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Monetary Policy, Asset Prices and Adaptive Learning*

Vicente da Gama Machado**

Abstract

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Following recent episodes of financial distress, the interaction between monetary policy and asset price fluctuations has gained renewed attention. Here, we assess the role of asset price misalignments in monetary policy in an adaptive learning context. Our model first extends Bullard and Mitra (2002), including an additional role for asset prices. From the point of view of the E-Stability criterion, commonly used in the learning literature, we find that a response to stock prices is not desirable under both a forward expectations policy rule and an interest rate rule responding to contemporaneous values. Heterogeneous beliefs about the dynamics of asset price fluctuations, inflation and the output gap are introduced and we also evaluate an optimal monetary policy rule including a weight on asset prices. Overall we find that the Taylor principle remains important over all interest rate rules analysed and that central banks should act cautiously when considering the introduction of stock prices in monetary policy.

Keywords: Interest rate rules, asset prices, adaptive learning, expectational stability

JEL Classification: E31, E52, D84

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1. Introduction

The issue of what role asset price inflation (or alternatively a measure of asset price gap) should have in the conduct of monetary policy has gained renewed momentum in the aftermath of the recent financial crisis and is still far from resolved. After a first wave of contributions to this debate in the first part of the last decade, mainly following the opposite views of Bernanke and Gertler (2000 and 2001) and Cecchetti et al (2000), there has been resurgent interest both among policymakers and academic researchers¹. At the same time, there is controversy about the actual behaviour of monetary authorities concerning the importance given to asset price fluctuations, as evidenced by divergent empirical conclusions found in the literature².

Forecasts of inflation and output have a key role in the standard new Keynesian framework of interest rate rules, and it is widely recognized that the expectations of economic agents influence the time path of the economy. Importantly, a great part of this literature still relies on the rational expectations hypothesis. However, as Evans and Honkapohja (2001) argued, the basic assumption implied by rational expectations, that all agents know the true structure of the economy, has proved too strong. The adaptive learning literature instead concedes that agents learn as they are endowed with information from the economy's structure, by updating their forecasting procedures. As a result, some apparently natural policy rules may not result in a stable equilibrium when agents' learning is considered, a point made by Bullard and Mitra (2002) and Evans and Honkapohja (2003a). Moreover, there is substantial evidence of heterogeneity in the formation of expectations of relevant economic variables³.

¹ See Greenspan (2005), Trichet (2005), Bullard (2009) and Smaghi (2009) for a general idea of the Federal Reserve and the European Central Bank main views. For recent theoretical contributions, see for example, Pfajfar and Santoro (2011), Assenza, Berardi and Gatti (2009), Ida (2011).

² Rigobon and Sack (2003) find a substantial policy response to stock prices, whereas Hayford and Maliaris (2004) conclude that, at least during the 90's, monetary policy tended to accommodate the apparent stock prices overvaluation. As an intermediary approach, Dupor and Conley (2004) argue that the FED response to stock prices tended to be more relevant in low inflation episodes. For results including more recent data, see Castelnovo and Nisticò (2010) and Castro (2011).

³ Branch (2007) and Pfajfar and Santoro (2010) claim that heterogeneity is pervasive in the process of expectation formation, while Milani (2011) identifies a learning process together with expectations shocks as important in economic fluctuations. Wieland and Wolters (2011) also arrive at similar conclusions, adding that heterogeneity of output growth and inflation forecasts tend to vary over time.

Concerning asset prices, there is a particularly vast literature highlighting that agents possess heterogeneous beliefs⁴.

In the present work, we consider the rather realistic points mentioned above in a combined framework. Specifically, we first assess the conditions for determinacy and stability under learning (or E-Stability⁵) of an extended monetary policy rule that accounts for asset prices. The starting point is a framework in which expectations are homogeneous and recursive least squares learning prevails, following to some extent Bullard and Mitra (2002). In this first step we generalise the chief result of Bullard and Mitra (2002) with regard to the Taylor principle to our extended framework. Comparing to Carlstrom and Fuerst (2007), we further investigate an interest rate rule responding to expectations of the key variables and assess E-Stability conditions. Differently from Pfajfar and Santoro (2011) and Assenza *et al* (2009), we focus on a standard new Keynesian macroeconomic model of monetary policy transmission, instead of allowing for cost-channel effects⁶.

Secondly, implications of heterogeneous beliefs about inflation expectations and forecasts of asset market developments are also explored, sharing ideas related to Guse (2005). Hence, we simultaneously address two concerns pointed out by Sims (2009): Referring to central banks (CB) research and practice, he criticises the lack of both a consistent treatment of asset markets and of frameworks that depart from the usual rational expectations approach⁷.

Finally, we derive optimal monetary policy allowing for asset prices, extending the expectations-based rule proposed in Evans and Honkapohja (2003a). By using the learning approach to evaluate alternative scenarios, we also provide an additional selection criterion, particularly vital for monetary policy, since equilibrium indeterminacy is usually present.

⁴ Hommes (2006) provides a comprehensive survey of heterogeneous agents in asset pricing and claims that the shift from the representative agent to a behavioural agent-based approach has been motivated, among other factors, by the increasing evidence of bounded rationality and by observations of excess volatility in stock prices.

⁵ For a formal definition of E-Stability, see Evans and Honkapohja (2001).

⁶ The literature has generally found mixed results regarding the importance of cost-channel effects. Using Bayesian methods to estimate parameters, Rabanal (2007) concludes that the traditional demand side effect dominates the supply side effect represented by the cost channel.

⁷ Besides the obvious “realistic” argument for modelling heterogeneity, there is also the point that since adaptive learning requires that agents behave as econometricians, these may also be subject to misspecification issues, as Evans and Honkapohja (2001) point out.

The remainder of this work is organized as follows. The next section examines the related literature in two directions: First, the debate on the introduction of asset price inflation in monetary policy and second, determinacy and stability of rational expectations equilibria (REE) under learning in general monetary economics. Section 3 describes the small macroeconomic model of households and firms. Section 4 presents the benchmark learning environment with homogeneous expectations and instrumental rules. In sections 5 and 6 we allow for heterogeneous beliefs and optimal monetary policy, and section 7 concludes.

2. Related literature

Monetary policy has succeeded reasonably well in stabilizing price index inflation in several countries over the last years, either targeting inflation strictly or a combination of inflation and output variability⁸. Nonetheless it is not so clear whether it is beneficial to include a response to asset prices in the monetary policy framework.

Earlier contributions to the debate on the optimal monetary policy response to atypical movements in asset prices dealt mainly with the trade-offs involved in including a weight for the volatility of these prices in a usual central bank reaction function. On the one hand, the rather orthodox view claims that the conduct of monetary policy should only be affected by shifts in asset prices as long as they signal future changes in inflation or output. Bernanke and Gertler (2000, 2001) for example, develop a New-Keynesian model with a role for frictions in the credit market – translated into a financial accelerator – and for financial bubbles, modelled as an endogenous stochastic process. Despite adopting a similar technique, Cecchetti et al (2000) reach opposite conclusions. They explore the fact that a countercyclical attitude from the monetary authority may, under particular conditions, soften the impact of abrupt shifts in asset prices and therefore enhance macroeconomic and financial stability.

Building on this debate, some papers modelled the possibility of a response to asset price deviations in a monetary policy rule. Gilchrist and Saito (2006) develop a dynamic stochastic general equilibrium (DSGE) model with a role for the financial

⁸ Although Ball and Sheridan (2005) argue that it is not clear whether the improved macroeconomic performance in a cross-country sample can be related to inflation targeting, they point out that targeters and non-targeters seem to have followed similar interest rate policies.

accelerator mechanism, as in Bernanke and Gertler (2000) in which both the private sector and the policymaker are allowed to learn about the trend growth rate of technology. Faia and Monaceli (2007) offer a similar model, but they further account for an agency issue associated with monitoring costs in the lending market, which implies that asset price fluctuations are a symptom of financial distortions. In both these models there is a scope for a response to asset price oscillations in the optimal policy. In the first article, such response is more beneficial when the central bank is more informed about the rate of technology than the private sector, whereas in the second one, it is true as long as the response to inflation is sufficiently small. Other contributions include, for example, Bean (2003), Haugh (2008) and Gruen, Plumb and Stone (2005). Detken and Smets (2004) offer a good survey of this literature⁹.

Concerning the interactions between asset prices and economic activity (which in turn would justify monetary policy movements), Gilchrist and Leahy (2002) mention three main channels which are explored by different authors in the literature: the wealth channel, Tobin's "q" theory and the financial accelerator.

The recent crisis has triggered more attention both from monetary authorities and from the academic literature. More specifically, a view that has been increasingly questioned is that central banks should not care about asset prices and should instead "clean up the mess" after an asset bubble bursts. Disyatat (2010) argues that a modification in the CB's objective function to include a measure of financial imbalances may be desirable, because it leads to a more practical alternative than introducing an explicit reaction to asset prices, as much of the previous literature had tried to do. By introducing doubts and pessimism in the standard new Keynesian model, Benigno and Paciello (2010) conclude that a flexible inflation targeting policy that includes a reaction to asset prices (represented by Tobin's "q") might be welfare improving in the case when doubts and pessimism about the true model play an important role¹⁰.

⁹ A closely related literature deals with the role central banks should have concerning the development of bubbles in financial markets. See, for example, Kent and Lowe (1997) and Bordo and Jeanne (2002).

¹⁰ Other types of similar departures in the standard macroeconomic model have also been proposed. Kannan, Rabanal and Scott (2009) argue that, provided there is some discretion, a monetary policy response to credit accelerating mechanisms and to distortions in asset prices may be beneficial to macroeconomic stability, together with macroprudential rules. Curdia and Woodford (2010) assess modifications of a Taylor rule to include a reaction to changes either in interest rate spreads or in the aggregate volume of credit.

In the literature of monetary economics, a growing set of studies has been focusing on the criteria of determinacy and stability of rational expectations equilibria under adaptive learning, following Evans and Honkapohja (2001)¹¹, as confirmed in surveys by Bullard (2006) and Evans and Honkapohja (2008). The implicit rationale is based on two pillars: First, the acknowledgment that expectations are a central part of monetary theory. As Woodford (2003) points out, “*not only expectations about policy matter, but at least under current conditions, very little else matters.*” Second, the need to provide an alternative to the rather strong assumptions implied by the rational expectations theory. Instead of knowing from the beginning the true macroeconomic structure, it is usually assumed that agents form expectations adaptively as they learn the real structure. Moreover, stability of such equilibria may present an alternative to the inherent instability problem of interest rate rules, as noted by Friedman (1968), based on self-fulfilling expectations. The central work of Bullard and Mitra (2002) assesses determinacy and E-Stability criteria for different types of interest rate rules. In their view, the so-called learnability of the equilibria (or E-Stability) arises as an additional criterion, which policymakers should take into account. Their results highlight the fact that some interest rate rules that otherwise would be considered desirable, may fail to be optimal under learning dynamics. Evans and Honkapohja (2003a) take a slightly different approach by studying E-Stability conditions for alternative types of interest rate rules derived from optimal monetary policy, for example fundamentals-based and expectations-based rules. McCallum (2008) further cites learnability of the equilibrium as a compelling necessary condition for a REE to be considered plausible¹². In the words of Bernanke (2007), many of the most interesting issues in contemporary monetary theory require an analytical framework that involves learning by private agents and possibly the central bank as well. Preston (2008) shows that if the CB implements monetary policy under the mistaken assumption that agents have purely rational expectations, severe instability follows. Moreover, Orphanides and Williams

¹¹ Evans and Honkapohja (2001) consolidated the theory of adaptive learning focusing mainly on the cobweb model. However, one of the main directions of academic research later proved to be monetary policy, as is evidenced in surveys like Bullard (2006) and Evans and Honkapohja (2008).

¹² Milani (2007) introduces empirical analysis in an adaptive learning setting and argues that such a setting manages to reproduce important features of observed expectations. More importantly, what he calls “mechanical sources of persistence” - like habit formation in consumption or indexation of past inflation in price-setting - are no longer necessary to match the data when the assumption of rational expectations is relaxed in favour of learning.

(2005) pointed out that imperfect knowledge of the structure may have important implications for monetary policy¹³.

Relatively few studies approached the more specific question of determinacy and stability criteria when a response to asset prices is considered in the interest rate rule, combining both pieces of literature just reviewed. Bullard and Schaling (2002) found that assigning more weight to asset price fluctuations in a Taylor rule, the probability of indeterminacy of the REE increases. Their work relies heavily on the findings of Bullard and Mitra (2002). A similar outcome is obtained by Carlstrom and Fuerst (2007), who furthermore consider patterns of money demand. However, they do not study E-Stability nor deal with a forward-expectations policy rule. Airaudò, Nisticò and Zanna (2007) develop a DSGE model, with overlapping generations that consider agents are non-Ricardian and there is a role for the wealth effect on the demand for consumption. Thus, they assume the supply effects in Bernanke and Gertler (2001) are complemented with demand effects¹⁴. Airaudò, Nisticò and Zanna (2007) find a special case where there could be a stable and determinate equilibrium in a rule with positive weight for asset price inflation. Pfajfar and Santoro (2011) also tackle the question of the optimal response to asset price deviations, in a DSGE model where a cost-channel is made explicit. They find that as this channel becomes more important, responding to stock prices increases the regions of determinacy and E-Stability. Assenza, Berardi and Gatti (2009) develop a DSGE model with an augmented Phillips curve – which, in their view, represents more clearly the cost-channel – to account for the impact of asset price misalignments in inflation. It is worth noting that none of these contributions account for heterogeneous expectations.

More recently, there has also been some work on important extensions to the learning framework. Backed by recent empirical evidence on expectation formation, heterogeneity in expectations has been a fruitful way of research. As Honkapohja and Mitra (2006) argue, introducing heterogeneity of expectations raises new challenges for policy. Honkapohja and Mitra (2005) focus on a new Keynesian model in which the central bank and private agents form their expectations differently, depending on initial

¹³ While the learning and the imperfect information literature have received much attention, there are further important alternatives to the paradigm of rational expectations in the theory. Sims (2003) focuses on rational inattention by the agents as a more realistic description of their behaviour. Hansen and Sargent (2001) derive important results in the robust control literature.

¹⁴ However, Milani (2008) finds weak evidence for the wealth effect in a both theoretical and empirical model with a role for learning.

conditions or learning algorithms. In the case of Muto (2010), heterogeneity arises as private agents learn from the central bank forecasts in an interactive way. Guse (2005) analyses heterogeneous expectations and learning in a univariate approach¹⁵.

3. Basic model

As a description of the economy, we adapt the theoretical general equilibrium model of Carlstrom and Fuerst (2007), with the difference that our model is not deterministic. The standard sticky price model economy is populated by households and firms. Households form decisions on consumption, asset holdings and labour supply while firms decide on the pricing of their goods, while using labour as input. Next we separately analyse such decisions and the resulting equilibrium.

3.1 Households

The infinitely-lived households are assumed to have an intertemporal discount factor β and decide on the amount of labour supplied N_t , consumption C_t and on their holdings of bonds and shares. The representative household's period-by-period utility function is represented by a CRRA function:

$$U\left(C_t, N_t, \frac{M_{t+1}}{P_t}\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1-\gamma}}{1-\gamma} + F\left(\frac{M_{t+1}}{P_t}\right) \quad (1)$$

where $\sigma > 0$, $\gamma > 0$, $F(\cdot)$ is increasing and concave, P_t denotes price level and $\frac{M_{t+1}}{P_t}$ denotes cash balances at the end of period t . As in Carlstrom and Fuerst (2007), cash balances assumed to enter the household's utility function at t are the cash balances that each household has after finishing period t transactions. Moreover, we assume that at period t each household's portfolio consists of M_t cash balances, B_{t-1} bonds paying r_{t-1} gross interest rate, wage revenue $W_t N_t$, a monetary injection X_t and S_{t-1} shares of stock that sell at price Q_t and pay D_t dividends, so that the household is subject to the following budget constraint:

¹⁵ Other forms of heterogeneity among private agents include, for instance, Branch and Evans (2011)'s predictor selection and Branch and McGough (2009) who assume a fraction of agents are boundedly rational, while the others form expectations rationally. As a general result, the determinacy conditions of the benchmark homogeneous situation are significantly altered. Another example of departure from homogeneous learning is Arifovic, Bullard and Kostyshyna (2007)'s social learning.

$$P_t C_t + P_t Q_t S_t + B_t + M_{t+1} \leq P_t W_t N_t + P_t S_{t-1} (Q_t + D_t) + r_{t-1} B_{t-1} + M_t + X_t \quad (2)$$

The representative household therefore maximises in period t :

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1-\gamma}}{1-\gamma} + F \left(\frac{M_{t+1}}{P_t} \right) \right] \quad (3)$$

subject to a sequence of budget constraints of the form (2). The first-order conditions lead to the usual optimal relations representing the Euler equations for consumption, labour supply and money demand:

$$\left(\frac{E_t C_{t+1}}{C_t} \right)^\sigma = \beta \frac{r_t}{E_t(\pi_{t+1})} \quad (4)$$

$$C_t^\sigma N_t^\gamma = W_t \quad (5)$$

$$F' \left(\frac{M_{t+1}}{P_t} \right) C_t^\sigma = \frac{r_t^{-1}}{r_t} \quad (6)$$

and an additional optimal relation for asset prices

$$\left(\frac{E_t C_{t+1}}{C_t} \right)^\sigma = \beta \frac{E_t(Q_{t+1} + D_{t+1})}{Q_t} \quad (7)$$

Equation (4) states that the intertemporal rate of substitution of consumption depends on the discount rate and on the real interest rate, equation (5) characterises the usual consumption-leisure trade off and (6) is a money demand function.

Finally, equation (7) can be understood as an Euler equation for asset prices, which expresses the equality between the utility gains of postponing consumption and the expected relative appreciation of shares and dividends. Note that combining (7) and the Euler equation for consumption yields:

$$\frac{r_t}{E_t(\pi_{t+1})} = \frac{E_t(Q_{t+1} + D_{t+1})}{Q_t} \quad (8)$$

which denotes a “no-arbitrage condition”, since it establishes the equivalence between the return on bonds on the left-hand side and the return on equities on the right-hand side. Equation (8) can also be rewritten as to make asset prices explicit:

$$Q_t = \frac{E_t(Q_{t+1} + D_{t+1}) E_t(\pi_{t+1})}{r_t} \quad (9)$$

3.2 Firms

Firms produce differentiated goods that are sold in a monopolistic competition market. Each firm’s output is a function of labour input and an aggregate productivity disturbance V_t :

$$Y_t = V_t N_t, \quad E(V_t) = 1 \quad (10)$$

where we also assume firms face constant returns to scale. The firms' cost minimization problem is

$$\min_{N_t} W_t N_t + Z_t (Y_t - V_t N_t)$$

where Z_t is the firm's real marginal cost. The first order condition implies

$$W_t = Z_t \quad (11)$$

As in Carlstrom and Fuerst (2007), firms distribute dividends in the same amount of their profits, i.e:

$$D_t = \Pi_t = Y_t - W_t V_t N_t = (1 - Z_t) Y_t \quad (12)$$

We consider nominal rigidities in the spirit of Calvo's (1983) staggered price setting. Each period a fraction $0 < \alpha < 1$ of the goods do not have their prices revised, or, in other words, with probability α a given price will be adjusted by the firm. Under reasonable assumptions on the profit function Π_t , Woodford (2003, pg 187) shows that the aggregate inflation rate and the output gap must satisfy the following aggregate-supply relation in any period t :

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \quad (13)$$

where $\kappa = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \zeta$ measures the degree of price-stickiness¹⁶.

3.3 Log-linearized equilibrium

The market clearing conditions are $S_t = 1$, $B_t = 0$ and the resource constraint $C_t = N_t$. Employing standard techniques, it is possible to represent the equilibrium in terms of log deviations from the respective steady state value of each variable.

$$(\sigma + \gamma)c_t = w_t \quad (14)$$

$$\sigma(E_t c_{t+1} - c_t) = r_t - r_t^n - E_t \pi_{t+1} \quad (15)$$

$$q_t = \beta E_t q_{t+1} + (1 - \beta) E_t d_{t+1} - (r_t - r_t^n - E_t \pi_{t+1}) \quad (16)$$

$$d_t = c_t - \frac{z}{1-z} z_t \quad (17)$$

$$w_t = z_t \quad (18)$$

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} \quad (19)$$

¹⁶ Since κ is negatively related to α , the longer prices are fixed on average, the less sensitive should be inflation to changes in the output gap. Here ζ describes the degree of strategic complementarity between the price-setting decisions of suppliers of different goods and β relates to the discount factor to which profits are discounted. For a more detailed derivation of (13) in a sticky-price environment, see Walsh (2003).

The small macroeconomic model can now be formalised. We begin with the new Keynesian Phillips curve (19) characterizing the aggregate supply relation. To keep notation more aligned with the learning and monetary policy literature, we substitute the real marginal cost by a measure of the output gap, $x_t = (Y_t - Y_t^n)$. Equation (15) and the resource constraint lead to the aggregate demand relationship, also called the intertemporal IS equation:

$$x_t = E_t x_{t+1} - \sigma^{-1}(r_t - r_t^n - E_t \pi_{t+1}) \quad (20)$$

where r_t is the nominal interest rate¹⁷. From (14) and (17),

$$d_t = -Az_t \quad (21)$$

where $A = \frac{z(1+\sigma+\gamma)-1}{(\sigma+\gamma)(1-z)} > 0$ for reasonable calibrations. The negative relationship between dividends and the output gap reflects the typical detrimental effect of marginal costs on firms profitability. Combining (16) and (21), the result is an equation relating the dynamics of stock price misalignments to their expected values, together with the expected values of inflation and output gap:

$$q_t = \beta E_t q_{t+1} - A(1 - \beta)E_t x_{t+1} - (r_t - r_t^n - E_t \pi_{t+1}) \quad (22)$$

The structural parameters σ , κ , and β respectively stand for the elasticity of intertemporal substitution, the degree of price stickiness and the discount factor.

The baseline model is complemented with interest rate policy. First we follow Bullard and Mitra (2002) and consider Taylor-type instrumental rules, augmented with a term corresponding to asset price misalignments. The first instrumental rule considers contemporaneous values of the output gap, inflation and asset price deviations:

$$r_t = \varphi_x x_t + \varphi_\pi \pi_t + \varphi_q q_t \quad (23)$$

where φ_y is the policy response to the expected future value of variable y . Differently from Carlstrom and Fuerst (2007), we start with a more general rule, which includes the output gap. We also extend Carlstrom and Fuerst (2007)'s analysis by examining an interest rate rule that responds to forward-looking expectations. Such a rule addresses McCallum's (1998) critique that monetary policy based on current values of inflation and the output gap may fail to be operational due to their non-availability. In our setting, the forward-looking rule can be represented by:

$$r_t = \varphi_x E_t x_{t+1} + \varphi_\pi E_t \pi_{t+1} + \varphi_q E_t q_{t+1} \quad (24)$$

¹⁷ Following Bullard and Mitra (2002), we assume the natural rate of interest to follow the stochastic process $r_t^n = \rho r_{t-1}^n + \varepsilon_t$, where ε_t is an i.i.d. noise with variance σ_ε^2 and $0 \leq \rho < 1$.

In section 6 we consider instead targeting rules, and analyse optimal monetary policy, which can be considered an extension to the analysis of Evans and Honkapohja (2003b).

4. Determinacy and E-Stability in the benchmark model

As previously mentioned, the adaptive learning literature has been developing considerably fast, following Evans and Honkapohja (2001), especially regarding its applications to monetary policy theory. In this section we present the basic concepts employed throughout the chapter with respect to desirable properties that an REE should have. As a starting point, we will examine determinacy conditions for the rational expectations equilibrium and then, in an adaptive learning environment, we proceed to the E-Stability requirements.

Combining equations (19), (20), (22) and an instrumental interest rate rule such as (23) or (24) we have the reduced form:

$$y_t = \alpha + BE_t y_{t+1} + \chi r_t^n \quad (25)$$

where $y_t = (x_t, \pi_t, q_t)'$ is the vector of endogenous variables forming the system, α and χ are 3×1 parameter vectors and B is a 3×3 parameter matrix, which are properly defined for each type of rule in the next subsections.

Determinacy conditions, or equivalently, conditions for the uniqueness of a REE are largely used as a desirable criterion in the rational expectations literature; see Woodford (2003), among others. A well-known result in Blanchard and Kahn (1980) states that if the number of eigenvalues of B inside the unit circle is equal to the number of non-predetermined variables, the solution to the system is unique. Otherwise, the system may have multiple solutions, which are commonly called “sunspot solutions”. In this case, some of these solutions may still be of interest if they can be learnable in the sense of Evans and Honkapohja (2001) and E-Stability thus arises as a useful additional selection criterion.

As argued before, our focus will be on an environment in which agents form expectations as they learn from the observed values of the system, instead of knowing from the outset the true structure of the economy. As much of the literature, here we

suppose agents adopt recursive least squares as a learning rule to update the parameters of their forecasting model¹⁸.

Let the minimal state solution (MSV) for the system (25) be $y_t = \bar{a} + \bar{c}r_t^n$, which stands for the fundamental equilibrium. Suppose agents believe that the solution is of the form

$$y_t = a + cr_t^n \quad (26)$$

where now vectors a and c are not known from the beginning but are estimated by private agents. Equation (26) is usually called the perceived law of motion (PLM), since it describes the intrinsic beliefs of the private agents about the relevant parameters in each period.

With this PLM, agents then form expectations as

$$E_t y_{t+1} = a + c\rho r_t^n \quad (27)$$

Inserting these expectations into the system (25), the result is an actual law of motion (ALM):

$$y_t = Ba + (Bc\rho + \chi)r_t^n \quad (28)$$

As Evans and Honkapohja (2008) point out, the ALM is a description of the temporary equilibrium for the expectations derived from the PLM.

The mapping from the PLM to the ALM then takes the form:

$$T(a, c) = (Ba, Bc\rho + \chi) \quad (29)$$

where the rational expectations solution (\bar{a}, \bar{c}) is a fixed point of this map. An important result is the E-Stability principle, as defined in Evans and Honkapohja (2001) and broadly employed in the adaptive learning literature. Given the differential equation

$$\frac{d}{d\tau}(a, c) = T(a, c) - (a, c) \quad (30)$$

if the particular REE (\bar{a}, \bar{c}) is locally asymptotically stable under (30), than the REE is said to be stable under learning (or E-Stable). Here, τ denotes “notional” or “artificial” time.

It is convenient to detail the exact sequence of events from the point of view of a private agent: at time t , agents have estimates (a_t, c_t) of the parameters of the PLM, computed with data available at $t - 1$. Then r_t^n is realised, and agents form expectations according to (27). The central bank sets the policy rate r_t following (23) or

¹⁸ Honkapohja and Mitra (2005) assess monetary policy in a context of heterogeneous agents where a fraction of agents use least squares while the other fraction uses the less sophisticated stochastic gradient learning.

(24), which generates the real y_t through the macroeconomic model. The whole process resumes at $t + 1$ as agents add new data to their information set to update their estimates (a_{t+1}, c_{t+1}) . The desirable E-Stability result is then achieved, if $(a_t, c_t) \rightarrow (\bar{a}, \bar{c})$ as $t \rightarrow \infty$.

Following Evans and Honkapohja (2001), it turns out that E-Stability of the REE obtains provided the eigenvalues of both B and ρB have real parts less than 1. Since $0 \leq \rho < 1$, it suffices for the roots of B to have real parts less than 1. These results apply to the benchmark case, where we consider homogeneous agents. As for the heterogeneous case, section 5 presents appropriate determinacy and E-Stability results.

4.1 Calibrated parameters

Whenever it is feasible and intuitive, our results are expressed in analytical language. Due to the complexity of the calculations involved, we additionally conduct numerical simulations¹⁹, which are illustrated in the figures next. We follow Bullard and Mitra (2002) by adopting Woodford (1999)'s baseline parameters $\sigma = 0.157$, $\kappa = 0.024$ and $\beta = 0.99$. As for the value of A , we follow Carlstrom and Fuerst (2007) for their case of lower marginal cost sensitivity, i.e., $z = 0.85$ and $\gamma = 0.47$, which yield $A = 4.072$. Importantly, alternative calibrations were also tested and proved not to alter significantly the results.

4.2 Benchmark case: homogeneous beliefs

We first consider an environment in which all private agents share the same beliefs about the correct form of the solution, which is reflected here as agents following the PLM defined in (26). The focus here is the macroeconomic model above comprising instrumental interest rate rules. This allows us to better compare our results to Bullard and Mitra (2002) and to papers that consider asset prices in a homogeneous setting, like Pfajfar and Santoro (2011) and Airaudo, Nisticò and Zanna (2007).

4.2.1 Contemporaneous data in the interest rate rule

A quite common policy rule, also analysed in Bullard and Mitra (2002), is an instrumental interest rate rule, where the central bank responds to contemporaneous

¹⁹ For all numerical calculations and graphs, as well as for some cumbersome matrix algebra, the computer software MAPLE 9.5 was used. Codes are available upon request.

values of inflation and the output gap. In our framework, with the addition of stock price misalignments, this is exactly equation (23). Considering the macroeconomic model (19)-(20) augmented with the stock price equation (22) and contemporaneous interest rate rule (23), the system can be expressed by $y_t = \alpha + BE_t y_{t+1} + \chi r_t^n$, where the relevant parameters are:

$$\alpha = 0, \chi = \frac{1}{1+\varphi_q} (\sigma^{-1}, \kappa\sigma^{-1}, 1 - \varphi_\pi\kappa\sigma^{-1})' \text{ and}$$

$$B = \frac{1}{\sigma + \sigma\varphi_q + \varphi_x + \varphi_\pi\kappa} \begin{bmatrix} -\sigma - \varphi_q(\sigma + A - A\beta) & -1 + \varphi_\pi\beta & -\varphi_q\beta \\ (-\sigma - \varphi_q(\sigma + A - A\beta))\kappa & \kappa\sigma^{-1}(1 - \varphi_\pi) + \beta & -\varphi_\pi\beta \\ -\sigma(A - A\beta) - (\varphi_x + \varphi_\pi\kappa)(\sigma + A - A\beta) & (-1 + \varphi_\pi\beta)\sigma & (\varphi_\pi\kappa + \varphi_x + \sigma)\beta \end{bmatrix} \quad (31)$$

Next we will show pertinent results supposing two extreme scenarios: first the response to stock price deviations is muted in the reaction function, that is, $r_t = \varphi_x x_t + \varphi_\pi \pi_t$. Then we allow for a response to contemporaneous values of inflation and stock price deviations: $r_t = \varphi_\pi \pi_t + \varphi_q q_t$. Through such simplifications it is possible to analytically assess the questions involved in the introduction of asset prices into rather standard instrumental monetary policy rules.

Proposition 1: *Under the contemporaneous data rule, assume $\varphi_q = 0$, i.e., there is no response to stock price misalignments. Then the necessary and sufficient condition for uniqueness and E-Stability of the MSV REE is*

$$\kappa(\varphi_\pi - 1) + \varphi_x(1 - \beta) > 0$$

Proof: See Appendix A.

The shaded area in Figure 1 shows the combinations of policy parameters φ_x and φ_π that lead to determinacy and E-Stability of the MSV solution, using well-known calibrations for the structural parameters. The fact that the results on both determinacy and E-Stability are exactly the same as Bullard and Mitra (2002)'s propositions 1 and 2 appears somewhat trivial, since our interest rate rule boils down to theirs. However, it is still an important result, since now there is stock price dynamics involved.

It is easy to see that, provided the response of interest rates to inflation is more than one-for-one, determinacy and E-Stability is achieved, regardless of the policy response to the output gap. This required condition corresponds to the Taylor principle, following the usual term in the literature, as in Woodford (2003). As the Central Bank

reacts to the output gap, the required reaction to inflation is loosened and becomes gradually lower than one.

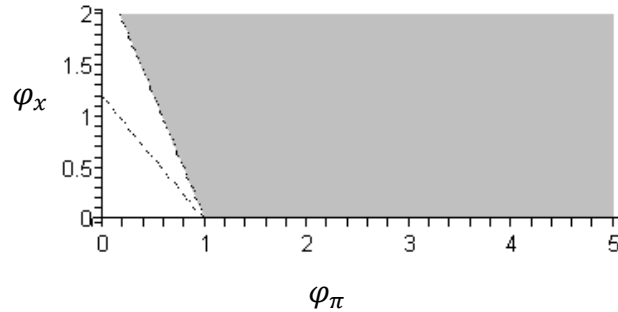


Figure 1: Determinacy and E-Stability regions for rule $r_t = \varphi_x x_t + \varphi_\pi \pi_t$

We now examine a setting where the monetary authority responds to inflation and stock price deviations, but not to the output gap, such that (23) boils down to $r_t = \varphi_\pi \pi_t + \varphi_q q_t$. Using the same basic framework, the following result obtains:

Proposition 2: *Under the contemporaneous data rule, Assume $\varphi_x = 0$, i.e, there is no response to the output gap. Then the necessary and sufficient conditions for uniqueness and E-Stability of the REE is*

$$\kappa(\varphi_\pi - 1) - \varphi_q A(1 - \beta) > 0$$

Proof: See Appendix A.

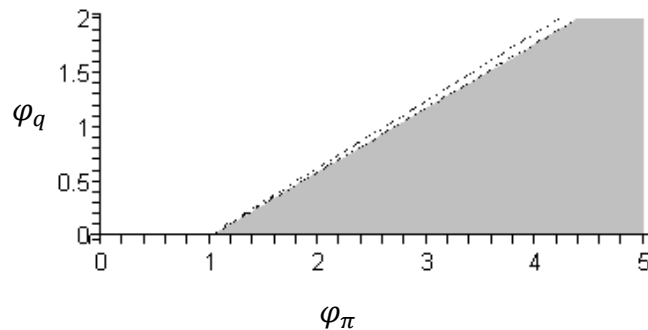


Figure 2: Determinacy and E-Stability regions for rule $r_t = \varphi_\pi \pi_t + \varphi_q q_t$

As expected, our result mirrors Carlstrom and Fuerst (2007)'s proposition 1, since their policy rule does not prescribe a reaction to the output gap. Notice that the result again resembles the Taylor rule, since it requires that nominal interest rates rise by more than the increase in the inflation rate, independently of the response to asset prices. However, as Figure 2 depicts, higher responses to asset prices now *deteriorate* determinacy and E-Stability conditions. This effect can be explained by the usual negative relationship between firm profits (which impact dividends and asset prices) and real marginal costs. As inflation rises, so do marginal costs, and consequently asset prices tend to fall. A response of the policy rate to asset prices in the same direction undermines the CB's response to inflation, as described by Carlstrom and Fuerst (2007).

4.2.2 Forward expectations in the interest rate rule

We now turn to a case not studied by Carlstrom and Fuerst (2007), which consists of a forward-looking expectations interest rate rule, where the CB responds to forecasts of output, inflation and asset price deviations. This alternative model which can be understood as an extension to Bullard and Mitra (2002), involves combining (19)-(20) together with (22) and the interest rate rule (24), leading to $y_t = \alpha + BE_t y_{t+1} + \chi r_t^n$, where:

$$\alpha = 0, \chi = (\sigma^{-1}, \kappa\sigma^{-1}, 1)' \text{ and}$$

$$B = \begin{bmatrix} 1 - \sigma^{-1}\varphi_x & \sigma^{-1}(1 - \varphi_\pi) & -\sigma^{-1}\varphi_q \\ \kappa(1 - \sigma^{-1}\varphi_x) & \kappa\sigma^{-1}(1 - \varphi_\pi) + \beta & -\kappa\sigma^{-1}\varphi_q \\ -A(1 - \beta) - \varphi_x & 1 - \varphi_\pi & \beta - \varphi_q \end{bmatrix} \quad (32)$$

Repeating the same two scenarios studied in the last subsection, we have propositions 3 and 4:

Proposition 3: *Under the forward expectations interest rate rule, assume $\varphi_q = 0$, i.e., there is no response to stock price misalignments. Then the necessary and sufficient conditions for uniqueness of the MSV REE are*

$$\kappa(\varphi_\pi - 1) + \varphi_x(1 + \beta) < 2\sigma(1 + \beta)$$

and

$$\kappa(\varphi_\pi - 1) + \varphi_x(1 - \beta) > 0$$

Furthermore, the MSV equilibrium is E-Stable if the latter condition is met.

Proof: See Appendix B.

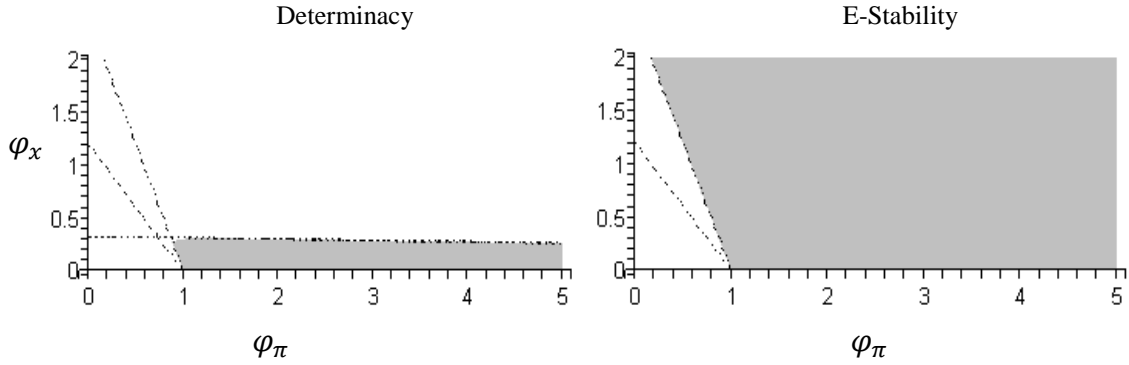


Figure 3: Determinacy and E-Stability regions for rule $r_t = \varphi_x E_t x_{t+1} + \varphi_\pi E_t \pi_{t+1}$

Again, our result is similar to Bullard and Mitra (2002)'s proposition 4. The key message here is that determinacy requirements are stricter under the forward-expectations rule, as becomes clear in Figure 3 above. The first constraint in proposition 3 means that there is an upper bound on both responses φ_x and φ_π ²⁰. On the other hand, the Taylor principle is enough to guarantee E-Stability and we have the same outcome as proposition 2. As a consequence, there may be some situations in which there is multiplicity of equilibria, but these equilibria may be learned by private agents, a possibility we do not study here.

As in the last subsection, we now examine a setting where the monetary authority responds to inflation and stock price deviations, but not to the output gap, so that (24) boils down to $r_t = \varphi_\pi E_t \pi_{t+1} + \varphi_q E_t x_{q+1}$. Using the same basic framework, the following result obtains:

Proposition 4: *Assume $\varphi_x = 0$, i.e., there is no response to the output gap in the instrumental interest rate rule. Then the necessary and sufficient conditions for uniqueness of the MSV REE are*

$$\kappa(\varphi_\pi - 1) + \varphi_q(A(1 - \beta) + 2\sigma) < 2\sigma(1 + \beta)$$

and

$$\kappa(\varphi_\pi - 1) - \varphi_q A(1 - \beta) > 0$$

Furthermore, the MSV equilibrium is E-Stable if the latter condition is met.

Proof: See Appendix B.

²⁰ Using the calibrated parameters, indeterminacy of the REE ensues for any $\varphi_x > 0.32$ or $\varphi_\pi > 27$.

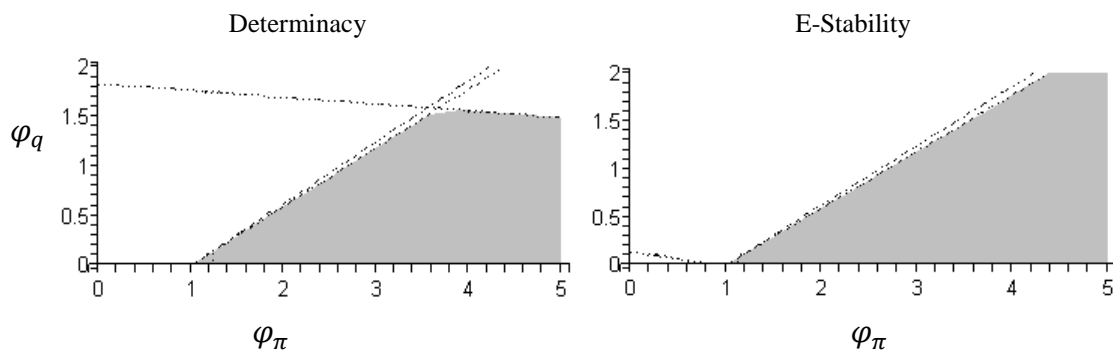


Figure 4: Determinacy and E-Stability regions for rule $r_t = \varphi_\pi E_t \pi_{t+1} + \varphi_q E_t q_{t+1}$

Similarly, when the central bank has to choose on the appropriate level of response between expected inflation and asset price deviations, an upper bound on φ_q emerges, represented by the first constraint on proposition 4. As for E-Stability, again the result we interpret as similar to the Taylor rule suffices.

A key difference between the rules is that, under the forward-expectations rule, equilibrium determinacy guarantees E-Stability, but the converse is not always true; on the other hand, under the contemporaneous rule, both regions are always equivalent.

Overall the desired response to inflation is guided by the Taylor principle, as in Bullard and Mitra (2002). However, differently from what follows the usual reaction to the output gap, as the response to asset prices increases the required response to inflation has to be even more aggressive in order to maintain uniqueness and E-Stability of the REE. These findings are further supported by numerical simulations showing that, under both rules (23) and (24), as φ_q increases, the area of determinacy and E-Stability in the $\varphi_\pi \times \varphi_q$ space shrinks²¹.

5. Heterogeneous beliefs

Maintaining the same macroeconomic framework with the forward expectation policy rule, we now turn to a more realistic assumption that not all private agents form their beliefs in the same fashion. Assume there are two types of agents in the economy:

²¹ Figures and calculations are available from the author on demand.

A fraction μ of type 1 agents form expectations according to the MSV perceived law of motion, exactly as in the benchmark case of section 3, that is:

$$PLM_1: y_t = a_1 + cr_t^n \quad (33)$$

The remaining $(1 - \mu)$ agents use a different form of PLM, which includes a lagged component. We call this overparameterized law of motion the AR(1) PLM:

$$PLM_2: y_t = a_2 + b_2 y_{t-1} + cr_t^n \quad (34)$$

where the estimated parameters a_1 , a_2 and c are 3×1 vectors and b_2 is a 3×3 matrix.

Taking expectations at $t + 1$ yields:

$$Ey_{t+1} = \mu a_1 + (1 - \mu)[(I + b_2)a_2 + b_2^2 y_{t-1} + b_2 cr_t^n] + cpr_t^n \quad (35)$$

Substituting it into system (25) and reorganising the terms:

$$y_t = \alpha + B[\mu a_1 + (1 - \mu)(I + b_2)a_2 + (1 - \mu)b_2^2 y_{t-1}] + B[(1 - \mu)b_2 c + c\rho]r_t^n + \chi r_t^n \quad (36)$$

This equation is precisely the ALM, which represents the stochastic process followed by y_t , given agents perceived law of motions. The next step is to construct the mapping from the PLM to the ALM, as in Evans and Honkapohja (2001). However, since there are 2 types of PLM, we follow closely Guse (2005). For type 1 agents, he assumes a ‘‘projected ALM’’, which is constructed by the mean of the implied y_t process. On the other hand, the ALM parameters for type 2 agents derive from the intercept and slope parameters of the implied y_t process.

Thus, the mapping from the PLMs to the ALM can be expressed by:

$$T \begin{pmatrix} a_1 \\ a_2 \\ b_2 \\ c \end{pmatrix} = \begin{pmatrix} [I - (1 - \mu)Bb_2^2]^{-1}[\alpha + \mu Ba_1 + (1 - \mu)B(I + b_2)a_2] \\ \alpha + \mu Ba_1 + (1 - \mu)B(I + b_2)a_2 \\ (1 - \mu)Bb_2^2 \\ (1 - \mu)Bb_2 c + \rho Bc + \chi \end{pmatrix} \quad (37)$$

The rational expectation equilibria are fixed points of this mapping and satisfy:

$$a_1 = [I - (1 - \mu)Bb_2^2]^{-1}[\alpha + \mu Ba_1 + (1 - \mu)B(I + b_2)a_2]$$

$$a_2 = \alpha + \mu Ba_1 + (1 - \mu)B(I + b_2)a_2$$

$$b_2 = (1 - \mu)Bb_2^2$$

$$c = (1 - \mu)Bb_2 c + \rho Bc + \chi$$

Following Evans and Honkapohja (2001), E-Stability is then determined by the differential equation:

$$\frac{d}{d\tau}(a_1, a_2, b_2, c) = T(a_1, a_2, b_2, c) - (a_1, a_2, b_2, c) \quad (38)$$

As for the MSV equilibria, since $b_2 = 0$, (37) boils down to:

$$T \begin{pmatrix} a_1 \\ c \end{pmatrix} = \begin{pmatrix} \mu B a_1 + (1 - \mu) B a_2 \\ \rho B c + \chi \end{pmatrix},$$

so that $\frac{d}{d\tau}(a_1, c) = (\mu B a_1 + (1 - \mu) B a_2 - a_1, \rho B c + \chi - c)$.

Regarding AR(1) equilibria, the equilibrium value for b_2 must be $\bar{b}_2 = (1 - \mu)^{-1} B^{-1}$, so that the differential equation (38) conditioned on (37) provide the following E-Stability result.

Proposition 5: *Under heterogeneous beliefs, the macroeconomic model with the forward-expectations rule has E-Stable MSV equilibria when the same conditions of Propositions 3 and 4 are met. However, AR(1) equilibria are not E-Stable, regardless of the proportion of agents of each type.*

Proof: See Appendix C.

As expected, MSV equilibria give rise to the same E-Stability results as in the case of propositions 3 and 4, where all agents homogeneously form beliefs. On the other hand, AR(1) equilibria are not E-Stable.

Even when some of the agents form beliefs that are not in accordance with the minimal state variable solution, the equilibrium will tend to converge to the MSV REE, since it leads to E-Stability over a quite broad range of parameters, as seen on the homogeneous case. However, AR(1) equilibria may also be reached and when it happens, no stability under learning at all is guaranteed.

The E-Instability result is unsurprisingly similar to the one represented in Honkapohja and Mitra (2004)'s proposition 3, since in both cases the same class of non-fundamental solutions was studied, that is, autoregressive solutions. This failure is related to the strong influence of past values of the endogenous variables on their current and expected values.

6. Heterogeneous beliefs

An interesting further step is to analyse the implications of the introduction of asset price misalignments in an optimised monetary policy setting. For this task, we borrow some ideas of Evans and Honkapohja (2003a,b). In their study of adaptive

learning applied to monetary policy in a standard new-Keynesian model, fundamental-based rules arguably entail instability under learning. Conversely, their proposal of expectations-based rules leads to convergence of the optimal REE to E-Stability results. This is valid not only when private expectations are observable, but also when key structural parameters can be estimated and used in the interest rate rule, supposing simultaneous learning by private agents and policymakers. Extending Evans and Honkapohja (2003a,b)'s main results, we focus here on the case of expectations-based rules.

Interest rate policy now consists of the optimisation of an intertemporal objective function. As usual it is assumed that the central bank faces the problem of minimising a quadratic loss function, adapted to account for asset price deviations, that is:

$$\min E_t \sum_{s=0}^{\infty} \beta^s [\pi_t^2 + \lambda_x x_t^2 + \lambda_q q_t^2] \quad (39)$$

The novelty here, compared to most of the literature on optimal monetary policy, is the CB's additional preference for stock price gap stabilization. We also assume the usual Phillips curve:

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \quad (40)$$

The cost-push shock u_t follows a random walk, that is, $u_t = \rho u_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$.²²

The first-order condition of (39) subject to the Phillips curve results in the following implied reaction function:

$$\kappa \pi_t + \lambda_x x_t + \lambda_q q_t = 0 \quad (41)$$

which can also be interpreted as a targeting rule, because it contains the desired relationship between inflation, the output gap and the stock price gap. Combining the IS relation, the Phillips curve in (40) and the implied reaction function (41), after some algebra we arrive at:

$$r_t = \delta_x E_t x_{t+1} + \delta_\pi E_t \pi_{t+1} + \delta_q E_t q_{t+1} + \delta_u u_t + r_t^n \quad (42)$$

where the coefficients are:

$$\delta_x = \sigma(\sigma + \gamma)^{-1}[\gamma - A(1 - \beta)] \quad (43)$$

$$\delta_\pi = 1 + \frac{\kappa\beta}{\lambda_q} \sigma(\sigma + \gamma)^{-1} \quad (44)$$

²² As Clarida, Gali and Gertler (1999) point out, this shock enables the model to generate variation in inflation that evolves independently of movement in excess demand, so that a trade-off between inflation and output stabilization arises.

$$\delta_q = \beta\sigma(\sigma + \gamma)^{-1} \quad (45)$$

$$\delta_u = \frac{\kappa}{\lambda_q}\sigma(\sigma + \gamma)^{-1} \quad (46)$$

and $\gamma = \frac{\kappa^2 + \lambda_x}{\lambda_q}$. Here, (42) is the expectations-based optimal rule, that implements the targeting rule (41) in every period.

An important result of the expectations-based rule is that a variant of the Taylor principle is again guaranteed, since $\delta_\pi > 1$ for plausible calibrations and for either λ_x or λ_q strictly positive.

6.1 Learning analysis

It is also interesting to check the resulting conditions of stability under adaptive learning, as we conducted for instrumental rules before. Taking into account the macroeconomic model above and the expectations-based rule, the following proposition states the desirable response to stock prices under optimal monetary policy.

Proposition 6: *Consider the expectations-based rule (42) together with the IS equation (20) and the Phillips curve (40). The REE of the obtained system is E-Stable if and only if $0 < \lambda_q < \frac{1}{A}\left(\lambda_x + \frac{\kappa^2}{A(1-\beta)}\right)$.*

Proof: See Appendix D.

According to this result, as the response to stock prices λ_q rises, λ_x has to rise even more for E-Stability to obtain. In other words, CB's preference for stock price gap stabilization is limited by its preference for output gap stabilization. As a consequence, if the Central Bank is rather inflation-targeter, it should respond very carefully to stock prices.

A relevant result is that, contrarily to Evans and Honkapohja (2003a,b), an expectations-based rule is no longer a guarantee that E-Stability of the REE will be reached. Confirming the previous results on instrumental interest rate rules, introducing asset prices is clearly not desirable under an optimal policy perspective as well.

7. Conclusions

We have assessed the conditions for determinacy and stability under learning of an extended monetary policy rule that accounts for asset price variations. Since we adopt instrumental interest rate rules in a normative way, our framework is closely related to Bullard and Mitra (2002) and Carlstrom and Fuerst (2007). Nevertheless, we depart from them in various ways: with respect to the latter, we consider both E-Stability of the REE in a learning environment as defined by Evans and Honkapohja (2001) and a forward expectations policy rule. We also take into account heterogeneous beliefs about the dynamics of variables, in a similar way to Guse (2005), and an optimal monetary policy rule, extending Evans and Honkapohja (2003a,b).

In the homogeneous expectations case, we have shown that introducing a response to stock prices in a contemporaneous interest rate rule does not generally lead to desirable outcomes with respect to determinacy, for the same reason as Carlstrom and Fuerst (2007) had already pointed out. The negative relationship between firm profits (and consequently dividends and asset prices) and marginal costs means that, as inflation rises, so do marginal costs, and consequently asset prices tend to fall. A response of the policy rate to asset prices in the same direction undermines the CB's response to inflation, leading to indeterminacy of REE. It turned out that this effect also threatens E-Stability, as we have shown both in a contemporaneous and in a forward expectations interest rate rule.

Heterogeneity leads to MSV equilibria being E-Stable at least over the same regions as in the homogeneous case, whereas AR(1) equilibria are unstable over all regions of the central bank responses. As an implication, if there is a considerable fraction of agents in the economy who form expectations looking at past values of key variables (in our case, stock price developments, inflation and output gap), non-fundamental equilibria which are not learnable may arise.

Our analysis of optimal interest rate policy in the presence of stock price misalignments showed that the Taylor principle is also important, at least when an expectations-based monetary policy rule in the sense of Evans and Honkapohja (2003a,b) is considered.

Given that stock price booms and busts have caused considerable damage to the financial stability of many industrialised countries over the last years, and since no

clear-cut stance of monetary policy is to be advised, further measures could also be taken into account. Indeed some authors and policymakers have recently mentioned macroprudential regulation as an alternative to cope with financial imbalances and the possibility of booms and busts (see IMF (2011)). Less discussed are the potential negative side effects of increased regulation. Further research could also extend on the ideas of heterogeneity in various ways. Agents may be allowed to switch between beliefs, depending on some measure of past performance of their forecast, as in Guse (2005), or Branch and Evans (2011). Alternatively, some agents may also form rational expectations, while others behave adaptively, as in Branch and McGough (2009).

Appendix: Proofs

A. Determinacy and E-Stability conditions: Contemporaneous rule

Proof of proposition 1

As mentioned in section 3.1, for determinacy of the REE of a system like (25), all of the 3 eigenvalues of B must lie inside the unit circle. Following LaSalle (1986, pg.28), given the characteristic polynomial of B , $p_B(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$, its roots lie inside the unit circle if and only if the following inequalities hold:

$$|a_0 + a_2| < 1 + a_1 \text{ and } |a_1 - a_0a_2| < 1 - a_0^2$$

where:

$$a_0 = \frac{\beta^2\sigma}{\sigma + \varphi_x + \varphi_\pi\kappa}$$

$$a_1 = \frac{\beta(2\sigma + \beta\sigma + \beta\varphi_x + \kappa)}{\sigma + \varphi_x + \varphi_\pi\kappa}$$

$$a_2 = -\frac{\beta(2\sigma + \varphi_\pi\kappa + 2\varphi_x + \kappa) + \kappa + \sigma}{\sigma + \varphi_x + \varphi_\pi\kappa}$$

The first inequality produces the result of the proposition, while the second one does not bind, as is shown in Figure 1 with numerical calculation using calibrated values.

For E-Stability of the REE we need all the eigenvalues of B to have real parts less than 1, which is equivalent to the condition that all eigenvalues of $C = B - I$ should have negative real parts (where I is an identity matrix). For the given

characteristic polynomial of C , $p_C(\lambda) = \lambda^3 + a_{E2}\lambda^2 + a_{E1}\lambda + a_{E0}$, the necessary and sufficient conditions follow from the Routh-Hurwitz Theorem (Gandolfo, 1997 pgs. 221-223):

$$\begin{aligned} a_{E1} &> 0 \\ a_{E0} &> 0 \\ a_{E2} &> 0 \\ a_{E1}a_{E2} - a_{E0} &> 0 \end{aligned}$$

Note that either the first or the second constraint can be suppressed, since either one is implied by the remaining three.

The polynomial coefficients are then:

$$\begin{aligned} a_{E0} &= (1 - \beta) \frac{\kappa(\varphi_\pi - 1) + \varphi_x(1 - \beta)}{\sigma + \varphi_x + \varphi_\pi\kappa} \\ a_{E1} &= \frac{\kappa((3 - 2\beta)\varphi_\pi - (2 - \beta)) + \sigma(1 - \beta)^2 + \varphi_x(1 - \beta)(3 - \beta)}{\sigma + \varphi_x + \varphi_\pi\kappa} \\ a_{E2} &= \frac{\kappa((3 - \beta)\varphi_\pi - 1) + 2\sigma(1 - \beta) + \varphi_x(3 - 2\beta)}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa} \end{aligned}$$

Condition $a_{E0} > 0$ implies the Taylor Principle result $\kappa(\varphi_\pi - 1) + \varphi_x(1 - \beta) > 0$. The remaining are non-binding conditions, as we again confirmed through numerical analysis. Note that the dotted lines on the plots represent these non-binding constraints.

Proof of proposition 2

We follow the proof of proposition 1. Coefficients a_0 , a_1 and a_2 are derived in the same way, with the difference that $\varphi_q = 0$ instead of $\varphi_x = 0$:

$$\begin{aligned} a_0 &= \frac{-\beta^2\sigma}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa} \\ a_1 &= \beta \frac{\varphi_q A(1 - \beta) + \sigma(2 + \beta) + \varphi_q\sigma + \kappa}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa} \\ a_2 &= \frac{\varphi_q A(1 - \beta) + \sigma(1 + 2\beta) + \varphi_q\sigma(1 + \beta) + \kappa(\beta\varphi_\pi + 1)}{\sigma + \varphi_q\sigma + \varphi_\pi\kappa} \end{aligned}$$

Here, algebraically solving $|a_0 + a_2| < 1 + a_1$ leads to the result of the proposition, while $|a_1 - a_0 a_2| < 1 - a_0^2$ turn out not to bind.

An analogous result is reached for the E-Stability conditions. Given the coefficients

$$\begin{aligned}
a_{E0} &= (1 - \beta) \frac{\kappa(\varphi_\pi - 1) + \varphi_q A(1 - \beta)}{\sigma + \varphi_q \sigma + \varphi_\pi \kappa} \\
a_{E1} &= \frac{\kappa((3 - 2\beta)\varphi_\pi - (2 - \beta)) + \sigma(1 - \beta)^2}{\sigma + \varphi_q \sigma + \varphi_\pi \kappa} + \\
&\quad + \frac{\varphi_q \sigma(1 - \beta) + \varphi_q A\beta(3 - \beta) + 2\varphi_q A}{\sigma + \varphi_q \sigma + \varphi_\pi \kappa} \\
a_{E2} &= \frac{\kappa((3 - \beta)\varphi_\pi - 1) - 2\sigma(1 - \beta) + \varphi_q \sigma(2 - \beta) - \varphi_q A(1 - \beta)}{\sigma + \varphi_q \sigma + \varphi_\pi \kappa}
\end{aligned}$$

it is easy to show that $a_{E0} > 0$ results in the corresponding Taylor principle like equation. At the same time, conditions $a_{E1} > 0$ and $a_{E2} > 0$ are not binding.

B. Determinacy and E-Stability conditions: Forward-looking rule

Proof of proposition 3

Using the same idea behind the proof of Proposition 1, under the forward-expectations interest rate rule the following coefficients of the characteristic polynomial of B obtain:

$$\begin{aligned}
a_0 &= \frac{\beta(\beta\varphi_x - \beta\sigma)}{\sigma} \\
a_1 &= \frac{2\beta\varphi_x - \beta\kappa(\varphi_\pi - 1) - \sigma(\beta^2 + 2\beta)}{\sigma} \\
a_2 &= \frac{\kappa(\varphi_\pi - 1) + \varphi_x + 2\sigma(1 - \beta)}{\sigma}
\end{aligned}$$

Solving for $|a_0 + a_2| < 1 + a_1$ we have both conditions shown in the text. The constraint $|a_1 - a_0 a_2| < 1 - a_0^2$ does not bind (see Figure 3).

For E-Stability of the REE again all eigenvalues of $C = B - I$ should have real negative parts. Given the characteristic polynomial of C , $p_C(\lambda) = \lambda^3 + a_{E2}\lambda^2 + a_{E1}\lambda + a_{E0}$, we showed the required conditions are

$$\begin{aligned}
a_{E0} &> 0 \\
a_{E2} &> 0 \\
a_{E1}a_{E2} - a_{E0} &> 0
\end{aligned}$$

where:

$$\begin{aligned}
a_{E0} &= (1 - \beta) \frac{\kappa(\varphi_\pi - 1) + \varphi_x(1 - \beta)}{\sigma} \\
a_{E1} &= \frac{\kappa(\varphi_\pi - 1)(2 - \beta) + \sigma(1 - \beta)^2 + 2\varphi_x(1 - \beta)}{\sigma} \\
a_{E2} &= \frac{\kappa(\varphi_\pi - 1) + \varphi_x + 2\sigma(1 - \beta)}{\sigma}
\end{aligned}$$

Condition $a_{E0} > 0$ implies $\kappa(\varphi_\pi - 1) + \varphi_x(1 - \beta) > 0$. The other conditions are not binding, as we again confirmed through numerical analysis.

Proof of proposition 4

As for determinacy, we follow the proof of proposition 1. The coefficients a_0 , a_1 and a_2 are derived in the same way, with the difference that $\varphi_q = 0$ instead of $\varphi_x = 0$:

$$\begin{aligned}
a_0 &= \beta \frac{\varphi_q A(1 - \beta) + \varphi_q \sigma - \sigma \beta}{\sigma} \\
a_1 &= \frac{\sigma \beta(\beta + 2) - \kappa \beta(\varphi_\pi - 1) - \varphi_q A(1 - \beta) - \varphi_q \sigma(\beta + 1)}{\sigma} \\
a_2 &= \frac{\kappa(\varphi_\pi - 1) + \varphi_q \sigma - \sigma(2\beta + 1)}{\sigma}
\end{aligned}$$

Here, algebraically solving $|a_0 + a_2| < 1 + a_1$ leads to both results of the proposition, while $|a_1 + a_0 a_2| < 1 - a_0^2$ does not to bind (see boundaries on Figure 4).

A similar pattern marks the E-Stability conditions. Given the coefficients

$$\begin{aligned}
a_{E0} &= (1 - \beta) \frac{\kappa(\varphi_\pi - 1) + \varphi_q A(1 - \beta)}{\sigma} \\
a_{E1} &= \frac{\kappa(\varphi_\pi - 1)(2 - \beta) + \sigma(1 - \beta)^2 \varphi_q(1 - \beta)(\sigma - A)}{\sigma} \\
a_{E2} &= \frac{\kappa(\varphi_\pi - 1) + \varphi_q \sigma + 2\sigma(1 - \beta)}{\sigma}
\end{aligned}$$

it is easy to show that $a_{E0} > 0$ results in the corresponding Taylor principle like equation. At the same time, conditions $a_{E2} > 0$ and $a_{E1} a_{E2} - a_{E0} > 0$ do not bind under the calibrated parameters, as shown in Figure 4.

C. Heterogeneous expectations

Proof of proposition 5

Let us first concentrate on MSV equilibria. Using the same notation as Evans and Honkapohja (2001), we have:

$$DT_{a_1}(\bar{a}_1, \bar{a}_2) = \mu B$$

$$DT_c(\bar{c}) = \rho B$$

$$DT_{a_2}(\bar{a}_1, \bar{a}_2) = DT_c(\bar{c}, \bar{b}_2) = 0$$

Since $0 < \rho \leq 1$ and $0 \leq \mu \leq 1$, E-Stability obtains whenever the eigenvalues of B have real parts less than one, or alternatively, the eigenvalues of $B - I$ have negative parts less than one. Note that this is equivalent to the conditions in Propositions 3 and 4. Moreover, when $\rho < 1$ and/or $\mu < 1$, E-Stability conditions for MSV equilibria are even less strict. Note also that since there is a discontinuity when $\mu = 0$, a strictly positive number of type 1 agents is required for the MSV REE to be E-Stable.

Now, turning to AR(1) equilibria, after solving for the differential equation (38) and substituting $\bar{b}_2 = (1 - \mu)^{-1}B^{-1}$, we arrive at:

$$DT_{a_1}(\bar{a}_1, \bar{b}_2) = [I - (1 - \mu)Bb_2^2]^{-1}\mu B = [I - (1 - \mu)^{-1}B^{-1}]^{-1}\mu B$$

$$DT_{a_2}(\bar{a}_2, \bar{b}_2) = (1 - \mu)B(I + b_2) = (1 - \mu)B + I$$

$$\begin{aligned} DT_{b_2}(\bar{b}_2) &= (1 - \mu)(b_2^T \otimes B + I \otimes Bb_2) = \\ &= (B^{-1})^T \otimes B + (1 - \mu)I \otimes (1 - \mu)^{-1}I \end{aligned}$$

$$DT_c(\bar{c}, \bar{b}_2) = (1 - \mu)Bb_2 + \rho B = I + \rho B$$

For E-Stability we need the eigenvalues of DT_{a_1} , DT_{a_2} , DT_{b_2} , and DT_c to simultaneously have real parts less than one. Using the property that the eigenvalues of a Kronecker product of two matrices A and B are the individual products of the eigenvalues of each matrix (Lancaster and Tismenetski, 1985), there is an unstable root, with value 2, among the eigenvalues of DT_{b_2} , independently of the proportion of agents of each type.

D. Expectations-based rule

Proof of proposition 6

Following the basic ideas explained in section 4, combining the expectations-based rule (42) and the IS and Phillips curve leads to $y_t = \alpha + BE_t y_{t+1} + \chi r_t^n$, where the relevant parameters are:

$$\alpha = 0, \chi = \frac{1}{1+\delta_q} (\sigma^{-1}, \kappa\sigma^{-1}, 1 - \delta_\pi \kappa\sigma^{-1})' \text{ and}$$
$$B = \begin{bmatrix} 1 - \sigma^{-1}\delta_x & \sigma^{-1}(1 - \delta_\pi) & -\sigma^{-1}\delta_q \\ \kappa(1 - \sigma^{-1}\delta_x) & \kappa\sigma^{-1}(1 - \delta_\pi) + \beta & -\kappa\sigma^{-1}\delta_q \\ -A(1 - \beta) - \delta_x & 1 - \delta_\pi & \beta - \delta_q \end{bmatrix}$$

Substituting (43)-(45) and solving for the condition that the roots of B must have real parts less than 1, it turns out that E-Stability obtains whenever

$$\frac{\kappa^2 - \beta\kappa^2 + 2A\beta\lambda_q - 2\beta\lambda_x - A\lambda_q + \lambda_x + \beta^2\lambda_x - A\beta^2\lambda_q}{\kappa^2 + \sigma\lambda_q + \lambda_x} > 0$$

which leads to

$$\lambda_q < \frac{1}{A} \left(\lambda_x + \frac{\kappa^2}{A(1-\beta)} \right)$$

Finally, $\lambda_q > 0$ is required in order to have the term $\gamma = \frac{\kappa^2 + \lambda_x}{\lambda_q}$ defined.

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