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Combining Hodrick-Prescott Filtering with a Production Function Approach to Estimate Output Gap*

Marta Areosa[†]

Abstract

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The proposed methodology combines two of the most important techniques to estimate output gap: the production function approach and the Hodrick-Prescott filtering. Three main advantages can be derived from this method: (i) it adds some economic structure to a filtering method, (ii) it can be easily adapted to incorporate new characteristics into the filter and (iii) it simultaneously produces estimates for potential output and its unobservable components.

JEL Classification: C13, E23, E32
Keywords: Hodrick-Prescott filter, Production function, Kalman filter

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1 Introduction

Estimation of output gap is a problem that has been debated for a long time. This intense discussion can be explained by the fact that output gap has become one of the most important unobserved economic time series. Its relationship with inflation, known as the Phillips curve, expresses a trade-off that is not only present in many macroeconomic models but has also been very useful for central banks that implicitly or explicitly target inflation.

Many methodologies have been proposed to estimate output gap. Two of the most popular techniques are Hodrick-Prescott (HP) filtering and the production function approach. Nevertheless, these methods present some flaws. While univariate statistical methods, such as filtering, deterministic trend extraction and latent variable models, lack economic content and impose statistical relations that are difficult to justify on a theoretical basis, a great uncertainty surrounds the estimates of the components that go into the growth-accounting formulas. [1]

In the present case, I combine HP filtering with a production function approach to overcome some of the drawbacks involving both methods. While the production function is used to decompose output gap in a weighted average of unemployment gap and installed capacity utilization gap, HP filtering is used in the estimation of these three gaps. This strategy creates a multivariable filter that simultaneously produces estimates for potential output and the unobservable components of a production function. Additionally, two methods to build the filter are presented.

2 The Filter

In line with the approach presented in Proietti et al (2007), I use a Cobb-Douglas production function with constant returns to scale to assess output gap and potential output, that is,[2]

[1] See Beveridge and Nelson (1981), Clark (1986) and Watson (1987) for univariate methods used on the estimation of output gap. Proietti et al (2007) uses the production function approach.

[2] In the present work z represents the series $\{z_t\}_{t=1}^{N}$ or the column vector $(z_1, \ldots, z_N)'$.

$$
\begin{aligned}
Y_t &= A_t(K_tC_t)^\alpha(L_t(1-U_t))^{1-\alpha} \qquad (1)\\
Y_t^n &= A_t(K_tC_t^n)^\alpha(L_t(1-U_t^n))^{1-\alpha} \qquad (2)
\end{aligned}
$$

where Y is the output, Y^n is the potential output, A is the productivity factor, K is the capital stock, L is the labor force, α is the income capital share, C is the installed capacity utilization, U is the unemployment rate, U^n is the natural unemployment rate and C^n is the natural installed capacity utilization.[3]

Equation (2) emphasizes that the uncertainty of estimating potential output is derived from the estimation of the unobserved components - U^n and C^n - and the errors obtained in the assessment of capital stock, labor force, and productivity. However, the Cobb-Douglas production function helps to eliminate unnecessary data problems and measuring errors generated in the estimation of both capital stock and productivity, since with it is possible to derive an expression for potential output that does not depend on A, K and L:

$$
Y_t^n = Y_t\left(\frac{C_t^n}{C_t}\right)^\alpha\left(\frac{1-U_t^n}{1-U_t}\right)^{1-\alpha} \qquad (3)
$$

Defining $E_t \equiv 1-U_t$ and $E_t^n \equiv 1-U_t^n$, it is possible to use the following equations to compute the output gap, x_t:[4]

$$
\begin{aligned}
x_t &\equiv \ln\left(\frac{Y_t}{Y_t^n}\right) = y_t - y_t^n \qquad (4)\\
y_t^n &= y_t + \alpha\left(c_t^n - c_t\right) + (1-\alpha)\left(e_t^n - e_t\right) \qquad (5)
\end{aligned}
$$

where the lower-case variables represent the logarithms of corresponding upper-case variables.

Alternatively, if the HP filter is used in the series y to estimate its trend, the

[3] The expression *potential output* does not refer to the concept used in the widespread New Keynesian literature. For further details see Clarida et al (1999). The term *natural* refers to the level that occurs when the economy is at its potential level.

[4] E stands for employment, considering that $(1-U_t)$ is the percentage of the labor force that is employed. Thus, $(1-U_t^n)$ is the natural rate of employment.

resultant series y^n will be the solution of the following problem:[5]

$$\min_{\{y_t^n\}_{t=1}^N} \left\{ \sum_{t=1}^{N} \left(y_t^n - y_t\right)^2 + \lambda_y \sum_{t=3}^{N} \left(\Delta^2 y_t^n\right)^2 \right\} \tag{6}$$

The methodology used to build a filter that combines HP filtering and the production function approach can be summarized in two steps: (i) adding a constraint derived from a production function - equation (5) - to the optimization problem known as the HP filter - equation (6) - and (ii) extending the objective function to estimate the unobserved variables that appear in the production function, that is,

$$\begin{array}{l} \min_{\{e_t^n\}_{t=1}^N, \{c_t^n\}_{t=1}^N} \left\{ \begin{array}{c} \beta_e Op_e + \\ \beta_c Op_c + \\ \beta_y \left[\sum_{t=1}^{N} \left(y_t^n - y_t\right)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta^2 y_t^n\right)^2 \right] \end{array} \right\} \\ s.t. \\ y_t^n = y_t + \alpha \left(c_t^n - c_t\right) + (1-\alpha)\left(e_t^n - e_t\right) \end{array} \tag{7}$$

where Op_c and Op_e represent the objective functions of any optimization process used to estimate c^n and e^n. The weights β_e, β_c, and β_y quantify the relative importance between optimization problems. If the HP objective function is also used to estimate these series, the resultant filter becomes

$$\begin{array}{l} \min_{\{e_t^n\}_{t=1}^N, \{c_t^n\}_{t=1}^N} \left\{ \begin{array}{c} \beta_e \left[\sum_{t=1}^{N} \left(e_t^n - e_t\right)^2 + \lambda_e \sum_{t=3}^{N} \left(\Delta^2 e_t^n\right)^2 \right] + \\ \beta_c \left[\sum_{t=1}^{N} \left(c_t^n - c_t\right)^2 + \lambda_c \sum_{t=3}^{N} \left(\Delta^2 c_t^n\right)^2 \right] + \\ \beta_y \left[\sum_{t=1}^{N} \left(y_t^n - y_t\right)^2 + \lambda_y \sum_{t=3}^{N} \left(\Delta^2 y_t^n\right)^2 \right] \end{array} \right\} \\ s.t. \\ y_t^n = y_t + \alpha \left(naicu_t - c_t\right) + (1-\alpha)\left(naire_t - e_t\right) \end{array} \tag{8}$$

This procedure generates a multivariate filter that simultaneously estimates po-

[5]See Hodrick and Prescott (1997), King and Rebello (1993) and Araújo et al (2003) for the definition of HP filtering and other relevant theoretical aspects.

tential output and its unobserved components, e^n and c^n. While the series e^n and c^n are the solution of the optimization problem, the series x and y^n are built from equations (4) and (5).

Without using equation (5) as a constraint, the series e^n, c^n, and y^n obtained from the optimization problem defined in (8) would be the HP trend of the series e, c and y. Nevertheless, the results change considerably when (5) is used as a constraint. For instance, setting $\beta_y = 0$ is equivalent to estimating y^n from a production function where e^n and c^n are the HP trend of e, and c. For other values of β_y, i.e., $\beta_y \in (0, \infty)$, the potential output will be within the HP trend and that given by the production function.

3 Implementation

I use the Kalman Filter to estimate (8) in a way similar to how Harvey (1995) uses it to estimate the HP filter. For this estimation, it is necessary to express (8) in the following state-space form.

Transition Equation

$$\begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \\ x_t^4 \\ x_t^5 \\ x_t^6 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} x_{t-1}^1 \\ x_{t-1}^2 \\ x_{t-1}^3 \\ x_{t-1}^4 \\ x_{t-1}^5 \\ x_{t-1}^6 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \bullet \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \\ \varepsilon_t^3 \\ \varepsilon_t^4 \\ \varepsilon_t^5 \end{bmatrix}$$

Measurement equation

$$\begin{bmatrix} e_t \\ c_t \\ y_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \\ x_t^4 \\ x_t^5 \\ x_t^6 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1-\alpha & 0 & \alpha & 0 & 0 \end{bmatrix} \bullet \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \\ \varepsilon_t^3 \\ \varepsilon_t^4 \\ \varepsilon_t^5 \end{bmatrix}$$

Similarly to Harvey(1995), I found it necessary to impose some restriction on the variances of the errors.[6] These variance relations are not necessary to make the Kalman Filter run, but only to induce it to converge to the result that would be given by the filter specified in (8) when a given parameter set $\{\lambda_e, \beta_e, \lambda_c, \beta_c, \lambda_y, \beta_y\}$ is considered. Running the Kalman filter without restrictions is equivalent to finding the set of parameters that best fit the data.

It is easy to interpret this state-space. The series x^1, x^3, and x^5 are associated with e^n, c^n, and y^n. Therefore the gaps $e - e^n$, $c - c^n$, and $y - y^n$ are given by ε^1, ε^3, and $(1-\alpha)\,\varepsilon^1 + \alpha\varepsilon^3$.

3.1 Incorporating the Phillips Curve

The non-accelerating inflation rate of unemployment - *nairu* - is usually regarded as the empirical counterpart of the natural rate of unemployment.[7] However, as commented in Boone et al (2002), *nairu* estimation processes that do not exploit information about inflation may result in inefficient historical measures of the *nairu*, biased parameter estimates, and inefficient forecasts of *nairu*.

Therefore, I have incorporated a Phillips curve to the filter proposed in (8) in order to be able to interpret the estimate of the natural rate of unemployment as being the *nairu*. Analogous to the way Boone (2000) implements the multivariable filter proposed by Laxton and Tetlow (1992), it is possible to modify the state space specified in Section 3 by adding the Phillips curve as another measurement equation to simultaneously estimate output gap and the *nairu*

4 Results

I used Brazilian quarterly data collected from 1995.Q1 to 2007.Q4 to generate the results presented in this section. The following series were used -[8]

1. unemployment (U) - quarterly average of the series "Taxa de desemprego - aberto - RMSP - Mensal - (%) - Seade e Dieese/PED - Seade12_TDAGSP12", available at www.ipeadata.gov.br.

[6] See Appendix B for details.

[7] The *nairu*, as defined by Mondigliani and Papademos (1978), is different from the concept of the natural rate of unemployment, as expressed in Friedman (1968) and Phelps (1967). Estrella and Mishkin (1999) show that these concepts may diverge.

[8] See Appendix C for details about each estimation.

2. installed capacity utilization (C) - quarterly average of the series "Utilização da capacidade instalada - indústria - (%) - CNI - CNI12_NUCAP12", available at www.ipeadata.gov.br.

3. seasonally adjusted quarterly GDP (Y) - series no. 1253, available at www.bcb.gov.br.

The results obtained when running the Kalman filter with no restriction on the variances are shown in the first line of Figure 1.

Additionally, the filter structure proposed in (7) can be modified to incorporate new features that may help on the output gap identification. A possible modification concerns the use of other objective functions to estimate e^n and c^n. Since each observation of E and C lies inside the interval $[0, 1]$ and by consequence does not incorporate any linear trend, the HP filter is not recommended to estimate e^n and c^n. Furthermore, I incorporated a Phillips curve into the filter. The results obtained from these modifications are shown on the second line of the Figure 1.

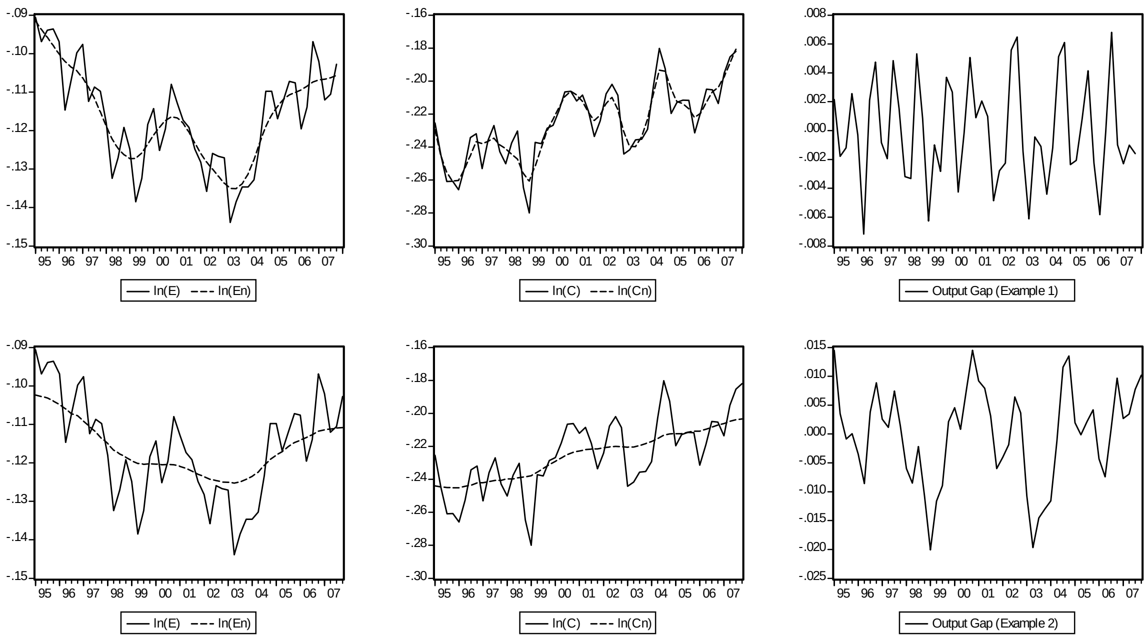

Figure 1: Results obtained on two different experiments

5 Conclusions and Extensions

To sharpen the identification of potential output, I generated a multivariate filter that imposes some economic structure onto an econometric method. In the present case, I combined HP filtering with a production function approach. The strategy used to build the filter can be summarized in two steps: (i) adding a constraint derived from a production function to the optimization problem known as the HP filter and (ii) extending the objective function to estimate the unobserved variables that appear in the production function. The ability to produce estimates not only for potential output but also for its unobservable components is one of the main advantages of this method. In addition to this, the filter does not require the specification of any productivity factor for this decomposition.

This basic structure can be easily adapted for situations. A possible modification concerns the use of other methods to estimate e^n and c^n. The optimization proposed in (8) implicitly imposes the utilization of the HP filter to estimate those series. If another estimation technique are preferred, HP objective functions can be replaced with other ones. It is also possible to modify the state-space proposed in Section 3 to estimate e^n and c^n as an ARMA process, or to incorporate a Phillips curve or any econometric relation as an additional measurement equation.

Nevertheless, the main idea is that results can change considerably when a statistical method incorporates some economic structure. Nevertheless, calibrating these models is not an easy task. Whenever a parameter - β_e, β_c,or β_y- changes, the whole problem is modified, since these parameters quantify the relative importance between optimizations problems.

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6 Appendix A

Besides the method described in Section 3, it is possible to implement the filter proposed in (8) by solving a linear system. For this, it is necessary to compute the first order conditions (FOCs) of the optimization problem proposed in (8). It is important to emphasize that the objective function of (8) is convex since it is a sum of convex functions defined in an open convex domain.[9] Therefore, the FOCs are not only necessary but also sufficient for the minimum.

The FOCs of the filter proposed in (8) can be expressed by the following linear system:

$$\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \cdot \begin{bmatrix} e^n \\ c^n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \end{bmatrix} \cdot \begin{bmatrix} e \\ c \\ y \end{bmatrix} \tag{9}$$

where X_{11}, X_{12}, X_{21}, X_{22}, Y_{11}, Y_{12}, Y_{13}, Y_{21}, Y_{22} and Y_{23} are $N \times N$ matrices given by[10]

$$\begin{array}{ll}
X_{11} = \beta_e(I + \lambda_e B_2) + \beta_y(1-\alpha)^2(I + \lambda_y B_2) & Y_{12} = Y_{21} = \beta_y \alpha(1-\alpha)(I + \lambda_y B_2) \\
X_{12} = X_{21} = \beta_y \alpha(1-\alpha)(I + \lambda_y B_2) & Y_{13} = -\beta_y(1-\alpha)\lambda_y B_2 \\
X_{22} = \beta_c(I + \lambda_c B_2) + \beta_y \alpha^2(I + \lambda_y B_2) & Y_{22} = \beta_c I + \beta_y \alpha^2(I + \lambda_y B_2) \\
Y_{11} = \beta_e I + \beta_y(1-\alpha)^2(I + \lambda_y B_2) & Y_{23} = -\beta_y \alpha \lambda_y B_2
\end{array}$$

In the previous expressions I is the $N \times N$ identity matrix, and B_2 is an $N \times N$ matrix that can be decomposed as $B_2 = A \cdot D \cdot A^T$ where A and D are two $N \times N$ diagonal matrices whose elements are given by the following formulas,[11]

$$a_{ij} = \begin{cases} 1 & \text{, if } i = j \text{ or } i = j+2 \\ -2 & \text{, if } i = j+1 \\ 0 & \text{, otherwise} \end{cases} \quad \text{and} \quad d_{ij} = \begin{cases} 1 & \text{, if } i = j \text{ and } i \le N-2 \\ 0 & \text{, otherwise} \end{cases}$$

[9] See Appendix B in Araújo, Areosa and Rodrigues Neto (2003) for the complete proof of the fact that each Hodrick-Prescott objective function is convex.

[10] Denoting F the objective function of (8), I obtain the first N lines of the system computing $\frac{\partial F}{\partial e_t^n} = 0$ for $t \in \{1, \ldots, N\}$. Analogously, I obtain the last N lines computing $\frac{\partial F}{\partial c_t^n} = 0$ for $t \in \{1, \ldots, N\}$.

[11] The matrix is B_2 is equal to the matrix $B\,(2)$ described in Araújo, Areosa and Rodrigues Neto (2003). In particular, see Appendix A of the referenced paper for the proof of this decomposition.

The linear system proposed in (9) has the same solution as (8).

7 Appendix B

In this appendix I derive the variance relations that must be imposed to the Kalman Filter in order to make the model stated in Section 3 converge to same solution of optimization problem proposed in (8).

Let ε_t^1, ε_t^2, ε_t^3, ε_t^4,and ε_t^5 be errors defined as

$$\begin{aligned}
\varepsilon_t^1 &= e_t - e_t^n \\
\varepsilon_t^2 &= \Delta^2 e_t^n = e_t^n - 2e_{t-1}^n + e_{t-2}^n \\
\varepsilon_t^3 &= c_t - c_t^n \\
\varepsilon_t^4 &= \Delta^2 c_t^n = c_t^n - 2c_{t-1}^n + c_{t-2}^n \\
\varepsilon_t^5 &= \Delta^2 y_t^n = y_t^n - 2y_{t-1}^n + y_{t-2}^n
\end{aligned}$$

where

$$\varepsilon_t \equiv \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \\ \varepsilon_t^3 \\ \varepsilon_t^4 \\ \varepsilon_t^5 \end{bmatrix} \sim iidN\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \sigma^2 V \right) \quad \text{and} \quad V = \begin{bmatrix} \rho_{11} & 0 & \rho_{13} & 0 & 0 \\ 0 & \rho_{22} & 0 & 0 & 0 \\ \rho_{13} & 0 & \rho_{33} & 0 & 0 \\ 0 & 0 & 0 & \rho_{44} & 0 \\ 0 & 0 & 0 & 0 & \rho_{55} \end{bmatrix}$$

For $t \in \{3, ..., T\}$ the period log-likelihood function is expressed as

$$\ln\left(f\left(\varepsilon_t^1, \varepsilon_t^2, \varepsilon_t^3, \varepsilon_t^4, \varepsilon_t^5\right)\right) = \ln\left(\frac{1}{(2\pi)^{5/2}\sigma^2\left(\det\left(V\right)\right)^{1/2}} \exp -\frac{1}{2\sigma^2}\varepsilon_t^T V^{-1} \varepsilon_t\right)$$

being V^{-1} given by

$$V^{-1} = \begin{bmatrix} \rho_{33}\left(\rho_{11}\rho_{33} - \rho_{13}^2\right)^{-1} & 0 & -\rho_{13}\left(\rho_{11}\rho_{33} - \rho_{13}^2\right)^{-1} & 0 & 0 \\ 0 & \rho_{22}^{-1} & 0 & 0 & 0 \\ -\rho_{13}\left(\rho_{11}\rho_{33} - \rho_{13}^2\right)^{-1} & 0 & \rho_{11}\left(\rho_{11}\rho_{33} - \rho_{13}^2\right)^{-1} & 0 & 0 \\ 0 & 0 & 0 & \rho_{44}^{-1} & 0 \\ 0 & 0 & 0 & 0 & \rho_{55}^{-1} \end{bmatrix},$$

while for $t \in \{1,2\}$ as

$$\ln\left(f\left(\varepsilon_t^1,\varepsilon_t^3\right)\right)=\ln\left(\frac{1}{(2\pi)\,\sigma^2\left(\det\left(\tilde{V}\right)\right)^{1/2}}\right)\exp\left(-\frac{1}{2\sigma^2}\begin{bmatrix}\varepsilon_t^1 & \varepsilon_t^3\end{bmatrix}\begin{bmatrix}\rho_{11} & \rho_{13}\\ \rho_{13} & \rho_{33}\end{bmatrix}^{-1}\begin{bmatrix}\varepsilon_t^1\\ \varepsilon_t^3\end{bmatrix}\right)$$

Therefore, maximizing the total log-likelihood generates the same solution as solving

$$\min_{e^n,c^n}\left\{\begin{array}{c}\left(\rho_{11}\rho_{33}-\rho_{13}^2\right)^{-1}\sum_{t=1}^T\left[\begin{array}{c}\rho_{33}\left(e_t-e_t^n\right)^2\\ -2\rho_{13}\left(e_t-e_t^n\right)\left(c_t-c_t^n\right)\\ +\rho_{11}\left(c_t-c_t^n\right)^2\end{array}\right]\\ +\sum_{t=3}^T\left[\rho_{22}^{-1}\left(\Delta^2e_t^n\right)^2+\rho_{44}^{-1}\left(\Delta^2c_t^n\right)^2+\rho_{55}^{-1}\left(\Delta^2y_t^n\right)^2\right]\end{array}\right\}$$

Considering that I can write the filter proposed in (8) as

$$\min_{e^n,c^n}\left\{\begin{array}{c}\left(\beta_e+\beta_y(1-\alpha)^2\right)\sum_{t=1}^N\left(e_t^n-e_t\right)^2+\beta_e\lambda_e\sum_{t=3}^N\left(\Delta^2e_t^n\right)^2+\\ \left(\beta_c+\beta_y\alpha^2\right)\sum_{t=1}^N\left(c_t^n-c_t\right)^2+\beta_c\lambda_c\sum_{t=3}^N\left(\Delta^2c_t^n\right)^2+\\ 2\beta_y\alpha(1-\alpha)\sum_{t=1}^N\left(c_t-c_t^n\right)\left(e_t-e_t^n\right)+\beta_y\lambda_y\sum_{t=3}^N\left(\Delta^2y_t^n\right)^2\end{array}\right\},$$

these two problems generate the same solution if

$$\begin{aligned}\rho_{11}&=\frac{\beta_c+\beta_y\alpha^2}{\beta_e\beta_c+\beta_c\beta_y(1-\alpha)^2+\beta_e\beta_y\alpha^2}\\ \rho_{13}&=-\frac{\beta_y\alpha(1-\alpha)}{\beta_e\beta_c+\beta_c\beta_y(1-\alpha)^2+\beta_e\beta_y\alpha^2}\\ \rho_{33}&=\frac{\beta_e+\beta_y(1-\alpha)^2}{\beta_e\beta_c+\beta_c\beta_y(1-\alpha)^2+\beta_e\beta_y\alpha^2}\\ \rho_{22}&=\frac{1}{\beta_e\lambda_e}\quad\rho_{44}=\frac{1}{\beta_c\lambda_c}\quad\rho_{55}=\frac{1}{\beta_y\lambda_y}\end{aligned}\tag{10}$$

These expressions turn into some variance relations for the Kalman filter since

$Var(\varepsilon_t) = \sigma^2 V$ and σ^2 is not known. That is,

$$\begin{aligned}
\sigma_2^2 &= \frac{\rho_{22}}{\rho_{55}}\sigma_5^2 = \frac{\beta_y\lambda_y}{\beta_e\lambda_e}\sigma_5^2 \\
\sigma_4^2 &= \frac{\rho_{44}}{\rho_{55}}\sigma_5^2 = \frac{\beta_y\lambda_y}{\beta_c\lambda_c}\sigma_5^2 \\
\sigma_1^2 &= \frac{\rho_{11}}{\rho_{33}}\sigma_3^2 = \frac{\beta_c + \beta_y\alpha^2}{\beta_e + \beta_y(1-\alpha)^2}\sigma_3^2 \\
\sigma_{13} &= \frac{\rho_{13}}{\rho_{33}}\sigma_3^2 = -\frac{\beta_y\alpha(1-\alpha)}{\beta_e + \beta_y(1-\alpha)^2}\sigma_3^2 \\
\sigma_3^2 &= \frac{\rho_{33}}{\rho_{55}}\sigma_5^2 = \frac{\beta_y\lambda_y\left[\beta_e + \beta_y(1-\alpha)^2\right]}{\beta_e\beta_c + \beta_c\beta_y(1-\alpha)^2 + \beta_e\beta_y\alpha^2}\sigma_5^2
\end{aligned}$$

8 Appendix C

8.1 Example 1

State-space estimated in E-views 6.

```
@state sv1 = 2*sv1(-1)-sv2(-1) + [var=exp(C(2))]
@state sv2 = sv1(-1)
@state sv3 = 2*sv3(-1)-sv4(-1) + [var=exp(C(4))]
@state sv4 = sv3(-1)
@state sv5 = 2*sv5(-1)-sv6(-1) + [var=exp(C(5))]
@state sv6 = sv5(-1)
@evar cov(e1,e3)=-exp(C(6))
lnempr = sv1+ [ename = e1, var=exp(C(1))]
lnuci = sv3+ [ename = e3, var=exp(C(3))]
lnpib_sa = sv5+ 0.6*e1+0.4*e3
```

Method: Maximum likelihood (Marquardt)

Sample: 1995.Q1 to 2007.Q4

Log likelihood 375.4109

Coefficient	Value	Std. Error	z-Statistic	Prob
C(1)	-10.12769	0.311508	-32.51186	0.0000
C(2)	-12.82040	0.621337	-20.63355	0.0000
C(3)	-9.328380	0.249306	-37.41741	0.0000
C(4)	-9.489584	0.535375	-17.72513	0.0000
C(5)	-8.200868	0.180615	-45.40527	0.0000
C(6)	-10.80437	0.779915	-13.85326	0.0000

8.2 Example 2

The objective of this example is to show that the basic filter structure can incorporate many modifications that might help on the estimation of output gap. This example not only includes a Phillips curve in the filter proposed in (7), but also replaces Op_e and Op_c with the objective function of the lowest order r-filter (r=1), that is,[12]

$$\sum_{t=1}^{N}\left(z_t^n - z_t\right)^2 + \lambda_z \sum_{t=3}^{N}\left(\Delta z_t^n\right)^2 \qquad z \in \{e, c\}$$

While the HP filter tries to identify a linear trend at the series z, this modified version tries do identify a steady-state level. The rationale behind this choice is that this modified filter causes less distortion in the border of the series.[13] Rewriting the errors stated in Appendix B as

$$\begin{aligned} \varepsilon_t^2 &= \Delta e_t^n = e_t^n - e_{t-1}^n \\ \varepsilon_t^4 &= \Delta c_t^n = c_t^n - c_{t-1}^n \end{aligned}$$

[12] Araújo et al (2003) studies the r-filters, a two-parameter family of filters in which the HP filter is considered as the second order member (r=2). While the HP filter converges to a linear time trend as the smoothing factor (λ) grows, the higher order members of the proposed family converge to higher order polynomial time trends, while the lowest order filter (r=1) converges to a constant.

[13] The so-called *border effect*, a problem concerning the use of the HP filter, has been widely discussed in the literature. The hole r-filter family presents this weakness. However, as stated in Araújo et al (2003), this problem grows with the filter order.

the error variances take the same form as (10). Nevertheless, the results presented in Section 4 consider the limit case when

$$\beta_y \to 0, \quad \text{and} \quad \beta_y \lambda_y = \bar{\lambda} \ (\text{cons}\tan\text{t})$$

In this case, it is possible to write

$$\begin{aligned}\rho_{11} &= \frac{1}{\beta_e}, \quad \rho_{13} = 0, \quad \rho_{33} = \frac{1}{\beta_c} \\ \rho_{22} &= \frac{1}{\beta_e \lambda_e}, \quad \rho_{44} = \frac{1}{\beta_c \lambda_c}, \quad \rho_{55} = \frac{1}{\bar{\lambda}}\end{aligned}$$

The variance relations previously obtained become $\sigma_1^2 = \lambda_e \sigma_2^2$, $\sigma_2^2 = \sigma_5^2 \bar{\lambda} / (\lambda_e \beta_e)$, $\sigma_3^2 = \lambda_c \sigma_4^2$ and $\sigma_4^2 = \sigma_5^2 \bar{\lambda} / (\lambda_c \beta_c)$, where σ_k^2 is the variance of ε_t^k. However, this example considers only the restrictions for σ_1^2 and σ_3^2 when $\lambda_e = \lambda_c = 40$. Setting $\lambda = 40$ on a filter with r=1 is equivalent to $\lambda = 1600$ on the HP filter (r=2).[14]

The following state-space was estimated in E-views 6.

```
@state sv1 = sv1(-1) + [var=exp(C(2))]
@state sv3 = sv3(-1) + [var=exp(C(4))]
@state sv5 = 2*sv5(-1)-sv6(-1) + [var=exp(C(5))]
@state sv6 = sv5(-1)
@state sv7 = sv6(-1)
lnempr = sv1+ [ename = e1, var=40*exp(C(2))]
lnuci = sv3 + [ename = e3, var=40*exp(C(4))]
lnpib_sa = sv5+ 0.6*e1+0.4*e3
lnipca = c(6)*lnipca(1)+(1-c(6))*lnipca(-1)+c(8)*lnpib_sa(–1) - c(8)*sv6
+[var=exp(C(11))]
```

[14] See Araújo et al (2003) for the concept of equivalence between r-filters.

Method: Maximum likelihood (Marquardt)

Sample: 1995.Q1 to 2007.Q4

Log likelihood 546.5115

Coefficient	Value	Std. Error	z-Statistic	Prob
C(2)	-12.88692	0.216206	-59.60478	0.0000
C(4)	-11.92240	0.264955	-44.99776	0.0000
C(5)	-8.154801	0.171000	-47.68878	0.0000
C(6)	0.521173	0.085575	6.090278	0.0000
C(8)	0.254124	0.142861	1.778814	0.0753
C(11)	-9.020204	0.198590	-45.42124	0.0000

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