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José Renato Haas Ornelas

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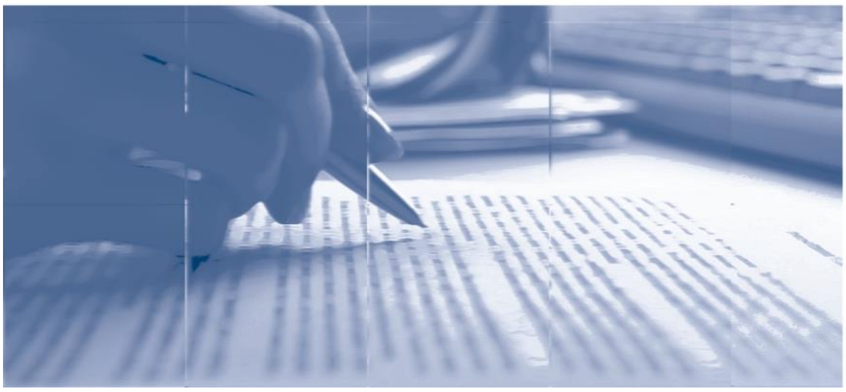

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Assessing the Forecast Ability of Risk-Neutral Densities and Real-World Densities from Emerging Markets Currencies♦

José Renato Haas Ornelas*

Abstract

The Working Papers should not be reported as representing the views of the Banco Central do Brasil. The views expressed in the papers are those of the author(s) and do not necessarily reflect those of the Banco Central do Brasil.

This paper empirically evaluates Risk-Neutral Densities (RND) and Real-World Densities (RWD) as predictors of future outcomes of emerging markets currencies. The dataset consists of volatility surfaces from 11 emerging market currencies, with approximately six years of daily data, using options with one-month expiration. Therefore, there is a strong overlapping in data, which is tackled with specific econometric techniques. Results of the out-of-sample assessment show that both RND and RWD underweight the tails of the actual distribution. This is probably due to the lack of options with extreme strikes. Although the RWDs perform better than RND in terms of Kolmogorov distance, they still have problems in fitting the tails of actual data. Thus, the risk-aversion adjustment may improve the forecast ability, but it does not solve the tails misfitting.
Keywords: Relative Risk Version, Risk-Neutral Density, Exchange Rate.
JEL Classification: C53, C13, E47, G17, F31.

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1. Introduction

Risk-Neutral Densities (henceforth RND) calculated from options prices have been used to infer market beliefs about future distributions of asset prices, such as exchange rates, interest rates and stock indexes. Market participants often use these RNDs to make decisions about asset allocation and risk management. In fact, any empirical application in finance that requires densities forecasts may also take advantage of RNDs. Regulators are also influenced by RNDs when taking policy decisions.

The empirical extraction of RNDs has started about 20 years ago with the papers of Shimko (1993) and Rubinstein (1994). During the 2000's, the relationship of Risk-Neutral Densities and Real-World Densities[1] (henceforth RWDs) has attracted the interest of researchers. One interesting point in this relationship is that we may calculate a relative risk aversion (henceforth RRA) measure by comparing both densities as was done originally by Jackwerth (2000). This risk aversion measure can be used then to transform RNDs into RWDs as was done by Bliss and Panigirtzoglou (2004) and Liu et al.(2007), among others. Empirical evidence has shown that these transformed densities have a better forecast ability than the pure RND.

Although the initial literature on RND and RWD had a focus on stock indexes options, other kinds of instruments have been analyzed such as interest rates, commodities and currencies. This paper focuses on emerging markets currencies.

Many papers have already extracted RNDs from emerging markets exchange rates (for instance, Abe et al., 2007, León and Casanova, 2004). Regarding RWD, the article of Fajardo et al. (2012) is the only one that estimates the relative risk aversion (RRA) and RWD for an emerging market currency, but only for the Brazilian Real.

The goal of this paper is evaluate the RND and RWD as predictors of future outcomes of emerging markets currencies. A set of 11 emerging market currencies is used in the empirical investigation. I use six years of daily data from over-the-counter (OTC) currency options with one-month expiration. Therefore, there is a strong overlapping in data. Most of previous papers filtered the data to get only non-overlapping times-series in order to avoid econometric problems arising from auto correlation. After this filtering, the number of observations is reduced.

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[1] In the literature, "real-world", "risk-adjusted", "physical", "subjective" and "historical" refers to price distributions in which market risk preferences are embedded.

In this paper, I use econometric techniques to deal with this data overlapping. Specifically, I tackle this issue by using the stationary bootstrap of Politis and Romano (1994) to t-test statistics and p-values on hypothesis testing regarding RRA. In this way, I am able to use overlapping times-series and then use all the data available, without having to discard overlapped data. On the density forecast out-of-sample performance evaluation, I follow Christoffersen and Mazzotta (2005) in testing the moments of the normal transformed variable *Z* using Newey-West (1987) correction for standard errors.

This research contributes to the literature in two ways: first by analyzing empirically risk-neutral densities, risk aversion and real-world densities on a set of eleven emerging markets currencies; and second, by employing econometric methods that make possible the use of overlapped data.

Results of the out-of-sample assessment show that both RND and RWD underweight the tails of the actual distribution. This is probably due to the lack of options with extreme strikes. This kind of issue was already faced by Christoffersen and Mazzotta (2005)[2]. Although the RWDs perform better than RND in terms of Kolmogorov distance, they still have problems in fitting the tails. Therefore, the risk-aversion adjustment may improve the forecast ability, but does not solve the tails misfitting.

The paper is organized as follows: Section 2 briefly revises the literature of Risk Neutral Density and risk-transformation methods; Section 3 shows the methodology used to estimate the relative risk-aversion parameter and the Risk-Neutral and Real-World distributions; Section 4 gives an overview of the dataset used; on section 5, the results of the RRA estimation are shown; Section 6 presents the results of the out-of-sample density forecast assessment; and finally section 7 concludes the paper.

2. Risk-Neutral Density and Risk Transformations Methods

The idea of extracting a RND comes from Ross (1976) article. Given a set of option prices for a specific expiration date, the risk-neutral probability distribution can be recovered. There are many methods for recovering this RND function implied in option prices. Jackwerth (1999) makes a review of this literature.

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[2] While Christoffersen and Mazzotta (2005) data had strikes with a minimum delta of 25, my deltas are as low as 10.

Jackwerth (1999) classifies methods into parametric and non-parametric. Parametric methods assume that the risk-neutral distribution can be defined by theoretical distribution and its parameters. Non-parametric methods consist of fitting CDF's to observed data by means of more general functions. Among the non-parametric methods are the kernel methods and the maximum-entropy methods. (see, for instance, Ait-Sahalia and Lo, 1998, and Buchen and Kelly, 1996).

This risk-neutral density, as the name itself suggests, represents market forecasts for the distribution of the underlying asset, under the view of a risk-neutral agent. However, when there is a gap between this forecast and subsequent realizations of the underlying asset, one may assume that investors are requiring a risk premium for this asset. Other explanations for this gap may be found on the behavior finance literature.

One way to measure this gap is to consider a utility function as in Bliss and Panigirtzoglou (2004), and then use the following relationship between risk-neutral and objective densities:

$$h(x) = \frac{\frac{g(x)}{u'(x)}}{\int_0^\infty \frac{g(y)}{u'(y)} dy} \tag{1}$$

where $h(x)$ is the RWD $g(x)$ is the RND and $u(x)$ the utility function.

Bliss and Panigirtzoglou (2004) use two types of utility function, the power and exponential utility functions, which contain only one parameter ρ. In the case of the power utility, this parameter is the relative risk aversion. Therefore, for a power utility function of the form $u(x) = \frac{x^{1-\rho}}{1-\rho}$, we would have:

$$h(x) = \frac{x^\rho g(x)}{\int_0^\infty y^\rho g(y) dy} \tag{2}$$

Once we have the risk-neutral density $g(x)$, we can estimate the risk-aversion parameter by using the actual realizations of the underlying asset.

3. Distribution Estimation Methodology

A wide range of methods for estimating the RND $g(x)$ is available. Parametric or non-parametric methods can be used. There is a natural trade-off between the flexibility and stability of functions. Obviously, the higher the flexibility, the higher is the in-sample goodness-of-fit. Micu (2005) analyzed several methods to extract RND for emerging markets currencies and suggests "… that there is a large scope for selection between these methods without essentially sacrificing the accuracy of the analysis".

On this paper, I use a Mixture of Log-Normals to model the Risk-Neutral Densities. This is a choice for stability of the distributions. Also, Liu et al. (2007) article provides an elegant transformation of the RND to the RWD, showing the relationship between the two set of parameters and the risk aversion.

Therefore, I model the exchange rate using a mixture of two lognormals densities:

$$g(x|w, F_1, \sigma_1, F_2, \sigma_2) = w * pdf_{LN}(x|F_1, \sigma_1) + (1 - w) * pdf_{LN}(x|F_2, \sigma_2) \tag{3}$$

with

$$pdf_{LN}(x|F, \sigma) = \left(x\sigma\sqrt{2\pi T}\right)^{-1} exp\left(-\frac{1}{2}\left[\frac{\log(x) - \ (\log(F) - \ 0.5\sigma^2 T)}{\sigma\sqrt{T}}\right]^2\right) \tag{4}$$

Where:

x is the exchange rate expressed as emerging market currency units per USD

T is the time to maturity (one month in our case) expressed in years

$\sigma, \sigma_1, \sigma_2$ are the volatility parameters

F, F_1, F_2 are the future exchange rate parameters

I use the forward exchange rate to reduce the number of free parameters of the MLN distribution. This is done by making the expectation of the distribution equal to forward exchange rate: $F = wF_1 + (1 - w)F_2$. In this way, F is the forward exchange rate quoted in the market. Therefore, from five overall parameters, only four are free parameters. The parameters F_1 and F_2 are the expectation of the two lognormal distributions of the mixture, while the $\sigma's$ set their volatility. The price of a European call option is the weighted average of two Black (1976) call option formulas. The parameters estimation of the Mixture of Lognormals was done using an adaptation of the algorithm of Jondeau and Rockinger[3] for the Emerging markets exchange rates characteristics and data. One special issue when dealing with mixture of log-normals is to avoid shapes with "spikes" (see Anderssona and Lomakka, 2005), i.e., one of the log-normals with an σ much lower than the other. This issue was avoided by limiting σ_i parameters to be at most twice the other. Regarding optimization criteria, the algorithm estimates parameters by minimizing the squared errors of the theoretical and actual option prices.

Once having the RND, one can calculate the RRA parameter following the Liu et al. (2007) Parametric Risk transformation. As seen on section 2, they consider the real-world density h defined by (1) when there is a representative agent who has constant RRA equal to ρ. If g is a single lognormal density then so is h. The volatility parameters for functions g and h are then equal but their expected values are respectively F and $F\, exp(\rho\ \sigma^2\ \mathrm{T})$ when g is defined by (3).

Thus, a transformed mixture of two lognormals is also a mixture of two lognormals. For a Mixture of Lognormals g (x| $w, F_1, \sigma_1, F_2, \sigma_2$) given by (3), it is shown by Liu et al. (2007) that the real-world density h is also a Mixture of Lognormals with the following density:

$$\tilde{g}(x|w, F_1, \sigma_1, F_2, \sigma_2, \rho) = h(x|w', F_1', \sigma_1', F_2', \sigma_2') \tag{5}$$

With the new set of transformed parameter given by:

$$F_1' = F_1 exp(\rho\sigma_1^2 \mathrm{T})$$

$$F_2' = F_2 exp(\rho\sigma_2^2 \mathrm{T})$$

$$\left(\frac{1}{w'}\right) = 1 + \left(\frac{1-w}{w}\right)\left(\frac{F_2}{F_1}\right)^{\rho} exp\Big((1/2)T(\rho^2 - \rho)(\sigma_2^2 - \sigma_1^2)\Big)$$

I have calculated a Relative Risk Aversion (RRA) for the full sample using the log-likelihood function as in Liu et al. (2007). For estimation of the RND parameters $(w, F_1, \sigma_1, F_2, \sigma_2)$, I minimize the squared errors of the actual option price and the theoretical option. For the relative risk aversion parameter (ρ), I maximize the log-likelihood function with the RRA being the only free parameter:

$$\sum_{\mathrm{i}=1}^{\mathrm{n}} log\left(\tilde{g}(S_{i+1}|\hat{\theta}_i, \rho)\right) \tag{6}$$

[3] The original algorithm of Jondeau and Rockinger is available at the website: http://www.hec.unil.ch/MatlabCodes/rnd.html.

Where n is the number of days with volatility surface data.

It is worth noting that all quotes are in terms of emerging market currency per U.S. Dollar, i.e., I am quoting the U.S. Dollar instead of the risky asset. In this context, a negative ρ means investors demand a premium to hold the EM currency. This would be the relative risk aversion if the U.S. Dollar were our risk asset. In order to have a RRA for the emerging market currency, I change the signal[4] of ρ. To avoid ambiguity, I call $\rho^{EM} = -\rho$ this emerging market RRA parameter.

4. Sample of Over-the-Counter Options

The exchange rate option prices used in this paper are over-the-counter (OTC). This data can be obtained from main data providers such as Thomsom Reuters and Bloomberg. Both providers conduct a daily pool with market participants asking their estimates about the volatility surface of OTC currency options. Data from Thomsom Reuters consists of 17 strikes with deltas varying from 10% to 45% for calls and puts. Data from Bloomberg consists of quotes for nine fixed moneyness. Therefore, data from Reuters have more cross-section granularity than Bloomberg. However, Bloomberg data were available for more currencies, and with a longer time-series. As the scope of this study is to calculate risk-aversion, I have decided to use data from Bloomberg, given the longer time series.

The original data from Bloomberg consists of four risk-reversals, four butterflies, besides the at-the-money volatility. Risk-reversals and butterflies have four different deltas: 10, 15, 25 and 35. So I have the following data for each day for each currency:

- Delta-neutral straddle implied volatility. A straddle is a set of a call option and a put option with the same strike. This quote corresponds to a strike that makes the Black-Scholes delta of the straddle equal to zero.
- Four different delta risk-reversals. The risk-reversal measures the difference in Black-Scholes implied volatilities between a specific delta out-of-the-money call option and an out-of-the-money put option with the same delta. Option traders use risk-reversal quotes to quantify the asymmetry of the implied volatility

[4] This can be viewed if we think in terms of log-returns. The log-return of the conventional quote exchange rate (EM currency per unit of USD) is equal to minus the log-return of the inverted quote exchange rate (USD per EM currency unit). Therefore, their returns distributions are mirrored.

smile, which reflects the skewness of the risk-neutral currency return distribution.

- Four different delta butterfly spreads. Butterfly spreads are defined as the average difference between out-of-the-money implied volatilities and the delta-neutral straddle implied volatility.

The above data describes the volatility surface for each day, for each currency. I have used only options with one-month expiration date. Data covers 11 emerging markets currencies: Brazilian Real (BRL), Chilean peso (CLP), Colombian peso (COP), Malaysian ringgit (MYR), Mexican peso (MXN), Indonesian rupiah (IDR), Israeli shekel (ILS), Philippine peso (PHP), Thai baht (THB) Turkish lira (TRY) and South African rand (ZAR). The surfaces cover the period from July 2007 to July 2013. There are some missing data during the period for some currencies, so that we have an unbalanced panel of surfaces. Table I shows the number of surfaces for each currency. Other Emerging Markets currencies have data available from Bloomberg, but starting later.

One important practical issue when dealing with OTC foreign exchange options is regarding the delta convention used. As mentioned by Reiswich and Wystup (2010), many academics overlook this issue. For emerging markets forex options, usually the premium-adjusted forward delta is used, so I follow this convention.

Besides the options data, I have collected also data from the spot exchange rate, one-month forward exchange rate and one-month deposit rate. All spot and forward exchange rates are from Bloomberg. For non-convertible currencies, where offshore delivery is not possible, I have used either on-shore forward or futures[5], or non-deliverable forwards.

Data from the interbank one-month deposit rate were taken from several sources. For the Brazilian and Mexican markets, I used data from the Swap market. For IDR, ILS, MYR, TRY, THB, PHP and ZAR data was taken from Libor-like deposits. For CLP, the Nominal Average Interbank Rate from "*Asociacion Nacional de Bancos*" was used, while the "*Tasa Basica de La Superintendencia Bancaria*" was the deposit rate for COP.

[5] For the Brazilian Real, I have used the dollar-Real futures contract traded at BM&FBovespa exchange. This contract is the most liquid instrument on the Brazilian Real currency market. In order to have a constant expiration of one-month, I interpolated the exchange rates of the first two contracts.

It is worth noting that all quotes in that markets are done in terms of emerging market currency per U.S. Dollar, which means that an appreciation (depreciation) of the EM currency decreases (increases) the exchange rate.

Table I shows the mean values for the deposit rate and volatilities. The one-month mean deposit rate was 5.2%. Brazil and Turkey had the highest rates, around 10.1%. Israel and Thai had the lowest rates, slightly above 2%. For the same period, the USD Libor rate was 0.91% on average.

Table I – Volatility and Deposit Rates Descriptive Statistics

	# Days	De-posit Rate	ATM Vola-tility	Mean Call Volatility				Mean Put Volatility			
				10-delta	**15-delta**	**25-delta**	**35-delta**	**10-delta**	**15-delta**	**25-delta**	**35-delta**
BRL	1,467	10.1	15.8	22.1	20.8	18.7	17.2	14.3	14.3	14.4	14.9
CLP	1,462	4.9	18.3	17.3	15.7	14.6	13.6	12.9	12.7	12.7	13.0
COP	1,448	2.7	14.7	20.0	18.9	17.0	15.8	14.0	13.7	13.8	14.1
IDR	1,331	6.3	11.4	17.1	16.1	14.2	12.7	10.4	10.1	10.2	10.7
ILS	1,518	2.2	9.9	12.1	11.6	10.8	10.3	10.2	10.1	9.8	9.7
MY	1,493	2.8	8.0	10.1	9.6	8.9	8.4	7.9	7.8	7.7	7.8
MX	1,519	5.1	13.6	18.6	17.5	15.8	14.6	12.6	12.5	12.6	12.9
PHP	1,378	4.4	11.5	10.9	9.7	9.0	8.3	8.5	8.2	8.0	8.1
THB	1,453	2.4	6.7	8.9	8.3	7.5	7.0	7.6	7.1	6.7	6.7
TRY	1,548	9.6	13.4	17.9	17.0	15.5	14.4	12.5	12.4	12.5	12.8
ZAR	1,386	6.7	18.3	23.9	22.7	20.9	19.5	17.1	17.1	17.1	17.5
Mea	1,455	5.2	12.9	16.3	15.3	13.9	12.9	11.6	11.4	11.4	11.6

This table shows descriptive statistics from 11 emerging markets currency options. The currencies are: Brazilian real (BRL), Chilean peso (CLP), Colombian peso (COP), Malaysian ringgit (MYR), Mexican peso (MXN), Indonesian rupiah (IDR), Israeli shekel (ILS), Philippine peso (PHP), Thai baht (THB) Turkish lira (TRY) and South African rand (ZAR). Data is from Bloomberg. The sample period is from July 2008 to July 2013. For each currency, the number of days with available data is shown on the "# Days" column. The deposit rate with one month maturity is shown on an annualized basis in percentage points. The ATM (at-the-money) volatility and the call and put volatilities are also expressed on an annualized basis and in percentage points. Calls and put deltas follow the forward delta premium-adjusted convention. The maturity of the options is one month. All options are quoted considering an exchange rate expressed as emerging market currency per U.S. Dollar.

The at-the-money volatilities range from 6.7% from THB to 18.3% from ZAR and CLP, with an average of 12.9%. All currencies had call volatilities higher than put volatilities, which suggest a skewness risk against emerging markets currencies.

Table II shows descriptive statistics about returns of exchange rates. Again, it is worth recalling that exchange rates quotes are expressed in terms of emerging market currency per U.S. Dollar. Therefore, a positive total return means that the emerging market currency has depreciate. This is the case of 8 out of 11 currencies. The largest depreciation is from TRY. The period of the sample starts before the financial crisis of 2008, when emerging market currencies suffer considerably. Positive skewness is consistent with investors expecting more negative surprises than positive for these EM currencies. The kurtosis data show fatter tails than the Normal distribution in all cases, with an average kurtosis of 11.6. The historical volatility is lower than the implied ATM from Table II for 7 out of 11 currencies. On average, the implied volatility is 0.9 percentage points higher than the historical. Some currencies like CLP and PHP have an implied volatility much higher than the historical.

Table II – Returns Descriptive Statistics

	Total Return	**Historical Volatility**	**Historical Skewness**	**Historical Kurtosis**	**Implied ATM – Historical Volatility**
BRL	19.1	19.0	0.05	10.8	-3.2
CLP	0.7	12.0	0.57	7.5	6.3
COP	-4.1	13.6	-0.39	16.9	1.1
IDR	11.4	9.8	-0.08	18.7	1.6
ILS	-14.4	9.9	0.12	5.5	0.0
MYR	1.1	6.7	-0.10	5.8	1.3
MXN	16.0	14.0	0.71	12.8	-0.4
PHP	4.3	6.4	0.20	3.8	5.0
THB	-8.1	5.0	0.13	7.3	1.7
TRY	48.5	15.0	0.13	14.5	-1.6
ZAR	26.0	19.6	1.66	23.4	-1.4
Mean	9.14	11.9	0.27	11.6	0.9

This table shows descriptive statistics for the returns of 11 emerging markets exchange rates. All exchange rates are quoted as emerging market currency per U.S. Dollar. The currencies are: Brazilian real (BRL), Chilean peso (CLP), Colombian peso (COP), Malaysian ringgit (MYR), Mexican peso (MXN), Indonesian rupiah (IDR), Israeli shekel (ILS), Philippine peso (PHP), Thai baht (THB) Turkish lira (TRY) and South African rand (ZAR). Data is from Bloomberg. The sample period is from July 2008 to July 2013. The total return is the percentage return during the all sample period, which may vary for each currency. The historical volatility is calculated based on daily continuously compounded returns, and then expressed on annualized basis in percentage points. Skewness and kurtosis are calculated based on daily continuously compounded returns.

5. Relative Risk Aversion Estimation Results

The main issue when estimating the RRA daily observations of one-month ahead volatility surfaces is the overlapping nature of the data. Although the point estimate might remain the same, we cannot use the traditional likelihood-ratio test approach to test if the RRA is statistically different from zero. Overlapping data induces autocorrelation in the log-likelihood function from equation (6), so that standard asymptotic distribution is not chi-square anymore.

This kind of problem has already occurred in other research settings with overlapped data. For instance, Patton and Timmermann (2010) tests the monotonicity of the term premium using overlapped data. This is possible thanks to the Stationary Bootstrap proposed by Politis and Romano (1994), which breaks the autocorrelation structure of the data. The idea is to generate several simulations using the stationary bootstrap on my data. Then, the RRA is calculated for each simulation. Finally, I sort the simulated RRA's in order to get a p-value for the desired hypothesis, i.e., RRA different from zero. This is a novel approach for RRA estimation with options, which is necessary because of the overlapped data. All similar previous studies used non-overlapping data (see Liu et al., 2007, and Fajardo et al., 2012).

Results for the RRA estimation and stationary bootstrap p-value are presented on Table III. Point estimates show the presence of risk aversion for 10 out of 11 currencies. The relative risk aversion coefficient of 3.1 is in line with previous literature. For instance, Bliss Panigirtzoglou (2004) and Liu et al. (2007) found RRA coefficients between 2 and 4 using UK stock index options. Fajardo et al. (2012) found a 2.7 RRA coefficient using exchange-traded BRL options.

Nevertheless, the stationary bootstrap p-values fail to reject the hypothesis that these coefficients are different from zero, except for the case of IDR. Most of the p-values are around 20%, showing very weak statistical evidence of a risk premium against theses currencies. However, as mentioned by Liu et al. (2007), it can be the case that these test conclusions are type II errors, reflecting the challenges to estimate risk premium accurately. As the risk premium is small relative to volatility, a large number of observations are needed to capture its true value. In fact, previous studies used more than 100 observations of volatility surfaces to obtain risk-aversion estimates.

It is worth to compare our results with those of Fajardo et al. (2012) using BRL with exchange-traded options over 143 non-overlapping months from 1999 to 2011. Their RRA estimate of 2.7 is statistically different from zero with a p-value equal to 7.45%. Although the RRA is similar to our average across all EM currencies, it is approximately the double of my estimate for the BRL, which is not statistically different from zero. I believe their longer time period could explain this result, and not the fact that I am using OTC instead of exchange-traded options. The key issue to estimate the RRA seems to be the size of the sample. One approach to increase the size of the sample keeping the time length is to pool several similar EM currencies into a single maximum likelihood estimation of the RRA parameter.

Following this line of reasoning, one may think of estimating a Latin American relative risk aversion using BRL, CLP, COP and MXN together; or a Southeast Asian RRA using IDR, MYR, PHP and THB together. The Latin American RRA estimate is 2.15, while the Southeast Asian RRA is 5.97.

Table III – Full Sample Relative Risk Aversion Estimates

	Relative Risk Aversion (ρ^{EM})	**Stationary Bootstrap p-value**
BRL	1.34	0.32
CLP	3.02	0.26
COP	3.78	0.20
IDR	8.74	0.03
ILS	5.43	0.21
MYR	1.23	0.44
MXN	0.93	0.40
PHP	5.33	0.20
THB	4.31	0.29
TRY	-0.43	0.52
ZAR	0.77	0.42
Mean	3.13	0.30

This table shows the relative risk aversion estimates of 11 emerging markets exchange rates. The currencies are: Brazilian real (BRL), Chilean peso (CLP), Colombian peso (COP), Malaysian ringgit (MYR), Mexican peso (MXN), Indonesian rupiah (IDR), Israeli shekel (ILS), Philippine peso (PHP), Thai baht (THB) Turkish lira (TRY) and South African rand (ZAR). The sample period is from July 2008 to July 2013. The Relative Risk Aversion (ρ^{EM}) is estimated using a risk-neutral distribution with mixture of log-normals density and the parametric-risk transformation of Liu et al. (2007). A positive risk aversion means investors charge a risk premium to carry the currency. The stationary bootstrap p-value is calculated using the Politis and Romano (1994) stationary bootstrap over the time series of volatility surfaces, and then calculating the RRA coefficient for each simulation. The p-value is the percentile of RRA = 0 of the sorted simulated RRA's.

6. Density Forecast Evaluation

In the previous sections, both the RND and RWD were estimated for a set of emerging markets currencies. In this section, I assess the out-of-sample goodness-of-fit of these densities. The goal is to evaluate if these densities give useful information about the future outcomes of the exchange rates.

I follow the density forecast evaluation literature and use the probability integral transformation in order to generate a time series U as follows:

$$U = \{U_i\} = \left\{\tilde{g}_{CDF}{}^{-1}\left(S_{i+30}|\hat{\theta}_i, \rho\right)\right\} \tag{7}$$

If the forecast densities are adequate, U must be a Uniform distribution with a domain in the range [0, 1]. The left side of Figure 1 shows the distribution of U for all 11 currencies RND. The right side shows U for the RWDs using the RRA calculated last section. In all cases, except IDR, the tails of the distribution are higher than the middle range. This means that option prices densities are underweighting the tails when compared to the actual distribution. This happens for both RND and RWD. However, in many cases the RWD reduce this overweighting, especially for the left tail. Therefore, although the parametric risk-transformation is not able to correct the thin tails of the RND, it softens a little the problem.

This pattern where RNDs are not able to fit well the fatter tails of the actual distribution has already been identified in the previous literature. Christoffersen and Mazzotta (2005) found similar problem with developed countries foreign exchange rate options, and Castren (2005) with Eastern European currencies.

Christoferssen and Mazzotta (2005) mention that the fatter tails of the actual distribution compared to the RND could be attributable to the lack of very out-of-the-money options. Their sample had options with strikes with a minimum delta of 25, and RND forecast ability only in the middle 70% range of the distribution was adequate. In fact, it is not possible to directly draw information for the distribution tails from options with strikes located after the most distant strikes. Some kind of extrapolating is needed.

In the case of my sample, options are available with strikes as low as 10-delta, going deeper into the tails of the distribution than Christoferssen and Mazzotta (2005). The question that arises is whether these deltas would be far enough to accurately model, for instance, the first and last deciles of the distributions, which show bad goodness-of-fit as seen on figure 1. The strikes of the 10-delta puts and 10-delta calls are located, on average, on the 10.3% and 93.2% cumulative distribution respectively. This means that the first and last deciles of the CDF are estimated based (almost) on extrapolation. So in this specific case, the functional parametric form of the distribution is relevant. Non-parametric densities estimations could not fit these deciles, since they are not able to extrapolate. Parametric functions should have heavier tails than those estimated by the Mixture of Log-Normals in this paper. I have also tried to estimate the parameters of the Log-Normal Mixture using a fitting method that gives emphasis on the tails of the distribution (see, for instance, Prause 1999, or Fajardo et al., 2005), so that fitting errors on the tails of the distribution would have more importance. However, this was not able to fix the problem. Other possible solutions would be to use time series data to shape the tails of the distribution, or use a more flexible parametric distribution such as the Generalized Hyperbolic.

The pattern of strikes to build the RND showed to be an important factor for the tail accuracy. OTC options data usually have fixed strikes, while exchange-traded options strikes vary over time. The empirical evidence seems to favor exchange-traded options when assessing the accuracy of the tails. While this paper, Christoferssen and

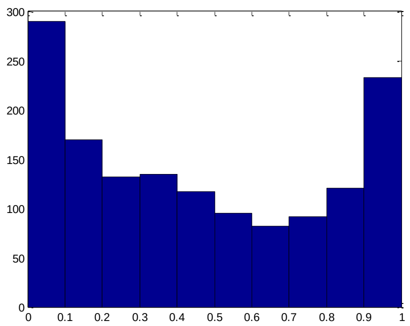
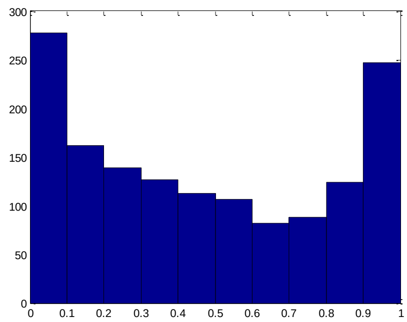
Mazzotta (2005) and Castren (2005) use OTC forex options and find a bad tail performance, Fajardo et al. (2012) and Craig and Keller (2005) use exchange-traded options and find good results.

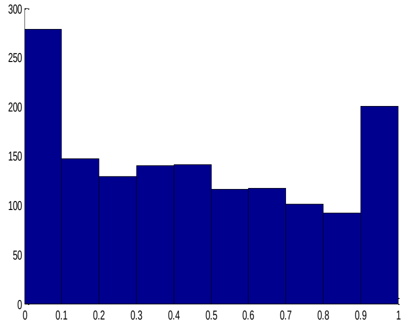
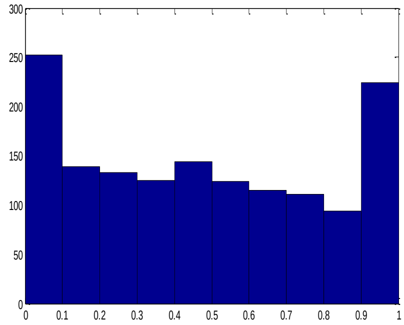
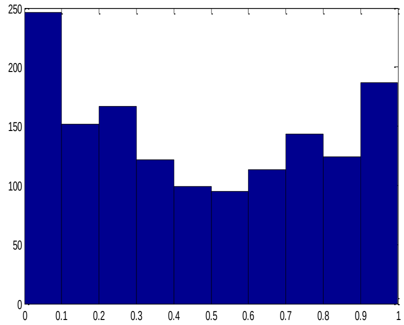
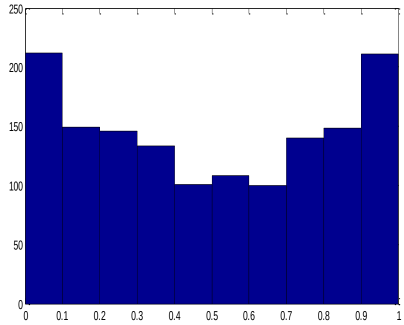
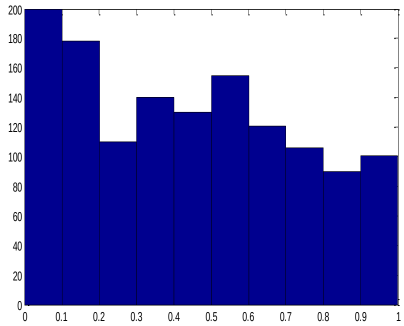
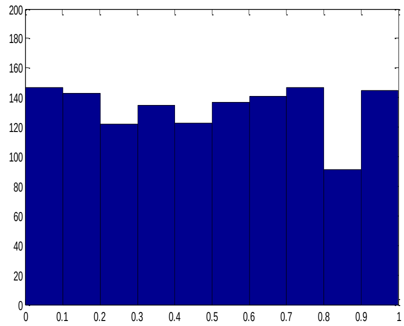
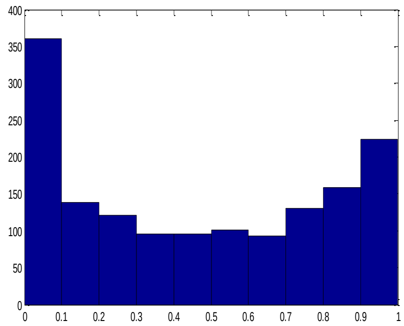
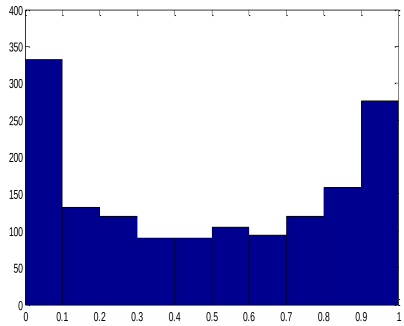
There are also analytical ways to further evaluate the forecast. Berkowitz (2001) goes further and "normalize" this U series using the inverse of the standard normal distribution, generating a Z series:

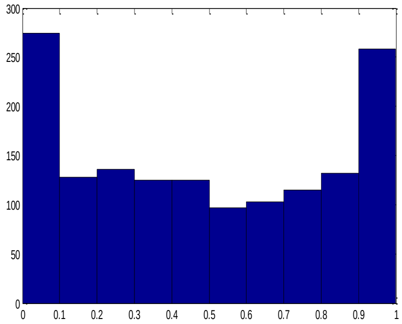
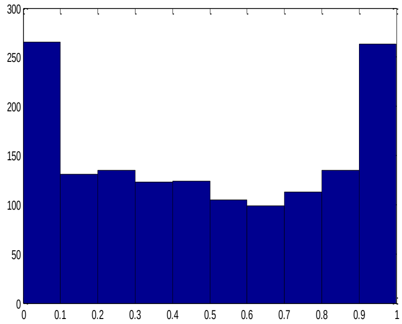
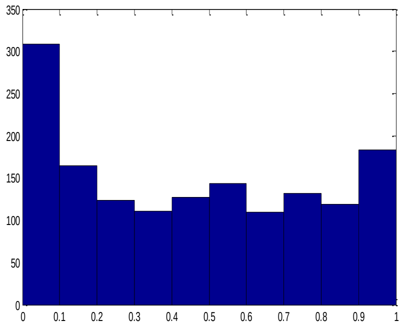
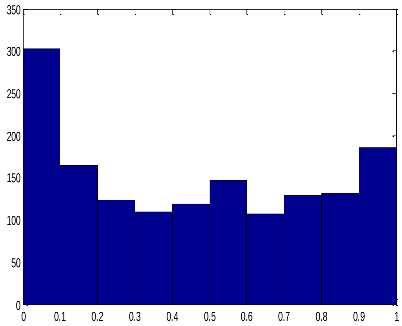
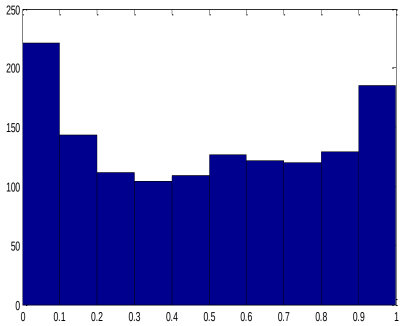
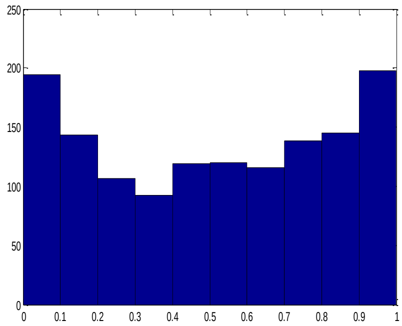
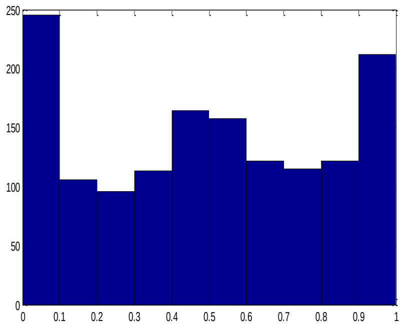
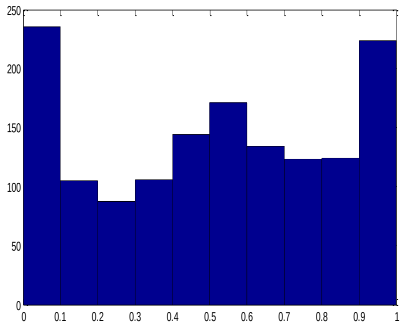
$$Z = \{Z_i\} = \{\Phi^{-1}(U_i)\} \tag{8}$$

If the forecast density models are good, this series Z should follow Standard Normal distribution. Then, it is possible to use a statistical test to check if the distributions Z are normally distributed. I calculate the Kolmogorov distance in these series Z in order to assess the quality of the density forecast (see Table IV). In all cases, the RWD provided a lower distance than the RND. Nevertheless, the Kolmogorov-Smirnov test rejects normality in all cases, except the RWD of IDR. Therefore, the only case in which the parametric risk transformation was able to improve the RND forecast enough to pass the Kolmogorov-Smirnov test was in the IDR.

Figure 1 – Out-of-sample Density Forecast Evaluation – *U_i*

Currenc y	Risk Neutral Densities	Real World Densities
BRL		

CLP		
COP		
IDR		
ILS		

MYR		
MXN		
PHP		
THB		

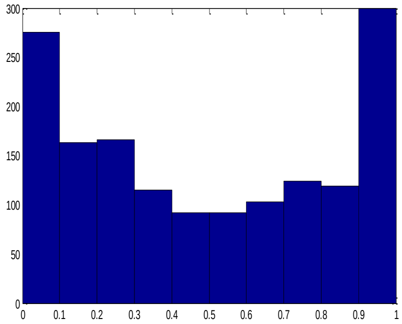
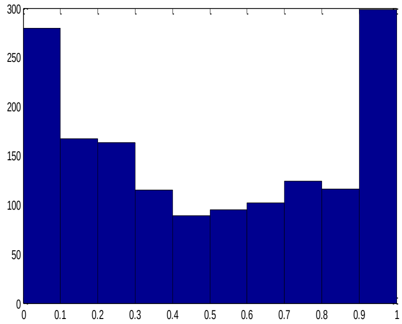
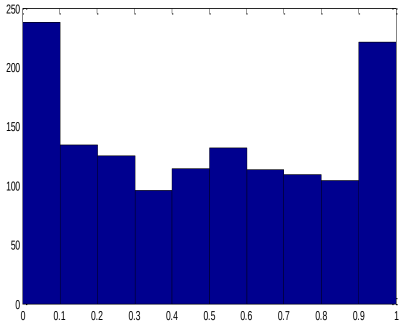
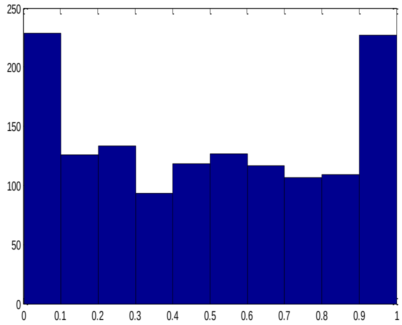
TRY		
ZAR		

Table IV – Kolmogorov Distances

	Risk-Neutral Densities (RND)	Real-World Densities (RWD)
BRL	12.0%	10.8%
CLP	9.8%	8.0%
COP	9.2%	5.8%
IDR	8.9%	2.7%
ILS	14.4%	12.5%
MYR	8.4%	8.2%
MXN	12.0%	11.5%
PHP	7.3%	6.2%
THB	7.8%	7.1%
TRY	9.6%	9.5%
ZAR	7.5%	7.0%
Mean	9.7%	8.1%

This table shows the Kologorov distance of the normal transform variable Z for RND and RWD of 11 emerging markets currencies. The variable Z is calculated as in equation (8). If the density forecast is adequate, Z should be normally distributed. The Kolmogorov distance measures the discrepancies of the Z variable from a Normal distribution, for each currency, and for the Risk-Neutral densities and Real-World Densities. The currencies are: Brazilian real (BRL), Chilean peso (CLP), Colombian peso (COP), Malaysian ringgit (MYR), Mexican peso (MXN), Indonesian rupiah (IDR), Israeli shekel (ILS), Philippine peso (PHP), Thai baht (THB) Turkish lira (TRY) and South African rand (ZAR). The sample period is from July 2008 to July 2013.

One issue with the above test is the autocorrelation in the series caused by data overlapping. Thus, I follow Christofferssen and Mazzotta (2005) in testing if the moments of Z are the same of a Standard Normal distribution, accounting for autocorrelation. So the first and third moments of Z should be zero, the second should be one and the fourth should be three. This test can be done using the following set of regressions, with Newey-West (1987) standard errors to deal with data overlapping:

$$
\begin{gathered}
Z_i = a_1 + \varepsilon_{1,t} \\
Z_i^2 - 1 = a_2 + \varepsilon_{2,t} \\
Z_i^3 = a_3 + \varepsilon_{3,t} \\
Z_i^4 - 3 = a_4 + \varepsilon_{4,t}
\end{gathered} \tag{9}
$$

The idea behind these regressions is that, if the forecast is adequate, all coefficients a should be zero so that moments of Z match those of a Normal distribution. This can shed a light on why the Kolmogorov-Smirnov test is rejecting normality, and what could be done to improve. Results are on Table V.

The point estimates of the first moment coefficients (a_1) are mostly negative for the RND, reflecting a risk premium against EM currencies ($\rho^{EM} = -\rho > 0$). The parametric risk transformation brings this coefficient closer to zero in most of the cases, as can be seen on the RWD a_1 coefficients. On average the a_1 coefficient is -0.06 for the RNDs and +0.01 for the RWD. However, in all cases the coefficient is statistically equal to zero. Thus, although RWDs may be better centered than RNDs, the first moment does not seem to be a major problem for the accuracy of the RNDs.

Nevertheless, the second moment shows strong evidence that densities derived option prices do not fit well the actual data. All a_2 coefficients are positive and statistically greater than zero, with the exception of the IDR. The fourth moment also shows performance problems with the forecasted densities. All a_4 coefficients are positive and most of them are statistically greater than zero. This evidence is consistent with fatter tails observed on my figure 1; and with the findings of Christoferssen and Mazzotta (2005) for major exchange rates, and Castren (2005) for Eastern European currencies.

Finally, the third moment shows mainly positive coefficients a_3, but none of them are statistically different from zero. This means that, on the negative EM currency return distribution side, the actual data shows a higher probability than that forecasted by RND and RWD. This is a puzzling result, since it would be consistent with a negative skewness risk premium. Anyway, as coefficients are not significant, this may not be serious problem.

Table V – Tests of *Z* Moments

	First Moment (a_1)		Second Moment (a_2)		Third Moment (a_3)		Fourth Moment (a_4)	
	RND	RWD	RND	RWD	RND	RWD	RND	RWD
BRL	**-0.06**	**-0.02**	**1.43**	**1.43**	**3.35**	**3.61**	**30.25**	**30.54**
	(-0.38)	(-0.12)	(2.95)	(2.93)	(1.14)	(1.22)	(1.65)	(1.65)
CLP	**-0.09**	**-0.01**	**1.46**	**1.46**	**3.67**	**4.15**	**38.06**	**38.84**
	(-0.56)	(-0.06)	(2.80)	(2.74)	(1.08)	(1.20)	(1.72)	(1.71)
COP	**-0.08**	**0.02**	**0.62**	**0.62**	**0.21**	**0.74**	**5.00**	**5.28**
	(-0.64)	(0.19)	(3.32)	(3.15)	(0.32)	(1.07)	(2.94)	(2.79)
IDR	**-0.16**	**0.03**	**0.33**	**0.29**	**0.49**	**1.30**	**5.26**	**5.83**
	(-1.37)	(0.25)	(1.45)	(1.16)	(0.54)	(1.28)	(1.52)	(1.38)
ILS	**-0.17**	**-0.06**	**0.92**	**0.92**	**-0.92**	**-0.30**	**6.73**	**6.70**
	(-1.30)	(-0.48)	(4.74)	(4.74)	(-1.22)	(-0.39)	(3.27)	(3.13)
MYR	**0.00**	**0.02**	**0.89**	**0.89**	**0.61**	**0.71**	**7.21**	**7.28**
	(0.02)	(0.16)	(4.05)	(4.04)	(0.75)	(0.88)	(3.45)	(3.44)
MXN	**-0.07**	**-0.04**	**1.55**	**1.55**	**5.90**	**6.04**	**51.93**	**52.20**
	(-0.41)	(-0.26)	(2.27)	(2.26)	(1.18)	(1.20)	(1.39)	(1.39)
PHP	**-0.02**	**0.06**	**0.68**	**0.69**	**0.49**	**0.91**	**7.46**	**7.73**
	(-0.18)	(0.51)	(2.99)	(2.97)	(0.52)	(0.95)	(2.16)	(2.14)
THB	**-0.06**	**0.00**	**0.88**	**0.89**	**-1.22**	**-0.93**	**10.22**	**10.17**
	(-0.44)	(-0.03)	(3.06)	(3.10)	(-1.08)	(-0.83)	(2.68)	(2.73)
TRY	**0.02**	**0.01**	**1.15**	**1.15**	**2.61**	**2.54**	**22.54**	**22.49**
	(0.15)	(0.07)	(3.11)	(3.12)	(1.11)	(1.09)	(1.44)	(1.44)
ZAR	**0.05**	**0.08**	**1.09**	**1.09**	**3.89**	**4.05**	**27.09**	**27.45**
	(0.33)	(0.52)	(2.52)	(2.50)	(1.43)	(1.47)	(1.50)	(1.49)

*This table shows results of the set of regressions described on equation (9). This set of regression is estimated for RND and RWD of 11 emerging markets currency options. The currencies are: Brazilian real (BRL), Chilean peso (CLP), Colombian peso (COP), Malaysian ringgit (MYR), Mexican peso (MXN), Indonesian rupiah (IDR), Israeli shekel (ILS), Philippine peso (PHP), Thai baht (THB) Turkish lira (TRY) and South African rand (ZAR). All options are quoted considering an exchange rate expressed as emerging market currency per U.S. Dollar. Point estimates of **a** coefficients are in bold, and 21 –lags Newey-West t-statistics are in parenthesis.*

Comparing RNDs and RWDs, we see that results are very similar, except for the first moment. Thus, the use of risk-transformation method could be justified just to fix the drift, but seems useless to cope with tails misspecification.

Overall, results of Table V show evidence against RND and RWD, especially on the even moments. This analysis focus on one moment at a time, i.e., tests each coefficient *a* individually. However, it is possible to test if the *a* coefficients are jointly different from zero using a Wald test. Results are on Table VI. As in the Kolmogorov-Smirnov tests, the only distribution that does not reject normality at 10% significance level of *Z* is the RWD of the IDR. However, lowering to 1% significance level, we would have six currencies being rejected and five not rejected.

Table VI – Joint Test of *Z* Moments

	All Moments Together $a_1 = a_2 = a_3 = a_4 = 0$	
	RND	**RWD**
BRL	**23.14**	**21.93**
	(0.000)	(0.000)
CLP	**17.57**	**14.74**
	(0.002)	(0.005)
COP	**12.73**	**10.56**
	(0.013)	(0.032)
IDR	**10.33**	**4.64**
	(0.035)	(0.326)
ILS	**33.37**	**33.97**
	(0.000)	(0.000)
MYR	**18.09**	**17.93**
	(0.001)	(0.001)
MXN	**18.31**	**18.51**
	(0.001)	(0.001)
PHP	**10.37**	**10.22**
	(0.035)	(0.037)
THB	**9.72**	**10.30**
	(0.045)	(0.036)
TRY	**23.95**	**24.1**
	(0.000)	(0.000)
ZAR	**13.22**	**12.7**
	(0.010)	(0.013)

*This table shows GMM estimation results of the regressions system described on equation (9). Point estimates of the joint (Wald) test that all **a's** coefficients are equal to zero are in bold, and Bartlett kernel with 21-days bandwidth p-value are in parenthesis. This set of regression is estimated for RND and RWD of 11 emerging markets currency options. The currencies are: Brazilian real (BRL), Chilean peso (CLP), Colombian peso (COP), Malaysian ringgit (MYR), Mexican peso (MXN), Indonesian rupiah (IDR), Israeli shekel (ILS), Philippine peso (PHP), Thai baht (THB) Turkish lira (TRY) and South African rand (ZAR). All options are quoted considering an exchange rate expressed as emerging market currency per U.S. Dollar.*

7. Final Remarks

This paper evaluates the forecast performance of Emerging Markets currencies option-implied densities. Results show that both RND and RWD fail to correctly forecast the tails of the realized distribution, specifically the first and last 10% of the distribution. The reason is probably the lack of option data with strikes on this region. The use of a parametric-risk transformation to build RWD from RND and a relative risk aversion parameter was not able to properly address the tails misspecification. The relative risk aversion estimation shows weak evidence that investors are willing to charge a premium to invest in emerging currencies. However, this weak statistical evidence may be due to the small size of the premium, if compared with its volatility.

Therefore, the use of RWD instead of RND brings only a limited advantage.

Despite this tail underweighting problem, these option-implied distributions could be used in applications in which the tails are not so important. For instance, they could be used in a mean-variance optimization process. The paper of Kostakis et al.(2011) uses S&P500 implied distributions to build optimal portfolios, and then evaluate the performance of this procedure. This approach could be used for a multicurrency portfolio of emerging markets using the data in this paper.

In order to tackle tails misspecification, I can see two possible solutions: to use time series data to shape the tails of the distribution; or use a more flexible parametric distribution such as the Generalized Hyperbolic. Thus, an estimation process that blends time-series and option-implied data through the use of a heavy-tails distribution such as Generalized Hyperbolic is a suggestion for further research.

Appendix I – Monthly Estimation Results for Relative Risk Aversion

An alternative approach to overcome data overlapping would be to use only non-overlapping data. The problem is that in my sample we would have only 72 non-overlapping months, which is relatively small for a risk aversion estimate. However, we may strengthen the analysis by repeating the procedure for each day of the month, so that approximately 30 sets of 72 non-overlapping data would be available for each currency. Thus, 30 risk aversion estimates can be calculated for each currency.

Results are on Table VII. The means of the risk aversion parameters calculated over all days of the month are very similar to the results of Table III. Most of the RRA's are positive: on average 22 out of 30. However, just some of them are statistically greater than zero with 10% significance.

Table VII – Relative Risk Aversion Estimates by Day of the Month

	Mean Relative Risk Aversion	**# postive RRAs**	**# negative RRAs**	**# statistically significant positive RRAs**
BRL	1.63	21	9	1
CLP	3.17	23	7	6
COP	3.68	25	5	5
IDR	8.77	28	2	15
ILS	5.58	25	5	5
MYR	1.48	19	11	0
MXN	0.90	17	13	0
PHP	5.32	24	6	1
THB	4.35	24	6	1
TRY	-0.49	13	17	2
ZAR	0.77	19	11	1
Mean	3.20	22	8	3

This table shows the mean relative risk aversion (ρ^{EM}) estimates of 11 emerging markets exchange rates using non-overlapping data. The Relative Risk Aversion (RRA) is estimated using a risk-neutral distribution with mixture of log-normals density and the parametric-risk transformation of Liu et al.(2007). A positive risk aversion means investors charge a risk premium to carry the currency. The currencies are: Brazilian real (BRL), Chilean peso (CLP), Colombian peso (COP), Malaysian ringgit (MYR), Mexican peso (MXN), Indonesian rupiah (IDR), Israeli shekel (ILS), Philippine peso (PHP), Thai baht (THB) Turkish lira (TRY) and South African rand (ZAR). The sample period is from July 2008 to July 2013. For each currency, 30 RRA were calculated, one for each day of the month. Each RRA calculation used a non-overlapping monthly time series starting in each of the 30 days of the month. The table shows also the number of positive and negative RRA's. For the positive RRA's, it shows the number that are significantly greater than zero at 10% significance level.

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