Testing the Liquidity Preference Hypothesis using Survey Forecasts

Jose Renato Haas Ornelas e Antonio Francisco de Almeida Silva Jr

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Abstract

We evaluate the Liquidity Preference Hypothesis (LPH) for the term structure of interest rates in a different way. Instead of using bond returns as traditional approaches, we use interest rate surveys with market expectations in order to evaluate LPH. This approach allows us to disentangle the effect of the changes in interest rate expectations from the liquidity premium. We found empirical support for the LPH with Brazilian data using both traditional and survey methods. However, the evaluation with interest rate surveys gives a higher statistical confidence level than the traditional approach when we perform tests for term premium monotonicity.

Keywords: Liquidity Preference Hypothesis, interest rates, term premium, survey forecast.
JEL Classification: C58, E43, E58.

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** Banco Central do Brasil and University of Miami.
*** Banco Central do Brasil and Universidade Federal da Bahia.
1. Introduction

There is a vast research on the term structure of interest rates, and one of the oldest issues in this literature is the liquidity preference hypothesis\(^1\) (LPH). This hypothesis states that the term premium increases monotonically over time to maturity. This means that the expected returns on government securities should monotonically increase over time to maturity. One problem with the empirical investigations of the term premium is that it is not directly measurable. The yield of long-term bonds embeds the market expectations for future short-term interest rates plus a term premium. Both can vary over time, and it creates a challenge for isolating term premium on a time series. In order to disentangle short-term interest rates expectations from term premium in bond yields, traditional literature usually use time series of ex post returns (see, for instance, Fama, 1984 or Patton and Timmermann, 2010). However, these time series must be long enough to mitigate the effects that arise from variations in future short-term interest rates expectations and term premium, as well as measurement errors. Also, the presence or absence of a rare event in the sample time period may induce a bias when using ex-post returns, if the sample size is not long enough\(^2\). The article of Elton (1999) describes the problems arising from the use of realized returns on asset pricing tests.

An alternative approach to investigate LPH is to use surveys with market expectations instead of using only data from security prices and returns. Several decades ago McCulloch (1975) mentioned this possible approach, but at that time the data were not reliable, nor long enough. Nowadays, data is no longer an issue. Kim and Orphanides (2007, 2012) and Swanson (2007) show how one can calculate the term premium using survey data for the US market. This approach mitigates problems from variations in the short-term interest rates expectation as we account for expectations in an explicit way. Therefore, we are able to isolate and extract the term premium from bond yields by using expectations of short-term interest rates over the bond life.

In this paper, we use this survey approach to test LPH, instead of relying on bond ex-post returns as the traditional literature. To the best of our knowledge, we are

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\(^1\) See McCulloch (1975), Fama (1984), Boudoukh et al (1999) and Kessel (1965). It is worth mentioning that the literature on testing LPH uses mainly US data.

\(^2\) This is related to the so-called peso-problem, which is the effect on statistical inference caused by a very low-probability event that do not occur in the sample.
the first to formally test LPH using survey data. We measure the term premium and test LPH for the Brazilian term structure using a survey of market expectations for the Brazilian monetary policy interest rate decisions carried out by the Central Bank of Brazil. This survey covers more than ten years of daily observations, and is therefore a valuable and unique source of information. We also use data from Brazilian term structures of interest rates. With this dataset, our approach consists of using the market expectations for future short-term interest rates to calculate the term premium for maturities up to 12 months. Our analysis strongly supports that term premia monotonically increase over time to maturity during our sample period. The term premium increases between two and four basis points for each additional month of maturity. These pairwise increases are statistically greater than zero, no matter whether we tested it separately or jointly. The traditional method produces much higher term premia, but also much wider standard errors. This suggests that the survey method is more precise. Overall, we found strong support for the Liquidity Preference Hypothesis in the Brazilian market using the survey method, and weaker evidence using the traditional method.

In order to use survey forecasts to calculate the term premium, we need to check for possible biases. Otherwise the bias may be “transmitted” to the term premium. If we have an upward bias on surveys, our term premium may be biased downward, and vice-versa. Our results show that survey forecasts are an unbiased estimator of future monetary policy interest rates for our sample period.

This paper is structured as follows: section 2 reviews the literature on LPH testing; section 3 describes the dataset; Section 4 presents the tests for the unbiasedness of survey forecasts; Section 5 describes the method for estimating term premium and presents some descriptive statistics; section 6 shows some interesting results when we condition term premium structure to the slope of term structure; section 7 formally tests for the monotonicity of the term premium structure; and section 8 concludes.

2. Literature Review

There are two fundamental hypotheses on interest rate term structures. According to the pure expectations hypothesis, the forward rates are unbiased estimates of future spot rates. A more general specification of the expectations hypothesis states
that forward rates are the expectations of future spot rates plus a liquidity premium. Therefore, the difference between the yield of a long-term bond and the market’s expectation of the future short-term rate is the liquidity or term premium (McCulloch, 1975). The liquidity preference hypothesis implies that the longer the term to maturity of a security, the higher its term premium. Therefore, the term structure of interest rates may be seen as the result of the combination of the expectations and liquidity preferences hypotheses (Kessel, 1965).

The term premium may also be defined as the expected return of holding a government bond for one period minus the return of a one-period bond. The LPH would imply that the ex-ante return on government bond should monotonically increase over time to maturity (Boudoukh et al, 1999). As highlighted by Kim and Orphanides (2007), these distinct definitions of the term premium have the same intuition.

There are many procedures to evaluate the expectation hypothesis such as those in Campbell and Shiller (1991), Hardouvelis (1994) and Engle, Lilien and Robis (1987). However, the dynamics of the interest rate term structure is a conundrum. Empirical tests are liable to sample errors, measurement errors and changes in the level and variance of spreads. Hardouvelis (1994) discussed two possible mechanisms to explain changes in the interest rates term structure. The first one assumes that risk premia are constant and long rates movements may be sluggish relative to the movements of the current short rates. On the other hand, it would also be possible that markets overreact to a central bank announcement raising future spot rates by more than necessary and making spreads between long and short rates higher than they should be. This explanation therefore considers that the market’s expectations violate rational expectations. The second mechanism considers that the market’s expectations are rational but the information in the spread is composite information about the variation of both expected future rates and risk premia.

One issue investigated by Kessel (1965) was the changes in liquidity premium in time. The liquidity premium can vary with the level of interest rates or can vary with the economic cycle. The author found evidence in favor of the liquidity premium as a function of the level of interest rates. On the other hand, McCulloch (1975) discussed some research refuting this relationship and found no empirical support for this assumption.

3 We use “liquidity premium” and “term premium” as synonyms.
The liquidity preference hypothesis states that the premium increases monotonically with maturity. Fama (1984) found empirical evidence for the liquidity preference hypothesis in shorter maturities, up to seven months. Nevertheless, the author argues that it is not possible to generalize this issue to longer maturities. McCulloch (1987) pointed out that there was a problem in the data sample with the selected time window due to the bid-ask spreads for the nine and ten months maturity in Fama’s work. As a result, McCulloch gave empirical evidence for the premium increasing monotonically with maturity. Richardson et al (1992) used a test of inequality constraints proposed by Wolak (1989) and their general results support the work of McCulloch (1987). Boudoukh et al (1999) reinvestigated Fama’s (1986) work and they showed that it is important to evaluate the joint inequality constraints and the type of conditioning information used in the estimate. Their findings, using a procedure similar to Wolak’s (1989), are consistent with the liquidity preference hypothesis. Patton and Timmermann (2010) evaluated the liquidity preference hypothesis by inspecting the term premia on T-Bills from 1964 to 2001. Regarding monotonicity, the Wolak test rejects the hypothesis of increasing term structure, while the monotonic relation (MR) test of Patton and Timmermann fails to find evidence in favor of a monotonically increasing term structure.

The above authors relied on ex-post bond returns in order to evaluate the liquidity preference hypothesis and basically they considered that the differences between long and short maturity bonds returns are due to the liquidity premium. For instance, Patton and Timmermann (2010) measured the term premium \( p_t^k \) in the following way:

\[
p_t^k = E[r_t^k] - E[r_t^1]
\]

where

\( E[r_t^k] \) is the expected one-period\(^4 \) return on a bond with maturity \( k \), at time \( t \);

\( p_t^k \) is the term premium at time \( t \), for a maturity \( k \).

On their empirical verification of the LPH, they estimated the expected return using monthly ex-post returns. Based on this set-up, the LPH implies that:

\(^4\) This one-period return is usually set to one-month.
\[ p_t^k > p_t^l \text{ for } k > l \]

Let us define \( \Delta_t^k = p_t^k - p_t^{k-1} \) as being the difference between the term premium of two adjacent terms. We may rewrite (2) with only the adjacent terms:

\[ \Delta_t^k > 0 \text{ for all } k's \]

Instead of using bond returns, several authors use interest rate surveys to estimate the term premium and then evaluate the expectations hypothesis. This is the case of Friedman (1979), Froot (1989), Cook and Hahn (1990), MacDonald and Macmillan (1994), Jongen, Verschoor and Wolff (2005). Nevertheless, no paper so far has formally tested the LPH using survey data.

Kim and Orphanides (2012) use survey forecasts of the short-term interest rate to improve results in the estimate of a three-factor pure-Gaussian model and the authors argue that problems encountered in “conventional” estimation without survey data are greatly alleviated. Therefore the model reproduces the well-known pattern of deviation from the expectations hypothesis regressions and generates an implied forecast of long-term interest rates that captures some of the deviations of survey forecasts of long-term interest rates from the expectations hypothesis. The authors also show evidence that documents the presence of a substantial bias and imprecision in the parameter estimates in the conventional estimation and the improvement brought by estimating with survey data. They discuss the importance to a dynamic term structure of a persistent factor with a long half-life longer than 5 years. Furthermore, they highlight that surveys are prone to some measurement error, for example due to differences in the information sets and precise timing when various participants may prepare their responses and therefore it is important to allow for the possible presence of substantial errors when using information from surveys.

It is worth mentioning that literature regarding LPH is concentrated on US data and therefore our paper contributes to empirical research in emerging markets literature as we deal with Brazilian term structure.
3. Sample

The Central Bank of Brazil has been conducting surveys of analysts’ forecasts of several financial and macroeconomic variables since December 2001. The forecasts come from different types of institutions, such as banks, brokers and consulting firms. One of the surveyed variables is the target for the monetary policy one-day interest rate which is called the Selic target rate. This target is set by the Brazilian monetary policy committee (Copom) at regular meetings. Up to the year 2005 these meetings were held every month, with intervals of four or five weeks. After 2006, the periodicity changed to every 45 days approximately, with intervals of 6 or 7 weeks. The meetings start on Tuesdays and finish on Wednesdays with the announcement of the target interest rate decision.

The Open Market Department of the Central Bank of Brazil is responsible for keeping the effective monetary policy interest rate (Selic rate) near the target set by Copom. The Selic rate is calculated based on one-day repo operations, using Brazilian Treasury Bonds as collateral. However, the effective rate is not exactly the same as the target rate, rather it oscillates some basis points around it\(^5\).

Our sample has the daily data of the survey forecasts for the Selic target rate from 12 months to 18 months ahead. Forecasts are done for almost every month. As the year of 2002 was very turbulent, including extraordinary meetings of Copom, we decided to exclude forecasts done in 2002 for the year of 2003. Thus, our sample starts with forecasts done in 2003 for the year of 2004. We have considered 86 Copom meetings from January, 21\(^{st}\) 2004 to August 28\(^{th}\) 2013. We limit the sample to 12-month forecasts to create a homogenous dataset, as 18 month forecasts are not available for the entire period. Forecasts are available on a daily basis, and a bulletin published every Friday night consolidates the results. We are the first to use such a detailed setup with interest rate surveys for LPH test. This is especially important when there is one or more monetary policy meetings in-between the three-month periods.

We evaluate the Liquidity Preference Hypothesis with the DI market interest rate term structure, which is the most liquid instrument in the Brazilian Fixed Income market. Besides the survey forecasts, we used daily time series for the Selic target rate

set by Copom, and the one-day DI rate, which is the uncollateralized interbank deposit rate used as a benchmark index by most mutual funds and certificate of deposits. We also used a zero-coupon yield curve. The zero-coupon curve was built with the one-day DI fixed-to-floating swaps with maturities from 1 to 12 months.

4. Tests for Bias on Survey Forecasts

If survey forecasts are used to calculate term premium, it is important to check for possible biases to ensure that no bias is included in our term premium calculation. Unfortunately, literature about forecast biases in Brazil tends to focus on inflation forecasts (see Carvalho and Minela, 2012, and Silva Filho, 2013). Therefore, we empirically test the unbiasedness of market surveys, considering changes in the monetary policy interest rates and survey forecasts on a daily basis. We tested the bias for the forecasts in each of the 86 Copom meetings in our sample with 1 day ahead and from 1 week to 52 weeks ahead. The one-day ahead is the mean forecast done on the Tuesday before the Copom meeting, which happens on Wednesdays. The one-week ahead is calculated with the forecast on the Friday before the Wednesday Copom meeting, and so on. This is done because the Friday forecasts are more visible due to the report published every Friday. Therefore, the one-week forecast is actually 5-days ahead, the two-week forecast is 12-days ahead and so on.

Table I shows the mean and standard deviation of the forecast errors as well as the mean absolute error and the t statistics for the null hypothesis that the forecast error is zero. We can see that the mean error oscillates from negative to positive with values under 3 basis points for forecasts up to 28 weeks. After that, the mean error is mainly positive, but under 20 basis points. In all cases, we cannot reject the null hypothesis that the forecast error is zero. Therefore, we have evidence that the forecasts are not biased.
Table I – Survey Forecast Errors

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<tr>
<th>Forecast Weeks Ahead</th>
<th>Mean Error</th>
<th>Standard Deviation of Error</th>
<th>Mean Absolute Error</th>
<th>t-stat</th>
<th>Forecast Weeks Ahead</th>
<th>Mean Error</th>
<th>Standard Deviation of Error</th>
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<td>2.36</td>
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This table shows survey forecast errors of 86 Copom meetings in our sample with survey forecasts one day ahead and from one week to 52 weeks ahead. The one-week ahead error is calculated with forecast on the Friday before the Wednesday Copom meeting, and so on. The zero-week ahead forecast is the error one day before the meeting. The error is calculated by subtracting the interest rate set by Copom from survey forecast. Table shows the mean and standard deviation of errors as well as the mean absolute error and the t statistics for the null hypothesis that error is equal to zero.
5. Method and Estimation of the Term Premium

We describe below how to calculate the term premium for the DI yield curve. We first define what would be the expectation for the one-day effective DI interest rate, compounded from time $t$ to time $k$, and then expressed on an annualized basis:

$$E_t[D_{t,k}] = \left( \prod_{i=t}^{k} \left( 1 + E_t[d_i] \right) \right)^{\frac{252}{k-t}} - 1$$

Where:

- $E_t[D_{t,k}]$ is the expected one-day effective DI interest rate compounded from time $t$ to time $k$, and then expressed in annualized terms;
- $E_t[d_i]$ is the expectation, at time $t$, of the effective DI rate for day $i$.

It is worth highlighting that $E_t[d_i]$ does not have to take into account any term premium because it considers only the expectation of the one-day interest rate behavior.

We can break down the yield of a zero-coupon bond into two components, the ex-ante term premium and the expected compounded short-term interest rates until the maturity of the bond, as follows:

$$y_t^k = E_t[D_{t,k}] + p_t^k$$

Where:

- $y_t^k$ is the annualized yield at time $t$ of a zero-coupon bond with maturity $k$ on the DI curve.
- $E_t[D_{t,k}]$ is the expected one-day effective DI interest rate compounded from time $t$ to time $k$, and then expressed in annualized terms (see equation 1).
- $p_t^k$ is the annualized ex-ante DI term premium at time $t$ for a $k$ maturity.

We used our method to estimate the term premia using daily data so as not to miss any policy interest rate (actual or expected) change.

In order to build the expected one-day DI interest rates $E[d_i]$, we define:

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6 The day counting practice in Brazil is based on working days, considering a year with 252 working days.
\[ E_t[g_i] = E_t[d_i] + E_t[s_i] \]

Where:

- \( g_i \) is the monetary policy target for the Selic interest rate for day \( i \)
- \( E_t[g_i] \) is the expectation, at time \( t \), of the monetary policy target for the Selic interest rate for day \( i \)
- \( E_t[s_i] \) is the expectation, at time \( t \), of the difference between the Selic target rate and the DI effective rate for day \( i \).

The expected Selic target rate \( E_t[g_i] \) is taken from the survey of market expectations carried out by the Central Bank of Brazil at time \( t \). We need the expected target rate for each day, but the actual forecast data is done 1 to 12 months ahead. It is worth remembering that the forecast is intended to be the target (and not the effective) interest rate at the end of the month. As the target rate only changes after the Copom meetings, we assume that for each month \( m \) for all days until the Copom meeting, the expected target rate \( E_t[g_i] \) is the one forecasted for the previous month \((m - 1)\), and after the Copom meeting, we consider the forecasted target rate for that month \( m \).

The expected difference, at time \( t \), between the target rate and the effective DI rate is estimated by simply taking the difference between the target Selic rate at time \( t \) and the effective DI rate at time \( t \):

\[ E_t[s_i] = g_t - d_t \]

Equation (7) assumes that the current difference between the DI effective rate and the Selic target rate is the best estimator for this difference in the future. This difference is reasonably stable. If this difference becomes very volatile, our term premium may experience an increase in the measurement error. A one-time level change as occurred in 2013 will affect our measure in just one day.

Using equations 4 to 7, we are able to calculate the ex-ante term premium \( p^k_t \) for the DI swap curve. We also calculated the term premium using the traditional method, which uses the ex post returns of bonds. Graph I shows the means for term premia using both methods. For the survey method, all premia are positive and statistically significant, except for the one-month term premium, which is not statistically
significant. For the traditional method, although the premia are positive and higher than using the survey method, only the maturities from 1 to 5 are statistically significant.

Graph I also shows the confidence intervals. We see that average term premia monotonically increase as predicted by LPH, for both methods. We will test this issue formally in section 7. The confidence intervals of the survey method are in general tighter. In most of the maturities, the confidence interval of the survey method lies entirely inside the confidence interval of the traditional method. This suggests that the survey method is able to measure the premium with more precision. This is expected as the traditional method absorbs errors coming from time-varying short-term rate expectations and premia. In our sample, these errors are likely to foster an upward bias on the premium calculated using the traditional method because there was a slightly downward trend on both premia and expectations of short-term rates. In fact, the premia using the traditional methods are higher, as we can see in graph I.

Graph I - Average Term Premium

This Graph shows the mean of the term premium ($P_t^m$) calculated using both traditional and survey methods. It shows the mean term premium for maturities ranging from 1 to 12 months. It also shows the confidence intervals using t statistics calculated with 8-lags Newey-West standard errors. Term Premia are on an annualized basis.
Graph I also suggests that term premium is a positive linear function of time to maturity. In fact, linear regressions of term premium and time to maturity on a daily basis show that nearly 80% of the linear coefficients are positive and statistically significant at 5%, for the survey method (see Table II). This is further evidence in favor of a monotonically increasing ex-ante term premium. However, when using the traditional method the results are not so strong, with only half of the days having a term premia curve with a positive statistically significant slope.

<table>
<thead>
<tr>
<th>Method</th>
<th>Percentage of Days with Positive Slope</th>
<th>Percentage of Days with Negative Slope</th>
<th>Percentage of Days with Statistically Significant Positive Slope</th>
<th>Percentage of Days with Statistically Significant Negative Slope</th>
<th>Percentage of Days with Slope statistically equal to zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>53.9%</td>
<td>46.1%</td>
<td>50.9%</td>
<td>44.3%</td>
<td>4.8%</td>
</tr>
<tr>
<td>Survey</td>
<td>79.9%</td>
<td>20.1%</td>
<td>64.9%</td>
<td>12.6%</td>
<td>22.5%</td>
</tr>
</tbody>
</table>

This table shows results of linear regressions of the Term Premium \( p_t^m \) against time to maturity \( m \), measure in months. We run the following regression for each day \( t \) of the sample: \( p_t^m = \alpha_t + \beta_t m \). Each regression has 12 data points. This is done for both traditional and survey methods. The coefficient \( \beta_t \) measures the slope of the premium term structure. Results show the percentage of the days that have positive and negative \( \beta \), percentage of the days with statistically significant, at 5%, positive and negative \( \beta \), and percentage of the days with \( \beta \) statistically equal to zero, also at 5%. Time to maturity \( m \) ranges from one to 12 months.

We now analyze the behavior of the term premia over time. Graph 2 shows the variation in the term premium over the sample period, using the survey method. We can see some spikes, e.g. the 2008 crisis. Graph 2 suggests that the term premium measure shown here should be treated with some smoothing filter.
6. Conditioning Term Premium to Spot Term Structure Slope

One empirical issue reported by Fama (1986) is that the slope of the term structure may influence the slope of the term premia structure. In his findings, monotonicity of the term premium disappears when the term structure is inverted, which is evidence against the LPH. Boudoukh et al (1999) revisited Fama’s (1986) findings using various econometric techniques, but found no evidence against LPH.

In order to investigate this issue, we fit linear regressions of bond yield ($y$) and time to maturity ($m$), for each day $t$. The results in Table III show that downward slopes are very frequent in the sample. The number of days with a statistically significant negative slope is approximately the same as those with a statistically significant positive slope.

Recall that, if LPH holds, the term premium should monotonically increase with time to maturity, no matter the slope of the yield curve. Therefore an important robustness test is to check if the results in Table III remain the same even if we consider only the days with a downward sloping spot term structure. We have to re-calculate the results in Table II conditional on negative slope of the yield curve, i.e., we consider only
the 45.1% of the days from Table III. The results can be found in Table IV. We can see they are qualitatively the same as in Table II. Therefore, we may say there is no evidence of bias in our results from the slope of the spot yield curve.

<table>
<thead>
<tr>
<th>Table III – Slope of the Spot Term Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percentage of Days with</td>
</tr>
<tr>
<td>Positive Slope</td>
</tr>
<tr>
<td>Negative Slope</td>
</tr>
<tr>
<td>Statistically Significant Positive Slope</td>
</tr>
<tr>
<td>Statistically Significant Negative Slope</td>
</tr>
<tr>
<td>Slope statistically equal to zero</td>
</tr>
</tbody>
</table>

*This table shows results of linear regressions of the yield (\(y_t^m\)) against time to maturity \(m\), measured in months. We run the following regression for each day \(t\) of the sample: \(y_t^m = \alpha_t + \beta_t m\). Each regression has 12 data points. The coefficient \(\beta_t\) measures the slope of the spot term structure. Results show the percentage of days that have positive and negative \(\beta\), percentage of days with statistically significant, at 5%, positive and negative \(\beta\), and percentage of days with \(\beta\) statistically equal to zero, also at 5%. Time to maturity \(m\) ranges from one to 12 months.*

<table>
<thead>
<tr>
<th>Table IV – Slope of the Premium Term Structure conditional on a downward yield curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method</td>
</tr>
<tr>
<td>Positive Slope</td>
</tr>
<tr>
<td>Negative Slope</td>
</tr>
<tr>
<td>Statistically Significant Positive Slope</td>
</tr>
<tr>
<td>Statistically Significant Negative Slope</td>
</tr>
<tr>
<td>Slope statistically equal to zero</td>
</tr>
</tbody>
</table>

*This table shows statistics of the slope of term premium (Table III) conditional on negative slope of the yield curve.*

7. Tests for Monotonicity of Term Premium

In this section, we formally test if term premiums monotonically increase with time to maturity. First, we test if the term premium of month \(m\) is higher than the term premium of month \(m-1\). We may also view this as a test if the difference of the term premium of two consecutive months is greater than zero:

\[
\Delta_t^m = p_t^m - p_t^{m-1} > 0
\]

Where:
\( p_t^m \) is the term premium of month \( m \) at time \( t \)

\( \Delta_t^m \) is difference of the term premium of two consecutive months \( m \) and \( m-1 \) at time \( t \)

Table V shows the descriptive statistics of \( \Delta_t^m \) time series for \( m \) varying from 2 to 12 months during our sample period calculated using traditional and survey methods. Results using the traditional method show very weak evidence in favor of LPH. On the other hand, the results from using the survey method show that in all cases we can reject the null hypothesis that \( \Delta_t^m \) is equal to zero with 5% significance. Therefore, we have very strong evidence in favor of the LPH using the survey method.

Data from Table V shows an average increase of 3.8 basis points for each additional month of maturity using the survey method, and 5.9 basis points using the traditional one.

<table>
<thead>
<tr>
<th># of Months (m)</th>
<th>Traditional Method</th>
<th>Survey Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean t statistics</td>
<td>Mean t statistics</td>
</tr>
<tr>
<td>2</td>
<td>0.047 2.49</td>
<td>0.021 2.35</td>
</tr>
<tr>
<td>3</td>
<td>0.051 1.92</td>
<td>0.026 6.59</td>
</tr>
<tr>
<td>4</td>
<td>0.057 1.44</td>
<td>0.034 7.55</td>
</tr>
<tr>
<td>5</td>
<td>0.057 1.40</td>
<td>0.037 10.26</td>
</tr>
<tr>
<td>6</td>
<td>0.058 1.26</td>
<td>0.038 11.28</td>
</tr>
<tr>
<td>7</td>
<td>0.060 1.19</td>
<td>0.037 12.39</td>
</tr>
<tr>
<td>8</td>
<td>0.060 1.15</td>
<td>0.041 13.94</td>
</tr>
<tr>
<td>9</td>
<td>0.062 1.11</td>
<td>0.044 14.97</td>
</tr>
<tr>
<td>10</td>
<td>0.065 1.08</td>
<td>0.046 16.24</td>
</tr>
<tr>
<td>11</td>
<td>0.065 1.05</td>
<td>0.049 18.04</td>
</tr>
<tr>
<td>12</td>
<td>0.067 1.01</td>
<td>0.050 18.98</td>
</tr>
</tbody>
</table>

This table shows the statistics of the difference between two adjacent months term premia \( \Delta_t^m = p_t^m - p_t^{m-1} \) calculated by both traditional and survey methods. It shows the mean term premium difference for \( m \) ranging from 2 to 12 months. It also shows the t statistics calculated with 8-lags Newey-West standard errors for the null hypothesis that the term premium difference is equal to zero. Premia are on an annualized basis.
The above procedure tests monotonicity independently for each pair of adjacent months. We could also jointly test if premia monotonically increase, i.e. $\Delta_t^m > 0$ for all $m$’s together.

In order to evaluate monotonicity we follow two procedures of testing for inequality constraints. The first one is presented by Wolak (1989). The null hypothesis is that all the estimated $\Delta$ are jointly greater than zero:

$$H_0: \Delta \geq 0$$

$$\hat{\Delta} = \Delta + \nu, \nu \text{ is a (N x 1) vector that is N}(0, \Omega)$$

And $\Omega$ is of full rank $N$ and known. Wolak built the likelihood ratio statistic for this problem as the optimal value of the objective function from:

$$\min_{\Delta} (\hat{\Delta} - \Delta)' \Omega^{-1} (\hat{\Delta} - \Delta)$$

Wolak defined $\hat{\Delta}$ as the solution of the likelihood ratio statistic problem and $IU$ as the shorthand for the null hypothesis of inequality constraints versus an unrestricted alternative:

$$IU = (\hat{\Delta} - \Delta)' \Omega^{-1} (\hat{\Delta} - \Delta)$$

The distribution of the likelihood ratio statistic satisfies:

$$\sup_{\Delta \geq 0} Pr_{\Delta, \Omega}[IU \geq c] = \sum_{k=0}^{N} Pr[\chi^2_k \geq c] w(N, N - k, \Omega)$$

Where:

$N$ is the number of restrictions.

The weight $w(N, N - k, \Omega)$ is the probability that $\hat{\Delta}$ has exactly $N-k$ positive elements. This weight is calculated by Monte Carlo simulation. The covariance matrix of the estimated parameters $\Omega$ may be calculated by the Newey-West (1987) procedure in order to correct to heteroskedasticity and autocorrelation.

The other procedure we use to test for inequality constraints is presented by Patton and Timmermann (2010). The authors defined the monotonic relation as MR and the null hypothesis in a different way than that presented above. The alternative hypothesis is that one the researcher wants to prove:

$$H_0: \Delta \leq 0$$

$$H_1 = \min_{i=1,\ldots,N} \Delta_i > 0$$
The statistic to be tested is:

\[ J_T = \min_{i=1,\ldots,N} \hat{\Delta}_i \]

The estimated parameter vector \( \hat{\Delta} \) asymptotically follows a normal distribution \( N(0, \Omega) \). Therefore the full set of the covariance matrix for the sample moments is necessary. The objective is to achieve the minimum value of a multivariate vector of estimated parameters and there are no tabulated critical values for such minimum values. Patton and Timmermann applied the stationary bootstrap procedure (Politis and Romano 1994) in the returns and a Monte Carlo simulation to implement the MR test.

It is worth noting that Wolak and Patton-Timmermann tests have inverted null hypothesis. While in Wolak the null hypothesis is that LPH holds, in the Patton-Timmermann approach the null hypothesis is that LPH does not hold.

Patton and Timmermann (2010) also proposed a test considering all pairs of maturity, and not only adjacent months. In this way the deltas would be calculated for all combinations of the 12 maturities that we have in our case.

We have performed these joint tests on our sample, and the results are in Table VI. The results obtained using the survey method strongly support LPH as the p-value for the Wolak test is very high and its null hypothesis is that LPH holds, and p-values for the Patton-Timmermann tests are under 1%, and its null hypothesis is that LPH does not hold. For the traditional method, the Wolak test result supports LPH, and for the Patton-Timmermann tests, the p-values of 4% and 3.5% provide some mild evidence in favor of LPH.

### Table VI – Joint Tests of Term Premium Differences

<table>
<thead>
<tr>
<th>Method</th>
<th>Wolak</th>
<th>Patton-Timmermann (adjacent maturities)</th>
<th>Patton-Timmermann (all pairs of maturities)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Survey</td>
<td>0.723</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>Traditional</td>
<td>0.248</td>
<td>0.040</td>
<td>0.035</td>
</tr>
</tbody>
</table>

This table shows the p-value for three joint tests of monotonicity of term premia. The Wolak test has a null hypothesis that the term premia difference are jointly greater than zero. The Patton-Timmermann test has a null hypothesis that the term premia differences are jointly lower than zero. The Patton-Timmermann has two versions: one considering only adjacent maturities, and another considering all combinations of the 12 maturities, two by two.
8. Final Remarks

This paper uses a novel approach to test the Liquidity Preference Hypothesis for the term structure of interest rates. Instead of using bond returns as the traditional approaches in the literature, we use interest rate surveys with market expectations in order to evaluate LPH. This approach allows us to split the effect of interest rate expectation movements (expectation hypothesis) from the liquidity premium in a clearer procedure. Thus, it mitigates problems from variations in the short-term interest rates expectations because we are accounting for expectations in an explicit way. As a result, we are able to extract the term premium from bond yields by using expectations of short-term interest rates over bond life.

We found empirical support for the LPH working with Brazilian interest rate data with both traditional and survey methods. Our analysis with survey information strongly supports the idea that term premia monotonically increase with time to maturity. The term premium increases between two and four basis points for each additional month of maturity. These pairwise increases are statistically greater than zero, no matter how we test it, separately or jointly. The traditional method produces much higher term premia, but also much wider standard errors. This suggests that the survey method is more precise. Furthermore, although there is some statistical evidence supporting LPH using the traditional method, the statistical evidence is much weaker than that found using the survey method.

The survey approach used in this paper may be used to evaluate the market sentiment regarding future steps in momentary policy. It also may be used to estimate the premium required by the market on Treasury bonds and bill auctions. In both cases, the levels of risk, uncertainty and disagreement for the future steps of the monetary and economic policies should affect the term premium.
9. References


