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Carlos Enrique Carrasco Gutierrez and Wagner Piazza Gaglianone

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Caixa Postal 8.670

70074-900 Brasília – DF – Brazil

Phones: +55 (61) 3414-3710 and 3414-3567

Fax: +55 (61) 3414-3626

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Evaluating Asset Pricing Models in a Fama-French Framework

Carlos Enrique Carrasco Gutierrez*

Wagner Piazza Gaglianone†

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Abstract

In this work we propose a methodology to compare different stochastic discount factor (SDF) proxies based on relevant market information. The starting point is the work of Fama and French, which evidenced that the asset returns of the U.S. economy could be explained by relative factors linked to characteristics of the firms. In this sense, we construct a Monte Carlo simulation to generate a set of returns perfectly compatible with the Fama and French factors and, then, investigate the performance of different SDF proxies. Some goodness-of-fit statistics and the Hansen and Jagannathan distance are used to compare asset pricing models. An empirical application of our setup is also provided.

Keywords: Asset Pricing, Stochastic Discount Factor, Hansen-Jagannathan distance.

JEL Classification: G12, C15, C22.

*Corresponding author. FUCAPE Business School. Vitória ES-Brazil and Graduate School of Economics, Getulio Vargas Foundation, Praia de Botafogo 190, s.1104, Rio de Janeiro, Brazil (e-mail: carlos.gutierrez@fucape.br).

†Research Department, Central Bank of Brazil (e-mail: wagner.gaglianone@bcb.gov.br).

1 Introduction

In this work, we propose a new methodology to compare different *stochastic discount factor* or *pricing kernel* proxies.¹ In asset pricing theory, one of the major interests for empirical researchers is oriented by testing whether a particular asset pricing model is (indeed) supported by the data. In addition, a formal procedure to compare the performance of competing asset pricing models is also of great importance in empirical applications.

In both cases, it is of utmost relevance to establish an objective measure of model misspecification. The most useful measure is the well-known Hansen and Jagannathan (1997) distance (or simply *HJ-distance*), which has been used both as a model diagnostic tool and as a formal criterion to compare asset pricing models. This type of comparison has been employed in many recent papers.²

As argued by Hansen and Richard (1987), observable implications of candidate models of asset markets are conveniently summarized in terms of their implied stochastic discount factors. As a result, some recent studies of the asset pricing literature have been focused on proposing an estimator for the SDF and also on comparing competing pricing models in terms of the SDF model. For instance, see Lettau and Ludvigson (2001b), Chen and Ludvigson (2008), Araujo, Issler and Fernandes (2006).

A different route to investigate and compare asset pricing models has also been suggested in the literature. The main idea is to assume a data generation process (DGP) for a set of asset returns, based on some assumptions about the asset prices and, then, create a *controlled framework*, which is used to evaluate and compare the asset pricing models.

In this sense, Fernandes and Vieira (2006) study through Monte Carlo simulations the performance of different SDF estimatives at different environments. For instance, the authors consider that all asset prices follow a geometric Brownian motion.

¹We use the term "stochastic discount factor" as a label for a state-contingent discount factor.

²For instance, by using the HJ-distance, Campbell and Cochrane (2000) explain why the CAPM and its extensions better approximate asset pricing models than the standard consumption based model; Jagannathan and Wang (2002) compare the SDF method with Beta method in estimating a risk premium; Dittmar (2002) uses the HJ-distance to estimate the nonlinear pricing kernels in which the risk factor is endogenously determined and preferences restrict the definition of the pricing kernel. Other examples in the literature include Jagannathan, Kubota and Takehara (1998), Farnsworth, Ferson, Jackson, and Todd (2002), Lettau and Ludvigson (2001a) and Chen and Ludvigson (2008).

In this case, one should expect that a SDF proxy based on a geometric Brownian motion assumption would have a better performance, in comparison to an asset pricing model that does not assume this hypothesis. The authors also study competing asset pricing models in a stationary Ornstein-Uhlenbeck process as done in Vasicek (1977).

However, a critical issue of this procedure is that the best asset pricing model inside these particular environments (i.e., when the asset prices are supposed to follow a geometric Brownian motion or a stationary Ornstein-Uhlenbeck process), might not be a good model in the real world. In other words, the best estimator for each *controlled framework* might not necessarily exhibit the same performance for observed stock market prices of a real economy.

In this paper, we use the controlled approach of Fernandes and Vieira (2006), but instead of generating the asset returns from an *ad-hoc* assumption about the DGP of returns, we use related market information from the real economy. Our starting point is the work of Fama and French, which evidenced that asset returns of the U.S. economy could be explained by relative factors linked to characteristics of the firms³.

Based on the Fama and French factors, we firstly construct a Monte Carlo simulation to generate a set of returns that is perfectly compatible with these factors. The next step is to create a framework to compare the competing asset pricing models. To do so, we consider two sets of returns: The first sample is used to estimate the different SDF proxies, whereas the remaining sample is used to analyze the out-of-sample performance of each asset pricing model. Although we do not directly use market returns data in this paper, we are able to compare different SDFs by using important market information provided by the Fama-French factors.⁴

Finally, because our approach enables us to construct a data generation process of the SDF provided by the Fama and French specification, it is possible to compare competing proxies through some goodness-of-fit statistics. In addition, it is relevant to test if a set of SDF candidates satisfy the law of one price, such that $1 = E_t(m_{t+1}R_{i,t+1})$, where m_{t+1} is referred to the investigated stochastic discount factor. Thus, we say that a SDF correctly "prices" the assets if this equation is (in fact) satisfied. In this sense, we test the previous restriction by evaluating, out-of-sample, the HJ-distance of each SDF candidate model.

³Fama and French (1993, 1995) argue that a three-factor model is successful because it proxies for unobserved common risk in portfolio returns.

⁴Notice that this procedure could also be adopted to compare models by using real data, but with some limitations since the DGP would be unknown.

As shown by Hansen and Jagannathan, the HJ-distance $\delta = \min_{m \in \mathcal{M}} \|y - m\|$, defined in the L^2 space, is the distance of the SDF model y to a family of SDFs, $m \in \mathcal{M}$, that correctly price the assets. In other interpretation, Hansen and Jagannathan show that the HJ-distance is the pricing error for the portfolio that is most mispriced by the underlying model. In this sense, even though the investigated SDF models are misspecified, in practical terms, we are interested in those models with the lowest HJ-distance.

The main objective here is not to propose a DGP process of actual market returns, but to provide a controlled environment that allows one to properly compare and evaluate different SDF proxies. This work follows the idea of Farnsworth et al. (2002), which study different SDFs by constructing artificial mutual funds using real stock returns from the CRSP data.

To illustrate our methodology, we present an empirical application, in which three SDF models are compared: a) The novel nonparametric estimator of Araujo, Issler and Fernandes (2006); b) The Brownian motion pricing model studied in Brandt, Cochrane and Saint-Clara (2006); and c) The (traditional) unconditional linear CAPM.

This work is organized as follows: Section 2 presents the Fama and French model and describes the Monte Carlo simulation strategy; Section 3 presents the results of the empirical application; and Section 4 shows the main conclusions.

2 The stochastic discount factor and the Fama and French model

A general framework to asset pricing is well described in Harrison and Kreps (1979), Hansen and Richard (1987) and Hansen and Jagannathan (1991), associated to the stochastic discount factor (SDF), which relies on the pricing equation:

$$p_t = E_t(m_{t+1}x_{i,t+1}), \quad (1)$$

where $E_t(\cdot)$ denotes the conditional expectation given the information available at time t , p_t is the asset price, m_{t+1} the stochastic discount factor, $x_{i,t+1}$ the asset payoff of the i -th asset in $t+1$. This pricing equation means that the market value today of an uncertain payoff tomorrow is represented by the payoff multiplied by the discount factor, also taking into account different states of nature by using the underlying probabilities.

The stochastic discount factor model provides a general framework for pricing assets. As documented by Cochrane (2001), asset pricing can basically be summarized by two equations:

$$p_t = E_t [m_{t+1}x_{t+1}], \quad (2)$$

$$m_{t+1} = f(\text{data, parameters}). \quad (3)$$

where the model is represented by the function $f(\cdot)$, and the (2) can lead to different predictions stated in terms of returns. For instance, in the Consumption-based Capital Asset Pricing Model (CCAPM) context, the first-order conditions of the consumption-based model, summarized by the well-known Euler equation: $p_t = E_t \left[\beta \frac{u'(c_{t+1})}{u'(c_t)} x_{t+1} \right]$. The specification of m_{t+1} corresponds to the intertemporal marginal rate of substitution. Hence, $m_{t+1} = f(c, \beta) = \beta \frac{u'(c_{t+1})}{u'(c_t)}$, where β is the discount factor for the future, c_t is consumption and $u(\cdot)$ is a given utility function. The pricing equation (2) mainly illustrates the fact that consumers (optimally) equate marginal rates of substitution to prices.

2.1 Fama and French framework

Fama and French (1992) show that, besides the market risk, there are other important factors that help explain the average return in the stock market. This evidence has been demonstrated in several works for different stock markets (see Gaunt (2004) and Griffin (2005) for a good review). Although there is not a clear link between these factors and the economic theory (e.g., CAPM model), these evidences show that some additional factors might (quite well) help to understand the dynamics of the average return.

These factors are known as the *size* and the *book-to-market equity* and represent special features about firms. Fama and French (1992) argue that size and book-to-market equity are indeed related to economic fundamentals. Although they appear to be "*ad hoc* variables" in an average stock returns regression, these authors justify them as expected and natural proxies for common risk factors in stock returns.

The factors

(i) *The SMB (Small Minus Big) factor* is constructed to measure the *size premium*. In fact, it is designed to track the additional return that investors have historically received by investing in stocks of companies with relatively small market capitalization. A positive SMB in a given month indicates that small cap stocks have outperformed the large cap stocks in that month. On the other hand, a negative SMB suggests that large caps have outperformed.

(ii) The *HML (High Minus Low)* factor is constructed to measure the *premium-value* provided to investors for investing in companies with high book-to-market values. A positive HML in a given month suggests that “value stocks” have outperformed the “growth stocks” in that month, whereas a negative HML indicates that growth stocks have outperformed.⁵

(iii) The *Market factor* is the market excess return in comparison to the risk-free rate. For instance, we proxy the excess return on the market ($R_M - R_f$), in the U.S. economy, by the value-weighted portfolio of all stocks listed on the New York Stock Exchange (NYSE), the American Stock Exchange (AMEX), and NASDAQ stocks (from CRSP) minus the one-month Treasury Bill rate.

The Model

Fama and French (1993, 1996) propose a three-factor model for expected returns (see also Fama and French (2004) for a good survey).

$$E(R_{it}) - R_{ft} = \beta_{im} [E(R_{Mt}) - R_{ft}] + \beta_{is} E(SMB_t) + \beta_{ih} E(HML_t), \quad i \in \{1, \dots, N\}, \quad (4)$$

where the betas β_{im} , β_{is} and β_{ih} are slopes in the multiple regression (4). Hence, one implication of the expected return equation of the three-factor model is that the intercept in the time-series regression (5) is zero for all assets i :

$$R_{it} - R_{ft} = \beta_{im} (R_{Mt} - R_{ft}) + \beta_{is} SMB_t + \beta_{ih} HML_t + \varepsilon_{it}. \quad (5)$$

Using this criterion, Fama and French (1993, 1996) find that the model captures much of the variation in the average return for portfolios formed on size, book-to-market equity and other price ratios.

Expected return - beta representation

The Fama and French approach is (in fact) a multifactor model that can be seen as an expected-beta⁶ representation of linear factor pricing models of the form:

$$E(R_i) = \gamma + \beta_{im} \lambda_m + \beta_{is} \lambda_s + \beta_{ih} \lambda_h + \alpha_i, \quad i \in \{1, \dots, N\}. \quad (6)$$

⁵Notice that, in respect to SMB, small companies logically are expected to be more sensitive to many risk factors, as a result of their relatively undiversified nature, and also their reduced ability to absorb negative financial events. On the other hand, the HML factor suggests higher risk exposure for typical value stocks in comparison to growth stocks.

⁶The main objective of the beta model is to explain the variation in terms of average returns across assets.

If we run this cross sectional regression of average returns on betas, one can estimate the parameters $(\gamma, \lambda_m, \lambda_s, \lambda_h)$. Notice that γ is the intercept and λ_m, λ_s and λ_h the slope in this cross-sectional relation. In addition, the β_{im}, β_{is} and β_{ih} are the unconditional sensitivities of the i -th asset to the factors⁷. Moreover, β_{ij} , for some $j \in \{m, s, h\}$, can be interpreted as the amount of risk exposure of asset i to factor j , and λ_j as the price of such risk exposure. Hence, the betas are defined as the coefficients in a multiple regression of returns on factors:

$$R_{it} - R_{ft} = \beta_{im}R_{Mt}^{ex} + \beta_{is}SMB_t + \beta_{ih}HML_t + \varepsilon_{it}, \quad t \in \{1, \dots, T\}, \quad (7)$$

where $R_{Mt}^{ex} = (R_{Mt} - R_{ft})$. Following the equivalence between this beta-pricing model and the linear model for the discount factor M , in an unconditional setting (see Cochrane, 2001), we can estimate M as:

$$M = a + b'f, \quad (8)$$

where $f = [R_M^{ex}, SMB, HML]'$, and the relations between λ e γ , and a and b , are given by:

$$a = \frac{1}{\gamma} \quad \text{and} \quad b = -\gamma [\text{cov}(ff')]^{-1} \lambda. \quad (9)$$

2.2 Evaluating the performance of competing models

In the asset pricing literature, some measures are suggested to compare competing asset pricing models. The most famous measure is the Hansen and Jagannathan distance. However, as long as the data generation process (DGP) is known in each specification of the Fama and French model, it is also possible to use some simple sample statistics. In addition, we use the Hansen and Jagannathan distance to test for model misspecification and to compare the performance of different asset pricing models.

The Hansen-Jagannathan (1997) distance measure is a summary of the mean pricing errors across a group of assets. It may also be interpreted as the distance between the SDF candidate and one that would correctly price the primitive assets. The pricing error can be written by $\alpha_t = E_t(m_{t+1}R_{i,t+1}) - 1$. Notice, in particular, that α_t depends on the considered SDF, and the SDF is not unique (unless markets are complete). Thus, different SDF proxies can produce similar HJ measures. In this sense, even though the investigated SDF models are misspecified, in practical terms, we are interested in those models with the lowest HJ-distance.

⁷An unconditional time-series approach is used here. The conditional approaches to test for international pricing models include those by Ferson & Harvey (1994, 1999) and Chan, Karolyi and Stulz (1992).

Goodness-of-fit statistics

We use two goodness-of-fit statistics to compare different SDF proxies. The \widehat{MSE}_s is merely a standardized version of the mean squared error of the SDF proxies, whereas the $\widehat{\gamma}_s$ compares the sample correlation between the actual and estimated stochastic discount factors. Let M_t be the stochastic discount factor generated by the Fama and French specification (DGP), and \widehat{M}_t^s the SDF proxy provided by model s in a family S of asset pricing models. The standardized mean squared error is computed as:

$$\widehat{MSE}_s = \frac{\sum_{t=1}^T (\widehat{M}_t^s - M_t)^2}{\sum_{t=1}^T M_t^2}, \quad \text{for } s \in S. \quad (10)$$

and the sample correlation between the actual and estimated SDF is given by:

$$\widehat{\gamma}_s = \text{corr}(\widehat{M}_t^s, M_t), \quad \text{for } s \in S. \quad (11)$$

2.3 Constructing the Fama and French environment

Based on the assumption that R_{Mt} , SMB_t and HML_t are known variables, we can reproduce a Fama and French environment following the three factors of the Fama and French model:

$$R_{i,t} - R_{ft} = \beta_{im}(R_{Mt} - R_{ft}) + \beta_{is}SMB_t + \beta_{ih}HML_t + \varepsilon_{it}. \quad (7)$$

The simulated asset returns are generated using equation (7). This way, we propose the following steps of a Monte Carlo simulation:

1) Firstly, calibrate each parameter β_{ij}^k , for $j \in \{m, s, h\}$ and $i \in \{1, \dots, N\}$ according to previous estimations of Fama and French (1992,1993). Therefore, we will generate for each j a N -dimensional vector of asset returns.

2) By considering β_{ij}^k created in step 1 for some $i \in \{1, \dots, N\}$ and using the known factors R_{Mt} , SMB_t and HML_t , we generate a vector of returns along the time dimension, through equation (7). The *iid* shock ε_{it} is assumed to be a white noise with zero mean and constant variance.

3) Repeating step 2 for each $i \in \{1, \dots, N\}$, we create the matrix \mathbf{R}^k of asset returns, in which rows are formed by different returns and columns represent the time dimension.

4) Evaluate the mean of \mathbf{R}^k across each row to generate a cross-section vector. Now, it is possible to estimate the parameters γ^k and λ^k through equation (6).

5) Estimate parameters a^k and b^k from the equivalence relation shown in equation (9). Finally, the stochastic discount factor can be estimated by using equation (8).

6) Repeat steps 1 to 5 for an amount of K replications in order to construct the Monte Carlo simulation.

7) Since our approach enables us to construct a data generation process of the SDF provided by the Fama and French specification (computed with N assets), it is possible to compare the competing SDF proxies, obtained in steps 1 to 6, through the goodness-of-fit statistics described in the previous section, as it follows:

7.a) Split the set of N assets into two groups (with the same number of time series observations in each group). Firstly, consider an amount of $\tilde{N} < N$ assets to estimate the SDF candidates (henceforth, this first group of assets will be denominated *in-sample*). Based on the estimated SDF proxies (\widehat{M}_t^s) we compute the *in-sample* goodness-of-fit statistics \widehat{MSE}_s and $\widehat{\gamma}_s$, in order to compare every SDF proxy with the correct SDF provided by the Fama and French setup. Secondly, the remaining ($N - \tilde{N}$) assets are used to generate the *out-of-sample* to compute the Hansen and Jagannathan distance. That is, we want to know how well the proxies are carried on when new information is considered.

3 Empirical Application

In this section, we present a simple empirical exercise of our proposed framework for the U.S economy. Three asset pricing models discussed in the literature are compared:

A. The Brownian motion pricing model (studied in Brandt et al., 2006)

Brandt, Cochrane and Santa-Clara (2006) consider that the asset prices follow a geometric Brownian motion (GBM). Such hypothesis is defined by the following partial differential equation:

$$\frac{dP}{P} = (R^f + \mu) dt + \Sigma^{\frac{1}{2}} dB, \quad (12)$$

where, $\frac{dP}{P} = \left(\frac{dP_1}{P_1}, \dots, \frac{dP_N}{P_N} \right)'$, $\mu = (\mu_1, \dots, \mu_n)'$, Σ is a $N \times N$ positive definite matrix, P_i is the price of the asset i , μ the risk premium vector, R^f the risk free rate, and B a standard GBM of

dimension N . Using Itô theorem, it is possible to show that:

$$R_{t+\Delta t}^i = \frac{P_{t+\Delta t}^i}{P_t^i} = e^{(R^f + \mu_i - \frac{1}{2}\Sigma_{i,i})\Delta t + \sqrt{\Delta t} \left(\Sigma_i^{\frac{1}{2}}\right)' Z_t}, \quad (13)$$

where Z_t is a vector of N independent variables with Gaussian distribution. Therefore, the SDF proposed by these authors is calculated as

$$M_{t+\Delta t} = e^{-(R^f + \frac{1}{2}\mu' \Sigma^{-1} \mu)\Delta t - \sqrt{\Delta t} \mu' (\Sigma^{-\frac{1}{2}})' Z_t}. \quad (14)$$

Thus, Brandt, Cochrane and Santa-Clara (2006) suggest the following SDF estimator:

$$\widehat{M}_t = e^{-(R^f + \frac{1}{2}\widehat{\mu}' \widehat{\Sigma}^{-1} \widehat{\mu})\Delta t - \widehat{\mu}' \widehat{\Sigma}^{-1} (R_t - \bar{R})}, \quad (15)$$

where, $\widehat{\mu}$, \bar{R} and $\widehat{\Sigma}$ are estimated by:

$$\widehat{\mu} = \frac{\bar{R} - R^f}{\Delta t}, \quad (16)$$

$$\widehat{\Sigma} = \frac{1}{\Delta t} \frac{1}{T} \sum_{t=1}^T (R_t - \bar{R}) (R_t - \bar{R})', \quad (17)$$

such that, $R_t = (R_t^1, \dots, R_t^N)'$ and $\bar{R} = \frac{1}{T} \sum_{t=1}^T R_t$.

B. Araujo, Issler and Fernandes (2006)

A novel estimator for the stochastic discount factor (within a panel data context) is proposed by Araujo, Issler and Fernandes (2006). This setting is slightly more general than the GBM setup put forth by Brandt, Cochrane and Santa-Clara (2006). In fact, this estimator assumes that, for every asset $i \in \{1, \dots, N\}$, $M_{t+1} R_{t+1}^i$ is conditionally homoskedastic and has a lognormal distribution. In addition, under asset pricing equation (1) and some mild additional conditions, they show that a consistent estimator for M_t is given by:

$$\widehat{M}_t = \left(\frac{\bar{R}_t^G}{\frac{1}{T} \sum_{t=1}^T \bar{R}_t^A \bar{R}_t^G} \right), \quad (18)$$

where $\bar{R}_t^A = \frac{1}{N} \sum_{i=1}^N R_{i,t}$ and $\bar{R}_t^G = \Pi_{i=1}^N R_{i,t}^{-\frac{1}{N}}$ are respectively the cross-sectional arithmetic and geometric average of all gross returns. Therefore, this nonparametric estimator depends exclusively on appropriate averages of asset returns that can easily be implemented.

C. Capital Asset Pricing Model - CAPM

Assuming the unconditional CAPM, the SDF is a linear function of market returns calculated as: $m_{t+1} = a + bR_{w,t+1}$, where $R_{w,t+1}$ is the gross return on the market portfolio of all assets. For instance, in the U.S. economy, in order to implement the static CAPM, for practical purposes, it is commonly assumed that the return on the value-weighted portfolio of all stocks listed on NYSE, AMEX, and NASDAQ is a reasonable proxy for the return on the market portfolio of all assets of the U.S. economy.

3.1 Monte Carlo design

In order to compare these three SDF proxies we construct the Monte Carlo experiment following the procedure showed in section 2.3. For the U.S. economy, the factors $(R_{Mt} - R_{ft})$, SMB_t and HML_t are extracted from the Kenneth R. French website⁸. Next, we calibrate the parameters β_{im} , β_{is} and β_{ih} according to previous estimations of Fama and French (1992,1993) and estimate the parameters $(\gamma, \lambda_m, \lambda_s, \lambda_h)$ from the cross-sectional regression (6), observing their significance through the F -statistic or the t -statistic for individual parameters.

We set $N = 36$ as our set of primitive assets, which are divided into two groups: The first one contains $\tilde{N} = 18$ assets that are used for the *in-sample* estimation. The second group has $(N - \tilde{N}) = 18$ assets, which are thus used for the *out-of-sample* analysis. We also consider, for each generated asset i , three sample sizes $T = \{200; 300; 400\}$.

This way, we estimate the stochastic discount factors for the three-factor model of Fama and French, and repeat the mentioned procedure for an amount of $K = 1,000$ replications. Some descriptive statistics of the generated SDFs are presented in appendix. Finally, the evaluation of the SDF proxies is conducted and the Monte Carlo results are summarized by two goodness-of-fit statistics (besides the HJ-distance), which are averaged across all replications.

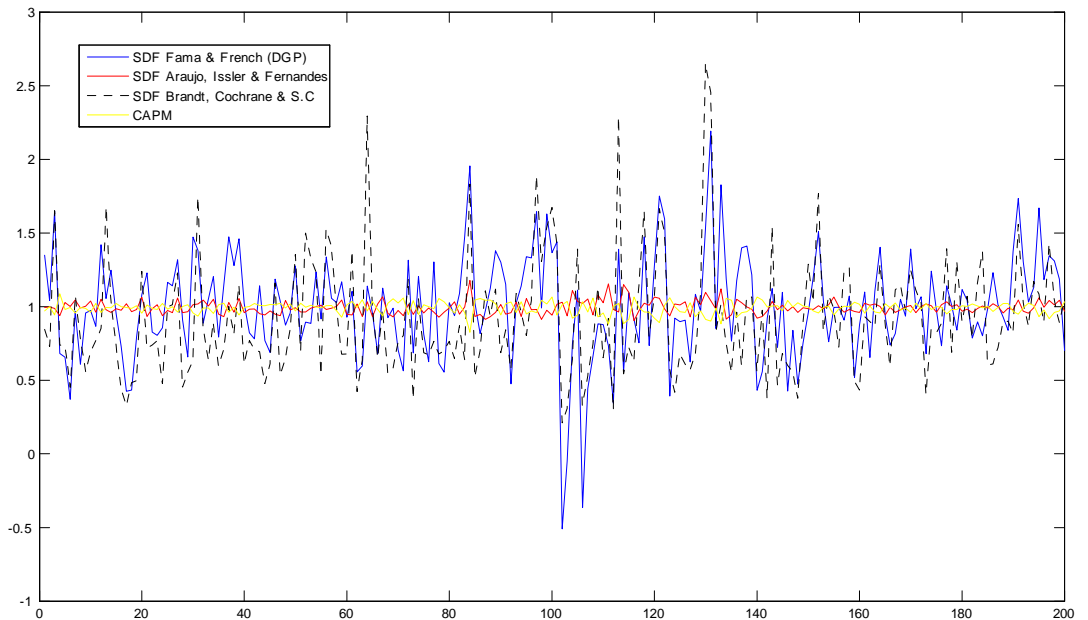
We denote the SDF proxies, estimated in each replication, as \widehat{M}_t^a , \widehat{M}_t^b and \widehat{M}_t^c to Araujo, Issler and Fernandes (2006), Brandt, Cochrane and Santa-Clara (2006) and the unconditional CAPM respectively. In addition, the stochastic discount factor implied by the Fama and French setup (DGP) is denoted by M_t .

⁸More information about data can be found in: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html For other economies, the factors can be constructed as showed in Fama and French (1992, 1993).

3.2 Results

In Figure 1, the estimates of the SDF proxies are shown for one replication of the Monte Carlo simulation, with a sample size $T = 200$. A simple graphical investigation reveals that the Brandt, Cochrane and Santa-Clara, \widehat{M}_t^b , and the CAPM proxy, \widehat{M}_t^c , are respectively the most and less volatile, which is a result confirmed by the descriptive statistics of Table 2 (in appendix). In addition, \widehat{M}_t^b appears to be the SDF proxy that best tracks the DGP M_t .

Figure 1 - Three factors, with a sample size $T = 200$



Notes: a) Figure 1 shows one replication out of the total amount of 1,000 replications.

b) We adopt $\tilde{N} = 18$ assets and $T=200$ observations.

Regarding the performance of the SDF proxies, Table 1 reports the evaluation statistics provided by the Monte Carlo simulation. Notice that results are robust to sample size. In all cases, the mean square error of Brandt, Cochrane and Santa-Clara (2006) SDF proxy (\widehat{MSE}_b) shows quite a good

performance, whereas the CAPM proxy seems to exhibit the worst one. Nonetheless, the magnitude of the standard deviation might suggest that all these values are quite close to each other.

In respect to the correlation of the true SDF with the considered SDF proxies, we have obtained the following ranking order for all sample sizes: $\widehat{M}_t^b \succ \widehat{M}_t^a \succ \widehat{M}_t^c$. This implies that the Brandt, Cochrane and Santa-Clara (2006) proxy (in general) best tracks the dynamic path of the true SDF. On the other hand, the CAPM model exhibits again the worst performance (with a negative correlation in some cases!)

Finally, in respect to the *out-of-sample* analysis, the HJ distance results⁹ (which should be as close as possible to zero in a correctly-specified model) indicate that for $T = 200$ and $T = 300$: $\widehat{HJ}_b < \widehat{HJ}_a < \widehat{HJ}_c$, revealing that the Brandt, Cochrane and Santa-Clara (2006) is the best proxy for forecasting purposes, followed by the Araujo et al. (2006) SDF estimator. For $T = 400$ we obtained similar results, except that in this case the CAPM model has a lower HJ-distance in comparison to the Araujo et al. (2006) proxy.¹⁰

Putting all together, the numerical results show that (in general) the Brandt, Cochrane and Santa-Clara (2006) has the best *out-of-sample* performance. Notice that Figure 1 already showed this tendency, since the referred SDF best tracked the respective Fama-French DGP.

Finally, the CAPM model shows a negative correlation with the true SDF, revealing its weakness in tracking the real dynamic of the true SDF. This result is because the linear CAPM only uses one single factor, out of the three factors correct-specification in the Fama-French setup. This way, our methodology allows one to rank the competing SDF models (according to different evaluation criteria), based on simulated data generated from U.S. market information.

⁹We compute the HJ distance based on the MatLab codes of Mike Cliff, available at: <http://mcliff.cob.vt.edu/>

¹⁰The standard error of the HJ-distance is estimated by a Newey & West (1987) HAC procedure, in which the optimal bandwidth (number of lags=5) is given by $m(T) = \text{int}(T^{1/3})$, where $\text{int}(\cdot)$ represents the integer part of the argument, and T is the sample size. The adopted kernel used to smooth the sample autocovariance function is given by a standard modified Bartlett kernel: $w(j, m(T)) = 1 - [j/\{m(T) + 1\}]$. See Newey & West (1994) for an extensive discussion about lag selection in covariance matrix estimation, and also Kan & Robotti (2008).

Table 1 - Monte Carlo Simulation Results

sample size: 200 (Over the time period from 09/1999 to 12/2007)					
\widehat{MSE}_a	\widehat{MSE}_b	\widehat{MSE}_c	$\hat{\gamma}_a$	$\hat{\gamma}_b$	$\hat{\gamma}_c$
0.0962	0.1070	0.1056	0.2645	0.6429	-0.0113
(0.0228)	(0.0374)	(0.0298)	(0.1106)	(0.0720)	(0.4387)
\widehat{HJ}_a -distance		\widehat{HJ}_b -distance		\widehat{HJ}_c -distance	
0.4114		0.3227		0.4207	
(0.0806)		(0.0760)		(0.0792)	
sample size: 300 (Over the time period from 05/1991 to 12/2007)					
\widehat{MSE}_a	\widehat{MSE}_b	\widehat{MSE}_c	$\hat{\gamma}_a$	$\hat{\gamma}_b$	$\hat{\gamma}_c$
0.0796	0.0722	0.0923	0.3301	0.6989	-0.1041
(0.0182)	(0.0221)	(0.0242)	(0.0895)	(0.0626)	(0.4399)
\widehat{HJ}_a -distance		\widehat{HJ}_b -distance		\widehat{HJ}_c -distance	
0.3489		0.2588		0.3631	
(0.0660)		(0.0606)		(0.0643)	
sample size: 400 (Over the time period from 09/1974 to 12/2007)					
\widehat{MSE}_a	\widehat{MSE}_b	\widehat{MSE}_c	$\hat{\gamma}_a$	$\hat{\gamma}_b$	$\hat{\gamma}_c$
0.0779	0.0608	0.0702	0.3423	0.7182	0.4319
(0.0153)	(0.0161)	(0.0160)	(0.0933)	(0.0551)	(0.2351)
\widehat{HJ}_a -distance		\widehat{HJ}_b -distance		\widehat{HJ}_c -distance	
0.3305		0.2275		0.3227	
(0.0553)		(0.0520)		(0.0556)	

Notes: a) We simulate a panel with 25 asset returns from a Fama and French model

of the form: $R_{i,t} - R_{ft} = \beta_{im}(R_{Mt} - R_{ft}) + \beta_{is}SMB_t + \beta_{ih}HML_t + \varepsilon_{it}$.

b) All results are averaged across the 1,000 replications. The MSE and γ are computed "in-sample", i.e., $N=18$, whereas the HJ-distance is calculated from the "out-of-sample" set of $(N-\tilde{N})=18$ assets.

The standard deviation is presented in parentheses.

c) The calibrated parameters varies from $\beta_{im} \in [0.1, 0.9]$; $\beta_{is} \in [-1.4, 1.6]$; $\beta_{ih} \in [-0.73, 8.7]$ in each replication of the Monte Carlo simulation.

4 Conclusions

In the present work, we propose a methodology to compare different stochastic discount factor models based on relevant market information. Based on the Fama and French factors, which are linked to characteristics of the firms in a particular economy, a Monte Carlo simulation strategy is proposed in order to generate a set of artificial returns that is perfectly compatible with those factors.

This way, we construct a *Fama-French world* through numerical simulations, in which SDF proxies are compared through some goodness-of-fit statistics and the Hansen and Jagannathan distance. An empirical application is provided to illustrate our methodology, in which returns time series are produced from factors such as the market portfolio return, size and book-to-market equity of the U.S. economy. The results reveal that the Brandt, Cochrane and Saint-Clara (2006) proxy dominates the other considered SDF estimators.

Therefore, the main contribution of this paper consists in a methodology to compare SDF models in a setup where the Fama and French factors are supposed to summarize the economic environment. This controlled framework allows one to use simple sample statistics to compare SDF candidates with the *true SDF* implied by the Fama and French DGP and, then, rank competing asset pricing models. In this case, the hypothesis of geometric Brownian motion, usually adopted in several empirical studies, seems to be quite reasonable for the simulated set of returns.

As a natural extension of this work, the proposed methodology could easily be adapted to compare asset pricing models based on real asset returns data. For instance, a principal component technique could be employed to generate factors from "real world" variables and, thus, these new factors could be used to generate a controlled environment in which SDF models are properly compared.

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Appendix

Table 2 - Descriptive statistics of the SDF

sample size = 200				
	Araujo	Saint Clara	CAPM	Fama & French DGP
Mean	0,9945	0,9185	0,9921	0,9967
Median	0,9900	0,8380	0,9927	1,0002
Maximum	1,1918	2,9764	1,1627	2,1010
Minimum	0,8860	0,1867	0,8121	-0,5184
Std. Dev	0,0482	0,4194	0,0531	0,3346
Skewness	0,7922	1,5141	-0,0567	-0,5456
Kurtosis	4,6444	7,4416	4,1835	6,0446
Freq. Jarque-Bera	0,0150	0,0000	0,0000	0,0000
sample size = 300				
Mean	0,9933	0,9196	0,9902	0,9959
Median	0,9889	0,8564	0,9917	0,9878
Maximum	1,2849	2,9480	1,1451	2,1842
Minimum	0,8728	0,2381	0,7905	-0,2985
Std. Dev	0,0506	0,3647	0,0426	0,3058
Skewness	1,1507	1,5303	-0,2606	-0,2345
Kurtosis	7,4321	8,2266	6,2824	5,3050
Freq. Jarque-Bera	0,0000	0,0000	0,0000	0,0000
sample size = 400				
Mean	0,9925	0,9181	0,9942	0,9952
Median	0,9887	0,8672	0,9875	1,0042
Maximum	1,2838	3,0148	1,5317	2,1668
Minimum	0,8661	0,1674	0,6924	-0,6743
Std. Dev	0,0504	0,3355	0,0998	0,3049
Skewness	0,9412	1,6279	0,5455	-0,9058
Kurtosis	6,4386	9,6505	5,4933	9,2686
Freq. Jarque-Bera	0,0000	0,0000	0,0000	0,0000

Notes: These statistics are computed in-sample. DGP (FF) means Data-Generating Process of the Fama & French model. The number of assets in-sample and out-of-sample is $N=18$. The descriptive statistics are averaged across the $K=1,000$ replications based on the sample sizes $T=\{200,300,400\}$. For instance, for $T=200$ the Jarque-Bera statistic indicates the frequency of rejection of the normality hypothesis across the 1,000 replications (based on a 5% significance level). In this case, $T=200$, for the Araujo et al. (2006) proxy, the statistic Freq. Jarque-Bera is equal to 0.015, which means that in 1.5% of the replications the normality hypothesis is rejected at a 5% significance level.

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