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Behavior Finance and Estimation Risk in Stochastic Portfolio Optimization

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Abstract

The objective of this paper is twofold. The first is to incorporate mental accounting, loss-aversion, asymmetric risk-taking behavior, and probability weighting in a multi-period portfolio optimization for individual investors. While these behavioral biases have previously been identified in the literature, their overall impact during the determination of optimal asset allocation in a multi-period analysis is still missing. The second objective is to account for the estimation risk in the analysis. Considering 26 daily index stock data over the period from 1995 to 2007, we empirically evaluate our model (BRATE – Behavior Resample Adjusted Technique) against the traditional Markowitz model.

Keywords: Behavior, Portfolio Optimization, Resampling

JEL Classification: G11, G12.

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In a standard asset allocation procedure, once the risk tolerance, constraints, and financial goals are set, the output is given by a mean-variance optimization (Markowitz, 1952; Feldman and Reisman, 2002). Unfortunately this procedure is likely to fail for individuals, who are susceptible to behavioral biases. For instance, in response to short-term market movements and to the detriment of the long-term investment plan, the individual investor may require his asset allocation to be changed. Fernandes et al. [2007] suggest that early liquidation of a long term investment may be the cause of momentum.

In terms of emotional biases, several empirical studies (Tversky and Kahneman, 1992) have shown that, when dealing with gains, agents are risk-averse, but when choices involve losses, agents are risk-seeking (asymmetric risk-taking behavior). Moreover, in a wide variety of domains, people are significantly more averse to losses than they are attracted to same-sized gains. Loss-aversion (Schmidt and Zank, 2005) is a relevant psychological concept that has been imported to financial and economic analysis, and it represents the foundation of prospect theory.

The current paradigm of individual behavior in finance theory is based on expected utility maximization and risk-aversion, which has been under attack in recent years due to its descriptive inaccuracy. Experimental psychologists have demonstrated that people systematically deviate from the choice predictions the classical paradigm implies as individuals are typically biased.

Behavioral biases can roughly be grouped in two categories: cognitive and emotional, though both types yield irrational decisions. Because cognitive biases (heuristics like anchoring, availability, and representative biases) stem from faulty reasoning, better information and advice can often correct them. Conversely, emotional biases, such as regret and loss-aversion, originate from impulsive feelings or intuition, rather than conscious reasoning, and are hardly possible to correct. Lo et al. [2005] investigated several possible links between psychological factors and trading performance, finding that subjects whose emotional reaction to monetary gains and losses was more intense on both the positive and negative side exhibited significantly worse trading performance.

Shefrin [2005] posits that the portfolios selected by investors whose choices conform to prospect theory will differ in key aspects from the portfolios selected by investors whose choices conform to expected utility theory. The general character of behavioral portfolios is that they feature a combination of securities that are very safe with securities that are very risky, with the overall portfolio failing to be well diversified. In this sense, an optimal solution to the asset allocation problem should guide investors to make decisions that serve their best interest. This could be the recommendation of an asset allocation that suits the investor's natural psychological preferences (emotional biases), even though it may not maximize expected return for a given level of risk. More simply, a client's best practical allocation may be a slightly under-performing long-term investment program to which the investor can comfortably adhere. From a mean-variance optimization perspective, behavioral investors select portfolios that are stochastically dominated. This does not mean that the individual investors are irrational in any sense: it is not irrational for people to anticipate emotional reactions and take them into account when making decisions that try to adjust their choices to their preferences. However, portfolio managers lack the guidelines necessary for incorporating these biases during the process of determining asset allocation. We address this issue by evaluating whether managers should moderate the way clients naturally behave to counteract the effects of behavioral biases so that they can fit a pre-determined asset allocation or they should create an asset allocation that adapt to clients' biases, so that clients can comfortable adhere to the fund.

In general terms, prospect theory and its latter version cumulative prospect theory[1] (Kahneman and Tversky, 1979, 1992) posits four novel concepts in the framework of individuals' risk preferences. First, investors evaluate assets according to gains and losses and not according to final wealth (mental accounting). Second, individuals are more averse to losses than they are attracted to gains (loss-aversion). Third, individuals are risk-seeking in the domain of losses and risk-averse in the domain of gains (asymmetric risk preference). Finally, individuals evaluate extreme probabilities in a way that overestimates low probabilities and underestimates high probabilities (probability weighting function). This study, as far as we know, is the first to consider all those aspects in the framework of portfolio choice.

There are conflicting results in the finance literature on how prior outcomes affect the risk-taking behavior of investors in subsequent periods. Loss-aversion would predict that traders with profitable mornings would reduce their exposure to afternoon risk, trying to avoid losses and thus guaranteeing the previous gains (Weber and Zuchel, 2003). Odean [1998] and Weber and Camerer [1998] have shown that investors are more willing to sell stocks that trade above the purchase price (winners) than stocks that trade below purchase price (losers) – a phenomenon termed the disposition effect

(Schefrin and Statman, 1985). Both works interpreted this behavior as evidence of decreased risk-aversion after a loss, and increased risk-aversion after a gain. The standard explanation for the previous behavior is based on prospect theory, and particularly on the fact that individuals are risk-seeking in the domain of losses and risk-averse in the domain of gains (asymmetric risk preference).

However, another stream of the literature found the opposite behavior. Thaler and Johnson [1990] name the house-money effect, the behavior of increasing risk appetite after a gain. Barberis et al. [2001] present a model where investors are less loss-averse after a gain while they become more loss-averse after prior losses. Our proposed model addresses and clarifies the previous contradiction between house-money and disposition effect.

Despite the vast literature confirming the behavioral biases associated with prospect theory, the consideration of all those biases in an asset allocation framework is still missing. Barberis and Huang [2001] and Barberis et al [2001] use loss-aversion and mental accounting (Thaler, 1999) to explain aspects of stock price behavior, but do not employ the full prospect theory framework and don't examine optimal asset allocation. Benartzi and Thaler [1995] consider prospect theory to solve the equity premium puzzle when investors are loss-averse and evaluate their portfolios myopically with a horizon of approximately one year. They also suggest an optimal allocation in equities from 30% to 55%. Magi [2005] uses behavioral preferences to numerically solve a simple model of international portfolio choice, providing a possible explanation for the equity home bias puzzle, the tendency of individual investors to prefer its home-country stocks despite the greater performance of foreign stocks.

Davies and Satchell [2004] provide a solution for the optimal equity allocation, and explore more thoroughly the cumulative prospect theory parameter space that is consistent with observed equity allocations given a financial market's returns distributions over a one-month horizon. Shefrin [2005] considers heterogeneous investors to see the impact of behavioral concepts in the framework of asset pricing.

The first main goal of this study is to incorporate mental accounting, loss-aversion, asymmetric risk-taking, disposition effect, and probability weighting in portfolio optimization in a multi-period setting for individual investors. We provide a solution for the asset allocation problem, taking into account all behavioral biases associated with prospect theory and using a utility function (suggested in Giorgi et. al., 2004) consistent with both the experimental results of Tversky and Kahneman, and also with the existence of equilibrium. We also shed more light on the issue of how prior outcomes affect subsequent risk-taking behavior, investigating the investor's risk-taking behavior following a rise, or a fall, in the price of the risky asset.

In line with prospect theory, investors derive utility from fluctuations in the value of their final wealth. In our framework, there is a financial market on which two assets are traded. A riskless asset, also called a bond, and a risky asset, also called a stock (under the assumption of normally distributed returns for the risky asset). As we are modeling the decision making process of an individual investor, short-selling is not allowed. In each period (we consider two periods), the investor chooses the weight of his endowment to be invested in the risky asset, in order to maximize his utility (prospect theory based). We assume that the investor acts myopically in a sense that he doesn't discount long-term welfare when evaluating his utility, and that the reference point relative to which he measures his gains and losses for the first period is his initial endowment. Although all agents solve the same maximization problem in the first period, the second period decision depends on the reference point relative to which the agent measures the second period outcomes (gains or losses). We consider two possible reference points: the initial wealth or the current wealth, and analyze both cases. St-Amour [2006] evaluates household portfolios and his results reveal that references are strongly relevant and state-dependent.

Another well-known issue in asset allocation problems, using Markowitz optimization, is that the output is strongly driven by the risk/return estimation, which usually generates very unstable portfolios. The most famous problem with this technique is the substitution problem, where two assets with the same risk but slightly different expected returns. The optimizer would give all the weight to the asset with the higher expected return, leading to a very unstable asset allocation. The second goal of this chapter is to incorporate estimation risk in the portfolio allocation behavioral problem.

Recent literature has tried to overcome the previous problem of leading to unfeasible portfolios. The main focus of those models is to find out how to create realistic portfolios considering that the values used for risk and return are not deterministic but instead just estimates (they are stochastic). It should be noted that the misspecification of expected returns is much more critical than that of variances (Zimmer and Niederhauser, 2003).

Jorion [1986] offers a simple empirical Bayes estimator that should outperform the sample mean in the context of a portfolio. His main idea is to select an estimator with average minimizing properties relative to the loss function (the loss due to estimation risk). Instead of the sample mean, an estimator obtained by "shrinking" the means toward a common value is proposed (the average return for the minimum variance portfolio), which should lead to decreased estimation error. Similar to Jorion, Kempf et al [2002] assumes that the prior mean is identical across all risky assets. However, Kempf's model considers estimation risk as a second source of risk, determined by the heterogeneity of the market and given by the standard deviation of the expected returns across risky assets.

Black and Litterman [1992] postulate that the consideration of the global CAPM (Capital Asset Pricing Model) equilibrium can significantly improve the usefulness of asset allocation models, as it can provide a neutral starting point for estimating the set of expected excess returns required to drive the portfolio optimization process. Horst et al. [2002] propose a new adjustment in mean-variance portfolio weights to incorporate the estimation risk. The adjustment amounts to using a pseudo risk-aversion, rather than the actual risk-aversion, which depends on the sample size, the number of assets in the portfolio, and the curvature of the mean-variance frontier. The pseudo risk-aversion is always higher than the actual one and this difference increases with the uncertainty in the expected return estimations. Maenhout [2004] also considers an adjustment in the coefficient of risk-aversion to insure the investor against some endogenous worst case.

Finally, Michaud [1998] suggests portfolio sampling as a way to allow an analyst to visualize the estimation error in traditional portfolio optimization methods, and Sherer [2002] posits that sampling from a multivariate normal distribution (a parametric method termed Monte Carlo simulation) is a way to capture the estimation error. Markowitz and Usmen [2003] compared the traditional approach to resampling and their results support the latter. Fernandes et al. [2008] evaluate several asset allocation models and suggest that resampling methods typically offer the best results.

This study presents a novel approach (BRATE – Behavioral Resample Adjusted Technique) to incorporate behavioral biases and estimation risk into mean-variance portfolio selection. In a paper close to ours, Vlcek [2006] proposes a model to evaluate portfolio choice with loss-aversion, asymmetric risk-taking behavior, and segregation of riskless opportunities. His findings suggest that the changes in portfolio weights crucially depend on the reference point and the ratio between the reference point and the current wealth, and thus indirectly on the performance of the risky asset. Our work differs from his study as we explicitly consider all novel aspects of prospect theory: mental accounting, loss-aversion, asymmetric risk-taking behavior, and probability weighting function. We also evaluate the inefficiency cost of the behavioral biases and consider a more general form for the risky asset return process, including estimation risk in the analysis.

Considering daily equity data from the period from 1995 to 2007, we empirically evaluate our model in comparison to the traditional Markowitz model. Our results support the use of BRATE as an alternative for defining optimal asset allocation and posit that a portfolio optimization model may be adapted to the individual biases implied in prospect theory.

The remainder of this paper contains the following sections. Section A discusses the behavioral biases considered and describes our model proposing the behavioral resampling adjusted technique (BRATE). Section B presents the empirical study, describing the data and implementation, and providing the results. Section C concludes the research by reviewing the main achievements.

A The Behavioral Model

We present a two period's model for portfolio choice in a stylized financial market with only two assets, where the investor's preferences are described by cumulative prospect theory as suggested by Kahneman and Tversky [1979] and Tversky and Kahneman [1992]. In our framework, there is a financial market in which two assets are traded. A riskless asset, also called the bond, and a risky asset, the stock. Let us consider the return of the stock in each period given by the following process: $R = \mu + \sigma n$, with $n \sim N(0,1)$. The riskfree bond yields a sure return of R_f. We assume that the time value of the money is positive, i.e. that interest rates are non-negative.

The preferences of the investor are based on changes in wealth and are described by prospect theory. We assume that he owns an initial endowment, W_0 (normalized to 1 monetary unit), and that he earns no other income. The agent invests a proportion θ of his wealth in the stock and (1 - θ) in the bond. Since we want to model the individual investor's behavior, we assume that short selling is not allowed ($0 \le \theta \le 1$). We also assume that the investor acts myopically, and the reference point relative to which he measures his gains and losses in the first period is his initial wealth. Then, the perceived gain or loss in the end of the first period is given by:

$$x = \Delta W = \lfloor (1-\theta)W_0(1+R_f) + \theta W_0(1+R) \rfloor - W_0$$
$$\therefore x = (1-\theta)R_f + \theta R$$
$$\therefore x = (1-\theta)R_f + \theta(\mu + \sigma n) \qquad \text{(Eq. 01)}$$

As pointed out in Vlcek [2006] the choice process under prospect theory starts with the editing phase, followed by the evaluation of edited prospects, and finally the alternative with the highest value is chosen. During the editing phase, agents discriminate gains and losses. They also perform additional mental adjustments in the original probability function $p = f(x)$, defining the probability weighting function $\pi(p)$. Based on experimental evidence, individuals adjust the likelihood of outcomes such that small probabilities are overweighted and large probabilities are underweighted. We will consider the probability weighting function, as in Giorgi et al. [2004] given by:

$$\pi(p) = \frac{p^{\gamma}}{\left(p^{\gamma} + (1-p)^{\gamma}\right)^{\frac{1}{\gamma}}}, \qquad \text{(Eq. 02)}$$

where γ is the adjustment factor. The following graph compares the values of p and $\pi(p)$, considering γ =0.80[2].

Figure 1

In the valuing phase, the agents attach a subjective value to the gamble. Let us assume the value function proposed by Giorgi et al. [2004], as follows:

$$v(x) = \begin{cases} \lambda^{+} - \lambda^{+}e^{-\alpha x}, & \text{if} \quad x \geq 0 \\ \lambda^{-}e^{\alpha x} - \lambda^{-}, & \text{if} \quad x < 0 \end{cases} \qquad \text{(Eq. 03)}$$

where α is the coefficient of absolute risk preference, $\lambda^{-} > \lambda^{+} > 0$ makes the value function steeper in the negative side (loss-aversion), and x is the change in wealth or welfare, rather than final states (mental accounting), as proposed by Kahneman and Tversky [1979]. Also, the value function is concave above the reference point and convex below it (asymmetric risk preference). It is useful to consider the previous form for the value function because of the existence of a CAPM equilibrium[3] and the ability to reach constant coefficients of risk preference (α). The previous formulation is also supported by the laboratory results from Bosh-Domènech and Silvestre [2003]. The following graph indicates $v(x)$ when α = 0.88, λ^- = 2.25 and λ^+ = 1 (Kahneman and Tversky suggested values).

Figure 2

In our two-period model for portfolio choice, the investor chooses a weight in the risky asset to maximize his expected utility (V). His preferences are based on changes in his wealth (x) and are described by prospect theory. The total expected value he addresses to a given choice of θ is given by:

$$V = \int_{-\infty}^{\infty} v(x) \frac{d}{dx} \pi(f(x)) dx \qquad \text{(Eq. 04)}$$

where $v(x)$ is the prospect value of the outcome x, and $\pi(f(x))$ is the weighted cumulative probability associated with that outcome. Prospect theory is a descriptive theory, postulating that, in comparing alternatives, the investor will choose the alternative that makes V as high as possible. Let us then evaluate the investor's problem in each period.

A.1 First Period

In the first period, the agent's problem consists of defining the allocation of his initial wealth between the two assets traded in the financial market. He maximizes his utility in t = 0 by allocating a fraction, θ_0, of his initial wealth[4], W_0, in the risky asset and (1 - θ_0) in the riskfree asset. We consider that the investor is a myopic optimizer in the sense that he takes into account only the first period result. For multi-period horizons, the choices at earlier dates impact the reference points at later dates. This feature allows for complex modeling. However, as pointed out in Shefrin [2005], prospect theory is a theory about investors who oversimplify, and so, assuming that individuals are sophisticated enough to perceive the link between their current choices and future reference points is something unreasonable. We also constrain short selling, as it is common for individual investors' models. Thus, his problem can be given by

$$\max_{0 \le \theta \le 1} V = \int_{-\infty}^{\infty} v(x) \frac{d}{dx} \pi(f(x)) dx \qquad \text{(Eq. 05)}$$

Let us make the following derivation: $x = (1-\theta_0)R_f + \theta_0(\mu + \sigma n)$. Rearranging the terms in x, we get $x = (1-\theta_0)R_f + \theta_0\mu + \theta_0\sigma n$. We call $(1-\theta_0)R_f + \theta_0\mu$ = B and $\theta_0\sigma = C$. Then, $x = B + Cn$, and so x > 0 implies $n > -\frac{B}{C}$. Then,

$$V = \int_{-\infty}^{\infty} v(x)\frac{d}{dx}\pi(f(x))dx$$

$$\therefore V = \int_{0}^{\infty}\left(-\lambda^{+}e^{-\alpha x} + \lambda^{+}\right)d\pi(f(x)) + \int_{-\infty}^{0}\left(\lambda^{-}e^{\alpha x} - \lambda^{-}\right)d\pi(f(x))$$

$$\therefore V = \int_{-\frac{B}{C}}^{\infty}\left(-\lambda^{+}e^{-\alpha(B+Cn)} + \lambda^{+}\right)d\pi(f(n)) + \int_{-\infty}^{-\frac{B}{C}}\left(\lambda^{-}e^{\alpha(B+Cn)} - \lambda^{-}\right)d\pi(f(n))$$

$$\therefore V = \lambda^{+}\left(1-\hat{\pi}(-\frac{B}{C})\right) - \lambda^{-}\pi(-\frac{B}{C}) + \lambda^{-}e^{\alpha B}\int_{-\infty}^{-\frac{B}{C}} e^{\alpha nC}d\pi(f(n)) - \lambda^{+}e^{-\alpha B}\int_{-\frac{B}{C}}^{\infty} e^{-\alpha nC}d\pi(f(n))$$

$$\therefore V = \lambda^{+} - \left(\lambda^{+} + \lambda^{-}\right)\pi(-\frac{B}{C}) + \lambda^{-}e^{\alpha B}\int_{\frac{B}{C}}^{\infty} e^{-\alpha nC}d\pi\left(f(n)\right) - \lambda^{+}e^{-\alpha B}\int_{-\frac{B}{C}}^{\infty} e^{-\alpha nC}d\pi(f(n))$$

$$\therefore V = \lambda^{+} - \left(\lambda^{+} + \lambda^{-}\right)\pi(-\frac{B}{C}) + e^{\frac{1}{2}\alpha^2C^2}\left[\lambda^{-}e^{\alpha B}\pi(-\frac{B}{C} - \alpha C) - \lambda^{+}e^{-\alpha B}\pi(\frac{B}{C} - \alpha C)\right] \quad \text{(Eq. 06)}$$

Where, for the last step, we used[5]:

$$\int_{z}^{\infty} e^{-\alpha\sigma x}d\phi(x) = e^{\frac{1}{2}\alpha^2\sigma^2}\hat{\phi}(-\alpha\sigma - z)$$

Observe that, if we were considering a standard utility function (risk-aversion over all possible outcomes), the value would be given by:

$$V^{S} = \lambda^{+} - \lambda^{+}e^{-\alpha B + \frac{1}{2}\alpha^2 C^2} \quad \text{(Eq. 07)}$$

Moreover, the partial derivatives of V (Eq. 06) are:

$$\frac{\partial V}{\partial \mu} = \{\alpha e^{\frac{1}{2}\alpha^2(\theta_0\sigma)^2}[\lambda^- e^{\alpha[(1-\theta_0)R_f+\theta_0\mu]}\pi\left(-\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}-\alpha\theta_0\sigma\right)+$$

$$\lambda^+ e^{-\alpha[(1-\theta_0)R_f+\theta_0\mu]}\hat{\pi}\left(\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}-\alpha\theta_0\sigma\right)]\}\cdot\theta_0 \qquad \text{(Eq. 08)}$$

$$\frac{\partial V}{\partial \sigma} = \{\alpha^2\theta_0\sigma e^{\frac{1}{2}\alpha^2(\theta_0\sigma)^2}[\lambda^- e^{\alpha B}\pi(-\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}-\alpha\theta_0\sigma)-$$

$$\lambda^+ e^{-\alpha([(1-\theta_0)R_f+\theta_0\mu])}\pi(\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}-\alpha\theta_0\sigma)]-\alpha(\lambda^- - \lambda^+)\pi\left(\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}\right)\}\cdot\theta_0 \quad \text{(Eq. 09)}$$

As a consequence, the following properties hold[6],

i) $\frac{\partial V}{\partial \mu} > 0$;

ii) $\frac{\partial V}{\partial \sigma} = 0$ for $\sigma = 0$ or $\sigma = \infty$;

iii) $\frac{\partial V}{\partial \sigma} < 0$ for $\sigma > 0$.

Equations 06 and 07 clearly yield different weights for the risky asset, considering the remaining parameters fixed. Thus, it is possible to evaluate the cost of inefficiency associated with the behavioral biases as compared to the standard utility solution.

$$Cost = [(1-\theta_0^S)R_f + \theta_0^S R] - [(1-\theta_0^{PT})R_f + \theta_0^{PT} R] \quad \text{(Eq.10)}$$

where θ_0^S is the risky asset weight given by the standard utility maximization problem, and θ_0^{PT} is the stock weight as defined in our model.

Proposition 1. *The optimal asset allocation in t = 0, for the risky asset* θ_0^* *is such that maximizes the value function given by:*

$$V = \lambda^+ - (\lambda^+ + \lambda^-)\pi(-\frac{B}{C}) + e^{\frac{1}{2}\alpha^2C^2}\left[\lambda^- e^{\alpha B}\pi(-\frac{B}{C}-\alpha C) - \lambda^+ e^{-\alpha B}\pi(\frac{B}{C}-\alpha C)\right]$$

where: $B = [(1-\theta_0^*)R_f + \theta_0^*\mu]$ *and* $C = \theta_0^*\sigma$.

If we were considering a standard utility function, the optimal allocation in t = 0, for the risky asset would then be given by:

$$\theta_0^* = \frac{\mu - R_f}{\alpha\sigma^2}$$

Let us first consider standard values for the model's parameters[7]. The riskfree rate equals the historical annual return of the US three-month Treasury Bill (R_f = 2.73%). The equity expected return and volatility equals the historical average of the MSCI global equity index and its standard deviation (μ = 7.61% and σ = 12.98%). The adjustment factor in the probability weighting function equals γ = 0.90. The coefficient of risk-aversion equals α = 3. Also, as suggested by Kahneman and Tversky, λ^- = 2.25 and λ^+ = 1. The individual's values (prospect theory and standard) as a function of the percentage of his wealth invested in the risky asset are given in Figure 3. The individual investor is expected to choose the allocation in the risky asset which maximizes his expected value.

Figure 3

As can be observed from the graph, using a standard utility function, the allocation in the risky asset approaches 100% (theta for which the value function reaches its maximum), while using prospect theory utility, the investor should allocate 81% of his wealth in the stock[8]. The shapes of the graphs are different, notably for large allocations in the stock. The value function using standard utility is equal to or greater than the one for prospect utility.

The reason for this difference comes from the fact that in prospect theory, negative outcomes are penalized more (as are risky portfolios) because individuals are loss-averse ($\lambda^- > \lambda^+$). In the loss-aversion literature evidence suggests that individuals are around twice more sensitive to losses than they are attracted to same size gains. For small allocations in stocks, the prospect of losses becomes less likely and the value functions tend to coincide.

Related to the effect of probability weighting, if we set γ = 1, thus canceling out its effect, we reach the following Figure representing the value function:

Figure 4

Note that the amount optimally invested by the behavioral investor in the risky asset decreases to 48%, and so probability weighting tends to increase the risk appetite. Kahneman and Tversky [1979] suggest that the overweighting of low probabilities has an ambiguous effect on risk-taking, as it can induce risk-aversion in the domain of losses, and risk-seeking in the domain of gains. In our case, the overestimation of the extreme positive outcomes probabilities, shown in Figure 3, is inducing investors to take more risk.

However, despite the effects of loss-aversion and probability weighting, even if we consider $\lambda^{-} = \lambda^{+} = 1$ and $\gamma = 1$, keeping constant the remaining parameters, the value functions wouldn't coincide, as can be seen in Figure 5:

Figure 5

Both models would predict that the investor should allocate 100% of his endowments in the stock. However, the value functions are different because, in prospect theory, individuals are risk-seeking in the loss domain (asymmetric risk preference). Thus, they would be more comfortable in allocating a greater part of their wealth in the risky asset. The prospect value function is greater than the standard utility function.

Observe that the effect of the asymmetric risk preference goes in the opposite direction of loss-aversion and probability weighting. When we diminish the coefficient of risk preference ($\alpha = 0.25$) in both utility functions, we reduce the effect of asymmetry, and so the value functions are much closer, as can be seen in the following figure.

Figure 6

The effects of the behavioral biases can thus be summarized as follows: loss-aversion reduces risk-taking, and asymmetric risk-taking behavior induces risky attitudes. Probability weighting has an ambiguous effect on risk. Our intuition is that, in the long run, as the value function parameters are changing, these biases tend to cancel out, eliminating the efficiency loss originated by each bias. That is why we argue that human biases do not need to be moderated to reach an efficient investment strategy. The experimental results of Blavatskyy and Pogrebna [2006] reveal that the effect of loss-aversion is largely neutralized by the overweighting of small probabilities and underweighting of moderate and high probabilities.

In order to verify property (*i*), Let us evaluate V while changing μ and keeping constant the other parameters (considering θ = 50%). Figure 7 presents the graph which indicates that over all positive values of μ, the slope of V is positive. The value function is increasing in μ. Thus, when the risky asset has a higher expected return, *ceteris paribus* implies a higher value for the investor:

Figure 7

Considering properties (*ii*) and (*iii*), Let us evaluate V while changing σ and keeping constant the other parameters (considering θ = 50%). Figure 8 presents the graph indicating that over all positive values of σ, the slope of V is negative, while for σ = 0, the slope is null. When σ tends to infinity, the slope tends to null. The value function is decreasing in σ.

The intuition is that, if the volatility of the risky asset is higher, for the same allocation, this implies a higher probability of losses reducing the value of the prospect. In line with traditional rational investor, behavioral individuals also prefer higher return and lower risk; mainly because they are risk-averse in the gain domain and also loss-averse.

Figure 8

Now let us evaluate the values of θ_0 when we change the riskfree rate and the expected return of the risky asset. Since many parameters are involved, it is not possible to find closed form solutions for θ_0. Therefore, we present numerical results for the optimal allocation of wealth in t = 0. Figure 9 presents the results for $0\% < \mu < 15\%$ and $0 < R_f < 6\%$. The remaining parameters are fixed (σ = 12.98%, α = 3, λ^- = 2.25, and λ^+ = 1).

Figure 9

As expected, when the risky asset offers more attractive returns, the agent gradually invests more in the stock. When the stock is very attractive, the investor chooses to allocate his entire wealth in the risky asset. Thus, we observe that θ_0 is increasing in μ and decreasing in R_f. Also, when R_f is higher, the changes in θ_0 due to a variation in μ are smoother, because in these cases losses are less likely and we approach the standard utility solution. When R_f is lower, the changes in θ_0 due to a variation in μ are more abrupt, giving rise to extreme portfolio allocations. If we consider that μ is not known with certainty, the resulting portfolio would be very unstable. Gomes [2003], in a model with loss-averse investors, has found that individuals will not hold stocks unless the equity premium is quite high.

We can evaluate the expected cost of inefficiency related to the behavioral biases associated to the prospect theory function, for the same parameters considered in the previous analysis, using equation 10. The result is presented in Figure 10, and its form is due to the fact that, in standard utility function, the investor is willing to take more risk than with the loss-averse prospect utility. The cost is due to the fact that the expected return of the stock is greater than the bond, and the standard utility investor is allocating a greater part of his wealth in the risky asset than the prospect utility individual. Thus, the cost is increasing in μ. However, it is worth noting that the previous cost is based on expected returns, which are stochastic in practice. The real cost can just be observed at the end of the first period with the realization of the stock's return.

An important insight can be made from Figure 10 in terms of the best practice for asset allocation. As long as the riskfree rate is lower and the expected return of the stock is higher, the optimal allocation should moderate the investor's biases in order to reach a better performance. On the other hand, if the risk premium is lower, the moderation is less relevant, and the optimal allocation may adapt to the individual's biases.

Figure 10

We can also analyze the change in the allocation of the stock when we vary the loss-aversion in the risk-taking behavior. The result is shown in Figure 11, for $2 < \lambda^{-} < 4$. Observe that, as long as the investor is much more averse to losses than he is attracted to gains, the allocation in the risky asset is lower. When $\lambda^{-} = 2.25$, the allocation in the risky asset corresponds to 81%, as previously mentioned.

Figure 11

Dimmock[2005] has already shown that a higher level of loss-aversion leads to lower equity exposure, and heterogeneity in the coefficient of loss-aversion has the ability to explain puzzling features of household financial behavior.

A.2 Second Period

In order to evaluate the second period allocation choice of the investor, Let us keep some parameters fixed: (σ = 12.98, α = 3, λ^{-} = 2.25 and λ^{+} = 1). After the investor has made his first period decision in t = 0, the state of nature realizes in t = 1, when he is faced with his second period problem. Again, he must allocate his wealth in the two possible assets in the financial market, bond and stock, to maximize his utility. Let us consider the same normal distribution for the return of the risky asset. The investor's wealth position at t = 2 equals his position in t = 1 plus the return of his portfolio in the second period.

While all agents solve the same maximization problem in the first period, in the second period, it will depend on the reference point to which he measures his gains and losses (in the framework of prospect theory). In our model, there are two candidates for the investor's reference point at t = 1: his initial wealth at t = 0 ($W_0 = 1$) or his wealth at the end of the first period, t = 1 (W_1). If he measures his gains and losses relative to his wealth at t = 1 (his current wealth), he treats each gain and loss separately. On the other hand, if he considers his initial wealth as the reference point, he adds up the outcomes (gains and losses), that is, he nets his positions. The previous distinction is relevant in prospect theory. The value function is concave in the domain of gains and convex in the loss domain (asymmetric risk behavior).

First, Let us consider as the investor's reference point his current wealth at t = 1. In this case, the maximization problem he will solve in the second period is the same as the one for the first period. Thus, we can state the following proposition.

Proposition 2. *The optimal asset allocation in t = 1, for the risky asset* θ_1^**, if the agent measures his gains and losses relative to his current wealth, is such that maximizes the same value function of the first period.* $\theta_1^* = \theta_0^*$

We can observe that an individual who measures his gains and losses relative to his current wealth is actually solving the same maximization problem in each period. That is why the allocation in the risky asset might be the same. This is not surprising; as he is not using past information to update his beliefs about the assets, his preferences are similarly unaffected.

Next, let us analyze the investor's maximization problem if he evaluates his gains and losses relative to his initial wealth. If he has an initial wealth position of W_0 = 100 and his wealth rises in the first period to W_1 = 110 and falls in the next period to W_2 = 105, he values his position at t = 2 as a gain of 5, and not as a gain of 10 followed by a loss of 5.

In the second period, the agent's problem consists of defining the allocation of his wealth (W_1) between the two assets traded in the financial market. He maximizes his utility in t = 1 by allocating a fraction, θ_1, of his wealth W_1 in the risky asset and 1- θ_1 in the riskless asset. As we did in the first period analysis, we also constrain short selling.

$$\max_{0 \le \theta \le 1} V = \int_{-\infty}^{\infty} v(x) \frac{d}{dx} \pi(f(x)) dx$$

Let us make the following derivation:

$$x = W_1 \left[(1-\theta_1) R_f + \theta_1 (\mu + \sigma n) \right] + W_0 \left[(1-\theta_0) R_f + \theta_0 R_1 \right]$$

and $W_1 = W_0 \left[1 + (1-\theta_0) R_f + \theta_0 R_1 \right]$, where R_1 is the return of the stock in the first period. So $x = W_0 \left[\left(1 + (1-\theta_0) R_f + \theta_0 R_1 \right) \cdot \left((1-\theta_1) R_f + \theta_1 (\mu + \sigma n) \right) + (1-\theta_0) R_f + \theta_0 R_1 \right]$.

Rearranging the terms in x and considering $W_0 = 1$, we get

$$x = [\theta_1 \sigma n \left(1 + \left((1-\theta_0) R_f + \theta_0 R_1 \right) \right) + \left(1 + \left((1-\theta_0) R_f + \theta_0 R_1 \right) \right) \cdot \left[(1-\theta_1) R_f + \theta_1 \mu \right] + \\ + \left((1-\theta_0) R_f + \theta_0 R_1 \right)]$$

Let us call

$$B = [\left(1 + \left((1-\theta_0) R_f + \theta_0 R_1 \right) \right) \cdot \left[(1-\theta_1) R_f + \theta_1 \mu \right] + \left((1-\theta_0) R_f + \theta_0 R_1 \right)]$$

and

$$C = \theta_1 \sigma \left(1 + \left((1-\theta_0)R_f + \theta_0 R_1\right)\right)$$

Then, $x = B + Cn$, so x > 0 implies $n > -\dfrac{B}{C}$. Then,

$$V = \lambda^+ - \left(\lambda^+ + \lambda^-\right)\pi(-\frac{B}{C}) + e^{\frac{1}{2}\alpha^2 C^2}\left[\lambda^- e^{\alpha B}\pi(-\frac{B}{C} - \alpha C) - \lambda^+ e^{-\alpha B}\pi(\frac{B}{C} - \alpha C)\right] \quad \text{(Eq. 11)}$$

Proposition 3. *The optimal asset allocation in t = 1, for the risky asset* θ_1^*, *if the agent measures his gains and losses relative to his initial wealth, is such that it maximizes the value function given by:*

$$V = \lambda^+ - \left(\lambda^+ + \lambda^-\right)\pi(-\frac{B}{C}) + e^{\frac{1}{2}\alpha^2 C^2}\left[\lambda^- e^{\alpha B}\pi(-\frac{B}{C} - \alpha C) - \lambda^+ e^{-\alpha B}\pi(\frac{B}{C} - \alpha C)\right]$$

where:

$B = W_0\left[1 + (1-\theta_0)R_f + \theta_0 R_1\right]\cdot\left[(1-\theta_1^*)R_f + \theta_1^*\mu\right]$, $C = W_0\left[1 + (1-\theta_0)R_f + \theta_0 R_1\right]\cdot\theta_1^*\sigma$,

θ_0 *is the amount allocated in the risky asset in the first period, and* R_1 *is the observed return of the risky asset in the previous period.*

Observe that the value function to be maximized is close to the one of the first period, but with changes in the parameters B and C, which account for the previous period outcome (gain or loss). As we are interested in the investor's risk-taking behavior after realizing a gain or a loss, let us evaluate the values of θ_1 when we change the total return obtained in the first period. Recall that the total return from t = 0 to t = 1 ($Rtot_1$), depends both on his allocation choice in t = 0 and on the realized return of the risky asset R_1.

$$Rtot_1 = (1-\theta_0^*)R_f + \theta_0^* R_1$$

Let us then, evaluate θ_1^* considering the realized return of the stock in the first period varying over the following range: $\mu - 2\sigma < R_1 < \mu + 2\sigma$. We present numerical results for the optimal allocation of wealth, θ_1^*, at t = 1. The remaining parameters are fixed (μ = 7.61%, σ = 12.98, α = 3, $W_0 = 1$, λ^- = 2.25 and λ^+ = 1). Figure 12 shows the results. Recall that the optimal allocation in the risky asset for the first period, considering the previous parameters, is 81%. Thus, we need to verify whether the allocation in the stock in the second period is greater or lower than 81%, indicating greater or lower risk appetite, respectively. First, observe that, for a total return in the first period equal to zero (no gains/losses), the situation replicates the same framework the investor faced in the first period. Then we reach the same optimal allocation in the risky asset (for $Rtot_1 = 0$ implies $\theta_1^* = 81\%$).

Figure 12

Consider the surroundings of the net value ($Rtot_1 = 0$). If the investor experiences a gain in the first period, the model predicts that he should optimally invest less in the risky asset in the second period. This behavior prevails up to the point where the loss-aversion effect is less pronounced. On the other hand, if a loss is observed in the first period, he should take more risk in the following period, allocating a greater part of his wealth in the stock. This prediction is in line with several experiments, which have shown that disposition effect dominates house-money in dynamic settings (Weber and Zuchel 2003). When the investor experiences a gain in the first period, he tends to reduce his risk appetite in order to guarantee the previous outcome. On the other hand, if he experiences a loss in the first period, he will increase his bets on stocks, trying to avoid the previous loss. In the model, the pattern holds for the whole gain domain; however, in the loss domain, high losses in the first period induce less risk appetite in the second period. The intuition is that if the investor is facing a huge loss, the loss aversion effect will dominate the risk-seeking behavior, inducing a reduction in the optimal allocation in the stocks.

When we evaluate the expected cost (Eq. 10) of the behavioral inefficiency in the second period as a function of the return of the risky asset in the first period (Figure 13), it is possible to observe that, depending on the previous outcome, the cost can be increasing or decreasing. If the value for R_1 is such that it implies a small loss in the first period, the cost is even negative, which means that the expected return in the second period under prospect theory is greater than the one associated with standard utility. This is related to a greater risk appetite of the prospect theory individual after a loss, implying a greater allocation in the stock, which has a greater expected return. If R_1 indicates a gain in the first period, then the cost is positive once the allocation in the stock for the standard utility investor is greater than for the prospect utility individual.

Figure 13

We can conclude that for losses in the first period, the optimal allocation should adapt to the individual's biases to reach better performance as the cost comes out to be negative in this domain. For gains in the previous outcome, the allocation should moderate the biases (observe a positive value for the expected cost). For extreme losses in the first period, the allocation should also moderate the investor's biases.

If we accumulate the cost results in periods 1 and 2, we get the graph represented in Figure 14. It indicates that, for a negative stock result in the first period, or even a slightly positive one, the prospect theory individual outperforms the standard utility investor. And so, the allocation strategy should be adapted to the individual biases. The previous results should be taken with care as they refer to expected values. In section A.3., we provide a more robust comparison, taking into account the performance of those individuals in an out-of-sample analysis.

Figure 14

A.3 Multi-Period Analysis

If we extend the two-period analysis to a multi-period one, by analogy, if the investor considers his current wealth as the reference to which he measures his gains/losses, he will solve the same maximization problem for each period and the optimal asset allocation is given as in proposition 1. In this situation, the agent acts myopically, just considering the following period possible gain/loss. In general, this result implies a smaller stock allocation if compared to a standard utility investor, generating an expected cost associated to the prospect theory biases.

On the other hand, if the individual's reference point is his initial wealth (or his wealth in some moment in time $t = t_1$), the allocation is defined as in proposition 3, but now considering the previous outcome as the total return obtained by him from $t = 0$ (or from $t = t_1$) to the current time. As discussed in the two-period model, the allocation in the risky asset will depend on the previous gains/losses, and can be greater or smaller than the one chosen by the standard utility investor. Observe that the standard utility investor always chooses the same allocation in the risky asset, no matter what the reference point, as neither his decisions nor his beliefs are affected by previous outcomes.

A.4 Resampling

In sections A.1, A.2 and A.3 we already evaluated the optimal asset allocation under prospect theory preferences and considering mental accounting, loss-aversion, asymmetric risk-taking behavior, and probability weighting. However, there is still an important issue in portfolio optimization missing: estimation error. Up to now, when solving the investor's problem, we considered the expected return known with certainty, which is not the case in reality (especially in emerging markets where the uncertainty is higher). The assumed return for the risky asset is just an estimate, and so the real value can be different. This problem is relevant in any model of portfolio optimization and is crucial under prospect theory, where for lower values of the riskfree rate, a slightly increase in the risk premium of stocks can lead to extreme allocations. If the real return of the risky asset is lower, the likelihood of facing a loss is greater and should significantly reduce the value of that prospect.

In an attempt to overcome this estimation problem, Michaud [1998] proposed the resampling technique. Portfolio sampling allows an analyst to visualize the estimation error in traditional portfolio optimization methods. Suppose that we estimated both the variance and the excess return by using N observations. It is important to note that the point estimates are random variables and so another sample of the same size from the same distribution would result in different estimates.

Sherer [2002] suggests that sampling from a multivariate normal distribution (a parametric method termed Monte Carlo simulation) is a way to capture the estimation error. In this sense, return and variance would just be the expected values for a multivariate normal distribution. If we just consider two assets, the probability density function for a multivariate normal distribution would be given by

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}-\frac{2\rho xy}{(\sigma_x\sigma_y)}\right)\right).$$

By repeating the sampling procedure *n* times, we get *n* new sets of optimization inputs, and then a different efficient allocation. The resampled weight for a portfolio would then be given by

$$\theta^{\text{Resamp}} = \frac{1}{n}\sum_{i=1}^{n}\theta_{\mathrm{i}}$$

The resampled portfolios should reflect a greater diversification (more assets enter in the solution) than the classical mean-variance efficient portfolio, and should also exhibit less sudden shifts (smooth transitions) in allocations as return requirements change. Both characteristics are desirable for investors.

Recent literature has shown unambiguous results in favor of resampled portfolios in out-of-sample analysis (Pawley, 2005; Markowitz and Usmen, 2003; Wolf, 2006; Jiao, 2003). However, Harvey et al. [2006], evaluating Bayes vs. resampling methods, posit that the choice of risk-aversion drives the results. Kohli [2005] concludes that, despite the fact that there are no conclusive advantages or disadvantages of using resampling as a technique to obtain better returns, resampled portfolios do seem to offer higher stability and lower transaction costs, two crucial features for long term investors' choices.

We then propose the BRATE (Behavior Resample Adjusted Technique) as a novel methodology to define asset allocation, which incorporates behavioral ideas and resampling techniques into portfolio optimization, thus adapting to the individual's preferences. In this case, the optimal asset allocation should be given by the previous propositions (1 and 2 or 3, depending on the reference point), but the procedure should be performed several times for different expected stock returns (given by a multivariate normal distribution). The final allocation is then given by the expected risky asset allocation. The procedure can be summarized as follows[9]

Step 1: Estimate variance-covariance and return from the historical inputs.

Step 2: Resample from inputs (created in Step 1) by taking n draws from the input distribution. The number of draws reflects the degree of uncertainty in the inputs. Calculate new variance-covariance and return from sampled series. Estimation error will result in estimations that are different from those obtained in Step 1.

Step 3: Calculate the optimal allocation for inputs defined in Step 2, using the appropriate propositions (1 and 2 or 3, depending on the reference point considered).

Step 4: After repeating Steps 2 and 3 many times, calculate average portfolio weights. This is the BRATE portfolio allocation.

In the next section, we provide an empirical analysis comparing the BRATE allocation performance to a standard utility allocation.

B. Empirical Study

B. 1 Data and Implementation

Our tests are based considering daily data from 26 countries' MSCI stock indices and riskfree rates, plus the MSCI World Index, for the period from April 4th, 1995 to January 5th, 2007. Developed countries and emerging markets (Brazil, Chile, South Africa, South Korea, Taiwan, Thailand, Turkey) were included in the analysis in order to find generalizable results. The total return time series are calculated on each country's currency and also in US-Dollars. Thus, we are considering both currency hedged and unhedged investors. Table I presents some descriptive statistics of each market considered, for the whole sample period.

Table I

From the table, we verify a risk premium associated with the stock market, both considering the values in each country's currency and in USD, with the mean return of stocks being higher than the one of the corresponding riskfree rate[10].

Let us first consider the values in each country's currency. The average annualized return of the riskfree rate varied from 0.151% (Japan) to 39.514% (Turkey), while for the stock index, it ranges from 0.076% (Thailand) to 47.804% (Turkey). The annualized volatility (standard deviation) of the stock market varied from 12.976% (World Index) to 45.171% (Turkey). As expected, emerging markets tend to be more volatile than developed markets. While in Brazil, South Korea, Thailand, and Turkey the volatility was above 30 %, in countries like United Kingdom and United States, its value was close to 16%. In terms of skewness and kurtosis, usual results appear, indicating that daily stock index returns are negative skewed and have excess kurtosis (greater than 3). Finally, Table 1 presents the annualized Sharpe Ratio, which was greater in developed markets (around 0.35) than emerging markets (0.19). Our results are in line with previous literature which gives 0.34 as an estimation of the long-term Sharpe Ratio for the U.S. economy.

When we consider the values in USD, say in the perspective of a US based international investor who doesn't currency hedge his investments, we find similar results. The average daily return in USD is close to the one in the country's currency, which is evidence of the mean reverting aspect of the foreign exchange market. However, the standard deviation in USD is slightly greater than the one in the country's currency, as the former includes both stock market risk and currency risk (the volatility of the foreign exchange rate). In terms of skewness and kurtosis, the previous results remain. However, now the Sharpe Ratios do not present relevant differences among emerging and developed markets (for instance it is 0.430 for Brazil and 0.422 for the United States). Thus it seems that emerging stock markets are less interesting for domestic investors than for foreign unhedged investors.

Next we analyze the performance of the following optimization strategies: an investor with a standard utility preference - STU; an investor with prospect utility preference, with reference point given by his current wealth – PTU; an investor with prospect utility preference, with reference point given by his wealth in the previous period – CPT; an investor with a standard utility preference (resampled) – RSTU; an investor with prospect utility preference, with reference point given by his current wealth (resampled) – BRATEa; and an investor with prospect utility preference, with reference point given by his wealth in the previous period (resampled) – BRATEb. The utility function parameters are fixed (α = 3, λ^- = 2.25 and λ^+ = 1). We vary the estimation period (*p*) in an out-of-sample analysis. The parameters are estimated using daily return observations of the past *p* days. We define the efficient portfolio and hold it for the next (*e*) months, then re-estimate the parameters and adjust the portfolio weights. To judge the financial performance of the strategies, we compute their average return and empirical Sharpe Ratios.

B. 2. Results

The Sharpe Ratios of the different strategies are presented in Table II for the World Index and for the total period from 1995 to 2007, considering *p* = 6 months, 1, 2, and 4 years, and *e* varying from 2 months to 1 year. We are evaluating the different strategies for a US based international stock investor. The riskfree rate considered was the 3 month T-Bill.

Table II

In general, we can state that the resampled models offered better results for a short selling constrained investor. It is an expected result as resampled models take into account the estimation risk, generating a more diversified portfolio which tends to outperform in out-of-sample studies. The highest Sharpe Ratio was reached by the BRATEb model for an estimation period of 2 years and evaluation period of 1 year (0.465). On average resampled models increase the Sharpe Ratio in around 0.10, when compared to the deterministic ones. Also, while the (R)STU investor seems to outperform (R)PTU, it doesn't happen with (R)CPT.

If we consider just the total return obtained by each strategy, we find the results presented in Table III. In this case, it's possible to infer an inefficiency cost related to the behavioral investors, who tend to underperform the results of the standard utility investor in around 10 bps[11]. However if take into account the increment in risk (a risk adjusted measure like Shape Ratio), the inefficiency disappears.

Table III

Based on the previous results, we can state that resampled models tend to outperform traditional models. Also, there is no clear advantage of standard utility investors over behavioral prospect theory investors at least to the CPT investor. Levy and Levy [2004] reached a similar result, positing that the practical differences between prospect theory and traditional mean-variance theory are minor. In this sense, behavioral biases should not be moderated, nor should standard models be adapted to include behavioral biases.

When we take into account each market separately, we find the results presented in Table IV (in each country's currency). Considering each country individually, there's no clear dominance of a single strategy. Resampled models tend to outperform traditional models in emerging markets (observe the results for Brazil, Chile, South Africa, South Korea, Taiwan, Thailand and Turkey), where the uncertainty over the risk/return estimation is higher.

Table IV

In terms of the comparison between the standard and the prospect utility investor, generally the former doesn't outperform the latter, indicating no clear dominance of the traditional rational model. In this sense, there is no need for moderating the behavioral biases as described by prospect theory, as no extra financial efficiency is gained.

Generally speaking, an interesting finding is the fact that all previous allocation models outperform the 100% risky strategy. The Sharpe Ratio of the 100% stock strategy was 0.383 while all resampled models reached, on average, a result above 0.509[12].

Finally, if we take into account the values in USD and so considering that the investor is facing foreign exchange risk, we reach the results presented in Table V.

Table V

Again, the results indicate a dominance of resampled models in emerging markets, while for developed countries, no clear dominance can be seen. The traditional rational model does not outperform the behavioral ones. Finally, all six dynamic models add value for the investor when compared to a 100% stock invested individual. Observe that the Sharpe Ratio found for the different markets (both in the country's currency and in USD) are notably higher than the ones presented in Table 1.

Summing up, resampled models, which take into account estimation risk, tend to outperform deterministic models, notably for emerging markets where the uncertainty of the expected return estimation is higher. Moreover, prospect theory utility investors don't reach worse returns if compared to the traditional rational ones, which indicates no need for addressing bias moderation in the portfolio allocation.

C. Conclusions

This study had two objectives: first to incorporate mental accounting, loss-aversion, asymmetric risk-taking behavior, and probability weighting in portfolio optimization for individual investors; and second to take into account the estimation risk in the analysis.

Considering daily index stock data from 26 countries over the period from 1995 to 2007, we empirically evaluated our model (BRATE – Behavior Resample Adjusted Technique) against the traditional Markowitz. Several estimation and evaluation periods were used and we also considered a foreign exchange hedged and an unhedged strategy.

Our results support the use of BRATE as an alternative for defining optimal asset allocation and posit that a portfolio optimization model may be adapted to the individual biases implied in prospect theory. Behavioral biases don't seem to reduce efficiency when we consider a dynamic setting. This result is robust for different developed and emerging markets. Also, the previous optimization models add value for the individual investor when compared to a naive 100% risky strategy.

As further extensions of the present research, we suggest the inclusion of several risky assets in the analysis. In this case, the issue of multiple mental accounting is a crucial issue to address the problem. An investor who evaluates every security in their own mental account will not necessarily view additional securities as redundant, which dramatically increases the complexity of the problem.

We also leave unanswered the question of how individuals arrive at the underlying return distribution. That is the model above is a proposed mechanism for how individuals might transform a given probability distribution (assumed to be an accurate representation of the underlying distribution) into decision weights. Once we introduce uncertainty, it can induce individual biases, subjectivity and error. There is evidence that people display considerable overconfidence when asked to provide a subjective assessment of a probability distribution[13]. Moreover, it is questionable whether the weightings provided by CPT truly reflect the process by which individuals evaluate continuous probability distributions.

Another suggestion is an analysis if the Sharpe Ratio is an appropriate performance measure when considering behavioral investors. Is the volatility capturing all the relevant risk for the individual behavioral investor ? The consideration of estimation error in the Sharpe Ratio estimation is also left for a further research.

The agent who measures his gains and losses always relative to his actual wealth solves the same maximization problem each period, therefore selecting a fix-mix strategy. An open question remains, if a fix-mix strategy, where the investor set a fixed proportion of stocks and bonds for his portfolio, can be the cause of the disposition effect.

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Footnotes:

1. Tversky and Kahneman's Cumulative Prospect Theory (CPT) [1992] combines the concepts of loss-aversion and a non linear rank dependent weighting of probability assessments.
2. Experiments suggest a value of γ between 0.80 and 0.90 (Tversky and Kahneman, 1992).
3. Under Cumulated Prospect Theory (CPT) with Tversky and Kahneman [1992] specifications, equilibria do not exist as at least one investor can infinitely increase his utility by infinitely leveraging the market portfolio (the utility index is almost linear for large stakes), while the Security Market Line Theorem holds (Giorgi et al. , 2004).
4. We will consider the investor's initial wealth equals to 1.
5. This last derivation is valid for the case where $\gamma = 1$.
6. See Appendix for the proofs.
7. The riskfree rate, the expected return of the risky asset and the volatility of the risky asset were calculated, using daily data, over the period from 1995 and 2007. The results were annualized.
8. Davies and Satchell [2004] found that the average proportion in domestic and foreign equities of large pension funds in 1993 was 83% in the UK, which is in line with the prospect theory results.
9. This methodology is an adaptation of the one proposed in Michaud [1998].
10. The only exception is Thailand where the Sharpe Ratio is negative (-0.109).
11. 1 bps = 0.01%.
12. A t-test over the Sharpe Ratio differences offered a significant result with a p-value of 0.0001.
13. Their subjective distribution is too tightly centered on their estimated mean.

Appendix : Proofs of the Value Function Properties

We want to prove that the following property hold:

i) $\dfrac{\partial V}{\partial \mu} > 0$;

The partial derivative of V (Eq. 06) is given by:

$$\frac{\partial V}{\partial \mu} = \{\alpha e^{\frac{1}{2}\alpha^2(\theta_0\sigma)^2}[\lambda^- e^{\alpha[(1-\theta_0)R_f+\theta_0\mu]}\pi\left(-\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}-\alpha\theta_0\sigma\right)+$$
$$\lambda^+ e^{-\alpha[(1-\theta_0)R_f+\theta_0\mu]}\pi\left(\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}-\alpha\theta_0\sigma\right)]\}\cdot\theta_0 \qquad \text{(Eq. 08)}$$

as

$$[\lambda^- e^{\alpha[(1-\theta_0)R_f+\theta_0\mu]}\pi\left(-\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}-\alpha\theta_0\sigma\right)+$$
$$\lambda^+ e^{-\alpha[(1-\theta_0)R_f+\theta_0\mu]}\pi\left(\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}-\alpha\theta_0\sigma\right)] > 0 \qquad \forall \quad \mu$$

so,

$$\frac{\partial V}{\partial \mu} > 0$$

Now, let's prove properties (ii) and (iii)

ii) $\dfrac{\partial V}{\partial \sigma} = 0$ for $\sigma = 0$ or $\sigma = \infty$;

iii) $\dfrac{\partial V}{\partial \sigma} < 0$ for $\sigma > 0$.

The partial derivative of V (Eq. 06) is given by:

$$\frac{\partial V}{\partial \sigma} = \{\alpha^2\theta_0\sigma e^{\frac{1}{2}\alpha^2(\theta_0\sigma)^2}[\lambda^- e^{\alpha`((1-\theta_0)R_f+\theta_0\mu)}\pi(-\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}-\alpha\theta_0\sigma)-$$
$$\lambda^+ e^{-\alpha([(1-\theta_0)R_f+\theta_0\mu])}\pi(\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}-\alpha\theta_0\sigma)]-\alpha(\lambda^- - \lambda^+)\pi'\left(\frac{[(1-\theta_0)R_f+\theta_0\mu]}{\theta_0\sigma}\right)\}\cdot\theta_0$$

It follows:

$\partial_\sigma V(\mu,0) = 0$ using that $\pi(-\infty) = 0, \pi(\infty) = 1$ and $\pi'(\infty) = 0$

Let us consider $f(\mu,\sigma) = \theta_0^{-1}\sigma^{-1}e^{-\frac{1}{2}\alpha^2(\theta_0\sigma)^2}e^{-\alpha[(1-\theta_0)R_f+\theta_0\mu]}\partial_\sigma \mathrm{V}(\mu,\sigma)$ for σ > 0. We show that $f(\mu,\sigma) < 0$ **.**

Suppose that for some $\mu*$ and $\sigma(\mu*)>0$, $f(\mu,\sigma(\mu*))>0$. Since $f(\mu,\cdot)$ is continuous, $\lim_{\sigma\to 0} f(\mu,\sigma) = -\lambda^{+}\ \mathrm{e}^{-2\alpha\left[(1-\theta_0)R_f+\theta_0\mu\right]} < 0$ and $\lim_{\sigma\to\infty} f(\mu,\sigma)=0$ for all μ > 0, we can assume without loss of generality that $\sigma(\mu*)>0$ is a local maxima of $f(\mu*,\cdot)$. We compute the partial derivative of f with respect to σ. We have

$$\partial_\sigma f(\mu,\sigma) = (\pi'\left(\frac{\left[(1-\theta_0)R_f+\theta_0\mu\right]}{\theta_0\sigma}+\alpha\theta_0\sigma\right)`(\lambda^{-}\left(\left[(1-\theta_0)R_f+\theta_0\mu\right](\theta_0\sigma)^{-2}-\alpha\right)+$$
$$+\lambda^{+}\left(\left[(1-\theta_0)R_f+\theta_0\mu\right](\theta_0\sigma)^{-2}+\alpha\right)-\alpha^{-1}\left(\lambda^{-}-\lambda^{+}\right)\left(\left[(1-\theta_0)R_f+\theta_0\mu\right]^2(\theta_0\sigma)^{-4}-\alpha^2-(\theta_0\sigma)^{-2}\right))\theta_0$$

Let $\eta=(\theta_0\sigma)^{-2}$ then

$$\partial_\sigma f(\mu,\sigma)=0 \Leftrightarrow \eta\left[-\frac{\lambda^{-}-\lambda^{+}}{\alpha}\left[(1-\theta_0)R_f+\theta_0\mu\right]^2\eta+\left(\lambda^{-}+\lambda^{+}\right)\left[(1-\theta_0)R_f+\theta_0\mu\right]+\frac{\lambda^{-}-\lambda^{+}}{\alpha}\right]=0$$
$$\Leftrightarrow \eta\in\{0,\eta*(\mu)\}$$

where $\eta*(\mu)=\dfrac{\alpha\left[(1-\theta_0)R_f+\theta_0\mu\right]\left(\lambda^{-}+\lambda^{+}\right)+\left(\lambda^{-}-\lambda^{+}\right)}{\left(\lambda^{-}-\lambda^{+}\right)\left[(1-\theta_0)R_f+\theta_0\mu\right]^2}$. Moreover, for $\eta>\eta*(\mu),\partial_\sigma f(\mu,\sigma)<0$ and for $0<\eta<\eta*(\mu),\partial_\sigma f(\mu,\sigma)>0$. It follows that $\sigma*(\mu)=\dfrac{\eta*(\mu)^{1/2}}{\theta_0}>0$ is the unique maximum/minimum of $f(\mu,\cdot)$ and since for $\sigma>\sigma*(\mu)$, $\partial_\sigma f(\mu,\sigma)>0$ and for $0<\sigma<\sigma*(\mu),\partial_\sigma f(\mu,\sigma)<0,\sigma*(\mu)$ is a minimum. This contradicts the existence of $\mu*$ and $\sigma(\mu*)$ local maxima of $f(\mu*,\cdot)$ such that $f(\mu*,\sigma(\mu*))>0$. Hence, $f(\mu,\sigma)<0$ and therefore $\partial_\sigma V(\mu,\sigma)<0$.

Also,

$\lim_{\sigma\to\infty}\partial_\sigma f(\mu,\sigma)=0$ for $\mu>0$ since

$$\lim_{\sigma\to\infty}\left((\theta_0\sigma)e^{\frac{1}{2}(\theta_0\sigma)^2\alpha^2}e^{\alpha\left[(1-\theta_0)R_f+\theta_0\mu\right]}\pi\left(-\frac{\left[(1-\theta_0)R_f+\theta_0\mu\right]}{\sigma}-\alpha(\theta_0\sigma)\right)\right)=$$
$$\lim_{\sigma\to\infty}\left((\theta_0\sigma)e^{\frac{1}{2}(\theta_0\sigma)^2\alpha^2}e^{-\alpha\left[(1-\theta_0)R_f+\theta_0\mu\right]}\pi\left(\frac{\left[(1-\theta_0)R_f+\theta_0\mu\right]}{\sigma}-\alpha(\theta_0\sigma)\right)\right)=\frac{1}{\alpha\sqrt{2\pi}}$$

And

$$\lim_{\sigma\to\infty}\pi'\left(\frac{\left[(1-\theta_0)R_f+\theta_0\mu\right]}{(\theta_0\sigma)}\right)=\frac{1}{\sqrt{2\pi}}$$

Tables

Table I – Descriptive Statistics

This Table provides descriptive statistics for the sample of world markets. For each market we present, the average risk free rate, the mean, standard deviation, skewness, and kurtosis of stock returns, as well as the Sharpe Ratio (annualized values). The values are presented in the countries' currency and also in USD. The risk free rate used to calculate the Sharpe Ratio in USD is the 3 month UST Bill rate for all markets.

	Currency						USD				
	Risk Free	Mean	Std.	Skew	Kurt	Sharpe Ratio	Mean	Std.	Skew	Kurt	Sharpe Ratio
T-Bill 3 month	2.722	2.722	0.076	-0.574	1.787	0.000	2.722	0.076	-0.574	1.787	0.000
'Australia'	3.856	8.971	13.022	-0.322	6.685	0.392	10.105	17.275	-0.125	6.389	0.428
'Austria'	1.537	11.844	15.790	-0.574	7.277	0.652	12.020	17.816	-0.319	5.626	0.522
'Belgium'	2.218	10.811	18.065	0.317	9.921	0.476	10.886	19.295	0.163	7.275	0.423
'Brazil'	16.531	23.184	30.022	0.972	23.667	0.222	17.590	34.552	0.035	8.387	0.430
'Canada'	2.722	11.516	16.686	-0.426	8.866	0.527	13.230	18.337	-0.532	8.046	0.573
'Chile'	2.470	7.636	15.309	0.166	7.188	0.337	5.670	18.370	-0.067	6.509	0.161
'Denmark'	2.218	14.767	17.256	-0.321	5.778	0.727	14.540	17.973	-0.281	5.292	0.657
'Finland'	2.293	21.269	37.629	-0.162	9.041	0.504	21.118	37.650	-0.101	9.202	0.488
'France'	2.243	11.516	20.800	-0.048	5.926	0.447	11.416	21.048	-0.012	5.332	0.413
'Germany'	2.772	10.987	23.172	-0.138	6.244	0.355	10.861	23.232	-0.097	5.337	0.350
'Ireland'	2.848	10.282	17.956	-0.528	8.877	0.414	10.458	19.676	-0.304	6.763	0.392
'Italy'	2.974	10.710	20.213	-0.064	6.000	0.383	10.660	20.726	-0.032	5.237	0.383
'Japan'	0.151	4.536	19.215	0.051	5.152	0.229	2.570	22.234	0.332	6.647	-0.008
'Netherlands'	2.092	10.156	21.489	-0.076	7.018	0.375	10.004	21.551	-0.006	6.177	0.338
'Norway'	3.326	12.121	19.635	-0.304	6.706	0.449	12.172	20.767	-0.318	7.104	0.454
'Portugal'	2.923	9.727	15.801	-0.261	8.097	0.430	9.878	17.664	-0.051	5.834	0.405
'SouthAfrica'	7.938	12.625	19.769	-0.437	9.002	0.237	8.039	24.810	-0.429	7.053	0.214
'SouthKorea'	2.318	12.676	34.524	0.271	6.664	0.300	13.709	41.728	1.336	26.151	0.263
'Spain'	2.797	16.405	21.118	-0.078	6.249	0.644	16.405	21.781	0.031	5.682	0.628
'Sweden'	2.696	15.473	24.896	0.187	6.700	0.513	16.380	26.850	0.120	6.322	0.509
'Switzerland'	1.058	12.197	18.051	-0.106	7.639	0.617	11.441	17.713	0.010	6.549	0.492
'Taiwan'	3.251	4.687	26.244	0.149	5.442	0.054	3.150	27.563	0.110	5.505	0.016
'Thailand'	3.654	0.076	32.851	1.415	17.779	-0.109	-1.865	36.002	0.984	13.281	-0.128
'Turkey'	39.514	47.804	45.171	0.324	8.017	0.184	21.521	50.887	0.219	8.094	0.369
UnitedKingdom'	3.704	6.779	16.476	-0.153	6.225	0.187	8.392	17.037	-0.100	5.213	0.332
'UnitedStates'	2.722	9.904	16.978	-0.024	6.598	0.422	9.904	16.978	-0.024	6.598	0.422
World Index	2.722	7.610	12.976	-0.144	5.763	0.376	7.610	12.976	-0.144	5.763	0.376

Table II – Sharpe Ratios

This Table presents the Sharpe Ratio of the efficient portfolio generated by each estimation model. The Sharpe Ratio is calculated by dividing the excess return observed by the standard deviation.

	STU	PTU	CPT	RSTU	BRATEa	BRATEb
6m-2m	0.189	0.134	0.136	0.207	0.154	0.156
6m-6m	0.101	0.080	0.083	0.125	0.102	0.114
1y-6m	0.439	0.392	0.392	0.438	0.400	0.401
2y-6m	0.462	0.426	0.421	0.464	0.434	0.423
4y-6m	-0.135	-0.023	-0.023	-0.122	-0.018	-0.019
1y-1y	0.413	0.347	0.389	0.420	0.354	0.393
2y-1y	0.456	0.428	0.431	0.461	0.444	0.465
4y-1y	-0.206	-0.126	-0.126	-0.193	-0.114	-0.113
mean	0.215	0.207	0.213	0.225	0.219	0.227

Table III – Average Total Return

This Table presents the Average Total Return of the efficient portfolio generated by each estimation model.

	STU	PTU	CPT	RSTU	BRATEa	BRATEb
6m-2m	4.302	3.781	3.822	4.447	3.935	3.976
6m-6m	3.654	3.449	3.476	3.883	3.643	3.749
1y-6m	6.670	6.211	6.211	6.632	6.238	6.242
2y-6m	7.377	6.987	6.875	7.345	6.910	6.775
4y-6m	1.065	2.083	2.083	1.064	1.992	1.993
1y-1y	6.711	5.981	6.295	6.630	5.979	6.341
2y-1y	7.247	6.935	6.966	7.289	7.068	7.194
4y-1y	0.419	1.226	1.226	0.530	1.287	1.297
mean	4.681	4.582	4.619	4.727	4.631	4.696

Table IV – Sharpe Ratios

This Table presents the Sharpe Ratio of the efficient portfolio generated considering an estimation period of 1 year and evaluation period of 6 months (in each country's currency). The Sharpe Ratio is calculated by dividing the excess return observed by the standard deviation.

	STU	PTU	CPT	RSTU	BRATEa	BRATEb
'Australia'	0.309	0.352	0.353	0.309	0.346	0.345
'Austria'	0.629	0.578	0.584	0.618	0.593	0.597
'Belgium'	0.977	0.982	0.973	0.982	0.986	0.977
'Brazil'	0.323	0.326	0.304	0.335	0.333	0.317
'Canada'	0.490	0.490	0.490	0.490	0.488	0.489
'Chile'	0.729	0.726	0.721	0.735	0.740	0.736
'Denmark'	0.914	0.914	0.914	0.908	0.910	0.909
'Finland'	0.696	0.685	0.638	0.691	0.658	0.665
'France'	0.778	0.790	0.755	0.780	0.785	0.764
'Germany'	0.619	0.614	0.619	0.619	0.616	0.616
'Ireland'	0.615	0.607	0.636	0.626	0.607	0.634
'Italy'	0.737	0.769	0.740	0.733	0.753	0.726
'Japan'	0.041	0.080	0.042	0.057	0.051	0.040
'Netherlands'	0.657	0.655	0.657	0.657	0.654	0.655
'Norway'	0.389	0.368	0.368	0.402	0.398	0.398
'Portugal'	0.751	0.685	0.728	0.764	0.716	0.738
'SouthAfrica'	0.161	0.206	0.218	0.167	0.208	0.224
'SouthKorea'	0.101	0.019	0.035	0.111	0.066	0.058
'Spain'	0.932	0.949	0.954	0.930	0.936	0.936
'Sweden'	0.634	0.631	0.631	0.643	0.634	0.633
'Switzerland'	0.773	0.720	0.739	0.773	0.739	0.748
'Taiwan'	-0.001	-0.003	0.000	-0.004	-0.005	-0.004
'Thailand'	0.041	-0.012	-0.048	0.055	0.033	0.018
'Turkey'	0.183	0.189	0.094	0.185	0.190	0.103
UnitedKingdom'	0.411	0.428	0.423	0.411	0.429	0.426
'UnitedStates'	0.618	0.624	0.626	0.615	0.623	0.616
World Index'	0.439	0.392	0.392	0.438	0.400	0.401

Table V – Sharpe Ratios

This Table presents the Sharpe Ratio of the efficient portfolio generated considering an estimation period of 1 year and evaluation period of 6 months (values in USD). The Sharpe Ratio is calculated by dividing the excess return observed by the standard deviation.

	STU	PTU	CPT	RSTU	BRATEa	BRATEb
'Australia'	0.589	0.567	0.564	0.591	0.595	0.591
'Austria'	0.896	1.015	1.015	0.882	0.971	0.977
'Belgium'	0.904	0.904	0.904	0.886	0.886	0.899
'Brazil'	0.656	0.653	0.653	0.669	0.675	0.668
'Canada'	0.421	0.421	0.421	0.423	0.420	0.420
'Chile'	0.752	0.757	0.757	0.774	0.776	0.775
'Denmark'	0.781	0.754	0.820	0.776	0.773	0.803
'Finland'	0.612	0.596	0.597	0.613	0.597	0.593
'France'	0.654	0.643	0.625	0.649	0.625	0.629
'Germany'	0.533	0.509	0.521	0.537	0.502	0.516
'Ireland'	0.654	0.593	0.600	0.641	0.620	0.618
'Italy'	0.674	0.614	0.640	0.681	0.638	0.664
'Japan'	0.195	0.181	0.194	0.198	0.175	0.182
'Netherlands'	0.645	0.655	0.656	0.645	0.653	0.654
'Norway'	0.351	0.387	0.387	0.361	0.381	0.380
'Portugal'	0.666	0.641	0.641	0.669	0.647	0.660
'SouthAfrica'	0.465	0.441	0.456	0.479	0.459	0.460
'SouthKorea'	0.226	0.222	0.189	0.233	0.230	0.191
'Spain'	0.858	0.899	0.899	0.862	0.892	0.894
'Sweden'	0.562	0.558	0.566	0.563	0.554	0.565
'Switzerland'	0.599	0.531	0.552	0.596	0.607	0.612
'Taiwan'	-0.090	-0.100	-0.086	-0.083	-0.095	-0.076
'Thailand'	0.104	0.040	0.030	0.114	0.108	0.096
'Turkey'	0.288	0.250	0.238	0.296	0.267	0.246
UnitedKingdom'	0.625	0.574	0.605	0.632	0.597	0.626
'UnitedStates'	0.618	0.624	0.626	0.612	0.615	0.612
World Index'	0.439	0.392	0.392	0.438	0.400	0.401

Figures

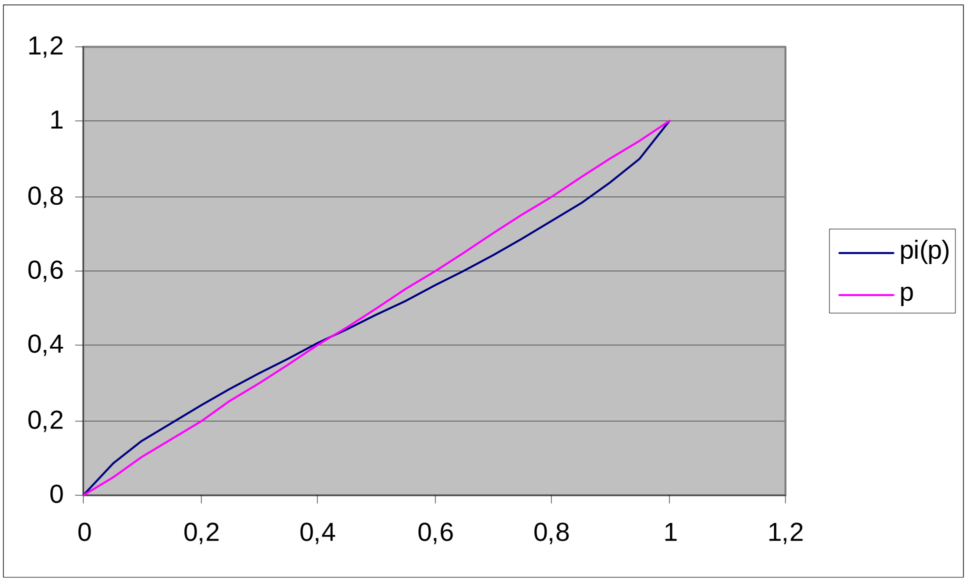

Figure 1 – Cumulative probability weighting function for γ =0.80.

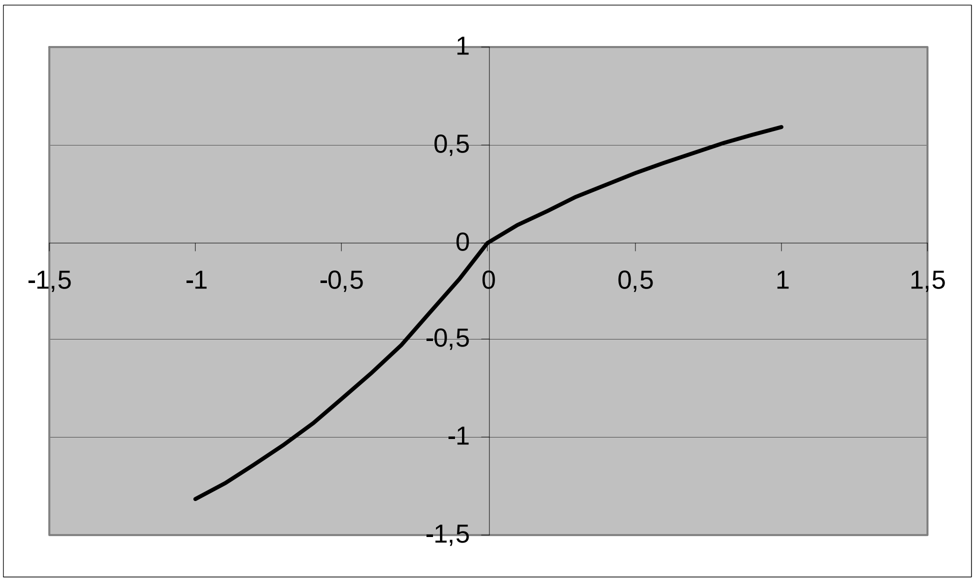

Figure 2 – Prospect theory value function for α = 0.88, λ^{-} = 2.25 and λ^{+} = 1

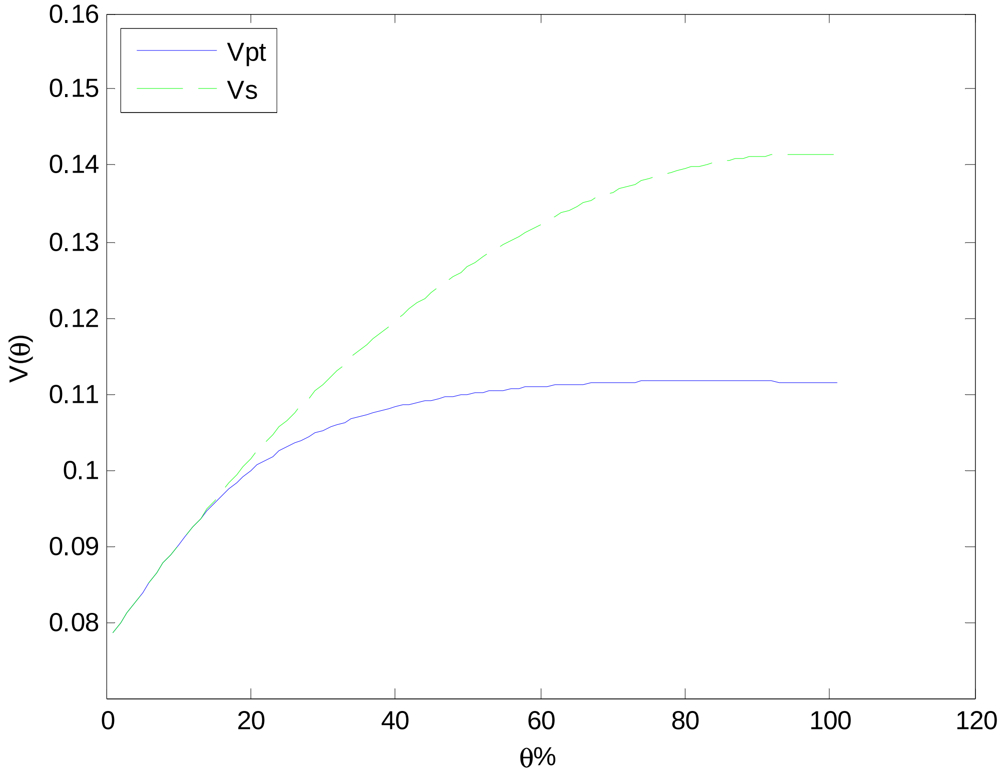

Figure 3 – Prospect value and standard utility value as function of θ

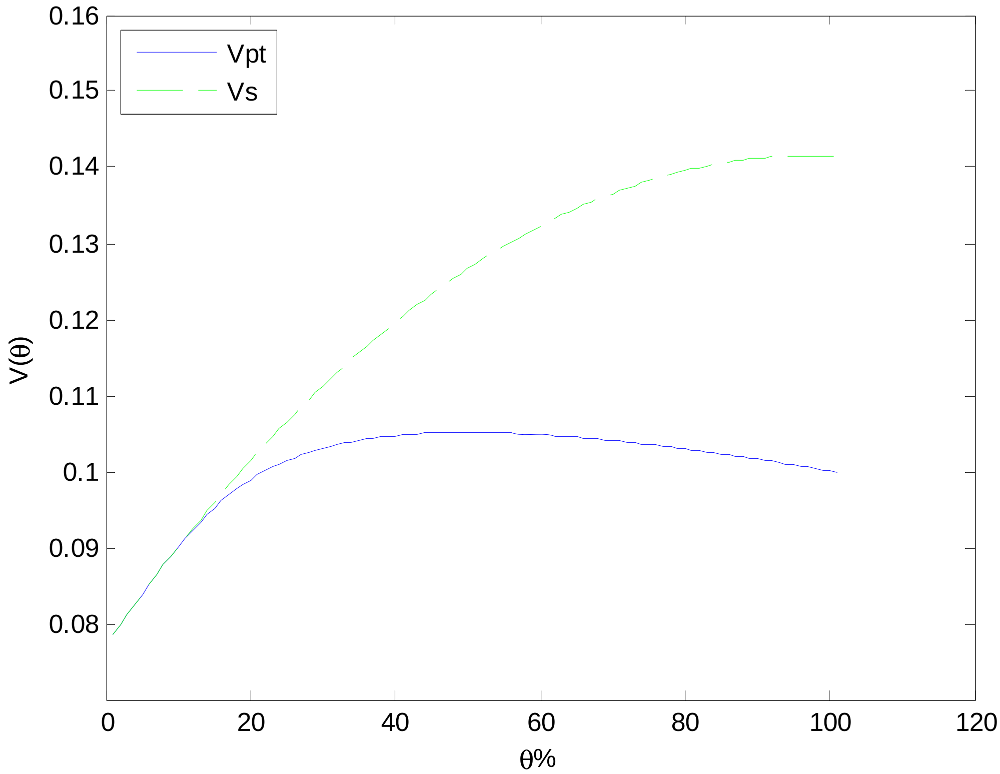

Figure 4 – Prospect value and standard utility value as function of θ

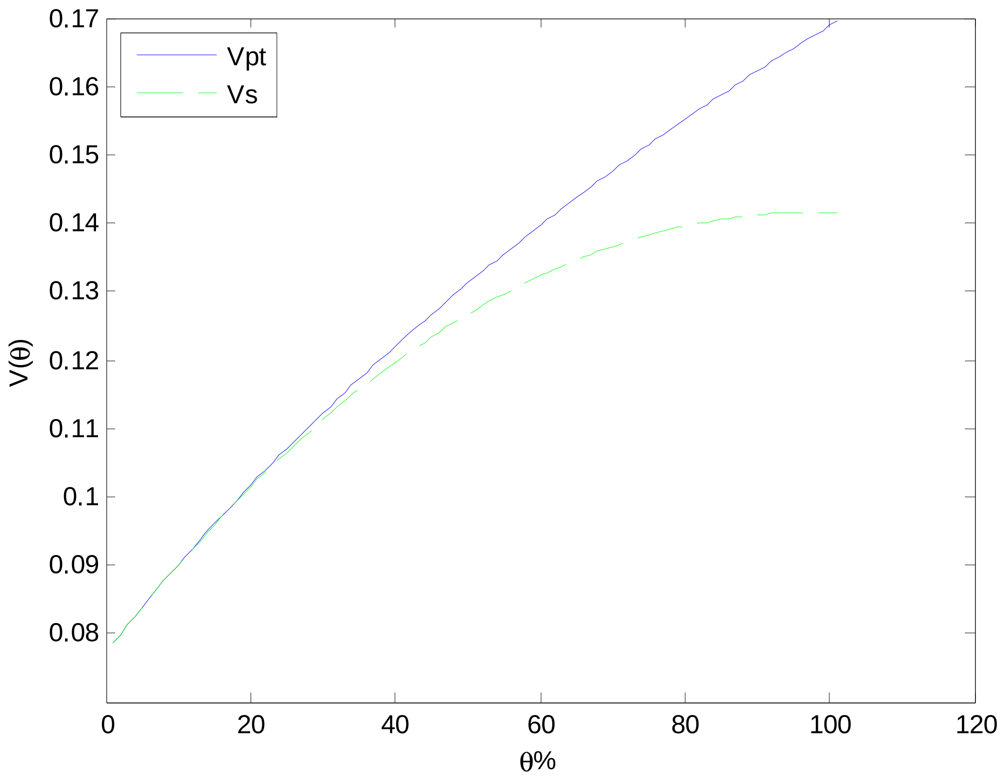

Figure 5 – Prospect value and standard utility value as function of θ

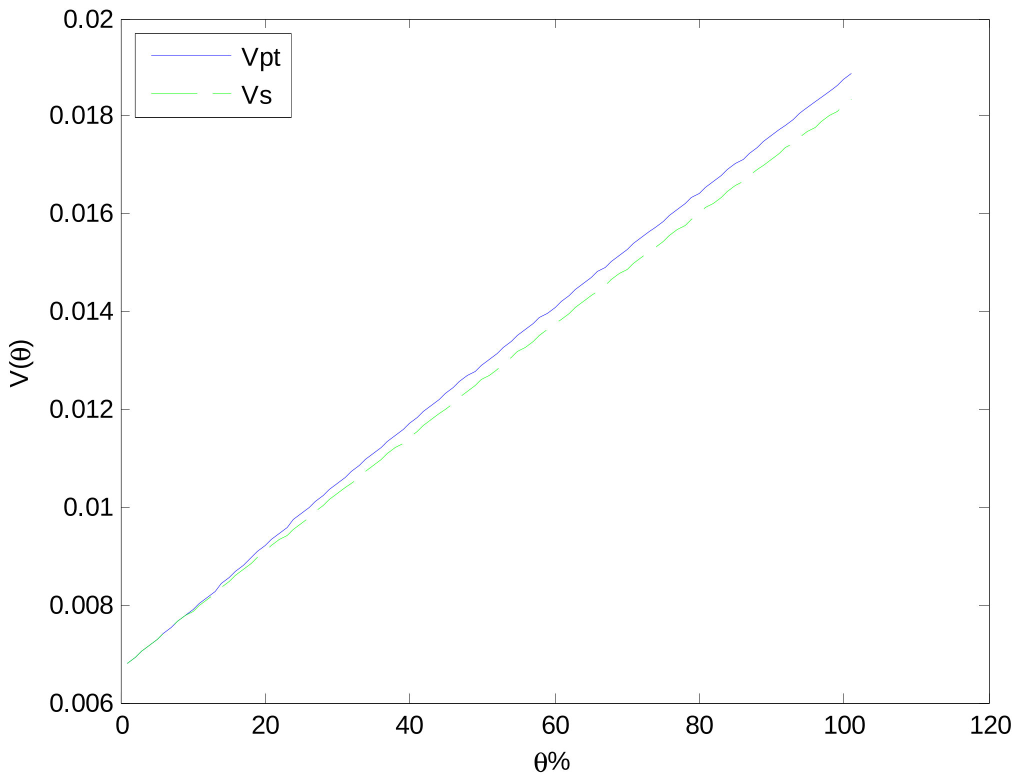

Figure 6 – Prospect value and standard utility value as function of θ

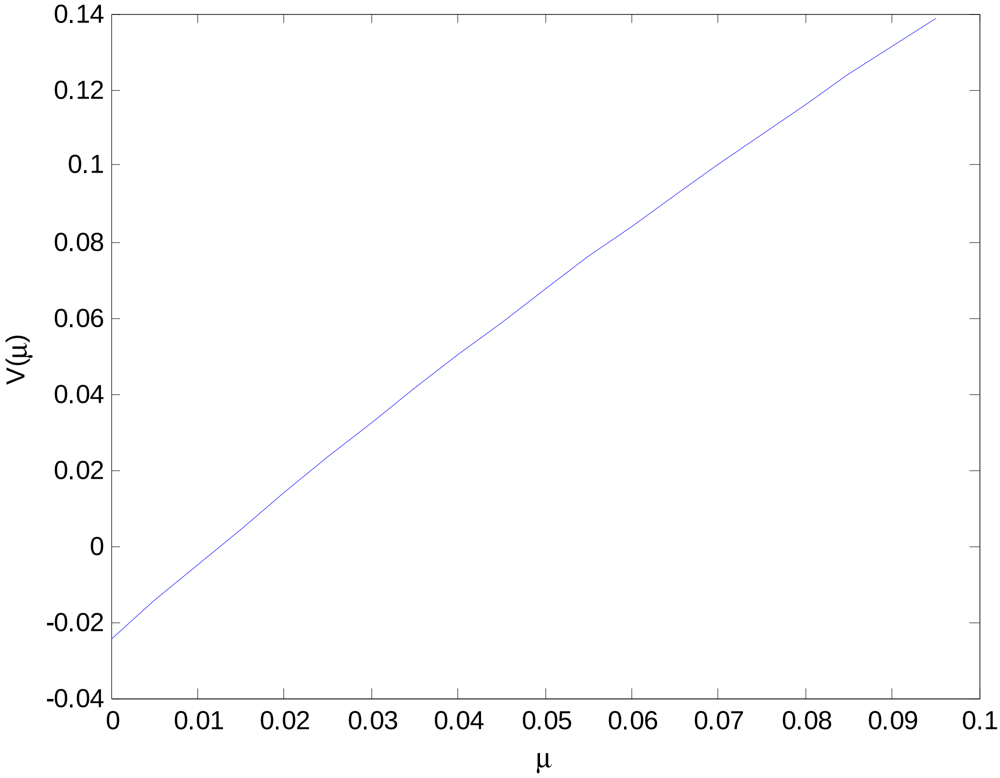

Figure 7 – Prospect value as function of μ

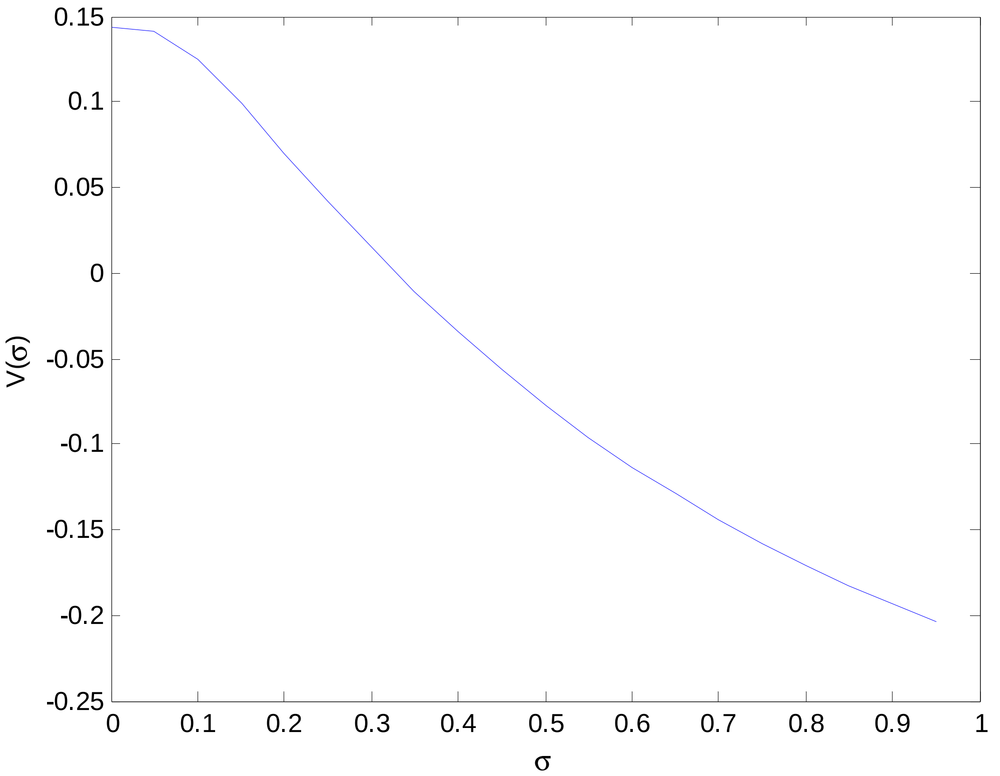

Figure 8 – Prospect value as function of σ

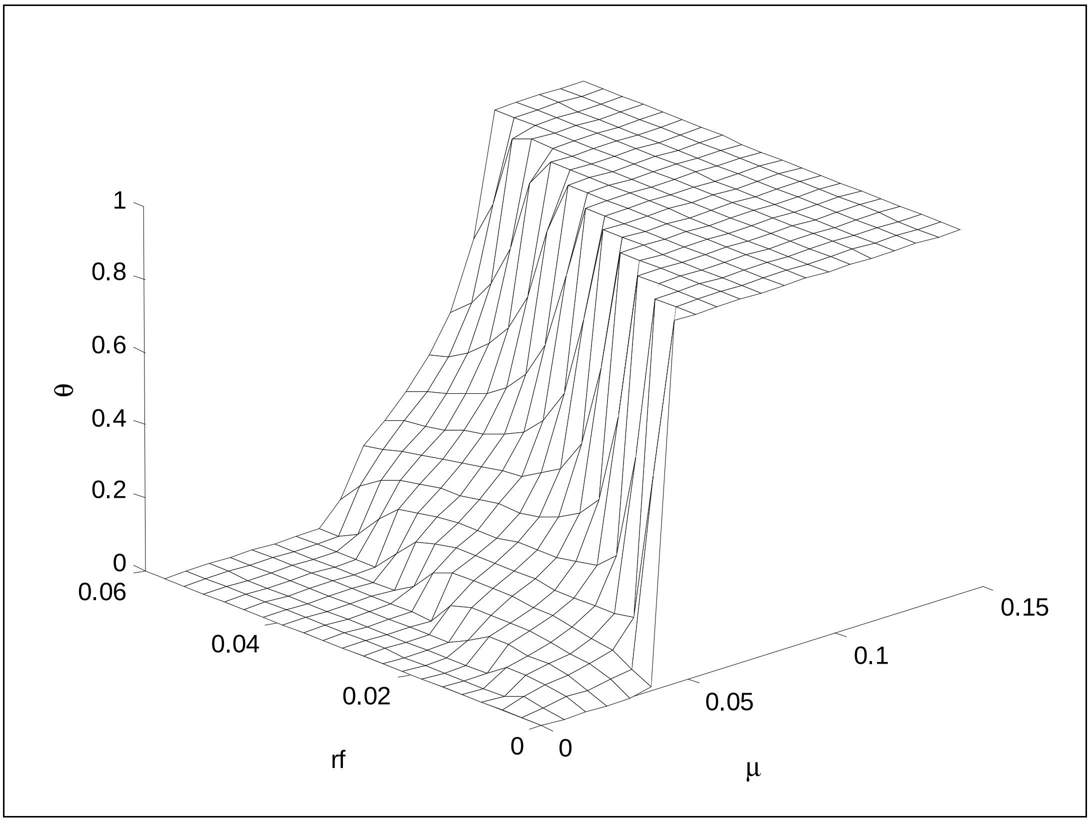

Figure 9 – Optimal equity allocation in the first period as function of μ and rf.

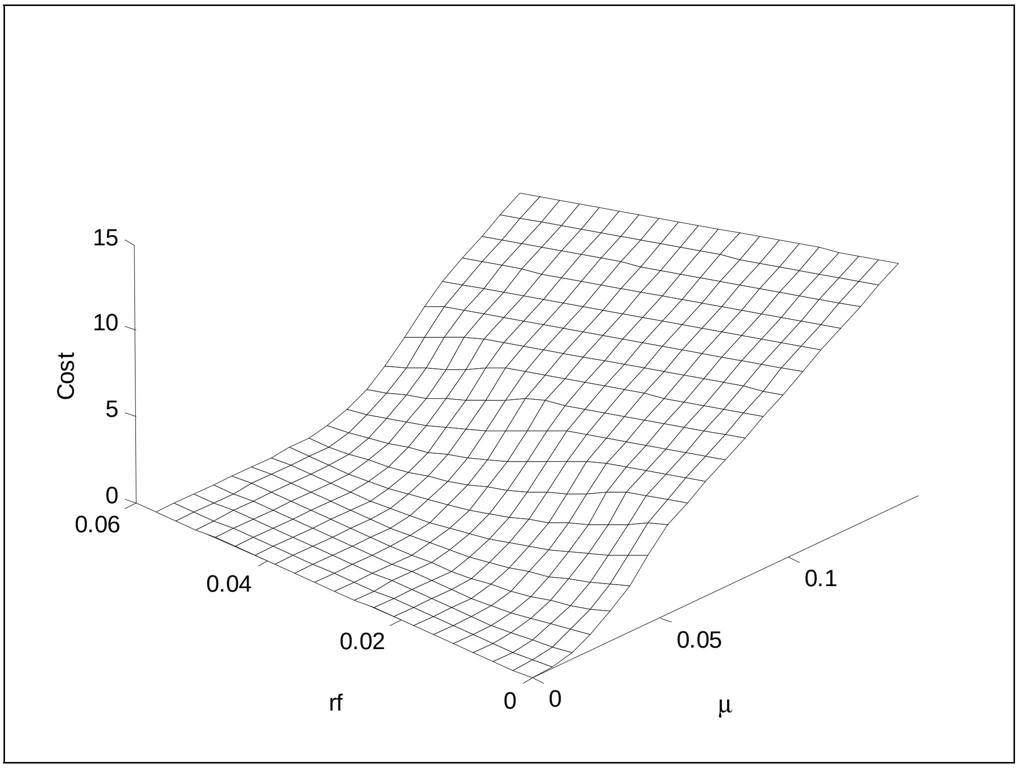

Figure 10 – Expected cost in the first period as function of μ and rf.

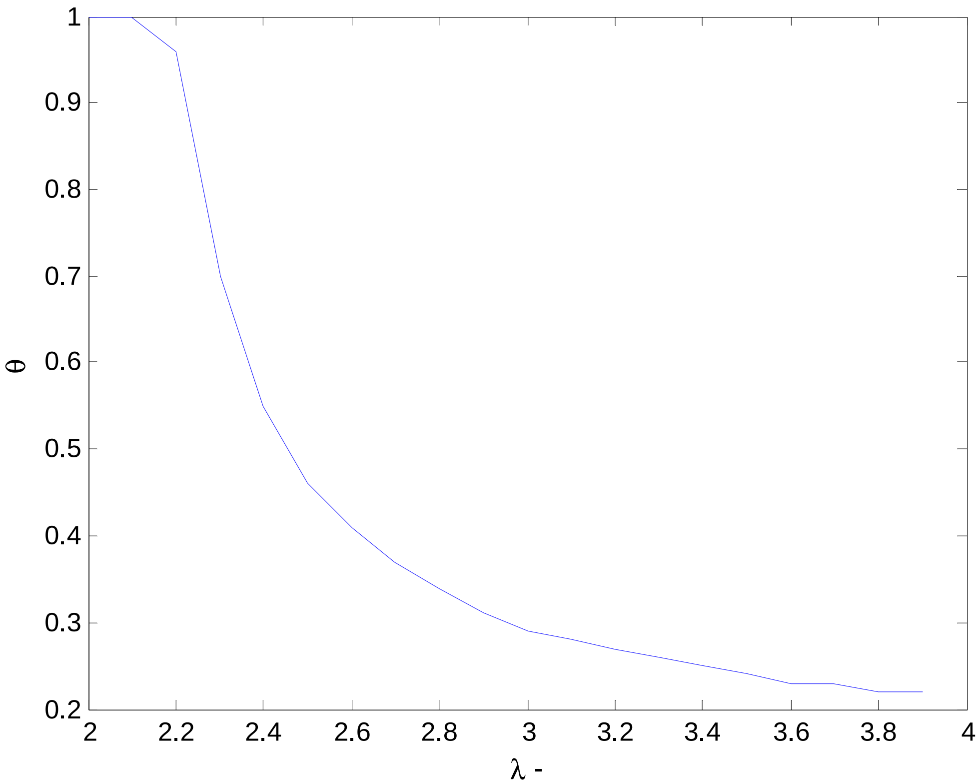

Figure 11 –Optimal equity allocation in the first period as function of λ^{-}.

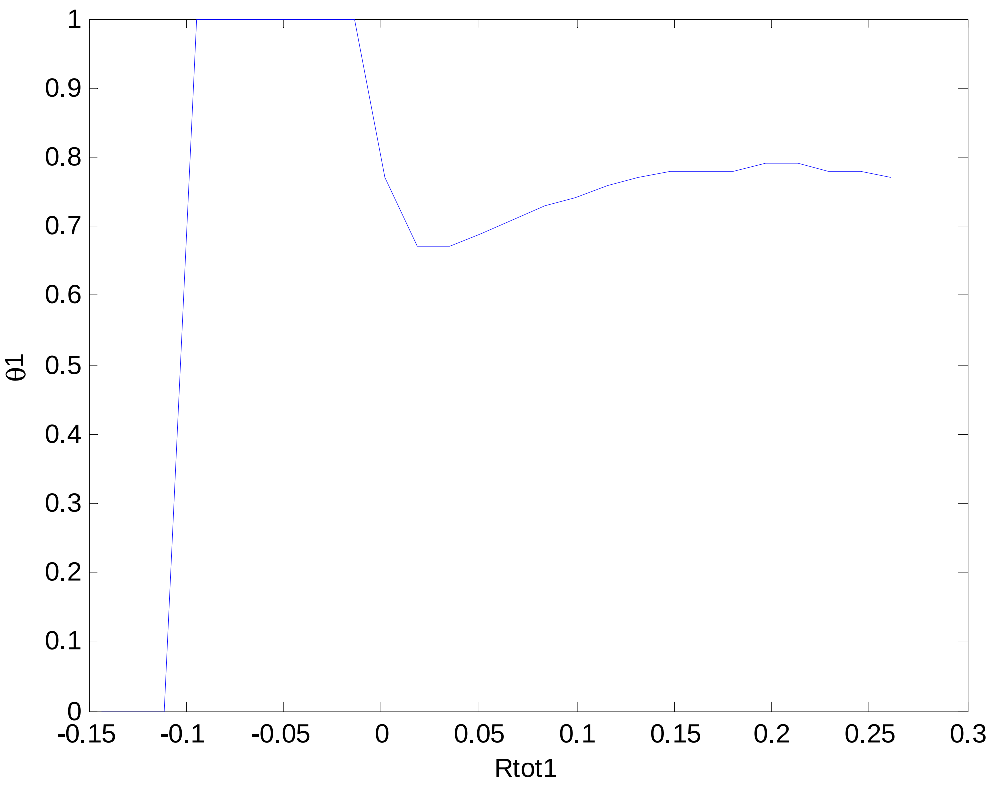

Figure 12 –Optimal equity allocation in the second period as function of the total return obtained in the first period.

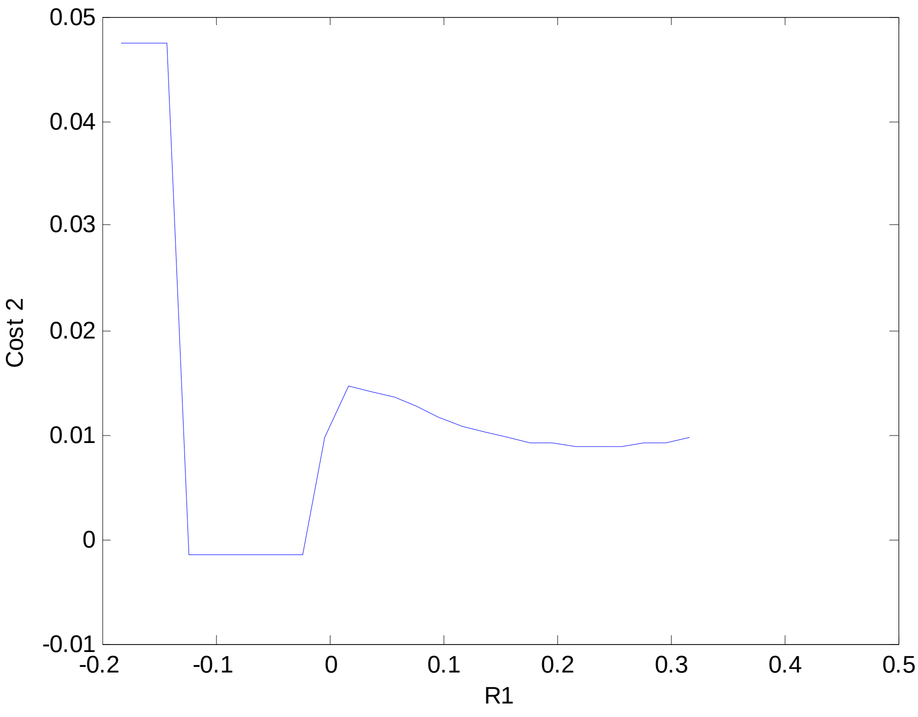

Figure 13 –Expected cost in the second period as function of the equity return obtained in the first period.

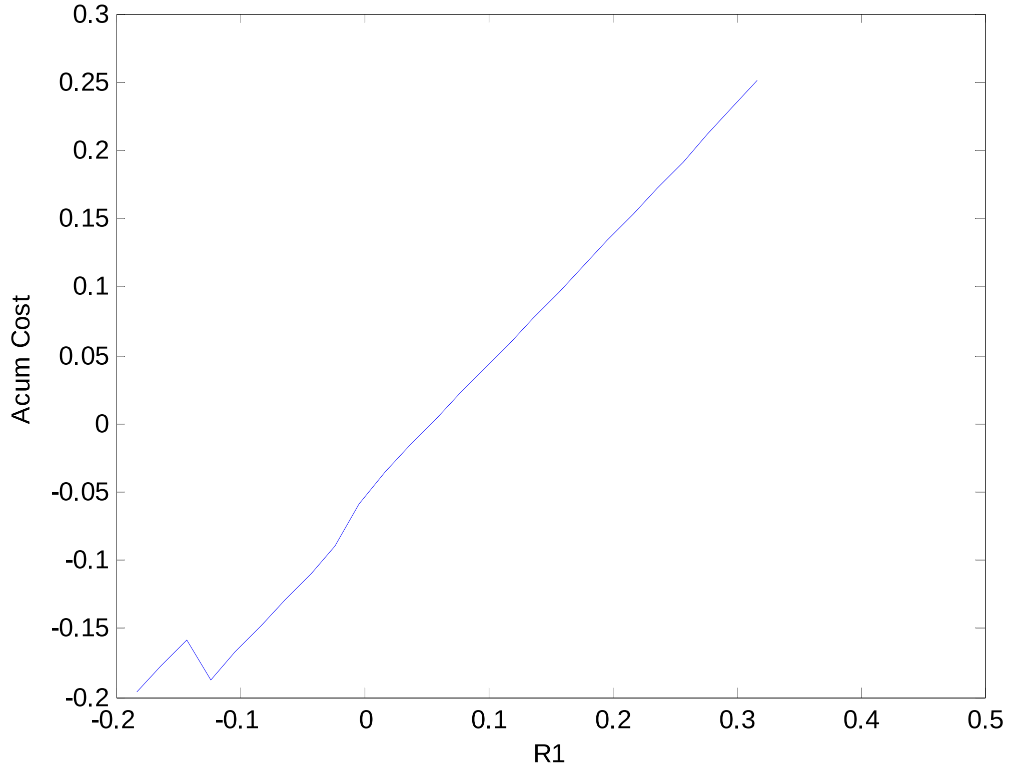

Figure 14 –Expected cumulative cost in the second period as function of the equity return obtained in the first period.

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