Directed Clustering Coefficient as a Measure of Systemic Risk in Complex Banking Networks

August, 2011
Directed clustering coefficient as a measure of systemic risk in complex banking networks

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Abstract

Recent literature has focused on the study of systemic risk in complex networks. It is clear now, after the crisis of 2008, that the aggregate behavior of the interaction among the agents is not straightforward and it is very difficulty to predict. Contributing to this debate, this paper shows that the directed clustering coefficient may be used as a measure of systemic risk in complex networks. Furthermore, using data from the Brazilian bank interbank network, we show that the directed clustering coefficient is negatively correlated with domestic interest rates.

Key Words: dynamic topology, clusters, interbank markets, systemic risk.

PACS: 64.60.aq, 89.65.Gh, 87.23.Ge

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1 Introduction

Recent literature has focused on the issue of modeling, measuring and avoiding systemic risk in complex networks Vespignani [2010]. The literature is divided in general approaches such as Watts [2002], Lorenz et al. [2009], Buldyrev et al. [2010], Cajueiro and Andrade [2010a, b, c] and techniques that deal with specific kinds of networks such as technological networks Motter [2004], Albert et al. [2004], Crucitti et al. [2004], Huang et al. [2008], social and biological networks Dodds and Watts [2004] and financial and economic networks Eisenberg and Noe [2001], Boss et al. [2004a], Elsinger et al. [2006], Iori et al. [2006]. In the particular case of financial and economic systems, based on the events that took place in the crisis of 2008, it is clear that the aggregate behavior of the interaction among the agents is not straightforward and it is very difficulty to predict Schweitzer et al. [2009a]. Furthermore, the challenging of understanding aggregate behavior of economic and financial systems require tools belonging to the field of econometrics of times series, complex systems, game theory and agent-based models Schweitzer et al. [2009b].

Banking lending networks are one of the most important financial systems that are subjected to systemic risk. In fact, small shocks constrained only to a few banks can be spread by contagion and affect the entire system Allen and Gale [2000]. These authors show that in a banking system with a homogeneous topology, the possibility of financial contagion depends strongly on the completeness of the structure of the system. It is worth mentioning that due to the development of the theory of complex networks Albert and Barabasi [2002], Boccaletti et al. [2006], Costa et al. [2007], it has been possible to improve our knowledge on banking networks. Now, knowing that banking networks have complex structure and dynamics Boss et al. [2004a], Müller [2003], Inaoka et al. [2004], Cajueiro and Tabak [2008], Wan et al. [2006], Masi et al. [2006], Lublòy [2006], Soramaki et al. [2007], models of heterogeneous banking networks have replaced the homogeneous ones. For instance, exploring the interconnections among banks in the Italian overnight market, Iori et al. [2008] have investigated potential implications of the current institutional system on the banking stability. In Nier et al. [2007], simulated banking systems show how systemic risk depends on their structures.

In this context, this paper shows that the directed clustering coefficient Fagiolo [2007] may be used as a measure of systemic risk in complex networks. In particular, exploring data from the Brazilian bank interbank network, we show that the way that banks make clusters of lending relationships have different impact in terms of systemic risk.

The remainder of this paper is structured in the following way. Section
2 revisits the measure known as directed clustering coefficient. Section 3 describes the data of the Brazilian interbank market used in this paper. Section 4 presents the main results of the paper. In particular, we show that the directed clustering coefficient is negatively correlated with interest rate chances and it varies strongly among banks. Finally, section 5 presents the main conclusions of this work.

## 2 Clustering coefficients for directed networks

In Fagiolo [2007], the standard clustering coefficient Watts and Strogatz [1998] used for unweighted and undirected networks were generalized for binary directed networks and weighted directed networks. Consider the following notation: Let $A$ and $W$ be respectively the directed adjacency matrix of the network and directed matrix of weights that represents the network. Let also $d_i^\text{in}$, $d_i^\text{out}$ and $d_i^\text{tot} = d_i^\text{in} + d_i^\text{out}$, be respectively the in-degree of node $i$, the out-degree of node $i$ the total degree of node $i$. Furthermore, let $d_i^- = \sum_{j \neq i} a_{ij}a_{ji} = A_{ii}^2$.

In binary directed networks, the clustering coefficient of node $i$ for a binary network may be defined as the ratio between all the possible triangles formed by $i$ and the number of all possible triangles that could be formed

$$C_i^D(A) = \frac{(A + A^T)^3_{ii}}{2[\sum_{i}^{} d_i^{\text{tot}}(d_i^{\text{tot}} - 1) - 2d_i^-]}.$$  \hspace{1cm} (1)

This clustering coefficient defined for the unweighted case can be easily extended to the weighted case by replacing the number of directed triangles formed with its weighted counterpart

$$\hat{C}_i^D(W) = \frac{[\hat{W} + (\hat{W}^T)]^3_{ii}}{2[\sum_{i}^{} d_i^{\text{tot}}(d_i^{\text{tot}} - 1) - 2d_i^-]}.$$  \hspace{1cm} (2)

where $\hat{W} = W^{\frac{1}{3}} = [w_{ij}^{\frac{1}{3}}]$.

However, as pointed in Fagiolo [2007], these two definitions (1) and (2) are not enough to characterize the richness of patterns that take place in a complex directed network. In fact, equations (1) and (2) treat all the possible triangles as if they were the same. However, in directed graphs, edges that point in different directions should be interpreted differently. Therefore, four more definitions are necessary, which are represented in Figure 1:

(a) cycle, when there is a cyclical relation among $i$ and its neighbors. In this case, the associated clustering coefficient for the binary case is
\[
C^{\text{cyc}}_i = \frac{(A)^3_{ii}}{d^{\text{in}}_i d^{\text{out}}_i - d^+_i}
\]

and for the weighted case is given by

\[
\tilde{C}^{\text{cyc}}_i = \frac{(\hat{W})^3_{ii}}{d^{\text{in}}_i d^{\text{out}}_i - d^+_i}.
\]

(b) Middleman, when one of the neighbor of node \(i\) holds two outward edges and the other holds two inward edges. In this case, the associated clustering coefficient for the binary case is

\[
C^{\text{mid}}_i = \frac{(AA^T A)_{ii}}{d^{\text{in}}_i d^{\text{out}}_i - d^+_i}
\]

and for the weighted case is given by

\[
\tilde{C}^{\text{mid}}_i = \frac{(\hat{W} \hat{W}^T \hat{W})_{ii}}{d^{\text{in}}_i d^{\text{out}}_i - d^+_i}.
\]

(c) In, when \(i\) holds two inward edges. In this case, the associated clustering coefficient for the binary case is

\[
C^{\text{in}}_i = \frac{(A^T A^2)_{ii}}{d^{\text{in}}_i (d^{\text{in}}_i - 1)}
\]

and for the weighted case is given by

\[
\tilde{C}^{\text{in}}_i = \frac{(\hat{W}^T \hat{W}^2)_{ii}}{d^{\text{in}}_i (d^{\text{in}}_i - 1)}.
\]

(d) Out, when \(i\) holds two outward edges. In this case, the associated clustering coefficient for the binary case is

\[
C^{\text{out}}_i = \frac{(A^2 A^T)_{ii}}{d^{\text{out}}_i (d^{\text{out}}_i - 1)}
\]

and for the weighted case is given by

\[
\tilde{C}^{\text{out}}_i = \frac{(\hat{W}^2 \hat{W}^T)_{ii}}{d^{\text{out}}_i (d^{\text{out}}_i - 1)}.
\]

Both unweighed and weighted clustering coefficients are interesting. Although the former uses less information, it counts the number of triangles of a given type. The latter uses more information, but it is strongly affected by the largest weights. Since our network is directed weighed we study here the
Figure 1: Representations of the triangles that can arise in a directed network: (a) cycle; (b) middleman; (c) in; (d) out.
dynamics of the cycles, middle, In and Out clustering coefficients using the weighed formulation.

In the following discussion, we assume that an edge that arrives to node \(i\) coming from node \(j\) mean that bank \(i\) borrowed money from node and bank \(j\) lent money to bank \(i\). Note that in terms of systemic risk, these four patterns presented in figure 1 offer different interpretations.

The first type of clustering that we present is the \(\tilde{C}_i^{yc}\), which is shown in Figure 1(a). In this case bank \(i\) lends to bank \(j\), which lends to bank \(h\), which in its turn lends back to bank \(i\). Therefore, large values do not represent a higher risk for the banking system.

The \(\tilde{C}_i^{mid}\) is presented in Figure 1(b) and represents the case in which the counterpart of bank \(i\), bank \(h\) and \(j\), are either borrowing or lending from the other two banks. In this case, large values imply a higher systemic risk. Figure 1(c) presents the case in which \(\tilde{C}_i^{in}\) bank \(i\) is borrowing from both banks. Therefore, it represents a situation in which bank \(i\) is increasing the risk of the banking system. If bank \(i\) fails then it will not pay some or all the loans that it has made and subsequently the other two banks may not be able to meet their own obligations with each other, increasing the losses within the system.

In Figure 1(d) we present the case in which \(\tilde{C}_i^{out}\) bank \(i\) is increasing it’s own exposure at it is lending to two counterparties. If one of these bank fails, as it may not pay the other bank the losses suffered from bank \(i\) may increase. Therefore, if this clustering coefficient is high we can say that bank \(i\) has a large exposure and higher risk within the interbank network. Overall, higher values for the coefficients \(\tilde{C}_i^{mid}\) and \(\tilde{C}_i^{in}\) imply higher systemic risk and higher values of \(\tilde{C}_i^{out}\) imply higher exposure of bank \(i\).

3 Data

All financial institutions report their counterpart in the interbank market and their size exposure. We have collected data on daily loans made between financial institutions within the Brazilian financial system for all banks and financial institutions that have exposures in the interbank market, for the period from January 2004 to November 2007.

Our sample, which consists of 86 banks and 23 non-bank financial institutions, allows us to analyze interbank lending between banks that do not belong to the same financial institution. The sample comprises public, private domestic and foreign banks. The role of these types of bank is examined through analyzing their relative importance in the interbank network.
Table 1: Descriptive statistics of the averaged clustering coefficients for the period of the sample and for each type of bank.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Jarque-Bera</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C^{cyc}$ Public</td>
<td>$2.64 \times 10^{-5}$</td>
<td>$3.60 \times 10^{-5}$</td>
<td>1.3846</td>
<td>3.9403</td>
<td>16.7490</td>
<td>0.0002</td>
</tr>
<tr>
<td>$C^{cyc}$ Domestic</td>
<td>$9.33 \times 10^{-5}$</td>
<td>$1.32 \times 10^{-5}$</td>
<td>-0.7146</td>
<td>4.0066</td>
<td>5.9846</td>
<td>0.0502</td>
</tr>
<tr>
<td>$C^{cyc}$ Foreign</td>
<td>$8.37 \times 10^{-5}$</td>
<td>$1.78 \times 10^{-5}$</td>
<td>-0.1479</td>
<td>1.7767</td>
<td>3.1021</td>
<td>0.2120</td>
</tr>
<tr>
<td>$C^{mid}$ Public</td>
<td>$7.31 \times 10^{-5}$</td>
<td>$6.57 \times 10^{-5}$</td>
<td>0.9123</td>
<td>2.8967</td>
<td>6.5410</td>
<td>0.0380</td>
</tr>
<tr>
<td>$C^{mid}$ Domestic</td>
<td>$6.75 \times 10^{-4}$</td>
<td>$9.88 \times 10^{-5}$</td>
<td>-0.5120</td>
<td>4.8929</td>
<td>9.0701</td>
<td>0.0107</td>
</tr>
<tr>
<td>$C^{mid}$ Foreign</td>
<td>$9.76 \times 10^{-4}$</td>
<td>$5.57 \times 10^{-5}$</td>
<td>1.2324</td>
<td>5.2336</td>
<td>21.6681</td>
<td>0.0000</td>
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<tr>
<td>$C^{out}$ Public</td>
<td>$8.54 \times 10^{-5}$</td>
<td>$8.78 \times 10^{-5}$</td>
<td>1.2668</td>
<td>3.8146</td>
<td>13.8711</td>
<td>0.0010</td>
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<tr>
<td>$C^{out}$ Domestic</td>
<td>$1.89 \times 10^{-4}$</td>
<td>$3.87 \times 10^{-5}$</td>
<td>-0.9146</td>
<td>2.5583</td>
<td>6.9347</td>
<td>0.0312</td>
</tr>
<tr>
<td>$C^{out}$ Foreign</td>
<td>$3.45 \times 10^{-4}$</td>
<td>$3.59 \times 10^{-4}$</td>
<td>-1.2701</td>
<td>3.9728</td>
<td>14.4901</td>
<td>0.0007</td>
</tr>
<tr>
<td>$C^{out}$ Public</td>
<td>$2.81 \times 10^{-5}$</td>
<td>$6.95 \times 10^{-5}$</td>
<td>3.1776</td>
<td>12.7655</td>
<td>265.8519</td>
<td>0.0000</td>
</tr>
<tr>
<td>$C^{out}$ Domestic</td>
<td>$2.09 \times 10^{-4}$</td>
<td>$6.74 \times 10^{-5}$</td>
<td>0.6892</td>
<td>2.5788</td>
<td>4.0685</td>
<td>0.1308</td>
</tr>
<tr>
<td>$C^{out}$ Foreign</td>
<td>$5.35 \times 10^{-4}$</td>
<td>$9.26 \times 10^{-5}$</td>
<td>1.9484</td>
<td>6.9723</td>
<td>60.6385</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

4 Results

We now study how frequent are the patterns of lending presented in figure 1 in the Brazilian interbank market. Since there is a relation between systemic risk and each pattern of lending, we want to know how each type of bank influences the systemic risk of the interbank market.

Table 1 presents the descriptive statistics of the averaged clustering coefficients for the period of the sample and for each type of bank. It is worth noticing that the distribution of these figures is not normal, which implies in a large heterogeneity between banks.

In Figure 2, we also show the evolution of these clustering coefficients over time reinforcing that different bank types have different strategies when dealing with the interbank market. Furthermore, although the clustering coefficients vary strongly over time, most of them vary around their mean value.

We also test for the correlation of clustering coefficients with domestic interest rates. In theory banks can change their exposure due to changes in interest rates. We find evidence of a negative correlation between the $C^{mid}$
coefficient and interest rates changes (CDI), which imply that as interest rates increases banks decrease their relative exposure within the network. The correlation coefficients for all, private, public and foreign banks with interest rates are -0.257423, -0.302239, -0.136349 and -0.370078, respectively.

Therefore, we also find that the effects are different depending on ownership. These results suggests that banks pursue different strategies within the interbank network, which may be due to diversity in obtaining funds domestically and internationally.

Figure 2: The types of banks are identified by the following notation: Public (solid), Domestic (dashes) and Foreign (dot-dashes). Different panels represent different clustering coefficients: (a) Cycle; (b) Middle; (c) In; (d) Out.
5 Conclusions

In this paper, we have interpreted the directed clustering coefficients as a measure of systemic risk. We have evaluated these clustering coefficients for the Brazilian interbank market data and we have shown that these measures vary strongly over the banks and they are negatively correlated with interest rate change. Therefore, banks change their risk exposure with changes in interest rates. Overall, systemic risk within this market is very limited.
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