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A Sticky-Dispersed Information Phillips Curve: a model with partial and delayed information*

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Abstract

This Working Paper should not be reported as representing the views of the Banco Central do Brasil. The views expressed in the paper are those of the authors and do not necessarily reflect those of the Banco Central do Brasil.

We study the interaction between dispersed and sticky information by assuming that firms receive private noisy signals about the state in an otherwise standard model of price setting with sticky-information. We compute the unique equilibrium of the game induced by the firms' pricing decisions and derive the resulting Phillips curve. The main effect of dispersion is to magnify the immediate impact of a given shock when the degree of stickiness is small. Its effect on persistence is minor: even when information is largely dispersed, a substantial amount of informational stickiness is needed to generate persistence in aggregate prices and inflation.

JEL Classification: D82, D83, E31

Keywords: Sticky information, Dispersed information, Phillips curve

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1 Introduction

Mankiw and Reis (2002) sticky information model achieves two important goals at once: (i) explains why prices fail to respond quickly to nominal shocks and (ii) reconciles the backward-looking behavior needed to generate the observed persistence in aggregate prices with the assumption that agents are fully rational. Nevertheless, in a world in which information becomes more frequently available, the assumption that information is sticky is less plausible. Consequently, one would expect that the power of a sticky-information model in explaining the observed degree of persistence in prices to diminish. Morris and Shin's (2006), however, conjecture that only a small incidence of informational stickiness is needed to generate substantial persistence in aggregate prices in a world of *differential information*. This paper puts such conjecture to test.[1]

We do so by studying how individual firms set prices when information is both sticky *and* dispersed, and analyze the resulting dynamics for aggregate prices and inflation rates. In our model, firms' optimal price is a convex combination of the current state of the economy and the aggregate price level. Nevertheless, as firms do not observe the current state nor other firm's pricing decisions, they have to use the available information to infer the optimal price. As in Mankiw and Reis (2002), only a fraction of firms update their information set at each period. Those who update receive two sources of information: the first piece is the value of all previous periods' states, while the second piece is a noisy, idiosyncratic, private signal about the current state of the economy. Since noisy signals are idiosyncratic, the firms that update their information set will have heterogenous information about the state (as in Morris and Shin (2002) and Angeletos and Pavan (2007)). Hence, in our model, *heterogenous* information disseminates *slowly* in the economy.

Firms must not only form beliefs about the current state but also form beliefs about the other firms' beliefs about the current state of the economy, and so on, so that higher-order beliefs play a key role in our model. Hence, the pricing decisions by firms induce

[1]Morris and Shin's (2006) last sentence is "The incidence of sticky information or rational inattention that is necessary to account for the observed degree of persistence may be quite small when embedded in a world of differential information. "

an incomplete information game among them.

In our main result, we prove that there exists a unique equilibrium of such game. The uniqueness of the equilibrium allows us to unequivocally speak about the sticky-dispersed-information (henceforth, SDI) aggregate price level and Phillips curve. The SDI aggregate price level we derive depends on all the current and past states of the economy. This is so for two reasons. First, there are firms in the economy for which the information set has been last updated in the far past. This is a *direct* effect of sticky information. Second, firms that have just received new information will behave, at least partly, as if they were backward-looking. This happens because of a *strategic* effect: their optimal relative price depends on how they believe all other firms (including those that have outdated information sets) in the economy are setting prices.

From aggregate prices, we are able to derive the SDI Phillips curve and show that inflation also depends on all the current and past states of the economy. This result is linked to the one obtained in Mankiw and Reis (2002), in which inflation depends on past expectations of current economic conditions, due to the fact that firms compute expectations based on outdated information. This is an implication of the stickiness of information in our model and was already present in Mankiw and Reis (2002). In our model, however, in addition to being sticky, information is also noisy and dispersed. The fact that information is noisy leads a firm that has its information set updated in t to find it optimal to place positive weight on the states from periods $t-j$, $j>0$, to predict the state in period t. Hence, in comparison to the model economy stated in Mankiw and Reis (2002), the adjustment of prices to shocks will be slower in an economy with noisy information. Through the complementarities in price setting, the fact that, on top of being noisy, information is dispersed magnifies such effect.

Hence, we establish analytically that dispersion magnifies the effects of stickiness on persistence. To evaluate the quantitative importance of dispersion in increasing the effect of stickiness, we perform some numerical simulations. These simulations suggest that, on the one hand, in settings with small incidence of sticky information, dispersion plays at most a minor role in generating persistence. However, for such cases, dispersion substantially magnifies the immediate impact of a given shock. Without a substantial amount of price stickiness, although much larger than what would have been if there were no dispersion, the effect of a shock tends to quickly vanish.

On the other hand, when there is a large amount of price stickiness, the effects of dispersion on both the immediate impact of a given shock and its persistence in the economy tend to be very small. Therefore, in a model of sticky-dispersed information, what is key for the generation of persistence is a substantial amount of price stickiness.

Related Literature. To the best of our knowledge, this is the first paper to build a dynamic model of pricing setting decisions where information is both sticky and dispersed. There is, however, a large number of papers that are connected to what we do. Among those papers, in addition to those that were already mentioned, our work follows a large number of papers that sheds new light into the tradition that dates back to Phelps (1968) and Lucas (1972) of considering the effects of imperfect information on price-setting decisions. Mankiw and Reis (2010) provide the most recent survey on the impact of informational frictions on pricing decisions, comparing a partial (dispersed) information model with a delayed (sticky) information model, and deriving their common implications.[2] In turn, Angeletos and La'O (2009) introduce dispersed information (and explicitly discuss the role of higher-order beliefs) in an otherwise standard setting with sticky prices *à la* Calvo (1983). Veldkamp (2009) covers a myriad of topics related to informational asymmetries and information acquisition in macroeconomics and finance.

Our paper connects to this broad literature through two strands. In our model, information is sticky (as in, for example, Mankiw and Reis (2002)) and dispersed (as in, among others, Morris and Shin (2002)). Also, by focusing on informational stickiness (rather than price stickness), we complement the analysis of Angeletos and La'O (2009).

The paper that is the closest to ours is Baeriswyl and Cornand (2010), who also combines in a single model both dispersed information and informational stickiness to

[2] The theories of "rational inattention" proposed by Sims (2009, 2003) and "inattentiveness" proposed by Reis (2006a, 2006b) have been used to justify models of dispersed information and sticky information.

study the signaling role of policy actions. There are two main differences between their paper and ours. First, and most important, the relevant fundamental in our economy evolves dynamically, whereas they assume that that the fundamental is i.i.d. over time. Dynamics change substantially the way agents form their beliefs. Over time, different agents have different pieces of information regarding the fundamental. In fact, since the fundamental evolves according to a process which has time dependence, any two agents which had their information sets updated in different points of time will have different beliefs regarding the state. As a consequence, at a given point in time, there is heterogeneity in beliefs regarding the fundamental (and aggregate prices) even within the agents who have outdated information sets. Such cross-section of beliefs has non-trivial impacts on both aggregate prices and inflation. Also, by explicitly incorporating dynamics, we are also able to obtain impulse responses of aggregate prices and inflation rates to structural and informational shocks that hit the economy. Second, we focus on a different question. While they are interested in understanding the signaling role of policy actions in a world of dispersed information, we wish to understand the interaction between informational stickiness and dispersion and their (combined) effect on aggregate prices and inflation.

Organization. The paper is organized as follows. We describe the set-up of the model in Section 2 and derive the unique equilibrium of the pricing game played by the firms in Section 3. In Section 4, we analytically derive the contributions of stickiness and dispersion on a measure of persistence of a shock in our setting and perform some numerical simulations to evaluate the quantitative importance of each of these factors. Section 5 draws the concluding remarks. All derivations that are not in the text can be found in the Appendix.

2 The Model

The model is a variation of Mankiw and Reis's (2002) sticky information model.[3] There is a continuum of firms, indexed by $i \in [0,1]$, that set prices at every period $t \in \{1, 2, ...\}$.

Although prices can be re-set at no cost at each period, information regarding the state of the economy is made available to the firms infrequently. At period t, only a fraction $\lambda \in (0,1)$ of firms is selected to update their information sets about the current state. For simplicity, the probability of being selected to adjust information sets is the same across firms and independent of history.

We depart from a standard sticky-information model by allowing information to be *heterogeneous* and *dispersed*: a firm that updates its information set receives public information regarding the past states of the economy as well as a *private* signal about the current state.

Pricing Decisions. Under complete information, any given firm $z \in [0,1]$ set its (log-linear) price $p_t(z)$ equal to the optimal price decision p_t^* given by

$$p_t^* \equiv rP_t + (1-r)\,\theta_t, \tag{1}$$

where $P_t \equiv \int_0^1 p_t(z)\,dz$ is the aggregate price level, and θ_t is the nominal aggregate demand, the current state of the economy. This pricing rule is standard, and, although we don't do it explicitly, can be derived from a firm's profit maximization problem in a model of monopolistic competition in the spirit of Blanchard and Kiyotaki's (1987).[4]

Information. The state θ_t follows a random walk

$$\theta_t = \theta_{t-1} + \epsilon_t, \tag{2}$$

[3] Subsequent refinements of the sticky information models can be found in Mankiw and Reis (2006, 2007, 2010) and Reis (2006a, 2006b, 2009).

[4] See Woodford (2002) for details.

with $\epsilon_t \sim N(0, \alpha^{-1})$.

If firm $z \in [0,1]$ is selected to update its information set in period t, it observes all *previous* periods realizations of the state, $\{\theta_{t-j}, j \geq 1\}$. Moreover, it obtains a noisy private signal about the current state. Denoting such signal by $x_t(z)$, we follow the literature and assume:

$$x_t(z) = \theta_t + \xi_t(z), \tag{3}$$

where $\xi_t(z) \sim N\left(0, \beta^{-1}\right)$, β is the precision of $x_t(z)$, and the error term $\xi_t(z)$ is independent of ϵ_t for all z, t.

As a result, if one defines

$$\Theta_{t-j} = \{\theta_{t-k}\}_{k=j}^{\infty}, \tag{4}$$

at period t, the information set of a firm z that was selected to update its information j periods ago is

$$I_{t-j}(z) = \{x_{t-j}(z), \Theta_{t-j-1}\}. \tag{5}$$

3 Equilibrium

Using (1), the best response for a firm z that was selected to update its information j periods ago – and, therefore, has $I_{t-j}(z)$ as its information set – is its forecast of p_t^*, given the available information $I_{t-j}(z)$:

$$p_t(z) = E\left[p_t^* \mid I_{t-j}(z)\right]. \tag{6}$$

Denoting by Λ_{t-j} the set of firms that last updated its information set at period $t-j$, we can express the aggregate price level P_t as

$$\begin{aligned} P_t &= \int_{\cup_{j=0}^{\infty}\Lambda_{t-j}} p_t(z)\, dz \\ &= \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E\left[p_t^* \mid I_{t-j}(z)\right] dz. \end{aligned} \tag{7}$$

Since the optimal price p_t^* is a convex combination of the state, θ_t, and the aggregate price level, firm z needs to forecast the state of the economy *and* the pricing behavior of the other firms in the economy. The pricing behavior of each of these firms, in turn, depends on their own forecast of the other firms' aggregate behavior. It follows that firm z must not only forecast the state of the economy but also, to predict the behavior of the other firms in the economy, must make forecasts of these firms' forecasts about the state, forecasts about the forecasts of these firms forecasts about the state, and so on and so forth. In other words, higher-order beliefs will play a key role in the derivation of equilibrium in our model.

Indeed, if one defines the average k-th order belief about the current state recursively as follows:

$$\bar{E}^k[\theta_t] = \begin{cases} \theta_t, & : k = 0, \\ \sum_{j=0}^{\infty} \int_{\Lambda_{t-j}} E\left[\bar{E}^{k-1}[\theta_t] \mid I_{t-j}(z)\right] dz, & : k \geq 1, \end{cases} \tag{8}$$

we have that, in equilibrium, the aggregate price level is

$$P_t = (1-r) \sum_{k=1}^{\infty} r^{k-1} \bar{E}^k[\theta_t]. \tag{9}$$

3.1 Computing the Equilibrium

In this section, we derive the unique equilibrium of the pricing game played by the firms. First, we obtain the higher-order beliefs. After, we compute the aggregate price level in period t as a weight average of all higher-order beliefs about the state θ_t.[5]

3.1.1 Posterior Distribution

In the Appendix, we show that, given the distribution of the private signals and the process $\{\theta_t\}$ implied by (2), a firm z that updated its information set in period $t-j$ makes use of the variables $x_{t-j}(z) = \theta_{t-j} + \xi_{t-j}(z)$ and $\theta_{t-j-1} = \theta_{t-j} - \epsilon_{t-j}$, to form

[5] We could also follow Morris and Shin (2002) and first derive an equilibrium for which the aggregate price level is a linear function of fundamentals. We could then establish, using (9), that this linear equilibrium is the unique equilibrium of our game.

the following belief about the current state θ_{t-j}:

$$\theta_{t-j} \mid I_{t-j}(z) \sim N\left((1-\delta)\, x_{t-j}(z) + \delta\theta_{t-j-1}, (\alpha+\beta)^{-1}\right), \tag{10}$$

where

$$\delta \equiv \frac{\alpha}{\alpha+\beta} \in (0,1). \tag{11}$$

Hence, a firm that updated its information set in $t-j$ expects the current state to be a convex combination of the private signal $x_{t-j}(z)$ and a (semi-) public signal θ_{t-j-1} – the only relevant piece of information that comes from learning all previous states $\{\theta_{t-j-k}\}_{k\geq 1}$.[6] The relative weights given to $x_{t-j}(z)$ and θ_{t-j-1} when the firm computes the expected value of state θ_{t-j} depend on the precision of such signals.

Using (2), one has that, for $m \leq j$,

$$\theta_{t-m} = \theta_{t-j} + \textstyle\sum_{k=0}^{j-m-1} \epsilon_{t-m-k}. \tag{12}$$

Thus, the expectation of a firm z that last updated its information set at $t-j$ about θ is

$$E\left[\theta_{t-m} \mid I_{t-j}(z)\right] = \begin{cases} E\left[\theta_{t-j} \mid I_{t-j}(z)\right] = (1-\delta)\, x_{t-j}(z) + \delta\theta_{t-j-1} & : m \leq j, \\ \theta_{t-m} & : m > j. \end{cases} \tag{13}$$

In words, a firm that last updated its information set in period $t-j$ expects that all future values of the fundamental θ will be the same as the expected value of the fundamental at the period $t-j$. Moreover, since at the moment it adjusts its information set the firm observes all previous states, the firm will know for sure the value of θ_{t-m} for $m > j$.

[6] θ_{t-j-1} is the only piece of information in $\Theta_{t-j} = \{\theta_{t-j-k}\}_{k=1}^{\infty}$ the firm needs to use because the state's process is Markovian.

3.1.2 Beliefs

We establish that there is a unique linear equilibrium in the game by computing the aggregate price level in period t as an weighed average of all (average) higher-order beliefs about the state θ_t, as stated in (9).

First-Order Beliefs. Using (13), we are able to compute (8) for the case $k = 1$.

$$\bar{E}^1 [\theta_t] = \lambda \textstyle\sum_{j=0}^{\infty} (1-\lambda)^j [(1-\delta)\theta_{t-j} + \delta\theta_{t-j-1}] . \tag{14}$$

Higher-Order Beliefs. In the Appendix, we use (14) and the recursion (8) to derive the following useful result:

Lemma 1 *The average k-th order forecast of the state is given by*

$$\bar{E}^k [\theta_t] = \lambda \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m [\kappa_{m,k}\theta_{t-m} + \delta_{m,k}\theta_{t-m-1}] , \tag{15}$$

with the weights $(\kappa_{m,k}, \delta_{m,k})$ are recursive defined for $k \geq 1$

$$\begin{bmatrix} \kappa_{m,k+1} \\ \delta_{m,k+1} \end{bmatrix} = [1-(1-\lambda)^m]^k \begin{bmatrix} (1-\delta) \\ \delta \end{bmatrix} + A_m \begin{bmatrix} \kappa_{m,k} \\ \delta_{m,k} \end{bmatrix},$$

where the matrix A_m is given by

$$A_m \equiv \begin{bmatrix} \left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right] + \delta\left[1-(1-\lambda)^m\right]\right] & 0 \\ \delta\left[\left[1-(1-\lambda)^{m+1}\right] - \left[1-(1-\lambda)^m\right]\right] & \left[1-(1-\lambda)^{m+1}\right] \end{bmatrix},$$

and the initial weights are $(\kappa_{m,1}, \delta_{m,1}) \equiv (1-\delta, \delta)$.

3.1.3 Equilibrium Price Level and SDI Phillips curve

We obtain the equilibrium aggregate price level by plugging (15) into expression (9) for P_t and the SDI Phillips curve by taking the first difference.

Proposition 1 *In an economy in which information is sticky and dispersed, and the state follows (2), there is a unique equilibrium in the pricing game played by the firms. In such equilibrium, the aggregate price level is given by*

$$P_t = \textstyle\sum_{m=0}^{\infty} K_m \left[(1-\Delta_m) \theta_{t-m} + \Delta_m \theta_{t-m-1} \right], \tag{16}$$

and the SDI Phillips curve is given by

$$\pi_t = \textstyle\sum_{m=0}^{\infty} K_m \left[(1-\Delta_m) (\theta_{t-m} - \theta_{t-m-1}) + \Delta_m (\theta_{t-m-1} - \theta_{t-m-2}) \right], \tag{17}$$

where

$$K_m \equiv \frac{(1-r)\lambda(1-\lambda)^m}{(1-r[1-(1-\lambda)^m])\left(1-r\left[1-(1-\lambda)^{m+1}\right]\right)}, \tag{18}$$

$$\Delta_m \equiv \frac{\delta[1-r[1-(1-\lambda)^m]]}{1-r\left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right]+\delta[1-(1-\lambda)^m]\right]}. \tag{19}$$

Note that the current aggregate price level P_t depends on current and past states of the economy. This is so for two reasons. First, there are firms in the economy for which the information set has been last updated in the far past. This is a *direct* effect of sticky information, captured by the term K_m. Second, even firms that have just adjusted their information set will be, at least partly, backward-looking. This happens because of a *strategic* effect: their optimal relative price depends on how they believe all other firms (including those that have outdated information sets) in the economy are setting prices. This strategic effects is captured by the terms Δ_m. While they depend on δ, they reflect a non trivial interaction between dispersion and stickiness due to strategic complementarity in pricing decisions. As the weight firms attach to other firms' behavior vanishes, $r \to 0$, the effects of dispersion and stickiness are completely disentangled in the resulting weights $\Delta_m \to \delta$ and $K_m \to \lambda(1-\lambda)^m$.

From aggregate prices, it is immediate to show that inflation also depends on the current and all past states of the economy. This result is linked to the one obtained in Mankiw and Reis (2002), in which inflation depends on past expectations of current economic conditions, due to the fact that in our model, as shown in (9), individual expectations about the current state *are* functions of the past states os the economy.

In our model, however, on top of being sticky, information is also *dispersed*. The effect of dispersion is captured by the positive weight given to the state in period θ_{t-m-1} by a firm that has its information set updated in $t-m$. If, instead of having a private signal of θ_{t-m}, firms knew the state, they would ignore the information given by θ_{t-m-1}. But, as the private signal the firm observes is noisy, it is always optimal to place some weight on past states to forecast the current state. Hence, in comparison to an economy à la Mankiw and Reis (2002), the adjustment of prices to shocks will be slower in an economy with disperse information. This result, in line with Morris and Shin (2006), shows that the introduction of differential information in an otherwise standard sticky information model tends to magnify the effect of stickiness on the persistence of aggregate prices and inflation rates. In the next section, we aim to quantify the potential contribution of dispersion for persistence.

As a side remark, we point out that the introduction of dispersion in a sticky information model leads to price and inflation inertia irrespective of assumptions regarding the firms' capacity to predict equilibrium outcomes. Indeed, although they may not have their information sets up to date, the firms in our model correctly predict the equilibrium behavior of their opponents. In spite of correctly predicting the *strategies* (i.e., contingent plans) adopted by the opponents in equilibrium, a firm cannot infer what is the actual price set by them (i.e., the action taken), since it cannot observe its opponents' private signals. Hence, a firm that hasn't updated its information set cannot infer the current state from the behavior of its opponents .This is in contrast to Mankiw and Reis (2002) who, in order to obtain price and information inertia in a model with sticky but non-dispersed information, (implicitly) assume that agents cannot condition on equilibrium behavior from the opponents. In fact, in their main experiment, there is a (single) nominal shock that only a fraction of the firms observe. Trivially, the prices set by those firms (as well as aggregate prices) will reflect such change in the fundamental. Hence, a firm that hasn't observed the shock but can predict the equilibrium behavior of the opponents will be able to infer the fundamental from such behavior.[7] It follows that *all* firms will adjust prices in response.

4 The Relative Importance of Stickiness and Dispersion of Information for Persistence

This section evaluates the importance of dispersion in a setting in which information is sticky. We first define a measure of persistence in our setting. We then (i) analytically derive the effects of the parameters that capture stickiness and dispersion in the model on our measure of persistence and (ii) perform some numerical simulations to evaluate the quantitative importance of each of these factors.

4.1 The Effects of Dispersion and Stickiness on Persistence

Note that we can rewrite (17) as

$$\pi_t = \textstyle\sum_{m=0}^{\infty} c_m \pi_{C,t-m}, \tag{20}$$

where the coefficients c_m, $m \geq 0$, are given by

$$c_m \equiv \begin{cases} \left(\frac{1-r}{r}\right)\left[\frac{1}{1-r(1-\rho)} - 1\right] & : m = 0, \\ \left(\frac{1-r}{r}\right)\left[\frac{1}{1-r[1-\rho(1-\lambda)^m]} - \frac{1}{1-r\left[1-\rho(1-\lambda)^{m-1}\right]}\right] & : m \geq 1, \end{cases} \tag{21}$$

and $\rho = 1 - \lambda(1-\delta)$.

Therefore, using equations (2), that describes the evolution of θ, and (17), one can write the SDI inflation as

$$\pi_t = \textstyle\sum_{k=0}^{\infty} c_k \varepsilon_{t-k}.$$

[7] The argument here is similar to the one in Rational Expectations Equilibrium models as in Grossman (1981).

The *direct* impact of a period t shock, ε_t, on the inflation rate in period $t+m$, π_{t+m}, is c_m. Hence, the *cumulative* impact of a shock ε_t on period π_{t+m} is

$$\begin{aligned} C_m &= \textstyle\sum_{k=0}^{m} c_k \\ &= \left(\frac{1-r}{r}\right)\left[\frac{1}{1-r+r\rho\,(1-\lambda)^m}-1\right]. \end{aligned}$$

Since

$$C_\infty = \lim_{m\to\infty} \textstyle\sum_{k=0}^{m} c_k = 1,$$

$1-C_m$ can be interpreted as the impact of a shock ε_t that still remains to be propagated after m periods. Hence, we take $1-C_m$ as the measure of persistence of a shock in our setting. We now evaluate how dispersion and stickiness affect persistence.

Not surprisingly, when information becomes less sticky, persistence falls

$$\begin{aligned} \frac{\partial\,(1-C_m)}{\partial\lambda} &= \left[\frac{1-r}{\left[1-r+r\rho\,(1-\lambda)^m\right]^2}\right]\frac{\partial\rho\,(1-\lambda)^m}{\partial\lambda} \\ &= -\left(\frac{1-r}{r}\right)\left[\frac{1}{\left[1-r+r\rho\,(1-\lambda)^m\right]^2}\right]\left[(1-\lambda)^m(1-\delta)+\rho m\,(1-\lambda)^{m-1}\right] < 0. \end{aligned}$$

It is natural to take δ, the weight placed by an agent on the previous values of the state vis à vis his private information, as a measure of information dispersion.[8] The higher δ, the more dispersed is the information. Hence, as expected, dispersion increases the persistence of a shock.

$$\frac{\partial\,(1-C_m)}{\partial\delta} = -\frac{\partial C_m}{\partial\delta} = \frac{(1-r)\,\lambda\,(1-\lambda)^m}{\left[1-r+r\rho\,(1-\lambda)^m\right]^2} > 0,$$

whenever $\lambda\in(0,1)$.

We are also interested in understanding how the effect of stickiness on persistence varies with the degree of dispersion as captured by δ. Notice that

$$\frac{\partial^2\,(1-C_m)}{\partial\lambda\partial\delta} = \left(\frac{1-r}{r}\right)\left[\frac{(1-\lambda)^m}{\left[1-r+r\rho\,(1-\lambda)^m\right]^2}\right] > 0,$$

[8] The weight δ depends on the relative precision of an agent's private information, β.

so that the effect of sticky information on persistence is higher in a world of differential information.

We summarize this discussion in the following result.

Proposition 2 *For a given degree of dispersion (stickiness), the higher the degree of stickiness (dispersion), the more persistent in the effect of a shock on inflation rates. Moreover, the effect of stickiness on persistence increases with the amount of dispersion in the economy.*

4.2 Numerical Simulations

Proposition 2 shows that dispersion magnifies the effects of stickiness. Through numerical simulations, we now aim to evaluate whether this effect is quantitatively relevant. In particular, we want to verify wether a small incidence of price stickiness can lead to a large amount of persistence in aggregate prices in a world of differential information.

A first hint of an answer comes from the fact that both $\frac{\partial(1-C_m)}{\partial\delta}$ and $\frac{\partial^2(1-C_m)}{\partial\lambda\partial\delta}$ are very close to zero when λ, the fraction of agent getting new information, is close to one. Our simulations confirm such impression.

Figure 1 deals with the case in which information stickiness is small ($\lambda = 0.95$) and one raises the amount of dispersion by moving from $\delta = 0.01$ to $\delta = 0.99$. The effect on persistence is minimal. While the shock fully vanishes after 3 periods when $\delta = 0.01$, it takes only an extra period to vanish when $\delta = 0.99$. It seems that, when the degree of price stickiness is low, the main effect of dispersion (in *any* degree) is to substantially magnify the immediate impact of a given shock. Indeed, when compared to a setting in which $\lambda = 0.05$, the price increase after a shock when $\lambda = 0.95$ is larger by a factor of 40 (see Figure 2).

Figure 2 analyzes the effect of dispersion when information stickiness is large ($\lambda = 0.05$). It is worth noticing that the effect of a substantial increase in the amount of dispersion (moving from $\delta = 0.01$ to $\delta = 0.99$) is minor both in terms of the size of the impact of a shock as well as in terms of its persistence.

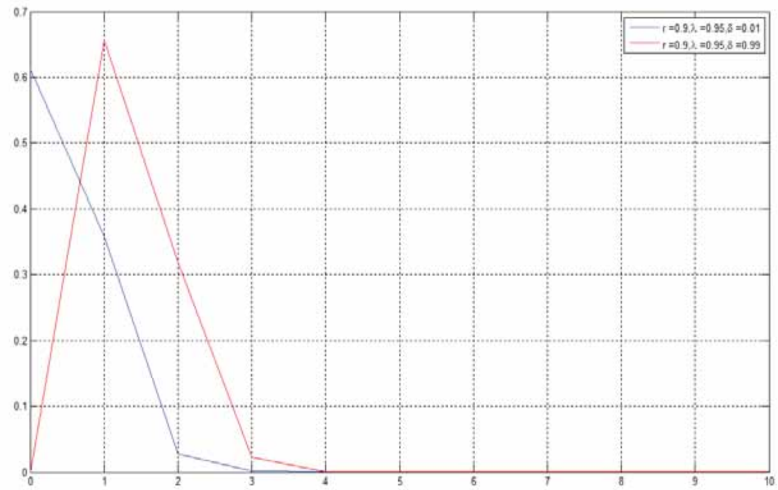

Figure 1: The effect of informaton dispersion in the persistence of aggregate prices when information stickiness is small.

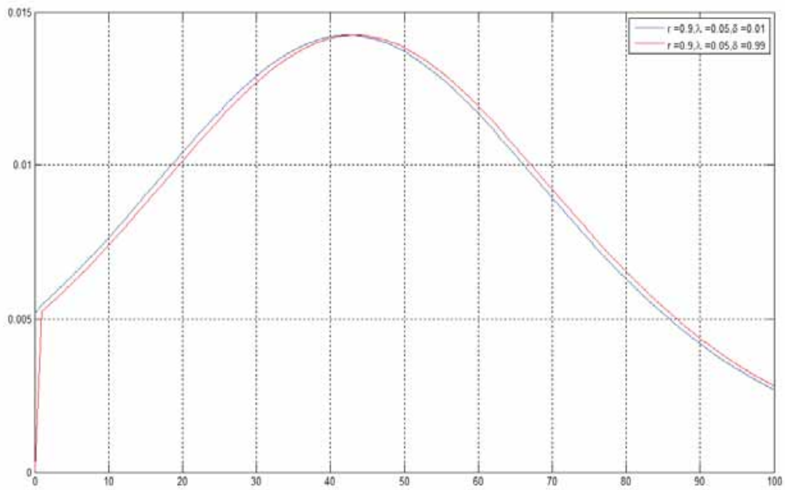

Figure 2: The effect of informaton dispersion in the persistence of aggregate prices when information stickiness is large.

This is true throughout all simulations we have performed: whenever the degree of price stickiness is large, the quantitative contribution of dispersion tends to be small.

5 Conclusion

In this paper, we have considered the impact of sticky and dispersed information on individual price setting decisions, and the resulting effect on the aggregate price level and the inflation rate. We also evaluated Morris and Shin's (2006) conjecture that, in a world of dispersed information, a small incidence of information stickiness can lead to large amounts of persistence in aggregate prices. Contrary to their conjecture, we show that, with a small incidence of price stickiness, while dispersion substantially magnifies the immediate impact of a shock, its effect on persistence tends to be minor.

The model we put forth nests the dispersed information model and the sticky information model as special cases, and can be extended in many directions. One could, for instance, consider the case in which agents receive public information from, say, a central banker about the state of the economy. In such a setting, it would be natural to evaluate what is the best disclosure policy for a benevolent central banker. One could also use our model to analyze how communication interacts with other policy instruments (e.g., interest rates) available to a central banker. We believe these extensions/applications are interesting avenues for future research.

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6 Appendix

6.1 Posterior Distribution

At this appendix, we calculate the distribution of the fundamental θ_{t-j} given that the firm updated its information set at period $t-j$. We can compute $f\left(\theta_{t-j} \mid \Theta_{t-j-1}, x_{t-j}\right)$ as

$$
\begin{aligned}
f\left(\theta_{t-j} \mid \theta_{t-j-1}, x_{t-j}\right) &= \frac{f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right)}{\int_{-\infty}^{\infty} f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right) d\theta_{t-j}} \\
&= \frac{f\left(\theta_{t-j-1}, x_{t-j} \mid \theta_{t-j}\right) f\left(\theta_{t-j}\right)}{\int_{-\infty}^{\infty} f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right) d\theta_{t-j}} \\
&= \frac{f\left(\theta_{t-j-1} \mid \theta_{t-j}\right) f\left(x_{t-j} \mid \theta_{t-j}\right) f\left(\theta_{t-j}\right)}{\int_{-\infty}^{\infty} f\left(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}\right) d\theta_{t-j}},
\end{aligned}
$$

where the last equality holds due to the independence of $\xi_t(z)$ and ϵ_{t-j}. As

$$
\begin{aligned}
x_{t-j}(z) &= \theta_{t-j} + \xi_{t-j}(z), \\
\theta_{t-j-1} &= \theta_{t-j} - \epsilon_{t-j},
\end{aligned}
$$

where $\xi_t(z) \sim N\left(0, \beta^{-1}\right)$ and $\epsilon_{t-j} \sim N\left(0, \alpha^{-1}\right)$, we have that $f\left(x_{t-j} \mid \theta_{t-j}\right) = N\left(\theta_{t-j}, \beta^{-1}\right)$ and $f\left(\theta_{t-j-1} \mid \theta_{t-j}\right) = N\left(\theta_{t-j}, \alpha^{-1}\right)$. If the dynamics of θ_t was

$$
\theta_{t-j-1} = \rho\theta_{t-j} - \epsilon_{t-j}
$$

we would have

$$
\begin{aligned}
E\left[\theta_{t-j}\right] &= E\left[\theta_t\right] = \frac{E\left[\epsilon_t\right]}{1-\rho} = 0, \\
Var\left[\theta_{t-j}\right] &= Var\left[\theta_t\right] = \frac{Var\left[\epsilon_t\right]}{1-\rho^2} = \frac{\alpha^{-1}}{1-\rho^2}.
\end{aligned}
$$

Therefore, the distribution of θ_{t-j} would be given by $f(\theta_{t-j}) = N(0, \Psi^{-1})$ where $\Psi = \alpha(1-\rho^2)$. Thus, we would obtain

$$\begin{aligned} f(\theta_{t-j}, \theta_{t-j-1}, x_{t-j}) &= c \exp\left\{-\frac{1}{2}\left[\frac{(x_{t-j}(z) - \theta_{t-j})^2}{\beta^{-1}} + \frac{(\theta_{t-j-1} - \rho^{-1}\theta_{t-j})^2}{(\rho^2\alpha)^{-1}} + \frac{\theta_{t-j}^2}{\Psi^{-1}}\right]\right\} \\ &= c \exp\left\{-\frac{1}{2}\left[(\beta+\alpha+\Psi)\theta_{t-j}^2 - 2(\beta x_{t-j}(z) + \alpha\rho\theta_{t-j-1})\theta_{t-j}\right]\right\} \\ &\quad \times \exp\left\{-\frac{1}{2}\left[\beta x_{t-j}^2(z) + \alpha\rho^2\theta_{t-j-1}^2\right]\right\} \\ &= cd\frac{1}{\sqrt{2\pi}\sigma\Sigma}\exp\left\{-\frac{1}{2}\frac{(\theta_{t-j}-\mu)^2}{\Sigma^2}\right\}, \end{aligned}$$

where

$$\begin{aligned} c &= (2\pi)^{-3/2}(\beta\alpha\Psi)^{1/2}, & d &= \sqrt{2\pi}\sigma\exp\left\{-\tfrac{1}{2}\left[-\mu^2\Sigma^{-2} + \beta x_{t-j}^2(z) + \alpha\rho^2\theta_{t-j-1}^2\right]\right\}, \\ \mu &= [\Delta x_{t-j}(z) + (1-\Delta)z_{t-j-1}], & \Delta &= \beta(\beta+\alpha+\Psi)^{-1}, \\ z_{t-j-1} &= \Lambda\rho\theta_{t-j-1}, & \Lambda &= \alpha(\beta+\alpha)^{-1}, \\ \Sigma^2 &= (\beta+\alpha+\Psi)^{-1}. \end{aligned}$$

As $\rho \to 1$, we have $\Psi \to 0$, $\Delta \to \delta$, and $\Sigma^2 \to (\beta+\alpha)^{-1}$. Thus $f(\theta_{t-j} \mid \theta_{t-j-1}, x_{t-j}) = N(\mu, \sigma^2)$ where $\mu = [\delta x_{t-j}(z) + (1-\delta)\theta_{t-j-1}]$, and $\sigma^2 = (\beta+\alpha)^{-1}$.

6.2 Higher-Order Beliefs

In this appendix we derive the general formula of the k-th order average expectation

$$\bar{E}^k[\theta_t] = \lambda \textstyle\sum_{m=0}^{\infty}(1-\lambda)^m[\kappa_{m,k}\theta_{t-m} + \delta_{m,k}\theta_{t-m-1}],$$

with the weights $(\kappa_{m,k}, \delta_{m,k})$ are recursive defined for $k \geq 1$

$$\begin{bmatrix}\kappa_{m,k+1} \\ \delta_{m,k+1}\end{bmatrix} = \begin{bmatrix}(1-\delta) \\ \delta\end{bmatrix}[1-(1-\lambda)^m]^k + A_m\begin{bmatrix}\kappa_{m,k} \\ \delta_{m,k}\end{bmatrix},$$

where the matrix A_m is given by

$$A_m \equiv \begin{bmatrix} \left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right]+\delta\left[1-(1-\lambda)^{m}\right]\right] & 0 \\ \delta\left[\left[1-(1-\lambda)^{m+1}\right]-\left[1-(1-\lambda)^{m}\right]\right] & \left[1-(1-\lambda)^{m+1}\right] \end{bmatrix},$$

and the initial weights are $(\kappa_{1,k}, \delta_{1,k}) \equiv (1-\delta, \delta)$.

We start by computing $\bar{E}^1[\theta_t]$ as

$$\begin{aligned} \bar{E}^1[\theta_t] &= \textstyle\sum_{j=0}^{\infty} \int_{\Lambda_j} E\left[\bar{E}^0[\theta_t] \mid I_{t-j}(z)\right] dz \\ &= \textstyle\sum_{j=0}^{\infty} \int_{\Lambda_j} E\left[\theta_t \mid I_{t-j}(z)\right] dz \\ &= \textstyle\sum_{j=0}^{\infty} \int_{\Lambda_j} \left[(1-\delta)\, x_{t-j}(z) + \delta\theta_{t-j-1}\right] dz \\ &= \lambda \textstyle\sum_{j=0}^{\infty} (1-\lambda)^j \left[(1-\delta)\,\theta_{t-j} + \delta\theta_{t-j-1}\right]. \end{aligned}$$

We can use this result to obtain $\bar{E}^2[\theta_t]$ as

$$\begin{aligned} \bar{E}^2[\theta_t] &= \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} E\left[\bar{E}^1[\theta_t] \mid I_{t-m}(z)\right] dz \\ &= \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{\infty} (1-\lambda)^j\, E\left[(1-\delta)\,\theta_{t-j} + \delta\theta_{t-j-1} \mid I_{t-m}(z)\right] dz. \end{aligned}$$

We know that

$$E\left[\theta_{t-j} \mid I_{t-m}(z)\right] = \begin{cases} (1-\delta)\, x_{t-m}(z) + \delta\theta_{t-m-1} & : m \geq j, \\ \theta_{t-j} & : m < j. \end{cases}$$

Thereafter

$$\begin{aligned} \bar{E}^2[\theta_t] = {} & \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{m-1} (1-\lambda)^j \left\{(1-\delta)\, E\left[\theta_{t-j} \mid I_{t-m}(z)\right] + \delta E\left[\theta_{t-j-1} \mid I_{t-m}(z)\right]\right\} dz \\ & + \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} (1-\lambda)^m \left\{(1-\delta)\, E\left[\theta_{t-m} \mid I_{t-m}(z)\right] + \delta\theta_{t-m-1}\right\} dz \\ & + \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=m+1}^{\infty} (1-\lambda)^j \left[(1-\delta)\,\theta_{t-j} + \delta\theta_{t-j-1}\right] dz \end{aligned}$$

$$
\begin{aligned}
= \;& \lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{m-1} (1-\lambda)^j \left[(1-\delta)\, x_{t-m}(z) + \delta\theta_{t-m-1}\right] dz \\
& +\lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} (1-\lambda)^m \left[(1-\delta)\left[(1-\delta)\, x_{t-m}(z) + \delta\theta_{t-m-1}\right] + \delta\theta_{t-m-1}\right] dz \\
& +\lambda \textstyle\sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=m+1}^{\infty} (1-\lambda)^j \left[(1-\delta)\,\theta_{t-j} + \delta\theta_{t-j-1}\right] dz
\end{aligned}
$$

$$
\begin{aligned}
= \;& \lambda^2 \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m \left[(1-\delta)\,\theta_{t-m} + \delta\theta_{t-m-1}\right] \sum_{j=0}^{m-1} (1-\lambda)^j \\
& +\lambda^2 \textstyle\sum_{m=0}^{\infty} (1-\lambda)^{2m} \left[(1-\delta)^2\,\theta_{t-m} + \left[1-(1-\delta)^2\right]\theta_{t-m-1}\right] \\
& +\lambda^2 \textstyle\sum_{j=1}^{\infty} (1-\lambda)^j \left[(1-\delta)\,\theta_{t-j} + \delta\theta_{t-j-1}\right] \sum_{m=0}^{j-1} (1-\lambda)^m
\end{aligned}
$$

$$
\begin{aligned}
= \;& \lambda \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m \left[(1-\delta)\,\theta_{t-m} + \delta\theta_{t-m-1}\right]\left[1-(1-\lambda)^m\right] \\
& +\lambda^2 \textstyle\sum_{m=0}^{\infty} (1-\lambda)^{2m} \left[(1-\delta)^2\,\theta_{t-m} + \left[1-(1-\delta)^2\right]\theta_{t-m-1}\right] \\
& +\lambda \textstyle\sum_{j=1}^{\infty} (1-\lambda)^j \left[(1-\delta)\,\theta_{t-j} + \delta\theta_{t-j-1}\right]\left[1-(1-\lambda)^j\right]
\end{aligned}
$$

$$
\begin{aligned}
= \;& \lambda \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m\, 2\left[1-(1-\lambda)^m\right]\left[(1-\delta)\,\theta_{t-m} + \delta\theta_{t-m-1}\right] \\
& +\lambda^2 \textstyle\sum_{m=0}^{\infty} (1-\lambda)^{2m} \left[(1-\delta)^2\,\theta_{t-m} + \left[1-(1-\delta)^2\right]\theta_{t-m-1}\right].
\end{aligned}
$$

We can write this expression as

$$
\bar{E}^2\left[\theta_t\right] = \lambda \textstyle\sum_{j=0}^{\infty} (1-\lambda)^j \left[\kappa_{j,2}\theta_{t-j} + \delta_{j,2}\theta_{t-j-1}\right],
$$

where

$$
\begin{aligned}
\kappa_{j,2} &= \left(1-\delta^2\right)\left[1-(1-\lambda)^j\right] + (1-\delta)^2\left[1-(1-\lambda)^{j+1}\right] \\
&= \left[1-(1-\lambda)^{j+1}\right]\kappa_{j,1}^2 + \left[1-(1-\lambda)^j\right]\left(1-\delta_{j,1}^2\right), \\
\delta_{j,2} &= \delta^2\left[1-(1-\lambda)^j\right] + \left[1-(1-\delta)^2\right]\left[1-(1-\lambda)^{j+1}\right] \\
&= \left[1-(1-\lambda)^{j+1}\right]\left(1-\kappa_{j,1}^2\right) + \left[1-(1-\lambda)^j\right]\delta_{j,1}^2.
\end{aligned}
$$

Note that

$$\kappa_{j,2} + \delta_{j,2} = \sum_{n=0}^{1} \left[1 - (1-\lambda)^{j}\right]^{n} \left[1 - (1-\lambda)^{j+1}\right]^{1-n}.$$

We use induction to obtain the general case. Suppose that (15) holds for $k-1$. Then

$$\bar{E}^{k-1}[\theta_t] = \lambda \sum_{m=0}^{\infty} (1-\lambda)^m \left[\kappa_{m,k-1}\theta_{t-m} + \delta_{m,k-1}\theta_{t-m-1}\right],$$

where

$$\sum_{j=0}^{m-1} (1-\lambda)^j \left(\kappa_{j,k-1} + \delta_{j,k-1}\right) = \frac{1}{\lambda}\left[1 - (1-\lambda)^m\right]^{k-1}.$$

As a result

$$\begin{aligned}
\bar{E}^{k}[\theta_t] &= \sum_{m=0}^{\infty} \int_{\Lambda_m} E\left[\bar{E}^{k-1}[\theta_t] \mid I_{t-m}(z)\right] dz \\
&= \sum_{m=0}^{\infty} \int_{\Lambda_m} E\left[\lambda \sum_{j=0}^{\infty} (1-\lambda)^j \left[\kappa_{j,k-1}\theta_{t-j} + \delta_{j,k-1}\theta_{t-j-1}\right] \mid I_{t-m}(z)\right] dz \\
&= \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{m-1} (1-\lambda)^j \left\{\kappa_{j,k-1} E\left[\theta_{t-j} \mid I_{t-m}(z)\right] + \delta_{j,k-1} E\left[\theta_{t-j-1} \mid I_{t-m}(z)\right]\right\} dz \\
&\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} (1-\lambda)^m \left\{\kappa_{m,k-1} E\left[\theta_{t-m} \mid I_{t-m}(z)\right] + \delta_{m,k-1}\theta_{t-m-1}\right\} dz \\
&\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=m+1}^{\infty} (1-\lambda)^j \left[\kappa_{j,k-1}\theta_{t-j} + \delta_{j,k-1}\theta_{t-j-1}\right] dz
\end{aligned}$$

$$\begin{aligned}
&= \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=0}^{m-1} (1-\lambda)^j \left(\kappa_{j,k-1} + \delta_{j,k-1}\right) \left[(1-\delta) x_{t-m}(z) + \delta\theta_{t-m-1}\right] dz \\
&\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} (1-\lambda)^m \left[\kappa_{m,k-1}\left[(1-\delta) x_{t-m}(z) + \delta\theta_{t-m-1}\right] + \delta_{m,k-1}\theta_{t-m-1}\right] dz \\
&\quad + \lambda \sum_{m=0}^{\infty} \int_{\Lambda_m} \sum_{j=m+1}^{\infty} (1-\lambda)^j \left[\kappa_{j,k-1}\theta_{t-j} + \delta_{j,k-1}\theta_{t-j-1}\right] dz
\end{aligned}$$

$$\begin{aligned}
&= \lambda^2 \sum_{m=0}^{\infty} (1-\lambda)^m \left[(1-\delta)\theta_{t-m} + \delta\theta_{t-m-1}\right] \sum_{j=0}^{m-1} (1-\lambda)^j \left(\kappa_{j,k-1} + \delta_{j,k-1}\right) \\
&\quad + \lambda^2 \sum_{m=0}^{\infty} (1-\lambda)^{2m} \left[\kappa_{m,k-1}(1-\delta)\theta_{t-m} + \left[\kappa_{m,k-1}\delta + \delta_{m,k-1}\right]\theta_{t-m-1}\right] \\
&\quad + \lambda^2 \sum_{j=1}^{\infty} (1-\lambda)^j \left[\kappa_{j,k-1}\theta_{t-j} + \delta_{j,k-1}\theta_{t-j-1}\right] \sum_{m=0}^{j-1} (1-\lambda)^m
\end{aligned}$$

$$
\begin{aligned}
&= \lambda \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m \left[1-(1-\lambda)^m\right]^{k-1} \left[(1-\delta)\,\theta_{t-m} + \delta\theta_{t-m-1}\right] \\
&\quad +\lambda^2 \textstyle\sum_{m=0}^{\infty} (1-\lambda)^{2m} \left[\kappa_{m,k-1}(1-\delta)\,\theta_{t-m} + \left[\kappa_{m,k-1}\delta + \delta_{m,k-1}\right]\theta_{t-m-1}\right] \\
&\quad +\lambda \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m \left[1-(1-\lambda)^m\right] \left[\kappa_{m,k-1}\theta_{t-m} + \delta_{m,k-1}\theta_{t-m-1}\right].
\end{aligned}
$$

We can rewrite the last three lines as

$$
\bar{E}^k[\theta_t] = \lambda \textstyle\sum_{m=0}^{\infty} (1-\lambda)^m \left[\kappa_{m,k}\theta_{t-m} + \delta_{m,k}\theta_{t-m-1}\right],
$$

where

$$
\begin{aligned}
\kappa_{m,k} &\equiv (1-\delta)\left[1-(1-\lambda)^m\right]^{k-1} + \left[(1-\delta)\,\lambda(1-\lambda)^m + \left[1-(1-\lambda)^m\right]\right]\kappa_{m,k-1} \\
&= (1-\delta)\left[1-(1-\lambda)^m\right]^{k-1} \\
&\quad + \left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right] + \delta\left[1-(1-\lambda)^m\right]\right]\kappa_{m,k-1}, \\
\delta_{m,k} &\equiv \delta\left[1-(1-\lambda)^m\right]^{k-1} + \delta\lambda(1-\lambda)^m\kappa_{m,k-1} + \left[\lambda(1-\lambda)^m + \left[1-(1-\lambda)^m\right]\right]\delta_{m,k-1} \\
&= \delta\left[1-(1-\lambda)^m\right]^{k-1} \\
&\quad +\delta\left[\left[1-(1-\lambda)^{m+1}\right] - \left[1-(1-\lambda)^m\right]\right]\kappa_{m,k-1} + \left[1-(1-\lambda)^{m+1}\right]\delta_{m,k-1},
\end{aligned}
$$

since

$$
\lambda(1-\lambda)^m = \left[1-(1-\lambda)^{m+1}\right] - \left[1-(1-\lambda)^m\right].
$$

Rewriting these weights in matrix format, we obtain

$$
\begin{bmatrix} \kappa_{m,k+1} \\ \delta_{m,k+1} \end{bmatrix} = \begin{bmatrix} (1-\delta) \\ \delta \end{bmatrix} \left[1-(1-\lambda)^m\right]^k + A_m \begin{bmatrix} \kappa_{m,k} \\ \delta_{m,k} \end{bmatrix},
$$

where the matrix A_m is given by

$$
A_m \equiv \begin{bmatrix} \left[(1-\delta)\left[1-(1-\lambda)^{m+1}\right] + \delta\left[1-(1-\lambda)^m\right]\right] & 0 \\ \delta\left[\left[1-(1-\lambda)^{m+1}\right] - \left[1-(1-\lambda)^m\right]\right] & \left[1-(1-\lambda)^{m+1}\right] \end{bmatrix},
$$

which is exactly our result.

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