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Are Interest Rate Options Important for the Assessment of Interest Rate Risk? *

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Abstract

Fixed income options contain substantial information on the price of interest rate volatility risk. In this paper, we ask if those options will provide information related to other moments of the objective distribution of interest rates. Based on a dynamic term structure model, we find that interest rate options are useful for the identification of interest rate quantiles. A three-factor model with stochastic volatility is adopted and its adequacy to estimate Value at Risk of zero coupon bonds is tested. We find significant difference on the quantitative assessment of risk when options are (or not) included in the estimation process of the dynamic model. Statistical backtests indicate that bond estimated risk is clearly more adequate when options are adopted, although not yet completely satisfactory.

Keywords: Dynamic term structure models, Value at risk, Back-testing procedures, Feller processes.

JEL: C13, G12, G13.

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1 Introduction

It is undeniable that options contain invaluable information on investors preferences and beliefs. Since the work of Breeden and Litzenberger (1978), a large number of researchers have used options to extract Arrow-Debreu prices for a variety of applications in financial economics (see for instance, Rubinstein (1994), Jackwerth and Rubinstein (1996), Ait Sahalia and Lo (1998, 2000), and Jackwerth (2000)).

More recently, some authors have adopted joint datasets of options and underlying assets to better identify risk premium factors in dynamic arbitrage-free models[1]. In such cases, the non-linear richer structure of option payoffs helps to econometrically identify the risk neutral dynamics of underlying assets. A better identification of risk neutral dynamics translates into more precise risk premia dynamics, suggesting that options data in the estimation of a dynamic model decrease the chances of mispricing risks.

Motivated by the idea above, in this paper, we analyze if interest rate options will capture more general probabilistic properties of the objective distribution of interest rates other than its conditional mean. In particular, we ask how helpful **interest rate options** are to determine the **quantiles of the distribution** of interest rates. Knowledge of these quantities can be specially important to assess interest rate risk, implying that our empirical results might be of importance to risk managers and portfolio managers.

In order to perform our tests, based on a dataset of bonds and interest rate options, we estimate two versions of a three-factor Cox et al. (1985) model (hereafter CIR model). The first adopts only bonds data (*bond version*), while the second includes bonds and options data (*option version*) on the estimation process. The multi-factor CIR model was chosen due to its ability to generate conditional probabilities with strong potential to depart from normality (non-central chi-square transition probability). In addition, the stochastic volatility implied by the three dynamic factors increases model ability to react to changes in conditional volatilities of interest rates.

We adopt Value at Risk (VaR) as a metric to analyze the dynamic term structure model[2]. VaR for different confidence levels and time-frequencies is estimated. Its

[1] This is, for instance, the case of Pan (2002) and Eraker (2004), who estimate stochastic volatility jump-diffusion processes based on joint options and equities data to analyze jump and volatility risk premium. See also Chernov and Ghysels (2000) and Johanes et al. (2007) for other papers based on equities data. On a fixed income context, Almeida et al. (2006) show that affine models estimated with joint bonds and interest rate options better predict excess bond returns than models estimated based on only bonds data. Almeida and Vicente (2006) and Joslin (2007) adopt joint datasets of fixed income options and interest rates to correctly identify interest rate volatility risk premium.

[2] Since the Bank of International Settlements (BIS) accord proposed in 1996, regulated banks are required to report VaR and also to control their levels of capital based on VaR. In addition, most financial institutions maintain a daily VaR control on their portfolios (see Duffie and Pan (2001)). However, despite being a very popular risk measure adopted in the financial industry, VaR is not a coherent measure of risk (see Artzner et al. (1999)), which means not preserving subadditivity. On the other hand, in a recent paper Garcia et al. (2007) show that proper conditioning information

statistical accuracy is verified based on tests that control the expected number of violations (Kupiec (1995) and Seiler (2006)) and tests for the independence of violations (Christoffersen (1998) and Ljung-Box (1978)). Considering the number of VaR violations, the *bond version* clearly overestimates risk. On the other hand, the performance of the *option version* is comparable to that of two popular benchmarks, namely, two historical simulations based on respectively 125 and 250 observations in a moving-window. In what regards independence tests, both versions of the model perform well but with performances clearly attributed to distinct reasons. The *option version* provides acceptable results when tested via Christoffersen's (1998) or Ljung-Box's (1978) statistics. On its turn, the *bond version* presents even better results on the very same tests, but to the extent that it generates an unacceptably small number of violations. Such small number of violations would, at first, invalidate its use as a risk management tool.

Results reported above indicate that interest rate options aggregate relevant information for the process of interest rate risk management. The implications here will be important for both capital allocation and internal risk control purposes. For instance, overestimation of risk directly implies an excessive capital allocation as collateral for trades. It may also mislead possible analysis of relative performance of traders [3].

Papers related to our work include Ait Sahalia and Lo (2000), and Egorov et al. (2006). Ait Sahalia and Lo (2000) propose an alternative way to estimate VaR, what they call *economic VaR*, which consists in obtaining the quantiles of the risk neutral distribution of returns extracted from option prices. Our approach consists in combining flavors of traditional VaR and economic VaR, in the sense that we estimate VaR under the objective probability measure but making use of information coming from option prices to estimate the dynamic model. Egorov et al. (2006) test the ability of affine models to forecast out-of-sample conditional probability densities of bond yields. Note that while we include options on the estimation process to test their importance on interest rate quantile forecasting, Egorov et al. (2006) adopt only bonds data to estimate the dynamic affine models on their forecasting exercises.

The rest of the paper is organized as follows. Section 2 describes the dataset adopted. Section 3 presents a theoretical description of the dynamic term structure model implemented. Section 4 introduces the VaR measure and all statistical backtesting procedures adopted. On Section 5, the results on the implementation of the CIR models are detailed and a comparison between the two versions is performed. Section 6 offers concluding remarks.

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may restore VaR subadditivity.

[3] Consider when the VaR of a particular instrument is overestimated. In this case, risk managers could incorrectly conclude that fixed income traders buying that product are violating required risk limits. Relative performance of traders operating in distinct markets would also be biased in cases like that.

2 Data and Market Description

We adopt data on bonds and interest rate options from one of the largest fixed income markets among all emerging economies, the Brazilian market. This particular data can potentially offer interesting insights due to at least two reasons. First, the payoff structure of its interest rate option is rather appropriate to analyze how volatility expectations may affect bond risk estimation. In this market, a call option gives the right to cap the accumulated short-term rate between its trade and expiration date. Therefore, it represents a bet on the expectation of the short-rate path directly determining a value for the *expected volatility* for this short-rate path[4]. A second reason comes from an econometric point: In emerging markets, volatility changes due to macroeconomic and political events usually happen in a much higher frequency than in developed markets. Thus, using options data from emerging economies we might better econometrically identify the transmission channel from volatility information to the probability distribution of interest rates.

In the next two subsections we describe the above mentioned market and its corresponding interest rate instruments.

2.1 ID-Futures and Reference Bonds

The One-Day Inter Bank Deposit Future Contract (ID-Future) with maturity T is a future contract whose underlying asset is the accumulated daily ID rate[5] capitalized between the trading time t ($t \leq T$) and T. The contract size corresponds to R$ 100,000.00 (one hundred thousand Brazilian Reals) discounted by the accumulated rate negotiated between the buyer and the seller of the contract.

This contract is very similar to a zero coupon bond, except that it pays margin adjustments every day. Each daily cash flow is the difference between the settlement price[6] on the current day and the settlement price on the day before corrected by the ID rate of the day before.

The Brazilian Mercantile and Futures Exchange (BM&F) is the entity that offers the ID-Future. The number of authorized contract-maturity months is fixed by BM&F (on average, there are about twenty authorized contract-maturity months for each day but only around ten are liquid). Contract-maturity months are the first four months subsequent to the month in which a trade has been made and, after that, the months that initiate each following quarter. Expiration date is the first business day of the contract-maturity month.

Corresponding to each ID-future there is a reference bond. Reference bonds are zero coupon bonds whose yields are equal to the accumulated daily ID rate implicit

[4]It will work like a usual U.S. cap on a swap, except that this swap would have a unique cash flow, paid at the maturity of the option.

[5]The ID rate is the average one-day inter bank borrowing/lending rate, calculated by CETIP (Central of Custody and Financial Settlement of Securities) every workday. The ID rate is expressed in effective rate per annum, based on 252 business-days.

[6]The settlement price at time t of an ID-Future with maturity T is equal to R$ 100,000.00 discounted by its closing price quotation.

in the settlement price of a corresponding ID-future (with the same maturity). We will adopt these reference bonds in our study as shall be seen in Section 2.3.

2.2 ID Index and its Option Market

The ID index (IDI) is defined as the accumulated ID rate. If we associate the continuously-compounded ID rate to the short term rate r_t then

$$IDI_t = IDI_0 \cdot \mathrm{e}^{\int_0^t r_u \mathrm{d}u}. \tag{1}$$

This index, computed on every workday by BM&F, has been fixed to the value of 100000 points in January 2, 1997, and has actually been resettled to its initial value a couple of times, most recently in January 2, 2003.

An IDI option with time of maturity T is an European option where the underlying asset is the IDI and whose payoff depends on IDI_T. When the strike is K, the payoff of an IDI option is $L_c(T) = (IDI_T - K)^+$ for a call and $L_p(T) = (K - IDI_T)^+$ for a put.

BM&F is also the entity that provides the IDI call options. Strike prices (expressed in index points) and the number of authorized contract-maturity months are established by BM&F. Contract-maturity months can happen to be any month, and the expiration date is the first business day of the maturity month. Usually, there are 30 authorized series within each day, from which about a third are liquid.

2.3 Data

Data consists on time series of yields of ID-Futures for all different liquid maturities, and values of IDI options for different strikes and maturities. The data covers the period from August 09, 2004 to August 06, 2007.

BM&F maintains a daily historical database with the price and number of trades of every ID-Future and IDI option that have been traded in any day. According to Section 2.1, a reference bond is a zero coupon bond with the same maturity and settlement price of an ID-future. We adopt reference bond yields with fixed times to maturity of 1, 63, 126, 189, 252, and 378 days[7]. We also adopt an at-the-money IDI call with time to maturity of 95 days. The prices of these options were obtained via an interpolation based on Black's implied volatilities[8].

After excluding weekends, holidays, and no-trade workdays, there is a total of 741 yields for each bond, and prices for at-the-money IDI call options.

[7]With the ID-Future database and a time series of ID interest rates, it is straightforward to estimate, by cubic interpolation, reference bonds interest rates for fixed maturities, and for all trading days.

[8]Our procedure is similar to that adopted to calculate VIX implied volatilities from S&P 500 index options for hypothetical at-the-money short-maturity (21 days) options.

3 The Model

Uncertainty in the economy is characterized by a filtered probability space $(\Omega, (\mathcal{F}_t)_{t\geq 0}, \mathcal{F}, \mathbb{P})$ where $(\mathcal{F}_t)_{t\geq 0}$ is a filtration generated by a standard N-dimensional Brownian motion $W^{\mathbb{P}} = \left(W_1^{\mathbb{P}}, \ldots, W_N^{\mathbb{P}}\right)$ defined on $(\Omega, \mathcal{F}, \mathbb{P})$ (see Duffie (2001)). We assume the existence of a pricing measure $\mathbb{Q}$ under which discounted security prices are martingales with respect to $(\mathcal{F}_t)_{t\geq 0}$.

A multi-factor CIR model directly represents the short-term rate $r(t)$:

$$r_t = \phi_0 + \sum_{n=1}^{N} X_n(t), \tag{2}$$

where ϕ_0 is a constant and the dynamics of X_n is given by

$$\mathrm{d}X_n(t) = \kappa_n \left(\theta_n - X_n(t)\right) \mathrm{d}t + \sigma_n \sqrt{X_n(t)} \mathrm{d}W_n^{\mathbb{Q}}(t), \quad n = 1, \ldots, N, \tag{3}$$

where $W^{\mathbb{Q}} = \left(W_1^{\mathbb{Q}}, \ldots, W_N^{\mathbb{Q}}\right)$ represents an N-dimensional Brownian motion under $\mathbb{Q}$, and κ_n, θ_n and σ_n represent positive constants satisfying Feller's condition that $2\kappa_n\theta_n > \sigma_n^2$ for all n.

Compatible with the studies by Cox et al. (1985) and Dai and Singleton (2000), we assume a simple functional form for the market price of risk process $\lambda^X(t) = \left(\lambda_1^X(t), \ldots, \lambda_N^X(t)\right)$, proportional to the instantaneous volatilities of the latent variables[9]:

$$\lambda_n^X(t) = \frac{\lambda_n}{\sigma_n} \sqrt{X_n(t)}, \quad n = 1, \ldots, N. \tag{4}$$

The connection between martingale probability measure $\mathbb{Q}$ and objective probability measure $\mathbb{P}$ is obtained via Girsanov's Theorem

$$\mathrm{d}W_n^{\mathbb{P}}(t) = \mathrm{d}W_n^{\mathbb{Q}}(t) - \lambda_n^X(t)\mathrm{d}t. \tag{5}$$

In fact, substituing (5) in (3) we obtain the dynamics of X_n under the objective probability measure:

$$\mathrm{d}X_n(t) = \overline{\kappa}_n \left(\overline{\theta}_n - X_n(t)\right) \mathrm{d}t + \sigma_n \sqrt{X_n(t)} \mathrm{d}W_n^{\mathbb{P}}(t), \quad n = 1, \ldots, N, \tag{6}$$

where $\overline{\kappa}_n = \kappa_n - \lambda_n$ and $\overline{\theta}_n = \frac{\kappa_n \theta_n}{\kappa_n - \lambda_n}$.

The probability density of X_n at time T under $\mathbb{Q}$, conditional on its value at the current time t, is given by (see Brigo and Mercurio (2001)):

$$f_{X_n(T)|X_n(t)}(x) = c_n f_{\chi^2(\nu_n, q_n)}(c_n x), \tag{7}$$

[9]Recently Cheredito et al. (2007) have proposed a functional form for the market prices of affine models that generate more general conditional expectations under the objective probability measure. However, as we are working with independent CIR models the gain with their specification in principle would be small. In addition, by specifying a richer class of models we would be favoring the *option version* since in this case, econometric identification becomes harder.

where

$$c_n = \frac{4\kappa_n}{\sigma_n^2\left(1 - \mathrm{e}^{-\kappa_n(T-t)}\right)},$$

$$\nu_n = \frac{4\kappa_n\theta_n}{\sigma_n^2},$$

$$q_n = c_n X_n(t)\mathrm{e}^{-\kappa_n(T-t)},$$

and $f_{\chi^2(\nu,q)}(\cdot)$ is the density function of a noncentral chi-square variable with ν degrees of freedom and non-centrality parameter q.

3.1 Bond Prices

The proposed model is in the class of affine term structure models (Duffie and Kan (1996))[10]. The price of a zero coupon bond is derived in Cox et al. (1985) under the single-factor case and in Brigo and Mercurio (2001), under the multi-factor case[11]:

$$P(t,T) = \mathrm{e}^{A(t,T)+B(t,T)'X_t}, \tag{8}$$

where

$$A(t,T) = -\phi_0\tau + \sum_{n=1}^{N}\frac{2\kappa_n\theta_n}{\sigma_n^2}\log\left(\frac{2\gamma_n\mathrm{e}^{\frac{(\kappa_n+\gamma_n)\tau}{2}}}{2\gamma_n + (\kappa_n+\gamma_n)\left(\mathrm{e}^{\tau\gamma_n}-1\right)}\right), \tag{9}$$

$\gamma_n = \sqrt{\kappa_n^2 + 2\sigma_n^2}$, $B(t,T)$ is a column vector with

$$B_n(t,T) = \frac{2\left(\mathrm{e}^{\tau\gamma_n}-1\right)}{2\gamma_n + (\kappa_n+\gamma_n)\left(\mathrm{e}^{\tau\gamma_n}-1\right)}, \tag{10}$$

in the n_{th} position, and $P(t,T)$ denotes the time t price of a zero coupon bond paying one monetary unit at time T.

3.2 IDI Option Prices

An IDI option is an interest rate asian option[12] whose payoff is a path-dependent function of the instantaneous short-term rate. Theoretical pricing of such options was provided by Longstaff (1995), Leblanc and Scaillet (1998), and Dassios and Nagaradjasarma (2003) under single factor interest rate models, and by Chacko and Das (2002) under general affine models.

In this section, we propose an alternative way of pricing these options under our multi-factor CIR model, which consists in an efficient implementation of the Laplace

[10] It is interesting to note that affine models have been frequently adopted in empirical fixed income studies. See, for instance, Dai and Singleton (2000, 2002), and Duffee (2002), among many others.

[11] See also Jagannathan et al. (2003).

[12] See Shreve (2004) for the definition and examples of asian options.

transform method of Chacko and Das (2002) via a smart numerical procedure proposed by Abatt and Whitt (1995).

Denote by $c(t,T)$ the time t price of a call option on the IDI index, with time to maturity T, and strike price K. Let $Y(t,T)=\int_t^T r_u \mathrm{d}u$. Then

$$\begin{aligned} c(t,T) &= E_t^{\mathbb{Q}}\left[\mathrm{e}^{-Y(t,T)}L_c(T)\right] \\ &= E_t^{\mathbb{Q}}\left[\mathrm{e}^{Y(0,t)}-K\mathrm{e}^{-Y(t,T)}\right] \\ &= IDI_t E_t^{\mathbb{Q}}\left[\left(1-\frac{K}{IDI_T}\right)^+\right] \\ &= \int_{y\geq\log\frac{K}{IDI_t}}\left(IDI_t-K\mathrm{e}^{-y}\right)f_{Y|\mathcal{F}_t}(y)\mathrm{d}y, \end{aligned} \tag{11}$$

where $f_{Y|\mathcal{F}_t}(y)$ is the probability density of $Y(t,T)|\mathcal{F}_t$. In order to obtain the price $c(t,T)$ it is necessary that we know the conditional distribution of $Y(t,T)$ under $\mathbb{Q}$. When r_t is a Gaussian process $Y|\mathcal{F}_t$ is normally distributed, and by using simple properties of normal distributions a closed-form formula for the option price can be promptly obtained. Considering the case of a non-central χ^2 distribution where there is no closed-form formula for $f_{Y|\mathcal{F}_t}$ (and neither for the option price), one can still calculate the Laplace Transform of $Y(t,T)|\mathcal{F}_t$

$$\mathcal{L}\left(Y|\mathcal{F}_t\right)(s)=\mathrm{e}^{\zeta\phi\tau}\prod_{n=1}^{N}\mathcal{L}\left(Y_n|\mathcal{F}_t\right)(s), \tag{12}$$

where $\mathcal{L}\left(Y_n|\mathcal{F}_t\right)(s)=E_t^{\mathbb{Q}}\left[\mathrm{e}^{-sY_n}\right]$ is the Laplace Transform of $Y_n(t,T)=\int_t^T X_n(u)\mathrm{d}u$ that is given by[13]

$$\mathcal{L}\left(Y_n|\mathcal{F}_t\right)(s)=\left[\frac{2\tilde{\gamma}_n\mathrm{e}^{\frac{(\kappa_n+\tilde{\gamma}_n)\tau}{2}}}{2\tilde{\gamma}_n+(\kappa_n+\tilde{\gamma}_n)\left(\mathrm{e}^{\tau\tilde{\gamma}_n}-1\right)}\right]^{\frac{2\kappa_n\theta_n}{\sigma_n^2}}\cdot\mathrm{e}^{\frac{2\left(\mathrm{e}^{\tau\tilde{\gamma}_n}-1\right)}{2\tilde{\gamma}_n+(\kappa_n+\tilde{\gamma}_n)\left(\mathrm{e}^{\tau\tilde{\gamma}_n}-1\right)}},$$

with $\tilde{\gamma}_n=\tilde{\gamma}_n(s)=\sqrt{\kappa_n^2+2\sigma_n^2 s}$.

It should be clear at this point that searching for efficient numerical procedures for inversions of Laplace transforms becomes central to solve our pricing problem. In this work, we decided to adopt and implement the Abate and Whitt (1995) method due to its precision under the CIR model and to its small processing-time, a fundamental factor for an efficient implementation of our dynamic model[14].

Observe that after inverting the Laplace transform (Equation (12)), in principle we would need to numerically integrate $\left(IDI_t-K\mathrm{e}^{-y}\right)f_{Y|\mathcal{F}_t}(y)$ on the interval

[13]The Laplace Transform of $Y_n(t,T)$ is just the time t bond price maturing at T when short term rate follows a one-dimensional CIR process with parameters $(\kappa_n, s\theta_n, \sqrt{s}\sigma_n)$ starting from sr_t.

[14]Nagaradjasarma (2003) reviews the main literature approach to the problem of pricing interest rate asian options with the CIR model.

$\left[\log \frac{K}{IDI_t}, +\infty\right)$ (see Equation (11)). Nevertheless, Dassios and Nagaradjasarma (2003) provide a useful lemma where they show that the call price can be computed in a single step:

Lemma 1 *Considering $\mu \geq 0$, denote by $\mathcal{G}(s, \mu)$ the inverse Laplace transform of*

$$\frac{E_t^{\mathbb{Q}}\left(\mathrm{e}^{-(\mu+s)Y(t,T)}\right)}{s} = \frac{\mathcal{L}\left(Y|\mathcal{F}_t\right)(\mu+s)}{s}$$

with respect to $s > 0$. Hence

$$c(t,T) = K\mathcal{G}\left(\log \frac{K}{IDI_t}, 1\right) - IDI_t\mathcal{G}\left(\log \frac{K}{IDI_t}, 0\right) - KP(t,T) + IDI_t. \quad (13)$$

Proof: See Dassios and Nagaradjasarma (2003).

We make direct use of their lemma to speed up our IDI option pricing procedure.

4 Measuring Market Risk

4.1 Value at Risk

It is fundamental for a financial institution to perform an active control of its portfolio risk. Among a number of risk measures available, Value at Risk (VaR) has become the most influential one[15]. Despite its documented drawbacks[16] it is largely adopted by financial institutions as a tool to control risk, and by regulatory institutions considering capital requirements to cover market risk.

VaR is usually defined as the loss that is exceeded with some given probability, α, over a given horizon. If we denote by M_t the value of a portfolio at time t, then this definition can be translated to

$$\mathbb{P}\left[M_{t+HP} - M_t \leq VaR_{t,HP}(\alpha)|\, \mathcal{F}_t\right] = \alpha, \qquad \alpha \in [0,1],$$

where $VaR_{t,HP}(\alpha)$ is the portfolio's VaR at time t for the horizon HP with confidence level $1 - \alpha$.

Certainly, the widespread use of VaR-based risk management models results from the fact that it is a risk metric easy to interpret. Note, however, that the estimation process of VaR when the probability distribution of M_{t+HP} is not known in closed-form, usually constitutes a task demanding high computational time. On the other hand, whenever that probability distribution is known, this task can be tremendously simplified, as in the case of a normal M_{t+HP}:

[15] For an overview of VaR see Duffie and Pan (2001).

[16] VaR is not a sub-additive measure of risk. For instance, Kerkhof and Melenberg (2004) show that for regulatory purposes, expected shortfall, which is a coherent measure of risk (Artzner et al. (1999)), produces better results than VaR. On the other hand, Garcia et al. (2007) indicate that using appropriate conditioning variables usually is enough to restore VaR sub-additivity.

$$VaR_{t,HP}(\alpha) = M_t - \mu\left(M_{t+HP}\right) - \sigma\left(M_{t+HP}\right)\Phi^{-1}(\alpha),$$

where $\mu\left(M_{t+HP}\right)$ is the mean of M_{t+HP}, $\sigma\left(M_{t+HP}\right)$ is the standard deviation of M_{t+HP} and $\Phi(\cdot)$ is the normal cumulative distribution function.

Unfortunately under our model, the distribution of M_{t+HP} is not known in closed-form, and as shown bellow we will make use of simulation techniques to estimate VaR.

4.2 Our experiment with VaR

In order to test the influence of option prices on the VaR estimation of fixed income instruments, we compute the VaR for zero coupon bonds with times to maturity of six and twelve months, respectively. We evaluate the VaR for two different horizons, 1-day and 10-day, and for four different confidence levels, 90%, 95%, 97.5%, and 99%, which are the VaR parameters usually adopted by market participants and regulatory institutions.

VaR risk measures are computed for each version of the dynamic model, the *bond version*, and the *option version*. We adopt a Monte Carlo Simulation, where simulated paths are obtained by using the fact that $X_n(t+HP)|X_n(t)$ is a multiple of a noncentral chi-square distribution (see Equation (7)). In order to generate random numbers chosen from the noncentral chi-square distribution we use a standard procedure: We first generate random values drawn from a uniform distribution on the unit interval. Then invert a noncentral chi-square cumulative distribution function evaluated at the previously obtained values. For each zero-coupon bond (6-month, 12-moth), a total of 20.000 random paths for its price process were generated using the Antithetic Variable Technique (see Glasserman (2003)).

4.3 The Accuracy of a VaR Model

Under each version of the CIR model, we verify the accuracy of VaR estimation via backtesting procedures. While there is a large number of backtesting methods to deal with validation of VaR models, we concentrate in particular on three important tests proposed respectively by Kupiec (1995), Christoffersen (1998) and Seiler (2006) (see Appendix A for details).

An accurate VaR model must jointly satisfied two properties (see Campbell (2005)). First, the *unconditional coverage property* i.e., number of violations[17] is on average equal one minus VaR confidence level. Second the *independence property*, i.e., previous violations have no impact on the probability of futures violations. The Kupiec test verifies only the former while Christoffersen's approach tests the latter.

The Kupiec and Christoffersen tests are the tests most widely adopted by market practitioners, but they work only within single-period forecasting (1-day VaR). Un-

[17] A violation occurs when profit and losses (P&L) in a day is lower than VaR

der the case of a 10-day VaR, it is necessary to use a more elaborated test that will comprise dependence among VaR violations. At this point, Seiler's (2006) method allowed us to test the unconditional coverage property for VaR horizon greater than one day.

Another important point to discuss is how general each test is when analyzing independence of VaR violations. For instance, Christoffersen's test considers only independence between successive days (probability of violating tomorrow's VaR depends on whether or not today's VaR was violated). However, there are innumerous ways for independence to fail[18]. In order to offer a meaningful complement to Christoffersen's test, we also examine dependence of violations up to lag 10 with the Ljung-Box (1978) test.

5 Empirical Results

In order to evaluate the importance of options on the estimation of future interest rate quantiles, we adopt the three-factor CIR model introduced in Section 3. The model is estimated based on two distinct strategies. The first strategy (*bond version*) uses only ID-Futures, while the second (*option version*) includes both ID-Futures and IDI calls in the estimation process. The estimation technique adopted here is based on the Maximum Likelihood procedure proposed by Chen and Scott (1993). When implementing the *bond version* latent factors are obtained from ID reference bonds with maturities of 1, 126 and 252 days, while remaining bonds are priced under assumption of i.i.d. Gaussian errors. The *option version* uses the same information set on ID reference bond yields adopted by the *bond version*, but in addition it prices the Black implied volatility of a 95-day maturity at-the-money call under assumption of Gaussian pricing errors. Technical details regarding the estimation process are described in the Appendix B. Table 1 presents parameters values under the *bond* and *option versions*. Standard deviations are obtained by the BHHH method[19]. All parameters whose ratio column presents bold values are significant at a 95% confidence level.

Figure 1 shows the historical evolution of the ID reference bond yields with maturities of 1, 126 and 256 days. In the beginning of the sample yields are increasing and the term structure is upward sloping (long-term rates higher than short-term rates). After July of 2005 yields decline over time and the shape of the term structure changes to downward sloping due to an improvement in the Brazilian economic conditions in recent years. Figure 2 show that both dynamic versions of the model keep the pricing errors at small levels[20]. Figure 3 presents the time series of the instantaneous implied volatility of the short-term rate under the two versions of the

[18] For instance, the likelihood of a violation tomorrow can depend on a violation happening a week ago.

[19] See Davidson and MacKinnon (1993).

[20] The *bond version* presents errors of less than 20 bps, while the *option version* presents slightly higher errors but still less than 30 bps for most of the time series. Note that errors are smaller under both versions in most recent periods, during the year of 2007.

model[21]. Note that volatility is much higher for the model without options, during the whole sample period. This overestimation of volatility will be one of the main factors contributing to the overestimation of VaR by the *bond version* of the model.

We estimate, for each model version, 1- and 10-day VaR's of zero-coupon bonds with time to maturities of 6- and 12- months, between August of 2004 and August of 2007. On each day, the present value of the portfolio is $M_t = P(t, t+T)$ with $T = 0.5$ or 1 years and its value at the end of the holding period is $M_{t+HP} = P(t+HP, t+T+HP)$ with $HP = 1$ or 10 days[22]. Figures 4 and 5 present the time series of the 95% VaR for 1- and 10- day VaR's, and the corresponding P&L for both zero coupon bonds. Qualitatively analyzing the pictures, we note that with a time series of 740 observations, for the 1-day VaR, we could expect on average around 5% of the period (about 37 observations) to have the P&L value crossing the VaR value. Observing Figure 4 we see that VaR for the one year bond was crossed 29 times under the *option version*, but only 8 times under the *bond version*. This is a first indication that the model without options would be overestimating VaR.

Table 2 confirms with formal tests the qualitative results described above. It represents the Kupiec tests with a size of 5% (type I error) for 1- and 10-day VaR's estimated based on four different confidence levels (90%, 95%, 97.5% and 99%). These are usually adopted values, both by regulatory institutions as well as by investment banks. Kupiec test for 10-day VaR were carry out ignoring dependence of violations. As explained in Appendix B it is equivalent to a more conservative test (a higher type I error). Each cell of this table presents the number of VaR violations observed. The last column shows the number of VaR violations that we have to observe to avoid rejection of the model at a 95% confidence level. We also present the number of violations when VaR is estimated by a Historical Simulation (HS) with respectively 125 or 250 days in a moving-window. HS is a very common method for the estimation of VaR adopted by market practioners. Undoubtedly, it is the simplest method to compute VaR. The aim of comparing quantile forecasts of the dynamic model to HS is to show that the CIR model provides results which are trustable according to a established benchmark. In other words, that the dynamic model elected to study the contribution of options on VaR estimation is not a spurious model.

The *option version* clearly outperforms the *bond version* in terms of unconditional coverage property: The *bond version* in fact consistently overestimates risk. If the *bond version* was adopted for capital allocation purposes it could be really demanding much more capital than the necessary to cover the risk of bond portfolios. On the other hand, the *option version* seems not to capture very well the 1-day quantiles for the 6-month bond and the 10-day quantiles for the 1-year bond. In the

[21]The instantaneous implied volatility of the short-term rate is given by $\sqrt{\sigma_1^2 X_1(t) + \sigma_2^2 X_2(t) + \sigma_3^2 X(t)}$.

[22]We could fix the future value as $M_{t+HP} = P(t+HP, t+T)$. But with this convention we have to correct the present value over the holding period to calculate P&L of a portfolio. Since it does not add any important information related to the quantile forecast, we opt to use the simpler convention described above.

former case this version overestimates risk while in the latter it underestimates risk. For a total of 16 tests, the *bond version* is not rejected in only 1 test while the *option version* is not rejected in 8 tests. It is interesting to note that the acceptance rate of the *option version* is similar to the one obtained by the two historical simulation benchmarks, which are not rejected in respectively 9 times for the 125 days-window HS, and 10 times for the 250 days-window HS.

Table 3 presents Seiler's test statistic for a 10-day VaR, and for both considered bonds. When the test size is 5%, a model is not rejected if Seiler's test statistic is lower than 1.96. Seiler's test is a simple way to take into account dependence due to overlapping forecasting horizons. Observing Figure 5 we can see that the *option version* generates some clustered violations. This is a sign for existence of temporal dependence between observations. Note that the acceptance rates of all models are greater when we use Seiler's test. This should be expected since under dependence assumptions we have less information to test model's performance. Dependence makes an effect which is equivalent to a decrease in sample size[23]. Formally, Seiler's test has a low statistical power[24]. Notwithstanding the tremendous improvement in acceptance rate of the *bond version*, when we jointly analyze 1-day (Table 2) and 10-day (Table 3) containing VaR backtesting results, the *option version* still outperforms the *bond version* (the *bond version* is accepted in 7 tests while the *option version* is accepted in 9 tests). Summarizing, when we use the most conservative test (Kupiec's test) the *option version* strongly outperforms the *bond version*, while when we adopt an appropriate test for multiple period dependence but that presents low power (Seiler's test), the *option version* still outperforms, but only by a mild advantage.

Moving from the analysis of unconditional coverage property to the independence property we first examine the results of Christoffersen's test. Table 4 reports Christoffersen's test statistic for a test size of 5%. A model is not rejected if the test statistic is lower than 3.84. The *bond version* is statistically accepted in 14 tests whereas the *option version* in 13 tests. In addition, both dynamic versions outperform historical simulations which do not show dependence of violations in 6 tests (125 days-window) and 11 tests (250 days-windows). It is not surprising that the *bond version* performs extremely well in independence tests. Such good performance is deceiving since it is due to a bad characteristic of the *bond version*: It presents an extremely low number of violations, usually under the lower critical value of the Kupiec test. This fact directly implies a very small probability that it ends up exhibiting clustered violations. Indeed, from a practical point of view, the *bond version* should already have been discarded due to its tremendously weak performance in unconditional coverage tests.

[23]We can easily translate Seiler's test statistic in terms of number of violations. For example, if we consider the 6-month bond and a test size of 2.5%, the critical values are around 0 and 38 violations (the exact critical values depend on the tested model, but they do not vary much), while Kupiec's critical values are 11 and 27 violations.

[24]Low statistical power is a common feature on all multi-period backtesting procedures that adopt small samples (see Dowd (2007) and Seiler (2006)).

Table 5 presents the Ljung-Box test statistic for a test considering an absence of serial correlation in the hit sequence[25] of VaR's of 6- and 12- month bonds. We verify whether a group of autocorrelations between lags 1 and 10 (for the 1-day VaR) and between lags 10 and 19 (for the 10-day VaR) are statistically different from zero. A model is not rejected if the test statistic is lower than 18.31. Here again the important analysis is the comparison between *option version* and the corresponding historical simulation benchmarks since the performance of the *bond version* will be biased by its too small number of VaR violations. The *option version* performs well being statistically accepted in 9 out of 16 tests. On the order hand, historical simulation seems to have some kind of problem with clustered violations. In fact, under both historical simulation methods violations are statistically not autocorrelated in only 5 out of 16 cases.

In summary, it appears that options indeed contain information that improve the ability of the dynamic term structure model to estimate VaR. Further analysis should be done in future research by considering models that accommodate the existence of jumps in data.

6 Conclusion

In this paper we test if interest rate options contain useful information that could help on the econometric identification of the quantiles of interest rate distributions. Identification of these quantiles should be useful for risk and portfolio management purposes. Adopting a three-factor CIR dynamic term structure model, we compare the performance of a version estimated with only bonds data (*bond version*) to a version estimated based on both bonds and options data (*option version*). While the *bond version* strongly overestimates risk (formally measured by a Kupiec statistical backtest), the *option version* demonstrates to be more adequate, presenting a performance comparable to historical simulation benchmarks. Tests of independence between violations (Christoffersen (1998) and Ljung-Box (1978)) confirm that the *option version* performs better than the historical simulation benchmarks (although the *bond version* also performs well in these tests due to its inadequately small number of VaR violations). Our results complement an existing literature that identifies risk premia based on the use of option data within dynamic models. Indeed, we show that interest rate options also contain information related to higher moments of interest rates that allow for a dynamic model to better estimate interest rates quantiles. These results suggest that banks and other financial institutions might consider interesting and possibly useful to perform tests with integrated datasets on linear and non-linear interest rate instruments in order to better manage their interest rate risks.

[25]The hit sequence is a time series of binary variables that indicates at each time if a violation has occurred or not. See Appendix A for more details.

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Appendix A - Backtesting Procedures

In this appendix we describe the backtesting procedures proposed by Kupiec (1995), Christoffersen (1998) and Seiler (2006), as well as the Portmanteau test developed by Ljung-Box (1978).

Suppose that we have a time series of VaRs with confidence levels of $1-\alpha$ and a time series of profits and losses (P&L) on a portfolio over a fixed time interval. A violation occurs when P&L is lower than VaR. Each variable $I_t(\alpha)$ in the hit sequence is equal to zero if there is no violation at time t or equal to one if there is a violation at time t, viz:

$$I_t(\alpha) = \begin{cases} 1 & \text{if } P\&L_t \leq VaR_{t-HP,t} \\ 0 & \text{else.} \end{cases}$$

First we consider the single-period case (1-day ahead quantile forecasting). Christoffersen (1998) points out that the problem of determining the accuracy of a VaR model is equivalent to the problem of determining whether the hit sequence satisfies the following two properties:

1. *Unconditional Coverage Property*, which means that the average number of violations is statistically equal to the expected one. Formally $\mathbb{P}\left[\, I_{t+1}(\alpha) = 1 \middle| \, \mathcal{F}_t \right] = \alpha$.

2. *Independence Property*, which means that violations are independently distributed. In other words, the history of VaR violations does not contain information about future violations. Formally $I_t(\alpha)$ and $I_{t+k}(\alpha)$ are independent for all t and $k \geq 1$.

The test developed by Kupiec (1995) examines only the unconditional coverage while Christoffersen (1998) considers both the unconditional coverage and independence. The number of violations observed in a sample of size L is

$$N_L(\alpha) = \sum_{t=1}^{L} I_t(\alpha).$$

If the model is accurate, N_L must present a binomial distribution. Using the Likelihood-Ratio test, the Kupiec's test statistic takes the form

$$2\ln\left(\left(\frac{1-\alpha}{1-\hat{\alpha}}\right)^{L-N_L(\alpha)}\left(\frac{\alpha}{\hat{\alpha}}\right)^{N_L(\alpha)}\right) \sim \chi^2(1), \quad \text{as } L \to \infty,$$

where $\hat{\alpha} = N_L(\alpha)/L$ and $\chi^2(\nu)$ stands for a chi-squared distribution with ν degrees of freedom.

The Kupiec test provides a necessary condition to classify a VaR model as adequate. But, not considering the arrival times of violations, it does not exclude models that do not satisfy the independence property. In other words, the Kupiec test can accept a model that creates successive violation clustering. In order to solve this problem, Christoffersen (1998) proposed a Markov test that takes into account the independence property. Within his test, if the probability of a violation in the current period has no impact on the probability of violation in the next period, then it must hold that

$$\frac{L_{10}}{L_{00}+L_{10}} = \frac{L_{11}}{L_{01}+L_{11}},$$

where L_{ij} is the number of observations of I_t with value i followed by j, $i, j = 0, 1$. Again, the test statistic is given by the Likelihood-Ratio test:

$$2\ln\left(\frac{(1-\pi_{01})^{L_{00}}\pi_{01}^{L_{01}}(1-\pi_{11})^{L_{10}}\pi_{11}^{L_{11}}}{(1-\pi)^{L_{00}+L_{10}}\pi^{L_{01}+L_{11}}}\right) \sim \chi^2(1), \quad \text{as } L \to \infty, \tag{14}$$

where $\pi_{ij} = \frac{L_{ij}}{\sum_j L_{ij}}$ and $\pi = \frac{L_{01}+L_{11}}{L}$. Note that this test does not depend on the true coverage value α, and thus, only tests the independence property. Indeed, Christoffersen (1998) also presents a test which simultaneously examines the unconditional coverage and the independence properties. We do not use this joint test because it does not disentangle which property has been violated.

Christoffersen's test considers only dependence from day to day, what implies that the independence property is not fully tested. In order to examine the existence of dependence with a time lag greater than one period, we can use the Ljung-Box test (see Ljung and Box (1978)). If $\hat{\gamma}_i$ is the empirical autocorrelation of order i, to test $\gamma_i = 0$ for the first K autocorrelations we can use the following statistic

$$L(L+2)\sum_{i=1}^{K}\frac{\hat{\gamma}_i^2}{L-i} \sim \chi^2(K), \quad \text{as } L \to \infty. \tag{15}$$

In order to validate the accuracy of a 10-day VaR we have to adjust the independence property, because the quantile forecasts are subject to common shocks. This can be done by requiring that $I_t(\alpha)$ and $I_{t+k}(\alpha)$ are independent for all t and $k \geq 10$. This fact complicates the accuracy problem since the variables I_t are not presumed to be i.i.d. There are different ways to handle the dependence of lag lower than 10 (see Dowd (2007) and Seiler (2006)). The first alternative is ignoring dependence. Although it can appear to be a naive choice, ignoring dependence gives valuable information since it is equivalent to implementing a very conservative test[26]. The problem is that we can reject "good" models. In others words, under this alternative probably the type I error is higher than the true type I error. Seiler (2006) and Dowd (2007) propose different tests that take into account dependence. Dowd (2007) uses a bootstrapping approach to yield an i.i.d. resample. On the other hand, Seiler's test considers that the model under analysis captures the true autocorrelations of

[26]The correlation between $I_t(\alpha)$ and $I_{t+k}(\alpha)$ for $k < 10$ are expected to be positive. Thus, the variance of N_L when ignoring dependence is lower than the variance when correlations are taken into account. Therefore the rejection region is larger if dependence is ignored.

lag lower than 10. Thus, based on an ARMA process, the dependence structure can be estimated from the hit sequence observations. If L is large enough, we have

$$N_L(\alpha) \sim \mathcal{N}(L\alpha, LA),$$

where $\mathcal{N}$ represents a normal distribution and the correlation matrix A is estimated using only the hit sequences $I_t(\alpha)$ and $I_t(0.5)$ (see Seiler (2006) for details about this estimation procedure)[27].

The adjusted independence property can be validated using the test in (14) with the number of "00", "01", "10" and "11" in the hit sequence counted from 10 to 10 days, that is

$$L_{ij} = \# \{2 \leq t \leq L | I_t(\alpha) = i, I_{t+10}(\alpha) = j\} \quad \text{for} \quad i, j = 0, 1.$$

Finally, to test autocorrelation with a time lag greater than one period we use the Ljung-Box test (15) with the summation extending from $i = 10$ to $i = K + 9$.

[27]We also experimented with the Bonferroni approach (see Dowd (2007)) to solve the problem of dependence, but the power of the resulting test was very low.

Appendix B - Maximum Likelihood Estimation

On this work, we adopt the Maximum Likelihood estimation procedure proposed by Chen and Scott (1993). We observe the following reference ID bonds yields through time: $rb_t(1)$, $rb_t(63)$, $rb_t(126)$, $rb_t(189)$, $rb_t(252)$, and $rb_t(378)$. Let rb represents the $H \times 7$ matrix containing these ID bonds yields for the whole time series of H points. Assume that model parameters are represented by vector ϕ and that the difference between times $t-1$ and t is Δt. From Equation (8), the relation between the yield of a reference ID bond with maturity τ and the state variables at time t is

$$R(t, \tau, \phi) = \phi_0 - \frac{A(\tau, \phi)}{\tau} - \frac{B(\tau, \phi)'}{\tau} X(t), \tag{16}$$

where $A(\tau, \phi) = \sum_{n=1}^{N} \log A_n(t, \tau)$, $B(\tau, \phi) = -\left(B_1(\tau, \phi), \ldots, B_N(\tau, \phi)\right)'$ and $X(t) = \left(X_1(t), \ldots, X_N(t)\right)'$. In accordance with the empirical term structure literature, we fix $N = 3$ and assume that the yields of the reference ID bond with maturities 1, 126 and 252 days are observed without error. As the state vector is three-dimensional, knowledge of functions $A(\tau, \phi)$ and $B(\tau, \phi)$ allows us to solve a linear system to obtain the values of the state vector at each time t:

$$\begin{aligned} rb_t(1) &= \phi_0 - \tfrac{A(1,\phi)}{1} - \tfrac{B(1,\phi)'}{1} X(t), \\ rb_t(126) &= \phi_0 - \tfrac{A(126,\phi)}{126} - \tfrac{B(126,\phi)'}{126} X(t) \quad \text{and} \\ rb_t(252) &= \phi_0 - \tfrac{A(252,\phi)}{252} - \tfrac{B(252,\phi)'}{252} X(t). \end{aligned} \tag{17}$$

For the reference ID bond with maturities 63, 189 and 378 days, we assume observation with gaussian errors u_t uncorrelated along time:

$$\begin{aligned} rb_t(63) &= \phi_0 - \tfrac{A(63,\phi)}{63} - \tfrac{B(63,\phi)'}{63} X(t) + u_t(63), \\ rb_t(189) &= \phi_0 - \tfrac{A(189,\phi)}{189} - \tfrac{B(189,\phi)'}{189} X(t) + u_t(189), \\ rb_t(378) &= \phi_0 - \tfrac{A(378,\phi)}{378} - \tfrac{B(378,\phi)'}{378} X(t) + u_t(378) \quad \text{and} \end{aligned} \tag{18}$$

When options are took account in the estimation procedure, in addition to yields errors, we have to consider options errors defined by

$$u_t^{\text{call}} = c_t - cs_t, \tag{19}$$

where cs_t is the observed Black volatility for the call in the sample at time t and c_t is the model Black volatility (see Section 3.2) to the same call at time t.

After extracting the corresponding state vector at the vector of parameters ϕ, we can write the log-likelihood function of the reference ID bond yields as

$$\begin{aligned} L(rb, \phi) &= \textstyle\sum_{t=2}^{H} \log p(X(t)|X(t-1); \phi) - \\ &- (H-1) \log |jac| - \tfrac{H-1}{2} \log |\Omega| - \tfrac{1}{2} \textstyle\sum_{t=2}^{H} u_t' \Omega^{-1} u_t, \end{aligned} \tag{20}$$

where:

1. $jac = \begin{bmatrix} -\frac{B(1,\phi)'}{1} \\ \\ -\frac{B(126,\phi)'}{126} \\ \\ -\frac{B(252,\phi)'}{252} \end{bmatrix}$;

2. Ω represents the covariance matrix for u_t, estimated using the sample covariance matrix of the u_t's implied by the extracted state vector along time;

3. $p(X(t)|X(t-1);\phi)$ is the transition probability density from $X(t-1)$ to $X(t)$ in the real world (that is, under the measure $\mathbb{P}$). Since as the dynamics of X in the real world is also a Feller process (see Equation (6)) and the X_n's are independents we have

$$p(X(t)|X(t-1);\phi) = \prod_{n=1}^{3} f_{X_n(t)|X_n(t-1)}(x)\Big|_{x=X_n(t)},$$

with $f_{X_n(t)|X_n(t-1)}(\cdot)$ given by Equation (7) changing κ_n by $\overline{\kappa}_n$ and θ_n by $\overline{\theta}_n$.

4. $u_t' = [u_t(63)\ u_t(189)\ u_t(378)]$ when we don't use options in the estimation procedure and $u_t' = \left[u_t(63)\ u_t(189)\ u_t(378)\ u_t^{\text{call}}\right]$ when we use.

Our final objective is to estimate the vector of parameters ϕ which maximizes function $L(rb,\phi)$. In order to try to avoid possible local minima we use several different starting values and search for the optimal point by making use of the Nelder-Mead Simplex algorithm for non-linear functions optimization (implemented in the MATLAB *fminsearch* function) and the gradient-based optimization method (implemented in the MATLAB *fminunc* function).

Parameter	Bond Version		Option Version	
	Value	$\frac{\lvert\text{Value}\rvert}{\text{Std Error}}$	Value	$\frac{\lvert\text{Value}\rvert}{\text{Std Error}}$
κ_1	11.1819	**29.94**	27.9618	**26.83**
κ_2	0.0003	0.30	0.0002	**7.42**
κ_3	2.0311	**29.37**	2.8889	**19.79**
θ_1	0.0291	**37.06**	0.0342	**23.39**
θ_2	4.7485	**2.45**	5.1643	**34.02**
θ_3	0.0430	**40.35**	0.0487	**32.90**
σ_1	0.1507	**27.86**	0.1367	**14.84**
σ_2	0.0518	**36.81**	0.0409	**26.71**
σ_3	0.1228	**31.65**	0.0650	**9.38**
λ_1	-14.0298	**2.34**	-22.5605	**4.93**
λ_2	-6.1960	1.32	-8.6856	**2.60**
λ_3	0.0296	0.00	0.037	0.00
ϕ_0	0	-	0	-

Table 1: Parameters values and standard errors.

This table presents parameters values and standard errors for both versions of the dynamic model. The model was estimated by Maximum Likelihood adopting the methodology proposed by Chen and Scott (1993). For the *bond version*, the 1-, 126-, and 252-days maturity ID reference bond were priced exactly, and the other bonds were priced with i.i.d. Gaussian errors. For the *option version*, the 1-, 126-, and 252-days maturity ID reference bond were priced exactly, and the other bonds and the Black volatility of the at-the-money IDI call maturing in 95 days were priced with i.i.d. Gaussian errors. Standard errors were obtained by the BHHH method. Bold value means that the parameter is significant at a 95% confidence level.

Confidence Levels	1-day VaR								
	6-month zero coupon bond				12-months zero coupon bond				Kupiec
	Bond	Option	HS 125	HS 250	Bond	Option	HS 125	HS 250	
90%	6	16	**59**	50	15	56	**66**	**59**	[59, 90]
95%	2	11	**32**	22	8	**29**	**40**	**27**	[26, 49]
97.5%	1	7	**19**	10	6	**17**	**20**	**11**	[11, 27]
99%	1	**5**	**11**	**4**	**5**	**10**	**11**	**6**	[3, 13]

Confidence Levels	10-day VaR								
	6-month zero coupon bond				12-months zero coupon bond				Kupiec
	Bond	Option	HS 125	HS 250	Bond	Option	HS 125	HS 250	
90%	9	**60**	**67**	46	40	101	94	**58**	[58, 89]
95%	1	**41**	55	18	19	76	60	**26**	[26, 48]
97.5%	0	**21**	42	**17**	7	51	38	**21**	[11, 27]
99%	0	**7**	29	**13**	**3**	32	28	18	[3, 13]

Table 2: Number of VaR violations in the Kupiec test.

This table presents the number of VaR violations for the 6- and 12-month zero coupon bonds. A VaR violation occurs when P&L in a day is lower than the estimated VaR for this day. VaR was estimated for two horizon period (1 and 10 days) and four confidence levels (90%, 95%, 97.5% and 99%). The columns represent the test statistics for *bond version*, *option version* and historical simulations with 125 and 250 scenarios. The last column represents the Kupiec critical values under a 5% type I error. Bold values means that the model pass Kupiec validation test.

Confidence Levels	6-month zero coupon bond			
	Bond	Option	HS 125	HS 250
90%	2.94	**0.59**	**0.29**	**1.16**
95%	2.38	**0.29**	**1.28**	**1.15**
97.5%	**1.81**	**0.26**	2.42	**0.12**
99%	**1.23**	**0.05**	3.75	**0.88**
Confidence Levels	12-month zero coupon bond			
	Bond	Option	HS 125	HS 250
90%	**1.79**	**1.52**	**1.07**	**0.75**
95%	**1.39**	3.16	**1.75**	**0.77**
97.5%	**1.33**	3.92	2.18	**0.29**
99%	**0.86**	5.07	3.88	1.97

Table 3: Seiler's test statistic for a 10-day VaR.
This table presents the Seiler test statistic for 10-day quantile forecast (10-day VaR) of 6- and 12-month bonds. Seiler test takes in account dependence between successive VaR violations. VaR was estimated for four different confidence levels (90%, 95%, 97.5% and 99%). The columns represent the test statistics for *bond version*, *option version* and historical simulations with 125 and 250 scenarios, respectively. Under a 5% type I error, a model is not rejected if Seiler's test statistic is lower than 1.96. Bold values means that the model is not rejected in Seiler validation test.

Confidence Levels	1-day VaR							
	6-month zero coupon bond				12-month zero coupon bond			
	Bond	Option	HS 125	HS 250	Bond	Option	HS 125	HS 250
90%	**0.10**	**0.87**	5.81	10.82	**0.62**	5.17	13.22	5.81
95%	**0.01**	**2.08**	10.20	5.23	**0.18**	**2.35**	**0.37**	**3.14**
97.5%	**0.00**	**0.13**	**2.87**	**2.43**	**0.10**	**0.80**	**0.33**	**2.08**
99%	**0.00**	**0.07**	**0.33**	**0.04**	**0.07**	**0.27**	**0.33**	**0.10**
Confidence Levels	10-day VaR							
	6-month zero coupon bond				12-month zero coupon bond			
	Bond	Option	HS 125	HS 250	Bond	Option	HS 125	HS 250
90%	**0.23**	11.77	30.49	9.67	**0.73**	5.29	8.33	4.13
95%	**0.00**	**1.06**	25.73	**0.56**	**1.03**	**3.82**	12.03	**2.35**
97.5%	-	**1.26**	11.68	**0.48**	**0.14**	**0.22**	8.48	**0.72**
99%	-	**0.14**	14.05	**0.29**	**0.03**	**2.69**	**1.61**	**0.51**

Table 4: Christoffersen's test statistic.

This table presents Christoffersen test statistic for the hit sequence of VaR of both bonds. VaR was estimated for 1 and 10 days horizons and for confidence levels of 90%, 95%, 97.5%, and 99%. The hit sequence is equal to one if there is a violation and zero otherwise. Christoffersen test examines the independence of violations. The columns represent the test statistics for *bond version*, *option version* and historical simulations with 125 and 250 scenarios. A dash entry means that is impossible to apply Christoffersen test (because $L_{01} = L_{10} = L_{11} = 0$). Under a 5% type I error, a model is not rejected if the test statistic is lower than 3.84. Bold values means that the model is not rejected in Christoffersen validation test.

Confidence Levels	1-day VaR							
	6-month zero coupon bond				12-month zero coupon bond			
	Bond	Option	HS 125	HS 250	Bond	Option	HS 125	HS 250
90%	**0.51**	35.44	**13.71**	24.52	**18.19**	**15.08**	32.56	24.29
95%	**0.06**	**14.51**	44.19	36.19	40.28	**10.59**	**15.02**	47.76
97.5%	**0.01**	**14.14**	27.45	29.58	38.54	**13.65**	28.13	44.64
99%	**0.01**	**0.35**	**5.99**	135.48	56.77	44.62	**14.60**	39.07

Confidence Levels	10-day VaR							
	6-month zero coupon bond				12-month zero coupon bond			
	Bond	Option	HS 125	HS 250	Bond	Option	HS 125	HS 250
90%	**1.21**	90.85	180.37	51.75	**11.82**	27.79	206.21	70.33
95%	**0.01**	55.55	180.83	**1.90**	**9.83**	22.75	70.06	23.96
97.5%	-	20.61	56.82	**1.57**	**0.73**	**6.21**	23.44	**5.87**
99%	-	**0.73**	53.64	**0.99**	**0.13**	**16.61**	**6.39**	**1.58**

Table 5: Ljung-Box test statistic.

This table presents Ljung-Box test statistic for the hit sequence of VaR of both bonds. VaR was estimated for 1 and 10 days horizons and for confidence levels of 90%, 95%, 97.5% and 99%. The hit sequence is equal to one if there is a violation and zero otherwise. Ljung-Box test examines the absence of serial correlation for the first ten autocorrelations after time lag of one (for the 1-day VaR) or ten (for the 10-day VaR). The columns represent the test statistics for *bond version*, *option version* and historical simulations with 125 and 250 scenarios. A dash entry means that is impossible to apply Ljung-Box test. Under a 5% type I error, a model is not rejected if the test statistic is lower than 18.31. Bold values means that the model is not rejected in Ljung-Box validation test.

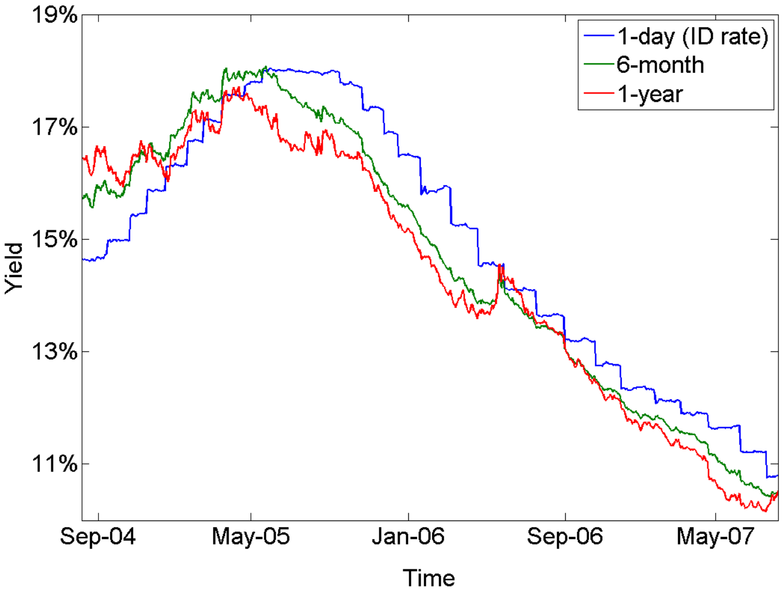

Figure 1: The term structure of Brazilian interest rates. This figure contains time series of the ID reference bond yields with time to maturity of 1-, 126-, and 252-day between August 9, 2004 and August 6, 2007. ID reference bonds are zero coupon bonds obtained by cubic interpolation from observed ID-Future rates.

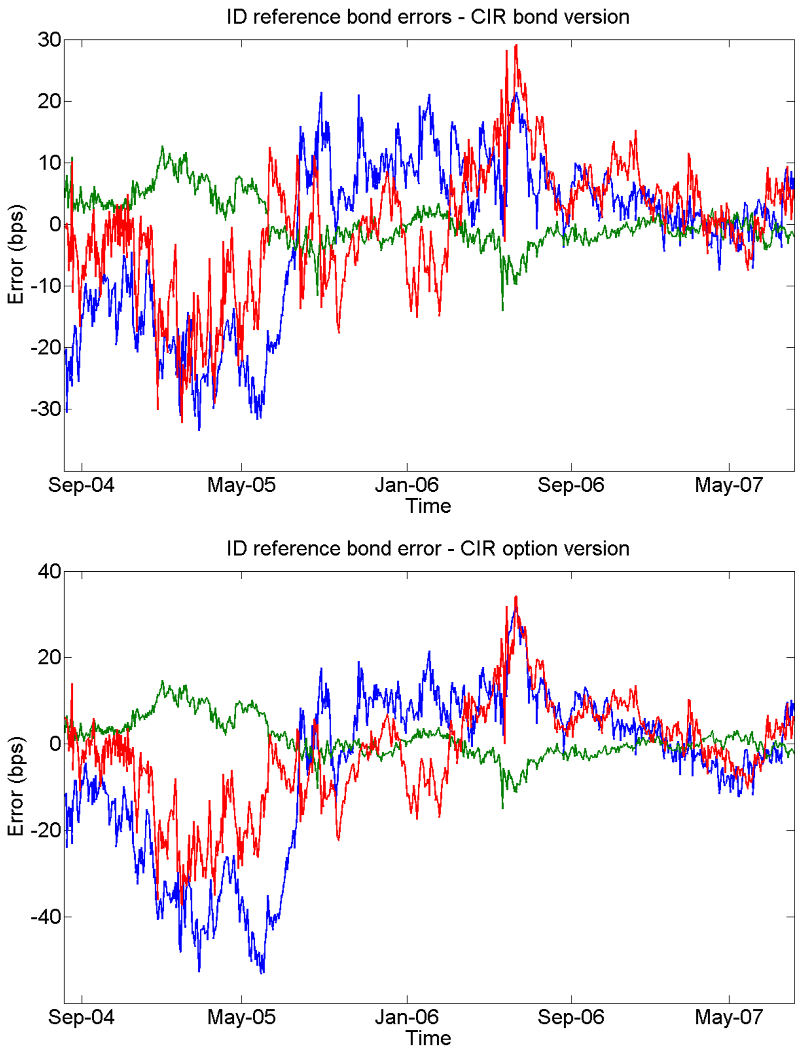

Figure 2: Pricing errors for yields observed with error.
This figure contains the time series, between August 9, 2004 and August 6, 2007, for the residuals of yields priced with error. Time to maturity of those particular yields were 63, 189, and 378 days. Error terms were estimated under the assumption of i.i.d. Gaussian distributions.

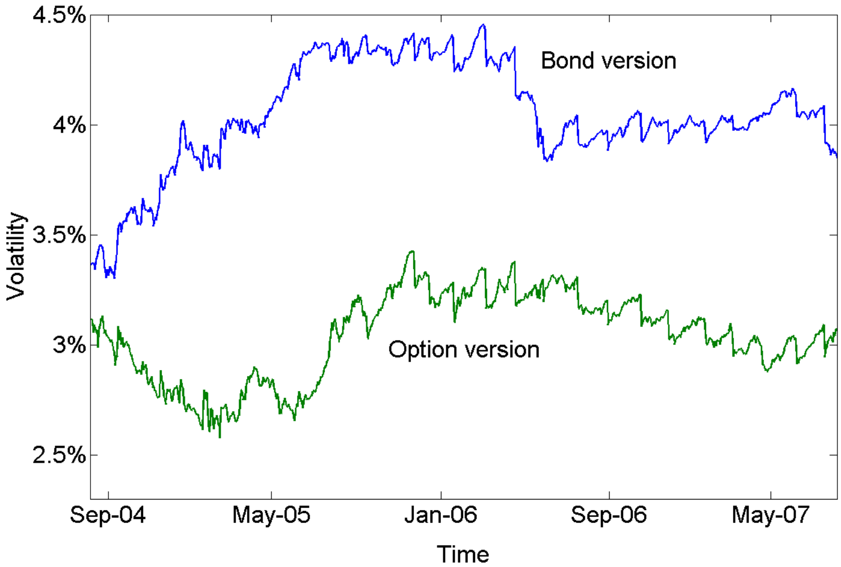

Figure 3: The instantaneous volatility of the short-term rate. This figure contains the instantaneous volatility of the short-term interest rate implied by each model version, between August 9, 2004 and August 6, 2007. The instantaneous implied volatility of short-term rate is given by $\sqrt{\sigma_1^2 X_1(t) + \sigma_2^2 X_2(t) + \sigma_3^2 X_3(t)}$.

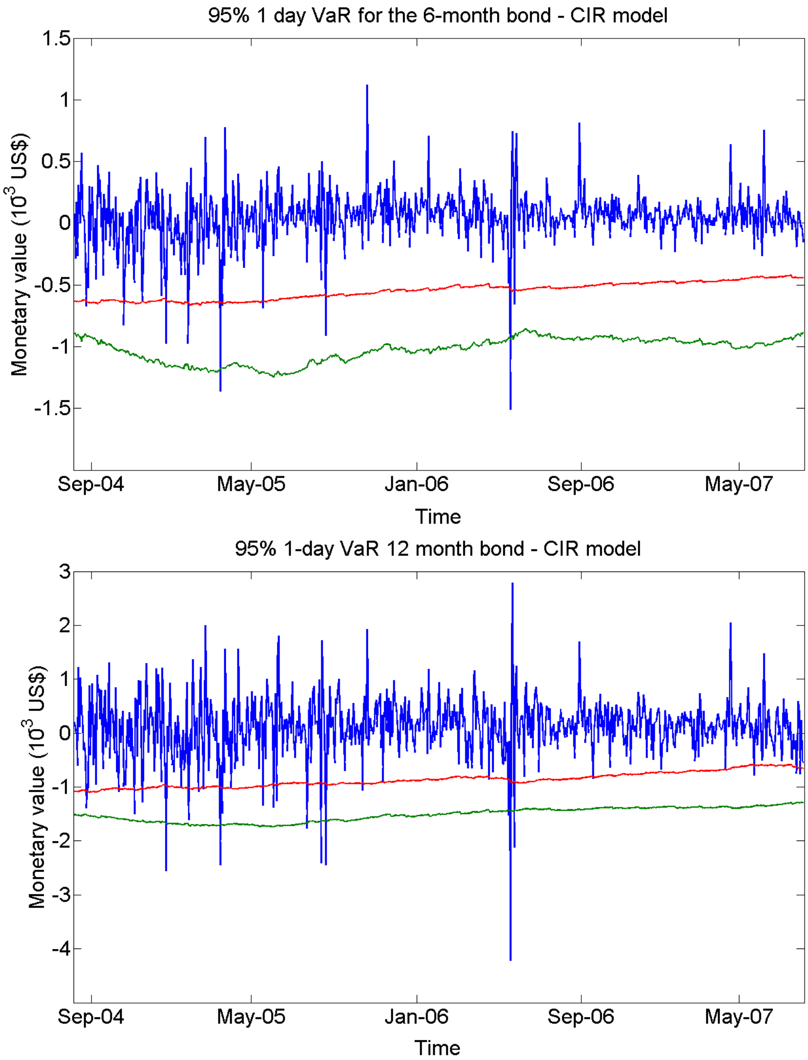

Figure 4: P&L and 95% 1-day VaR for zero coupon-bonds. This figure shows the time series of 95% 1-day VaR and P&L for Brazilian zero coupon bonds between August, 2004 and August, 2007. VaR were estimated using both versions of the dynamic term structure model. The top panel presents the evolution of the VaR for the 6-month bond and the bottom panel presents the evolution of VaR for the 12-month bond.

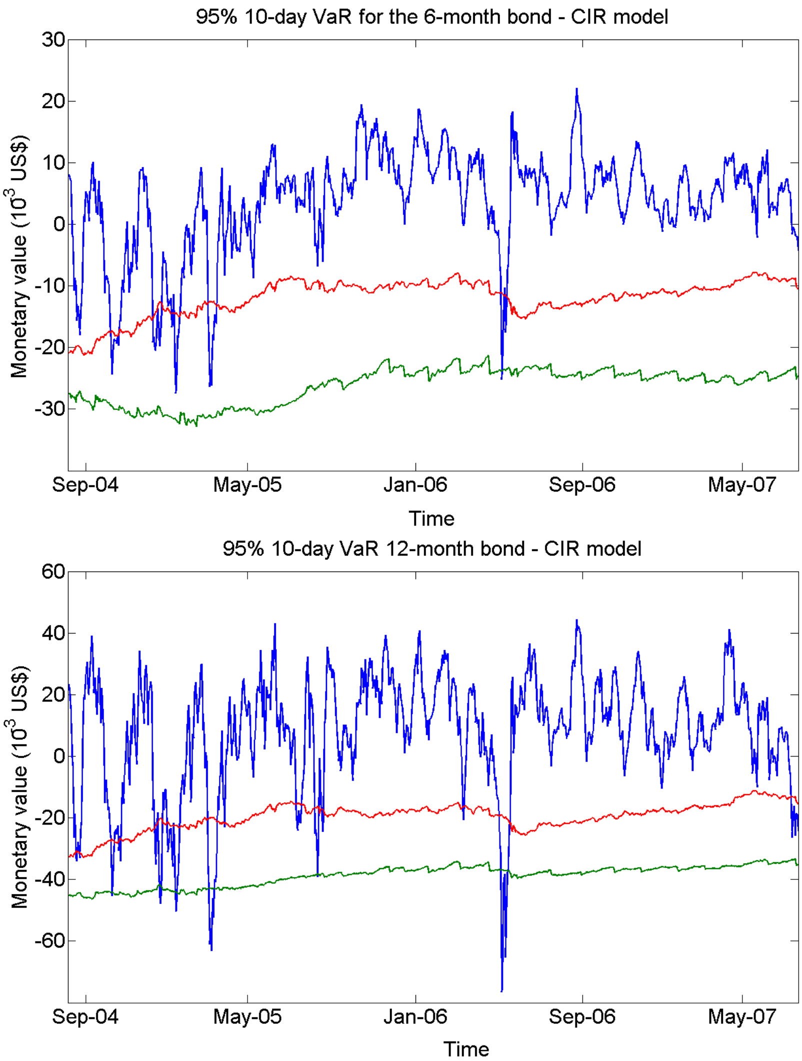

Figure 5: P&L and 95% 10-day VaR for zero coupon-bonds. This figure shows the time series of 95% 10-day VaR and P&L for Brazilian zero coupon bonds between August, 2004 and August, 2007. VaR were estimated using both versions of the dynamic term structure model. The top panel presents the evolution of the VaR for the 6-month bond and the bottom panel presents the evolution of VaR for the 12-month bond.

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