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Optimal costs of sovereign default*

Leonardo Pio Perez†

Abstract

I apply a standard model of sovereign debt in order to identify the optimal costs of default from the *ex-ante* point of view of the borrower. I depart from the literature by distinguishing events of strong economic crises from standard business cycles. Crisis events seem to be appropriate moments in which the option to default might be welfare improving by providing state contingency in the debt contract. The quantitative analysis shows that the costs of default should be limited, leaving default as an option, but at much higher levels than the ones consistent with the observed debt-output and default ratios of emerging economies. The results in this paper have implication on the evolution of sovereign debt workout procedures and the potential demand for new debt instruments.

Keywords: Sovereign debt; Default; Economic crises; Welfare.
JEL Classification: F33; F34.

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1 Introduction

A particular feature of sovereign debt is the fact that no international court has effective power to enforce repayment. It is a common view in the literature that its sustainability depends on the belief that a country that defaults will incur losses directly or indirectly imposed by their creditors.[1] The fact that default is costly is the mechanism that makes unsecured sovereign debt possible.

I apply a standard incomplete markets model of sovereign debt along the lines of Aguiar and Gopinath (2006) and Arellano (2008)[2] in order to identify the optimal costs of default from the *ex-ante* point of view of the borrower. I depart from those papers by distinguishing events of strong economic crises from standard business cycles in the output process. Those events seem to be appropriate moments in which the option to default might be welfare improving by providing some state contingency in the debt contract.[3]

Since the borrowing country cannot commit to repay its debt, it always chooses to default whenever it is profitable to do so. The lender, which is assumed to be risk neutral, adjusts the interest rate of the debt contract according to the probability of repayment, such that the expected return on lending to the sovereign country equals the risk-free rate. Under these standard assumptions, the lender is indifferent to the frequency of default or the level of default costs whereas the borrower faces a trade-off.

Among the reasons for a sovereign country to issue debt, impatience and self-insurance motives ask for a harsh punishment for default, such that it becomes a less attractive option. As a result, higher levels of debt would be sustainable and the interest rate would be closer

[1]See Dooley (2000) and Cohen and Sachs (1986).

[2]Those models are based on the framework proposed by Eaton and Gersovitz (1981).

[3]State contingency in the debt contract is discussed in Grossman and Van Huyck (1988). They differentiate excusable default, which does not trigger default costs, from debt repudiation, which never happens in equilibrium.

to the risk-free rate. Weaker costs of default would favor another reason to issue debt, insurance. If there is a state of the world in which paying back the debt becomes a huge burden, without having default as an option, the borrower would anticipate it and would not hold much debt. Lower costs of default would increase the country's demand for debt considering the option to avoid repayment during economic crises. Reasonable default costs should allow enough debt to provide transfers of wealth from the future to the present and, at the same time, leave default as an option, which would act as an insurance component in the case of bad shocks.

I briefly discuss the literature about the interaction of default and insurance in section 2, and also provide a simple two-period example that illustrates the exercise performed in this paper. The example confirms that the default costs are necessary for unsecured debt to be sustainable, but if those costs are excessive, they reduce borrowers' demand for debt due to precautionary motives. As a result, welfare is maximized at intermediate levels of default costs.

This paper is motivated by a set of evidence suggesting a recent trend to reducing default costs for debtor countries.[4] In terms of access to external finance, Gelos et al. (2004) find that, on average, defaulters regained access to international financial markets in more than four years in the 1980's, whereas in the 1990's, this average reduced to less than four months.[5] Confirming this movement, Dooley (2000) refers to an IMF press release from 1999, in which the director of the fund recommended the approval of Ecuador's request for financial assistance before an agreement with its creditors had been reached.[6] This rec-

[4] According to Tomz and Wright (2007), there has been much debate in recent years about policies to reduce the frequency and costs of sovereign default.

[5] This reduction can be partially explained by a change in the composition of debt instruments, from syndicated bank lending to bonds.

[6] The justification was that Ecuador was "making *good faith efforts* to reach a collaborative agreement with its creditors".

ommendation revealed some leniency, since the fund used to require the agreement before providing financial assistance.

Also, many proposals to rule sovereign debt restructurings focus on reducing the losses associated with this process. One example is the IMF's Sovereign Debt Restructuring Mechanism (SDRM), proposed in Krueger (2001).[7] Considering contractual approaches, the recent introduction of collective action clauses (CACs) on sovereign bonds, a contractual feature that facilitates debt renegotiation, is another movement towards lower default costs.[8] After Bulow and Rogoff (1989), which showed that reputational-based debt is not sustainable under a broad range of assumptions, the current consensus is that reputation cannot be the only cost of default. Lowering its additional costs could increase the likelihood of default for a given debt level, which would increase the costs of borrowing and lower capital flows.

The results in this paper indicate that decreasing default costs would leave borrowers worse off in *ex-ante* terms. The quantitative analysis shows that the costs of default should be limited, leaving default as an option for states of high debt and bad shocks, but the optimal level is much higher than the one consistent with the observed debt levels and default rates of emerging economies. Those results are presented in section 7. The pictures therein clearly illustrate that the optimal level of default costs are far beyond the ones corresponding to the original calibration of the parameters. Those optimal costs would sustain an average debt level of five times the country's output, whereas the target for the calibration is an average debt level of 44.1% of output. With such higher costs, emerging economies would be able to borrow much more and, more interestingly, default more frequently.[9]

In the empirical section, I use the classification of rare disasters of Barro (2006) to define

[7]The evolution of ideas that anteceded the SDRM is discussed in Rogoff and Zettelmeyer (2002).

[8]Those clauses allow for a previously defined majority of bondholders to agree on a change in payment terms that is binding on all bondholders. An analysis of this feature is developed in Haldane et al. (2005).

[9]Default is more frequent because the option of discharging such higher levels of debt compensates for its higher costs.

the events of strong economic crisis. Those events, reported in section 3, were characterized by a huge drop in real per capita output in a short period of time. Most of those events were related to the World Wars or to the Great Depression, which affected most of the countries. Other events, specific to some countries or regions, were mostly experienced by developing economies. This empirical fact should reinforce the role of default as insurance because it makes the income process closer to a binary process. Since the model also considers default a binary choice, it would respond better to this kind of shock than to the gradual output shocks representing business cycles.

A detailed description of the model is provided in section 4. In the model, the default decision generates two consequences for the debtor country. First, the country cannot borrow from international financial markets, which is the reputation cost. Also, there are output losses, often assumed to be due to problems in domestic financial markets during the process of debt renegotiation or even as an effect of being cut off from trade finance. Those costs last for a stochastic number of periods and act as a punishment for default. The parameters related to the output cost and the probability of redemption, meaning the moment when the consequences of default are eliminated, are calibrated and then compared to their optimal values.

The calibration procedure is described in section 5. The *benchmark* calibration of the model is intended to replicate a typical emerging economy. In order to generate a countercyclical trade balance and more volatile consumption, this calibration requires a highly persistent output shock.[10] A lower autocorrelation parameter is considered in the *low persistence* calibration as a robustness check. In both calibrations, the default costs are much lower than optimal values, which provide welfare levels close to the elimination of default. In

[10] Those are stylized facts of emerging economies. See Neumeyer and Perri (2005) and Aguiar and Gopinath (2006).

order to reinforce this result, the *severe crisis* calibration is also considered. Under the assumption that emerging economies may face economic crises stronger than the ones already observed, but similar to the ones faced by OECD economies, which include effects of the World Wars; the optimal costs of default are lower, but never close to the calibrated values.

Statistics generated from simulations of the model are provided in section 6. I also compare the behavior of the model with the data around crisis events. The simulated time series generated by the model must be similar to the ones corresponding to emerging economies facing economic crises in order to show that the model is a good representation of reality and therefore generates reliable results in terms of optimal costs of default. The model replicates the strong reaction of the trade balance to an economic crisis, a stylized fact of emerging economies, but the correct timing of default choices is affected by the assumption that default is never partial.

2 Discussion on default and insurance

In the theoretical literature, default plays two distinct roles. When market incompleteness is an endogenous effect of the limited commitment represented by the option to default, it reduces risk sharing. Agents limit their wealth transfers, keeping them within a level that would not leave their counterparts willing to break the risk sharing scheme. However, if the set of available assets is exogenously assumed incomplete, default may improve risk sharing by allowing borrowers to enter contracts that they may not be able or willing to repay in some states.

The first case, in which risk sharing is reduced, is considered in Kehoe and Levine (1993) and Alvarez and Jermann (2000). They study economies with a complete set of contingent claims in which default is punished with permanent exclusion from financial markets. The full set of contingent claims allows the definition of the debt contract as a portfolio of such assets such that the repayment values given each future state will never leave the borrower willing to default. The presence of potential default, even though it never happens in equilibrium, generates endogenous borrowing limits.[11]

In the second case, in economies with exogenously incomplete markets, default actually happens in equilibrium. Zame (1993) and Dubey et al. (1988, 2005) show that, in this case, default may generate the opposite effect, improving risk sharing. Zame (1993) argues that default allows agents to enter into contracts that they will be able to execute with high probability, but not with certainty. This situation is observed when some contract allows the agent to transfer income between two states, but leaves a high negative payoff for some improbable third state. If default were not allowed, the agent would anticipate the possibility of the third state and reduce his position in the contract which, in turn, would reduce his otherwise optimal transfer of wealth. The option to default allows the optimal transfer of wealth given that, in the case of the realization of the low probability state, the agent will optimally choose to default accepting a reasonable punishment. This argument is made clear by the example that follows.

Example with non-contingent bond

I present a simple example in order to illustrate the trade-off generated by the costs related to the default decision. Assume that the economy lasts for two periods in order to abstract from reputation effects. There is a fixed endowment for the first period $y_1 = 1$; and two possible endowments for the second period, $y_2 = 2$ with probability 98% or $y_2 = 0.1$ with

[11]Default may also reduce risk sharing in an economy with a single uncontingent bond, but not in a natural way. In Zhang (1997), for example, a borrowing limit is defined at the point in which the uncontingent repayment value is such that the agent is indifferent between defaulting or not in at least one state, whereas he strictly prefers not defaulting in all remaining states.

probability 2%. Therefore, the economy is expected to grow, since the expected value of the endowment in the second period is almost twice its value in the first period, but the low probable bad shock generates a strong precautionary savings effect.

Consumption in the first period is given by $c_1 = y_1 + q(b)b$, where $q(b)$ is the price of the country's debt, or the inverse of the gross interest rate, and this value is a function of the amount of debt taken. In the second period, the agent has the option to default on his debt. If he does not choose to do so, his consumption is equal to $c_2 = y_2 - b$. If he defaults, he avoids debt repayment, however there is an output cost ϕ, so consumption in this case is given by $c_2 = (1 - \phi)y_2$.

The expected utility can be written as:

$$EV(b) = u(c_1) + \beta \left[pu(c_{2,1}) + (1 - p)u(c_{2,2})\right]$$

where $p = 0.98$.

Given a risk-free interest rate, the price of debt is the present value of the expected value of repayment, which is equal to one minus the probability of defaulting, which depends on the stock of debt b:

$$q(b) = \frac{1 - D(b)}{1 + r}$$

This pricing kernel assumes risk neutral lenders, so that only the expected value of repayment matters.

The intertemporal discount factor β, the risk-free rate and the utility function are parameterized as:

$$\beta = 1 + r = 1$$
$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$
$$\sigma = 2$$

The parameter ϕ, which represents the output loss if default is chosen in the second period, defines an endogenous borrowing limit, a value $\bar{b}$ such that $(1-\phi)y_2 \geq y_2 - \bar{b}$ for all states in the second period with strict equality for one of them. This value is given by $\bar{b} = 2\phi > 0.1\phi$, so that the agent defaults on the bad state and it is assumed that he does not default in the good state, in which he is indifferent.

In figure 1, the debt level and the corresponding expected utility are illustrated for three scenarios: when the agent can commit to repay his debt in any case, so there is no default; when the agent commits to repay only if hit by the good shock in the second period, then default only happens in the low state; and when the agent cannot commit and decides *ex-post* whether or not to default. The last scenario corresponds to the actual objective function of the agent.

The upper-left graphic, $\phi = 0$, illustrates the case in which there is no output cost of default, therefore the borrowing limit is zero since the borrower would always choose to default. Since no borrowing is sustainable, the agent chooses to save for precautionary motives. He would be better off if he could commit to default only in the bad shock. Since *ex-post* the agent would default in both states, his inability to commit prevents this allocation.

In the upper-right graphic, $\phi = 0.10$, the agent is able to borrow up to a borrowing limit $\bar{b} = 0.20$. The agent would like to borrow more if he could but the borrowing limit is binding, resulting in the corner solution $b^* = \bar{b}$. An interior solution is obtained in the lower-left graphic, with $\phi = 0.30$, the higher borrowing limit $\bar{b} = 0.60$ is not binding. At this point, an increase in the output cost of default, although improving the agent's ability to borrow, does not generate any benefit since the agent does not take advantage of it. Actually, the only effect of a higher cost is hurting more the agent when he chooses to default, which is still the optimal choice when the bad endowment shock is realized.

Finally, the insurance role of default is eliminated when ϕ is set to 0.45, which is observed in the lower-right graphic. In this case, the interior solution obtained with $\phi = 0.30$ turns out to be a local maximum. The cost of default is so high that, rather than issuing debt in the first period, the agent's optimal choice turns back to be the same level of savings $b^* < 0$, observed when he was unable to borrow ($\phi = 0$). The precautionary savings motive dominates: the agent anticipates the possibility of the realization of the bad endowment shock, in which either paying back the debt or defaulting generates a very low utility level, and decides not to borrow.

Summarizing all cases, the expected utility and the optimal choice of debt of the agent unable to commit are represented in figure 2, both as a function of the output cost. For lower values of ϕ, a small increase allows for higher levels of debt but leaves default as an option in the case of the bad shock, increasing the expected utility. For higher values of ϕ (above 0.2), a further increase lowers expected utility. When ϕ lies in the interval between 0.2 and 0.25, although the debt level increases from roughly 0.4 to 0.5, the utility loss due to the expected default costs dominates the gains from front-loading consumption, decreasing expected utility. When ϕ is greater than 0.25, there is no benefit from relaxing the endogenous borrowing constraint by further increases of this parameter, the only effect is a higher expected default cost. The agent does not borrow when ϕ is greater than 0.4, therefore there is no role for default and further increases of ϕ turn out to be inocuous.

This simple example illustrates the objective of this paper. In a model calibrated in order to match observed business cycles statistics and stylized facts of emerging economies,

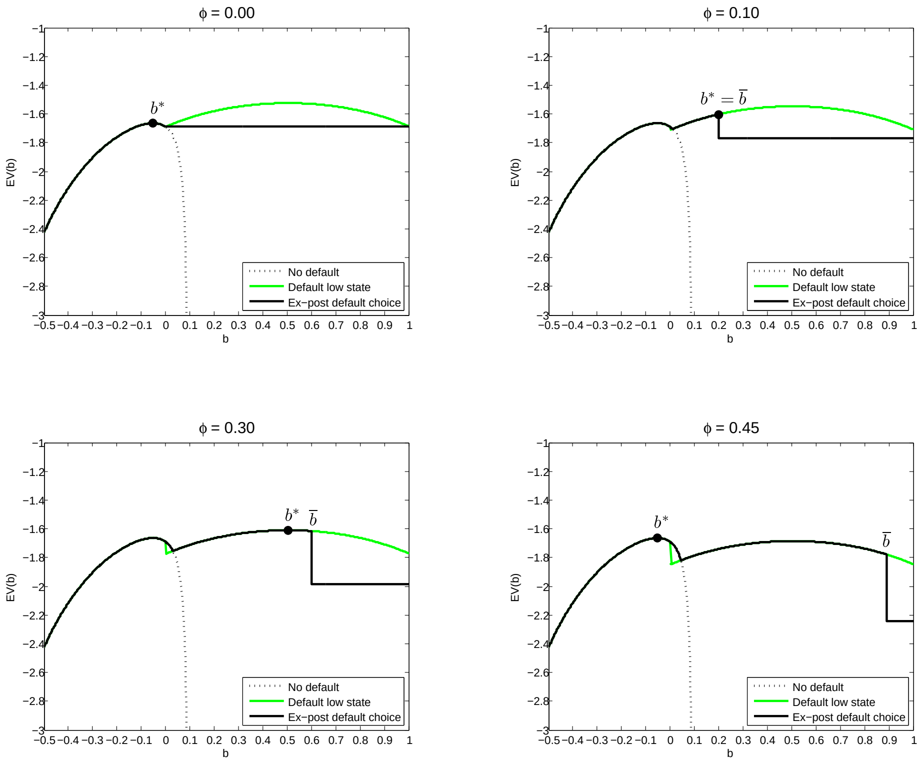

Figure 1: Optimal debt level given different output costs of default ϕ. When $\phi = 0$, no borrowing is sustainable. Increasing it relaxes the borrowing limit. Borrowing limit is binding when $\phi = 10\%$ and is not binding when $\phi = 30\%$. When $\phi = 45\%$ the cost of default is so high that it is not optimal to default in any case, the optimal level of debt is the same as when $\phi = 0$.

in which events of economic crisis are explicitly considered, I perform a similar exercise. Comparing the optimal costs of default with the ones obtained in the calibrations, I intend to identify which effect is dominating: if the costs are high, weakening the insurance role of default, or low, generating borrowing limits that decrease welfare in sovereign countries by limiting their capacity to front-load consumption.

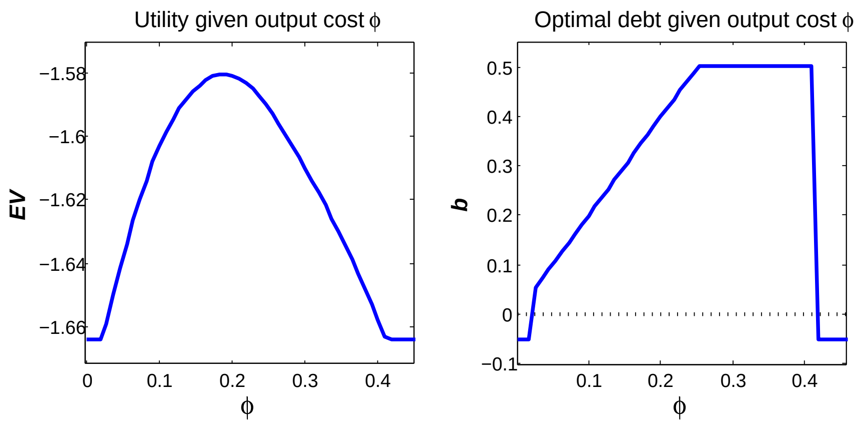

Figure 2: Left: Expected utility given output cost of default ϕ. Right: Amount of debt optimally chosen by the agent given ϕ. For lower values, an increase in ϕ relaxes the borrowing limit and improves welfare. At some point, the loss of output generated by the cost of default reduces welfare. When a threshold value is reached, default is never chosen and the agent does not borrow.

3 Events of economic crises

In general, the business cycle literature relies on well-behaved processes to model output movements. Those processes may be successful in replicating moments of output time series, however, they ignore rare but strong shocks that sometimes hit economies and could potentially affect agent's decisions. Robert Barro tries to identify such events, which he calls *rare disasters*. In Barro (2006), he measures the frequency and size of international economic disasters during the twentieth century. Those events are part of the history of many economies, independently of their level of development or geographic location. The set of events considered are related to the World War I, the Great Depression and the World War II among others specific to countries or regions. In figure 3 there are plots of the times series of log real per capita GDP of four countries, two European developed economies: Austria and Spain; and two Latin American emerging economies: Argentina and Chile. The events of economic crisis are identified by the shaded areas.

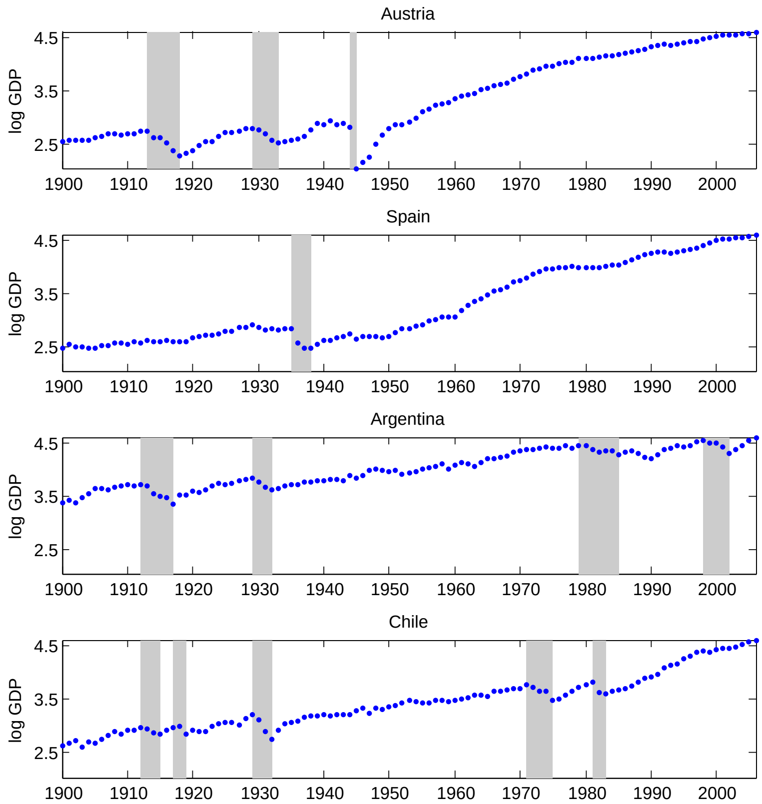

Figure 3: Real per capita GDP normalized to 100 in 2006 (in logs) for Austria, Spain, Argentina and Chile from 1900 to 2006. The shaded areas represent the economic crisis described in Barro (2006), corresponding to periods of declines of 15% or more in this variable.

The effects of the Great Depression can be observed for Austria, Argentina and Chile. Austria's output was also strongly affected by both World Wars. Spain, which was doing relatively well during those periods, had an event of its own: the Spanish Civil War generated a huge per capita output contraction in the 30's. According to the criterion used in the paper, the developed world seems to be free of crises after the World War II. This is not, however, the case for emerging economies. Economic crises have been present in each emerging country or region in its own time, as illustrated for Chile in the 70's and 80's and for Argentina more recently.

The distribution of the size of contractions is illustrated in figure 4 and reveals that those shocks can be extremely strong, usually as a result of wars.

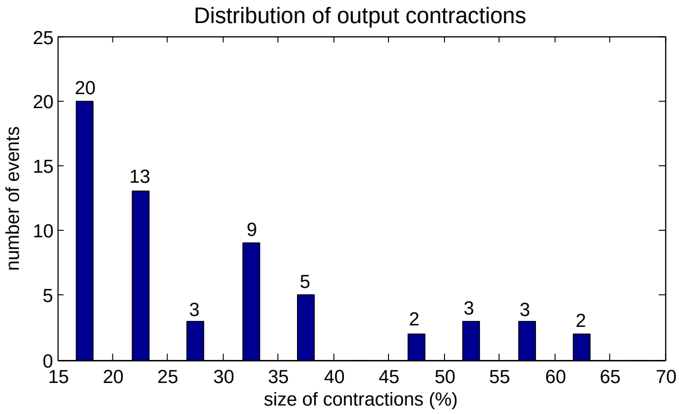

Figure 4: Distribution of the size of contractions in economic disasters described in Barro (2006). Some declines of more than 50% could be observed.

In the model described in the next section, the rare events of economic crisis are represented by permanent shocks to output. The economy is assumed to follow a constant growth path in normal times and, with a fixed probability, switches to a crisis state in which the growth rate is negative. The government, aware of the possibility of such events, has to choose its debt stock balancing its desire to front-load consumption from expected output growth and its precautionary savings motive.

4 Model

The sovereign country is represented by a small open economy, whose benevolent government seeks to maximize the expected discounted lifetime utility of a risk-averse representative agent, which is represented by:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s)$$

where $\beta \in (0,1)$ is the intertemporal discount factor. Each period's utility is represented by a standard constant relative risk aversion utility function with coefficient σ:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

In each period, the output is given by a stochastic endowment. Following Aguiar and Gopinath (2006, 2007), the endowment process is defined as a function of two shocks and is represented by the following expression:

$$y_t = \Gamma_t e^{z_t}$$

where Γ_t is a trend and e^{z_t} is a transitory component. The trend component accumulates each period's log growth rate g_t:

$$\Gamma_t = \Gamma_{t-1} e^{g_t} = \prod_{s=0}^{t} e^{g_s} \tag{1}$$

The growth shock g_t and the temporary shock z_t are assumed to be independent of each other and both follow their own Markov processes. The log of the transitory component of output, z_t, follows an AR(1) process given by:

$$z_t = (1-\rho)\mu_z + \rho z_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2)$$

where μ_z is the unconditional mean, ρ is the autocorrelation parameter and ε_t is an i.i.d. shock of variance σ_ε^2.

The realization of the growth shock defines whether the economy is in normal times or in a crisis state. In normal times the economy grows at the fixed log growth rate $\overline{g}$. During crisis, the economy grows at a negative log growth rate $\widetilde{g}$. Those states switch between each other following a transition matrix which is a function of the parameters χ and $\widetilde{\chi}$, corresponding to the probability of switching from normal to crisis times and, once in the crisis state, of remaining in crisis in the subsequent period, respectively.

$$\begin{bmatrix} 1-\chi & \chi \\ 1-\widetilde{\chi} & \widetilde{\chi} \end{bmatrix}$$

Due to the presence of growth, output is not stationary. In order to have a stationary problem that can be solved with the usual recursive dynamic optimization techniques, a transformation has to be applied to the variables: they are normalized by the accumulated trend growth up to period $t-1$ times the growth rate in normal times $e^{\overline{g}}$. For each original variable x_t, the corresponding normalized variable is given by:

$$\widehat{x}_t = \frac{x_t}{e^{\overline{g}}\Gamma_{t-1}}$$

Given the homothetic structure of the utility function, utility as a function of detrended consumption is given by:

$$u(c_t) = u(e^{\overline{g}}\Gamma_{t-1}\widehat{c}_t) = \left(e^{\overline{g}}\Gamma_{t-1}\right)^{1-\sigma} u(\widehat{c}_t)$$

In the following sections, I deal only with normalized variables and never their full counterparts so, for ease of notation, I do not use the hat to identify them. The value functions are also normalized by the term $\left(e^{\overline{g}}\Gamma_{t-1}\right)^{1-\sigma}$. Details of the value function transformation are provided in the appendix.

4.1 Sovereign government's problem

There are three state variables in the government's problem: the stock of debt b, for which positive values represent debt and negative values represent savings; the temporary shock z and the growth shock g. Given the current stock of debt and both shocks to output, the sovereign government chooses whether or not to default:

$$V^o(b,z,g) = \max\left\{V^c(b,z,g), V^d(0,z,g)\right\} \tag{2}$$

where $V^c(b,z,g)$ represents the expected discounted utility of staying in the debt contract for one more period, and $V^d(0,z,g)$ corresponds to the default decision. The stock of debt turns out to be zero in the case of default because it is assumed that the agent defaults on his whole stock of debt and never renegotiates. Also, defaulting is a strictly dominated strategy for an agent with a positive net stock of assets (negative b) and the fact that $V^d(0,z,g)$ is the continuation utility directly implies that the agent entered the period with some debt.

Considering the case in which default is not chosen, the sovereign government has to choose the level of consumption and next period's debt. The problem is represented by the following Bellman equation:

$$V^c(b,z,g) = \max_{c\geq 0, b'}\left\{u(c) + \beta e^{g(1-\sigma)} E_{z,g} V^o(b',z',g')\right\} \tag{3}$$

$$s.t. \qquad c = e^{(g-\overline{g})} e^z - b + q(b',z,g) e^g b'$$

The term $e^{(g-\overline{g})}$ is equal to one in normal times, which makes detrended output equal to e^z, which gives the value function a usual balanced growth path structure. In times of crisis

$e^{(g-\overline{g})}$ is lower than one in order to account for the negative growth shock.[12]

If the government chooses to default, the positive debt stock is set to zero and the country temporarily loses its ability to borrow from international financial markets. In addition to not being able to borrow, the country faces an output cost of default ϕy. The country regains total access to international financial markets with probability θ. The government's problem when in autarky is represented by the following Bellman equation:

$$V^d(b,z,g) = \max_{c\geq 0, b'\leq 0} \left\{ u(c) + \beta e^{g(1-\sigma)} E_{z,g}\left[\theta V^o(b',z',g') + (1-\theta)V^d(b',z',g')\right]\right\} \quad (4)$$
$$s.t. \qquad c = (1-\phi)e^{(g-\overline{g})}e^z - b + \frac{e^g b'}{1+r}$$

The country is still allowed to save during the autarky period, therefore b' is still a choice variable, but constrained to be lower or equal to zero. In the budget constraint, the price of the bond is explicitly represented as the inverse of the gross risk-free interest rate, since it is assumed that the country saves in riskless assets.

4.2 Pricing sovereign bonds

The international financial markets consist of risk-neutral lenders that discount the future at a fixed risk-free interest rate r. Since the country's government cannot commit to the debt repayment, it has the choice of defaulting and suffering the associated costs, the market price of the bonds issued is calculated taking into account the probability of such event happening.

Once it is assumed that lenders are risk neutral, the current price of a future payment is based exclusively on the expected value of such payment. In order to account for the future

[12] To be precise, the full value of output is given by $y_t = \Gamma_t e^{z_t}$, and its detrended counterpart is $\widehat{y}_t = \frac{\Gamma_t e^{z_t}}{\Gamma_{t-1}e^{\overline{g}}}$. Since $\frac{\Gamma_t}{\Gamma_{t-1}} = e^g$, we have $\widehat{y}_t = \frac{e^g}{e^{\overline{g}}}e^{z_t} = e^{(g-\overline{g})}e^{z_t}$.

states in which the debt is repaid, it is useful to define the repayment set $C(b, g)$:

$$C(b,g) = \left\{ z \in Z \quad | \quad V^c(b,z,g) \geq V^d(0,z,g) \right\} \tag{5}$$

This set contains the values of z for which, given the amount of debt b and the growth shock g, the utility of staying in the contract is greater or equal to the utility of defaulting. Given this set, the bond prices are calculated as follows:

$$q(b',z,\overline{g}) = \frac{1}{1+r}\left[(1-\chi)\int\limits_{z'\in C(b',\overline{g})} dF(z'|z) + \chi \int\limits_{z'\in C(b',\widetilde{g})} dF(z'|z)\right] \tag{6}$$

$$q(b',z,\widetilde{g}) = \frac{1}{1+r}\left[(1-\widetilde{\chi})\int\limits_{z'\in C(b',\overline{g})} dF(z'|z) + \widetilde{\chi} \int\limits_{z'\in C(b',\widetilde{g})} dF(z'|z)\right] \tag{7}$$

The price is defined for normal times and crisis times, the only difference being the probabilities associated with those states in the subsequent period, χ and $\widetilde{\chi}$. The price of debt of face value one is given by the probability that this face value is paid by the debtor in the future discounted by the risk-free rate r.

When the value of b is negative, which means that the country is saving, the repayment set comprehends all possible values of the shock z, defined by the set Z, which means that, for $b' \leq 0$, $q(b', z, g) = \dfrac{1}{1+r}$ for any values of z and g.

4.3 Definition of equilibrium

The definition of equilibrium for the transformed model, with deflated variables, is stated as follows.

Definition 1. *A* recursive equilibrium *is defined as a set of policy functions for: (i) con-*

sumption $c(b, z, g)$; (ii) next period's debt stock $b(b, z, g)$; a no default set $C(b, g)$ and a price function $q(b', z, g)$ such that:

1. *Taking the price function as given, the policy functions satisfy the government's optimization problem represented by (2), (3) and (4); and the corresponding value functions define the repayment sets according to (5);*

2. *The price function reflects the default probabilities given the repayment set and the output process, such that lenders break even in expected value, which is represented by equations (6) and (7).*

5 Calibration

For evaluating the effect of different costs of default, the model is calibrated in order to replicate some statistics and stylized facts of a real economy. The focus is on emerging economies, since those are the ones usually involved in episodes of debt crises and default.

The coefficient of relative risk aversion in the utility function is set to 2, a standard value widely used in the macro literature. The risk-free interest rate is calibrated to 4%, based on the U.S. annual interest rate. The remaining parameters are set in order to match some empirical observations of emerging economies data.

The time series of real per capita GDP are taken from Barro and Ursúa (2008).[13] In order to compare their statistics, the countries are included in five non-exclusive groups: all countries, emerging economies, Latin America, Asia and OECD. The countries and the groups they are included in are listed in the appendix.

[13] The dataset in Barro and Ursúa (2008) covers 40 countries, but only the 35 countries considered in Barro (2006) are used here.

The time series are split between normal times and crisis times. For the statistics of normal times, observations corresponding to the crisis periods listed in Barro (2006) are dropped from the sample. Periods of five years after the end of each crisis are also discharged in order to avoid a possible different behavior during recovery periods. For the remaining data, only continuous periods of at least 10 years are considered and then have their trend removed with the Hodrick-Prescott (HP) filter.[14] Business cycle statistics during crises are not calculated since the crisis periods are very short for a trend to be identified. Those statistics are listed on table 1.

Table 1: Statistics for each country group

	# of countries	σ_y	ρ	σ_ε	$\overline{g}$	$\widetilde{g}$
Emerging economies	12	3.71%	0.4056	3.37%	2.59%	−6.86%
Latin America	8	3.64%	0.4611	3.22%	2.37%	−6.90%
Asia	7	3.70%	0.4060	3.37%	2.90%	−8.99%
OECD	23	3.78%	0.4202	3.43%	2.45%	−9.46%
all countries	35	3.67%	0.4121	3.33%	2.46%	−8.75%

Business cycles statistics, respectively, standard deviation of the cycle and, for the AR(1) process, autocorrelation and standard deviation of the error term. Also, average growth rates for normal and crisis times.

The statistics for normal times, represented by the first four columns of values, are very similar across groups. Differences are evident for the growth rate during crisis times, $\widetilde{g}$, with values ranging from −6.86% to −9.46%. Differences in those values should be expected since there are only few observations of those events. Most of the economies that belong to the OECD were the ones more involved in the World Wars, which is the reason for their very low $\widetilde{g}$. This value is also considered for the calibration of emerging economies as a robustness exercise, since those economies have been increasing their financial integration with the world and, in addition to the benefits it may bring, it also leaves them more exposed

[14]The HP-filter smoothing parameter λ is set to 100.

to international crises which could cause similar economic disasters.

Emerging economies' detrended output has a standard deviation of 3.71% in normal times. It is assumed that it follows an AR(1) process, which is estimated, yielding an autocorrelation coefficient of 0.41 and a standard deviation of the error term equal to 3.37%. The AR(1) process for the model's output is approximated using the algorithm in Tauchen (1986). The parameters used in the model should generate simulated time series with the same statistics obtained in the data after the trend is extracted by the same procedure, the HP-filter.

When the model is simulated with an autocorrelation parameter $\rho = 0.86$ and an error term standard deviation of $\sigma_\varepsilon = 4.1\%$, it is able to yield the same statistics obtained in the data. However, the parameter ρ is not precisely determined, since setting it to 0.96, and keeping the same standard deviation of the error term, changes the estimated autocorrelation to 0.43. This value is within one standard deviation (0.03) of the original estimated value. The problem with setting $\rho = 0.86$ is that the model generates a positive correlation of output and trade balance and an excessive consumption smoothing: the volatility of consumption being lower than the one for output. Both of these features are counterfactual for emerging economies.[15] Setting $\rho = 0.96$, however, generates statistics qualitatively right: a correlation of output and trade balance of -18.3% and a ratio of standard deviations of consumption and output of 1.12. Thus, the value of 0.96 is adopted in the *benchmark* calibration whereas the other one is used for robustness check.

A higher persistence of the output shock increases its effect on the level of debt in which the government is indifferent between defaulting or not. This level of debt defines the limit of the default region, illustrated in figure 5. It is almost close to a vertical line with $\rho = 0.86$ and it is less steep with $\rho = 0.96$. The debt price $q(b', z, g)$ will also be more sensitive to the

[15] An extensive analysis of business cycles of emerging economies is found in Neumeyer and Perri (2005).

current shock z: with persistence, a higher z' in the following period is expected given a high z today, lowering the probability of default. This feature allows for more borrowing in good times due to a lower interest rate (higher $q(\cdot)$), which allows for more consumption than what would be given exclusively by a higher current output. It generates more volatility of consumption with respect to output and a countercyclical trade balance.

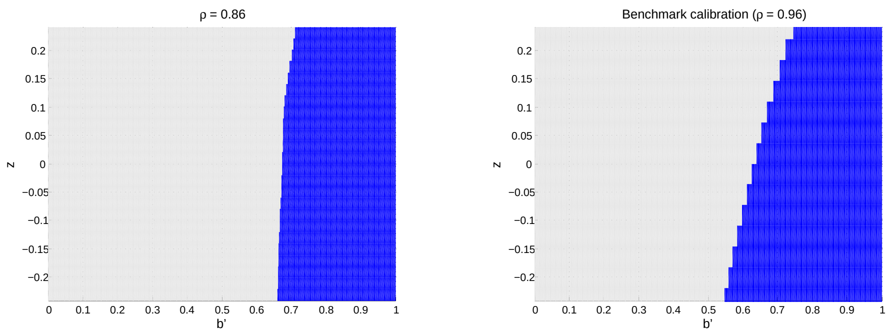

Figure 5: Default region for different calibrations of the model. Output shock has almost no effect on the default decision when $\rho = 0.86$. The effect of the shock increases with persistence, which is observed when $\rho = 0.96$.

The log growth rates are set to the average first-difference of the log real per-capita output of all countries considered in the sample. The average is calculated separately for normal times and crisis times and the corresponding values are used to calibrate the parameters $\overline{g}$ and $\widetilde{g}$, respectively. The average log growth rate of emerging economies during normal times is equal to 2.59%, whereas the corresponding value for crisis periods is of -6.86%.

The parameter χ determines the probability of switching from normal times to a crisis event. In order to calibrate it, I consider the number of crisis observed in the sample, independent of their duration, since the event of entering the crisis is what is relevant. This number is divided by the number of years in normal times for all countries considered. The duration of each crisis is used to calibrate $\widetilde{\chi}$, the probability of remaining in crisis. The average duration of observed crisis should be equal to $\frac{1}{1-\widetilde{\chi}}$. For emerging economies, crisis events start with probability 1.97% and last for 5 years on average. Those values are presented in table 2.

Table 2: Frequency and duration of crisis

	Probability of crisis	Probability of remaining in crisis
Emerging economies	1.97%	80.16% (5.04 years on average)
Latin America	2.58%	78.43% (4.63 years on average)
Asia	1.22%	80.00% (5.00 years on average)
OECD	1.44%	79.77% (4.94 years on average)
all countries	1.74%	79.23% (4.81 years on average)

The parameter θ, which determines the probability of redemption after a default decision, is set to 21.28%, which results in an average period of 4.7 years in which the country is unable to borrow. This autarky spell was found by Gelos et al. (2004) to be the mean time for a defaulting country to regain access to international financial markets in the 80's.

The output cost of default ϕ and the intertemporal discount factor β are set in order to match the probability of default conditional on a crisis and the average debt level of emerging economies. A lower β increases the average debt level, due to impatience, but it also increases the incidence of default on crisis. While β is chosen to match the latter, the output cost ϕ is used to adjust the debt level. A higher value of β would make the representative agent more patient, remaining only the insurance motive to justify holding debt with the option to default. In this case however, instead of holding debt, the country self-insures by saving.

The crisis events related in Barro (2006) are listed in the appendix. The last column of the table signalizes when a default decision coincided with those crisis.[16] Considering the emerging economies only, there are 25 crisis events, 10 of them coinciding with a default

[16]The default events are listed in the appendix. The list considers three sources and was extracted from Borensztein and Panizza (2008).

decision: 40% of the cases. This number decreases to 28% if all countries are considered and increases to 50% when restricted to Latin America. This incidence of default is consistent with the finding of Tomz and Wright (2007) that only one-third of debtors lapsed into default during extremely hard times. The parameters are chosen to match the value of 40%, corresponding to emerging economies.

The model, calibrated to match the incidence of default on crisis, generates an overall probability of default around 0.75%, which is low compared to the data. The low incidence of default events is a known problem of sovereign debt models.[17] One explanation is that many default events happen during political turnovers and are due to opportunistic reasons or myopic governments.[18] The model abstracts from those motives, therefore generating a probability of default which is low compared to historical observation.

The target for the average debt level is based on the observations documented by Reinhart et al. (2003). They document the fact that emerging market economies cannot sustain debt-GDP ratios higher than 50%, citing as examples the debt crisis of Mexico, in 1982, and Argentina, in 2001, which occurred at ratios of 47% and 50% respectively. The value of 44.1% used in the calibration is the average of the period from 1970 to 2000 for emerging economies which have defaulted in the past. All *benchmark* calibration parameters are reported on table 3.

Two alternative calibrations are also considered. The *low persistence* calibration keeps the autocorrelation ρ equal to 0.86, value that matches the autocorrelation estimated from the data but, as explained before, generates counterfactual features: a procyclical trade

[17] Arellano (2008) considers asymmetric default output costs, in which the cost is lower in recessions, in order to match a probability of default of 3% for Argentina. Aguiar and Gopinath (2006) show that an output process driven by growth shocks to increase the probability of default.

[18] Kohlscheen (2010) mentions that some default episodes, such as in Peru in 1989 and Argentina in 2001, have actually coincided with inauguration speeches of presidents. Hatchondo et al. (2008) and Cuadra and Sapriza (2008) consider political turnover in their models.

Table 3: Model parameters - *benchmark* calibration

Risk aversion	$\sigma = 2$	standard
Interest rate	$r = 4\%$	U.S. annual interest rate
Output process	$\rho = 0.96$	$\sigma_c/\sigma_y > 1$ and $corr(y, tb/y) < 0$
	$\sigma_\varepsilon = 4.1\%$	st. dev. of detrended output $\sigma_y = 3.71\%$
Normal times growth	$\overline{g} = 2.59\%$	average log growth rate in normal times
Crisis growth	$\widetilde{g} = -6.86\%$	average log growth rate during crisis
Probability of redemption	$\theta = 21.28\%$	average autarky period of 4.7 years
Output cost of default	$\phi = 11.4\%$	incidence of default on crisis of 40%
Discount factor	$\beta = 0.95$	average debt level of 44.1%
Prob. of entering a crisis	$\chi = 1.97\%$	from emerging economies data
Prob. of remaining in crisis	$\widetilde{\chi} = 80.16\%$	from emerging economies data

balance and excess consumption smoothing. After setting $\rho = 0.86$, the other parameters are set in order to match the same targets as in the *benchmark* calibration. The remaining calibration considers the lowest value for the growth rate during crisis, $\widetilde{g} = -9.46\%$, which corresponds to OECD countries. In this calibration it is assumed that stronger crisis are expected, but they were not realized for emerging economies during the time period covered by the sample. Another view could be that the development of those economies and their integration with the world, in addition to bringing benefits, also leaves them exposed to shocks to which they were not in the past. In this calibration, only the parameter $\widetilde{g}$ is changed. Instead of attempting to match the targets as in the previous calibrations, the idea is to observe the effect of this change on the results.

Table 4: Alternative calibrations

	low persistence	*severe crisis*
ρ	0.86	0.96
ϕ	11.8%	11.4%
β	0.9635	0.95
$\widetilde{g}$	-0.0686	-0.0946

Remaining parameters keep their original values.

6 Simulation

The model is simulated and, for each calibration, 5000 samples of 1000 years are generated. Only the last 100 years of each sample are considered, in order to avoid the effect of initial conditions. The simulation statistics are listed on table 5.

Table 5: Simulation results

	benchmark	*low persistence*	*severe crisis*
σ_y	3.7%	3.7%	3.7%
σ_c/σ_y	1.13	0.92	1.12
$corr(y, tb/y)$	-0.219	0.347	-0.179
average b	0.440	0.442	0.458
default on crisis	40.01%	40.55%	70.94%
share of default on crisis	98.73%	100%	97.99%
total default incidence	0.75%	0.75%	1.35%

Standard deviation of output and consumption taken after trend is removed by the HP-filter. Average debt measured as fraction of GDP.

Both the *benchmark* and the *low persistence* calibrations were defined in order to match target values of the average debt level and the probability of default conditional on a crisis event. As mentioned before, the *low persistence* calibration generates counterfactual procyclical trade balance and excess consumption smoothing ($\sigma_c/\sigma_y < 1$). The *severe crisis* differs from the *benchmark* calibration in just one parameter, which is the growth rate during crisis events, it is equal to -6.86% in the *benchmark* and changes to -9.46% in the *severe crisis* calibration. It is not aimed at matching any other statistic. Using this calibration, the consumption volatility and the correlation of output and trade balance is roughly the same and there is a small increase in the average debt level. The incidence of default on crisis events increases to 70%.

Besides observing the model's statistics, it is important to understand its behavior around crises, since they are the main trigger of default, and verify if it is similar to what is observed in the data. To this end, I present a simulated time series of output, consumption, debt-output ratio and trade-balance-output ratio in figure 6; and some time series comprehending periods of crises listed in Barro (2006) in figure 7. The latter includes examples of output contractions in Chile and Philippines, at the beginning of the 80's; and Argentina and Indonesia, at the end of the 90's. In all cases, the shaded areas correspond to crisis events and the periods of default and the subsequent autarky are circled.

The simulation includes two events of crisis. The dark-shaded area corresponds to periods with the realization of the negative growth shock. The first one did not trigger a default event whereas the second one generated a default. The first crisis starts in period 9, in which the debt-output ratio is around 40%. Due to the negative growth rate, the country starts saving by accumulating large trade surpluses, which initially corresponds to a reduction in the stock of debt, until period 12, when it starts to be a foreign creditor, with negative debt.

The second crisis starts in period 37 and, in this case, the debt-output ratio is greater than 50%. As shown in figure 5, default is an attractive option when the level of debt is high and the cycle is closer to a recession. Those conditions trigger a default in period 38: the total stock of debt is discharged and the country loses part of output and access to international financial markets.

There is a strong positive movement in the trade-balance-output ratio at the beginning of the first crisis. This variable closely follows the current account. This phenomenon is similar to the current account reversals observed in some episodes for emerging economies, the so-called *sudden stops*: strong capital outflows together with falls of output and consumption.[19]

In the second crisis, the default generates a delay in this movement. Although being allowed to save after default, the country does not save because of the temporary output loss generated by the default decision. With redemption, there is an increase in output and,

[19]More details about the *sudden stops* in Calvo (1998).

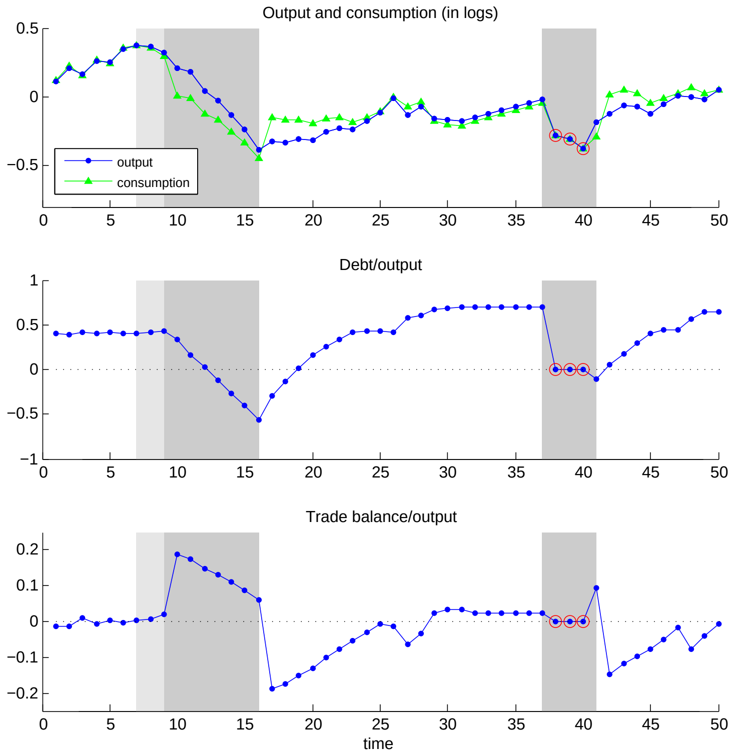

Figure 6: Sample of simulated time series. Crisis events are represented by the dark-shaded areas. The light-shaded areas extend the crisis to the point in which output starts decreasing.

at this point, the country starts saving because the crisis period is not over yet.

At the end of both crises, by again experiencing positive growth rates and having already recovered access to international financial markets, the country starts borrowing again in order to front-load consumption out of expected higher future output.

In order to make the analysis of model simulations and data compatible, I add a light-shaded area at the beginning of the model's crises, extending the crisis period to the last peak of output, as they are defined in Barro (2006). In the second crisis it does not make a difference once that the negative growth shock corresponds to the output peak. This way, we can justify the fact that, in the data, the strong movement in the current account does not happen at the beginning of a crisis period in some cases. It is immediate in the cases of Indonesia and Chile whereas there is some lag with respect to the beginning of their crises in the cases of Argentina and the Philippines. These lags could be compared with the light-shaded areas of figure 6, in which the output is declining due to temporary shocks.

In the data, default does not generate the same effect in the current account as what happens with the trade balance in the model. The model assumes full default, eliminating the country's total stock of debt, and only short-term debt. In the real world, debt is composed of different maturities, from different creditors, and default is often partial or sequential. This simplification is common in this literature and is not believed to significantly affect the results.

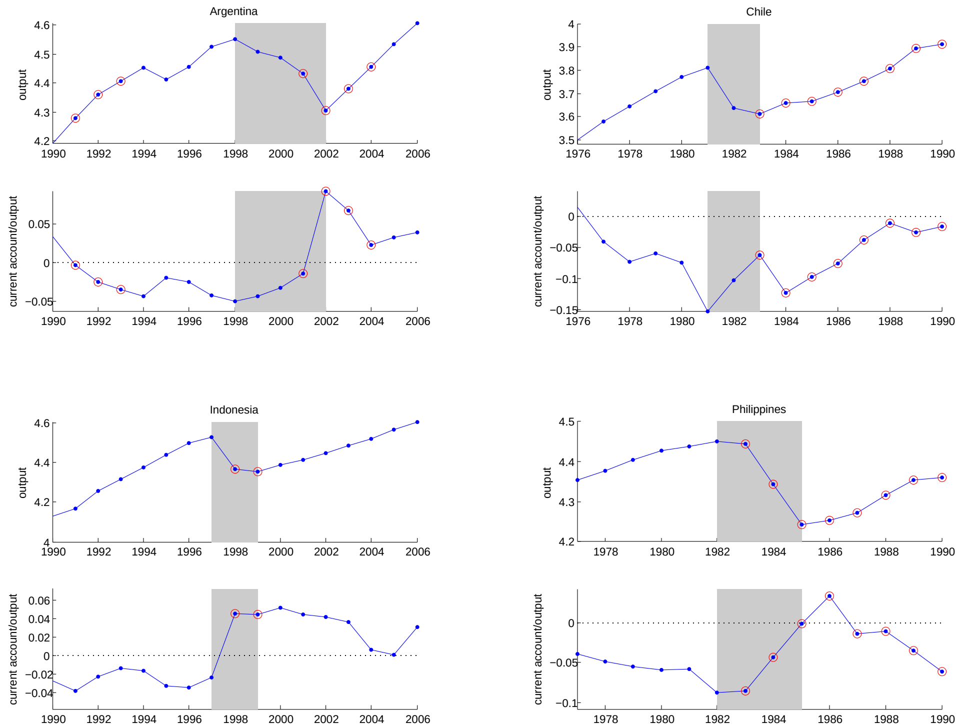

Figure 7: Time series of output and current account around economic crises (shaded areas).

7 Optimal level of default costs

After calibrating the model, identifying the parameters consistent with the observed emerging economies' statistics related to debt and default, I search for the optimal costs of default, the ones that maximize the expected utility of the borrowing country's representative agent.

The main result of the paper is illustrated in figures 8 to 10. For each calibration, four variables are plotted as functions of the output cost ϕ: the welfare gain measured in terms of the consumption compensation,[20] the average debt level, the unconditional probability of default and the probability of default conditional on a crisis event. Also, they are plotted for different values of the probability of redemption θ. The plots of the consumption compensation and the average debt level correspond to figure 2 in the two-period example presented in section 2.

The lowest probability of redemption considered is 10%, which generates an average of 10 years exclusion from international credit markets and incurring output losses after default. This average period corresponds to the value found in the historical dataset of Tomz and Wright (2007), covering the period 1820—2004, and is used as an upper bound. The value of 40% corresponds to $2\frac{1}{2}$ years, which is an average of the mean autarky periods observed in the 80's and 90's. The highest probabilities considered are 70% and 100%, corresponding respectively to $1\frac{1}{2}$ years and to an immediate redemption. In each picture, the point corresponding to the original values of ϕ and θ is indicated.

The output cost of default ϕ assumes values in the interval between zero and 90%. For high values of ϕ, the results converge to the case in which default is ruled out. The lower is the probability of redemption θ, the lower is the threshold value of the output cost ϕ for which default is never chosen. For example, considering the *benchmark* calibration, it can

[20]The relative increase in the expected consumption path under the original default costs necessary to generate the same expected utility.

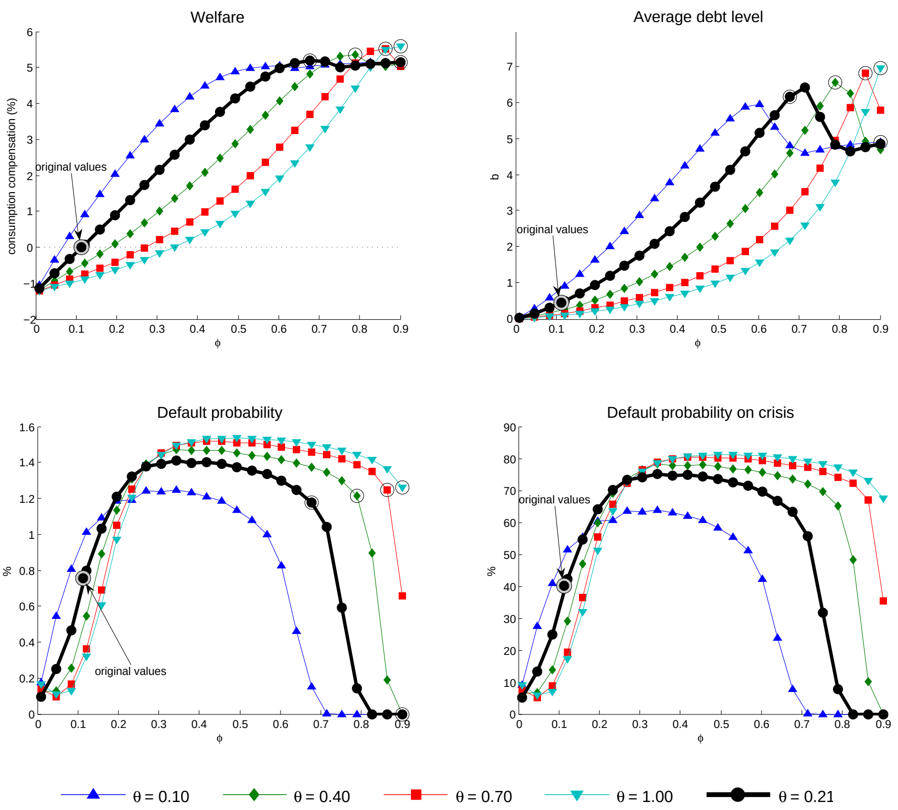

Figure 8: Results for different values of the output cost ϕ and the probability of redemption θ based on the *benchmark* calibration.

be observed in the plots of the default probability that, with $\theta = 0.10$, it goes to zero with $\phi = 0.7$, whereas it needs ϕ greater than 0.8 when θ assumes the original value of 21.28%.

The convergence of the variables to the level corresponding to the no-default case is not monotonic. The probabilities of default, for example, are hump-shaped functions of the output cost ϕ. Initially, the probability of default increases with the amount of debt held by the country. It happens because the benefit of default is increasing in the amount of debt that can be discharged, whereas its cost is independent of it. At some point, the cost of default starts inhibiting its occurrence until it completely rules it out.

Default can be welfare improving as explained by Zame (1993). If its costs to the borrower are enough to sustain an optimal level of debt but not so high to eliminate it, default will be optimally chosen after bad shocks. The maximum welfare level and the corresponding values of debt and default probabilities for each θ are circled in each picture. In all cases, those points correspond to positive probabilities of default and to average debt levels greater than their no-default values.

In the *benchmark* calibration, illustrated in figure 8, the average debt level in the no-default case is around 4.5 times output, with some default it reaches 6 times output. However, the higher amount of debt does not translate into a significant welfare gain when compared to the no-default case. The results are very similar to the *low persistence* calibration, which can be observed in figure 9. The main difference of this calibration is that lower levels of the output cost ϕ are required to rule out default. For those calibrations, eliminating default is very close to optimal for the sovereign borrower in *ex-ante* terms.

The insurance role of default is more evident in the *severe crisis* calibration. If we assume that emerging economies are subject to crisis events similar to the ones observed on OECD countries, corresponding to an average negative growth rate of 9.46%, the option to default leads to better allocations, improving welfare, in comparison to its elimination. The hump-shaped consumption compensation plots in figure 10 best resemble the expected utility plot in the two-period example illustrated in figure 2.

In this calibration, default indeed improves welfare, allowing the country to borrow more relying on default as an alternative to the burden of paying back the debt during bad times. In the no-default case, the stronger precautionary motive brings the average debt level down to a value around three times output. When θ has the original value of 21.28%, a value of ϕ of 0.7 allows the country to double this average debt level, with a default rate of 1.2%. The ability to borrow more increases the *ex-ante* utility of the sovereign country. The higher welfare level is reached in a level of debt higher than the no-default value but lower than

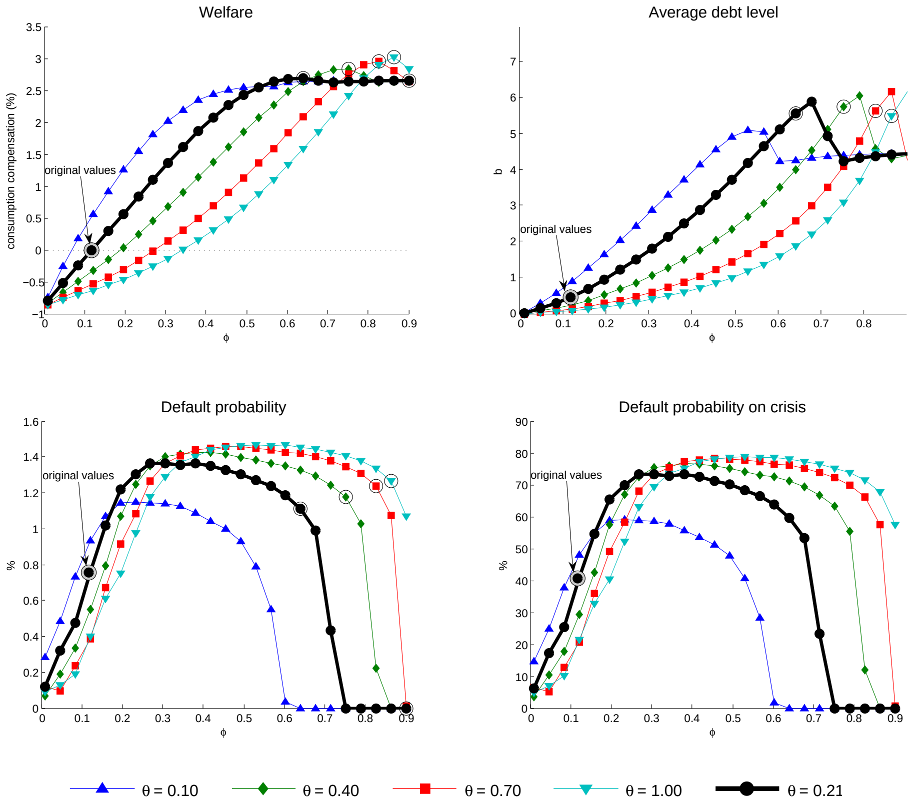

Figure 9: Results for different values of the output cost ϕ and the probability of redemption θ based on the *low persistence* calibration.

the maximum level, as in the two-period example. With the original value of θ, the higher welfare level is reached when the average debt level is around 4.5 times output.

In all calibrations, both the optimal level of debt and the corresponding no-default value are extremely high for the standards of emerging economies' sovereign debt. Given that emerging economies hold a stock of debt of around one half of their output, the option to default just hurts them by generating higher interest rates and lower borrowing limits. Even in the *severe crisis* calibration, for which default can significantly improve welfare, the model suggests that emerging economies cannot reach the optimal level of debt due to weak default punishments. Considering the original values of default costs indicated in the pictures, either

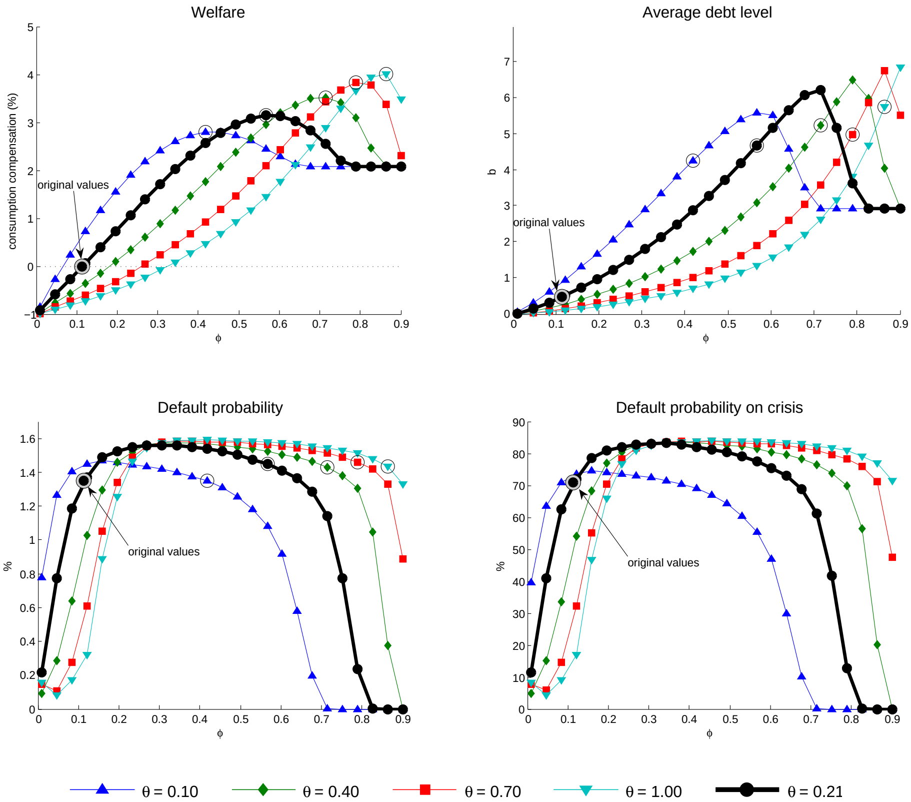

Figure 10: Results for different values of the output cost ϕ and the probability of redemption θ based on the *severe crisis* calibration.

increasing the output cost ϕ or decreasing the probability of redemption θ would generate a welfare gain to the sovereign borrower.

8 Conclusion

I apply a standard incomplete markets model of sovereign debt in order to investigate the potential welfare improving properties of the option to default. The new feature of the model is the introduction of events of strong negative shocks representing economic crises, which are distinguished from standard business cycles. Those events should be the opportunity in which the option to default could be welfare improving by providing insurance. The insurance provided by the option to default can reduce the precautionary savings motive of a sovereign government that anticipates the huge burden of debt repayment after output contractions.

Given the observed levels of debt held by emerging economies and the incidence of default on crisis events, the results suggest that the option to default and its current costs are actually limiting the countries' ability to borrow. The welfare of the emerging economy's representative agent would be improved if the default decision generated harsher punishments. Higher levels of debt would then be sustainable, effect that is strongly dominating the insurance role of default. For the option to default to be welfare improving, it must result on the elimination of a higher stock of debt, in a level not reached by emerging economies.

Those results have implication on the evolution of debt workout procedures and the potential demand for new debt instruments.[21] Both should lead to an increase in expected default costs from the borrower's *ex-ante* point of view. In a debt renegotiation process aligned with this objective, the debtor country must have a limited bargaining power, guaranteed by conditional access to alternative sources of financing. Concerning debt instruments, they must have standardized contracts, with rules conditioning their issuance to the absence of previous debt arrears. Besides penalties, mechanisms that enforce repayment by limiting the sovereign's ability to default should also be considered. A good example is the securitization of future flow receivables, in which future revenues, such as those from oil exports, are directed to debt service payments.[22]

Justifiable or even unavoidable defaults may not be exclusively a consequence of bad shocks, but actually a result of moral hazard. Having default as an easy way out of high

[21]Those issues are discussed in Shleifer (2003).

[22]The borrower's ability to default is limited because the corresponding funds are managed by an international trustee, responsible for making debt service payments and then transferring the remaining funds to the debtor. For more details see IMF (2003).

indebtedness may stimulate bad debt management and fiscal policies. Even assuming that lower costs are a result of changes in the sovereign debt market structure, beyond the control of any country or international institution, those costs can, at least marginally, be affected by agent's decisions. It seems puzzling though that recent efforts have moved in the direction of lower exclusion time, for instance, by the introduction of collective action clauses in debt instruments; and of increasing the availability of alternative sources of funds, such as precocious bailouts. Understanding these movements requires considering not only operational but also political aspects, which is out of the scope of this paper.

The results presented here are in line with Athreya et al. (2009) who investigate how default penalties on personal loans affect credit access, consumption smoothing and welfare of households. They show that a positive probability of catastrophic events is necessary to justify lax default penalties. In the consumer credit literature, those events come usually in the form of expense shocks. Here, that role is played by the permanent shocks representing economic crises.

The analysis performed in this paper abstracts from possible effects of debt service and on the way it is financed on output, such as distortionary taxation. Also, partial default and the renegotiation process are not considered. Extending the model with the introduction of those features is an interesting topic for future research. Also, the costs of default are introduced exogenously in the model. Welfare analysis could be improved by endogenizing them, which would require a more complicated model, along the lines of Mendoza and Yue (2008).

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A Appendix

A.1 Value function transformation

Detrending consumption in t by the accumulated growth until $t-1$ times the growth rate in normal times, $e^{\bar{g}}\Gamma_{t-1}$, results in detrending utility by the factor $\left(e^{\bar{g}}\Gamma_{t-1}\right)^{1-\sigma}$. This factor is used to detrend the value function, defining a Bellman equation with the detrended variables.

Define:

$$\widehat{V^c}(\widehat{b}, z, g) = \frac{V^c(b, z, g, \Gamma)}{\left(e^{\bar{g}}\Gamma_{-1}\right)^{1-\sigma}}$$

where $\widehat{b} = \dfrac{b}{e^{\bar{g}}\Gamma_{-1}}$ and $\Gamma = e^g\Gamma_{-1}$.

The transformation of the Bellman equation is performed as follows:

$$\begin{aligned}
V^c(b, z, g, \Gamma) &= \max_{c\geq 0, b'} \{u(c) + \beta E V^o(b', z', g', \Gamma')\} \\
\frac{V^c(b, z, g, \Gamma)}{\left(e^{\bar{g}}\Gamma_{-1}\right)^{1-\sigma}} &= \max_{c\geq 0, b'} \left\{ \frac{u(c)}{\left(e^{\bar{g}}\Gamma_{-1}\right)^{1-\sigma}} + \beta E \frac{V^o(b', z', g', \Gamma')}{\left(e^{\bar{g}}\Gamma_{-1}\right)^{1-\sigma}} \right\} \\
\frac{V^c(b, z, g, \Gamma)}{\left(e^{\bar{g}}\Gamma_{-1}\right)^{1-\sigma}} &= \max_{c\geq 0, b'} \left\{ u\left(\frac{c}{e^{\bar{g}}\Gamma_{-1}}\right) + \beta e^{g(1-\sigma)} E \frac{V^o(b', z', g', \Gamma')}{\left(e^{\bar{g}}\Gamma\right)^{1-\sigma}} \right\} \\
\frac{V^c(b, z, g, \Gamma)}{\left(e^{\bar{g}}\Gamma_{-1}\right)^{1-\sigma}} &= \max_{c\geq 0, b'} \left\{ u\left(\widehat{c}\right) + \beta e^{g(1-\sigma)} E \frac{V^o(b', z', g', \Gamma')}{\left(e^{\bar{g}}\Gamma\right)^{1-\sigma}} \right\} \\
\widehat{V^c}(\widehat{b}, z, g) &= \max_{c\geq 0, \widehat{b}'} \left\{ u\left(\widehat{c}\right) + \beta e^{g(1-\sigma)} E \widehat{V^o}(\widehat{b}', z', g') \right\}
\end{aligned}$$

The corresponding budget constraint must also be detrended:

$$\begin{aligned}
c &= \Gamma e^{z} - b + q(b', z, g, \Gamma) b' \\
\frac{c}{e^{\overline{g}}\Gamma_{-1}} &= \frac{\Gamma e^{z}}{e^{\overline{g}}\Gamma_{-1}} - \frac{b}{e^{\overline{g}}\Gamma_{-1}} + q(b', z, g, \Gamma)\frac{b'}{e^{\overline{g}}\Gamma_{-1}} \\
\frac{c}{e^{\overline{g}}\Gamma_{-1}} &= \frac{e^{g} e^{z}}{e^{\overline{g}}} - \frac{b}{e^{\overline{g}}\Gamma_{-1}} + q(b', z, g, \Gamma) e^{g}\frac{b'}{e^{\overline{g}}\Gamma} \\
\widehat{c} &= e^{g-\overline{g}} e^{z} - \widehat{b} + \widehat{q}(\widehat{b'}, z, g) e^{g}\widehat{b'}
\end{aligned}$$

The price function $q(b', z, g, \Gamma)$, which is based on the expected value of repayment given by the probability of having an output shock inside the no-default set $C(b, g, \Gamma)$. Repaymend is given by the condition $V^{c}(b, z, g, \Gamma) \geq V^{d}(0, z, g, \Gamma)$, which is not affected by the transformation once that both sides are divided by $\left(e^{\overline{g}}\Gamma_{-1}\right)^{1-\sigma}$. The term Γ is irrelevant since the growth effect is given by the current growth shock g. The inequality is then equivalent to $\widehat{V^{c}}(\widehat{b}, z, g) \geq \widehat{V^{d}}(0, z, g)$, which makes the no-default set $\widehat{C}(\widehat{b}, g)$, defined in (5), equal to $C(b, g, \Gamma)$ for any Γ. Therefore, the price function $\widehat{q}(\widehat{b'}, z, g) = q(b', z, g, \Gamma)$.

A.2 Additional tables

Table 6: List of default events

country	Standard & Poor's begin	Standard & Poor's end	Beim & Calomiris begin	Beim & Calomiris end	Sturzenegger & Zettelmeyer begin
Indonesia	1998	1999			
Indonesia	2000				
Indonesia	2002				
Japan			1942	1952	
Korea					
Philippines	1983	1992	1983	1992	1983
Austria			1914	1915	1914
Austria			1932	1952	1932
Austria					
Austria					
Germany					1932
Germany			1932	1953	
Greece			1932	1964	
Italy			1940	1946	1940
Argentina			1956	1965	
Argentina	1982	1993	1982	1992	1982
Argentina	2001	2004			2001
Brazil					
Brazil			1914	1919	1914
Brazil					1931
Brazil			1931	1943	
Brazil			1961	1964	
Brazil	1983	1994	1983	1992	1983

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Table 6: List of default events (continued)

	Standard & Poor's		Beim & Calomiris		Sturzenegger & Zettelmeyer
country	begin	end	begin	end	begin
Chile			1931	1948	1931
Chile			1965		
Chile			1972	1975	
Chile	1983	1990	1983	1990	1983
Colombia					1900
Colombia			1932	1944	1932
Colombia					
Mexico			1914	1922	1914
Mexico			1928	1942	
Mexico	1982	1990	1982	1990	1982
Peru			1931	1951	1931
Peru			1968	1969	
Peru	1976				
Peru	1978				
Peru	1980				
Peru	1984	1997	1978	1992	1978
Peru					1983
Uruguay			1915	1921	1915
Uruguay			1933	1938	1933
Uruguay	1983	1985			
Uruguay	1987				

This table is a subset of table A1 of Eduardo Borensztein and Ugo Panizza's "The Costs of Sovereign Default"corresponding to the countries in the sample used in this paper and the same time period. Standard & Poor's lists defaults on foreign currency bank debt in the period from 1824 to 2004. Beim and Calomiris (2001) use a different methodology and cover the period from 1800 to 1992. Sturzenegger and Zettelmeyer (2006) is based on data from Beim and Calomiris (2001) and Lindert and Morton (1989) and covers the period from 1874 to 2003.

Table 7: List of crisis events

country	begin	end	related to	default
Argentina	1912	1917	WW I	
Argentina	1929	1932	Great Depression	
Argentina	1979	1985	Post-WW II Depression	•
Argentina	1998	2002	Post-WW II Depression	•
Australia	1928	1931	Great Depression	
Austria	1913	1919	WW I	•
Austria	1929	1933	Great Depression	•
Austria	1944	1945	WW II	
Belgium	1916	1918	WW I	
Belgium	1939	1943	WW II	
Canada	1917	1921	Aftermaths of WW I	
Canada	1929	1933	Great Depression	
Chile	1912	1915	WW I	
Chile	1917	1919	WW I	
Chile	1929	1932	Great Depression	•
Chile	1971	1975	Post-WW II Depression	•
Chile	1981	1983	Post-WW II Depression	•
Denmark	1914	1918	WW I	
Denmark	1939	1941	WW II	
Finland	1913	1918	WW I	
France	1916	1918	WW I	
France	1929	1932	Great Depression	
France	1939	1944	WW II	
Germany	1913	1919	WW I	
Germany	1928	1932	Great Depression	•
Germany	1944	1946	WW II	
Greece	1939	1945	WW II	
Indonesia	1941	1949	WW II	
Indonesia	1997	1999	Post-WW II Depression	•
Italy	1918	1921	Aftermaths of WW I	
Italy	1940	1945	WW II	•
Japan	1943	1945	WW II	•

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Table 7: List of crisis events (continued)

country	begin	end	related to	default
Korea	1938	1945	WW II	
Malaysia	1929	1932	Great Depression	
Malaysia	1942	1947	WW II	
Mexico	1926	1932	Great Depression	•
Netherlands	1913	1918	WW I	
Netherlands	1929	1934	Great Depression	
Netherlands	1939	1945	WW II	
New Zealand	1929	1932	Great Depression	
Norway	1939	1944	WW II	
Peru	1929	1932	Great Depression	•
Peru	1941	1943	WW II	
Peru	1981	1983	Post-WW II Depression	•
Peru	1987	1992	Post-WW II Depression	
Philippines	1940	1946	WW II	
Philippines	1982	1985	Post-WW II Depression	•
Portugal	1934	1936	Spanish Civil War	
Spain	1935	1938	Spanish Civil War	
Sri Lanka	1929	1932	Great Depression	
Sri Lanka	1943	1946	WW II	
Sweden	1913	1918	WW I	
Taiwan	1942	1945	WW II	
United Kingdom	1918	1921	Aftermaths of WW I	
United Kingdom	1943	1947	Aftermaths of WW II	
United States	1929	1933	Great Depression	
United States	1944	1947	Aftermaths of WW II	
Uruguay	1912	1915	WW I	•
Uruguay	1930	1933	Great Depression	•
Uruguay	1981	1984	Post-WW II Depression	•
Uruguay	1998	2002	Post-WW II Depression	
Venezuela	1913	1916	WW I	
Venezuela	1929	1932	Great Depression	
Venezuela	1939	1942	WW II	
Venezuela	1977	1985	Post-WW II Depression	

This list was extracted from table 1 of Robert J. Barro's "Rare Disasters and Asset Markets in the Twentieth Century". The crisis events are configured as periods of a decline of 15% or more in real per capita GDP.

Table 8: Countries included in the sample

country	Emerging economy	Latin America	Asia	OECD
Argentina	•	•		
United Kingdom				
Australia				•
Austria				•
Belgium				•
Brazil	•	•		
Canada				•
Chile	•	•		
Colombia	•	•		
Denmark				•
Finland				•
France				•
Germany				•
Greece				•
India	•		•	
Indonesia	•		•	
Italy				•
Japan			•	•
Korea	•		•	•
Malaysia	•		•	
Mexico	•	•		•
Netherlands				•
New Zealand				•
Norway				•
Peru	•	•		
Philippines	•			
Portugal				•
Spain				•
Sri Lanka			•	
Sweden				•
Switzerland				•
Taiwan			•	
United Kingdom				•
United States				•
Uruguay		•		
Venezuela	•	•		

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