowment $e_i$, the total debts vector $\boldsymbol{d}$ is given by $d_i = \sum_j x_{ij}$, and the liabilities matrix $\boldsymbol{X}$ is normalized with respect to each bank's debt, resulting in a relative liabilities matrix $\mathbf{\Pi}$, whose elements are: $\pi_{ij} = X_{ij}/d_i$ if $d_i > 0$ and $\pi_{ij} = 0$ otherwise.

   Eisenberg and Noe (2001) propose a clearing payment vector $\boldsymbol{x}^*$ computation method for a banking system defined by $(\mathbf{\Pi}, \boldsymbol{e}, \boldsymbol{d})$, valid under the hypotheses: a) Limited liability; b) Absolute priority, and c) Payment proportionality in case of default. The bank $i$'s payment is given by $\boldsymbol{x}_i^* \in [0, d_i]$, computed as:

$$\boldsymbol{x}_i^* = \min(\sum_j \pi_{ji} x_j^* + e_i, d_i) \tag{5}$$

   The banking system clearing payment vector is the fixed point $\boldsymbol{x}^*$ of the map $\Phi(\,\cdot\,; \mathbf{\Pi}, \boldsymbol{d}, \boldsymbol{e}) : [\mathbf{0}, \boldsymbol{d}] \to [\mathbf{0}, \boldsymbol{d}]$, given by:

$$\Phi(\boldsymbol{x}; \mathbf{\Pi}, \boldsymbol{d}, \boldsymbol{e}) \equiv (\mathbf{\Pi}^T \boldsymbol{x} + \boldsymbol{e}) \wedge \boldsymbol{d} \tag{6}$$

   Eisenberg and Noe (2001) state that the fixed point of Equation (6) exists and is unique when the financial system is regular[1].

3. Asset prices

   We assume, following Cifuentes et al. (2005), that the prices of assets that are subject to fire sales are exponential functions of the total quantities of assets

[1] A regular financial system is one in which every bank risk orbit (the set of its direct or indirect creditors) is a surplus set, i.e. it is a set of banks in which none of them has liabilities to a bank that does not belong to the set and, at least one bank has a strictly positive endowment.

that are sold. The market prices $p_L$, of liquid assets $A_i^L$, and $p_I$, of illiquid assets $A_i^I$, are different and decay differently, following exponents $\alpha_L$ and $\alpha_I$. We consider that illiquid assets decay faster, due to their lower liquidity. On a given cycle $t$, the asset prices are given by:

$$p_L^t = p_L^{t-1} \exp\left(-\alpha_L \sum\nolimits_i s_i^{L,t} / \sum\nolimits_i A_i^L\right) \tag{7}$$

$$p_I^t = p_I^{t-1} \exp\left(-\alpha_I \sum\nolimits_i s_i^{I,t} / \sum\nolimits_i A_i^I\right) \tag{8}$$

In which $s_i^{L,t}$ and $s_i^{I,t}$ stand for liquid and illiquid assets sold by bank $i$ in $t$.

### 2.2.2 Solvency assessment

To assess their solvency, banks compute their net worth using an approach based on David and Lehar (2011). In that approach, banks estimate the payments they can afford based on their balance sheet assets and on the estimates their debtors made the previous cycle. They do not really pay these amounts, as they refer to exposures of different maturities. Bank $i$ computes its endowment at cycle $t$ as follows:

$$\begin{aligned} e_i^t =& C_i + R_i^{L,t-1} + R_i^{I,t-1} + p_L^{t-1}\left(A_i^L - \sum\nolimits_{k=1}^{t-1} s_i^{L,k}\right) + A_i^I - \sum\nolimits_{k=1}^{t-1} s_i^{I,k} + \\ & F_i^A - \varepsilon_i \end{aligned} \tag{9}$$

In which $\sum_{k=1}^{t-1} s_i^{L,k}$ and $\sum_{k=1}^{t-1} s_i^{I,k}$ are the liquid and illiquid asset totals sold from the beginning to the cycle $t-1$, $R_i^{L,t-1}$ and $R_i^{I,t-1}$ are the cash amounts received from the sales of liquid and illiquid assets from the beginning to the cycle $t-1$, $p_L^{t-1}(A_i^L - \sum_{k=1}^{t-1} s_i^{L,k})$ is the liquid assets' marked-to-market value and $\varepsilon_i$ is the additional loss (shock) suffered by the bank at the beginning of the simulation. The bank $i$ net worth is given by:

$$V_i^t = e_i^t(1 - \psi_{[i\in\mathcal{L}]}) + \sum\nolimits_j \pi_{ji} x_j^{t-1} - D_i - L_i^0 - d_i \tag{10}$$

In which $\psi_{[i \in \mathcal{L}]}$ is the factor we apply the endowment to compute the liquidation cost. $\psi \neq 0$ if the bank $i$ is liquidated, that is, $i \in \mathcal{L}$. We exclude network assets from the liquidation cost calculation for simplification, following David and Lehar (2011) and Gauthier et al. (2012). The estimated interbank payment affordable to bank $i$ at cycle $t$ is:

$$x_i^t = \frac{d_i}{d_i + L_i^0} \min\left(d_i + L_i^0, \max\left(0, e_i^t(1 - \psi_{[i \in \mathcal{L}]}) + \sum\nolimits_j \pi_{ji} x_j^{t-1} - D_i\right)\right) \quad (11)$$

Equation(11) differs from Eisenberg and Noe (2001)'s framework, as in Equation (5), in the following. *a)* if bank $i$ is liquidated by the end of cycle $t-1$, it incurs a liquidation cost; *b)* the deposits $D_i$ have higher seniority than interbank debts, which requires that they are subtracted from the endowment $e_i$ prior to calculation; *c)* the interbank debts have the same seniority as the debt towards creditors outside the network. To address this assumption, we consider that the debt to be cleared, which appears in the $\min(\cdot,\cdot)$ is the sum $d_i + L_i^0$. In case of default, the amounts paid to network creditors and to outside creditors are proportional to the debts to each of them as shown by the term $d_i/(d_i + L_i^0)$.

If bank $i$ was not liquidated after the previous cycle and its interbank payment estimate $x_i^t$ is lower than its debt $d_i$, it is considered to be insolvent and is liquidated, being, thus, included into the liquidated banks set $\mathcal{L}$. In this case, the bank $i$ payment estimate is updated considering the liquidation cost using Equation (11). The liquidation cost is computed by:

$$C_i^{liq} = e_i^t \psi_{[i \in \mathcal{L}]} \quad (12)$$

Liquidated banks do not assess their capital adequacy and do not negotiate their risky assets along the next cycles. They continue to compute how much debt they can pay but do not reassess their condition regarding liquidation. However, when a bank becomes insolvent, we consider that, in the current cycle, it sells all its risky assets, as in Cifuentes et al. (2005). Bank $i$'s risk-weighted assets are given by:

$$RWA_i^t = w_i \left( A_i^I - \sum\nolimits_{k=1}^{t-1} s_i^{I,k} + p_L^{t-1} \left( A_i^L - \sum\nolimits_{k=1}^{t-1} s_i^{L,k} \right) + \sum\nolimits_j \pi_{ji} x_j^t - \varepsilon_i \right) \tag{13}$$

In Equation (13) above, $w_i$ is the risk weight applied to risky assets, which is assumed to be the same for all asset classes. The capital adequacy is assessed by:

$$\frac{V_i^t}{RWA_i^t} = r \geq F \tag{14}$$

Above, $V_i^t$ is the regulatory capital, $r$ is the capital ratio and $F$ is the capital requirement ratio. We assume that banks are bound, by regulatory enforcement, to keep their capital adequacy as their net worth (regulatory capital) falls. According to Equation(14), the liquidated bank $i$ has to reduce the value of its risk-weighted assets, which is achieved by the sale of risky assets. It sells firstly its liquid assets and, if necessary, the illiquid ones. The amount of liquid assets to be sold at cycle $t$ is computed using the asset's market prices at the end of the previous cycle, $p_L^{t-1}$:

$$s_i^{L,t} = \max \left( 0, \min \left( A_i^L - \sum\nolimits_{k=1}^{t-1} s_i^{L,k}, RWA_i^t / (w_i p_L^{t-1}) \right) \right) \tag{15}$$

If $RWA_i^t / (w_i p_L^{t-1}) > A_i^L - \sum_{k=1}^{t-1} s_i^{L,k}$, the fire sale of liquid assets will not have been enough to achieve the capital adequacy. In this case, the illiquid assets are sold at the market price $p_I^{t-1}$:

$$s_i^{I,t} = \min \left( A_i^{IS} - \sum\nolimits_{k=1}^{t-1} s_i^{I,k}, (RWA_i^t / w_i - p_L^{t-1} s_i^{L,t}) / p_I^{t-1} \right) \tag{16}$$

In the Equation (16) above, $A_i^{IS}$ is the illiquid asset class considered to be marketable, for instance, the credit portfolio.

#### 2.2.3 Capital adequacy assessment

In each cycle, solvent banks assess their capital adequacy. The banks found to be non-compliant (i.e. those for which $r < F$ in Equation (14)), define the measures to be taken to restore their adequacy. To do this, they initially compute how much they need to reduce the value of their risk-weighted assets, taking into account the current asset prices, that is, the $t-1$ prices. They calculate the risk-weighted assets value reduction by:

$$\Delta RWA_i^t = (RWA_i^t - V_i/F)/w_i \tag{17}$$

The amount of external liquid assets to be sold at cycle $t$ is computed by:

$$s_i^{L,t} = \max\left(0, \min\left(A_i^L - \sum\nolimits_{k=1}^{t-1} s_i^{L,k}, \Delta RWA_i^t/p_L^{t-1}\right)\right) \tag{18}$$

If $\Delta RWA_i^t/p_L^{t-1} > A_i^L - \sum_{k=1}^{t-1} s_i^{L,k}$, it will be necessary to negotiate illiquid external assets. The capital adequacy condition, after the liquid assets fire sale, is given by:

$$\frac{V_i^t - s_i^I(1 - p_I^{t-1})}{RWA_i^t - w_i(s_i^{L,t} p_L^{t-1} + s_i^{I,t})} \geq F \tag{19}$$

in which $s_i^I$ is the amount of illiquid external asset that the bank needs to sell and $RWA_i^t$ is computed using Equation (13). The numerator of Equation (19) is not affected by the liquid assets fire sale as we estimate their sale price as equal to the mark-to-market price. Rearranging the previous equation:

$$s_i^I = \frac{V_i^t - F(RWA_i^t - w_i s_i^{L,t} p_L^{t-1})}{1 - p_I^{t-1} - F w_i} \tag{20}$$

Bank $i$ has to negotiate illiquid assets to restore its capital adequacy if the numerator of Equation (20) is lower than zero. The denominator defines the need of buying or selling these assets. If $p_I^{t-1} > 1 - Fw_i$, the bank must sell its illiquid assets, otherwise, it must buy them[2]. The amount to be effectively bought or sold is computed considering the availability of resources. In case of a sale, the amount of required sales $s_i^I$ is compared to the asset balance in the balance sheet, while in the case of a purchase, we assume that the bank pays the operation with Federal Securities $FS_i$ and availabilities received previously as cash payments related to asset sales ($R_i^{L,t-1}$ and $R_i^{I,t-1}$). The amounts to be negotiated are given by:

$$\text{If } s_i^I > 0 \text{ (sales)}, \ s_i^{I,t} = \min\left(s_i^I, A_i^{IS} - \sum\nolimits_{k=1}^{t-1} s_i^{I,k}\right) \tag{21}$$

$$\text{If } s_i^I < 0 \text{ (purchases)}, \ s_i^{I,t} = \max\left(s_i^I, -\frac{R_i^{L,t-1} + R_i^{I,t-1} + FS_i}{p_I^{t-1}}\right) \tag{22}$$

If the amount of required sales of a given bank $i$ is greater than its stock of external illiquid marketable assets ($s_i^I > A_i^{IS} - \sum_{k=1}^{t-1} s_i^{I,k}$), or if its amount of required purchases is greater than the available resources it is able to give as payment ($|s_i^I| > (R_i^{L,t-1} + R_i^{I,t-1} + FS_i)/p_I^{t-1}$), the bank will not be able to restore compliance with capital requirements. In this case, it is liquidated and included into the set $\mathcal{L}$ of liquidated banks.

### 2.2.4 Negotiation of Assets

Once banks define the amounts of assets to be negotiated, they place their offers in markets outside the network. We consider that asset prices fall with the increase of the quantities of assets to be sold, given by $\sum_i s_i^{L,t}$ and $\sum_i s_i^{I,t}$, according to Equations (7) and (8). The prices computed from these equations will be used in the computations in the next cycle, and to compute the cash received (paid) for the amounts sold (purchased) in the present cycle, given by:

[2]Illiquid assets usually have a relatively high value of $w$, thus, a bank has to buy these assets, for capital adequacy purposes, only if they are sold at a high discount. This effect occurs in this model because this type of asset is not marked to market.

$$R_i^{L,t} = s_i^{L,t}(p_L^t + p_L^{t-1})/2 + R_i^{L,t-1} \tag{23}$$

$$R_i^{I,t} = s_i^{I,t}(p_I^t + p_I^{t-1})/2 + R_i^{I,t-1} \tag{24}$$

In Equations (23) and (24) above, we assume that assets are sold (purchased) at the average of the prices at the current and previous periods.

### 2.2.5 Capital adequacy equilibria

When all solvent banks are compliant with legal capital requirements, they do not need to take any action regarding the capital adequacy process described in this paper. In this case, we say that the banking system is in equilibrium regarding the capital adequacy process, i.e., no bank needs to negotiate risky assets by legal enforcement. We show that whatever the initial state of the banking system, the capital adequacy process always reaches an equilibrium. However, this equilibrium is not unique. Unlike the setting proposed by Eisenberg and Noe (2001), which present a unique equilibrium for a regular financial system, the capital adequacy process we present here may have infinite equilibria. In the Eisenberg and Noe (2001)'s model, whichever the initial value of the clearing vector $\boldsymbol{x}$ in Equation (5), the model converges to its unique equilibrium; on the other hand, in the capital adequacy process, it is usually possible to sell more risky assets than legally required. If this sales excess does not lead the seller bank into default, it gets compliant and thus, will be in an equilibrium. This equilibrium is different from that achieved if just the legally required risky assets are sold.

We propose and provide a demonstration draft for the existence and non-uniqueness of equilibrium states regarding the capital adequacy process in Appendix A.

## 3 Data

We analyze the Brazilian banking system, which is formed by financial conglomerates and individual banks that do not belong to a conglomerate. We take two types of conglomerates: a Type-I conglomerate, which has at least one bank that has a commercial portfolio (it can hold demand deposits), and a Type-II one, that does not have banks with a commercial portfolio but has at least one bank with an investment portfolio (it cannot take demand deposits, and raise funds from time deposits and other sources). From now on, we will refer to these entities, indistinctly, as banks. We do not include credit unions in the analysis. Data is monthly, from February 2011 to December 2013.Along this period, the number of banks has varied from 124 to 134. In December 2013, date for which we perform a more detailed analysis, the banking system has 124 banks.

We get data for the last working day of each month from three sources: a) accounting data; b) supervisory variables; and c) network exposures among banks. The accounting data comes from the Accounting Plan of the National Financial System Institutions database. This database has monthly records with standardized balance sheet information provided by banks to the Central Bank of Brazil. We get from this source information on cash equivalent assets, federal securities stock, deposits, and aggregated credit portfolio. We take regulatory capital information, liquid assets, fixed assets and total assets data from the supervisory variables database provided by the Central Bank of Brazil Financial System Monitoring Department. Finally, we get banks network exposures from a monthly dataset provided by CETIP[3]. We take from this dataset the open positions of banks, against others, in interfinancial deposits, bank deposit certificates, interbank onlending, credit and credit assignment operations, instruments eligible as capital, real state credit bills, financial letter and swap operations. These exposures are not netted out, as in case of a bank liquidation, the liquidated bank continues to receive its claims even if its payments are suspended, including those to a possible debtor.

Before performing the simulations, we present monthly information on the balance sheet composition computed for the aggregate banking system from February 2011 to December 2013. Figure 8 presents the share of the system's total assets for each category of balance sheet items related to the model we use during this period.[4] These shares are roughly stable along the period, presenting a small transfer from

[3]CETIP is an open capital company that offers services related to registration, custody, trading and settlement of assets and bonds.

[4]In the computation of the total assets for each category, we subtract the interbank assets mentioned above to avoid double-counting.

liquid to illiquid assets along 2013. There is a preponderance of illiquid assets, especially credit operations, which correspond to about 45% of total assets. Interbank assets, for which we have network data, are of reduced relevance vis a vis the total assets, and represent, on average, 19% of the regulatory capital and 2.6% of total assets. At first, a default in this market mostly originates compliance needs rather than a default cascade. On the liabilities side, deposits from counterparties external to the banks' network play an important role, corresponding in size to credit operations.

The liquid external assets and external illiquid tradable assets represent, on average, 48% of total assets, which gives banks room to get compliant by selling these assets. On the other hand, these assets expose banks to market risk. External liquid assets, by being marked to market, expose continually their owners to price falls due to fire sales. Fortunately, they correspond to about only 5% of total assets. External illiquid tradable assets, by not being marked to market, expose to risk only banks that need to sell them.

Figure 9 presents distributions of banks' capital buffers, and of liquid and illiquid assets subject to sales during the capital adequacy process for December 2013, date for which we perform the simulations presented in next sections. In all these distributions, there are about five banks with a share of at least 6% of the variable's total. This indicates that there is a group of banks whose sales decisions produce stronger externalities upon the entire system. If one or more banks of the group sell part or all of its stock of external liquid assets, the resulting price fall will affect the majority of the other banks, which own this type of asset. These banks will have their net worth reduced by the mark to market process and possibly need to perform a fire sale to remain compliant with capital requirements. If, on the other hand, they sell external illiquid tradable assets, which in this paper we assume to be the credit portfolio, the other banks in the system are not immediately affected, but become more vulnerable as, in case they need to sell this type of asset, they would suffer additional losses due to the asset's price fall, which would feed back into the capital adequacy process and increase the amount of assets to be sold and, consequently, the corresponding losses.

# 4 Analyses and Results

In the next sections, we simulate the effects of different types of shocks to the banking system, comparing them to the simulation performed in the absence of shocks, i.e., for the system as it is. For data of December 2013, the adjustment process is rather small, once the largest banks are compliant and only one small bank needs to adjust its risky assets to get compliant.

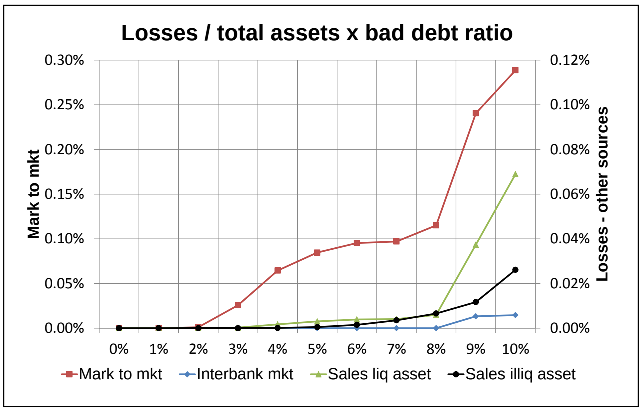
The parameters we employ in the simulations are, unless differently specified, the following: liquidation cost factor $\psi = 0.1$, as in Alessandri et al. (2009), capital requirement ratio $F = 0.11$, price decay is $\alpha_L$, such that the minimum possible liquid asset price = 80% initial price and price decay $\alpha_I$ is such that the minimum illiquid asset price = 70% initial price. We also compute a proxy coefficient to relate bank $i$ risky assets, given by $\lambda_i - C_i + A_i^I$ with risk-weighted assets given by $V_i^{req}/F$, in which $V_i^{req}$ is the regulatory capital requirement taken from the supervisory variables database. For simplicity, we adopt a unique weight for the whole set of bank assets, given by:

$$w_i = \frac{V_i^{req}}{F(\lambda_i - C_i + A_i^I)} \tag{25}$$

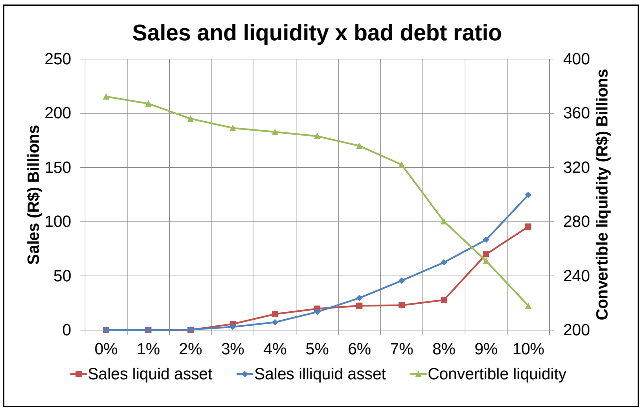
## 4.1 Bad debt ratio increases

The first simulation we perform aims at measuring the impact of bad delinquency rate increases on the banking system. Banks' capital is computed taking into account loss provisions for expected losses related to all asset classes. In this simulation, we increase bad debt ratio related to banks' lending operations and fund providing operations to levels above the expected loss provisions so that the exceeding loss reduces their capital. We compute this loss excess as the product of a bad debt ratio increase, beyond the expected bad debt, and the sum of bank's lending and funding operations. We vary the bad debt ratio increase from 0 p.p. to 10 p.p. and apply each of the resulting shocks simultaneously to all banks. We intend to simulate the worsening of macroeconomic conditions effect on the banking system through the bad debt channel.

The process mechanics we simulate is as follows. We apply the mentioned shocks to all banks capital. Some banks may become insolvent, while others will be weakened, some of them losing compliance with capital requirements. This situation triggers a capital adequacy process in which banks strive to adequate their risky assets base by the sale of external assets, both liquid and illiquid. These sales affect banks' net worth due to asset prices reduction, which feeds back into the process until an equilibrium is reached. We study the features of this new equilibrium using the charts that follow.




(a)




(b)

Figure 1: Effect of a simultaneous increase in the delinquency rate of credit operations in the whole banking system. In (a), we compare the banking system's aggregated losses from different sources. In (b), we compare the convertible liquidity to assets sales. Computed for December 2013.

In Figure 1a we present the effects of bad debt ratio increases on the aggregate loss of the banking system. We also present results in the absence of shocks (0 p.p. bad debt ratio increase) to allow a comparison. We decompose these losses into additive components by source: we have losses within the interbank market, that is, losses caused by contagion from a bank's default, losses caused by the process of marking to market the unsold liquid external assets computed in the final equilibrium, losses due to the sale of liquid external assets at a price lower than the initial and losses due to sales of external illiquid tradable assets.

We note firstly that the interbank market losses are comparatively very low. From Figure 10, we note that the threshold for insolvent banks to cause this type of loss is a bad debt ratio increase of 9 p.p.. The defaulting banks are small-sized and cause a negligible impact on the system. Regarding the other types of loss, the mark to market losses are initially larger compared to the others as: 1) the illiquid assets are sold only by banks that have sold their whole stock of external liquid assets, and 2) a relatively small share of the system's aggregate external liquid assets had to be sold. Regarding the liquid external assets, starting from a 0% sale to a 100% sale, initially a price decrease induces a mark to market loss larger than the loss due to fire sales. This happens because: 1) the same price fall applies both to the unsold assets share, to which the marking to market applies, and to the sold ones, and 2) initially, the amount of unsold assets is larger than that of sold ones. If the amount sold increases, both losses increase, but from a given point, the sales loss exceeds the mark to market ones given the decreasing stock of unsold assets. In this simulation, this happens for a debt debt ratio of above 10 p.p.. About this point, the mark to market losses begin to decrease until reaching zero, when a 100% sale is performed.

Up to a 10 p.p. bad debt ratio increase, losses are not so severe. If we increase the delinquency rate further, there will be a shock magnitude for which losses from sales of external illiquid tradable assets outperform the others, because both the illiquid asset price is lower and the stock of these assets in the whole banking system is much higher than the stock of external liquid assets.

Figure 1b compares the amount of liquid and illiquid assets sold to the banking system's convertible liquidity. Here, we compute convertible liquidity for a bank $i$ as:

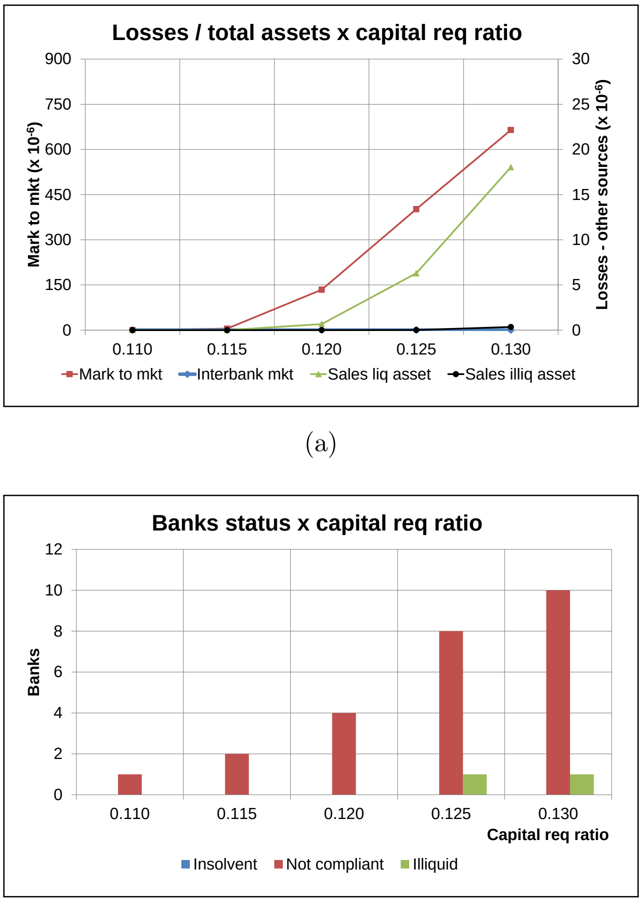
$$Liq_i = \min(C_i + R_i^{L,t} + R_i^{I,t}, (V_i^t - F\ RWA_i^t)/(F\ w_i)) \tag{26}$$

In Equation (26), $t$ is the last cycle of the capital adequacy process, i.e., the cycle in which the banking system reaches equilibrium. We aggregate $Liq_i$ for all compliant banks, that is, for banks for which $r_i \geq F$.

We define convertible liquidity as the minimum between risky assets that can be purchased, given the regulatory capital excess over the capital requirement (the term $(V_i^t - F\ RWA_i^t)$) and cash equivalent assets, i.e., the initial cash equivalent assets plus the proceeds of possible fire sales. If a bank is not compliant, it is illiquid once it cannot use its cash equivalent assets to purchase risky assets (for instance, for lending operations). We also note that in this model, these banks sell enough assets to become compliant, with a zero capital buffer. Thus, the available liquidity comes from banks that didn't sell assets. In this model, we consider that fire sales are performed outside the banks' network, however, in this simulation, these sales could be absorbed by the banking system for a bad debt ratio increase up to 9 p.p.. For a 10 p.p. increase, a share of the sold assets would have to be bought in the market outside the network.

We also note that in Figure 10a, the net worth of the banking system decreases almost linearly with the increase of the shocks magnitude, reaching 70% of its original value. The decrease of asset prices is lower. According to the decays given as data, the liquid asset price can decrease up to 80% of the initial price and illiquid asset's up to 70%, but they decreased, respectively to 93% and 98%, which indicates that prices remained relatively high. On the other hand, the decrease in the banking system net worth indicates a fall of banking sector shares in the stock market of about 30%, which is relevant.

Finally, we show in Figure 14a that shock amplification tends to increase with the magnitude of the initial shock for this type of shock, i.e., a distributed shock. This increase is non-monotonic and is mostly related with the sale of illiquid assets and its impact on their prices, which affect subsequent sales.




(a)



(b)

Figure 2: Effect of an increase in the capital requirement ratio $F$. In (a), we compare the aggregate losses from different sources, and in (b), the number of insolvent and illiquid banks at the end of the adjustment process. Computed for December 2013.

## 4.2 Capital requirement ratio increases

In this simulation, we increase the capital requirement ratio $F$ from 0.11 to 0.13 in 0.05 steps, with the purpose of simulating, from the point-of-view of this model, the consequences, for the banking system, of possible increases in the capital requirement ratio $F$ such as those that may result from decisions related to the operation of countercyclical capital buffers.

Table 2 shows data at the beginning of the simulation. Even for a capital ratio increase to 0.14, few banks become not compliant. The 9 banks that lose compliance represent 6.43% of the banking system's total assets. In December 2013, the larger banks had capital ratios from 0.14 to 0.17. These 27 banks correspond to 76.04% of the banking system's total assets and wouldn't need to adjust their portfolios for

Table 2: Capital ratio and total assets in December 2013.

| Capital ratio | Number of banks | Total Assets share (%) |
|---|---|---|
| 8 − 11 | 1 | 0.01 |
| 11 − 11.5 | 1 | 0.18 |
| 11.5 − 12 | 2 | 2.90 |
| 12 − 12.5 | 4 | 2.56 |
| 12.5 − 13 | 2 | 0.79 |
| 13 − 14 | 10 | 2.01 |
| 14 − 15 | 14 | 25.71 |
| 15 − 16 | 9 | 18.18 |
| 16 − 17 | 4 | 32.15 |
| 17 − 18 | 7 | 2.29 |
| 18 − 19 | 7 | 1.96 |
| 19 − 20 | 5 | 9.68 |
| 20 − more | 58 | 1.58 |

compliance. No bank becomes insolvent even for a capital ratio of 0.13 and one very small bank becomes illiquid for a capital ratio of 0.125 as can be seen in Figure 2b. Figure 2a shows that the mark to market losses are the highest, since a small share of system's available external liquid assets is sold (about 5%). In this case, the losses suffered by the set of all holders of unsold assets due to the price fall are much larger than those suffered by the banks that sold them. Regarding the external illiquid tradable assets, the sales are comparatively negligible as are the corresponding losses.

## 4.3 Mark to market effects

In this simulation, we test the reaction of the banking system to market prices falls. We simulate the banking system considering the initial prices of both liquid and illiquid assets as a fraction of the real price, varying from 100% to 90%. The simulation for 100% prices considers the banking system as it is, without shocks, situation in which only 1 small bank is not compliant.

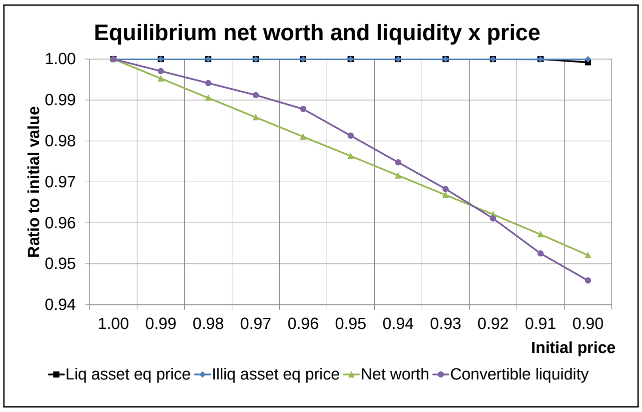
Figures 3 and 11 present results for these simulations. Figure 3a shows that prices of both external liquid assets and external illiquid tradable assets almost do not fall, which means that the sales of these assets are negligible. Figure 11 shows that only 4 banks lose compliance as a consequence of these shocks, for a price fall of 10%. From these, one cannot recover and becomes illiquid. Additionally, one of the banks gets insolvent from these price falls as its net worth is lower than the mark to market losses it suffers. We attribute the banks resilience to these shocks to the capital buffer held by the majority of these banks, specially the larger ones (see Table 2). Additionally, the external liquid assets share is about 6% while the net worth is 13% in December 2013 (see Figure 8a), which reduces the influence of marking to market these assets. This reduced influence and the small amount of liquid assets sales (see Figure 3b) explain the relative price inelasticity of the net worth of the banking system, which we see as a proxy for the prices of banks shares in the stock market. For a prices fall of 10%, the net worth of the banking system falls about 5%.

Figure 3b also shows that the mark to market process causes a convertible liquidity loss much higher than the sales amount, as it affects the whole system. However, these sales volumes can be easily absorbed by the banking system. The comparison among losses by source presented in Figure 3c shows that the mark to market ones are more than 100 times the sum of the others.

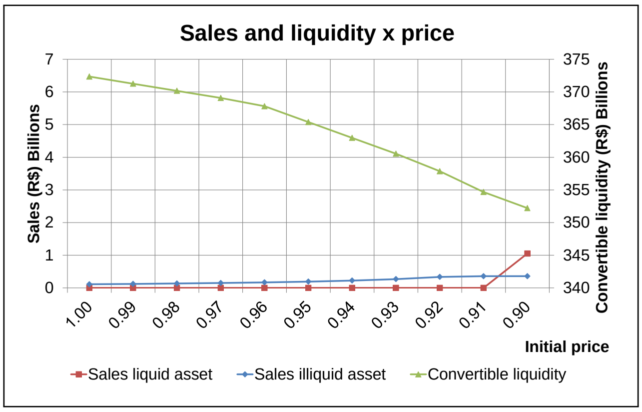
The comparatively little effects caused by prices fall up to 10% highlight the resilience brought to the banking system by capital buffers banks have. It would be necessary sharper simultaneous price falls to lead the banking system to a crisis.

## 4.4 Bank defaults

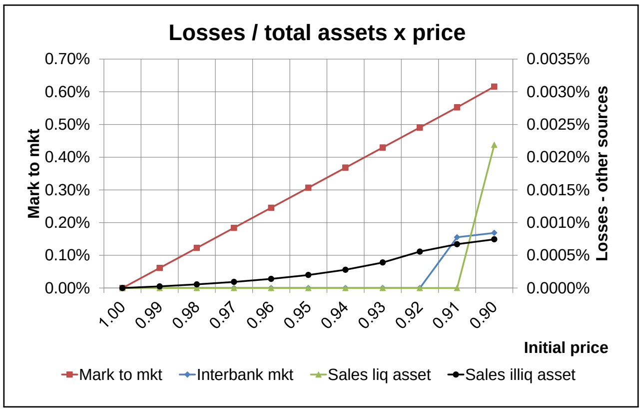
This simulation is an extension of the Eisenberg and Noe (2001)'s fictitious default algorithm. Here, we simulate, as in Eisenberg and Noe (2001), the default of an individual bank and check the equilibrium reached after this shock. To perform this simulation, we start by imposing a shock to the bank which default's impact we intend to assess. The shock magnitude is enough to prevent that the bank performs payments of any amount of interbank liabilities to its creditors. These creditors suffer the losses due to the original default and may default, as a consequence, become non-compliant with capital requirements, or, if these losses are smaller than their capital buffer, they remain compliant. The non-compliant banks put their assets to sale, triggering a capital adequacy process.




(a)




(b)




(c)

Figure 3: Effect of external assets prices shock in the banking system. In (a), we present effects of these shocks on prices, convertible liquidity and aggregate net worth. In (b), we compare the convertible liquidity to assets sales occurred during the capital adequacy process, and in (c), we compare the banking system's aggregated losses from different sources. Computed for December 2013.

Figure 4 presents the number of banks which default results in each category of

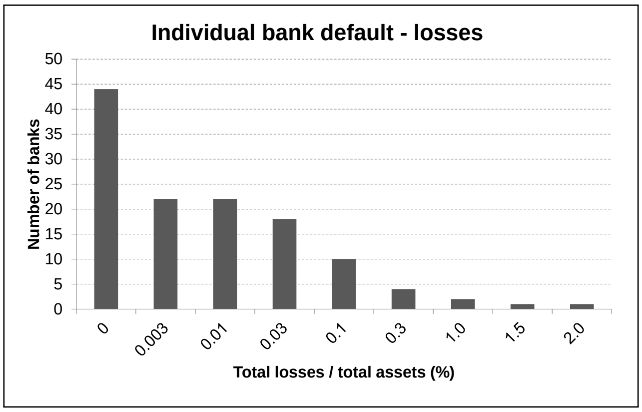



Figure 4: Distribution of banking system losses caused by the default of an individual bank. These losses includes those suffered by the defaulting bank, but not the idiosyncratic loss that originated the initial default. Computed for December 2013.

total losses / total assets ratio. These total losses are the sum of losses from default costs, mark to market losses, interbank market losses and losses from liquid and illiquid assets sales. These losses include those from fire sales and default costs of the defaulting bank. We include losses from fire sales for defaulting banks considering that they try to keep compliant as they suffer losses. These sales are an additional channel by which a bank's default affects the banking system. We also include losses from default costs for all banks including the originally defaulting one, as these costs are non-negligible and reduce the amounts they will be able to pay its creditors, for instance, depositors. In Figure 4 we see that the bank with the greatest impact originates losses about 2.0% total assets. The 4 banks that cause the largest losses together amount to about 5.1% total assets losses. On the other hand, the remaining 120 banks originate together about a 3.2% total assets loss. Investigating the contagion chains that transmit losses generated by these initial defaults to the other banks, we find that the insolvency chains are short: only 6 big banks originate additional liquidations (insolvency or illiquidity) of up to 2 banks. This happens because in this banking system interbank payments are low compared to the net worth of their creditors. In the 6 cases mentioned, the additional default was caused by a conjunction of interbank market losses and mark to market losses. Figure 12 presents assets fire sales by the banking system excluding the bank that defaulted initially. We see that these idiosyncratic defaults mostly originate near zero additional illiquid assets fire sales while originating liquid assets fire sales of up to 20% of the stock of these assets in the banking system. The 3 banks which idiosyncratic defaults trigger fire sales from about 1% of the stock of liquid assets and higher are big-sized, so that the fire sales due to their default induce a non-negligible fall on the prices of these assets, originating significant mark to market losses for the other banks. The maximum number of banks that become non-compliant is 6, which indicates that most banks' capital buffers are enough to protect them individually and as a system against idiosyncratic defaults.

In Figure 5 we compare loss amplifications due to contagion with sizes of the individual banks defaulting initially. We compute these amplifications as the ratios of the banking system losses to the idiosyncratic loss that caused the initial bank default. The banking system losses include those suffered by the defaulting bank along the simulation, but not the idiosyncratic loss that caused the initial default. We categorize bank sizes initially taking total assets logarithms for all banks in the sample. We then form a scale between the largest and the smallest logs with 4 equal divisions and locate all bank logs in this scale. Figure 5 shows that, in December 2013, no bank default causes additional systemic losses greater then the loss that lead this bank to default. For idiosyncratic defaults, bank size do not seems to influence much loss amplification. Thus, the importance of big banks is mostly related with their loss amounts than with their amplification.

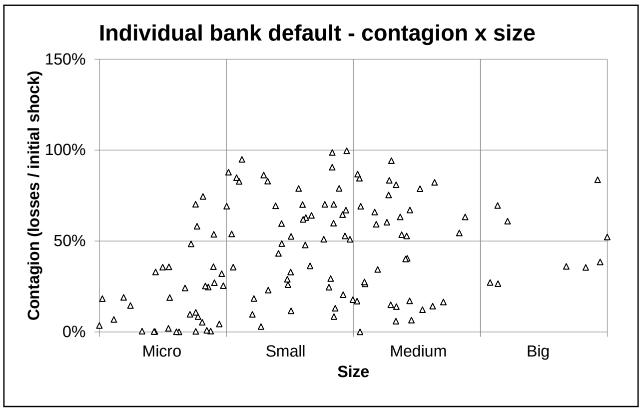



Figure 5: Loss amplifications due to contagion by sizes of the individual banks defaulting initially. Computed for December 2013.

# 5 Banks risk contribution

In this section we use this model to assess the risk contribution of individual banks, i.e., their contribution to systemic losses. Initially, we choose, as a scenario for which we wish to know the risk contributions, the simultaneous delinquency rate increase of 10 p.p. in the whole banking system dealt with in section 4.1. We compute the risk contribution of a given bank firstly protecting it against becoming insolvent or not compliant. The protection is performed before the simulation's start and consists on adding cash equivalent assets to the protected bank's balance sheet, increasing its net worth without increasing its risky assets amount. As the bank neither gets insolvent nor puts assets to sale, it remains in the banking system, but no more contributes with any event that would cause loss to the other banks. We compare, for a given variable, the change it suffers in a normal simulation and that it suffers in a simulation in which we protect the bank. For instance, we compute the external liquid asset price change in both cases and assign as the bank $i$ contribution for this variable change:

$$C_i^{p_L} = \frac{p_L^0 - p_L^{t,i}}{p_L^0 - p_L^t} \tag{27}$$

In Equation (27), $p_L^{t,i}$ is liquid asset price computed in a simulation in which we protect bank $i$. In Figure 13, we present the banks contributions to the liquid and illiquid assets price fall. In both cases, most of contributions are around zero while there are a few banks with contributions of 10% or more. One explanation for the very low contributions is that there is a significant share of well capitalized banks (see Table 2). These banks almost are not affected by the bad debt ratio increase of 10 p.p., thus, their fire sales amounts are low and taking them out of the process causes very little impact. Other explanation is that most of these banks are small, which reduce the potential impacts they can originate.

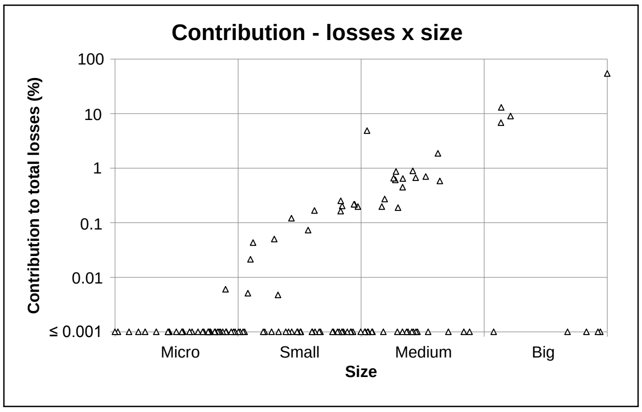
The large contributions tend to come from larger banks; even in case they are not severely affected, their risk assets adjustments require significant asset sales. We take 3 banks that appear in the list of the 5 banks that most contribute to the variables presented in Figure 13 (excluding contributions to banks insolvencies) and find that their relevance is due to a combination of a lower capital buffer and the share of the banking system's stock of external liquid assets. Another important factor, although not so important, is the bank's share of the banking system's stock of external liquid assets. From these 3 banks, 2 are big-sized and 1 is medium-sized, and all are initially compliant with regulatory capital requirements, although their capital buffers are among the smallest. Regarding contributions to bank insolvencies, we note that in the shock scenario we analyze, only 1 bank defaults (see Figure 10b). In this particular case, the bank's insolvency is due to the shock it suffers and cannot be prevented by the protection of other banks.

We also perform analyses of banks contribution to total systemic losses. Figure 6 presents information to support two types of analyses. Figure 6a shows that, for banks that contribute to systemic losses, it is possible that there is a positive correlation between size of the bank and its contribution to systemic losses. Banks that do not lose compliance due to the shock in the original simulation do not contribute to systemic losses. Figure 6b presents data for a bank protection cost $\times$ benefit analysis for each bank. It compares, for each bank, the cost to protect it from losses with the decrease of systemic losses that result from this protection, i.e., the bank's contribution to systemic losses. The protection cost is computed for banks that at least lose compliance with capital requirements in the scenario under study. It is given by the cash amount to be added to the bank's balance sheet such that it prevents the bank of losing compliance along the simulation:

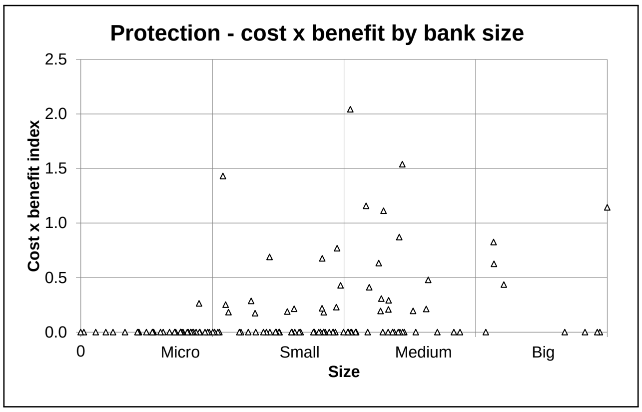
$$C_i^{prot} = \varepsilon_i + l_i^{IB} + l_i^{MM} + l_i^{L} - l_i^{MAX} \tag{28}$$

In Equation (28), $C_i^{prot}$ is the protection cost of bank$i$, $\varepsilon_i$ is the shock suffered by bank $i$, $l_i^{IB}$ is its loss from defaults in the interbank market, $l_i^{MM}$, its loss due to liquid assets marking to market, and $l_i^{L}$, its loss resulting from liquid assets fire sales. $l_i^{IB}$, $l_i^{MM}$ and $l_i^{L}$ are computed in the simulation without banks protection. $l_i^{MAX}$ is the maximum loss that bank $i$ can suffer without losing compliance with capital requirements, given by:

$$l_i^{MAX} = \min(V_i\ ,\ V_i(F/r_i^0 - 1)/(w_iF - 1)) \tag{29}$$




(a)




(b)

Figure 6: Analysis of banks' contributions to systemic losses: a) contributions to total losses by size of the protected bank, and b) cost × systemic benefit of protecting a bank by size of the protected bank. Computed for December 2013.

The cost × benefit index of protecting bank $i$ is given by:

$$I_i^{CB} = \frac{l_0^T - l_i^T}{C_i^{prot}} \tag{30}$$

In Equation (30), $l_0^T$ is the total systemic loss without banks protection and $l_i^T$ is the total systemic loss if bank $i$ is protected. Figure 6b shows that systemic losses prevented by individual banks protection are usually lower than the protection cost, with the exception of a few banks, of which the largest bank, according to its total assets, deserves more attention. It is the only big bank for which protection benefits exceeds its costs. The figure also shows that there is not a remarkable relationship between cost × benefit indexes and bank protection costs.

This simulation and the banks defaults simulation in section 4.4 provide elements to a comparison between contagion channels: the direct exposure channel and the prices channel. Given that the liquid assets interbank exposures are relatively low (see section 4.4), in the present scenario and configuration, the prices channel appears to be much more important as a contagion channel. For the prices channel, network structure mostly is not important, except in the case of insolvency, in which prices affect the net worth of the defaulting bank via a mark to market process, therefore affecting the expected payments value to their creditors. The prices channel is directly affected by the prices decay, thus we investigate the effect of different decays on the simulation outcomes on next section.

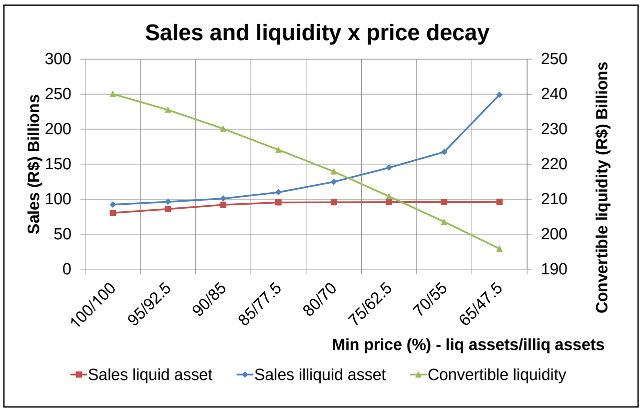
# 6 Robustness

In this section, we perform a robustness check on the simulation results. In section 5, we find that for this banking system configuration, the shocks transmission through the prices channel dominates the direct contagion transmission channel. However, the prices channel depends strongly on price decays that we assume for assets as they are sold. In this model, we assume that both "liquid" and illiquid assets are not perfectly liquid, being, thus subject to prices decay during a fire sale. On the other hand, cash equivalent assets and federal securities are assumed perfectly liquid. To distinguish external liquid and illiquid tradable assets, we assume the price decay is higher for the last one: in all simulations before, we consider that the illiquid assets' price can decay 50% more than the liquids'. We simulate the system's reaction to the simultaneous delinquency rate increase of 10 p.p. dealt with before, assuming different decay rates, including that used on previous simulations, for comparison (80/70). We denote the rates of decay by the minimum price that they lead to. We keep the rate assumed before of a 50% difference between liquid and illiquid prices decay rates.

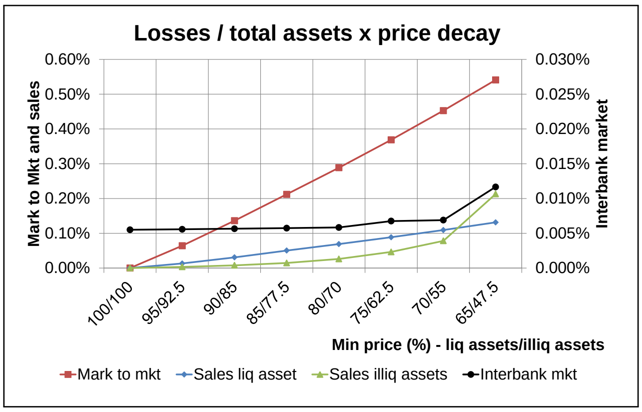
Figure 7 presents results for this robustness check. In Figure 7b, we note that if there is no decay (100/100), losses are zero, because fire sales only replace risky assets for cash without any loss as prices are 1. In the present analysis, mark to market losses are the most significant, for all price decays. From decays 70/55 to 65/47.5, sales of external illiquid assets increase significantly (see Figure 7a), corresponding to an increase that more than double the related losses (see Figure 7b). Two factors have a major contribution to this increase. Firstly, 2 big banks had to increase their illiquid asset sales to get compliant. This increase alone was more than a half of the increase experienced by the whole banking system from the first to the second scenario. The second factor was an increase in the number of banks that are liquidated in both scenarios. In the first scenario, 3 banks are liquidated (i.e., become insolvent or illiquid), while in the second, 5 are in this situation. The additional liquidation of these 2 banks also causes an increase in fire sales.

In Figure 14b, we present shock amplifications for increasingly more severe scenarios. The initial shock is the same, but the price decays are increasing, resulting in increasing additional losses (i.e., shock amplifications). Additional losses increase with price decay rates because, in case of fire sales, the same amount of sold assets produce larger price falls, resulting in higher losses.

These results suggest that price decay is itself a risk factor. If price decays are not much steep, losses are lower and it is easier for the banking system to recover from losses. However, for higher price decays, the system losses increase faster, indicating that the banking system in its present configuration and the process modeled has a knife-edge property, that is, for a given type of shock, there is a region beyond which relatively small increases in the shock's intensity produce a much larger losses increase. For this process, the reason for this seems to be related with the big banks; because while they are compliant, assets sales remain comparatively low; however, if they need to sell a greater amount of assets, this may affect the entire banking system. Therefore, identifying sources of high price decays and being effective in controlling them can mitigate systemic risk.




(a)




(b)

Figure 7: Robustness check: dependence on price decay rates of the equilibrium adjustment occurred after a simultaneous delinquency rate increase of 10 p.p. for all banks. In (a) we compare the convertible liquidity to assets sales occurred during the capital adequacy process, and in (b), we compare the banking system's aggregated losses from different sources. Computed for December 2013.

# 7 Final remarks

In this paper, we present a model of a banking system that follows the spirit of Cifuentes et al. (2005). The model considers the banking system as a network in which banks are mutually exposed by interbank market operations and are legally bound to comply with capital requirements. Non-compliant banks sell risky assets to recover compliance. Assets are categorized in two classes: liquid assets, which we to mark to market, and illiquid assets, which we do not. Prices of both types of assets decay as a function of the amount sold.

Our analysis suggests: 1) the reactions of a banking system to a shock depend jointly on banks capital buffer, size and assets owned, and type of shock: targeted to a single bank or affecting more banks; 2) Under a severe shock (bad debt ratio increase of 10 p.p. of lending and funding operations), medium-sized banks contribute significantly to system losses. However, if this shock becomes more severe, the most significant contributions will come from big banks only. This is in line with the idea of requiring an additional capital buffer for larger banks: if a bank increases its capital buffer, it will require stronger shocks to make it lose compliance and put assets to sale, which reduce assets prices. 3) In our model, the banking system seems to present a loss amplification increasing with the shock's magnitude. 4) The asset prices decay rates are critical. If they are high enough, they make it easier for a knife-edge region to be reached. Crises turbulence is associated with high asset prices decays. If it is possible to control these decays, it will be more difficult that the banking system reaches the region with larger procyclicality and amplification.

Future research can follow these lines. This model is yet a stylized one. One possible research line is to detail this model to use it as a stress test tool, as in Elsinger et al. (2006), who identify assets classes within each bank's balance sheet and generate joint shocks for them in a Monte Carlo simulation. Another possibility is to endogenize the fire sales process, allowing that banks with available liquidity decide over asset purchases. Another possibility yet is to build a model in which this one interacts with a macroeconomic model, along the lines of Alessandri et al. (2009).

# Appendix

## A Capital adequacy equilibria characterization

In this section, we characterize capital adequacy equilibria regarding existence and uniqueness and provide a demonstration draft to support our claims. To do this, it is convenient to associate these equilibria with fixed points of a map that represents the capital adequacy process, that is, a map that computes the state of the banking system in a cycle as a function of this state in the previous one. We define this map rewriting the process equations in vector form and rearranging them conveniently.

The map has the form $\Phi(\mathcal{A};\mathcal{S};\mathcal{P}) : \boldsymbol{D} \to \boldsymbol{D}$, in which $\mathcal{A}$ is the set of control variables, which refer to the actions that banks can take, $\mathcal{S}$, the state variables set, i.e., the set of variables that, at the same time, are a basis for decision making by banks and are affected by these decisions, $\mathcal{P}$, the set of map parameters, and $\boldsymbol{D}$, the map's domain, which refers to its control and state variables. We define these sets in more detail as follows, using bold characters for vectors and matrices, and regular ones for scalars:

$\mathcal{A} \equiv \{\boldsymbol{x}, \boldsymbol{B}^L, \boldsymbol{B}^I\}$, in which $\boldsymbol{x}_{[N\times 1]}$ is the bank payments vector, $\boldsymbol{B}^L_{[N\times 1]}$ is the banks liquid assets stock vector, and $\boldsymbol{B}^I_{[N\times 1]}$, the banks illiquid assets stock vector.

$\mathcal{S} \equiv \{\boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I\}$, in which $\boldsymbol{I}_{[N\times 1]}$ is a vector of indicators of the liquidation state of banks, with $I_i = 1$ if the bank is liquidated, and $I_i = 0$ otherwise, $\boldsymbol{R}^L_{[N\times 1]}$ is a vector with amounts received by banks due liquid assets sales, and $\boldsymbol{R}^I_{[N\times 1]}$, one with amounts received by banks due illiquid assets sales.

$\mathcal{P} \equiv \{\boldsymbol{\Pi}, \boldsymbol{d}, \psi, \boldsymbol{C}, \boldsymbol{A}^L, \boldsymbol{A}^{IS}, \boldsymbol{F}^A, \boldsymbol{D}, \boldsymbol{L}^0, \boldsymbol{w}, \boldsymbol{\varepsilon}, F, p^0_L, p^0_I, \alpha_L, \alpha_I\}$, in which $p^0_L$ and $p^0_I$ are the initial market prices of liquid and illiquid assets. The other scalars have already been defined; vectors have dimensions $[N \times 1]$ and correspond to the same variables already defined for individual banks.

Without loss of generality, we set as boundary condition, that before the beginning of the process, all banks are solvent and no sale has been performed, i.e., $\boldsymbol{I} = \boldsymbol{0}, \boldsymbol{x} = \boldsymbol{d}, \boldsymbol{B}^L = \boldsymbol{A}^L, \boldsymbol{B}^I = \boldsymbol{A}^{IS}$, and $\boldsymbol{R}^L = \boldsymbol{R}^I = \boldsymbol{0}$. If a bank is insolvent or not compliant to capital requirements, this will be detected by the first solvency assessment (cf section 2.2.2) or capital adequacy assessment (cf section 2.2.3).

The map's domain refers to its control variables and state variables and is given by:

$$\begin{aligned}\boldsymbol{D} \equiv &\{(\boldsymbol{x}, \boldsymbol{B}^L, \boldsymbol{B}^I, \boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I) : (\boldsymbol{x}, \boldsymbol{B}^L, \boldsymbol{B}^I, \boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I) \in \\ &[\boldsymbol{0}, \boldsymbol{d}] \times [\boldsymbol{0}, \boldsymbol{A}^L] \times [\boldsymbol{0}, \boldsymbol{k} \cdot \boldsymbol{A}^{IS}] \times \{0,1\}_{[N\times 1]} \times [\boldsymbol{0}, \boldsymbol{L}^L_u] \times [\boldsymbol{L}^I_l, \boldsymbol{L}^I_u]\}\end{aligned} \tag{A.1}$$

In the definition above, $\boldsymbol{k}_{[N\times 1]}, K_i \geq 1$ is a vector of multipliers used to provide an upper limit to each $B^I_i, i = 1, \ldots, N$. $\boldsymbol{L}^L_{u\ [N\times 1]}$ and $\boldsymbol{L}^I_{u\ [N\times 1]}$ have the same use concerning $R^L_i, i = 1, \ldots, N$ and $R^I_i, i = 1, \ldots, N$, and $\boldsymbol{L}^I_{l\ [N\times 1]}$ provides lower limits to $R_i^I, i = 1, \ldots, N$. Given these elements, we can represent the capital adequacy process by the map in Equation (A.2). In what follows, we compute variables in cycle $t$ as a function of their previous values.

$$\begin{aligned}
&\Phi(\boldsymbol{x}, \boldsymbol{B}^L, \boldsymbol{B}^I; \boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I; \mathcal{P}) : \boldsymbol{D} \to \boldsymbol{D} \\
&\Phi(\boldsymbol{x}, \boldsymbol{B}^L, \boldsymbol{B}^I; \boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I; \mathcal{P}) \equiv (\Phi_1(\boldsymbol{x}; \boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I; \mathcal{P}), \\
&\qquad\qquad \Phi_2(\Phi_1(\boldsymbol{x}; \boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I; \mathcal{P}), \boldsymbol{B}^L, \boldsymbol{B}^I; \boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I; \mathcal{P}))
\end{aligned} \tag{A.2}$$

In which:

$$\Phi_1(\boldsymbol{x}; \boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I; \mathcal{P}) : \boldsymbol{D}_1 \to \boldsymbol{D}_1 \tag{A.3}$$

$$\begin{aligned}
&\boldsymbol{D}_1 \equiv \{(\boldsymbol{x}, \boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I) : \\
&\qquad (\boldsymbol{x}, \boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I) \in [\boldsymbol{0}, \boldsymbol{d}] \times \{0, 1\}_{[N \times 1]} \times [\boldsymbol{0}, \boldsymbol{L}_u^L] \times [\boldsymbol{L}_l^I, \boldsymbol{L}_u^I]\} \\
&\Phi_1(\boldsymbol{x}; \boldsymbol{I}, \boldsymbol{R}^L, \boldsymbol{R}^I; \mathcal{P}) \equiv \\
&\qquad \Phi_1(x_i^{t-1}; I_i^{2,t-1}, R_i^{L,t-1}, R_i^{I,t-1}; \mathcal{P})_{i=1,\ldots,N}, \text{ given by:}
\end{aligned}$$

$$\begin{cases}
x_i^t \text{ computed from Equation (11), assuming } i \in \mathcal{L}, \text{ that is,} \\
\psi_i = \psi \neq 0 \text{ if } I_i^{2,t-1} = 1 \\
I_i^{1,t} = \max(I_i^{2,t-1}, I_i^{1,t0}) \text{with } I_i^{1,t0} = 1 \text{ if } d_i > x_i \text{ and 0 otherwise.} \\
R_i^L = R_i^{L,t-1} \\
R_i^I = R_i^{I,t-1} \\
\text{If } I_i^{1,t} = 1 \text{ and } I_i^{2,t-1} = 0, \text{ include } i \text{ in } \mathcal{L} \text{ (make } \psi_i = \psi \neq 0) \text{ and} \\
\text{recalculate } x_i^t \text{ from Equation (11)}
\end{cases}$$

Using:

$$p_L^{t-1} = p_L^0 \exp\left(-\alpha_L \sum_i B_i^{L,t-1} / \sum_i A_i^L\right)$$

And:

$$\Phi_2(\Phi_1(\boldsymbol{x};\boldsymbol{I},\boldsymbol{R}^L,\boldsymbol{R}^I;\mathcal{P}),\boldsymbol{B}^L,\boldsymbol{B}^I;\boldsymbol{I},\boldsymbol{R}^L,\boldsymbol{R}^I;\mathcal{P}):\boldsymbol{D}\to\boldsymbol{D} \tag{A.4}$$

$$\begin{aligned}\Phi_2(\Phi_1(\boldsymbol{x};\boldsymbol{I},\boldsymbol{R}^L,\boldsymbol{R}^I;\mathcal{P}),\boldsymbol{B}^L,\boldsymbol{B}^I;\boldsymbol{I},\boldsymbol{R}^L,\boldsymbol{R}^I;\mathcal{P}) \equiv\\ &\Phi_2(\Phi_1(x_i^{t-1};I_i^{2,t-1},R_i^{L,t-1},R_i^{I,t-1};\mathcal{P}),\\ &\quad B_i^{L,t-1},B_i^{I,t-1},I_i^{1,t},R_i^{L,t-1},R_i^{I,t-1};\mathcal{P})_{i=1,\ldots,N}, \text{ given by:}\end{aligned}$$

If $I_i^{1,t}=1$:

$$I_i^{2,t}=1; B_i^L=B_i^{L,t-1}; B_i^I=B_i^{I,t-1}; R_i^L=R_i^{L,t-1}; R_i^I=R_i^{I,t-1}$$

Else:

$$\begin{cases} B_i^{L,t}=B_i^{L,t-1}-s_i^{L,t}\\ B_i^{I,t}=B_i^{I,t-1}-s_i^{I,t}\\ I_i^{2,t}: \begin{cases}\text{If } I_i^{2,t-1}=1 \text{ (bank defaulted before cycle } t\text{): } I_i^{2,t}=1\\ \text{If } x_i^t<d_i \text{ and } x_i^{t-1}=d_i \text{ (bank defaulted in cycle } t\text{): } I_i^{2,t}=1\\ \text{If } x_i^t=d_i \text{ (bank is solvent in cycle } t\text{) and}\\ \qquad ((s_i^I<0 \text{ and } |s_i^I|>(R_i^{L,t-1}+R_i^{I,t-1}+FS_i)/p_I) \text{ or}\\ \qquad (s_i^I>0 \text{ and } s_i^I>A_i^{IS})): I_i^{2,t}=1\\ \text{Otherwise: } I_i^{2,t}=0.\end{cases}\\ R_i^L=R_i^{L,t-1}+s_i^L(p_L^t+p_L^{t-1})/2\\ R_i^I=R_i^{I,t-1}+s_i^I(p_I^t+p_I^{t-1})/2\end{cases}$$

Using:

$p_L^t = p_L^{t-1} \exp\left(-\alpha_L \sum_i s_i^{L,t} / \sum_i A_i^L\right)$

$p_I^t = p_I^{t-1} \exp\left(-\alpha_I \sum_i s_i^{I,t} / \sum_i A_i^{IS}\right)$

If $t = 1$, $p_L^0$ and $p_I^0$ are the initial prices, given as parameters

If $I_i^{2,t-1} = 1$ (bank defaulted before cycle $t$)

$s_i^{L,t} = s_i^{I,t} = 0$ (bank does not negotiate assets)

Else if $x_i^t < d_i$ and $x_i^{t-1} = d_i$ (bank defaulted in cycle $t$)

$s_i^{L,t}$ given by Equation (15)

$s_i^{I,t}$ given by Equation (16)

Else if $x_i = d_i$ (bank is solvent in cycle $t$)

$s_i^{L,t}$ given by Equation (18)

$s_i^{I,t}$ given by Equation (21) if $s_i^I > 0$ (illiquid assets sale)

$s_i^{I,t}$ given by Equation (22) if $s_i^I < 0$ (illiquid assets purchase)

$s_i^I$ given by Equation (20)

We use this map to characterize the capital adequacy process equilibria by the following proposition:

**Proposition 1.** *The capital adequacy process represented by the map* $\Phi(\mathcal{A}; \mathcal{S}; \mathcal{P})$ *as defined in Equation (A.2):*

*(a) Has at least one equilibrium point, and*

*(b) The equilibrium point is not unique.*

*Proof.* Demonstrating that the adequacy process has at least an equilibrium point is equivalent to prove that the map $\Phi(\mathcal{A}; \mathcal{S}; \mathcal{P})$ has at least one fixed point. To do this, we use the Brouwer's fixed-point theorem, that states that if a map $\Phi(\cdot)$ of a compact convex set into itself is continuous, then it has a fixed point, that is, a point $x$ for which $\Phi(x) = x$. Firstly we analyze the map's domain, given by Equation (A.1). The domain $\boldsymbol{D}$ is the product of five compact and convex subsets in $\mathbb{R}^N$, corresponding to the vectors $\boldsymbol{x}, \boldsymbol{B}^L, \boldsymbol{B}^I, \boldsymbol{R}^L$ and $\boldsymbol{R}^I$, with a product of $N$ binary sets $\{0, 1\}$ that corresponds to the vector of indicator variables $\boldsymbol{I}_{[N \times 1]}$. Our strategy is to divide this problem into ones for which we have a fixed value of $\boldsymbol{I}$ and the full domain for the other variables.

Before proceeding, we remember that Eisenberg and Noe (2001) demonstrate the existence of the fixed point of its map using lattice theory, however, as the map is continuous in its domain, the Brouwer's fixed point theorem assures the existence of its fixed point.

We sketch the proof as follows:

(a) We assume $\boldsymbol{I}$ is fixed and does not change along time. In this case, we show that the map is defined in every point of its domain. Next, we show that it is continuous in its domain.

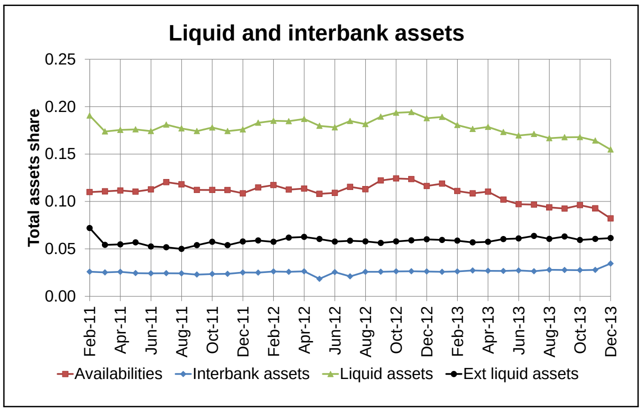
(b) According to the Brouwer's fixed point theorem, the map $\Phi$ with a fixed $\boldsymbol{I}$ has at least one fixed point.

(c) Now, we cope with the discontinuous variable $\boldsymbol{I}$. We claim that $\Phi$ is monotonic regarding $\boldsymbol{I}$. We see this noting that in Equation (A.3), $I_i^{1,t}$ is given by $\max(\cdot)$ function, which involves $I_i^{2,t-1}$, which is greater or equal to $I_i^{1,t-1}$ (see Equation (A.4)). This means that if a bank is liquidated in $t-1$, it remains liquidated in $t$, whatever happens.

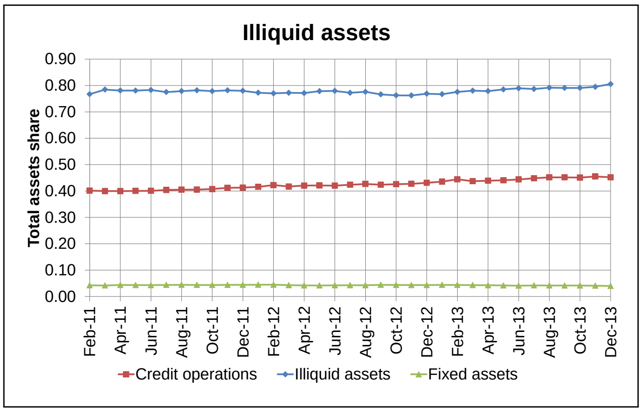
(d) If $\Phi$ is monotonic regarding $\boldsymbol{I}$, and, for a given $\boldsymbol{I}$, $\Phi$ is continuous, then $\Phi$ has a fixed point. In our model, the vector of variables $\boldsymbol{I}$ is associated with modifications on the map $\Phi$ due to the incidence of default costs and to the sale of risky assets and subsequent freeze on asset transactions in case of default of a given bank, whereas in the Eisenberg and Noe (2001)'s model, the map $\Phi$ remains the same until the fixed point is attained. To show that $\Phi$ is monotonic regarding $\boldsymbol{I}$, we first note that in the beginning of a simulation, we assume no bank has defaulted (thus, $\boldsymbol{I} = \boldsymbol{0}$), and that each bank attempts to pay its debt in full. Whenever a bank defaults, its payments to the other banks suffer a further decrease due to the incidence of default costs. Additionally, its risky assets are sold, which reduces the prices of these assets, potentially reducing the net worth of the other banks. Thus, the map $\Phi$ associated with a vector $\boldsymbol{I}$ that includes this default in cycle $t$ will yield banks payments and asset stocks less or equal to those obtained from the map $\Phi$ used just before this default, in cycle $t-1$. Thus, we have $\boldsymbol{I}^t > \boldsymbol{I}^{t-1}$ and $\Phi^t \leq \Phi^{t-1}$, i.e., $\Phi$ is monotonic regarding $\boldsymbol{I}$, which proves *(a)*.

To prove *(b)*, it is enough to present an example. We initially build an example for which $\Phi$ has a fixed-point for $\boldsymbol{I} = \mathbf{0}$. To do it, it is sufficient to define a vector of cash equivalent assets $\boldsymbol{C}$ high enough for each bank so that these banks pay their debts in full and do not need to sell risky assets to become compliant with capital requirements. Banks in this situation are not liquidated, therefore, $\boldsymbol{I}$ remain equal to $\mathbf{0}$. We have shown in *(a)* that, as $\boldsymbol{I}$ does not change, the map has a fixed point. Suppose now that for the same initial state, bank $i$ decides to sell part of its liquid assets. Suppose additionally, that the price does not lower much as the amount sold was not significant compared to the banking system total liquid assets, and that the mark to market process do not lead any other bank into insolvency or to lose its compliance. In this case, after the sale, the banking system will be in an equilibrium, different from the first, which proves *(b)*.

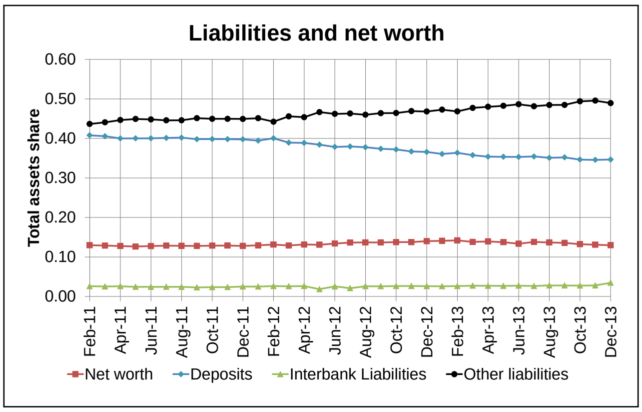
$\square$




(a) Liquid assets.

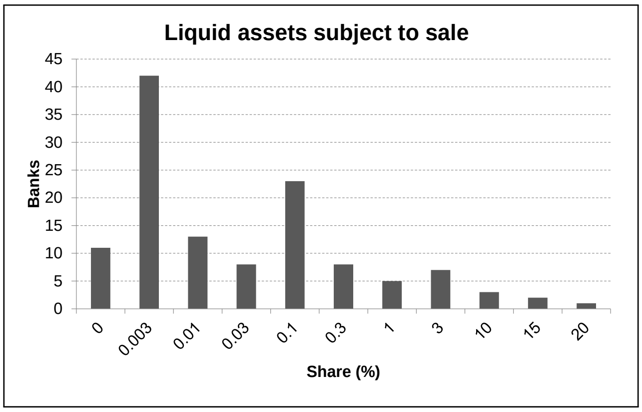



(b) Illiquid assets.

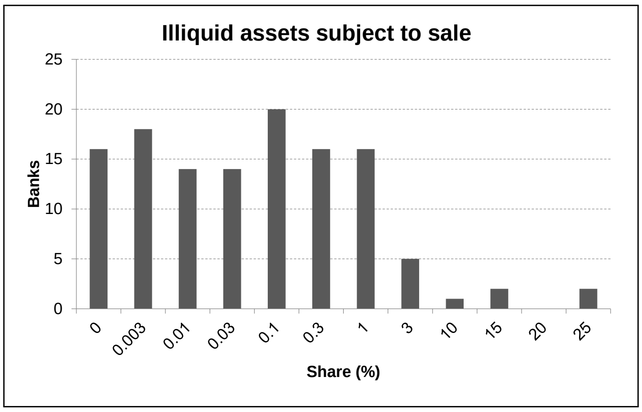



(c) Liabilities and net worth.

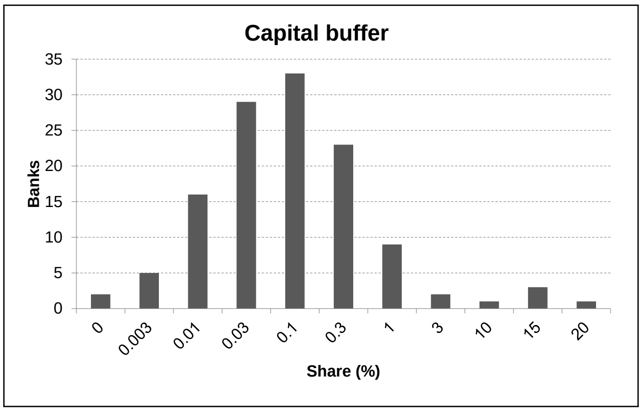
Figure 8: Balance sheet average composition for the entire banking system, from February 2011 to December 2013.




(a) Liquid assets that can be put to sale.

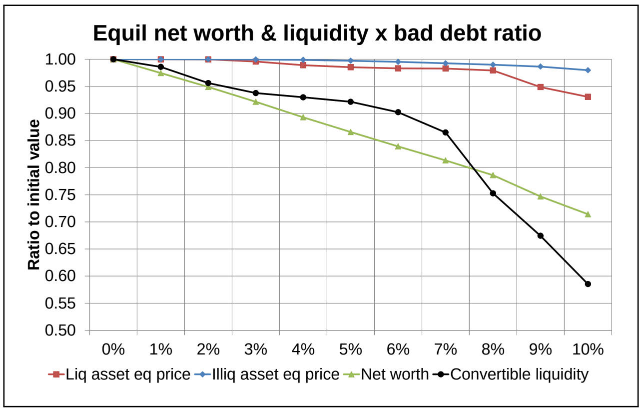



(b) Illiquid assets that can be put to sale.

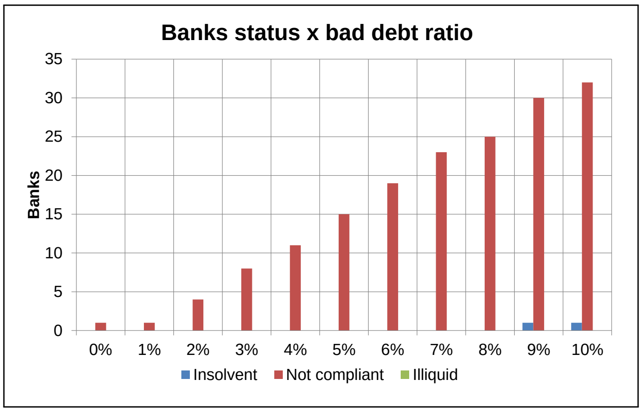



(c) Capital buffer.

Figure 9: Balance sheet variables distributions in December 2013.




(a)




(b)

Figure 10: Effect of a simultaneous increase in the delinquency rate of credit operations in the whole banking system. In (a), we present effects of these shocks on assets prices, on convertible liquidity and on the net worth of the banking system. In (b), we compare the number of insolvent and illiquid banks at the end of the capital adequacy process. We also present the number of banks that initially became non-compliant due to these shocks. Computed for December 2013.

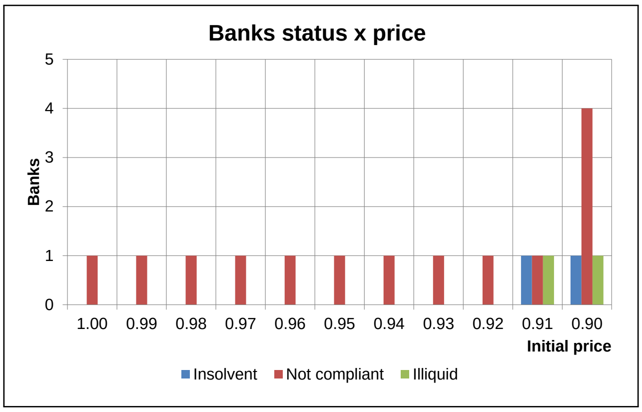
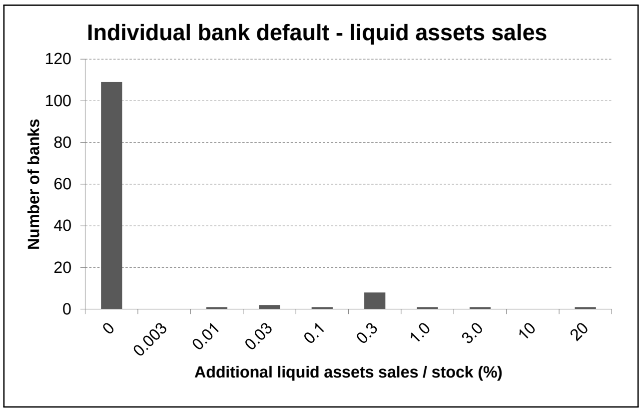
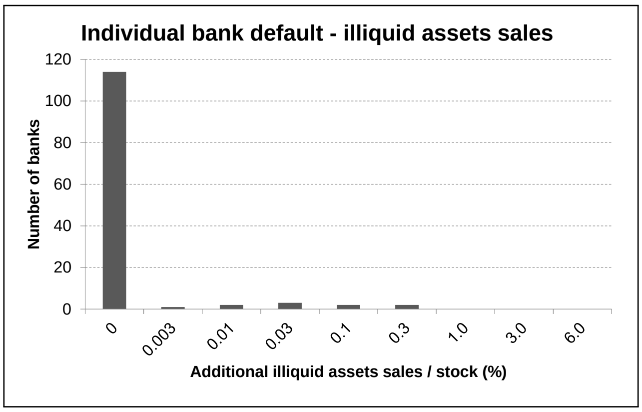



Figure 11: Effect of external assets prices shock in the banking system. We compare the number of insolvent and illiquid banks at the end of the capital adequacy process. Additionally, we present the number of banks that initially became non-compliant due to these shocks. Computed for December 2013.

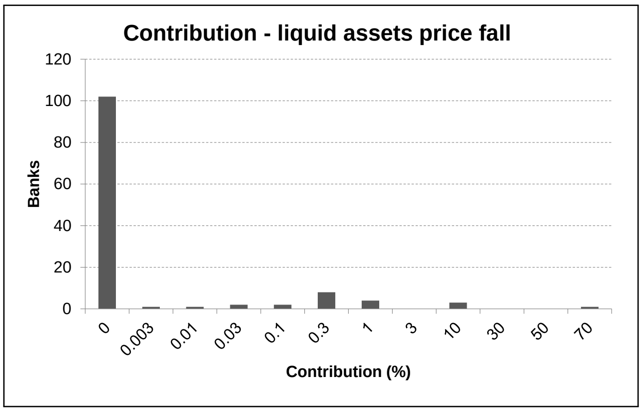



(a)

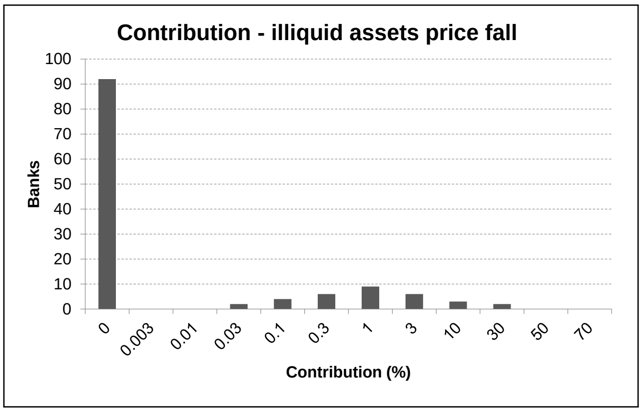



(b)

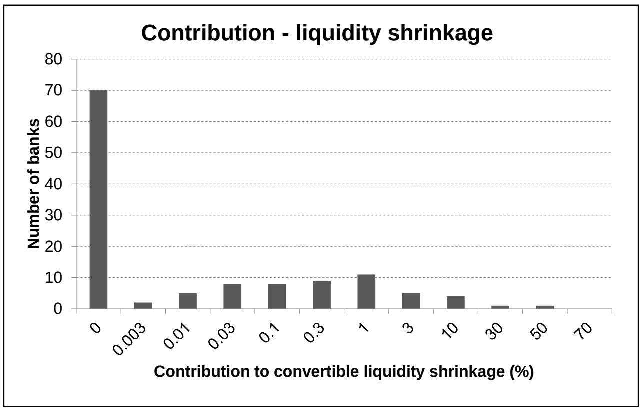
Figure 12: Distribution of banking system sales caused by the default of an individual bank. We do not include sales by the defaulting bank. The charts present the number of banks which default originates each sales amount. In (a), external liquid assets sales, and in (b), external illiquid tradable assets sales. Computed for December 2013.




(a)

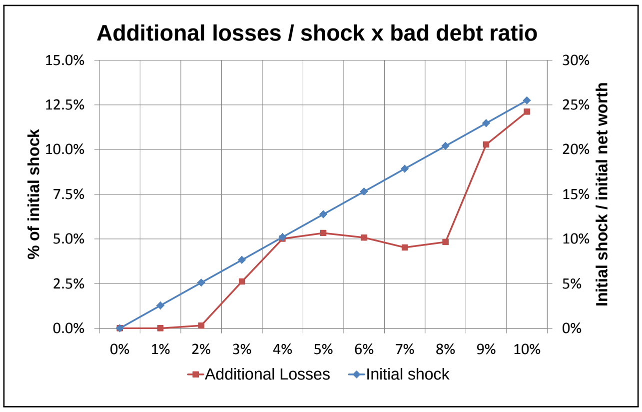



(b)

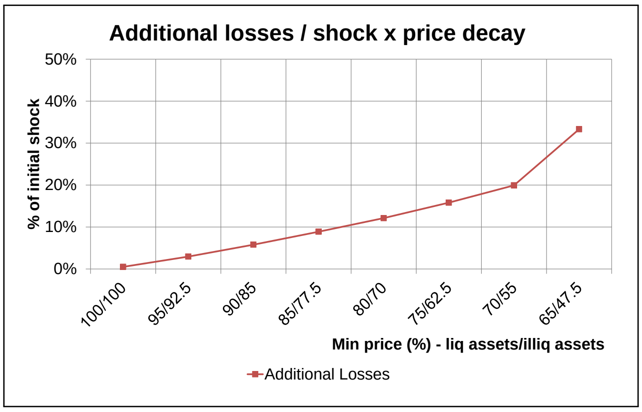



(c)

Figure 13: Contribution of individual banks to the equilibrium adjustment value of the banking system after a simultaneous delinquency rate of 10 p.p. for all banks, computed for: a) liquid assets price fall, b) illiquid assets price fall, and c) convertible liquidity shrinkage. Computed for December 2013.




(a)




(b)

Figure 14: Shock amplifications for increasingly more severe scenarios. In (a), we compute additional losses for for delinquency rates from 0 p.p. to 10 p.p.. In (b), we compute them for the same initial shock and increasingly steeper assets price decay rates. Computed for December 2013.

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