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Ricardo Schechtman

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Edited by Research Department (Depep) – E-mail: workingpaper@bcb.gov.br

Editor: Benjamin Miranda Tabak – E-mail: benjamin.tabak@bcb.gov.br
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SBS – Quadra 3 – Bloco B – Edifício-Sede – 1º andar

Caixa Postal 8.670

70074-900 Brasília – DF – Brazil

Phones: +55 (61) 3414-3710 and 3414-3565

Fax: +55 (61) 3414-3626

E-mail: editor@bcb.gov.br

Consumer Complaints and Public Enquiries Center

Banco Central do Brasil

Secre/Surel/Diate

SBS – Quadra 3 – Bloco B – Edifício-Sede – 2º subsolo

70074-900 Brasília – DF – Brazil

Fax: +55 (61) 3414-2553

Internet: http//www.bcb.gov.br/?english

From Default Rates to Default Matrices: a complete measurement of Brazilian banks' consumer credit delinquency*

Ricardo Schechtman**

Abstract

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Despite the manifold utilities of monitoring credit default rates, little attention is usually devoted to the underlying default definition. This paper proposes working simultaneously with different default severities, related to several past-due ranges, by means of transition matrices (to be named default matrices). In this way, default, as well as recovery, is depicted in a multidimensional way with the purpose of avoiding missing relevant information. The challenge lies on performing comparisons between default matrices, which requires specific metrics. In this paper, the default matrices are built to measure consumer credit delinquency at four large Brazilian banks. The study is able to draw relevant information from comparisons between estimations techniques, between default criteria, between banks and over time, as well as with recent applied literature on matrices of rating agencies.

JEL classification: C13; C41; G21; G32
Keywords: default rates; credit delinquency; transition matrices; banks.

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**Research Department, Central Bank of Brazil. E-mail: ricardo.schechtman@bcb.gov.br.

1. Introduction

Default rate is a term frequently used in financial and economic circles to designate the percentage of borrowers of a given universe (e.g. a specific bank portfolio) that have not or will not comply to their credit obligations. Measuring and monitoring historic loan default rates is important for several reasons. Based on past default data, expectations of future delinquency is one of the components that usually explains the level of bank spreads (see BCB, 1999). Also, the monitoring of default rate time series makes it possible to draw relationships with business cycles (e.g., Bangia et al, 2002) and may assist in constructing anti-cyclical regulations dealing with bank provision or capital (e.g., Jiménez and Saurina, 2006). Further, measuring default rates is a problem closely related to that of estimating PDs (probabilities of default) in credit rating models, which is required by Basel II (e.g. BCBS, 2004). Finally, monitoring default rates is generally part of the financial stability task of supervisory authorities and Central Banks. In Brazil in particular, the last years of economic expansion (up to the financial turmoil) have observed a sharp increase in credit volumes and in the number of borrowers, with little research devoted to the consequences of that to the behavior of default rates.

Generally, default rates can be measured either following a stock approach or a flow approach. In the stock approach, both the numerator and the denominator of the default rate refer to quantities of borrowers at the same point in time (e.g. a selected month). An example of such measurement could be the percentage of outstanding borrowers that is 90 days past-due in a specific month. However, default stock rates are affected by non-default events such as variations on the number of borrowers being granted loans or on the maturity of new loans. For example, an increase in the number of borrowers in the early stages of a credit boom could reduce default stock rates simply due to an increase in the denominator base, a phenomenon not related necessarily to any improvement in individual credit risk. Therefore, though computationally more demanding, this study favors the measurement of default rates using a flow approach.

In the flow approach, the numerator and the denominator of the default rate refer to different points in time but to the same group of borrowers. An example could be the percentage of borrowers that become 90-days past-due during the course of a specific year. This is an example of a univariate flow that considers only one criterion of default

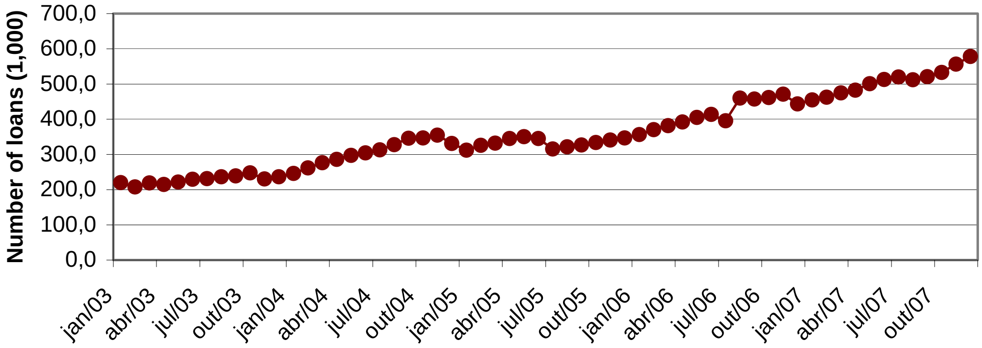
(90 days). If, instead, several default severities are simultaneously considered to avoid missing relevant information (e.g. several past-due ranges), one arrives at a multivariate flow description of default. That is precisely the notion of transition matrices where the underlying states being transited are the different default severities (besides the non-default states). This paper proposes such matrices (to be named default matrices) as a more complete measurement of default, as well as of recovery, than solely default rates, and investigates their use. Results based on default matrices reveal indeed that measurement of credit delinquency may vary considerably depending on the measurement tool used.

The applied literature on credit risk transition matrices basically concentrates on matrices of rating agencies (where the states are the external credit ratings). Initial works have been Bangia et al. (2002) and Nickell et al. (2000), both discussing the sensitivities on these matrices to phases of the business cycle. Point estimation and confidence interval estimation of rating matrices have been discussed by Lando and Skodeberg (2002), Christensen et al. (2004), Hanson and Schuermann (2004) and Gagliardini and Gouriéroux (2005). Comparisons between transitions matrices (e.g. over time) is much more complicated than the trivial comparison between default rates and requires specific metrics. Gewecke et al.(1986) and Jafry and Schuermann (2004) discuss such metrics. On the other hand, the applied literature is scarce on matrices whose underlying states are not external agencies' ratings. Mahlmann (2006) represents an exception that deals with matrices derived from banks' internal ratings but, to best of the author's knowledge, this is the first paper to work with matrices built based on different default severities, the so-called default matrices.

This study employs the proposed default matrices to measure default risk in consumer credit at large Brazilian banks[1]. Consumer credit is well suited to the purposes of this study since it is a typical form of retail credit, where the number of borrowers is large and the management practices are more uniform across banks. As with other types of credit, consumer credit has experienced a large increase over the last years. Figure 1 shows that the number of loans at four large Brazilian banks has almost tripled over a period of five years.

[1] At this paper, consumer credit refers to non-revolving, non-payroll guaranteed credit and excludes auto, house and other types of financing. Consumer credit represents the largest percentage stock of Brazilian bank retail credit, ranging from 16% to 25% since 2004.

Figure 1: Time series of the number of consumer credit loans at four large Brazilian banks



In measuring the default risk of Brazilian consumer credit through the use of default matrices, this paper has both methodological and practical (or policy-oriented) goals. Included in the former, there are the questions of how default matrices compare to rating agencies' matrices and how the different methods of estimation compare to each other specifically in the case of default matrices. From a practical point of view, this study aims at extracting relevant information from comparisons between the different default criteria underlying the default matrices, from the time evolution of default in Brazilian consumer credit over the last years and from how it behaved differently between banks along this period.

Section 2 presents the data used in the estimation. Section 3 discusses several approaches to estimating default matrices and comparing them. Section 4 contains a varied selection of the results produced while section 5 concludes.

2. Data

The database used in this study is based on data drawn from the Brazilian Public Credit Register. It consists of time series of regulatory credit risk classifications of consumer loans at four large Brazilian banks from January 2003 until January 2008[2]. The database includes loans started before January 2003, but still in effect during the time span of the study, or started within that period.[3] Almost all of the loans do not stay in the database until January 2008, for a series of reasons presented below. For estimation purposes, the database is consolidated by borrower within each bank, taking the worst loan classification as the borrower classification when distinct contemporaneous classifications are found.[4]

In order to increase comparability between classifications and, therefore, between default matrices of different banks, this study is restricted to borrowers with small loans (e.g. smaller than R$50,000 in the initial month), that, according to Brazilian regulation, can be subject to review solely as a result of arrears. Besides, those reviews must happen on a monthly basis, so that the problem raised by Mahlmann (2006), relative to the non-observation of the precise months in which classification transitions indeed occur, is not relevant to the present study. Finally, I carry out reclassifications of the original regulatory classifications in order to increase the interpretation of the former as occurrences of brackets of arrears, according to table 1. At the end of the process, the resulting classifications are expected to portray information on arrears according to table 1 plus, although to a lesser extent, any additional bank private information on loan delinquency.

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[2] Regulatory classifications are regulated by Resolution 2682/99 of the Central Bank of Brazil. Only consumer loans without payroll-deduction and without earmarked funds are considered. The four large banks refer to four large financial conglomerates with their constitutions restricted to their most representative financial institutions in consumer credit (typically two institutions per conglomerate).

[3] The creation of a database on a time-series format, suitable to this study, is a very lengthy and demanding computational task.

[4] That multiplicity is, however, very rare (≈2%).

Table 1 – Interpretation of classifications as arrears[5]

Classification	A	AR	B	C	D	E	F	G	H
Arrears (days)	-	renegotiated	15-30	31-60	61-90	91-120	121-150	151-180	>180 or written-off

In the database, the continuous observation of consumer loans and of their borrowers can be censored prior to January 2008 for a variety of reasons: 1) loan paid and not renovated, 2) loan sold or transferred without guarantee to outside the bank, 3) loan that was written-off removed from balance-sheet, 4) outstanding loan balance falls below threshold required for the loan to be reported on an individual basis,[6] 5) consumer loan is replaced by another form of credit (e.g. overdraft).

As it is practically impossible to infer what was the case that really happened, this study regards the right censorship as non-informative for modeling purposes. In doing so, the empirical results of this paper should be understood as conditional on the manifestation of default risk while derived from within the banks analyzed and restricted to the form of consumer credit[7]. Accordingly, in order to avoid unrealistic representations of the default risk experienced by the banks, I adopt the time horizon of one semester for the default matrices, notably less than the typical 1-year horizon of rating agencies' matrices. The one-year horizon was found to be longer than the typical consumer borrower lifetime in some of the banks, and particularly greater than the remaining lifetime of borrowers with loans already past due at the starting point of the horizon.

Default matrices are estimated on section 4 for each bank, so that it is interesting to have an idea of the database size on a bank level. For the sake of brevity, size numbers are reported only for bank 1. During the time span of the study, it encompasses 343,616 borrowers, a number significantly higher than the corresponding number underlying matrices of rating agencies, of around 10,000. Borrower-month observations are at the figure of 3,228,401. Transitions to a different classification, including the appearance

[5] It's imperative to recall that those classifications are distinct from those present in Resolution 2682/99. AR refers to consumer loans in no arrears but that have been the result of renegotiation of past loans, possibly in arrears.
[6] Only loans above R$ 5,000 are reported on an individual basis in the Brazilian Credit Register, so that the database in this study is restricted to them.
[7] Therefore, difference of delinquency behavior between banks, besides being the result of distinct credit market niches, could also be related to distinct credit management policies adopted by the banks (e.g. risk transfer policy).

and disappearance of borrowers, amount to 1,129,385 transitions, while excluding them, add up to 273,248. The last observation points out to the sparcity of the database and adds to the argument in favor of working with shorter time horizons.

3. Methodology

3.1 Estimating matrices

This section discusses the estimation of banks' consumer credit default matrices. To accomplish that, the time series of classifications of each bank is seen as a realization of a Markov chain of nine states ("A" through "H", according to table 1) in discrete or continuous time, depending on the estimation technique employed.

The simplest and most used estimation technique is the cohort method, based on discrete time. The technique is widely employed by rating agencies and the academic literature (e.g. Cantor and Hamilton, 2007). Given N_i borrowers with a given classification i at the start of the time horizon considered, suppose that N_{ij} of these end up in classification j at the horizon end T. Then, the transition probability is estimated by:

$$\hat{p}_{ij} = \frac{N_{ij}}{N_i} \tag{1}$$

If the transition process is also assumed time-homogeneous, one can use the multinomial estimator, in which N_i and N_{ij} are collected over the course of various sample periods of duration T. In this case:

$$\hat{p}_{ij} = \frac{\sum N_{ij}}{\sum N_i} \tag{2}$$

Estimators of discrete type permit the construction of analytical confidence intervals for the elements of the default matrices. Due to the significant number of borrowers upon which this study is based, it is safe to adopt the normal approximation to the binomial distribution (below) for the construction of such intervals. Also the independence assumption underlying the binomial distribution is easier to be imposed at the short horizon of 1-semester (see Hanson and Schuermann, 2006).[8]

$$\hat{p}_{ij} \sim \mathrm{N}\left(p_{ij}, \sqrt{\frac{p_{ij}(1-p_{ij})}{N}} \right) \tag{3}$$

On the other hand, the continuous time estimation based on survival analysis (also called duration) makes use of the transitions observed at shorter frequencies than horizon T, assuming a Markov process homogeneous or not. In the homogeneous case, estimation by survival analysis turns into estimation of the generator matrix **G** of the chain, which allows the production of transition matrices for any forecasting horizon t=αT, α>0, according to the equation below.

$$\mathbf{P}(t)=\exp(\mathbf{G}\alpha), \tag{4}$$

in which $\mathbf{P}(t)\equiv(P_{ij}(t))$ is the transition probability matrix for horizon t.

The elements of **G** satisfy $g_{ij} \geq 0$ for $i\neq j$, $g_{ii} = -\sum g_{ij}$ and are estimated through maximum likelihood by:

$$g_{ij} = \frac{N_{ij}M}{\int_0^T Y_i(t)\,dt}, \tag{5}$$

where M is the number of months in horizon T, N_{ij} is the total number of transitions from i to j observed in the base and $Y_i(t)$ is the number of borrowers of classification i in month t.

Finally, the non-homogeneous continuous time case is equivalent to applying the cohort method for the shortest observation frequency, monthly in the case at hand, in order to estimate monthly transition matrices. Then, a horizon-T matrix is formed by appropriately multiplying T previously estimated monthly matrices. This is, in fact, an application of the Aalen-Johansen estimator and the resulting matrix so obtained is specific to the time period used in the estimation.

[8] Here N can mean N_i or ΣN_i, depending on the point estimator used.

By using all the information available in the database, the continuous time estimations have three major advantages in relation to the discrete methods, as discussed by Lando and Skodeberg (2002). First, non-null probabilities are generated for transitions that have not occurred for any fixed set of borrowers, but that are plausible through intermediate transitions that have occurred for different sets of borrowers. Second, transitions of borrowers that do not remain in the base during all the months of horizon T, either due to withdrawal prior to the final month or entry subsequent to the initial month, are used in the method, producing more efficient estimations. Third, transition matrices are generated for arbitrary time horizons with greater ease, particularly in the homogeneous case.

Yet, Gagliardini and Gouriéroux (2005) propose a procedure that is somewhat different from the estimators described above. In a context in which the horizon-T matrices are assumed themselves stochastic, albeit i.i.d., the authors demonstrate that it is the average of the various sample matrices of different consecutive periods of duration T that produces the appropriate estimator[9]. In particular, when each of those is estimated by cohort, the simple average, instead of the weighted average given by the multinomial estimator, is the appropriate estimator. That observation may be particular importance to the Brazilian case, where the number of borrowers has displayed a sharp increase pattern lately.

3.2 Comparing matrices

In order to compare how different are delinquencies and their dynamics among various banks, metrics for transition matrices must be considered. Jafry and Schuermann (2004) examine alternative proposals of metrics, with the goal of measuring the average quantity of “mobility” embedded into the matrices. Mobility is understood as the probability of migration to a classification different from the original one and the aurthors suggest a metric (denoted hereafter as Mob) based on the singular value decomposition of the matrix to be measured.

[9] Gagliardini and Gouriéroux (2005) make use of those assumptions to discuss estimation of migration correlation, so that their proposed matrix estimator is consistent with a cross-section correlation modeling.

$$\mathrm{Mob}(\mathbf{P}) \equiv \frac{\sum_{i=1}^{D} \sqrt{\lambda_i\left((\mathbf{P}-\mathbf{I})^T (\mathbf{P}-\mathbf{I}) \right)}}{D} \tag{6}$$

where D is the dimension of P, λ_is are the autovalues of the matrix in parentheses, **P** is the transition matrix to be measured and **I** is the identity matrix.

The authors' preference towards Mob is based on the properties of mononicity (M($\mathbf{P_1}$) > M($\mathbf{P_2}$) if $p_{1ij} \geq p_{2ij}$ $\forall i \neq j$ and $p_{1ij} > p_{2ij}$ for some $i \neq j$) and distribution discrimination (M($\mathbf{P_1}$) ≠ M($\mathbf{P_2}$) if $p_{1ii} = p_{2ii}$ $\forall i$ and $p_{1ij} \neq p_{2ij}$ $\forall i \neq j$). Two matrices $\mathbf{P_1}$ and $\mathbf{P_2}$ can then be compared through[10]:

$$\Delta\mathrm{Mob} \equiv |\mathrm{Mob}(\mathbf{P_1}) - \mathrm{Mob}(\mathbf{P_2})| \tag{7}$$

However, even for the proposed metric, it is difficult to capture all the dimensions underlying the concept of mobility in a single scalar. Indeed, Mob is not able to distinguish between migrations to better classifications and migrations to worse classifications. Note, for example, that Mob generates the same value for **P** and $\mathbf{P}^t$. To cope with this issue, this study proposes additionally the concepts of improvement and worsening mobilities. First, two new transition matrices are generated from the original default matrix **P**, an upper triangular $\mathbf{P^u}$ and a lower triangular matrix $\mathbf{P^l}$, where:

$$p_{ij}^{u} = \begin{cases} p_{ij} & if\ i > j \\ \sum_{k \leq j} p_{kj} & if\ i = j \\ 0 & if\ i < j \end{cases} \quad and \quad p_{ij}^{l} = \begin{cases} p_{ij} & if\ i < j \\ \sum_{k \geq i} p_{ik} & if\ i = j \\ 0 & if\ i > j \end{cases} \tag{8}$$

Worsening migrations have the same probabilities in **P** and $\mathbf{P^u}$, but borrowers are not allowed to strictly improve in $\mathbf{P^u}$. Analogous note is valid for $\mathbf{P^l}$ and improvement migrations. Now a worsening mobility metric and an improvement mobility metric are defined based on $\mathbf{P^u}$ and $\mathbf{P^l}$**,** respectively.

$$\text{Mob\_worsening } (\mathbf{P}) \equiv \ \mathrm{Mob}(F_4(\mathbf{P^u}))/ \text{ worsening\_constant}, \tag{9}$$

[10] To be precise, it is ΔM, not M, that represents a metric (or better yet, a pseudo-metric because it can be null for a pair of distinct matrices) in the space of the transition probability matrices.

$$Mob\_improvement\ (\mathbf{P}) \equiv Mob(L_5(\mathbf{P}^{\mathbf{l}}))/\ improvement\_constant\ , \tag{10}$$

where F_m and L_m are matrix operators that replace respectively the first or last m lines of a matrix by the corresponding lines of a identity matrix.

Mob_worsening is defined based only on the (worsening) behavior of classifications A, AR, B and C, since the behavior of other states, representative of more severe past-due ranges, is typically thought of as containing information on recovery. Analogously, Mob_improvement is defined based only on the (improvement) behavior of classifications D until H. The cut-off between C and D is somewhat arbitrary but based on the fact that the discussion about proper default definitions in Brazil lie generally between 60 and 90 days. The denominator constants have a normalizing function so that the new metrics measure the "average" mobility to a state better (in the case of the improvement metric) or worse (in the case of the worsening metric) than the original one[11]. The new metrics help disentangle the good and bad parts of the concept of mobility.

Jafry and Schuermann (2004) further note that Mob is not able to distinguish between extreme and short migrations. They show that Mob may fail to generate larger values for matrices with migration probability distributed further away from the diagonal and suggest that incorporating such desired property may indeed require some ad-hoc weighting of the elements of the matrix to be measured. Using that observation as a starting point, this study proposes an additional metric based on the concept of expected opportunity cost of the operations in arrears. First, an opportunity cost matrix **Cost**, 9x9, is defined as below, where i is the average monthly rate of return of Brazilian consumer credit and d_j is the number of days in past-due relative to default classification j (see table 1).[12]

[11] More specifically, worsening_constant is defined so that:

$$Mob\_worsening\begin{pmatrix} 1-p & \frac{p}{N-1} & \cdots & \frac{p}{N-1} & \frac{p}{N-1} \\ 0 & 1-p & \frac{p}{N-2} & \cdots & \frac{p}{N-2} \\ . & . & & & . \\ . & . & .... & \cdots & . \\ . & . & & & . \\ 0 & 0 & \cdots & 0 & 1-p \end{pmatrix} = p$$

For N=9, worseing_constant=1.0763. Analogous definition is valid for the improvement_constant. Its value is 1.0847 for N=9.

[12] The lower bounds of the intervals are used. d_1=d_2=0, relative to classifications A and AR, respectively.

$$
\textbf{Cost} \equiv \begin{pmatrix}
(1+\mathrm{i})^{\frac{d_1}{30}}-1 & (1+\mathrm{i})^{\frac{d_2}{30}}-1 & \ldots & \ldots & (1+\mathrm{i})^{\frac{d_9}{30}}-1 \\
(1+\mathrm{i})^{\frac{d_1}{30}}-1 & (1+\mathrm{i})^{\frac{d_2}{30}}-1 & \ldots & \ldots & (1+\mathrm{i})^{\frac{d_9}{30}}-1 \\
0 & 0 & (1+\mathrm{i})^6-1 & \ldots & (1+\mathrm{i})^6-1 \\
. & . & . & . & . \\
. & . & . & (1+\mathrm{i})^6-1 & . \\
. & . & . & . & . \\
0 & 0 & (1+\mathrm{i})^6-1 & \ldots & (1+\mathrm{i})^6-1
\end{pmatrix} \tag{11}
$$

The opportunity cost of every transition from A or AR to a classification in arrears is approximated by the missed return i compounded the number of months in arrears given in table 1.[13] Transitions between classifications in arrears produce all the same cost, assuming nothing is paid throughout the semester or that the part paid is insignificant, while recovery migrations from classifications in arrears back to A or AR do not generate opportunity cost, as if they are paid immediately. These are arbitrary but necessary assumptions to come up with a measure of opportunity cost that only assesses transition probabilities. The cost metric is then defined as the expected opportunity cost of the average portfolio of default classifications.

$$\text{Cost\_metric }(\mathbf{P}) \equiv \mathbf{weight} \bullet \mathbf{expected\ cost}, \tag{12}$$

where $\mathbf{expected\ cost} \equiv (\mathbf{cost} \times \mathbf{P}) \bullet [1, 1, \ldots 1]^{\mathrm{T}}$, (13)

with the symbol × denoting element wise matrix multiplication and **weight** the vector containing the composition of default classifications found in the data.

Although Cost_metric makes arbitrary assumptions about the exact moments when migrations occur and, therefore, is not a precise measure of missed opportunity cost, it serves the purposes of penalizing more both default transitions to brackets of higher arrears and recovery transitions to brackets of lesser arrears. To reflect average Brazilian financial conditions during period 2003-2007, rate i is fixed at 4.5% a.m. and

[13] The metric abstracts from exposure considerations.

the vector of weights in percentage format estimated from the data is [75.47 4.95 1.02 2.05 1.35 1.12 1.02 0.96 12.06].

4. Results

Time-homogeneous estimates of 1-semester matrices using the whole 5-year period database (to be denoted time-unrestricted estimates) as well as semester-restricted estimates are produced for every bank, using the estimation techniques discussed in section 3. Representative results are shown and discussed in this section. They include comparisons between default criteria, between estimations techniques, between banks and over time.

4.1 Default matrices and default classifications

Table 2 shows the time-unrestricted multinomial estimate of bank 1 one-semester default matrix. It illustrates the general pattern of default matrices found in this study. Compared to matrices of rating agencies (see for example estimates in Lando and Skodeberg, 2002), default matrices display much less probability on the diagonal and strong probability concentration on the extreme columns A and H. That strong mobility of default matrices derives from the fact that most states represent past due ranges, in which borrowers are not likely to stay for long (generally not more than one month). In table 2 particularly, it is interesting to note that the probability of migration to H increases continuously with the departing classification, with a violation of monotonicity ocurring only between AR and B. Similarly migration probability to A also decreases continuously, with monotonicity violation among departing states AR, B and C. Both observations mean that, for bank 1, renegotiated consumer loans are riskier than loans less than 30-days past-due.

Table 2: Time-unrestricted multinomial estimate of bank 1 one-semester default matrix

	A	AR	B	C	D	E	F	G	H
A	87,6	1,2	2,3	3,2	1,9	1,5	1,3	0,9	0,2
AR	22,4	46,6	0,5	1,4	3,7	3,0	3,2	2,4	16,8
B	34,6	1,3	18,7	11,4	5,7	4,7	4,5	16,0	3,2
C	23,2	3,0	2,8	12,2	6,0	4,5	4,1	6,0	38,3
D	5,6	3,1	1,0	2,6	4,0	2,9	3,4	3,5	74,0
E	1,7	1,5	0,6	1,1	0,6	1,7	1,5	1,3	90,1
F	1,0	1,3	0,2	0,5	0,5	0,4	0,7	0,5	94,9
G	0,4	0,7	0,2	0,2	0,2	0,3	0,0	0,4	97,6
H	0,3	1,1	0,1	0,1	0,1	0,0	0,0	0,0	98,4

Default matrices are also useful to compare different default definitions. However, the classifications of table 1 are not proper default criteria because their specifications contain upper bounds for the number of days in arrears. Instead, default criteria can be formed by considering the union of worse classifications starting from a given classification. For example, the 90-days past-due criterion is recovered from the union of states E, F, G and H and will be denoted “>=E” throughout the paper.[14] Similar notations are adopted for other default or recovery definitions. Default probabilities estimates according to different default criteria are obtained from table 2 by adding up the appropriate columns of the matrix. The results in table 3 illustrate the sensitivity of PD (derived from departing state A) to different default definitions and may be useful to the task of choosing a particular definition to work with in the context of internal credit risk models.

Table 3 – Bank 1 default probabilities derived from table 2

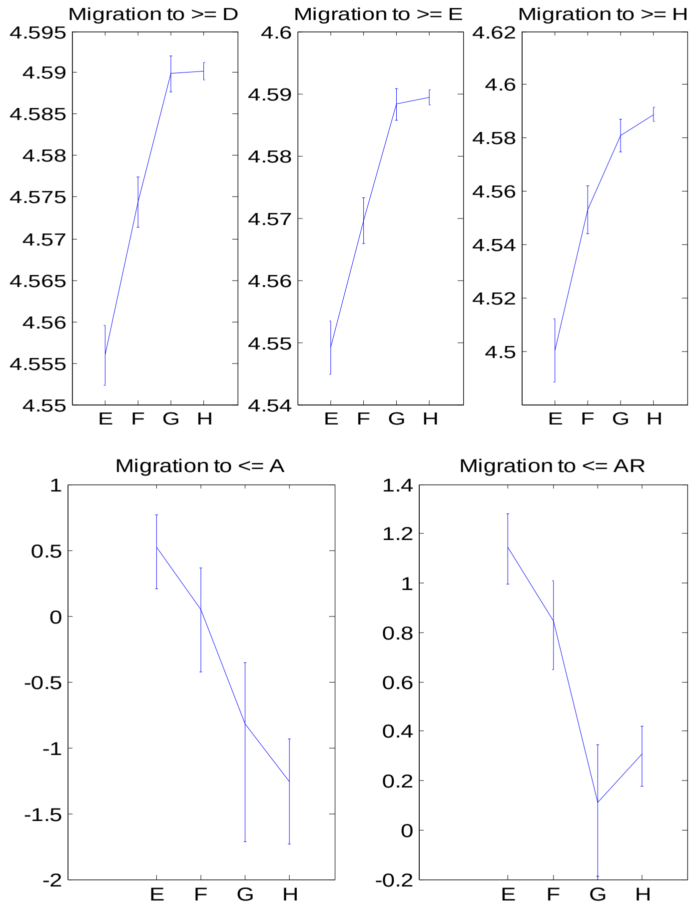
Migration	A→A	A→>=D	A→>=E	A→>=F	A→>=G	A→H
Probability estimate (%)	87.6	5.8	3.9	2.4	1.1	0.2

Based on confidence intervals for migration probabilities, it is possible to check whether default classifications are statistically distinguishable. Even in the optimistic case, without assumption of time heterogeneity and making use of the 5-year data period,

[14] The term default definition or default criterion refer, throughout the paper, to this concept, while the term default classification or default severity refer to the classes of table 1.

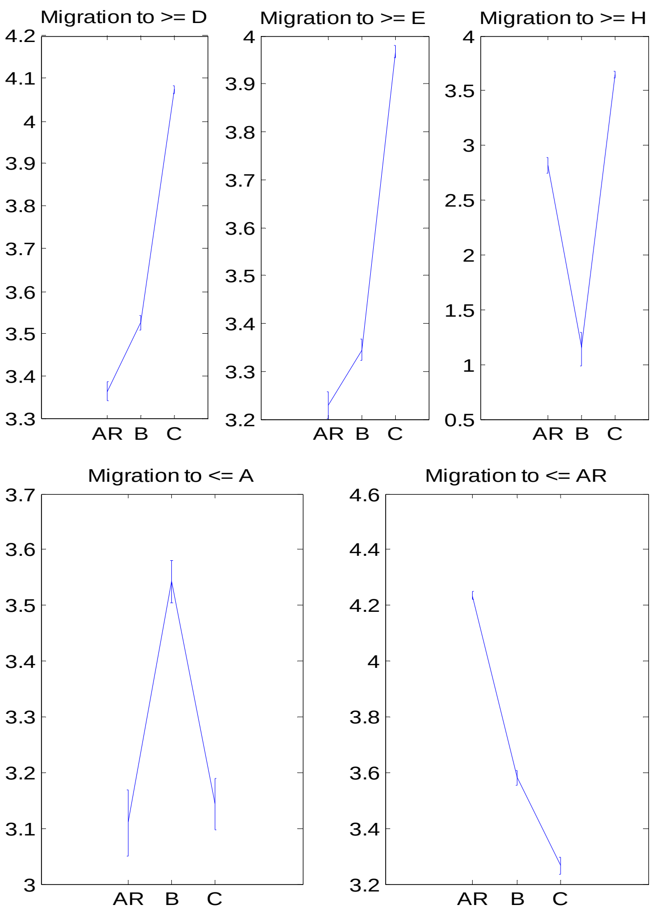
figure 2 shows that this is not the case for states G and H at bank 1. Probability confidence intervals for both improvement and worsening migrations that depart from G and H overlap. For other banks, this phenomenon occurs at pairs (E,F), (G,H) or (F,G), implying, in general, the presence of a discrimination problem between classifications related to large number of days in arrears. From a statistical standpoint, that means that the number of default classifications considered in this study may be excessive. As a policy implication, it results that the goal of risk discrimination for loans with significant past due (e.g. implicit in requirements of different regulatory provisions) may be unfeasible.

Figure 2: Probability confidence intervals for selected default and recovery migrations departing from classifications E, F, G, and H at bank 1. Transition probabilities estimated by the multinomial method. Confidence intervals are analytical based on the normal approximation to the binomial distribution. Y-axes are on the log scale.



Confidence interval results based on departure from states AR, B and C illustrates another interesting issue at Figure 3. By considering less stringent default or recovery definitions (e.g. moving from H to >=D or from <=A to <=AR), it is possible to recover probability monotonicity (with no interval overlap) across classifications. The non-monotone behaviour at the extreme migrations (shown already in table 2) disappears once new classifications are added to the target definition, in great part due to a sample size increase effect.

Figure 3: Probability confidence intervals for selected default and recovery migrations departing from classifications AR, B and C at bank 1. Transition probabilities estimated by the multinomial method. Confidence intervals are analytical based on the normal approximation to the binomial distribution. Y-axes are on the log scale.



Hereafter, for the sake of brevity, attention will be restricted to transitions to default definitions >=D, >=E and >=H and to recovery definitions <=A and <=AR.

4.2 Comparison between estimation techniques

I now focus on the comparison between estimation techniques. The bank 1 one-semester default matrix is also estimated by the homogeneous survival method (again using the whole 5-year period) and table 4 shows the probability ratios between the duration and the multinomial estimates for some selected migrations of interest. Compared to the multinomial estimation, the survival estimation implies higher probabilities of transition from non-arrears classifications (A and AR) to typical default definitions such as >=D, >=E and H, but also higher probabilities for typical recovery migrations, such as from D until H back to A or <=AR (typical migrations in blue). Those findings are qualitatively consistent with what is found for default transitions of top quality ratings of external agencies (e.g. Hanson and Schuermann, 2006) and with the discussion of section 3 regarding the efficiency gains of survival estimation for rare transitions. Nevertheless, it is impressive the sizable difference in estimates for the extreme migrations: for A→>=H, the duration estimate is 10 times the multinomial estimate, going from 0.2% to 2.2%.

Table 4: Probability ratios between the homogeneous duration and the multinomial estimates for selected migrations at bank 1.

	A	<=AR	>=D	>=E	H
A	0,96	0,97	1,40	1,53	10,31
AR	1,30	0,74	1,59	1,72	1,93
B	1,69	1,70	0,88	0,90	3,85
C	1,53	1,50	0,95	0,99	0,86
D	2,30	1,95	0,93	0,95	0,85
E	3,43	2,78	0,95	0,95	0,91
F	3,24	2,47	0,97	0,97	0,95
G	5,16	3,89	0,97	0,97	0,96
H	5,76	2,67	0,98	0,98	0,97

It is also interesting to note that the survival estimator gives lower probabilities for transitions between classifications in arrears (the right low corner of table 4). A possible explanation could be the presence of downward momentum, a violation of the Markov property in which borrowers who have been downgraded have a higher chance of a further downgrade. Since the survival estimator makes more use of the Markov property than its multinomial counterpart, it is generally less affected by the presence of downward momentum. See similar discussion in Hanson and Schuermann (2006) for the case of rating agencies. If downward momentum is the cause (and it could be clearly intuitive when classifications are based on past-due ranges), its effect is relatively small: the survival estimates are as close as 95% of the multinomial ones for the transitions considered. Therefore, the issue is no further investigated in this paper. All observations related to table 4 are qualitatively similar to all the banks analyzed.

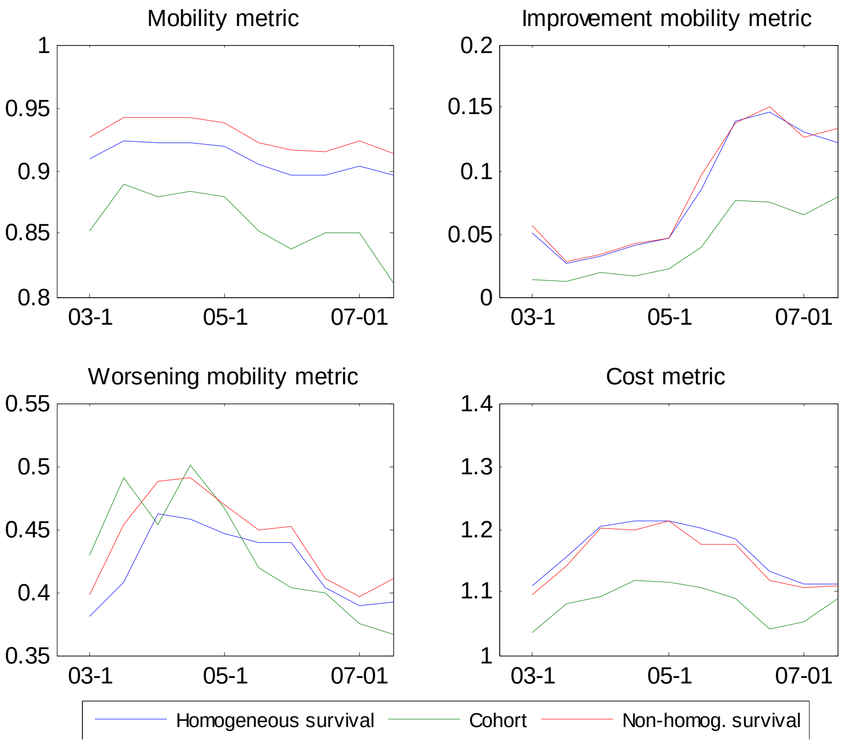
Similarly to table 4, table 5 displays a comparison between the non-homogeneous and homogeneous estimators of the 1-semester default matrix of bank 1, the former relative to the first semester of 2007. The ratios here are generally closer to 1 than in table 4, particularly for the typical default and recovery transitions, in blue. For those migrations it is valid, as in Jafry and Schuerman (2004), that the efficient gains of survival estimation are more important than a hypothesis of homonegeneity (and this holds for other banks and semesters as well). On the other hand, for all migrations in general, the differences between the two survival estimators are far greater than in the case of rating agencies (see, Lando and Skodeberg, 2002, for the latter). That suggests that time specific shocks, for example related to discrete movements in the credit policy of the bank, have a material impact on the results and demonstrate that the Aalen-Johansen estimator may be a useful tool for closely monitoring the behavior of delinquencies on a bank level. Indeed, specific ratios contained in table 5 may vary significantly dependending on the bank and the semester considered, the larger time variations between semesters generally found for extreme migrations (e.g. A→H and H→A) and for migrations involving the AR renegotiated state.

Table 5: Probability ratios between the non-homogeneous and the homogeneous estimates for selected migrations at bank 1.

	A	<=AR	>=D	>=E	H
A	1,00	1,00	1,01	1,03	0,65
AR	1,17	1,03	0,95	0,94	0,94
B	1,04	1,05	1,02	1,11	0,98
C	0,98	0,99	1,04	1,09	1,38
D	0,92	0,95	1,02	1,03	1,25
E	0,83	0,95	1,01	1,01	1,10
F	0,97	1,07	1,00	0,99	1,02
G	0,75	0,94	1,01	1,01	1,01
H	0,84	1,00	1,00	1,00	1,00

The variations over time between the different estimation techniques for bank 1 are analyzed in figure 4. For that, the metrics discussed in section 3 are helpful in avoiding the ungrateful task of understanding the behaviour of 81 matrix elements over time. For all metrics but the worsening mobility, figure 4 reafirms the previously mentioned result that the difference between the survival and the discrete estimation (in this case cohort) are larger than those found between the two types of survival estimation, regardless of the bank analyzed (the latter not shown). For some metrics, the homogeneity assumption is almost irrelevant (e.g. metrics cost and improvement mobility for bank 1). On the other hand, the decision to adopt or not an homogeneity assumption is important to the worsening mobility for most banks (clearly important for bank 1 at figure 4). Results not shown indicate that this is in large part due to the inclusion of non-typical default migrations departing from B and C in the worsening metric (and therefore do not stay in contrast with the previous observations about typical transitions). Hereafter, for the sake of brevity, only the homogeneous case of the survival estimation is reported.

Figure 4: Comparison between default matrix estimation techniques over time via metrics. Results for bank 1.



Besides the estimation techniques, figure 4 allows an analysis of the metrics themselves. Note that the mobility metric for 1-semester default matrices vary at ranges much higher than the typical mobility values achieved by 1-year rating agencies' matrices (e.g. ≈[0.12 0.24] as found by Jafry and Schuerman, 2004)[15]. That is just another result pointing to the greater mobility of default matrices, already observed in table 1. However, a great part of that mobility of default matrices is on the worsening direction, given the values assumed by the worsening and improvement mobility metrics. Only the improvement mobility of 1-semester default matrices is already of comparable order to the whole mobility of 1-year rating agencies' matrices.[16]

4.3 Time-paths of transition probilities

The most immediate evidence of figure 4 was left uncommented so far: that default matrices are far from constant over time. In this subsection time heterogeneity along semesters is properly investigated at the transition level, which permits the incorporation of analytical confidence intervals to the analysis. Figure 5 shows for bank 1 the time paths of selected migration probabilities, estimated by cohort, together with their confidence intervals. The blue horizontal line is the multinomial estimator (equivalent to the weighted average of cohort estimators by the number of borrowers at each semester start) while the green line represents the simple average of cohorts, motivated by Gagliardini and Gouriéroux (2005).

Results indicate pronounced time variation of transition probabilities. The confidence intervals do not include the horizontal lines for most of the 5-year period for transitions A→ >=D and A→>=E and the paths of these transtions are clearly not derived from just white noise (the assumption underlying the simple average estimator). It means that default matrices for bank 1 are neither constant nor i.i.d.. On the other hand, not much can be concluded about time variation for migration A→H and the recovery migrations from the use of analytical intervals[17]. At the same time, note that, for all transitions, the range of time variation is much greater than the difference between the simple and weithed average estimators, implying that the choice of the particular homogeneous

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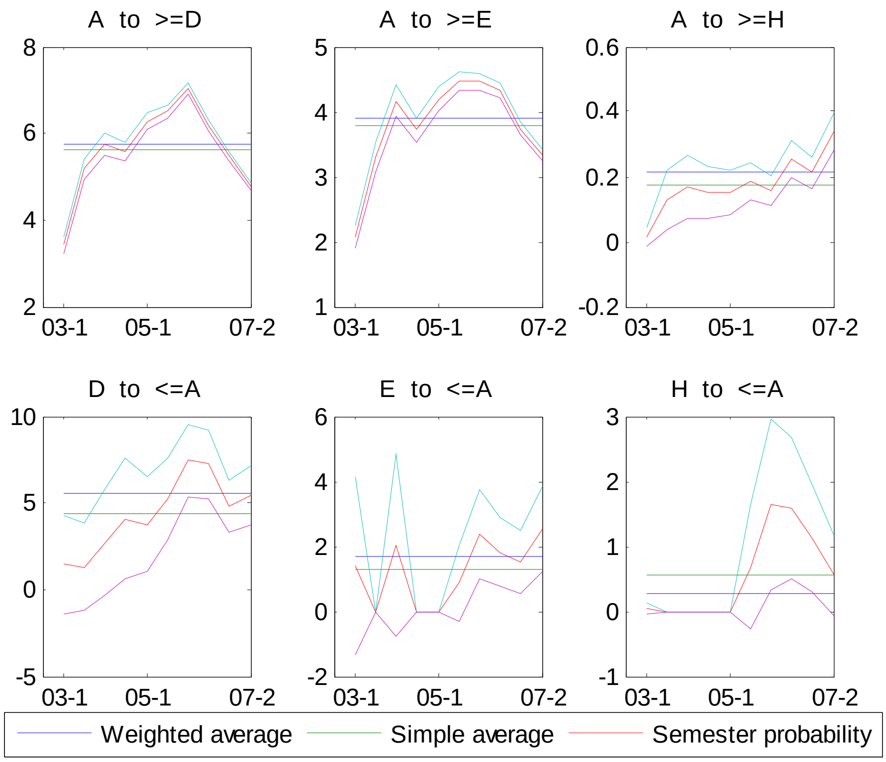
[15] 1-year default matrices will have even higher mobility.

[16] Given the simplifications underlying the cost metric, this paper refrains from interpreting its absolute values and will prefer to investigage cost distances instead, on section 4.4.

[17] Here bootstrap intervals (e.g. Hanson and Schuermann, 2006) could be of some utility. They are, however, too computational intensive for the large dataset of this study and, therefore, out of the scope of this paper.

estimator becomes less relevant in the Brazilian data. Finally, note that probability intervals degenerate at some semesters for improvement migrations E→A and H→A. That represents a deficiency of analytical intervals when the probability estimate is zero and harms the analysis of those semesters[18].

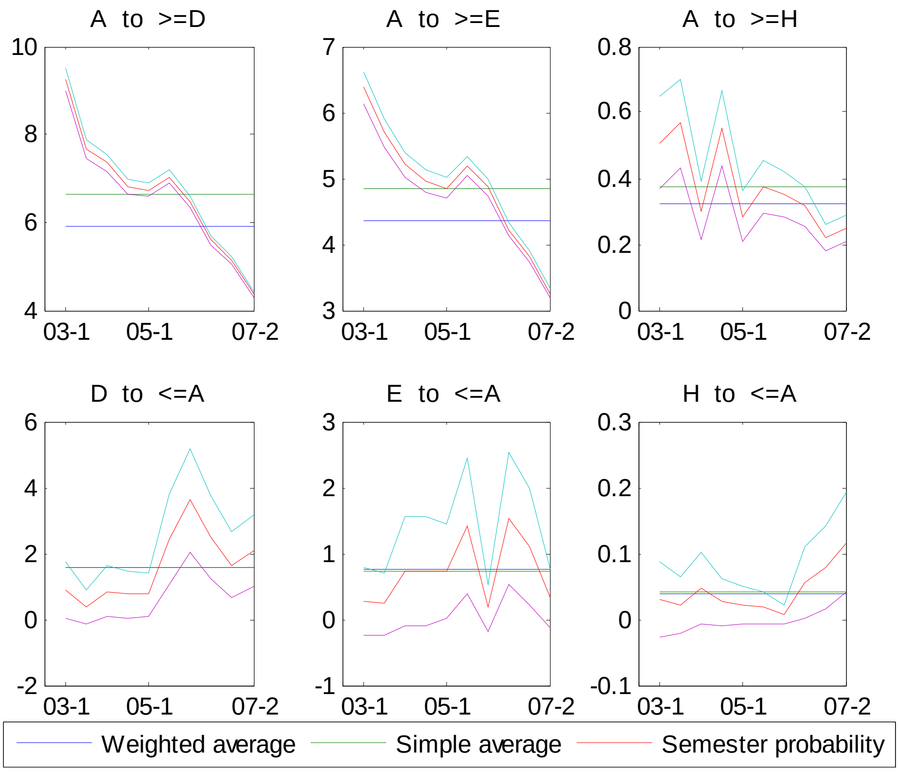
Figure 5: Time paths of selected migration probabilities (in red), estimated by cohort, together with their confidence intervals (in pink and light blue). Results for bank 1. The blue horizontal line is the multinomial estimator (equivalent to the weighted average of cohort estimators by the number of borrowers at each semester start) while the green line represents the simple average of cohort estimators.



[18]That is another disadvantage of analytical intervals when compared to bootstrap intervals.

Figure 6 shows analogous results for bank 2. The important distinction lies on the fact that bank 2 shows a sharp decrease in default rates over time (according to the transitions A→>=D and A→>=E) whereas bank 1 shows an increase pattern until the end of 2005. The comparison between banks including the time dimension is further addressed at subsection 4.5.

Figure 6: Time paths of selected migration probabilities (in red), estimated by cohort, together with their confidence intervals (in pink and light blue). Results for bank 2. The blue horizontal line is the multinomial estimator (equivalent to the weighted average of cohort estimators by the number of borrowers at each semester start) while the green line represents the simple average of cohort estimators.



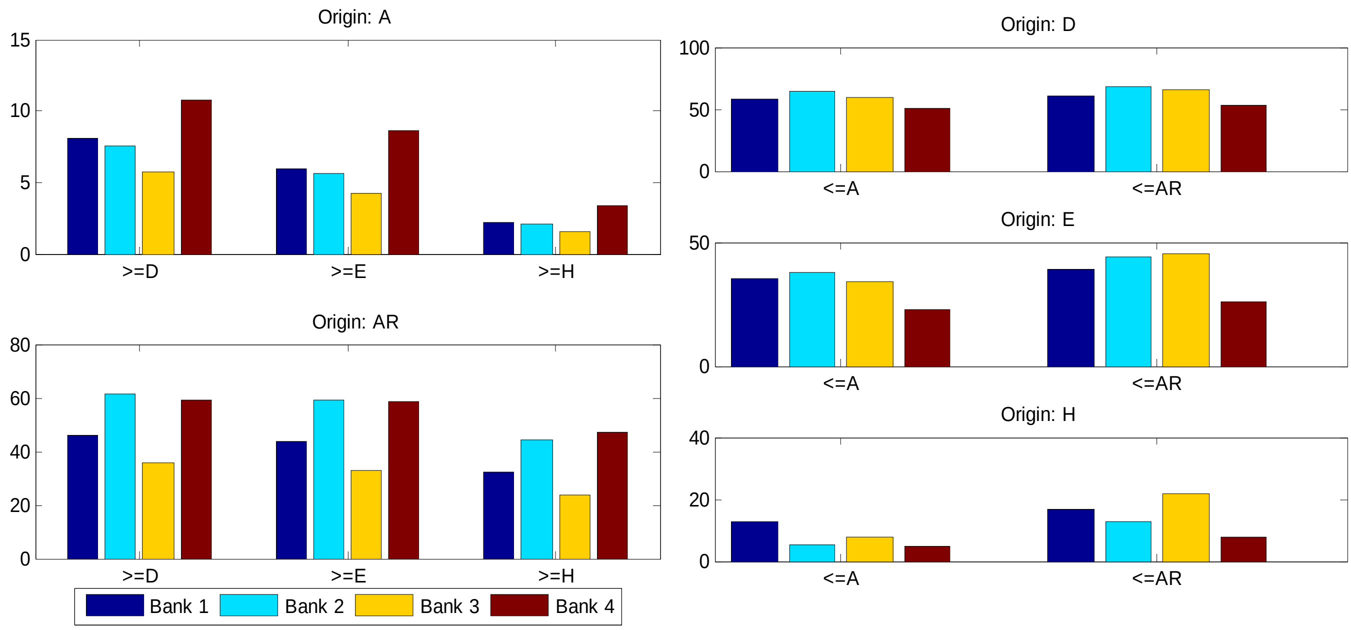
4.4 Static comparison between banks

Supervisory authorities are often interested in making comparisons between banks in terms of credit risk ranges in which they operate. For analytical purposes, sometimes banks are grouped together based on similar credit risk characteristics or other attributes and monitored jointly over time, in search for the outbreak of any within-group bank discrepancy at some point in time. In the context of this paper, grouping of banks based on similar (or distinct) credit risk profiles can be obtained from the analysis of either selected migrations or matrix metrics.

I start the analysis on the transition level.[19] Regarding default transitions, figure 7 shows that, among the four banks analyzed, bank 4 presents the highest default probabilities from state A and among the two highest from AR, whereas bank 3 displays always the lowest default rates. These banks are likely to operate in different ranges of credit risk and price accordingly. On the other hand, banks 1 and 2 generally lie in-between and seem more similar in default behavior when renegotiated loans are not considered. As far as recovery transitions are concerned, the general picture is more entangled, possibly reflecting smaller differences in recovery efficacies or policies. Banks 1, 2 and 3 experience close rates, at least starting from classifications D and E, while bank 4 generally presents the lowest improvement probability. As a net result from both default and recovery aspects, it may be appropriate to identify banks 1 and 2 as constituting a pair of banks with more similar risk behaviors and banks 3 and 4 as a pair with notably distinct risk profiles. Indeed, tests on equality of means find 54 different transition probabilities for the first pair of banks and 68 differences for the second.

[19] All the results of this subsection are based on time-unrestricted homogeneous duration estimates.

Figure 7: Comparison between banks' selected migration probabilities. On the right, default migrations and on the left recovery migrations. Probabilities estimated by time-unrestricted homogeneous duration.



Here again, default matrices' metrics are useful in consolidating the results of many transitions, this time to provide a meaningful cross-bank comparison. In table 6, metrics make it easier to explore how each pair of banks compare to each other. Table 6 also explains similarity or distinction between banks' credit risks based on the concepts of mobility, worsening mobility, improvement mobility and opportunity cost. Banks 1 and 2 are the closest pair of banks for every metric but improvement (but also near to the closest according to improvement) while banks 3 and 4 are the most dissimilar in every aspect. Note, in particular, that both the respective similarity and dissimilarity of pairs (1,2) and (3,4) in terms of mobility can be largely attributed to worsening mobility, in which they display values far from the rest of the banks' pairs. Finally, the cost metric shows that differences between default matrices are of economic significance. Assuming an average bank difference in rates charged for consumer credit of 1% a.m. within the universe of large Brazilian banks, table 6 reveals that as high as a quarter of that could be derived from bank differences in default opportunity costs[20]. All in all, table 6 is a powerful tool for immediate cross-bank comparison.

[20] It is possible that bank differences in opportunity costs are subestimated, since the same return rate i is applied to every bank in the cost metric. It is reasonable to expect that banks that normally operate with riskier default matrices will also charge higher and will, therefore, incur in higher opportunity costs from defaults.

Table 6: Metric distances between banks' default matrices. Matrices are estimated by homogeneous duration.

		Bank 1	Bank 2	Bank 3	Bank 4
Mobility	Bank 1	-	-	-	-
	Bank 2	0,01	-	-	-
	Bank 3	0,04	0,05	-	-
	Bank 4	0,02	0,01	0,06	-
Improvement mobility	Bank 1	-	-	-	-
	Bank 2	0,03	-	-	-
	Bank 3	0,03	0,06	-	-
	Bank 4	0,06	0,02	0,08	-
Worsening mobility	Bank 1	-	-	-	-
	Bank 2	0,01	-	-	-
	Bank 3	0,07	0,08	-	-
	Bank 4	0,12	0,11	0,18	-
Cost metric	Bank 1	-	-	-	-
	Bank 2	0,03	-	-	-
	Bank 3	0,11	0,14	-	-
	Bank 4	0,15	0,12	0,26	-

4.5 Dynamic comparison between banks

I now investigate what new information can be derived from the inclusion of the time dimension in the cross-bank comparison. I start the analysis on the transition level, focusing on the representative default transition A→>=E (estimated by the homogeneous survival method), and then turn to metrics for a more complete delinquency description. Figure 8 shows, for the selected migration and for all banks, a large time heterogeneity along the semesters, reaffirming therefore the general evidence of figures 5 and 6 and generalizing them to banks 3 and 4. More striking, however, are the sharp differences in the probability trajectories among the four banks. In particular, note that banks 1 and 2, when observed along time, no longer seem so similar as before. The high dissimilarity between banks' trajectories can be attributed to differences in market niches, growth strategies, renegotiation policies, among others. Bank analysts in possession of specific bank information can use the results of figure 8 to link their knowledge of banks' policies and decisions to the resulting time variations of credit delinquencies. The explanation of such variations is not within the scope of this paper.

Figure 8: Trajectories of banks' probabilities of default transition A→>=E. Probabilities estimated by semester-restricted homogeneous duration. The blue horizontal line is the time-unrestricted homogeneous survival estimator (kind of a weighted average) while the green line is the simple average of the semester-restricted survival estimators.

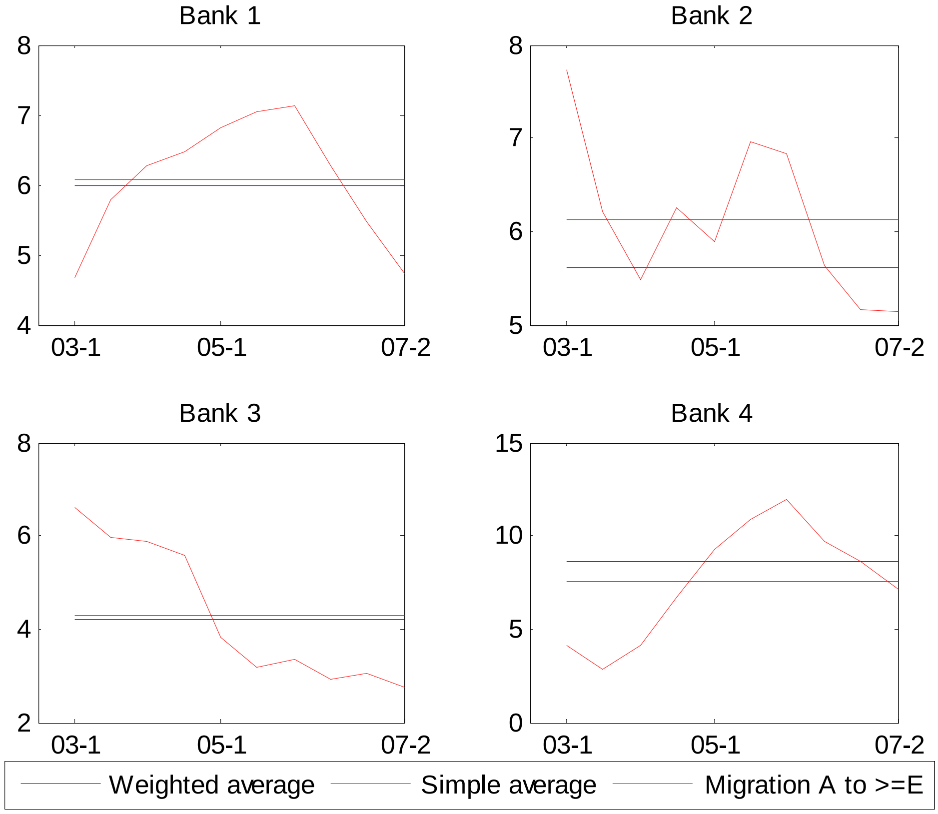
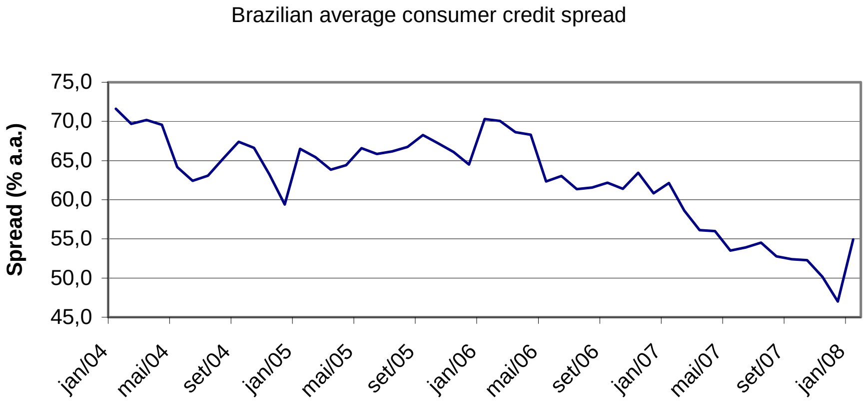


Figure 8 still reveals that, apart from the aforementioned differences, there seems to be, from 2006 until the end of 2007, a decreasing move in default rates common to all banks. It could reflect a new phase in the risk dynamics of the Brazilian consumer credit market. That would be consistent with the general descending trend of the average Brazilian consumer credit spread observed during the same period, as depicted in Figure 9. However, a closer investigation of the relationship between spread and default risk (measured on a flow approach) would require longer time series of default matrices and is left to future research.

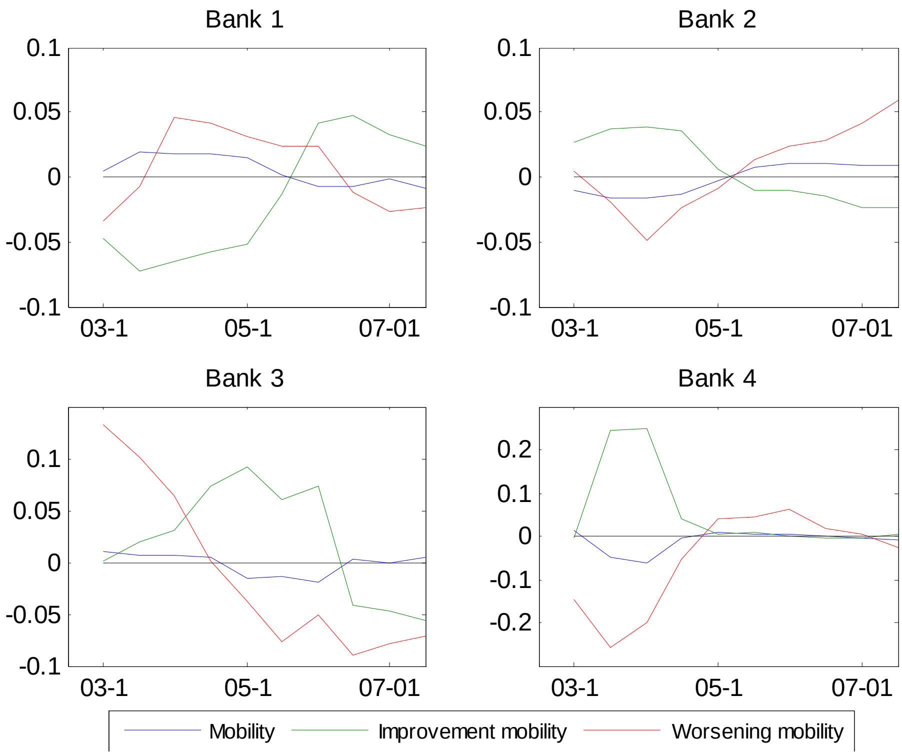
Figure 9: Trajectory of average spread charged on non-payroll-deducted Brazilian consumer credit with non-earmarked funds and preset rates.



In order to incorporate other transitions to the comparative analysis of banks' time evolution, matrix metrics results are now investigated. Figure 10 shows the distances along time between semester-restricted default matrices and time unrestricted matrices (all estimated by homogeneous survival), for each bank and each metric. Distances are used to allow the analysis to better focus on the relative movements of banks' metric trajectories, apart from the average levels in which banks operate. The trajectories confirm the high dissimilarity among banks' credit risk dynamics. During the passage from the first to the second half of the time period analyzed, bank 1 is experiencing a decrease in worsening mobility and an increase in improvement mobility, moving then to a less risky net position, while bank 2 displays an opposite behavior. At the same time, bank 3 shows a large decrease in improvement mobility and a moderate stabilization after a sharp reduction in worsening mobility. Bank 4 displays relatively stable metric trajectories, apart from sharp movements in the worsening and improvement mobilities in the first half of the period.[21] Finally, note that the variation of the whole mobility is shorter than of other mobilities, since it averages out the effects of many mobility directions that possibly behave in different ways.

[21] Perhaps these sharp movements are caused by a data problem. The study was not able to determine the specific cause.

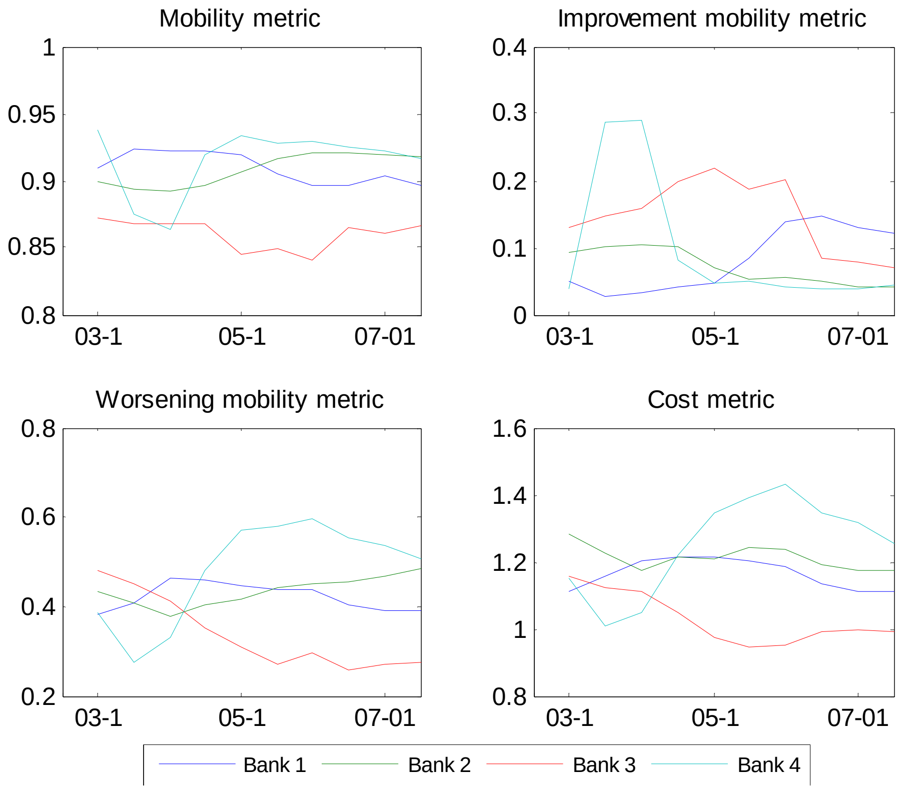
Figure 10: Trajectories of banks' distances between semester-restricted default matrices and time-unrestricted matrices. All matrices are estimated by homogeneous survival.



The worsening metric trajectories display important distinctions with regard to the paths of migration A→>=E depicted in figure 8. For bank 1, the peak in default occurs sooner according to the metric than according to the referred transition, so that the worsening metric works in this case as a warning indicator to the default definition >=E. That shows that other default transitions (to less severe default definitions than >=E or starting from already more severe states than A) add information to the general default behavior of bank 1. For bank 2, the worsening metric serves to smooth the ascending part of the default transition path A→>=E in the first half of the period and, then, to drastically reverse the decreasing pattern of the transition in the second half. That shows again the new information provided by metrics of default matrices. Banks 3 and 4 seem to face smaller differences between the metric paths and the transition paths.

I now reinsert the average levels into the bank comparative analysis. Figure 11 displays, for each metric, the absolute trajectories of all banks together. Here, it is possible to reaffirm that, among the four banks analyzed, banks 1 and 2 are indeed the closest and lie between the extremes represented by banks 3 and 4. (The general picture is again a little more entangled on the improvement dimension) There is notably, however, an inversion in relative positions, from the first to the second halves of the period, of both worsening and improvement mobilities of banks 1 and 2. Bank 2 surpasses bank 1 in credit risk at the second half, with higher worsening and smaller improvement. Note that this also translates into the opening of an opportunity cost gap between two banks at the second half. That is the sort of within-group bank comparative analysis that can be useful to prompt closer investigations of specific banks.

Figure 11: Trajectories of banks' metrics of semester-restricted default matrices. All matrices are estimated by homogeneous survival.



In figure 11, it is possible to observe that both the cost and the mobility metrics resemble slightly the general pattern of the worsening mobility (at least in terms of banks' relative positions). That is not surprising since the cost metric weights much more departing classifications in no arrears, from which improvement is not a possibility, and since a great part of the mobility of default matrices, composed of many *bad* states, is on the worsening direction.

5. Conclusion

This paper investigates the measurement of credit delinquency through a flow approach instead of the easier and more popular stock methodology. A flow approach does not let measurement of credit risk to be distorted by non-default events, such as the increase in the number of loans, as observed in Brazil in recent years. In order to avoid missing relevant information, this paper further proposes considering many default severities together, related to several past-due ranges, and the transition rates between them. The suggested approach becomes then a multivariate flow description of default risk (as well as of recovery risk), called default matrix, and metrics are proposed to compare different matrices. Besides the mobility metric suggested in the literature, this paper proposes an improvement and a worsening mobility metrics, in order to disentangle the good and bad parts of mobility, as well as a cost metric that penalizes more some transitions to more severe default classifications.

One-semester default matrices are estimated for consumer credit at four large Brazilian banks. Default matrices present very high mobility, more located on the worsening direction. Only their improvement mobility is already of similar order to the whole mobility of 1-year rating agencies' matrices. Default matrices are also shown useful to explore or compare different default severities, focusing on aspects such as probability monotonicity and risk discrimination. In particular, this paper points out problematic risk discrimination between default classifications related to large number of days in arrears.

As far as estimating techniques are concerned, this paper indicates the efficiency gains of survival compared to discrete estimation, reflected in the larger survival estimated probabilities for typical default and recovery migrations. The effect of the homogeneity assumption is shown, in general, less important than the difference between survival and discrete estimation, but, contrary to the case of rating agencies, it is far from insignificant and, for some banks, quite pronounced through the lens of the worsening mobility metric. The non-homogeneous survival estimator could be, therefore, a useful tool for closely monitoring within-semester time specific shocks on a bank level.

As far as time evolution of default risk is concerned, empirical results of this study show that the sharp increase in Brazilian consumer credit during the period from 2003 until 2007 was followed by strong heterogeneity of credit risk over time and across banks. That heterogeneity is observed not only in some default classification transitions but also through the consolidated credit risk behavior reflected in default matrices' metrics. That indicates that time and bank variations in growth strategies, in renegotiation policies, among other credit policies, have also been high in recent years. On the other hand, from 2006 until the end of 2007, a common decreasing move across banks is noted in transition from the no-arrears, non-renegotiated state to the 90-days past-due default definition, although that trend is not identified from the trajectories of the bank worsening metrics. In fact, default matrices metrics can display important distinctions in their trajectories with regard to particular migration paths. The paper finds examples where the worsening metric works as an early warning indicator to a particular default definition or radically reverses the behavior of a migration path.

Metrics also provide credit risk distances between banks in a straightforward manner that can be useful to supervisory purposes. Among the four banks analyzed, metrics identify the most similar and the most dissimilar pairs of banks. The respective similarity and dissimilarity of the two pairs are more pronounced according to worsening rather than to improvement mobility. Also, these characteristics are valid not only on a static comparison but also over time. Nevertheless, the banks forming the similar pair inverse their relative metric positions from the first to the second halves of the period analyzed, a fact more clearly noted through the use of metrics again.

Two final notes about applications of this work are worth mentioning. First, it should be remarked that the sort of results produced, coupled with other sources of bank risk information available to the supervisory authority, such as on-site supervision, should allow it to better understand the behavior of realized default over time on a bank level and prompt closer investigations when necessary. Second, longer time series of Brazilian default matrices could assist in drawing relationships between multivariate flow measured default risk and credit spreads, business cycle indicators and/or macroeconomic variables, shedding new light on past studies that usually employ default stock rates.

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