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João Barata R. B. Barroso

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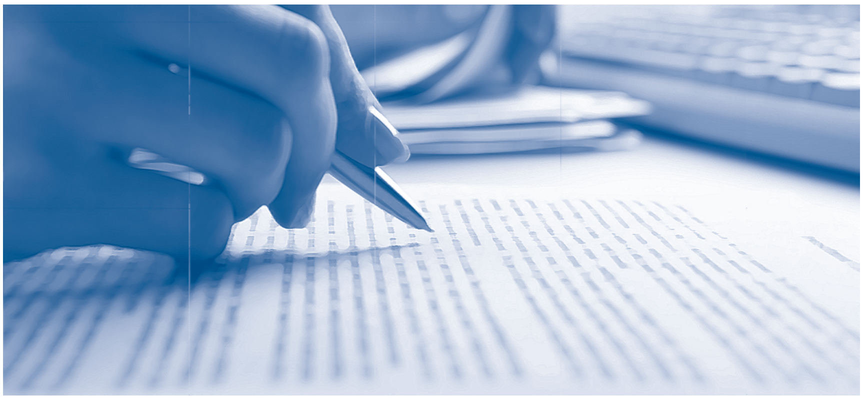

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Does trade shrink the measure of domestic firms?[‡]

João Barata R. B. Barroso[*]

Abstract

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Does international trade shrink the steady state measure of domestic firms? The most recent models with heterogeneous firms suggest it does (Melitz (2003), Chaney (2007) and Arkolakis (2008)). The main force at work in such models is the selection of the fittest, with the least efficient firms exiting the market. Within the same class of models with heterogeneous firm productivity and strong selection effects, both in the consumption goods and the intermediate goods sectors, this paper shows that the measure of domestic firms may actually expand. The result is robust to the particular production function used to bundle labor and intermediate goods.

Keywords: trade; selection effect; intermediate goods; measure of firms.

JEL Classification: D50, D60, F12, F13, F16

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1. Introduction

Does international trade shrink the steady state measure of domestic firms? The most recent models with heterogeneous firms suggest it does (Melitz (2003), Chaney (2007) and Arkolakis (2008)). The main force at work in such models is the selection of the fittest, with the least efficient firms exiting the market[1]. As it appears, in spite of positive productivity and welfare effects, exit is strong enough to shrink the measure of domestic firms after liberalization, and, in some cases, even the sum of domestic and foreign firms which export to the domestic economy (Baldwin and Forslid (2004), Arkolakis et.al (2008)).

Within the same class of models with heterogeneous firm productivity and strong selection effect, both in the consumption goods and the intermediate goods sectors, this paper shows that the measure of domestic firms may actually expand. Our result follows from endogenous improvements in labor productivity which increases the size of the market and, therefore, the measure of varieties that the equilibrium may support. The labor productivity effect results from the selection effect in the intermediate goods sector.

There are not many papers addressing the possibility of an increasing measure of domestic firms in environments with strong selection effects from trade. Bache (2012) considers a two sector model with capital accumulation where the measure of domestic varieties of intermediate goods (i.e. the measure of firms producing them) may increase in the free trade steady state in spite of selection effects. However, unlike our model, the consumption sector in Bache (2012) is actually homogeneous and competitive, and, therefore, with no selection effect. Other papers with pro-variety effects (for example, Melitz and Otaviano (2008)) focus only on the sum of the measures of domestic and imported varieties, with perverse effects on the domestic varieties still occurring on the free trade steady state.

There is a large literature of endogenous growth driven by increasing varieties of intermediate goods (Grossmann and Helpman, 1990). However, this literature does not have heterogeneous firm productivity, which means it does not explore the selection effect from trade. It also does not consider varieties of final goods consumption,

[1] We may quote Arkolakis et.al (2008), according to which “it seems reasonable to think that an increase in import competition would cause a decline in domestic variety as domestic firms exit”.

focusing on intermediate goods varieties. As a result, this literature is not able to address the problems highlighted here.

Regarding the plausibility of productivity gains derived from imported goods, which is the theoretical channel explored in this paper, we refer to the positive empirical evidence of Kasahara and Rodrigue (2008) and Amiti and Konings (2007). Also related, the partial equilibrium effect of intermediate goods variety on the domestic goods variety has been found to be significant by Goldberg et.al. (2008). Overall, there are sufficient empirical grounds to motivate the construction of counterexamples to the anti-variety effects emphasizing intermediate goods as the key channel of productivity gains. Considering the selection effect, there is robust empirical evidence supporting it (Pavcnik, 2002; Bernard et al, 2006). However, the associated anti-variety effect has not been conclusively supported by empirical evidence. For example, Lewrick et.al. (2011) and Ardelea and Lugovskyy (2009) provide evidence of decreasing domestic varieties with more trade openness. In contrast, the empirical results of Goldberg et.al. (2008) and Brambilla (2006) point to the expansion of domestic varieties.

The main contribution of the paper is to provide a simple counterexample to a significant theoretical result according to which trade would have strong anti-variety effects in the domestic consumption goods sector, therefore shrinking the measure of domestic firms. The counterexample is obtained with minimal departures from the standard assumptions of the well known Melitz (2003) model.

2. Single sector model

We briefly review Melitz (2003). Consider two identical economies without labor mobility. Consumers supply labor inelastically at the aggregate level L, and labor is the numeraire good. There is a continuum of goods with a Dixit-Stiglitz demand system with elasticity of substitution $\sigma > 1$. Each good is produced by a single firm, with marginal costs $1/\varphi$ and fixed costs f. Firms sunk an entry cost f^e without information on their productivity φ, which is has distribution $\mu(\varphi)$. Each period, firms face an exogenous probability δ of exiting the market and evaluate future streams without discounting. Export activity involves an additional fixed cost f^*, and is subject to an iceberg transportation cost $\tau > 1$. The measure of domestic firms M^d is endogenous.

Firms with draws above some minimum cutoff productivity ϕ enter the market. Given the higher fixed costs of the export activity, only firms with productivities at least above an even higher cutoff $\phi^* > \phi$ become exporters, and it can be shown that marginal entrants productivity in the domestic and export market have a one to one relation $\phi^* = \phi^*(\phi)$.

Free entry drives the ex-ante expected profit stream to the sunk cost of entry, that is $\bar{\pi} = \delta f^e / \mu(\varphi > \phi)$, an equation often referred to as the free entry condition. Since the profit of the marginal entrant is zero, the average profit must respect the so called zero cut off profit condition $\bar{\pi} = k(\phi)f + \mu_s(\varphi > \phi^*(\phi))k(\phi^*(\phi))f^*$ where $k(\phi) = \left(\left(\tilde{\varphi}_\phi/\phi\right)^{\sigma-1} - 1\right)$, and $\tilde{\varphi}_\phi = \left(E_{\varphi>\phi}\varphi^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$. Intersection of both conditions defines the steady state average profit and cutoff productivity $(\bar{\pi}^\circ, \phi^\circ)$. Trade results in higher steady state average profit and cutoff productivity, i.e. the selection effect from trade. Both variables are determined independently of the measure of domestic firms. The equilibrium measure follows from economy wide resource and output constraints. Trade shrinks the measure of firms.

3. Intermediate goods extension

The economy consists of a final goods sector and an intermediate goods sector both formally identical to the sector described in the previous section[2]. We subscript all the variables and sector specific parameters with an index of the sector $s = x, y$ with x for final goods and y for intermediate goods. The steady state average profit and cutoff productivity for each sectors are well defined as before $(\bar{\pi}_s^\circ, \phi_s^\circ)_{s=x,y}$. The comparative statics of free trade for these variables is the same as before. We explore the additional structure to determine the measure of firms.

Production

Costs are expressed in terms of the numeraire, which is defined as the minimum cost to obtain a *composite factor* of production. This composite factor is assembled by firms in a preliminary production stage from labor and intermediate goods varieties. The

[2] Of course, demand system represents preferences in the case of consumption goods, while it represents technology in the case of intermediate goods.

technology at this stage has constant returns to scale, so we may think of a sector producing the aggregate amount of the composite factor. The total output of the composite factor is $Z = Z(L, Q_y)$, where L is the aggregate labor inelastically supplied and Q_y is the aggregate quantity index of intermediate goods. Given the price of labor w_l and of intermediate goods P_y, the minimum cost to assemble and therefore the price to obtain the composite factor is obtained. All the fixed and marginal costs introduced in the previous section are now measured in this numeraire.

Cost minimization equalizes marginal productivities to the real factor prices: $\partial_L Z(L, Q_y) = w$ and $\partial_{Q_y} Z(L, Q_y) = P_y$. Moreover, by Euler theorem, factor payments exhaust the value of the output $wL + P_y Q_y = Z$. We refer to *factor price relations* when referencing the equations in this paragraph.

Steady state

Integrating the profit functions for each sector and adding both sectors, we conclude that the aggregate composite factor used in production is $Z^p = \mathrm{R} - \Pi$, that is, total revenue minus total profit. Steady state requires successful entrants in each sector to equal the measure of exiting firms $\mu(\varphi_s > \phi_s) M_s^e = \delta M_s^d$. Therefore, composite factor used up in the entry process is $Z_s^e = M_s^e f_s^e = M_s^d \delta f_s^e / \mu(\varphi_s > \phi_s) = M_s^d \bar{\pi}_s = \Pi_s$, and adding across sectors $Z^e = \Pi$. Therefore, in the steady state, the total composite factor $Z = Z^p + Z^e$ is such that,

(1) $$Z = R = R_x + R_y = P_x Q_x + P_y Q_y$$

Using factor price relations and the average revenue in the sector, we obtain

(2) $$wL = R_x = M_x^d \bar{r}_x.$$

This is a central equation, because it establishes the relation between real wage w, average revenue in the consumption sector $\bar{r}_x$ and domestic varieties of consumption goods M_x^d. It is important for comparative statics, since the change in average revenue in the consumption sector relative to the change in real wage will determine the effect on the measure of domestic consumption varieties.

Average revenue is already determined by the Melitz system, and is in fact a function of average profit by $\bar{r}_s = \sigma_s(\bar{\pi}_s^d + f_s) + \mu(\varphi_s > \phi_s^*)\sigma_s(\bar{\pi}_s^* + f_s^*)$. Using the factor price relations we can rewrite (2) as the *labor constraint*:

(3) $$M_x^d \bar{r}_x = \partial_L Z(L, Q_y) L$$

Similarly, using the factor price relations in the identity $R_y = P_y Q_y$ we obtain the following constraints for intermediate goods[3]:

(4) $$M_y r_y^d(\tilde{\varphi}_y^t) = M_y^d \bar{r}_y = \partial_{Q_y} Z(L, Q_y) Q_y$$

We can solve the aggregate price equation $P_y = (M_y)^{\frac{1}{1-\sigma_s}} p_y^d(\tilde{\varphi}_y^t)$ to the measure M_y. Substituting the result into (4), and using $P_y = \partial_{Q_y} Z(L, Q_y)$, we obtain

(5) $$(\rho_y \tilde{\varphi}_y^t)^{1-\sigma_y} r_y^d(\tilde{\varphi}_s^t) = \partial_{Q_y} Z(L, Q_y)^{\sigma_y} Q_y$$

From zero profit for the marginal entrant ϕ_y we have $r_y^d(\tilde{\varphi}_y^t) = (\tilde{\varphi}_y^t/\phi_y)^{\sigma_y - 1}\sigma_y f_y$. Substituting into (5) we obtain the *intermediate goods constraint:*

(6) $$({\rho_y}^{1-\sigma_y}\sigma_y f_y)(\phi_y)^{1-\sigma_y} = \partial_{Q_y} Z(L, Q_y)^{\sigma_y} Q_y$$

Since equilibrium ϕ_y° is determined by the Melitz system, the intermediate goods constraint determines equilibrium aggregate intermediate goods Q_y°. All other endogenous variables are thereafter determined. In particular, real wages follow from factor price relations and steady state measure of domestic firms follows from the labor constrain. Finally, prices and wages determine per capita welfare, $W = w_l/P_x$.

[3] Define the trade adjusted productivity $\tilde{\varphi}^t = \left[M^d/M(\tilde{\varphi}_\phi)^{\sigma-1} + M^*/M\left(\frac{\tilde{\varphi}_{\phi^*}}{\tau}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}$ where $M = M^d + M^* = M^d + \mu(\varphi > \phi^*)M^d$ is the measure of goods in the domestic market including imported ones.

4. A simple counter-example

The properties of the system just defined depend on the properties of the composite factor production function and the productivity distributions. In this section, we consider simple assumptions to make our point.

Assumption 1. The composite factor production function is Cobb-Douglas, that is $Z = L^a Q^{1-a}$, with $a\sigma_y > 1 > a > 0$

Assumption 2. The productivity densities are Pareto, $\mu_s(\varphi) = \begin{cases} \theta_s/\varphi^{1+\theta_s} & \varphi \geq 1 \\ 0 & \varphi < 1 \end{cases}$, with $\theta_s > \sigma_s - 1$ for each $s = x, y$.

Using the Cobb-Douglas assumption in the intermediate goods constraint (6), we obtain the following equation:

$$Q_y = g_y \phi_y^{\frac{\sigma_y - 1}{a\sigma_y - 1}} L^{\frac{a\sigma_y}{a\sigma_y - 1}} \tag{7}$$

where the factor g_y depends only on structural parameters[4]. Since the cutoff productivity is determined in the Melitz system and since intermediate goods determine all the other variables, we have a complete solution for the model. In particular, intermediate goods increase after trade, in response to a higher productivity cutoff, increasing real wages along the way. Since average revenue also increases, it is not clear what the net effect on the measure of firms is. We construct particular examples.

Consider the log difference between the trade and no trade steady states of the labor constraint (3). Using the equilibrium intermediate goods, this is

$$\Delta ln M_x^d + \Delta ln \bar{r}_x = (1 - a) \frac{\sigma_y - 1}{a\sigma_y - 1} \Delta ln \phi_y \tag{8}$$

The log difference in average revenue and productivity cutoffs is determined in the Melitz system. We may use the Pareto assumption to obtain closed form solutions. After some fairly standard algebra, we may rewrite (8) as

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[4] $g_y = \left[(1-a)/\left({\rho_y}^{1-\sigma_y} \sigma_y f_y\right)\right]^{\frac{1}{a\sigma_y - 1}}$

(9) $$\Delta ln M_x^d + \ln\left(1 + {\phi_{x,open}^*}^{-\theta_x}\frac{f_x^*}{f_x}\right) = (1-a)\frac{\sigma_y - 1}{a\sigma_y - 1}\frac{1}{\theta_y}\ln\left(1 + {\phi_{y,open}^*}^{-\theta_y}\frac{f_y^*}{f_y}\right)$$

The important point to observe is that the second term on the left hand side depends only on the parameters of the consumption goods sector. Indeed,

(10) $${\phi_{s,open}^*}^{-\theta} = \tau^\theta \left(\frac{f_s^*}{f_s}\right)^{\frac{\theta_s}{\sigma_s - 1}} \left(\frac{1}{\delta_s f_s^e} k_s f_s + Re_s\right)^{-\theta_s}$$

where $k_s = \theta_s/(\theta_s - (\sigma_s - 1)) - 1$ and $Re_s = \frac{1}{2\delta_s f_s^e}\left(((k_s f_s)^2 + k_s d_s)^{1/2} - k_s f_s\right)$[5]. Therefore, we can make the second term on the left hand side arbitrarily small, for instance, when θ_x is sufficiently close to $\sigma_x - 1$. Since the right hand side is positive and constant for any such θ_x, we have just established that the measure of firms in the consumption goods sector may increase. We summarize this result as follows:

Proposition 1. The measure of domestic firms in the consumption goods sector may increase after trade. In particular, this is the case when θ_x is sufficiently close to $\sigma_x - 1$.

Consider now the log difference between the trade and no trade steady states of the intermediate goods constraint (4). By the same algebra as in the labor resource constraint, we obtain:

(11) $$\Delta ln M_y^d + \ln\left(1 + {\phi_{y,open}^*}^{-\theta_y}\frac{f_y^*}{f_y}\right) = \frac{\sigma_y - 1}{a\sigma_y - 1}\frac{1}{\theta_y}\ln\left(1 + {\phi_{y,open}^*}^{-\theta_y}\frac{f_y^*}{f_y}\right)$$

Notice that the second term on the left hand side, which is positive, also appears in the right hand side. The following proposition is immediate:

Proposition 2. The measure of domestic firms in the intermediate goods sector increases or decreases after trade when $\frac{\sigma_y - 1}{a\sigma_y - 1}\frac{1}{\theta_y}$ is greater or lower than 1, respectively.

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[5] $d_s = 4\delta_s f_s^e \tau^\theta (f_s^*/f_s)^{\frac{\theta_s}{\sigma_s - 1}} f_s^*$

5. A simple generalization

In this section we relax the Cobb-Douglas assumption as follows:

Assumption 1*. The composite factor production function Z, which has constant returns to scale, also satisfies:

(i) Z is C^2 with $\partial_L Z > 0$, $\partial_Q Z > 0$, $\partial_{QQ} Z < 0$, $\partial_{LQ} Z > 0$

(ii) $\lim_{q\to 0} \partial_Q Z(L,Q)^{\sigma} Q = \infty$ and $\lim_{q\to\infty} \partial_Q Z(L,Q)^{\sigma} Q = 0$

(iii) $\sigma_y |\varepsilon| > 1 > |\varepsilon|$ for all Q > 0, where $\varepsilon = \frac{\partial_{QQ} Z(L,Q) Q}{\partial_Q Z(L,Q)}$

It is clear the Cobb-Douglas function satisfies the new set of assumptions. Since we cannot solve the model explicitly, existence is established by the intermediate value theorem, and the elasticity condition ensures the steady state is unique. The same elasticity condition also guarantees that intermediate goods increase after trade. Finally, the algebra of the comparative statics exercise is exactly the same, but with $|\varepsilon|$ in place of the Cobb-Douglas parameter a. We summarize these results in the following proposition, with further details in the appendix.

Proposition 3. There is one and only one steady state equilibrium which has a positive level of intermediate goods, such that trade increases this level. Moreover:

(i) The measure of domestic firms in the consumption goods sector may increase after trade. In particular, this is the case when θ_x is sufficiently close to $\sigma_x - 1$.

(ii) The measure of domestic firms in the intermediate goods sector may increases or decreases when $|\varepsilon| \frac{\sigma_y - 1}{|\varepsilon|\sigma_y - 1} \frac{1}{\theta_y}$ is greater or lower than 1.

6. Welfare analysis

Per capita welfare is simply the aggregate quantity index on final goods divided by the labor force. Using the labor constraint and the inverse of aggregate prices, this is

$$W = \frac{R_x/L}{P_x} = w_l M_x^{\frac{1}{\sigma_x - 1}} \rho_x \tilde{\varphi}_x^t \tag{12}$$

Using the definition of average revenue and the zero profit condition for marginal entrants, we can see that

$$\left(\frac{\tilde{\varphi}_x^t}{\phi_x}\right)^{\sigma_s - 1} = \frac{r_x^d(\tilde{\varphi}_x^t)}{r_x^d(\phi_x)} = \frac{R_x/M_x}{f_x}$$

Solving for $\tilde{\varphi}_x^t$ and substituting in (12),

$$W = w_l \rho_x \left(\frac{R_x}{f_x}\right)^{\frac{1}{\sigma_x - 1}} \phi_x = w_l \rho_x \left(\frac{M_x^d \bar{r}_x}{f_x}\right)^{\frac{1}{\sigma_s - 1}} \phi_x$$

Consider the log difference of the trade and no trade steady states of this expression. The selection effect means the contribution from $\Delta ln\phi_x$ is positive. We have established before that the wage effect Δlnw_l and the payroll effect $\frac{1}{\sigma_x - 1}\Delta lnR_x$ are positive. We can decompose the payroll effect term ΔlnR_x into a final good average expenditure effect $\ln\Delta\bar{r}_x$ and a final good measure of firms effect ($\Delta\ln M_x^d$), with positive effects from expenditure and possibly positive effects from the measure of firms according to Proposition 1 and Proposition 3.i. In this decomposition, only the selection effect is already in the Melitz (2003).

Proposition 4. Trade liberalization has a selection effect ($\Delta ln\phi_x > 0$), a real wage effect ($\Delta lnw_l > 0$) and a payroll effect ($\Delta lnR_x > 0$), which may be further decomposed into an final good average expenditure effect ($\Delta ln\bar{r}_x > 0$) and a measure of firms effect (ΔlnM_x^d), with possibly positive welfare consequences resulting from the measure of firms.

7. Conclusion

This paper provides a counterexample to the frequent intuitive argument and apparently robust theoretical result according to which trade would shrink the steady state measure of domestic firms in the consumption goods sector as a result of strong selection effects. We also provide examples where the steady state measure of firms in the intermediate goods sector also expands. Both results are robust to the particular production function used to bundle labor and intermediate goods. In summary, in our model, selection effects in the intermediate goods sector increase labor productivity and associated real wages, therefore increasing the size of the market and the measure of firms the economy may support.

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Appendix A

In this appendix we provide more details on the propositions stated in the text.

Proposition 1

The only difficulty here is the algebra. We start from equation (8). For a closed economy, dropping the sector subscripts,

$$\bar{r}_{closed} = E_{\varphi>\phi} r(\varphi) = E_{\varphi>\phi}\left(\frac{\varphi}{\phi}\right)^{\sigma-1} r(\phi) = r(\phi)\left(\frac{1}{\phi}\right)^{\sigma-1} E_{\varphi>\phi}\varphi^{\sigma-1}$$
$$= r(\phi)\left(\frac{\tilde{\varphi}_\phi}{\phi}\right)^{\sigma-1} = \sigma f\left(\frac{\tilde{\varphi}_\phi}{\phi}\right)^{\sigma-1} = \sigma f\frac{\theta}{\theta-(\sigma-1)}$$

where the last equality uses the Pareto assumption, since

$$\left(\tilde{\varphi}_\phi\right)^{\sigma-1} = E_{\varphi>\phi}\varphi^{\sigma-1} = \int_\phi^\infty \varphi^{\sigma-1}\left(\theta\phi^\theta/\varphi^{1+\theta}\right)d\varphi = \frac{\theta}{\theta-(\sigma-1)}\phi^{(\sigma-1)}$$

For an open economy,

$$\bar{r}_{open} = r^d\left(\tilde{\varphi}_\phi\right) + \mu(\varphi > \phi^*)r^*\left(\tilde{\varphi}_{\phi^*}\right) = \frac{\theta}{\theta-(\sigma-1)}\sigma f + \phi^{*-\theta}\frac{\theta}{\theta-(\sigma-1)}\sigma f^*$$
$$= \bar{r}_{closed}\left(1+\phi^{*-\theta}\frac{f^*}{f}\right)$$

Therefore,

$$ln\left(\bar{r}_{open}\right) - ln\left(\bar{r}_{closed}\right) = \ln\left(1+{\phi^*_{open}}^{-\theta}\frac{f^*}{f}\right)$$

All that remains is calculating the cutoffs. Substituting the average revenues in the zero cutoff profit conditions and then back into the free entry condition, we get, for the closed economy

$$\phi^\theta_{closed} = \frac{1}{\delta f^e}k_\theta f,$$

and for the open economy,

$$\phi^\theta_{open} = \frac{1}{\delta f^e}k_\theta f + {\phi^*_{open}}^{-\theta}\frac{1}{\delta f^e}k_\theta f^* = \phi^\theta_{closed}\left(1+\frac{f^*}{f}{\phi^*_{open}}^{-\theta}\right)$$

Therefore,

$$ln(\phi_{open}) - ln(\phi_{closed}) = \frac{1}{\theta}\frac{f^*}{f}\phi_{open}^{*}{}^{-\theta}$$

Finally, we can substitute ϕ_{open}^{*}=$\phi_{open}\tau\left(\frac{f^*}{f}\right)^{\frac{1}{\sigma-1}}$ in the open economy equation and solve a quadratic equation, from which

$$\phi_{open}^{*}{}^{-\theta} = \tau^{\theta}\left(\frac{f^*}{f}\right)^{\frac{\theta}{\sigma-1}}\left(\frac{1}{\delta f^{e}}kf + Re\right)^{-\theta}$$

Proposition 2

The algebra is the same as in the previous proposition.

Proposition 3

First consider existence. Let $f(Q) = \partial_Q Z(L,Q)^{\sigma}Q$, and let h denote the left hand side of equation (6) with ϕ_y determined in the Melitz system. We must show that the equation $f(Q) = h$ has a positive solution. From the limit at zero condition, there is a $Q_a > 0$ such that $f(Q_a) > h > 0$. Similarly, from the limit at infinity condition, there is a $Q_b > Q_a$ such that $f(Q_b) < h$. Continuity for f follows from continuity assumption for Z. From the intermediate value theorem, there is $Q_y \in [Q_a, Q_b]$, with $f(Q_y) = h$. For uniqueness, note that $f' = \sigma_y \partial_Q Z(L,Q)^{\sigma_y - 1}\partial_{QQ}(L,Q)Q + \partial_Q Z(L,Q)^{\sigma_y} = \partial_Q Z(L,Q)^{\sigma_y}(1 - \sigma_y|\varepsilon|) < 0$. Therefore, Q_y is the only value that solves the equation. Notice also that $df/dL = \sigma_y \partial_Q Z(l,q)^{\sigma_y - 1}Z_{yl}(l,q)q$>0, so that an increase in L shifts up the f schedule and requires an increase in Q_y to restore the equilibrium.

Now consider the comparative statics for the measure of firms. We rewrite equation (8) as

$$\Delta ln M_x^d + \Delta ln \bar{r}_x = \Delta ln \partial_L Z(L, Q_y)$$

Along equation (6), which solely determines the relation between Q_y and ϕ_y, we have

$$\frac{dlnQ_y}{dln\phi_y} = \frac{\sigma_y - 1}{|\varepsilon|\sigma_y - 1} > 1$$

Therefore, the right hand side of equation (8) is positive. Since the second term on the left hand side can be varied independently, we establish (i), as in the Cobb-Douglas case. The analogous equation (11) reads

$$\Delta lnM_y^d + \Delta ln\bar{r}_y = \Delta ln\partial_Q Z(L, Q_y)$$

Taking Taylor approximations, we have

$$\Delta lnZ_Q(L, Q_y) = |\varepsilon| \frac{\sigma_y - 1}{|\varepsilon|\sigma_y - 1} \Delta ln\phi_y + Re$$

In this case, (8) becomes

$$\Delta lnM_x^d + \ln\left(1 + {\phi_{y,open}^*}^{-\theta_y} \frac{f_y^*}{f_y}\right) = |\varepsilon| \frac{\sigma_y - 1}{\sigma_y|\varepsilon| - 1} \frac{1}{\theta_y} \ln\left(1 + {\phi_{y,open}^*}^{-\theta_y} \frac{f_y^*}{f_y}\right) + Re$$

Since the remainder term is bounded, if the difference $|\varepsilon| \frac{\sigma_y - 1}{\sigma_y|\varepsilon| - 1} \frac{1}{\theta_y} - 1$ is sufficiently large in absolute value it will eventually dominate, and therefore the possibility result.

Proposition 4

This follows directly from the comparative statics of the Melitz system and from the results of the previous propositions.

Appendix B

For convenience, we summarize in somewhat greater detail the algebra and logic of single sector Melitz (2003) model with the unfamiliar reader in mind.

Demand

Let Q and P denote quantity and price indexes over bundles of goods $\{q(v)\}_{v\in\Omega}$. Assuming Dixit-Stiglitz preferences, with elasticity of substitution $\sigma = 1/(1-\rho) > 1$, the indexes are $Q = \left(\int_{v\in\Omega} q(v)^{\rho} dv\right)^{\frac{1}{\rho}}$ and $P = \left(\int_{v\in\Omega} p(v)^{1-\sigma} dv\right)^{\frac{1}{1-\sigma}}$. The required expenditure to obtain Q unites of consumption is $R = PQ$. The expenditure in a particular variety is $r(v) = p(v)q(v)$. From the cost minimization problem defining the price index we obtain demand and expenditure functions: $q(v) = Q(p(v)/P)^{-\sigma}$ and $r(v) = R(p(v)/P)^{1-\sigma}$, respectively.

Production

Costs are expressed in terms of the numeraire, which is labor in this section. Consumers supply labor inelastically at the aggregate level L. Each variety is produced by a single firm. The firm has marginal costs $1/\varphi$ and fixed costs f, so the total cost to obtain q units of its variety is $z(\varphi) = q(\varphi)/\varphi + f = r(\varphi)/(\varphi p(\varphi)) + f$, where the second equation used the demand system from the previous paragraph.

The firm may sell to domestic or foreign consumers. Export activity involves an additional fixed cost f^*. It is also subject to an iceberg transportation cost $\tau > 1$, so that the actual marginal cost for goods sold abroad is τ/φ. Let χ be an indicator of export activity, which is constrained at zero in a closed economy. We may decompose production costs as $z = z^d + \chi z^*$ where z^d and z^* are the costs incurred in selling for the domestic and foreign consumers. Firms optimize profits given the demand structure, leading to the pricing relations: $p^d(\varphi) = 1/\rho\varphi$ and $p^*(\varphi) = \tau p^d(\varphi)$. The profit is $\pi(\varphi) = \pi^d(\varphi) + \chi(\varphi)\pi^*(\varphi) = r^d(\varphi) - z^d(\varphi) + \chi(\varphi)[r^*(\varphi) - z^*(\varphi)]$

Entry and Exit

Before starting production firms must sunk an entry cost f^e without information on their productivity φ, which is only known to have distribution $\mu(\varphi)$ on the support

$(0, \infty)$ with a continuous distribution function. Upon entry, firms learn their productivity and decide if profit levels are sufficient to cover the relevant fixed costs of their choices. Since only firms with positive profit would choose positive production levels, only firms with draws above some *minimum cutoff productivity* ϕ would be successful entrants. These firms are able to charge low enough prices to capture a relevant fraction of the sectors' revenue while also saving on production costs. Given the higher fixed costs of the export activity, only firms with productivities at least above an even higher cutoff $\phi^* > \phi$ would choose to become exporters. Since productivity is unbounded, cutoff productivities for entrants and exporters are well defined. In the case of a closed economy we impose $\phi^* = \infty$. Note that $r(\phi) = \sigma f$ and $(\phi^*) = \sigma f^*$.

Each period, incumbent firms face an exogenous probability δ of exiting the market. As a result, the expected profit stream conditional on successful entry is $\sum_t (1-\delta)^t \bar{\pi} = \bar{\pi}/\delta$ where $\bar{\pi} = E_{\varphi > \phi} \pi(\varphi)$ is the average incumbent profit. Moreover, the ex-ante expected profit stream is $\mu_s(\varphi_s > \phi_s)\bar{\pi}_s/\delta$, which is the crucial variable in firm entry decision.

Aggregation

There are two useful notions of average productivity. The first is simple average productivity, defined as $\tilde{\varphi}_\phi = \left(E_{\varphi > \phi} \varphi^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$. With this notation, average incumbent profit is $\bar{\pi} = \pi^d\left(\tilde{\varphi}_\phi\right) + \mu(\varphi > \phi^*)\pi^*\left(\tilde{\varphi}_{\phi^*}\right)$. The second is the trade adjusted average productivity $\tilde{\varphi}^t = \left[M^d/M\left(\tilde{\varphi}_\phi\right)^{\sigma-1} + M^*/M\left(\frac{\tilde{\varphi}_{\phi^*}}{\tau}\right)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}$ where $M = M^d + M^* = M^d + \mu(\varphi > \phi^*)M^d$ is the measure of varieties in the domestic market including imported ones.

There are two equivalent expressions to represent aggregate indexes. Consider the price index. With the trade adjusted average productivity, $P = (M)^{\frac{1}{1-\sigma}} p^d(\tilde{\varphi}^t)$. With the simple average productivity, $P = (M^d)^{\frac{1}{1-\sigma}} \bar{p}$, where $\bar{p} = \left[p^d\left(\tilde{\varphi}_\phi\right)^{1-\sigma} + \mu(\varphi > \phi^*)p^*\left(\tilde{\varphi}_{\phi^*}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$. Analogous representations may be deduced for aggregate quantities Q and revenues R. In particular, $R = Mr^d(\tilde{\varphi}^t) = M^d \bar{r}$, where $\bar{r} = r^d\left(\tilde{\varphi}_\phi\right) + \mu(\varphi > \phi^*)r^*\left(\tilde{\varphi}_{\phi^*}\right)$.

The important point is that marginal entrant productivities ϕ and ϕ^* are the only determinants of all aggregate variables. As a matter of fact, only domestic marginal productivity matter. Exporter marginal productivity is actually a function of the domestic cutoff $\phi^* = \phi^*(\phi)$. Indeed, $(\phi^*/\phi\tau)^{\sigma_s - 1} = r^*(\phi^*)/r^d(\phi) = f^*/f$

Steady State

Free entry drives the ex-ante expected profit to the sunk cost of entry, that is $\bar{\pi} = \delta f^e/\mu(\varphi > \phi)$, the *free entry condition* (FE). Average incumbent profit is given by the zero cutoff profit condition (ZCP) $\bar{\pi} = \pi^d(\tilde{\varphi}_\phi) + \mu_s(\varphi > \phi^*)\pi^*(\tilde{\varphi}_{\phi^*}) = k(\phi)f + \mu_s(\varphi > \phi^*(\phi))k(\phi^*(\phi))f^*$ where $k(\phi) = \left((\tilde{\varphi}_\phi/\phi)^{\sigma-1} - 1\right)$. Intersection of both conditions defines steady state average profit and cutoff productivity $(\bar{\pi}^\circ, \phi^\circ)$. Existence is established in Melitz (2003). Trade liberalization shifts the zero cutoff profit condition up because the second term turns positive for some productivity levels. Note this condition is decreasing in productivity while free entry is increasing. As a result, equilibrium average profit and cutoff productivity are higher than in autarky, which amounts to the selection effect from trade.

An important feature of the Melitz (2003) model is that cutoffs are determined independently of the measure of firms (varieties). To obtain the equilibrium measure we need economy wide resource constraints and aggregate factor payments. We continue the steady state analysis along this line in the context of our extensions.

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