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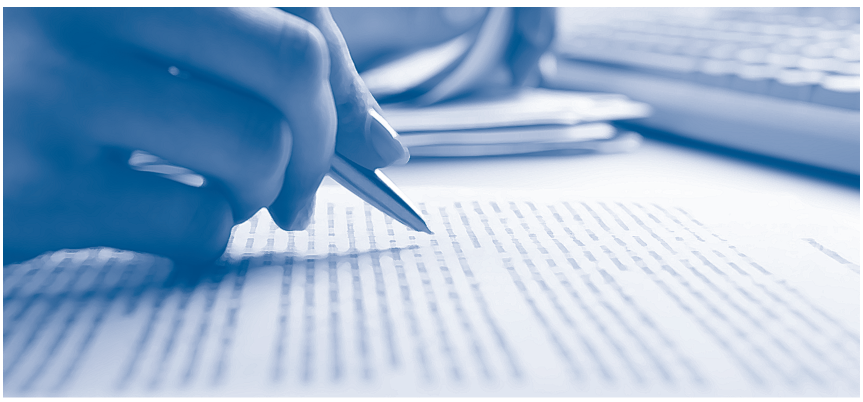

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Abstract

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We build a model of credit card payments where the retailers are allowed to charge differential prices depending on the instrument of payment chosen by the consumer. We follow the Rochet and Wright (2010) approach, but assuming a credit card system without a no-surcharge rule or any type of price differentiation disincentive. In a Hotelling competition framework at the retailers level, the competitive equilibrium prices are computed assuming that the store credit provided by the retailer is less cost efficient than the one provided by the credit card. In accordance with the literature, we obtain that the interchange fee becomes neutral if we eliminate the no-surcharge rule, when the interchange fee loses its ability to distort the individual consumer's decisions displacing the aggregated consumers' welfare from its maximum. We prove that the average price obtained under price differentiation is smaller than the single retail price under the no-surcharge rule, despite the retailer's margins being the same in both scenarios. Furthermore, we show how some cross subsidies are eliminated when price differentiation is allowed. In addition, we introduce menu costs to prove that there is a threshold value for the interchange fee such that price differentiation is equilibrium if that fee is above this value. The threshold may be interpreted as an endogenous cap for the interchange fee fixed by the credit card industry. Finally we conclude that, even with menu costs associated to price differentiation, the consumers' welfare can be greater in the price differentiated equilibrium than in the single price equilibrium under the non-surcharge rule.

Keywords: Credit cards, payments, two-sided markets.
JEL Classification: L11; E42; G18.

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1. Introduction

There is an intensive international debate involving industry members, market regulators and consumers representatives about the structure of the credit card market, the behavior of its players and the consequences on competitiveness and, most importantly, on the social welfare (Weiner and Wright (2005), Bradford and Hayashi (2008)). Actually, the social welfare maximization should be the ultimate goal of any regulator. However, this is a difficult challenge that encompasses the assessment of distributive aspects, like defining relative importance of the welfare of each segment of the society.

Among the most instigating issues in the debate about the credit card market regulation is the one concerning the effects of the no-surcharge rules, or any other disincentive on price differentiation, on consumer's welfare. We can find in the literature that surcharging may have positive effects for merchants and consumers (Chakravorti and Emmons (2003), Bolt and Chakravorti (2008)) as well as studies showing ambiguous or positive impacts on the system (Rochet (2003), Rochet and Tirole (2008)). A distinctive feature of the card payment system is that, despite the fact that the cardholders make the choice of their payment instruments, the transaction costs is only incurred by the merchants, which, in general, recover those costs through the single price strategy. In practice, the fee structure has triggered the use of merchant fees to reward the issuance (interchange fee) and the usage of cards (card rewards), which is a typical behavior in two-side market structures. The central question is if the credit card industry can exert market power by imposing scheme rules prohibiting surcharging of credit card purchases by merchants. In other words, no-surcharge rules could prevent price signaling to cardholders about the relative costs of different payment methods, reinforcing the pattern where the higher the merchant fee, the greater the capacity to reward cardholders, leading to a less efficient allocation of resources in the payment system ("excess" of card usage).

Another important issue is that, under the no-surcharge rule, merchants recover the average cost of different instruments of payment charging all consumers equally. Consequently, consumers who do not use credit cards pay more than they would otherwise. In other words, consumers who use credit cards are subsidized in their purchases (Chakravorti and Emmons (2003), Chakravorti and To (2007)). There are empirical studies that measure these cross subsidies in some jurisdictions, generally indicating that these are not negligible[1].

Because of its anti-competitive nature, the no-surcharge rule has been prohibited in some jurisdictions. For instance, in United Kingdom since 1991, in the Netherlands since 1994, in Sweden since 1995 and in Australia since 2003[2]. The authorities judged that merchant pricing freedom is essential for an effective price competition, in particular, for the competition between payment schemes.

In Australia, the prohibition on no-surcharge rules is stated in the Standards as "Neither the rules of the Scheme nor any participant in the Scheme shall prohibit a merchant from charging a credit cardholder any fee or surcharge for a credit card transaction". Further, an assessment of interchange-fee capping in this country showed that issuers had recovered part of the loss of interchange fees in the short run (Chang et. al. (2005)). Those authors also showed that merchants definitely had a benefit which was not substantially passed to the consumers. Despite all this, regulators still recognize that the surcharging reforms in Australia have been successful and have provided significant public benefits. Notwithstanding, they have become concerned about cases where surcharges seem to be higher than the acceptance costs. Since the evidence obtained shows that in some instances surcharging has developed in a way that potentially compromises price signals and reduces the effectiveness of the reforms, regulators are currently reviewing the no-surcharge standards in order to provide card schemes with the ability to constraint the level of surcharges to something close to the merchant acceptance costs, whose main component is the merchant fees[3].

When analyzing the convenience of eliminating the no-surcharge rule for credit card payments, it is quite reasonable to affirm (as we prove in this paper) that some merchants will have incentives to unilaterally surcharge credit card transactions above

1 Schuh, S.; Stavins, J., (2010) and Central Bank of Brazil (2011a) and (2011b) are examples of empirical studies that estimate cross subsidies in the United States and Brazil, respectively.
2 See, respectively, United Kingdom Parliament (1990), Vis, E.;Toth, J. (2000), MA Market Development AB (2000) and Reserve Bank of Australia (2012).
3 Consultation documents of the Reserve Bank of Australia (2011a) and (2011b).

the single price level, without any reduction of prices of other types of transactions. Notwithstanding, it is far from being a valid argument against differentiation, because this assertion should assume the merchant has market power. The correct analysis should need to take into account the existence of new equilibrium prices in the absence of the no-surcharge rule, as well as, to compare consumer welfares in both equilibria. This is exactly one of the main goals of this paper.

As we will illustrate in the next section, through our theoretical analysis of a simple model, the fact that each merchant has the possibility to obtain an extra profit when he individually deviates from the single price does not guarantee that such strategy is sustainable. Actually, a complete and coherent analysis needs, first of all, to find the new equilibrium prices, which will depend on the specific competitive environment and their effects on profit possibilities. Then, we can to measure the welfare gains, or losses, when comparing both equilibria.

In the literature we can find papers focusing on the economic role of the credit card interchange fee, as well as on its determination and possible regulation. In a economic environment of profit-seeking firms which are imperfectly competitive, Schmalensee (2002) concludes that the interchange fee shifts the costs between issuers and acquirers and as a consequence, also shifts the distribution of charges on merchants and consumers. This allows enhancing the value of the payment system as a whole to its owners, due to the network externality. Rochet and Tirole (2002) analyze the welfare implications of a cooperative determination of the interchange fee by member banks, in a framework in which banks and merchants may have market power, as well as, consumers and merchants decide rationally on whether to buy or accept a payment card. Wright (2003) evaluates the social optimality of privately set interchange fees under the no-surcharge rule in two extremes of merchant pricing, namely monopolistic pricing and perfect competition. In addition, the positive aspect of the no-surcharge rule in preventing the excessive merchant surcharging is assessed. Rochet and Tirole (2006) analyze the welfare effects of the externalities inherent in the card payment system and discuss whether consumer surplus or social welfare is the proper benchmark for the study of the regulation in the card payment industry. They bring all the theoretical analysis to unravel the recent antitrust actions taken by regulators and merchants against card associations in Australia, the UK and US. Wang (2010) suggests that the card networks demand higher interchange fees to maximize member issuers' profits as card payments become more efficient and convenient. He also discusses positive and negative features of policy interventions.

Our work is closely related to Rochet and Wright (2010). They model the credit card explicitly, allowing a separate role for the credit functionality of credit cards, which is modeled apart from other payment cards (i.e., debit cards). They assume impossibility (or lack of incentives) of retailers to differentiate prices according to the instrument of payment chosen by consumers. Under those assumptions, they showed how a monopoly card network could select an interchange fee high enough to promote the utilization of credit cards in a level that exceeds the one that maximizes the aggregated consumer surplus. They show how a regulatory cap for the interchange fee could be used to increase consumer surplus.

Our work aims to extend the Rochet and Wright (2010) model, giving a distinctive subsidy to the debate and helping to clarify, through a simple theoretical model, the implications of price differentiation and of menu costs incurred by merchants in credit card payments. With this extension we are able to illustrate how the absence of a no-surcharge rule could generate equilibrium prices capable of improving consumer welfare and reduce the market power of banks through the interchange fee[4], even under the assumption that retailers face menu costs associated to price differentiation. In this case, we prove that retailers will differentiate prices as long as the menu costs are not high enough.

The paper is divided into four sections. Section 2 describes the model. In Section 3, we present the main results of the paper, first considering the absence of friction given by the menu costs of price differentiation and secondly including such costs to analyze the effects of that market imperfection. In section 5, we summarize the main conclusions. The Appendix contains the detailed proofs of all the results enunciated in the paper.

4 See Gans and King (2003) about the neutrality of the interchange fee under price differentiation.

2. The model with price differentiation

Two distinctive changes in the model proposed by Rochet and Wright (2010) are introduced. In a first version of our model, we only allow retailers to differentiate the price of credit card payments from the price of the other payment instruments (store credit, cash, debit cards and others). The second specification introduces, in the former version of the model, menu costs incurred by merchants, which represent any costs, pecuniary or not, associated to the adoption of the price differentiation by the merchant.

As in Rochet and Wright (2010), we assume here that there is a continuum of consumers, all distributed uniformly in a unitary length interval. All consumers have identical quasi-linear preferences, spending their income on retail goods costing γ for the merchant[5]. There are two payment technologies. The first one corresponds to a group of 'cash' payment technologies, which could include money, checks, debit cards or other instruments not involving any credit functionality. The second one corresponds, exclusively, to the credit cards' payment technology. As an alternative to both technologies, each retailer can directly provide credit to the consumer, which is called 'store credit'.

Credit cards are held by a constant fraction x of consumers and assumed to be more costly than cash. Without loss of generality, the costs of all payments are expressed relatively (normalized) to the cost of the cash payment, which is assumed to have zero cost. Credit cards allow consumers to purchase on credit and entail a cost (or benefit, if negative) f for the consumer (buyer)[6], which is received (or paid) by the issuer, and entail a cost m (merchant fee) for the retailer (seller)[7].

[5] As usual in partial equilibrium models, quasi-linearity allows us to measure the utility in monetary terms.

[6] There might be other net costs/benefits associated to credit card payments with respect to cash and not linked with the relationship with the issuer (for instance, those coming from the privacy, agility, safety, financial planning). If such costs/benefits are lower than f, then the qualitative analysis remains unchanged.

[7]. If retailers receive their payments from the acquirer with some delay, they will incur a cost of the time value of money (interest rate on anticipated receivables). If this cost is lower than the merchant fee m, the qualitative results obtained here remain unchanged.

The store credit is an alternative to the credit function of the credit card and entails a random transaction specific cost (or benefit, if negative) c_B for the consumer and cost c_S to the retailer.

Each consumer purchases one unit of the retail good, called 'ordinary purchases', providing him utility $u_0 > \gamma$, but, in addition, with probability θ, he also receives utility $u_1 > \gamma$ from consuming another unit of the retail good called ''credit purchases". It is assumed that merchants cannot bundle the two transactions nor distinguish between "ordinary" and "credit" purchases.

When making ordinary purchases, all consumers can choose between cash or store credit, but only a fraction x of them have the possibility to choose credit cards. On the other hand, when making a credit purchase, cash is not an option for any consumer. Additionally, it is assumed that each consumer always has sufficient cash to pay for his ordinary purchases, but must rely on credit for credit purchases.

The transaction specific cost c_B of a store credit is observed by the consumer only when he is in the store[8], which is drawn from a continuous distribution with the cumulative distribution function *H*. We assume the distribution has full support over some range $\left(\underline{c_B}, \overline{c_B}\right)$, where $\underline{c_B}$ is sufficiently negative, such that cardholders will sometimes choose to use store credit even if cash can be used instead, and $\overline{c_B}$ is positive but not too high (in comparison with $u_1 - \gamma$), such that consumers will always prefer to make the credit purchase, even if they have to pay with store credit, rather than not buy at all. The draw c_B is the net cost of using store credit rather than credit cards or cash. A negative draw of c_B could represent a situation where a cardholder needs to preserve his cash or credit card balance for some other contingencies and thus values the use of store credit. The assumption that there are situations when consumers see more benefits in using the store credit than credit cards is a key aspect of our model, since it will justify

[8] In contrast, the costs/benefits of a credit card payment are assumed to be known by the consumers before arriving to the store. Actually, the credit card service's contractual conditions (i.e., interest, annual fees and rewards) are usually posted in advance to credit card users.

the existence of differential price equilibria, even in the presence of menu costs associated with the differentiation.

If the merchant fee m of a credit card purchase is smaller than the cost of a store credit c_S ($m<c_S$), accepting credit cards is a potential mean for merchants to reduce their transaction costs of accepting credit purchases. But if $m>c_S$, acceptance of credit card increases the merchant's transaction costs.

In general, consumers will prefer credit cards to store credit when $c_B > f$, for both ordinary and credit purchases. In particular, when issuers give benefits ($f < 0$, i.e., through rewards or cash back bonuses) to consumers in each credit card purchase, consumers will prefer to use their credit cards rather than cash for ordinary purchases. It was proved in Rochet and Wright (2010) that, from the point of view of aggregated consumers, excessive incentives for credit card use could be socially wasteful.

The bank of the merchant, or acquirer of the transaction, incurs in an acquiring cost c_A, as well as incur in an interchange fee a (which is paid to the bank of the consumer) for each credit card transaction. It is assumed, without loss of generality, that only acquirers are perfectly competitive, which implies that the merchant fee m is equal to the sum of the acquiring cost c_A and the interchange fee a,

$$m = c_A + a \tag{1}$$

The bank of the cardholder, or issuer of the card, incurs in an issuing cost c_I and receives the interchange fee a from the acquirer[9]. It is assumed that issuers are imperfectly competitive, which implies that the cardholder fee f is equal to the issuer cost that exceed the interchange fee revenue ($c_I - a$) plus a constant profit margin π,

[9] Beside the interchange fee and the annual fees, the interest revenue might be an important component of the issuers' total revenues. This is the case in some jurisdictions, as in Brazil. Notice that the annual fees and the interest revenues may be included in the net cardholder fee *f*.

$$f = c_I - a + \pi \tag{2}$$

Thus, the total cost of a credit card transaction is

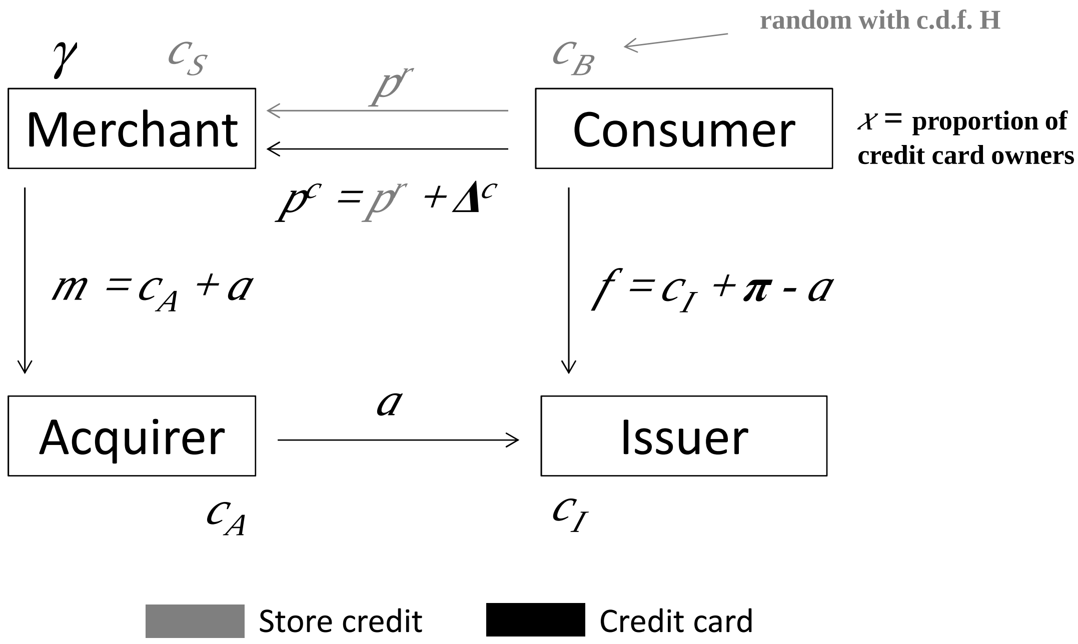
$$c := c_A + c_I \tag{3}$$

Denote by δ the excess cost of the store credit with respect to the total cost to provide a credit card transaction, including issuer's profit, which is defined by

$$\delta := c_S - c - \pi \tag{4}$$

We will restrict our analysis to the situation where, from the point of view of the suppliers of the credits (merchants or credit card industry), a credit card transaction is more cost efficient than the store credit, or, equivalently, $\delta > 0$.

Figure 1 – Prices, costs and fees of instruments of payment



Competition between retailers occurs as in the standard Hotelling model: consumers are uniformly distributed on an interval of unit length, with one retailer

$(i = 1,2)$ located at each extremity of the interval. There is a transportation cost t for consumers per unit of distance. Unlike Rochet and Wright (2010) we are interested here in the situation where retailers have the option to charge different retail prices according to the instruments of payment. To simplify, we restrict ourselves to the particular situation where the retailer is allowed to charge a price p^c for a credit card transaction which may be different from the price p^r charged for a cash or store credit transaction. We denote by Δ^c the spread between these two prices, namely:

$$\Delta^c := p^c - p^r \qquad (5)$$

Figure 1 illustrates the interconnection between participants of the credit card market, as well as the respective prices, costs and fees charged by each one.

The timing of the decisions is as in the model of Rochet and Wright (2010), which can be divided into 9 steps, grouped in two periods: 5 steps before the arrival of the consumer to the store and 4 steps once the consumer is in the store.

Before arriving at the store:

1. The card network sets the interchange fee a ;
2. Banks set their fees: f for cardholders and m for retailers;
3. Retailers independently choose their card acceptance policies: $L_i^r = 1$ if retailer i accepts credit cards, 0 otherwise;
4. Retailers independently set retail prices p_i^r and $p_i^c = p_i^r + \Delta_i^c$;
5. Consumers select one retailer to patronize, after observing the observed retail prices, retail's acceptance policies, issuer's fee, the distribution of store credit cost and transportation cost.

Once the consumer is in the store:

6. Consumer buys a first unit of the retail good (‘‘ordinary purchase”), and pays for it using cash or credit card (if he has one);

7. Nature decides whether the consumer has an opportunity for an additional credit purchase, which will occur with probability θ ;
8. The cost c_B of using store credit for the buyer is drawn according to the c.d.f. H, with full support on $\left(\underline{c_B}, \overline{c_B}\right)$;
9. Cardholders then select their mode of payment. We set $L_i^c = 1$ if the consumer prefers credit cards over cash when buying at the retailer i, or 0 otherwise. In other words:

$$L_i^c := \begin{cases} 1 & \text{if } \ f + \Delta_i^c \leq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

3. Analysis and results

Assuming the merchant charges price p_i^r for cash and store credit, and charges an additional spread Δ_i^c specifically on credit card transactions, we obtain (see Appendix) that the expected margin of the retailer i is given by

$$M_i = (1+\theta).(p_i^r - \gamma) - (H(0)+\theta).c_S - x.L_i^r.\overline{\Gamma}(a, \Delta_i^c) \tag{7}$$

where

$$\overline{\Gamma}(a, \Delta_i^c) := [1 - H(0)].L_i^c.c_S + \left[1 - H\left(f + \Delta_i^c\right)\right](L_i^c + \theta).\left(m - c_S - \Delta_i^c\right) \tag{8}$$

The first two terms at the right hand side of (7) correspond to the expected revenue of the retailer i, when there are not credit card users ($x = 0$) or the retailer i decides not to accept credit cards ($L_i^r = 0$). These terms are, respectively, the net cost of the products and the net cost of the instruments of payment. The third term corresponds to the expected margin reduction associated to the use of credit cards by credit card owners.

We obtain (see Appendix) that the utility of the consumer that chooses to purchase the good from the retailer i is given by

$$U_i = u_0 + \theta . u_1 - (1+\theta).p_i^r - \int_{\underline{c}_B}^{0} c_B.dH(c_B) - \theta.E(c_B) + x.L_i^r.\overline{S}(a,\Delta_i^c) \qquad (9)$$

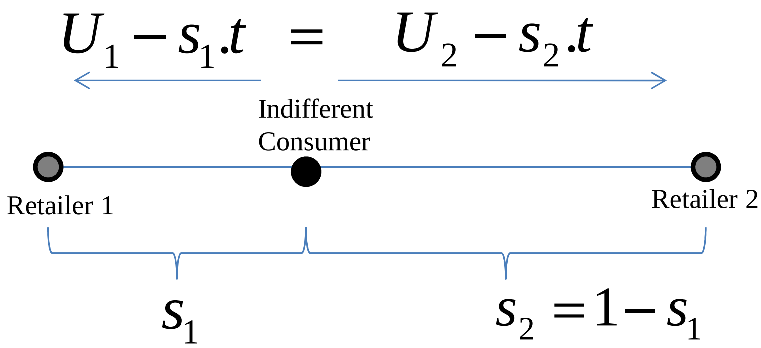
where

$$\overline{S}(a,\Delta_i^c) := (L_i^c + \theta).\left(\int_{f+\Delta_i^c}^{\overline{c}_B} (c_B - f - \Delta_i^c).dH(c_B) \right) - L_i^c.\int_0^{\overline{c}_B} c_B.dH(c_B) \qquad (10)$$

The first five terms at the right side of (9) correspond to the net expected utility of the consumption, taking into account the product cost and the store credit cost, when there are no credit card users or the retailer decides not to accept credit cards. The last term corresponds to the additional welfare associated exclusively to the use of credit cards.

The market share of both retailers is determined by computing the position of the indifferent consumer in the region (interval of size one) where all consumers are uniformly distributed. Since there is a cost t for every unit of displacement, the utility minus the displacement cost of a consumer who decides to purchase the good from retailer i is $U_i - s_i.t$. Therefore, the distance s_i between the indifferent consumer and the retailer i is equal to the proportion of consumers choosing retailer i. Figure 2 shows net utilities and the market shares of both retailers.

Figure 2 – Indifferent consumer in the Hotelling model



Then the market share s_i of the retailer i depends on the interchange fee, prices and spreads, and is given by the following expression

$$s_i = \frac{1}{2} + (1+\theta).\left(\frac{p_j^r - p_i^r}{2.t}\right) + x.\left(\frac{L_i^r.\overline{S}(a,\Delta_i^c) - L_j^r.\overline{S}(a,\Delta_j^c)}{2.t}\right) \qquad (11)$$

Note that, for a fixed interchange fee a, the (Nash) equilibrium price $\overline{p}$ when differentiation is not allowed, as defined by equation (5) in Rochet and Wright (2010), satisfies the following equation

$$(1+\theta).\overline{p} = t + (1+\theta).\gamma + (H(0)+\theta).c_S + \frac{x}{3}.(L_j^r - L_i^r).\overline{\phi}(a,0) + x.L_j^r.\overline{\Gamma}(a,0) \qquad (12)$$

where

$$\overline{\phi}(a,\Delta_i^c) := \overline{S}(a,\Delta_i^c) - \overline{\Gamma}(a,\Delta_i^c) \qquad (13)$$

is the difference between the additional welfare of the consumer and the additional cost of the retailer associated to each credit card transaction.

For each $\delta > 0$, as defined in (4), consider the following parameter definition

$$\overline{\phi}_{\delta} := (1+\theta).\left(\int_{-\delta}^{\overline{c}_B} (c_B + \delta).dH(c_B) \right) - \int_{0}^{\overline{c}_B} (c_B + c_S).dH(c_B) \qquad (14)$$

and note that $\overline{\phi}_{\delta} = \overline{\phi}(a, a + c_A - c_S)$.

Note that, if we $\overline{\phi}_{\delta} > 0$ and the spread is equal to $m - c_S \ (= a + c_A - c_S)$, the benefit of the credit card transactions for consumers is greater than the cost of the same transactions for retailers. Note that if $\delta > 0$ is sufficiently small, the assumption $\overline{\phi}_{\delta} > 0$ is equivalent to the condition

$$\theta.\left(\int_{0}^{\overline{c}_B} (c_B + c_S).dH(c_B) \right) > (1+\theta).\int_{0}^{\overline{c}_B} (c + \pi).dH(c_B) \qquad (15)$$

where the left-hand side term above corresponds to the costs savings in using credit cards for extraordinary purchases and the right-hand side term corresponds to the cost of using credit cards in both types of purchases.

The results in this paper are obtained under the four basic assumptions defined below. The first one relaxes the non-surcharge rule allowing retailers to charge different prices for credit card transactions, and represents the main assumption.

Assumption 1: Price differentiation of credit cards transactions is allowed.

The second assumption is related to the cost efficiency of the credit card industry compared to the store credit instrument.

Assumption 2: The parameter δ, defined by (4), is strictly positive.

Assumption 2 above means that, from the point of view of the lenders, the sum of the total bank costs and profits of credit card transactions $c_A + c_I + \pi$ is lower than the retailer's cost of providing store credit c_S. In this specific sense, the credit card industry is more cost efficient than retailers in providing credit.

As demonstrated in Rochet and Wright (2010), under the no-surcharge rule, if the consumer's benefit of a credit card transaction is equal to the cost savings of generating credit through a credit card transaction instead of a store credit ($f = \delta$), consumers obtain the maximum aggregate welfare. They proved that any other level of credit cards' costs/benefits f (which depends on the interchange fee, since $f = c_I + \pi - a$) will generate a loss in consumers' welfare. In other words, despite consumers are individually deciding their instruments of payments in an optimal way, from the aggregate point of view, those decisions generate an collectively inefficient level of credit card usage, if compared to the optimal situation when $f = \delta$.

The third assumption has a more sophisticated interpretation, and it essentially imposes restrictions on the retailers' average cost and the consumers' average benefits of a credit card transaction.

Assumption 3: The parameter $\bar{\phi}_\delta$, defined in (14), is strictly positive.

As noticed formerly, Assumption 3 above is equivalent to imposing that, when the spread charged by both merchants is equal to $m - c_S$, the consumers' average benefit from credit card transactions is greater than the retailers' average cost from the same credit card transactions. Note that, if retailers recover those costs through the average price paid by consumers, Assumption 3 implies that consumers have positive average benefits in using credit cards.

Assumption 4: The transportation cost t is greater than $\frac{x.(1+\theta)}{2} \int_{\underline{c}_b}^{-\delta} (-\delta - c_B).dH(c_B)$

Assumption 4 above guarantees that the transportation cost t is greater than

$$\varepsilon(a) := \frac{x.(1+\theta)}{2} \int_{-\delta + c_S - c_A - a}^{-\delta} (-\delta - c_B).dH(c_B) \qquad (16)$$

for all values of a greater than $c_S - c_A$. Note that $2.\varepsilon(a)$ corresponds to the aggregate welfare benefits of all consumers and retailers obtained from differentiation equilibrium if compared with the non surcharge single price equilibrium. The condition means that the transportation cost is greater than the benefit obtained from the price differentiation.

We prove (Proposition 2) that greater the transportation cost greater the profit of a retailer when he deviates unilaterally from differential prices equilibrium. We also prove that his market share and the margin are both zero when $t \leq \varepsilon(a)$, since even his most closest consumer will prefer to incur in transportation costs, since these costs are compensated by the benefits of choosing the differential prices offered by the other retailer.

3.1. Equilibrium prices under price differentiation

In this subsection we provide some results that allow us to analyze the impacts of ruling out the no-surcharge rules in the credit card systems. All the results below are obtained under Assumptions 1, 2, 3 and 4.

Proposition 1: If both retailers charge the single price $\bar{p}$, as defined in equation (12), the merchant fee is greater than the cost of the store credit ($m > c_S$) and the density of consumers that are indifferent to the cost of a store credit or a credit card ($c_B = f$) is positive ($h(f) > 0$), retailers have incentives to impose a surcharge over the single price.

Proof: See Appendix.

The first additional condition in Proposition 1 ($m > c_S$) is satisfied if the interchange fee is high enough ($a > c_S - c_A$), since (1) gives us the equilibrium merchant fee. An immediate consequence of Proposition 1 above is that the single price $\bar{p}$ charged for every instrument of payment is not the (Nash) equilibrium price under price differentiation.

The proof of Proposition 1 employs the fact that the profit function is strictly increasing in the price spread in a neighborhood of $\Delta_i^c = 0$, meaning that it is desirable for the merchant to surcharge credit card transactions above $\overline{p}$. However, it is worth noting that this result is only a comparative static assessment, whose utility is exclusively to prove that the single price strategy cannot be sustained in equilibrium under the price differentiation assumption.

Any policy assessment needs to address more relevant questions like: is there a competitive equilibrium with that price differentiation characteristic? And if the response is positive, how does the consumer welfare in that equilibrium fare when compared to that of a single price? The results in the following theorem help us to clarify these questions.

Theorem 1: For each interchange fee *a* defined by the banks, there is a pair of prices $(\overline{p}^r, \overline{p}^c)$, respectively, the price charged for cash/store credit transactions and the one charged exclusively for credit card transactions, where this pair is a Nash equilibrium. Specifically, if both retailers are charging those prices, none of them, has incentives to deviate from those prices. The prices are given by

$$\overline{p}^r = \gamma + \frac{t + \left[H(0) + x.\left(1 - H(0)\right) + \theta\right].c_S}{(1+\theta)} \tag{17}$$

and

$$\overline{p}^c = \overline{p}^r + m - c_S \tag{18}$$

Proof: See Appendix.

Note that the price $\overline{p}^r$ does not depend on interchange fee a. Actually, only the price of credit cards transactions $\overline{p}^c$ depends on it (through (1)) and any increase of the interchange fee is totally transferred to the product price when the payment is made using the credit card.

From equations (18) and (1), we obtain that the equilibrium spread is given by

$$\overline{\Delta}^c = c_A + a - c_S \tag{19}$$

and, consequently, using (19) and (2), we conclude that $f + \overline{\Delta}^c = \delta$. In other words, the interchange fee loses its capability of affecting the consumers' net benefit of a credit card transaction ($f + \overline{\Delta}^c$), under differentiation, which becomes constant and equal to the optimal benefit value (δ) under the no-surcharge rule. This is a remarkable difference with respect to the Rochet and Wright (2010) findings.

The average equilibrium price is

$$\overline{p}^m = (1 - \alpha_\Delta)\,.\,\overline{p}^r + \alpha_\Delta\,.\,\overline{p}^c \tag{20}$$

where $\alpha_\Delta := x.[1 - H(f + \overline{\Delta}^c)]$ is the proportion of card owners that, under price differentiation, prefer to use credit cards rather than store credit or cash.

Corollary 1: The price $\overline{p}$ is a convex combination of the prices $\overline{p}^r$ and $\overline{p}^c$. More specifically,

$$\overline{p} = (1 - \alpha_0)\,.\,\overline{p}^r + \alpha_0\,.\,\overline{p}^c \tag{21}$$

where $\alpha_0 := x.[1 - H(f)]$ corresponds to the proportion of credit card owners that, under no-surcharge rule, prefer credit cards to any other instrument.

Proof: See appendix.

An important and immediate consequence of Corollary 1 is that, under a strictly positive surcharge $\overline{\Delta}^c > 0$, we have $\alpha_\Delta = x.[1 - H(f + \overline{\Delta}^c)] < x.[1 - H(f)] = \alpha_0$. Therefore, using (20) and (21) we obtain that the average price $\overline{p}^m$ under price differentiation is lower than the single price $\overline{p}$ under the no-surcharge rule.

To finalize the prices analysis, we show how the cross subsidies arising in a framework with the no-surcharge rule are eliminated under this new scenario with price differentiation. Figure 3 below illustrates how the price $\overline{p}$ can be decomposed into the new equilibrium prices and the subsidy components that are eliminated. In cases 1 and 2, the terms in the right side of both equalities, not included in the gray boxes, correspond to the price $\overline{p}^r$ charged using cash or the store credit. In case 3, the terms in the right side of the equality, not included in the gray box, correspond to the price $\overline{p}^c$ charged using credit card.

Notice that part of the total subsidy is eliminated with the price differentiation of credit card transactions, but one component of the subsidy remains. This component is associated to the group of consumers without credit cards (fixed proportion $1-x$) that has not benefit using the store credit. Thus, they use cash. This particular group of consumers pays a subsidy to the other consumers. The subsidy occurs because they have fewer options of payment instruments and as a consequence less competitiveness in their payment technology.

Figure 3 – Decompositions of the single price equilibrium

1) Cash:

$$\overline{p} = \underbrace{\gamma}_{\text{Product cost}} + \underbrace{\frac{t}{1+\theta}}_{\text{Transportation markup}} + \underbrace{\left(1-\frac{(1-x).[1-H(0)]}{1+\theta}\right).c_S + \boxed{x.[1-H(f)].(m-c_S)}}_{\text{Subsidy paid}}$$

Eliminated with price differentiation

2) Store credit:

$$\overline{p} = \gamma + \frac{t}{1+\theta} + \underbrace{c_S}_{\text{Store credit cost}} - \underbrace{\frac{(1-x).[1-H(0)]}{1+\theta}.c_S}_{\text{Subsidy received}} + \underbrace{\boxed{x.[1-H(f)].(m-c_S)}}_{\text{Subsidy paid}}$$

3) Credit card:

$$\overline{p} = \gamma + \frac{t}{1+\theta} + \underbrace{m}_{\text{Merchant fee}} - \underbrace{\frac{(1-x).[1-H(0)]}{1+\theta}.c_S}_{\text{Subsidy received}} - \underbrace{\boxed{\{1-x.[1-H(f)]\}.(m-c_S)}}_{\text{Subsidy received}}$$

Notice that the remaining subsidy from cash users to the others is eliminated if we suppose that every consumer has a credit card ($x = 1$). However, since we are not allowing for price differentiation between cash and store credit transactions, a subsidy between them persists. This is because the price $\overline{p}^r$, as an average price, does not reflect the different costs of each instrument (cash has zero cost and store credit has cost c_S).

The following theorem shows that there is a positive impact on consumer welfare as a consequence of the elimination of barriers to price differentiation of credit card transactions.

Theorem 2: The consumer welfare in the equilibrium under price differentiation is greater than the corresponding under the no-surcharge rule. They are equal only if the interchange rate a is equal to $c_S - c_A$.

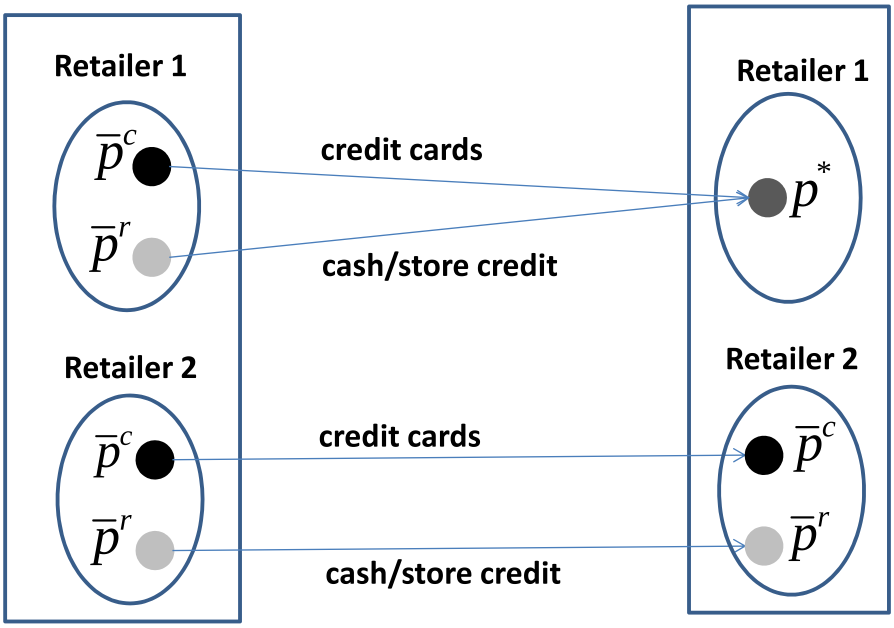
Proof: See appendix.

With respect to the merchants' profits the following corollary shows that retailers are indifferent with respect to the no-surcharge rule. In a model which considers both convenience users and interest-paying users of credit cards, Chakravorti and Emmons (2003) showed that retailers prefer to charge different prices.

Corollary 2: The equilibrium merchant profit is the same under price differentiation and under the no-surcharge rule.

Proof: See appendix.

Figure 4 – Retailer unilateral movement to single price strategy



To analyze the effect of the interchange fee a, fixed by banks, on the incentives of retailers to deviate to a single price, it will be useful to compute the profit of a retailer resulting from this deviation whilst the other retailer continues to charge the differentiated prices $\overline{p}^r$ and $\overline{p}^c$. Figure 4 illustrates the situation where Retailer 1 moves unilaterally to the single price strategy, whilst Retailer 2 continues to charge two prices.

The following proposition shows that, provided that both merchants are charging differentiated prices, none of them has incentive to unilaterally change the maximum profit single price p^*, which is not necessary equal to the no surcharge single price equilibrium $\overline{p}$. In the next subsection, the same type of analysis is revisited, but in a context of existence of menu costs when considering the possibility of differentiated prices (see Theorem 3).

Proposition 2: Assume that both retailers are initially charging differentiated prices $\overline{p}^r$ and $\overline{p}^c$, with a positive spread $\overline{\Delta}^c$. If one of them decides to charge the single price p^* that maximizes its profit compared with all alternative single prices, the profit of this retailer will decrease the amount $\varepsilon(a).\left(1-\frac{\varepsilon(a)}{2t}\right)$, whereas the profit of the competitor will augment in the amount $\frac{\varepsilon(a)}{2}$.

Proof: See the proof of Theorem 3 in appendix, the case without menu costs is a particular case of the more general demonstration presented there.

A consequence of Corollary 2 and Proposition 2 is that, despite both retailers having the same profit under price differentiation and under a single price, no one has, without cooperation, incentives to move unilaterally to the single price. Figure 5 illustrates the profits of both retailers where the strategies are "apply surcharge" and "apply single price". As usual, each entry of the matrix represents the profits of each retailer corresponding to the adopted strategies. Thus, the gray cell indicates the equilibrium profits of Retailer 1 and Retailer 2, which are equal to $t/2$, when both decide to apply surcharge. The profits of both retailers are $t/2$ again if they apply the single price. However that strategy profile is not equilibrium.

Figure 5 – Profits under price differentiation assumption

<table>
<tr><th colspan="2" rowspan="2">Retailers' profits</th><th colspan="2">Retailer 2</th></tr>
<tr><th>Differential prices</th><th>Single price</th></tr>
<tr><td rowspan="4">Retailer 1</td><td rowspan="2">Differential prices</td><td rowspan="2">$t/2$; $t/2$</td><td>$t/2 + \varepsilon/2$; $t/2 - \varepsilon.(1-\varepsilon/2t)$</td></tr>
<tr><td>$t/2 + \varepsilon.(1+\varepsilon/2t)$; $t/2 - \varepsilon/2$</td></tr>
<tr><td rowspan="2">Single price</td><td>$t/2 - \varepsilon.(1-\varepsilon/2t)$; $t/2 + \varepsilon/2$</td><td rowspan="2">$t/2$; $t/2$</td></tr>
<tr><td>$t/2 - \varepsilon/2$; $t/2 + \varepsilon.(1+\varepsilon/2t)$</td></tr>
</table>

Notice that, if both retailers are at the differential price equilibrium and Retailer 1 unilaterally decides to charge the single price, his profit falls from $t/2$ to $t/2-\varepsilon(a).(1-\varepsilon(a)/2t)$ and Retailer 2 profit grows from $t/2$ to $t/2+\varepsilon(a)/2$ However, if both retailers are charging single prices and Retailer 2 individually decides to charge the differential prices, his profit increases from $t/2$ to

$t/2+\varepsilon(a).(1+\varepsilon(a)/2t)$ and Retailer 1 profit decrease from $t/2$ to $t/2-\varepsilon(a)/2$. Therefore, since each movement represents a unilateral deviation from a specific initial price scenario, each one generates distinct profits scenarios. The reasoning is analogous for when Retailer 1 moves unilaterally from single price to differential prices strategy and when Retailer 2 moves unilaterally from differential prices to single price strategy.

3.2. Menu costs

In this last subsection we analyze the effects of the introduction of the menu costs in the model of price differentiation that we are considering. In this framework, we will call a menu cost to any cost, pecuniary or not, faced by the retailer as a consequence of charging different prices according to the instrument of payment choosed by the consumers. As usual, the reasons to consider this type of friction include: the costs of developing pricing strategies for differentiation or the costs of implementing and updating the systems with the information of differentiated prices.

Another reason for including menu costs can be related to the legal and regularory insecurity regarding eventual penalties that the retailer could suffer as a result of applying price differentiation. In jurisdictions where there is not a clear rule with respect to the price differentiation or even where there are institutional conflicts regarding to an existing rule, the sellers attribute high costs to price differentiation.[10]

Suppose that retailers face menu costs per transaction μ_1 and μ_2. So the margin of the Retailer *i* is given by

$$M_i^{\mu} := (1+\theta).(p_i^r-\gamma)-(H(0)+\theta).c_S - x.L_i^r.\overline{\Gamma}(\Delta_i^c)-\mu_i.I(\Delta_i^c) \qquad (22)$$

[10] The Syndicate of Retailers of Belo Horizonte, the capital of the State of Minas Gerais (Brazil), appealed to the Court of Justice against fines applied by the Institute of Consumer Protection of Minas Gerais (Procon/MG) to retailers who differentiate prices of credit cards transactions. In July 9th, 2012 the 6th Chamber of Court of Justice announced the sentencing in favor of the Syndicate. In fact, there is not explicit legal basis for the applied fines; however the Procon/MG argues that the prohibition is supported in a regulatory act issued by the Ministry of Finance (No. 118 of March 11th, 1998). Indeed, that regulatory act refers to the implementation of the transition rules for a new currency unit of that era. Since it is not a final sentence, the Procon/MG is appealing to the Court of Justice of the State of Minas Gerais.

where $I(\Delta_i^c) := \begin{cases} 0 & ; \text{if } \Delta_i^c = 0 \\ 1 & ; \text{if } \Delta_i^c \neq 0 \end{cases}$

The following theorem asserts that, even in the presence of menu costs, the price differentiation strategy remains as Nash equilibrium. The spreads are the same obtained in the case without menu cost. The equilibrium prices are greater than the ones without menu costs, as well as, they are different if their respective menu costs are different.

Theorem 3: If $\mu_1 \geq \mu_2$, $t \geq \frac{\mu_1 - \mu_2}{3}$ and $\varepsilon(a) > \frac{\mu_1}{2}$, then the surcharges on credit card purchases remain being equilibrium, where the equilibrium spreads are equal the one for the case without menu cost, $\overline{\Delta}_1^{c,\mu} = \overline{\Delta}_2^{c,\mu} = \overline{\Delta}^c$, and the equilibrium base prices

$$\overline{p}_1^{r,\mu} = \overline{p}^r + \frac{1}{1+\theta}.\left(\frac{2.\mu_1 + \mu_2}{3}\right) \quad \text{and} \quad \overline{p}_2^{r,\mu} = \overline{p}^r + \frac{1}{1+\theta}.\left(\frac{\mu_1 + 2.\mu_2}{3}\right).$$

are greater than corresponding one for the case without menu costs.

Proof: See appendix.

Particularly, when there is not menu cost dispersion, $\mu_1 = \mu_2$, it is easy to see that, despite the presence of cost associated to differentiation, the retailers are capable to recover the full menu costs increasing their prices, since $\frac{2.\mu_1 + \mu_2}{3} = \mu_1 = \mu_2 = \frac{\mu_1 + 2.\mu_2}{3}$

However, if there is menu cost dispersion, when $\mu_1 > \mu_2$, Retailer 1 is not capable to recover its full menu cost, since $\left(\frac{2.\mu_1 + \mu_2}{3}\right) < \mu_1$. However, Retailer 2 recover more than its menu cost, since $\left(\frac{\mu_1 + 2\mu_2}{3}\right) > \mu_2$. As a consequence (see the proof of Theorem 3 at appendix) Retailer 1 loses market share and margin, and Retailer 2 increases his respective market share and margin, if compared with the corresponding ones in case without menu cost dispersion.

Figure 6 below generalizes the results at Figure 5 above. It shows the profits of both retailers depending on their individual decisions of pricing strategies, when we assume the existence of menu costs. We define $\alpha := \frac{1}{t}.\left(\frac{\mu_1 - \mu_2}{3}\right)$ and $\beta_i(a) := \frac{1}{t}.\left(\varepsilon(a) - \frac{\mu_i}{2}\right)$. Note that Assumption 4 and the additional assumptions of Theorem 3 above are sufficient to guarantee that $0 \leq \alpha < 1$ and that, for every interchange fee *a*, $0 < \alpha + \beta_i(a) < 1$, for $i = 1, 2$.

Figure 6 shows the effects of the menu costs on the retailers' decision about the application, or not, of surcharges on credit card purchases. Note that, as consequence of the unilateral deviation from the differential price to the single price, the Retailer 1's profit decreases by $\frac{t}{2}.\beta_1(a).\{\,2.[1-\alpha]-\beta_1(a)\,\}$. By the other side, the unilateral deviation of Retailer 2 from the differential price to the single price decreases his profit by $\frac{t}{2}.\beta_2(a).\{\,2.[1+\alpha]-\beta_2(a)\,\}$. Essentially, the greater the interchange fee the greater is the profit loss and, consequently, the more stable is the equilibrium with differential prices. Additionally, the greater the menu costs the less stable is the equilibrium with differential prices.

Figure 6 – Profits under price differentiation and menu costs assumptions

<table>
<tr><th colspan="2" rowspan="2">Retailers' profits</th><th colspan="2">Retailer 2</th></tr>
<tr><th>Differential prices</th><th>Single price</th></tr>
<tr><td rowspan="4">Retailer 1</td><td rowspan="2">Differential prices</td><td rowspan="2">$t/2.(1-\boldsymbol{\alpha})^2$; $t/2.(1+\boldsymbol{\alpha})^2$</td><td>$t/2.(1-\alpha).(1-\alpha+\beta_2)$; $t/2.(1+\boldsymbol{\alpha}-\boldsymbol{\beta}_2)^2$</td></tr>
<tr><td>$t/2.(1+\boldsymbol{\beta}_1)^2$; $t/2.(1-\beta_1)$</td></tr>
<tr><td rowspan="2">Single price</td><td>$t/2.(1-\boldsymbol{\alpha}-\boldsymbol{\beta}_1)^2$; $t/2.(1+\alpha).(1+\alpha+\beta_1)$</td><td rowspan="2">$t/2$; $t/2$</td></tr>
<tr><td>$t/2.(1-\beta_2)$; $t/2.(1+\boldsymbol{\beta}_2)^2$</td></tr>
</table>

In this vein, Rochet (2003) argued that surcharging is seldom used by sellers, probably because of transaction costs, even when the system does not prohibit it. Our results reveal that, if there are not many consumers obtaining benefits from the use of store credits, then $\beta_i(a)$ could be negative, for $i=1$ or 2, and, consequently the differential price strategy is not equilibrium. Therefore, an interesting topic of applied research, could be to assess the disposability and convenience for consumers of the store credit compared with the credit of credit card, in order to analyze if this could explain why in some economies, despite surcharge is explicitly allowed, retailers do not charge differential prices.

The corollary below shows that, despite the existence of menu costs due to price differentiation, consumers and retailers obtain greater welfare and profits respectively when the non-surcharge rule is avoided. In particular, when there is not menu costs dispersion between retailers, the corollary shows that the consumers and retailers aggregated welfare gain with differentiation is the difference between the net welfare benefits of the replacing store credit for credit cards by part of consumer as a result of the implementation of the price differentiation, minus the menu cost incurred by retailer.

Corollary 3: The aggregated consumer's welfare and the aggregated retailers' profits in the differential price equilibrium with menu costs are greater than the corresponding ones obtained under the non-surcharge single price equilibrium. In particular, if both costs are equal, where $\mu := \mu_1 = \mu_2$, the aggregated gain of welfare of consumer and retailers with the differentiation is given by $2.\varepsilon(a) - \mu$.

Proof: See appendix.

Therefore, our results described a theoretical situation where the abolishment of the non-surcharge rule can reduce the market power of the credit card system and increase the aggregated welfare of consumers and retailers. These results have an important link with the findings of Rochet and Wright (2010). Let $\tilde{a} := \min\{\varepsilon^{-1}(\mu_1/2); \varepsilon^{-1}(\mu_2/2)\}$. If the banks decide to set a high interchange fee, say $a > \tilde{a}$, the retailers would be willing to set the equilibrium differential prices (see Figure 6). In particular, suppose that the threshold value $\tilde{a}$ is lower than the interchange fee $\bar{a}$, that maximizes the banks profit under the no-surcharge rule, as obtained in Rochet and Wright (2010). In this situation, it is possible that some retailer decides to differentiate prices, which can destabilize the former equilibrium price $\overline{p}$, enforcing banks to set the interchange fee at $\tilde{a}$. In this way, the market power and the profits of the credit card issuers are reduced. Consequently, the consumers and retailers welfare are augmented, if compared with the no-surcharge single price equilibrium.

4. Conclusions

In this paper, we adapt the framework of Rochet and Wright (2010), to the absence of no-surcharge rules for prices of credit cards transactions. In this setting we prove that the equilibrium prices for the purchases using credit cards and using cash or store credit are not the same. In particular, we obtain that the equilibrium surcharge spread is the difference between the merchant fee and the cost he has to provide the store credit.

The result regarding the equilibrium price spread is remarkable, especially in jurisdictions where the debate agenda is the necessity of defining a merchant's surcharge cap (as in Australia). Our results assert that the surcharge cap should not exceed the competitive equilibrium price spread that we found, namely, that the surcharge cap must be lower or equal to the difference between the merchant fee and the store credit cost faced by merchants. In particular, this result implies that only if the cost for the retailers of a store credit transaction is equal to or greater than the credit card merchant fee, the no-surcharge rule should be acceptable from the point of view of those who seek to preserve the welfare of the consumers. This is a contrasting result with the model analyzed in Rochet (2003).

Initially, we prove that the single price is not equilibrium when the differentiation is allowed. It is a consequence of each retailer being willing to unilaterally surcharge the credit card payments and deviate from the single price equilibrium stated by the no-surcharge rules.

The result given above leads us to the following question: if price differentiation is allowed, might it provide some degree of market power to the merchants so that they would be able to keep a surcharge on credit card transactions with an average price of all transactions greater than the single price found under the no-surcharge rule? The answer is definitely no. In order to show that, we computed the new equilibrium prices when price differentiation of credit card payments is allowed, and proved that the average price is lower than the single price under the no-surcharge rule. Moreover, the new aggregated consumer welfare in the price differentiation equilibrium is, in general, greater than the corresponding one in the single price equilibrium. Equality would only take place if the interchange fee was at the level that maximizes the consumers' surplus under the no-surcharge rule framework.

We also obtain that the merchants' profits under price differentiation are equal to those under the single price equilibrium. This result raised the following question: would the retailer have incentives to unilaterally deviate towards the single price? We found situations where he would not. In fact, we proved that, if a representative group of consumers see benefits in using store credit instead of credit cards, none of the retailers have incentives to unilaterally deviate from the equilibrium with price differentiation to the single price.

In the last exercise we introduce menu costs associated with the price differentiation to analyze whether this sort of friction may inhibit retailers' incentives to differentiate prices. In this new context, we conclude that there are situations when the strategy of differential pricing may turn out to be equilibrium. In fact, if both retailers charge equilibrium differential prices, the profit of the retailer that incurs in the greater (smaller) menu cost will fall below (increase above) the levels attained under a single price.

Consequently, we identity situations when, despite the existence of menu costs, if both retailers are charging differential prices none of them has incentives to unilaterally change to the single pricing strategy. Actually, if only one merchant charges the single price, he loses market share and margin, reducing his profit. Therefore, there will be a loss of retailers welfare associated to the decision of unilaterally moving toward the single price strategy.

If the menu costs are the same, the both retailers profits are equal the one attained under the differential prices equilibrium without menu costs, which is the same one under non surcharge single price equilibrium. Therefore, retailers are indifferent between these three scenarios. Despite of this indifference of the retailers, consumers can have a higher aggregated utility compared with the one obtained under no surcharge single price equilibrium.

In summary, by using a simple credit card market model we were able to illustrate how the absence of a no-surcharge rule could generate equilibrium prices capable of improving consumer welfare. This may reduce the market power that banks have using the interchange fee, even in the presence of menu costs associated to price differentiation that the retailers may face. We also illustrate a situation when retailers that face smaller menu costs associated to price differentiation have a competitive advantage in a context where differentiation is allowed. Thus, it could be a possible theoretical justification for the empirical evidences of retailers that refuse to accept the imposition of non-surcharge rules.

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Appendix

To prove that, under price differentiation, the single price $\bar{p}$ is not equilibrium and, to find the new equilibrium prices, we first derive the merchant margin, the consumer utility and merchant market share, and use them to find the profit function. Subsequently, we compute the derivatives of the profit function with respect to the base price $\bar{p}_i^r$ and the spread $\bar{\Delta}_i^r$.

Figure A1 shows a decomposition of the merchant's expected margin into several components. The first group of components corresponds to the margin obtained by merchants from consumers that cannot use credit cards, either by not having them, or because the retailer did not adhere to the credit card system. The second group of components corresponds to the retailer margins from the consumers who have a credit card conditioned to his adherence to the credit card system.

Figure A1 – Merchant's expected margin

$$
\begin{aligned}
M_i = \overbrace{(1-x.L_i^r)}^{\text{Cannot use credit cards}}.&\left[\underbrace{\overbrace{H(0).(p_i^r-\gamma-c_S)}^{\text{store credit}} + \overbrace{(1-H(0)).(p_i^r-\gamma)}^{\text{cash}}}_{\text{ordinary purchase}} \right.\\
&\left. + \underbrace{\overbrace{\theta.(p_i^r-\gamma-c_S)}^{\text{store credit}}}_{\text{credit purchase}}\right]\\
+ \underbrace{x.L_i^r}_{\text{can use credit cards}}.&\left[\underbrace{\overbrace{H\left(L_i^c.(f+\Delta_i^c)\right).\left(p_i^r-\gamma-c_S\right)}^{\text{store credit}} + \overbrace{\left(1-H\left(L_i^c.(f+\Delta_i^c)\right)\right).\left(p_i^r-\gamma+L_i^c.(\Delta_i^c-m)\right)}^{\text{credit card and cash}}}_{\text{ordinary purchase}}\right.\\
&\left. + \underbrace{\theta.\left[\overbrace{H(f+\Delta_i^c).\left(p_i^r-\gamma-c_S\right)}^{\text{store credit}} + \overbrace{\left(1-H(f+\Delta_i^c)\right).\left(p_i^r-\gamma+\Delta_i^c-m\right)}^{\text{credit card}}\right]}_{\text{credit purchase}}\right]
\end{aligned}
$$

The formula in Figure A1 can be simplified to derive equation (7). Then the partial derivatives with respect to the base price and the spread are given by

$$\frac{\partial M_i}{\partial p_i^r} = (1+\theta) \tag{A1}$$

and

$$\frac{\partial M_i}{\partial \Delta_i^c} = x.L_i^r.(L_i^c + \theta).\left\{h\left(f + \Delta_i^c\right).(m - c_S - \Delta_i^c) + \left[1 - H\left(f + \Delta_i^c\right)\right]\right\} \tag{A2}$$

Consumers decide which retailer to patronize computing its utility and subtracting the transportation costs of each choice. To calculate the market share we need to identify the indifferent consumer in the interval.

Analogously, Figure A2 below shows a decomposition of the consumers expected utility. The first group of components corresponds to the utility obtained by consumers that cannot use credit cards, either by not having them, or because the retailer did not to adhere to the credit card system. The second group of components corresponds to the utility of consumers that have a credit card and the chosen retailer adhered to the system.

Figure A2 – Consumer's utilitity

$$U_i = \underbrace{(1 - x.L_i^r)}_{\text{cannot use credit card}}.\left(\underbrace{u_0 + \theta.u_1}_{\text{utility of consumption}} - \underbrace{(1+\theta).p_i^r}_{\text{cost of product}} - \underbrace{\int_{\underline{c}_B}^{0} c_B.dH(c_B) - \theta.E(c_B)}_{\text{cost of store credit (ordinary and credit purchases)}}\right)$$

$$+ \underbrace{x.L_i^r}_{\text{can use credit card}}.\left(\begin{array}{l} \underbrace{u_0 + \theta.u_1}_{\text{utility of consumption}} - \underbrace{(1+\theta).p_i^r}_{\text{cost of product}} - \underbrace{\int_{\underline{c}_B}^{L_i^c.(f+\Delta_i^c)} c_B.dH(c_B)}_{\text{cost of store credit (ordinary purchases)}} - \underbrace{\int_{L_i^c.(f+\Delta_i^c)}^{\overline{c}_B} L_i^c.(f + \Delta_i^c).dH(c_B)}_{\text{cost of credit card (ordinary purchase)}} \\ -\theta.\left(\underbrace{\int_{\underline{c}_B}^{f+\Delta_i^c} c_B.dH\left(c_B\right)}_{\text{cost of store credit (credit purchase)}} + \underbrace{\int_{f+\Delta_i^c}^{\overline{c}_B} (f + \Delta_i^c).dH(c_B)}_{\text{cost of credit card (credit purchase)}}\right) \end{array}\right)$$

The expression in Figure A2 can be simplified to derive equation (9), which is used to obtain the market share equation (11). The derivatives of the merchant's market share (11), with respect to the base price and the spread are given by

$$\frac{\partial s_i}{\partial p_i^r} = -\frac{(1+\theta)}{2.t} \tag{A3}$$

and

$$\frac{\partial s_i}{\partial \Delta_i^c} = -(L_i^c + \theta).\frac{x.L_i^r}{2.t}.\left[1 - H\left(f + \Delta_i^c\right)\right] \tag{A4}$$

The merchant's expected profit is the product of the margin and the market share. As we can see below, the expressions (A5) to (A8) are useful to compute the derivatives of the profit function.

We can use the equations (7), (11), (A1) and (A3) to derive the expressions below:

$$\frac{2.t}{(1+\theta)}.\frac{\partial M_i}{\partial p_i^r}.s_i = t + (1+\theta).\left(p_j^r - p_i^r\right) + x.\left(L_i^r.\overline{S}(\Delta_i^c) - L_j^r.\overline{S}(\Delta_j^c)\right) \tag{A5}$$

and

$$\frac{2.t}{(1+\theta)} M_i.\frac{\partial s_i}{\partial p_i^r} = -(1+\theta).(p_i^r - \gamma) + \left(H(0) + \theta\right).c_S + x.L_i^r.\overline{\Gamma}(\Delta_i^c) \tag{A6}$$

We can use the equations (7), (11), (A2) and (A4) to derive the expressions below:

$$\begin{aligned} \frac{2.t}{(L_i^c + \theta)} \frac{\partial s_i}{\partial \Delta_i^c}.M_i = {} & -x.L_i^r.\left[1 - H\left(f + \Delta_i^c\right)\right] \\ & .\left[(1+\theta).(p_i^r - \gamma) - \left(H(0) + \theta\right).c_S - x.L_i^r.\overline{\Gamma}(\Delta_i^c)\right] \end{aligned} \tag{A7}$$

and

$$\begin{aligned} \frac{2.t}{(L_i^c + \theta)}.\frac{\partial M_i}{\partial \Delta_i^c}.s_i = {} & x.L_i^r.\left\{ h\left(f + \Delta_i^c\right).(m - c_S - \Delta_i^c) + \left[1 - H\left(f + \Delta_i^c\right)\right] \right\} \\ & .\left\{ t + (1+\theta).\left(p_j^r - p_i^r\right) + x.\left(L_i^r.\overline{S}(\Delta_i^c) - L_j^r.\overline{S}(\Delta_j^c)\right) \right\} \end{aligned} \tag{A8}$$

Summing up (A5) and (A6), we obtain the derivative of the profit function with respect to the base price

$$\frac{2.t}{(1+\theta)}.\frac{\partial \pi_i}{\partial p_i^r} = \begin{cases} t+(1+\theta).\left(p_j^r - p_i^r\right) + x.\left(L_i^r.\overline{S}(a,\Delta_i^c) - L_j^r.\overline{S}(a,\Delta_j^c)\right) - (1+\theta).(p_i^r - \gamma) \\ + \left(H(0)+\theta\right).c_S + x.L_i^r.\overline{\Gamma}(a,\Delta_i^c) \end{cases} \tag{A9}$$

Analogously, we find the derivatives of the profit function with respect to the spread from (A7) and (A8)

$$\frac{2.t}{(L_i^c+\theta)}.\frac{\partial \pi_i}{\partial \Delta_i^c} = \begin{cases} -x.L_i^r.\left[1-H\left(f+\Delta_i^c\right)\right].\left[(1+\theta).(p_i^r-\gamma) - \left(H(0)+\theta\right).c_S - x.L_i^r.\overline{\Gamma}(a,\Delta_i^c)\right] \\ + x.L_i^r.\left\{ h\left(f+\Delta_i^c\right).(m-c_S-\Delta_i^c) + \left[1-H\left(f+\Delta_i^c\right)\right] \right\} \\ \quad .\left\{ t+(1+\theta).\left(p_j^r - p_i^r\right) + x.\left(L_i^r.\overline{S}(a,\Delta_i^c) - L_j^r.\overline{S}(a,\Delta_j^c)\right) \right\} \end{cases} \quad \text{(A10)]}$$

Proof of Proposition 1:

Substituting (12) in (A10), with $p_i^r = p_j^r = \overline{p}$, $\Delta_i^c = 0$, $L_i^c = 1$and $L_i^r = 1$, we obtain that the derivative of the merchant's expected profit with respect to the spread satisfy the following equation

$$\frac{2.t}{(1+\theta)}.\frac{\partial \pi_i}{\partial \Delta_i^c} = t.x.h\left(f\right).(m-c_S) \tag{A11}$$

where *h* represents the density function of the cumulative distribution function *H*.

Notice that the right hand side of (A11) is positive if all the following conditions are satisfied:

- There are transportation costs ($t > 0$);
- There are card users ($x > 0$) ;

- The density of consumers that are indifferent to the cost of a store credit or a credit card ($c_B = f$) is positive ($h(f) > 0$);
- The merchant fee is greater than the cost of the store credit ($m > c_S$).

We conclude that, if price differentiation is allowed, merchants have incentives to surcharge and, consequently, the single price $\overline{p}$ is not equilibrium.

Proof of Theorem 1:

The first order conditions with respect to the base price and the spread, using equations (A9) and (A10), are given, respectively, by

$$\begin{aligned} & t + (1+\theta).(p_j^r - p_i^r) + x.\left(L_i^r.\overline{S}(a,\Delta_i^c) - L_j^r.\overline{S}(a,\Delta_j^c)\right) \\ & = (1+\theta).(p_i^r - \gamma) - (H(0)+\theta).c_S - x.L_i^r.\overline{\Gamma}(a,\Delta_i^c) \end{aligned} \tag{A12}$$

and

$$\begin{aligned} & x.L_i^r.[1 - H(f+\Delta_i^c)].\{(1+\theta).(p_i^r - \gamma) - (H(0)+\theta).c_S - x.L_i^r.\overline{\Gamma}(a,\Delta_j^c)\} \\ & = x.L_i^r.\{\, h(f+\Delta_i^c).(m - c_S - \Delta_i^c) + [1 - H(f+\Delta_i^c)] \,\} \\ & \qquad .\{\, t + (1+\theta).(p_j^r - p_i^r) + x.\left(L_i^r.\overline{S}(\Delta_i^c) - L_j^r.\overline{S}(a,\Delta_j^c)\right) \,\} \end{aligned} \tag{A13}$$

Substituting (A12) in (A13) we obtain $h(f+\Delta_i^c).(\Delta_i^c - m + c_S) = 0$ and the spread in equilibrium is $\overline{\Delta}_i^c = \overline{\Delta}^c = m - c_S$.

Using (A12) we obtain the following expressions containing the base prices of both retailers

$$\begin{aligned} (1+\theta).(2.p_i^r - p_j^r) &= t + (1+\theta).\gamma + (H(0)+\theta).c_S \\ &\quad + x.\overline{S}(a,\overline{\Delta}^c).(L_i^r - L_j^r) + x.L_i^r.\overline{\Gamma}(a,\overline{\Delta}^c); \end{aligned}$$

Interchanging *i* and *j* and multiplying by 2 (two), we obtain

$$(1+\theta).\left(4.p_j^r - 2.p_i^r\right) = 2.t + 2.(1+\theta).\gamma + 2.\left(H(0)+\theta\right).c_S$$
$$+ 2.x.\overline{S}(a,\overline{\Delta}^c).\left(L_j^r - L_i^r\right) + 2.x.L_j^r.\overline{\Gamma}(a,\overline{\Delta}^c)$$

Summing up the equations above and rearranging we obtain that the equilibrium price satisfies the following equation

$$(1+\theta).p_j^r = t + (1+\theta).\gamma + \left(H(0)+\theta\right).c_S + \frac{x}{3}.\left(L_j^r - L_i^r\right).\overline{\phi}(a,\overline{\Delta}^c) + x.L_j^r.\overline{\Gamma}(a,\overline{\Delta}^c) \qquad \text{(A14)}$$

From equation (A14) above, we obtain

$$(1+\theta).(p_j^r - p_i^r) = -\frac{2.x}{3}.\left(L_i^r - L_j^r\right).\overline{\phi}(a,\overline{\Delta}^c) - x.\left(L_i^r - L_j^r\right).\overline{\Gamma}(a,\overline{\Delta}^c)$$

which can be included in the market share equation (11) to obtain

$$s_i = \frac{1}{2} + \frac{x.\overline{\phi}(a,\overline{\Delta}^c).\left(L_i^r - L_j^r\right)}{6.t} \qquad \text{(A15)}$$

If $\overline{\phi}(a,\overline{\Delta}^c) = \overline{\phi}_\delta > 0$, we conclude from formula (A15) that in equilibrium both retailers adhere to the credit card system, $L_i^r = 1$. This occurs because if one of them decides the contrary, he will lose market share. Consequently, the market share of both retailers are the same, $s_i = s_j = \overline{s} = \frac{1}{2}$.

Therefore, using $L_i^r = L_j^r = 1$ and $\Delta_j^c = \overline{\Delta}^c = m - c_S$, we conclude from (A14) and (8) that the equilibrium value for the base price is given by equation (17).

Actually, to conclude the proof, and show that those prices are not only a local equilibrium, but really a global Nash equilibrium, we need to assess the profit variation when each one of the retailers decides to deviate from the differential prices equilibrium to another set of prices. Since we do this demonstration in more general context in the Proof of Theorem 3 below, we omit here this part of the proof, but we comment there how analyze this case without menu costs as a particular case of the more general case.

Proof of Corollary 1:

From (12) and (A13), since $L_i^r = L_j^r = 1$ and $\Delta_j^c = \overline{\Delta}^c = m - c_S$, we conclude that

$$\overline{p} - \overline{p}^r = x.[1 - H(f)].(m - c_S) \tag{A16}$$

which can be rearranged, using equation (17), to obtain equation (21).

Proof of Theorem 2:

We can use equation (9) to show that the difference between the consumers' aggregated utilities (surplus) in both equilibria is given by

$$\Delta U_i = (1+\theta).(\overline{p} - \overline{p}^r) - x.(1+\theta).\left[\int_f^{f+\overline{\Delta}^c} (c_B - f).dH(c_B) + \overline{\Delta}^c.[1 - H(f + \overline{\Delta}^c)]\right]$$

Substituting equation (A16) into the equation above we obtain

$$\Delta U_i = x.[H(f + \overline{\Delta}^c) - H(f)].(1+\theta).\overline{\Delta}^c - x.(1+\theta).\left[\int_f^{f+\overline{\Delta}^c} (c_B - f).dH(c_B)\right]$$

Rewriting the equation above we obtain that

$$\Delta U_i = x.(1+\theta).\left[\int_f^{f+\overline{\Delta}^c} (f + \overline{\Delta}^c - c_B).dH(c_B)\right] \rightarrow \begin{cases} = 0 & \text{if } \overline{\Delta}^c = 0 \\ > 0 & \text{if } \overline{\Delta}^c \neq 0 \end{cases}$$

or

$$\Delta U_i = x.(1+\theta).\left[\int_{-\delta}^{-\delta+c_S-c_A-a}(c_B+\delta).dH(c_B)\right] \rightarrow \begin{cases} =0 & \text{if } \ a = c_S - c_A \\ >0 & \text{if } \ a \neq c_S - c_A \end{cases}$$

Proof of Corollary 2:

We use equation (7) to show that the difference between the retailers' margins in both equilibria is given by

$$\Delta M_i = (1+\theta).\left\{x.[1-H(f)].\overline{\Delta}^c - (\overline{p} - \overline{p}^r)\right\}$$

which is equal to zero by equation (A16). Since the market shares are equal in both equilibria ($\overline{s} = 1/2$), we conclude that the merchants' profits are also the same. In other words, merchants are indifferent between both equilibria.

Proof of Theorem 3:

From equation (22) we obtain that, if both retailers charge differential prices, the menu costs (μ_i) will decrease the margins.

The first order conditions with respect to the base price and the spread are given, respectively, by

$$\begin{aligned} & t + (1+\theta).\left(p_j^r - p_i^r\right) + x.\left(L_i^r.\overline{S}(a,\Delta_i^c) - L_j^r.\overline{S}(a,\Delta_j^c)\right) \\ & = (1+\theta).(p_i^r - \gamma) - \left(H(0)+\theta\right).c_S - x.L_i^r.\overline{\Gamma}(a,\Delta_i^c) - \mu_i.I(\Delta_i^c) \end{aligned}$$

and

$$\begin{aligned} & x.L_i^r.\left[1-H\left(f+\Delta_i^c\right)\right].\left\{(1+\theta).(p_i^r-\gamma) - \left(H(0)+\theta\right).c_S - x.L_i^r.\overline{\Gamma}(a,\Delta_j^c) - \mu_i.I(\Delta_i^c)\right\} \\ & = x.L_i^r.\left\{ h\left(f+\Delta_i^c\right).(m - c_S - \Delta_i^c) + \left[1-H\left(f+\Delta_i^c\right)\right] \right\} \\ & \quad .\left\{ t + (1+\theta).\left(p_j^r - p_i^r\right) + x.\left(L_i^r.\overline{S}(\Delta_i^c) - L_j^r.\overline{S}(a,\Delta_j^c)\right) \right\} \end{aligned}$$

Denoting by $\overline{\Delta}_i^{c,\mu}$, $\overline{\Delta}_j^{c,\mu}$, $\overline{p}_i^{r,\mu}$ and $\overline{p}_i^{r,\mu}$ the spreads and prices that solve the first order conditions above. We use the same technique applied to prove Theorem 1, to prove that $\overline{\Delta}_i^{c,\mu} = \overline{\Delta}_i^{c,\mu} = \overline{\Delta}^c$ ($\overline{\Delta}^c := m - c$), and that $\overline{p}_i^{r,\mu} = \overline{p}^r + \frac{1}{1+\theta}.\left(\frac{2.\mu_i + \mu_j}{3}\right)$ and $\overline{p}_j^{r,\mu} = \overline{p}^r + \frac{1}{1+\theta}.\left(\frac{\mu_i + 2.\mu_j}{3}\right)$.

Assuming, without loss of generality, that $\mu_i \geq \mu_j$, the retailer i, that incurs in a greater menu cost, charge a equilibrium price greater than retailer j. More precisely, we obtain that $\overline{p}_i^{r,\mu} - \overline{p}_j^{r,\mu} = \frac{1}{1+\theta}\left(\frac{\mu_i - \mu_j}{3}\right) \geq 0$.

If the transportation cost (of one distance unity) is bigger, or equal, the one third of the difference between the price (of $1+\theta$ goods) of both retailers, $t \geq \frac{(\mu_i - \mu_j)}{3}$, and if we define $\alpha := \frac{(\mu_i - \mu_j)}{3.t}$, we have $0 < \alpha < 1$. Under all the conditions above, and using equation (11), we also prove that the equilibrium market shares are, respectively, $\overline{s}_i^{\mu} = \frac{1}{2}.(1-\alpha)$ and $\overline{s}_j^{\mu} = \frac{1}{2}.(1+\alpha)$. Additionally, we use equation (22) to prove that the equilibrium margins are, respectively, $\overline{M}_i^{\mu} = t.(1-\alpha)$ and $\overline{M}_j^{\mu} = t.(1+\alpha)$. We conclude that the equilibrium profits under menu costs are, respectively, $\overline{\pi}_i = \frac{t}{2}.(1-\alpha)^2$ and $\overline{\pi}_j = \frac{t}{2}.(1+\alpha)^2$. Note that $\overline{\pi}_j - \overline{\pi}_i = 4.\alpha > 0$, meaning that profit of the retailer with the biggest menu cost is smaller the profit of retailer with the smaller menu cost.

After solving the first order conditions, we now need to prove that the retailers do not have incentives to unilaterally deviate from those differential prices. Note that the function

$$\varepsilon(a, \Delta_i^c, \Delta_j^c) := \frac{x.(1+\theta)}{2}.\int_{f+\Delta_j^c}^{f+\Delta_i^c} (f + \overline{\Delta}^c - c_B).dH(c_B)$$

is a generalization of the function of $\varepsilon(a)$, since $\varepsilon(a) = \varepsilon(a, 0, \overline{\Delta}^c)$, and that

$$\varepsilon(a, \Delta_i^c, \Delta_j^c) = \frac{x}{2}.\left[\left(\overline{\Gamma}(a, \Delta_j^c) - \overline{\Gamma}(a, \Delta_i^c)\right) - \left(\overline{S}(a, \Delta_j^c) - \overline{S}(a, \Delta_i^c)\right)\right].$$

All the computations below can be reproduced in an analogous way from the point of view of each retailer. Particularly, we fix the price charged by the retailer *j*, p_j, and both spreads, Δ_i and Δ_j, we can write the profit of the retailer *i* as

$$\pi_i(p_i^r) = \left[-\frac{(1+\theta)^2}{2.t}\right].(p_i^r)^2 + \frac{(1+\theta)}{2.t}.\begin{bmatrix} t + (1+\theta).p_j^r + x.\left(\overline{S}(a, \Delta_i^c) - \overline{S}(a, \Delta_j^c)\right) + \\ (1+\theta).\gamma + \left(H(0)+\theta\right).c_S + x.\overline{\Gamma}(\Delta_i^c) + \mu_i.I(\Delta_i^c) \end{bmatrix}.p_i^r$$
$$-\left[(1+\theta).\gamma + \left(H(0)+\theta\right).c_S + x.\overline{\Gamma}(\Delta_i^c) + \mu_i.I(\Delta_i^c)\right]\left[\frac{1}{2} + (1+\theta).\left(\frac{p_j^r}{2.t}\right) + x.\left(\frac{\overline{S}(a, \Delta_i^c) - \overline{S}(a, \Delta_j^c)}{2.t}\right)\right]$$

Note that the price that maximize, globally, the profit function above satisfies the following equation

$$(1+\theta).p_i^{\max} = \frac{1}{2}.\left[(1+\theta).p_j^r + t + (1+\theta).\gamma + \left(H(0)+\theta\right).c_S + x.\overline{\Gamma}(\Delta_i^c) + \mu_i.I(\Delta_i^c) + x.\left(\overline{S}(a, \Delta_i^c) - \overline{S}(a, \Delta_j^c)\right)\right]$$

We can use the expression above to define as $p_i^{\mu,\overline{\Delta}}(a, \Delta_i^c)$ the price of maximum profit for the retailer *i*, when he deviates unilaterally from base price $\overline{p}_i^{r,\mu}$ and the spread $\overline{\Delta}^c$, so the retailer *j* stay charging base price $\overline{p}_i^{r,\mu}$ and the spread $\overline{\Delta}^c$, and decide to charge the spread Δ_i^c. Consequently, the price variation is given by

$$(1+\theta).\left[p_i^{\mu,\overline{\Delta}}(a, \Delta_i^c) - \overline{p}_i^r\right] = x.\left(\overline{S}(a, \Delta_i^c) - \overline{S}(a, \overline{\Delta}^c)\right) - \frac{\mu_i.\left(1 - I(\Delta_i^c)\right)}{2} + \varepsilon(a, \overline{\Delta}^c, \Delta_i^c)$$

Defining $s_i^{\mu,\bar{\Delta}}(a,\Delta_i^c)$ and $M_i^{\mu,\bar{\Delta}}(a,\Delta_i^c)$, respectively, as the market share and the margin after the movement, we can compute the corresponding market share variation and margin variation, repectively,

$$s_i^{\mu,\bar{\Delta}}(a,\Delta_i^c)-\bar{s}_i^{\mu}=\frac{1}{2.t}\left\{\frac{\mu_i.\left(1-I(\Delta_i^c)\right)}{2}-\varepsilon(a,\overline{\Delta}^c,\Delta_i^c)\right\}$$

and

$$M_i^{\mu,\bar{\Delta}}(a,\Delta_i^c)-\overline{M}_i^{\mu}=\frac{\mu_i.\left(1-I(\Delta_i^c)\right)}{2}-\varepsilon(a,\overline{\Delta}^c,\Delta_i^c)$$

Since $\varepsilon(a,\overline{\Delta}^c,\Delta_i^c)>0$ for all Δ_i^c's and $\overline{\Delta}^c=\underset{\Delta_i^c}{\operatorname{argmin}}\left\{\varepsilon(a,\overline{\Delta}^c,\Delta_i^c)\right\}$, and we conclude from the expressions above that the maximum profit is globally attained by the retailer *i* at the spread $\overline{\Delta}^c$ and base price $\bar{p}_i^{r,\mu}$, concluding here the proof of Theorem 3.

Note that the result above has as a particular case, which is the case without menu costs. Then, the proof of the existence of global equilibrium of Theorem 1 can be concluded using the results above assuming $\mu_1=\mu_2=0$.

Below, we will use the expressions above to prove Proposition 2 and to compute the profits in Figure 5 and 6. Particularly, we can use the same expressions above to obtain the market share and margin variations when retailer *i* decides to charge a single price, respectively,

$$s_i^{\mu,\bar{\Delta}}(a,0)-\bar{s}_i^{\mu}=-\frac{\beta_i}{2} \qquad \text{and} \qquad M_i^{\mu,\bar{\Delta}}(a,0)-\overline{M}_i^{\mu}=-t.\beta_i$$

where, under our assumption, we have $0<\beta_i:=\frac{1}{t}.\left(\varepsilon(a)-\frac{\mu_i}{2}\right)<1$. Therefore, we can conclude the profit variation, given by

$$\pi_i^{\mu,\bar{\Delta}}(a,0)-\bar{\pi}_i^{\mu}=-\frac{t}{2}.\beta_i.\left[2.(1-\alpha)-\beta_i\right]$$

which is negative, since, under our assumptions, $0 \le \alpha := \dfrac{(\mu_i - \mu_j)}{3.t} < 1$.

Using the same technique we can compute the market share and margin variations when retailer *i* deviates from the non surcharge single price equilibrium $\overline{p}$, respectively,

$$s_i^{\mu,0}(a,\Delta_i^c) - \overline{s}_i^{\overline{p}} = \frac{1}{2t}\left\{\varepsilon(a,\Delta_i^c,0) - \frac{\mu_i.I(\Delta_i^c)}{2}\right\}$$

and

$$M_i^{\mu,0}(a,\Delta_i^c) - \overline{M}_i^{\overline{p}} = \varepsilon(a,\Delta_i^c,0) - \frac{\mu_i.I(\Delta_i^c)}{2}$$

Now, since $\overline{\Delta}^c = \underset{\Delta_i^c}{\operatorname{argmax}}\{\varepsilon(a,\Delta_i^c,0)\}$, we obtain positive market share and margin variations, respectively,

$$s_i^{\mu,0}(a,\overline{\Delta}^c) - \overline{s}_i^{\overline{p}} = \frac{\beta_i}{2} > 0 \quad \text{and} \quad M_i^{\mu,0}(a,\overline{\Delta}^c) - \overline{M}_i^{\overline{p}} = t.\beta_i > 0$$

and, consequently, positive profit variation

$$\pi_i^{\mu,0}(a,\overline{\Delta}^c) - \overline{\pi}_i^{\overline{p}} = \frac{t}{2}.\beta_i.(2+\beta_i) > 0$$

From those expressions above for the profit variations is straightforward to compute the profits in the Figure 6, as well as, assuming $\mu_1 = \mu_2$, to demonstrate Proposition 2 and to compute all the profits showed in Figures 5.

Proof of Corollary 3:

Denote by $\overline{U}_i^{\overline{p}}$ and $\overline{U}_i^{\mu}$ the utility of the consumers that choose retailer *i*, respectively, under the non surcharge single price equilibrium and under differential price equilibrium with menu costs. To compare both scenarios, we compute their utility variation

$$\overline{U}_i^{\mu} - \overline{U}_i^{p} = 2.\varepsilon(a) - \frac{2\mu_i + \mu_j}{3}$$

Analogously, for consumers that choose retailer *j*. The difference between both utilities reflects the differences on prices, and is given by $\overline{U}_j^{\mu} - \overline{U}_i^{\mu} = \frac{\mu_i - \mu_j}{3}$.

Since the market shares are different depending on the equilibrium, the aggregated utility variation is given by

$$\overline{U}^{\mu} - \overline{U}^{p} = \left[\frac{(1-\alpha)}{2}\overline{U}_i^{\mu} + \frac{(1+\alpha)}{2}\overline{U}_j^{\mu}\right] - \left[\frac{1}{2}\overline{U}_i^{p} + \frac{1}{2}\overline{U}_j^{p}\right]$$

We can rewrite the aggregated utility variation as

$$\overline{U}^{\mu} - \overline{U}^{p} = t.\left[\beta_i + \beta_j + \frac{\alpha^2}{2}\right]$$

where $\alpha := \frac{(\mu_i - \mu_j)}{3.t} > 0$ and $\beta_i := \frac{1}{t}.\left(\varepsilon(a) - \frac{\mu_i}{2}\right) > 0$. Thus, we conclude that the utility increase. However, to conclude the consumer welfare analysis, we also need to compute the aggregated transportation cost variation

$$t.\left(\int_0^{(1-\alpha)/2} s.ds + \int_{(1-\alpha)/2}^{1} (1-s).ds\right) - t.\left(\int_0^{1/2} s.ds + \int_{1/2}^{1} (1-s).ds\right)$$

which is equal to $t.\dfrac{\alpha^2}{4}$. Finally, if we subtract transportation cost variation from the utility variation, we obtain the total consumer welfare variation $t.\left[\beta_i + \beta_j + \dfrac{\alpha^2}{4}\right]$, which is also positive.

Now we analyze the retailer's welfare. We compute the profit variation

$$\left[\frac{t}{2}.(1-\alpha)^2 + \frac{t}{2}.(1+\alpha)^2\right] - t$$

which is equal to $t.\alpha^2$. Then, there is a profit increase if the retailers' menu costs are different.

Particularly, assuming that retailers have the same menu costs, we can define $\mu := \mu_i = \mu_j$ and $\beta := \beta_i = \beta_j$. In this case, it is interesting to note that summing up the aggregated welfare variation of consumers (utility minus transportation cost) and retailers (profit) computed above, we obtain the total welfare gain of consumer and retailers with the differentiation is given by $t.[2.\beta] = 2.\varepsilon(a) - \mu$.

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