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# An integrated model for liquidity management and short-term asset allocation in commercial banks.

Wenersamy Ramos de Alcântara*



**Abstract**

This work develops an integrated model for optimal asset allocation in commercial banks that incorporates uncertain liquidity constraints that are currently ignored by RAROC and EVA models. While the economic profit accounts for the opportunity cost of risky assets, what may even incorporate a market liquidity premium, it neglects the risk of failure due to the lack of sufficient funds to cope with unexpected cash demands arising from bank runs, drawdowns, or market, credit and operational losses, what may happen along with credit rationing episodes or systemic level dry ups. Given a liquidity constraint that can incorporate these factors, there is a failure probability $P_f$ that the constraint will not hold, resulting in a value loss for the bank, represented by a stochastic failure loss $\widetilde{L}_f$. By assuming that bankers are risk neutral in their decision about the size of the liquidity cushion, the economic profit less the possible losses due to the lack of liquidity is optimized, resulting in a short-term asset allocation model that integrates market, credit and operational risks in the liquidity management of banks. Even though a general approach is suggested through simulation, I provide a closed form solution for $P_f$, under some simplifying assumptions, that may be useful for research and supervision purposes as an indicator of the liquidity management adequacy in the banking system. I also suggest an extreme value theory approach for the estimation of $\widetilde{L}_f$, departing from other liquidity management models that use a penalty rate over the demand of cash that exceeds the availability of liquid resources. The model was applied to Brazilian banks data resulting in gains over the optimization without liquidity considerations that are robust under several tests, giving empirical indications that the model may have a relevant impact on the value creation in banks.

**Keywords:** Liquidity risk, liquidity management, asset allocation, RAROC, EVA.
**JEL:** G21, G32.

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# 1 Introduction

Risk management in financial institutions is a subject that has been refreshed by the recent and increasing interest following Basel I and Basel II Accords discussions. Nevertheless, even if an institution maintains a proper level of economic capital to protect its depositors and creditors, adequately accounting for market, credit and operational risks, two variables may fall far from expected: the operations' demand for cash, and the availability of funding and short-term resources. The first may be affected by sudden cash needs, such as bank runs, drawdowns, and market, credit or operational losses. Unexpected values for the latter may occur in the case of credit rationing episodes among banks[1] or systemic level dry ups. Both situations lead to losses that may come from the need to sell assets at unfavorable prices, to obtain funding at unreasonably high costs or to forego profitable investment opportunities. In extreme cases, when the failure scales up to bankruptcy, the shareholders may loose all equity plus bankruptcy costs. Thus, liquidity, or the lack of it, brings a failure risk that is neglected by solely marking banks' assets to market.

The liquidity risk in financial institutions is usually categorized as *market liquidity risk* or *funding liquidity risk*[2]. The first one is the risk of loss due to changes in the value of assets caused by the low liquidity of the markets where they are traded. The funding liquidity risk is the risk of loss due to the bank's lack of enough liquid resources to cope with cash needs. This lack of resources may come either from the unavailability of funding sources, or from the impossibility to realize an asset for cash. This is usually the case of credit portfolios, except, of course, in the case of securitization or loan sales, but these processes take time and if they are not yet in an advanced stage, they

[1] Before the US mortgage crisis started in 2007, credit rationing episodes were usually discussed with respect to banks' customers and not among banks, so that this statement may need some clarification: credit rationing among banks is the limitation of the amount of money that a bank would accept to lend to another bank, regardless of the interest rate charged, in the spirit of Bhattacharya and Fulghieri (1994). In their work, Battacharya and Fulguieri present a model in which there can be a maximum amount for interbank lending, regardless of the interest rate, as the short-term allocation of liquid assets in the bank that borrows money is a private information. The recent credit rationing episodes triggered by the mortgage crisis offer further evidence that the interbank credit market provides only partial protection against liquidity risk.

[2] There are some divergencies in literature about the exact meaning of funding liquidity risk, as between Gup and Kolari (2005, chap. 11) and Koch and MacDonald (2000, chap. 3). Saunders and Cornett (2006, chap. 17), on the other hand, use a classification that considers the impact on liquidity caused by the asset and liability structure, dividing liquidity risks in asset-side liquidity risks and liabilities-side liquidity risks. The definition used in this work follows Koch and MacDonald (2000), Jorion (2001), and BIS (2006b).

cannot be executed in an arbitrarily small amount of time. In less developed credit markets, such as in Brazil, this is an even bigger concern. Anyway, in many situations the factors that cause losses in both cases are common (BIS, 2006b), since the bank would not sell assets at unfavorable prices unless there was a good reason, such as the payment of pressing liabilities.

While much attention has been paid to the modeling and pricing of market liquidity risk, and to the development of sophisticated capital adequacy models, the same has not happened with the funding liquidity risk and the liquidity management in financial institutions. Most of the literature about practical issues in liquidity management is in financial institutions and risk management textbooks,[3] or in regulators' documents, such as those issued by the Bank for International Settlements (BIS) or by Central Banks.

Most textbooks and regulators' recommendations, such as BIS (2006b) and BIS (2000), despite being able to guide a minimum level of liquidity and a responsible liquidity management, do not indicate an optimal liquidity level because they are dissociate from investment decisions, as opposed to theoretical liquidity management models that frequently use an objective function associated to the financial performance in order to find the ideal liquidity level. Traditional liquidity management involves the mapping, estimation and simulation of inflows and outflows within some time horizon, including safety margins and contingency plans to deal with exceptional losses and disbursements. The separation from the investment decision makes it difficult to assess objectively how much cash is too much, hindering bank's ability to seize profitable opportunities, and how much is too few, making the risk of losses higher than acceptable in exchange for the increased returns on illiquid assets. As a result, there is a gap between theoretical developments in liquidity management and what is actually used in practice by commercial banks, and the decision about the optimal liquidity level relies much more on art and professional experience than on science and well specified decision processes.

According to Xavier and Rochet (1999, chap. 8), there are two theoretical paradigms to liquidity management in financial institutions. One of them, proposed by Ho and Saunders (1981), applies to banks the analysis of Ho and Stoll (1980) for the management of market makers' inventory of assets. In the second paradigm, the bank's profit is modeled as a function of the total resources that are kept as reserves, $R$, that pay a return $r$. The remaining resources, which are obtained through the deposits $D$, are used to make

[3]Among the several texts available, it is possible to cite: Hastings (2006), Saunders and Cornett (2006), Gup and Kolari (2005), Banks (2004), Van Greuning and Bratanovic (2003), Bessis (2002), Koch and MacDonald (2000), Kidwell et al. (1997), Rose (1996), Hempel et al. (1994).

illiquid loans that receive the interest rate $r_L$. If the total withdrawals after some time, represented by the stochastic variable $\widetilde{x}$, are bigger than the value of reserves, the bank pays a punitive rate $r_P$ over the difference between withdrawals and reserves. This way, the bank's profit would be given by $\Pi(R) = r_L(D - R) + rR - r_P E[Max(0, \widetilde{x} - R)]$, and the optimal level of reserves is obtained from the maximization of $\Pi(R)$.

Particularly in the second paradigm, the greatest difficulty is to correctly incorporate the opportunity costs associated to a great variety of operations realized by banks, with exposition to several risks that stem from uncertain returns. Other issue that can be raised is the use of a linear relation between the margin by which the cash demands exceed the availability of resources and the cost incurred by the lack of liquidity. It is possible to argue that the risk of extreme losses, or even bank's bankruptcy, may significatively influence the expected loss because of the lack of liquidity.

The model proposed in this work contributes to the evolution of the liquidity management practice by adopting the second paradigm, but with the use of the underlying concept of economic profit on RAROC and EVA models, largely used by financial institutions, as the objective function of the optimization, since RAROC and EVA models are recognized by their ability to adequately incorporate opportunity costs throughout all operations of a financial institution (Schroeck, 2002, chap. 6). The idea is to develop a consistent, flexible and comprehensive model that correctly incorporates the risks and complexities of banking activities, but still allows for a gradual and evolutive implementation, resulting in lower costs to introduce the new model, including training and disturbances to normal activities of the institution. A great deal of care was taken throughout the description of the model in order to use variables that can be directly related to managerial information, instead of using stylized and abstract variables, what also facilitates the conversion of the theoretical results into real applications.

In addition, the expected loss was modeled in two ways: one of them through a linear function of the lacking liquid resources, as in the basic paradigm described above, and the other based on the Extreme Value Theory, making it possible to capture the effects of extreme losses that may come from bank fragility.

# 2 The Model

## 2.1 General Setting

The liquidity constraint faced by commercial banks depends on the chosen short-term allocation between liquid and illiquid assets. The bank will possibly lose money if the lack of sufficient resources results in the violation of the liquidity constraint, even if the bank is still solvent. Tradable or liquid assets, $A_L$, for the purposes of this work, are all those assets that could be realized for money (i.e., that can be used to make payments and redeem obligations) at reasonable prices in the time horizon $\delta t$. Non-tradable or illiquid assets, $A_I$, are all those that cannot be realized for money in $\delta t$, or that could be realized at punitive discounts (fire-sales). As a simplification, one can think $A_L$ as the trading book and $A_I$ as the banking book,[4] but the definition used in this work does not imply that some classes of assets are liquid and some are not. Simply put, $A_L$ is the total amount of assets that can be amassed to redeem obligations in the time $\delta t$. One should notice that if a large institution tries to negotiate too much of an otherwise liquid security, it will suffer rather high discounts marginally, so that not all of the security's holdings will be considered liquid. Even cash may be considered illiquid if it cannot be timely available for logistics reasons, for example, as it is the case of cash in ATMs. Therefore, the banks' asset allocation decision in the short-term should consider the choice of a portfolio of liquid and illiquid assets, given some cash and cash equivalent assets available for allocation after reasonably predictable cash demands are provisioned for use in $\delta t$. Part of this available cash must be used to protect the institution from liquidity compromising events, and part will enhance profits at increased liquidity, credit and market risks.

The expected economic profit of an investment $I$ with stochastic return $\widetilde{r}$ is given by $E(\widetilde{\Pi}) = I \times [E(\widetilde{r}) - R]$, where $E(\cdot)$ represents the mathematical expectation of a random variable and $R$ is the risk adjusted opportunity cost of the investment. If the expected economic profit is positive, $E(\widetilde{\Pi}) > 0$, the investment should be done and it increases the bank's value. Conceptually, this is the analysis done in EVA models, which is equivalent to require that $E(\widetilde{r}) > R$, which, by its turn, is the acceptance criterium of investments in

[4] In regulation and bank accounting terminology, "a trading book consists of positions in financial instruments and commodities held either with trading intent or in order to hedge other elements of the trading book" (BIS, 2006a, p. 158). The banking book corresponds to the remaining assets that are not in the trading book, usually related to traditional bank activities, as loans. As the trading book corresponds to tradable positions, in many countries, including Brazil, it is registered in the balance sheet by its market value, while the banking book is registered by its cost.

RAROC model, given a correct assessment of expected return and opportunity cost. The risk of loss due to the prompt unavailability of liquid resources is related to a failure probability, $P_f$, which is a function of the short-term allocation between liquid and illiquid assets. Additionally, once the failure takes place, there is a stochastic loss given by $\widetilde{L}_f$.

It is necessary to establish some hypothesis about the behavior of economic agents in order to justify the choice of the investment selection criterium that will be modified to include the effects of the lack of liquidity. It is assumed that such economic agents represents shareholders of commercial banks, which ultimately define the behavior of the bank as economic agent,[5] and that they are rational and risk neutral regarding short-term allocation decisions. Long-term resource allocation and strategic decisions, which are related to how the bank makes business and compete in the market, are not reached by this model and can include risk aversion. Implicitly, it means that the investment opportunities in illiquid assets are those in usual "projects" of the bank, as loan offers to target markets. The allocation decision of risk adjusted economic capital among divisions incorporates strategic decisions and reflects the institution's appetite for risk, but once defined, short-term investments in liquid assets represent simply a liquidity cushion or reservoir that the bank keeps exclusively because of the liquidity risk or the lack of profitable investment opportunities. In summary, the only function of this liquidity cushion is the protection against unexpected cash demands. It is not used to adjust the risk and return profile of the institution, and the definition of this profile is considered a strategic decision that is exogenous to this model, what makes the risk-neutral behavior a reasonable assumption. This assumption is only necessary because the expected loss due to the lack of liquidity will be deducted from the expected economic profit.

At the beginning o the $i$-th period, the bank is assumed to have $A_L^i$ liquid assets and $A_I^i$ illiquid assets. Nevertheless, it is necessary to distinguish from all liquid assets those whose allocation is discretionary in the short-term, $A_{L,D}^i$, after one subtracts from $A_L^i$ the cash and cash equivalent securities required to take strategic and long-term positions in liquid assets and to make predictable disbursements in $\delta t$, net from all predictable cash inflows. This includes adjustments for:

- the cash received from fees and matched positions, which earn fixed spreads;
- operational costs, hedging costs and interest expenses;

[5] For the purposes of this work, agency problems are being disregarded.

- taxes that will be effectively collected in $\delta t$;
- required deposits, exchange margins, guarantees, indemnity deposits, and any other effective pledges of liquid resources;
- the cash committed to either ongoing or new long-term investments, which are evaluated as usual[6];
- strategic and long-term positions in tradable securities, usually set by an Asset and Liability Committee (ALCO), $A^i_{L,ALCO}$;
- the cash required (received) to redeem (increase) outstanding debt, whether required by contract or as a long-term funding decision;
- the cash that will be used to pay dividends (or any other cash transfer to equity holders) or will be received from the issuance of new shares.

The economic profit of the chosen allocation of the resources $A^i_{L,D}$ by the end of $\delta t$ may be given by:

$$\widetilde{\Pi}^i = A^i_{L,D}\mathbf{w}^i\left(\widetilde{\mathbf{r}}^i - \mathbf{R}^i\right)^\top, \tag{1}$$

where:

- $\mathbf{w}^i$ is a vector of weights corresponding to the proportions of $A^i_{L,D}$ invested in liquid assets, $w^i_L$, or in illiquid assets, $w^i_I$, during the $i$-th period: $\mathbf{w}^i = \begin{bmatrix} w^i_I & w^i_L \end{bmatrix}$;
- $\widetilde{\mathbf{r}}^i$ is the vector of stochastic total returns of illiquid, $\widetilde{r}^i_I$, and liquid assets, $\widetilde{r}^i_L$, in the $i$-th period: $\widetilde{\mathbf{r}}^i = \begin{bmatrix} \widetilde{r}^i_I & \widetilde{r}^i_L \end{bmatrix}$. It is assumed that the market liquidity risk is embedded in the stochastic behavior of the returns on liquid assets;
- $\mathbf{R}^i$ is the vector of risk adjusted opportunity costs of investments in illiquid, $R^i_I$, and liquid assets, $R^i_L$: $\mathbf{R}^i = \begin{bmatrix} R^i_I & R^i_L \end{bmatrix}$.

It is also assumed that $\widetilde{\mathbf{r}}^i$ is independent from the choice of $\mathbf{w}^i$. This assumption has distinct consequences for liquid and illiquid assets. In the case of liquid assets, this means that the bank does not individually affect market clearing prices, i.e. it is price taker. In the case of illiquid assets, as loan portfolios, this means that the increase in credit operations does not occur at the expense of the credit quality of new clients.

[6] Long-term investments are usually evaluated through discounted cash flow and real options models.

Additionally, the vector of stochastic total returns may be split in two parts: one representing capital gains, and other representing the cash yield in the period, so that we may write:

$$\widetilde{\mathbf{r}}^i = \widetilde{\mathbf{r}}^i_V + \widetilde{\mathbf{r}}^i_C, \tag{2}$$

where:

- $\widetilde{\mathbf{r}}^i_V$ is the vector of stochastic capital gains, the percent change in the value of the asset after any cash payments, which is given by: $\widetilde{\mathbf{r}}^i_V = \begin{bmatrix} \widetilde{r}^i_{V,I} & 0 \end{bmatrix}$, where it is assumed that the total return on liquid assets can be completely represented as a cash yield;
- $\widetilde{\mathbf{r}}^i_C$ is the vector of stochastic cash yields (cash generated in the $i$-th period by invested monetary unit) on liquid assets, $\widetilde{r}^i_{C,L}$, and on illiquid assets, $\widetilde{r}^i_{C,I} : \widetilde{\mathbf{r}}^i_C = \begin{bmatrix} \widetilde{r}^i_{C,I} & \widetilde{r}^i_{C,L} \end{bmatrix}$.

The problem of economic profit maximization would be trivial in this setting without a liquidity constraint. One should expect that liquid markets have more participants that make a greater deal of effort to collect information, so that they are more efficient in the sense of the Market Efficiency Hypothesis, at least in its semi-strong form (Fama, 1991), in such a way that positive economic profit opportunities should be rapidly spotted. The resulting increase in the demand for "cheap" assets would rapidly make them more expensive and eliminate profit opportunities, making these opportunities scarce. The opacity and low accessibility of less liquid markets should allow the availability of more positive economic profit opportunities. Therefore, the chances are that $E(\widetilde{r}^i_L - R^i_L) < E(\widetilde{r}^i_I - R^i_I)$. Under these conditions, a risk neutral agent would see no trade-off between liquid and illiquid assets, and the optimization would have a corner solution: the bank should invest resources on illiquid assets as long as positive economic profit opportunities exist, keeping only the excess of available resources over profitable opportunities as liquid assets. This is, in fact, the result that would be obtained by using RAROC and EVA models without any liquidity constraint.

Nevertheless, given the possibility of losses due to the lack of resources to fulfill short-term obligations or to make long-term investments, the economic profit maximization should take into account a liquidity constraint, that may be written as:

$$\begin{aligned} A^i_{L,D}\mathbf{w}^i \left\{ \begin{bmatrix} 0 & 1 \end{bmatrix} + \widetilde{\mathbf{r}}^i_C \right\}^\top + A^i_{L,ALCO}\widetilde{r}^i_{C,L} + A^i_I \widetilde{r}^i_{C,I} + \widetilde{F}^i &\geq \widetilde{U}^i_C \\ \Rightarrow \left\{ A^i_{L,D}\mathbf{w}^i + \begin{bmatrix} A^i_I & A^i_{L,ALCO} \end{bmatrix} \right\} (\widetilde{\mathbf{r}}^i_C)^\top + w^i_L A^i_{L,D} + \widetilde{F}^i &= \widetilde{U}^i_C, \end{aligned} \tag{3}$$

where:

- $\widetilde{F}^i$ represents the stochastic funding opportunities in the short-therm during the $i$-th period;
- $\widetilde{U}_C^i$ is the uncertain demand for cash in $\delta t$.

Expression (3) means that the total liquid resources available at the beginning of the period, plus those generated in $\delta t$, and plus the funding opportunities in the $i$-th period, $\widetilde{F}^i$, must be at least equal to the immediate and unexpected cash demand $\widetilde{U}_C^i$. For simplicity, it is assumed that the return on the inventory of illiquid assets is equal to the return on new illiquid assets. This assumption may be relaxed simply by using a different stochastic return for $A_I^i$. Likewise, it is assumed that the return on ALCO positions is equal to the return on other liquid positions.

In order to further clarify the meaning of this liquidity constraint, some comments are worthwhile. As a general rule, the terms on the left hand side of the constraint are cash sources, while the terms on the right hand side represent the consumption of or demand for cash. As changes in the inventory of assets and liabilities may represent either sources or demands for cash, and as even some financial instruments may generate positive as well as negative cash flows, such as swaps, the side of the inequality in which these flows are modeled are not important from a numeric point of view. What should dictate the choice is the ease in modeling and the coherence. As a suggestion, $\widetilde{F}^i$ may represent funding opportunities through interbank lending (including funds provided by the Central Bank), nonredeemable short term deposits, short-term debt, and repurchase agreements. Funding through savings accounts, demand accounts, and other redeemable short-term debt are considered potential sources of immediate and unexpected cash demands, and, exactly because of this, should be included in $\widetilde{U}_C^i$. As a matter of fact, it is the potentially extreme value of $\widetilde{U}_C^i$, stemming from bank fragility, that gives relevance to the liquidity management in commercial banks.

Several sources of risk are embedded in restriction (3). Unexpected losses with credit or market operations would be reflected in the realization of $\widetilde{\mathbf{r}}_C^i$. Bank runs, margin calls and drawdowns would be accounted for in the realization of $\widetilde{U}_C^i$, while interbank credit rationing episodes would reduce the realized value of $\widetilde{F}^i$. Systemic level dry ups can affect both $\widetilde{\mathbf{r}}_C^i$ and $\widetilde{F}^i$. Finally, given the broad definition of operational losses, they may be reflected in the realization of any of the uncertainty sources.[7]

[7]For example, the realized value of $\widetilde{\mathbf{r}}_C^i$ may be lower because of a faulty credit rating system, $\widetilde{U}_C^i$ may be affected by a fraud or by the unexpected loss of a law suit, and image compromising problems may affect both $\widetilde{U}_C^i$ and $\widetilde{F}^i$.

It is important to point out that the losses from operational, credit, and market risks are not, by themselves, the main concern. Capital allocation models already deal with these risks. The focus of this work is the loss that comes from the lack of liquidity triggered by a harmful event, that may be related to market, credit or operational losses, but is not restricted to them. There is a failure probability $P_f^i$ that the relation (3) will not hold, and in this case the bank will fail to pay some of its obligations, will have to sell assets or obtain short-term funding at unfavorable conditions, will forego profitable investment opportunities, or will be forced to change long-term strategies. If the failure occurs, the bank's value should be reduced by a certain amount, corresponding to a stochastic failure cost $\widetilde{L}_f^i$, and there may be also the possibility of bankruptcy if the loss is too extreme. Even if the loss itself does not make the bank insolvent, it can affect the institution's market position, blocking the access to short-term resources, or causing panic among depositors. The resulting catastrophic run would lead the bank to insolvency.

The probability $P_f^i$ changes according to the short-term allocation decision $\mathbf{w}^i$, so that the liquidity management problem, in the context of resource allocation, may be seen as an economic profit maximization problem throughout time horizons $\delta t$. Figure 1 shows the allocation decision faced by a commercial bank:

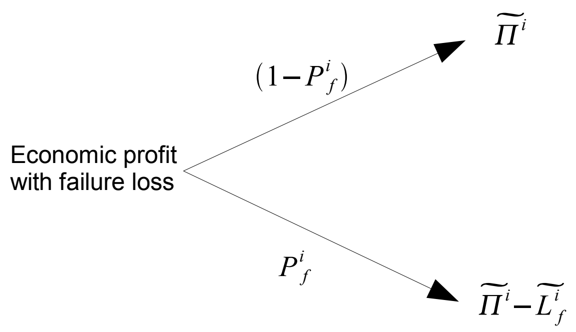



Figure 1: Allocation decision with liquidity risk.

It is also important to identify some restrictions to the allocation. Initially, it is not always possible to assign any volume of investments to any asset. There may be restrictions related to market opportunities and tradable volumes. These restrictions may be expressed as:

$$\mathbf{O}^i \geqq A_{L,D}^i \mathbf{w}^i. \tag{4}$$

This means that the invested volume cannot surpass available investment opportunities with value creation potential, $\mathbf{O}^i = [O_I^i \quad O_L^i]$ . $O_I^i$ represents the restriction on the volume invested in illiquid assets, and $O_L^i$ represents the restriction on the volume invested in liquid assets, even though, most probably, the restriction will be stronger to illiquid assets. For example, it is much harder to expand credit operations to prime clients than to increase positions on government issued securities, since the first depends on the demand for credit, market share, product development strategies and so on, while the latter depends only on a buying order.

Finally, it is also necessary that:

$$\mathbf{w}^i \mathbf{1}^\top = 1 \tag{5}$$

and

$$\mathbf{w}^i \geqq \mathbf{0}, \tag{6}$$

where $\mathbf{1}$, $\mathbf{0} \in \mathbb{R}^2$ are, respectively, vectors of ones and zeros.

Conditions (5) and (6) apply because any allocation must add up to exactly the total amount of available resources and because funding is treated separately. Though, leveraged or short positions are allowed inside the class of liquid assets as long as they are part of an investment strategy.

## 2.2 The Failure Probability

Given the generality of the liquidity constraint, the most flexible and simple way to estimate $P_f^i$ is through simulation methods, either Monte Carlo, based on multivariate models for the statistic distribution of the random variables in (3); or historical, through the re-sampling of historical data about returns, funding opportunities and unexpected demands for liquid resources, assuming that there is enough data available. Such simulation must be done for a finite set $\mathbf{W} \subset \mathbb{R}^2$ of chosen values for the vector $\mathbf{w}^i$, so that it is possible to establish a functional relation $P_f^i(\mathbf{w}^i) : \mathbf{W} \rightarrow \mathbf{P}^i$, where $\mathbf{P}^i \subset \mathbb{R}$ is the set of failure probabilities simulated for each weight vector $\mathbf{w}^i$ in $\mathbf{W}$. Though, under certain simplifying assumptions, it is possible to obtain a closed expression for the failure probability.

In order to obtain this closed expression, it is necessary to limit the quantity of continuous distributions that need to be parameterized in (3). Discrete distribution models, on the other hand, may be added without making it impossible to obtain a closed solution, even though they add to the final expression complexity. In this simplification, deposits and returns are continuous random variables, while the funding capacity in the short-term follows a discrete model. The assumptions used in this work are:

- the exceptional demands for cash correspond to decreases in the volume of deposits;

- the returns on assets, in annualized and continuous rates, follow a multivariate normal distribution;
- the variation in the volume of deposits is independent on the current volume of deposits, in such a way that the variation itself, and not the percent variation, has a normal distribution, even though negative jumps are allowed;
- the availability of funding resources in the short-term is a Bernoulli experiment, with availability $\overline{F}_+$ in case of success, and $\overline{F}_-$ in case of failure;
- the correlation between a negative jump in the variation of the volume of deposits and the lack of funding opportunities in the short-term is 1;[8]
- the distribution of the variation in the volume of deposits is independent on the distributions of assets' cash yields;
- the stochastic processes generating the random variables are stationary.

If the matrix notation is removed, $\widetilde{U}_C^i$ is exchanged for the negative variation in the volume of deposits between this period and the next, $-\delta\widetilde{D}^i = D^i - \widetilde{D}^{i+1}$, and remembering that the total return on liquid assets, $\widetilde{r}_L^i$, is assumed to be equal to the cash yield on liquid assets, $\widetilde{r}_{C,L}$, then the liquidity constraint may be rewritten as:

$$\left(w_I^i A_{L,D}^i + A_I^i\right)\widetilde{r}_{C,I}^i + \left(w_L^i A_{L,D}^i + A_{L,ALCO}^i\right)\widetilde{r}_L^i + w_L^i A_{L,D}^i + \widetilde{F}^i \geq -\delta\widetilde{D}^i. \quad (7)$$

In order to obtain the probability that the restriction is violated, it is necessary to estimate the parameters of the distributions of $\widetilde{r}_L^i$, $\widetilde{r}_{C,I}^i$, $\widetilde{F}^i$ and $\delta\widetilde{D}^i$, according to the assumptions above.

In the case of $\widetilde{r}_L^i$ and $\widetilde{r}_{C,I}^i$, as a normal distribution is assumed, the parameters to me estimated are mean and variance. In order to facilitate the interpretation of the results and the algebraic manipulations, these parameters will be estimated in annualized continuous rates, such that the time horizon $\delta t$ represents a fraction of the year.

[8]The simultaneous occurrence of negative jumps in deposits and restrictions in the funding opportunities tries to mimic the effects of a liquidity crisis, either systemic of specific to a particular bank, in which the best informed creditors and depositors react, promptly and simultaneously, restricting credit and withdrawing resources.

The mean $\mu^i_{C,I}$ of the random variable $\widetilde{r}^i_{C,I}$, in an annualized continuous rate, may be estimated by:[9]

$$\mu^i_{C,I} = \frac{1}{n\delta t} \sum_{j=1}^{n} \ln\left(1 + \frac{rev_I^{i-j}}{A_I^{i-j-1}}\right), \tag{8}$$

where $n$ is the number of periods used in the estimation, and $rev_I^{i-j}$ represents the historical values of the revenues, prior to the period $i$, that are accrued to the illiquid assets.[10] As discussed in footnote 4, page 7, liquid assets are usually marked to market, while illiquid assets are registered by its cost value. This way, it is possible to use historical values of revenues over the total of illiquid assets by the time the revenues were generated to represent the cash yield of illiquid assets.

Thus, the standard deviation $\sigma^i_{C,I}$ of the distribution of continuous cash yields from illiquid assets in the $i$-th period may be estimated as:

$$\widehat{\sigma}^i_{C,I} = \sqrt{\frac{1}{(n-1)\delta t} \sum_{j=1}^{n} \left[\ln\left(1 + \frac{rev_I^{i-j}}{A_I^{i-j-1}}\right) - \widehat{\mu}^i_{C,I}\delta t\right]^2}. \tag{9}$$

As the stochastic processes that generate each period's random variables are assumed to be stationary in order to simplify the exposition, there is no reason why to analyze the composition of the portfolio. Though, it is obvious that a practical implementation of this model should take this composition into account. If the asset allocation inside the general class of illiquid assets changed substantially throughout the estimation window, it will be necessary to estimate the parameters for each asset in the portfolio of illiquid assets, as well as the correlation matrix, such that it is possible to obtain an estimate of global parameters for the current proportion of each asset, according to the Modern Portfolio Theory (Markowitz, 1952). The process is analogous to what will be done later, when the parameters of the portfolio of liquid and illiquid assets will be estimated.

Other adjustment that should be done when the stationarity assumption is ruled out is the use of an autoregressive estimation of the standard deviation through, for example, EWMA (*Exponentially Weighted Moving Aver-*

[9]The results discussed in this section are well known from the theory of measure, probability, and stochastic processes. A basic, but classic reference is Hull (2003). Other more advanced, but still very accessible references are Albanese and Campolieti (2006), Klebaner (2004), and Bingham and Kiesel (2004).

[10]These revenues are, basically, bank's credit portfolio income, but may include rents, dividends from strategic long-term positions in equity, among others.

*age*) or GARCH (*Generalized Autoregressive Conditional Heteroscedasticity*) models.[11]

Assuming that the value of liquid assets is marked to market, the average $\mu_L^i$ of annualized continuous returns on liquid assets may be estimated as:

$$\widehat{\mu}_L^i = \frac{1}{n\delta t}\sum_{j=1}^{n} \ln\left(\frac{A_L^{i-j}}{A_L^{i-j-1}}\right) \tag{10}$$

and the standard deviation $\sigma_L^i$ as:

$$\widehat{\sigma}_L^i = \sqrt{\frac{1}{(n-1)\delta t}\sum_{j=1}^{n}\left[\ln\left(\frac{A_L^{i-j}}{A_L^{i-j-1}}\right) - \widehat{\mu}_L^i \delta t\right]^2}. \tag{11}$$

The comment about changes in the composition of the portfolio during the estimation window also applies to the broad class of liquid assets. Actually, in this case it is even more likely that the portfolio has changed substantially, even for a smaller estimation window.

As by hypothesis the returns are multivariate normally distributed, the return distribution of a portfolio of these assets is also normal. In order to obtain the distribution parameters of a portfolio of liquid and illiquid assets, it is necessary to estimate the correlation between the annualized continuous returns, $\sigma_{L,I}^i$:

$$\widehat{\sigma}_{L,I}^i = \frac{1}{n\delta t}\sum_{j=1}^{n}\left[\ln\left(1+\frac{rev_I^{i-j}}{A_I^{i-j-1}}\right) - \widehat{\mu}_{C,I}^i\delta t\right]\left[\ln\left(\frac{A_L^{i-j}}{A_L^{i-j-1}}\right) - \widehat{\mu}_L^i\delta t\right]. \tag{12}$$

By defining:

$$w_{C,I}^i = \frac{w_I^i A_{L,D}^i + A_I^i}{w_I^i A_{L,D}^i + A_I^i + w_L^i A_{L,D}^i + A_{L,ALCO}^i} = \frac{w_I^i A_{L,D}^i + A_I^i}{A_{L,D}^i + A_I^i + A_{L,ALCO}^i} \tag{13}$$

and

$$w_{C,L}^i = \frac{w_L^i A_{L,D}^i + A_{L,ALCO}^i}{A_{L,D}^i + A_I^i + A_{L,ALCO}^i} \tag{14}$$

as the weights of illiquid and liquid assets, respectively, in the generation of cash, it is finally possible to estimate the parameters of mean, $\mu_C^i$, and standard deviation, $\sigma_C^i$, of the resulting cash yield:

[11] There are several good texts that discuss the volatility estimation in detail, such as Gourieroux and Jasiak (2001), Campbell et al. (1997), and Hamilton (1994).

$$\widehat{\mu}_C^i = w_{C,I}^i \widehat{\mu}_{C,I}^i + w_{C,L}^i \widehat{\mu}_L^i \tag{15}$$

and

$$\widehat{\sigma}_C^i = \sqrt{\left(w_{C,I}^i \widehat{\sigma}_{C,I}^i\right)^2 + \left(w_{C,L}^i \widehat{\sigma}_L^i\right)^2 + 2 w_{C,I}^i w_{C,L}^i \widehat{\sigma}_{L,I}^i} \tag{16}$$

Moreover, for the following discussion $A_C^i$ will be defined as the portfolio of cash generating assets:

$$A_C^i = w_I^i A_{L,D}^i + A_I^i + w_L^i A_{L,D}^i + A_{L,ALCO}^i = A_{L,D}^i + A_I^i + A_{L,ALCO}^i. \tag{17}$$

Given an interest rate $r$ continuously compounded during a period $T$, it is possible to approximate it, for a sufficiently large number of capitalization intervals $m$ on $T$, by an interest rate at each capitalization interval with the value of $\frac{r}{m}$, since $\lim_{m\to\infty}\left(1+\frac{r}{m}\right)^{mT} = e^{rT}$. Thus, if the time horizon $\delta t$ is sufficiently small, the number of $\delta t$ intervals throughout a year, given by $\frac{1}{\delta t}$, will be sufficiently big and we will have $(1+r\delta t)^{\frac{1 \text{ ano}}{\delta t}} \approx e^{r\times 1 \text{ ano}}$, that is, the rate at each interval may be approximated by $\frac{r}{m} = r\delta t$.

This means that the cash yield of $A_C^i$ in the interval $\delta t$, that will be denoted by $\frac{\delta \widetilde{A}_C^i}{A_C^i}$, may be modeled through a mean variation $\mu_C^i \delta t$, given by the continuously compounded annual cash yield of $A_C^i$ multiplied by a fraction of the year $\delta t$. To this mean variation it is possible to add a random component $\sigma_C^i \widetilde{\varepsilon}^i \sqrt{\delta t}$, where $\widetilde{\varepsilon}^i$ is random variable with standard normal distribution, such that $\frac{\delta \widetilde{A}_C^i}{A_C^i}$ will have a normal distribution with mean given by $\mu_C^i \delta t$ and standard deviation given by $\sigma_C^i \sqrt{\delta t}$. In other words, it is possible to model the cash yield on $A_C^i$ as a generalized Wiener process in discrete time:

$$\frac{\delta \widetilde{A}_C^i}{A_C^i} = \mu_C^i \delta t + \sigma_C^i \widetilde{\varepsilon}^i \sqrt{\delta t}, \tag{18}$$

or equivalently,

$$\delta \widetilde{A}_C^i = A_C^i \mu_C^i \delta t + A_C^i \sigma_C^i \widetilde{\varepsilon}^i \sqrt{\delta t}, \tag{19}$$

such that:

$$\delta \widetilde{A}_C^i \sim N(A_C^i \mu_C^i \delta t, A_C^i \sigma_C^i \sqrt{\delta t}). \tag{20}$$

By using (19) in (7), the liquidity restriction becomes:

$$\delta \widetilde{A}_C^i + \delta \widetilde{D}^i \geq -w_L^i A_{L,D}^i - \widetilde{F}^i. \tag{21}$$

Regarding the models of $\delta \widetilde{D}^i$ and $\widetilde{F}^i$, as it is assumed that a negative jump of deposits and the reduction of funding opportunities occur simultaneously (what, in discrete time, means that they both take place in the same interval $\delta t$), both processes will be modeled together through the process $\delta \widetilde{F}_S^i$, which represents all short-term funding opportunities in $\delta t$: savings accounts, demand accounts, issuance of short-term debt, repurchase agreements, interbank lending, and so on, in such a way that:

$$\delta \widetilde{F}_S^i = \delta \widetilde{D}^i + \widetilde{F}^i. \tag{22}$$

By hypothesis, the change in the volume of deposits $\widetilde{D}^i$ follows a simple jump-diffusion process, where jumps are constant and negative, representing bank runs. In order to jointly model the decrease in funding opportunities and negative jumps in deposits, the jump $J^i$ in $\delta \widetilde{D}^i$ will include a component that represents a decrease in deposits, $J_D^i$, plus a component that represents the decrease in funding opportunities, departing from the normal conditions value $\overline{F}_+^i$:

$$J^i = J_D^i + \overline{F}_+^i - \overline{F}_-^i. \tag{23}$$

Therefore, the stochastic differential of deposits, including the funding squeeze effect, will be:

$$d\widetilde{D}^t = \mu_D^i dt + \sigma_D^i d\widetilde{W}^t - J^i d\widetilde{Q}^t, \tag{24}$$

where $\mu_D^i$ is the drift component of the model, $\sigma_D^i$ is the diffusion component and $d\widetilde{Q}^t$ is the stochastic differential of a Poisson process with intensity $\lambda$, representing the average number of jumps in the period of one year.

In the time interval $\delta t$, from its start at $t_0$ to its end at $t_1$, the change in deposits will be:

$$\begin{aligned} \int_{t_0}^{t_1} d\widetilde{D}^t &= \int_{t_0}^{t_1} \mu_D^i dt + \int_{t_0}^{t_1} \sigma_D^i d\widetilde{W}^t - \int_{t_0}^{t_1} J^i d\widetilde{Q}^t \\ \Rightarrow \delta \widetilde{D}^i &= \mu_D^i \delta t + \sigma_D^i \widetilde{\varepsilon}^i \sqrt{\delta t} - J^i (\widetilde{Q}^{t_1} - \widetilde{Q}^{t_0}), \end{aligned} \tag{25}$$

Because of (23) and (25), (22) may be rewritten as:

$$\delta \widetilde{F}_S^i = \delta \widetilde{D}^i + \widetilde{F}^i = \delta \widetilde{D}^i + \overline{F}_+^i, \tag{26}$$

as the possibility of decrease in funding opportunities was incorporated to $\delta \widetilde{D}^i$.

If the time interval $\delta t$ is small enough so that the probability of more than one jump is negligible, it is possible to rewrite (25) as:

$$\delta \widetilde{D}^i = \begin{cases} \mu_D^i \delta t + \sigma_D^i \widetilde{\varepsilon}^i \sqrt{\delta t}, \text{ with probability } e^{-\lambda t}; \\ \mu_D^i \delta t + \sigma_D^i \widetilde{\varepsilon}^i \sqrt{\delta t} - J^i, \text{ with probability } (1 - e^{-\lambda t}). \end{cases} \tag{27}$$

Note that the probability of exactly one jump is $\lambda\delta t \times e^{-\lambda\delta t}$, and not $1 - e^{-\lambda\delta t}$, but in order to both probabilities sum up to exactly one, it is assumed that the difference is negligible for the time interval $\delta t$ used.

If we define $\widetilde{Q}^{\delta t} = \widetilde{Q}^{t_1} - \widetilde{Q}^{t_0}$, the probability of more than one jump is $P(\widetilde{Q}^{\delta t} \geq 2) = 1 - e^{-\lambda\delta t} - e^{-\lambda\delta t}\lambda\delta t$, which is less than $1/5000$ for any $\lambda$ up to 1 and any $\delta t$ up to $1/52$, and less than $1/1000$ for any $\lambda$ up to 0.5 and $\delta t$ up to $1/12$. If the likelihood of more than one jump is greater than desired, say, more than one in one thousand, it is possible to include a second jump in (27):

$$\delta\widetilde{D}^i = \begin{cases} \mu_D^i\delta t + \sigma_D^i\widetilde{\varepsilon}^i\sqrt{\delta t}, \text{ with probability } e^{-\lambda t}; \\ \mu_D^i\delta t + \sigma_D^i\widetilde{\varepsilon}^i\sqrt{\delta t} - J^i, \text{ w. p. } \lambda\delta t \times e^{-\lambda\delta t}; \\ \mu_D^i\delta t + \sigma_D^i\widetilde{\varepsilon}^i\sqrt{\delta t} - 2J^i, \text{ w. p. } (1 - e^{-\lambda t} - \lambda\delta t \times e^{-\lambda\delta t}), \end{cases} \tag{28}$$

and again, it is assumed that the probability of three of more jumps is negligible. The same logic can be used to include as many jumps as desired, simply by using

$$P(\widetilde{Q}^{\delta t} = n) = \frac{e^{-\lambda\delta t}(\lambda\delta t)^n}{n!}. \tag{29}$$

There is an extensive literature about jump-diffusion stochastic processes, with various levels of sophistication, as well as about the estimation of its parameters (see Cai and Hong, 2003). In this simplified model, with constant jump, a moment matching estimation will be used. It is only assumed that a historical series of the changes on the short-term obligations is available.

From (25) and (26), it follows that:

$$\delta\widetilde{F}_S^i = (\mu_D^i\delta t + \overline{F}_+^i) + \sigma_D^i\widetilde{\varepsilon}^i\sqrt{\delta t} - J^i\widetilde{Q}^{\delta t} = \mu_F^i\delta t + \sigma_F^i\widetilde{\varepsilon}^i\sqrt{\delta t} - J^i\widetilde{Q}^{\delta t}, \tag{30}$$

where

$$\mu_F^i\delta t = \mu_D^i\delta t + \overline{F}_+^i, \tag{31}$$

and

$$\sigma_F^i = \sigma_D^i. \tag{32}$$

The expected value of $\delta\widetilde{F}_S^i$ is, therefore:

$$E(\delta\widetilde{F}_S^i) = \mu_F^i\delta t - J^iE(\widetilde{Q}^{\delta t}) = \mu_F^i\delta t - J^i\lambda\delta t, \tag{33}$$

and the standard deviation is:

$$\sigma^2(\delta\widetilde{F}_S^i) = (\sigma_F^i)^2\delta t + (J^i)^2\lambda\delta t. \tag{34}$$

The third and fourth central moments, $\mu_3(\cdot)$ and $\mu_4(\cdot)$ respectively, are given by:

$$\begin{aligned}
\mu_3(\delta\widetilde{F}_S^i) &= E\left\{\left[\mu_F^i\delta t + \sigma_F^i\widetilde{\varepsilon}^i\sqrt{\delta t} - J^i\widetilde{Q}^{\delta t} - E(\delta\widetilde{F}_S^i)\right]^3\right\} \\
\Rightarrow \mu_3(\delta\widetilde{F}_S^i) &= E\left[\left(\mu_F^i\delta t + \sigma_F^i\widetilde{\varepsilon}^i\sqrt{\delta t} - J^i\widetilde{Q}^{\delta t} - \mu_F^i\delta t + J^i\lambda\delta t\right)^3\right] \\
\Rightarrow \mu_3(\delta\widetilde{F}_S^i) &= E\left\{\left[\sigma_F^i\widetilde{\varepsilon}^i\sqrt{\delta t} + J^i(\lambda\delta t - \widetilde{Q}^{\delta t})\right]^3\right\} \\
\Rightarrow \mu_3(\delta\widetilde{F}_S^i) &= -(J^i)^3 E\left[\left(\widetilde{Q}^{\delta t} - \lambda\delta t\right)^3\right] = -(J^i)^3\lambda\delta t, \qquad (35)
\end{aligned}$$

and

$$\begin{aligned}
\mu_4(\delta\widetilde{F}_S^i) &= E\left\{\left[\mu_F^i\delta t + \sigma_F^i\widetilde{\varepsilon}^i\sqrt{\delta t} - J^i\widetilde{Q}^{\delta t} - E(\delta\widetilde{F}_S^i)\right]^4\right\} \\
\Rightarrow \mu_4(\delta\widetilde{F}_S^i) &= E\left\{\left[\sigma_F^i\widetilde{\varepsilon}^i\sqrt{\delta t} - J^i(\lambda\delta t - \widetilde{Q}^{\delta t})\right]^4\right\} \\
\Rightarrow \mu_4(\delta\widetilde{F}_S^i) &= (J^i)^4\lambda\delta t + 3\left[(\sigma_F^i)^2\delta t + (J^i)^2\lambda\delta t\right]^2 \\
\Rightarrow \mu_4(\delta\widetilde{F}_S^i) &= (J^i)^4\lambda\delta t + 3\sigma^2(\delta\widetilde{F}_S^i), \qquad (36)
\end{aligned}$$

given that for $\widetilde{\varepsilon}^i$: $E(\widetilde{\varepsilon}^i) = 0$, $E[(\widetilde{\varepsilon}^i)^2] = 1$, $E[(\widetilde{\varepsilon}^i)^3] = 0$, and $E[(\widetilde{\varepsilon}^i)^4] = 3$, and for $\widetilde{Q}^{\delta t}$: $E(\widetilde{Q}^{\delta t}) = \lambda\delta t$, $E[(\widetilde{Q}^{\delta t})^2] = \lambda\delta t$, $E[(\widetilde{Q}^{\delta t})^3] = \lambda\delta t$, and $E[(\widetilde{Q}^{\delta t})^4] = 3(\lambda\delta t)^2 + \lambda\delta t$. In order to obtain the above results, it was also necessary to assume the independence between $\widetilde{\varepsilon}^i$ and $\widetilde{Q}^{\delta t}$, such that $E(\widetilde{\varepsilon}^i\widetilde{Q}^{\delta t}) = E(\widetilde{\varepsilon}^i)E(\widetilde{Q}^{\delta t})$.

Given the first four sample central moments:

$$m_1 = \frac{1}{n}\sum_{j=1}^{n}\delta F_S^{i-j}; \qquad (37)$$

$$m_2 = \frac{1}{n}\sum_{j=1}^{n}(\delta F_S^{i-j} - m_1)^2; \qquad (38)$$

$$m_3 = \frac{1}{n}\sum_{j=1}^{n}(\delta F_S^{i-j} - m_1)^3; \qquad (39)$$

$$m_4 = \frac{1}{n}\sum_{j=1}^{n}(\delta F_S^{i-j} - m_1)^4, \qquad (40)$$

where $\delta F_S^{i-j}$, $1 \leq j \leq n$, represents a sample of $n$ historical values of the change in short-term liabilities, it is possible to use Fisher's $k$ statistics (Fisher, 1928) as non-biased estimators of the central moments of $\delta \widetilde{F}_S^i$:

$$\begin{aligned}
\widehat{E}(\delta \widetilde{F}_S^i) &= k_1 = m_1; & (41)\\
\widehat{\sigma}^2(\delta \widetilde{F}_S^i) &= k_2 = \frac{n}{n-1} m_2; & (42)\\
\widehat{\mu}_3(\delta \widetilde{F}_S^i) &= k_3 = \frac{n^2}{(n-1)(n-2)} m_3; & (43)\\
\widehat{\mu}_4(\delta \widetilde{F}_S^i) &= k_4 = \frac{n^2[(n+1)m_4 - 3(n-1)m_2^2]}{(n-1)(n-2)(n-3)}. & (44)
\end{aligned}$$

By combining estimates (41) to (44) with the results (33) to (36), it is possible to obtain the following estimates for the parameter of the distribution of $\delta \widetilde{F}_S^i$:

$$\begin{aligned}
\widehat{J}^i &= -\frac{\widehat{\mu}_4(\delta \widetilde{F}_S^i) - 3\widehat{\sigma}^2(\delta \widetilde{F}_S^i)}{\widehat{\mu}_3(\delta \widetilde{F}_S^i)}; & (45)\\
\widehat{\lambda} &= -\frac{\widehat{\mu}_3(\delta \widetilde{F}_S^i)}{(\widehat{J}^i)^3 \delta t}; & (46)\\
\widehat{\sigma}_F^i &= \sqrt{\frac{\widehat{\sigma}^2(\delta \widetilde{F}_S^i) - (\widehat{J}^i)^2 \widehat{\lambda} \delta t}{\delta t}}; & (47)\\
\widehat{\mu}_F^i &= \frac{\widehat{E}(\delta \widetilde{F}_S^i) + \widehat{J}^i \widehat{\lambda} \delta t}{\delta t}. & (48)
\end{aligned}$$

From (21), (26), and (27), and using the independence between $\delta \widetilde{D}^i$ and $\delta \widetilde{A}_C^i$, the liquidity restriction becomes:

$$\begin{cases} \delta \widetilde{A}_C^i + \delta \widetilde{F}_{S,NJ}^i \geq -w_L^i A_{L,D}^i \text{ w. p. } e^{-\lambda \delta t}; \\ \delta \widetilde{A}_C^i + \delta \widetilde{F}_{S,J}^i \geq -w_L^i A_{L,D}^i \text{ w. p. } (1 - e^{-\lambda \delta t}), \end{cases} \quad (49)$$

where $\delta \widetilde{F}_{S,NJ}^i$ and $\delta \widetilde{F}_{S,J}^i$ represent random variables whose distributions correspond to the distribution of $\delta \widetilde{F}_S^i$ conditional to $\widetilde{Q}^{\delta t} = 0$ (no jump), and $\widetilde{Q}^{\delta t} = 1$ (exactly one jump), respectively. As $\delta \widetilde{A}_C^i$, $\delta \widetilde{F}_{S,NJ}^i$, and $\delta \widetilde{F}_{S,J}^i$ are normal and independent random variables, the distribution of the sum of random variables in (49) is also normal, with parameters given by:

$$\delta \widetilde{A}_C^i + \delta \widetilde{F}_{S,NJ}^i \sim N\left(A_C^i \mu_C^i \delta t + \mu_F^i \delta t, \sqrt{(A_C^i \sigma_C^i)^2 \delta t + (\sigma_F^i)^2 \delta t}\right), \quad (50)$$

and

$$\delta \widetilde{A}_C^i + \delta \widetilde{F}_{S,J}^i \sim N\left(A_C^i \mu_C^i \delta t + \mu_F^i \delta t - J^i, \sqrt{(A_C^i \sigma_C^i)^2 \delta t + (\sigma_F^i)^2 \delta t}\right). \quad (51)$$

Finally, for sufficiently small optimization intervals, as discussed previously, the probability of a violation in the liquidity restriction will be:

$$\begin{aligned} P_f^i(\mathbf{w}^i) = e^{-\lambda \delta t} \Phi \left[ \frac{-w_L^i A_{L,D}^i - (A_C^i \mu_C^i \delta t + \mu_F \delta t)}{\sqrt{(A_C^i \sigma_C^i)^2 \delta t + (\sigma_F^i)^2 \delta t}} \right] + \\ +(1 - e^{-\lambda \delta t}) \Phi \left[ \frac{-w_L^i A_{L,D}^i - (A_C^i \mu_C^i \delta t + \mu_F \delta t - J^i)}{\sqrt{(A_C^i \sigma_C^i)^2 \delta t + (\sigma_F^i)^2 \delta t}} \right], \end{aligned} \quad (52)$$

where $\Phi(x)$ is the cumulative probability function of the standard normal distribution.

## 2.3 The Loss Given a Failure

The modeling of the failure loss in the case of violation of the liquidity constraint brings additional challenges. The information needed to obtain the failure probability, either through simulation or through a parametric model, is used in a daily basis and is largely available to the financial institution's management, and even to regulators in less detail. The idea that a loss, by the time it is recorded, should be assigned to the lack of liquidity, though, is not usual, and it becomes even more complex because failure losses have multiple distinct origins and natures, as the losses because of foregone investments and the forced realization of assets. If it is added to this set of issues the fact that considerable losses are hopefully rare, one can conclude that the difficulties to estimate failure losses resemble those involved in the estimation of operational losses. One of the main concerns in deploying operational losses models is the lack of enough historical information or reliable models for its statistical distribution (Marshall, 2001, chap. 6 and 7). In this context, the use of the Extreme Value Theory[12] looks like a natural path to establish a statistical model for $\widetilde{L}_f$.

The Extreme Value Theory seeks to find limiting distributions to the tails of unknown distributions, just like the Central Limit Theorem ensures that the limiting distribution of the mean of a sequence of independent and identically distributed random variables is the normal distribution. According to McNeil et al. (2005, chap. 7), there are, roughly, two main approaches.

[12] A classical reference on Extreme Value Theory applied to finance is Embrechts et al. (1997). A good reference for the statistical aspects described here is McNeil et al. (2005).

The first one describes the distributions of maximums (or minimums) of a block of data, such that if there is a non-degenerate limiting distribution to the maximum of a sequence of $n$ independent realizations drawn from the same distribution, then the cumulative probability function of the maximum of the sequence, as $n$ increases, tends to a distribution with the form:

$$H_\xi(x) = \begin{cases} \exp[-(1+\xi x)^{-\frac{1}{\xi}}], \xi \neq 0; \\ \exp(-e^{-x}), \xi = 0, \end{cases} \tag{53}$$

which is called *Generalized Extreme Value Distribution*, GEV, where $\xi$ is a shape parameter.

The second approach models the limiting distribution of losses above a certain level $u$ in a group of $n$ losses. As $n$ increases, if there is a GEV for the distribution of the block maximum, then the cumulative probability distribution of losses that exceed $u$ converges to a *Generalized Pareto Distribution*, GPD:

$$G_{\xi,\beta}(x) = \begin{cases} 1 - \left(1 + \dfrac{\xi x}{\beta}\right)^{-\frac{1}{\xi}}, \xi \neq 0; \\ 1 - e^{-\frac{x}{\beta}}, \xi = 0, \end{cases} \tag{54}$$

where $\beta > 0$ and $x \geq 0$ when $\xi \geq 0$, and $0 \leq x \leq -\frac{\beta}{\xi}$ when $\xi < 0$.

McNeil et al. (2005) argue that this second approach is more economic in the use of data and suggest a maximum likelihood estimation method of the parameters. As discussed at the beginning of this section, the availability of data about the losses due to the lack of liquidity is, at best, limited, and therefore, the second approach seems to be the best choice.

In order to estimate the parameters in (54), consider the set of historical data about $n$ losses due to the lack of liquidity, given by $\{L_1, L_2, \ldots, L_n\}$, and a subset $\{L_{u,1}, L_{u,2}, \ldots, L_{u,N_u}\}$ of this set, containing the $N_u$ losses that surpassed the level $u$. Define $Y_j = L_{u,j} - u$ as the loss in excess of the level $u$ and $g_{\xi,\beta}(x)$ as the probability density function of the GPD. The logarithm of the likelihood function is given by:

$$\begin{aligned} \ln L(\xi, \beta, Y_1, \ldots, Y_{N_u}) &= \sum_{j=1}^{N_u} \ln g_{\xi,\beta}(Y_j) \\ &= -N_u \ln \beta - \left(1 + \frac{1}{\xi}\right) \sum_{j=1}^{N_u} \ln \left(1 + \xi \frac{Y_j}{\beta}\right), \end{aligned} \tag{55}$$

and should be maximized subject to $\beta > 0$ and $1 + \xi \frac{Y_j}{\beta} > 0$ for every $j$, resulting in estimations of the parameters $\xi$ and $\beta$. It is assumed that the losses are independent and identically distributed, but according to McNeil et al. (2005), it is still possible to obtain good point estimations, even though standard errors may be too small.

The result of this discussion is that even if it is not possible to distinguish every failure losses, it is possible to establish a threshold $u$ above which every loss due to the lack of liquidity should be accounted for. Given the risk neutrality assumption, the only information needed for optimization purposes is the expected value of the failure losses, which is given by:

$$E(\widetilde{L}_f) = \frac{\beta}{1-\xi}, \tag{56}$$

if it has a finite value, what occurs for $\xi < 1$.

It is important to point out that the use of (56) requires the assumption that the distribution of failure losses is stationary. The rationale of this assumption is that, once all liquid assets have been used, the distribution of losses does not depend anymore on how the resources have been allocated in the short-run, because only illiquid assets remain. If there is some dependence it would be on the type of illiquid assets, on bank's business practices, on bank's reputation, or on some other factor that is exogenous to the model and could remain constant or change slowly throughout the estimation period, and in this case, rare, but extreme effects related to bank fragility could dominate the result of the mathematical expectation.

Without enough historical data it is not possible to estimate the expected loss, let alone to test the stationarity hypothesis. Thus, the usual assumption that $E(\widetilde{L}_f)$ is a function of the asset allocation is also discussed in this work, allowing for the estimation of $E(\widetilde{L}_f)$ with available data. In the empirical section of this work, I describe the methodology that was used to apply the model with real data, and the expected loss is assumed to be the mathematical expectation of the cash demands in excess of the availability of liquid resources, conditional to the occurrence of a failure. In general, this approach assumes a linear relation between the conditional expectations of the failure loss and the cash demand in excess to available liquid resources, given a failure. The argument, in this case, is that more value destroying actions would take place when the lack of liquid resources is more severe. The coefficient of the linear relation is the punitive cost of the value destroying sources of cash.

## 2.4 The Economic Profit

Due to banks' particularities, the opportunity cost of investments should be given in terms of the acceptable return on the economic capital required by operations (Schroeck, 2002, chap. 6). Thus, the expected return to be estimated is the return to equity holders. This implies that the cost of funding through debt, reflected in transfer prices, must be deducted together with operational expenses. As a result, the expected returns on liquid and illiquid assets may be estimated as:

$$E(\widetilde{r}_L^i) = rev_L^i - exp_L^i - EL_L^i - TP^i, \quad (57)$$
$$E(\widetilde{r}_I^i) = rev_I^i - exp_I^i - EL_I^i - TP^i, \quad (58)$$

where

- $rev_L^i$ and $rev_I^i$ are the expected revenues per monetary unit invested in the portfolios of liquid and illiquid assets, respectively, in the $i$-th period;
- $exp_L^i$ and $exp_I^i$ are costs and operational expenses per monetary unit invested[13] in the portfolios of liquid and illiquid assets, respectively, in the $i$-th period;
- $EL_L^i$ and $EL_I^i$ are the expected losses per monetary unit invested in the portfolios of liquid and illiquid assets, respectively, in the $i$-th period;
- $TP^i$ represents the transfer price of resources in the $i$-th period.

In the case of liquid assets, as market prices are available, it is possible to use these prices to estimate returns, which should already reflect expected losses, so that (57) may be written as:

$$E(\widetilde{r}_L^i) = E\left(\frac{A_L^{i+1}}{A_L^i} - 1\right) - exp_L^i - TP^i. \quad (59)$$

Regarding illiquid assets, mainly loans, the biggest challenge is not to estimate revenues, but expected losses. This can be done with the use of historical values of losses on credit contracts or through more sophisticated credit risk models.

Moreover, the opportunity cost must reflect the economic capital required by the investment. Thus, in the case of banks, the economic profit will be given by how much the investment return to shareholders exceeds the required return on the economic capital needed to make the investment:

[13]Such costs and expenses refers only to incremental values in $\delta t$, since fixed and predictable costs have already been excluded from the amount of resources available to allocation.

$$
\begin{aligned}
E(\widetilde{\Pi}) = w_L A_{L,D} E(\widetilde{r}_L) - ROE \times K(w_L A_{L,D}, f_{\widetilde{r}_L}) + \\
+ w_I A_{L,D} E(\widetilde{r}_I) - ROE \times K(w_I A_{L,D}, f_{\widetilde{r}_I}),
\end{aligned} \tag{60}
$$

where $ROE$ represents the required return on equity. $K(w_L A_{L,D}, f_{\widetilde{r}_L})$ and $K(w_I A_{L,D}, f_{\widetilde{r}_I})$ represent the economic capital required to invest in liquid and illiquid assets, respectively, as a function of the invested amounts and the return distributions $f_{\widetilde{r}_L}$ and $f_{\widetilde{r}_I}$. So, by (1):

$$
R_L = ROE \times \lim_{w \to w_L^+} \frac{K(wA_{L,D}, f_{\widetilde{r}_L})}{wA_{L,D}} \tag{61}
$$

and

$$
R_I = ROE \times \lim_{w \to w_I^+} \frac{K(wA_{L,D}, f_{\widetilde{r}_I})}{wA_{L,D}}, \tag{62}
$$

where the limit was used only to make treatable the cases in which $w_L$ or $w_I$ are zero.

If it is assumed that the distribution of returns does not change with the volume invested, what in practice means that the bank will not change its investment profile, neither in liquid nor in illiquid assets, then the required economic capital will grow linearly with the volume invested. This will be true for any methodology to assess the economic capital requirement that uses a coherent risk measure (Artzner et al., 1999), or even measures that are not coherent, but shows the positive homogeneity property, as the *Value at Risk*, (VaR). In this conditions, the relations above become:

$$
R_L = ROE \times \lim_{w \to w_L} \frac{wK(A_{L,D}, f_{\widetilde{r}_L})}{wA_{L,D}} = ROE \times \frac{K(A_{L,D}, f_{\widetilde{r}_L})}{A_{L,D}} \tag{63}
$$

and

$$
R_L = ROE \times \lim_{w \to w_I} \frac{wK(A_{L,D}, f_{\widetilde{r}_I})}{wA_{L,D}} = ROE \times \frac{K(A_{L,D}, f_{\widetilde{r}_I})}{A_{L,D}} \tag{64}
$$

Equations (63) and (64) simply establish that the opportunity cost will be given by the required return on equity, weighted by the leverage of investments, that is, by the proportion of equity that will be used.

The discussion about the details of the implementation of economic capital models is beyond the scope of the work. Many banks have already their own models and there are excellent texts that provide broad coverage of the theoretical and practical aspects involved, such as Crouhy et al. (2001, chap. 14), Schroeck (2002, chap. 5) and Bessis (2002, sections 15 and 16). Nonetheless, there are some important aspects related to risk aggregation that should be considered.

As discussed in Saunders (1999), the RAROC "has been historically calculated on individual basis, despising correlations", but such correlations may exist and the economic capital would not be, necessarily, simply the weighted sum of $K(A_{L,D}, f_{\tilde{r}_L})$ and $K(A_{L,D}, f_{\tilde{r}_I})$. In general, the RAROC would be a function of the allocation between liquid and illiquid assets. If only the correlations between the new portfolios of liquid and illiquid assets are taken into account, after the investment $A^i_{L,D}$ we should have:

$$w_L R_L + w_I R_I = ROE \times \frac{K(A_{L,D}, f_{w_L\tilde{r}_L + w_I\tilde{r}_I})}{A_{L,D}}, \tag{65}$$

and the economic profit to be maximized would become:

$$E(\widetilde{\Pi}^i) = A_{L,D}\mathbf{w}^i E\left[(\tilde{\mathbf{r}}^i)^\top\right] - ROE \times K(A_{L,D}, f_{w_L\tilde{r}_L + w_I\tilde{r}_I}). \tag{66}$$

According to Artzner et al. (1999), in the case of coherent risk measures, or at least sub-additive risk measures, $K(A_{L,D}, f_{w_L\tilde{r}_L + w_I\tilde{r}_I}) \leq w_L K(A_{L,D}, f_{\tilde{r}_L}) + w_I K(A_{L,D}, f_{\tilde{r}_I})$.

That said, I assume that the risk measure used to assess the economic capital can correctly incorporate correlations (see Saita, 2004). Though, in order to simplify the exposition of more fundamental aspects of the optimization, which will be discussed in the next section, I will use the capital costs as in (63) and (64).

In the case of the $ROE$, even though it can be estimated by the average of historical results on shareholders equity, accounting measures are more prone to manipulations and distortions. One alternative is to use pricing models, as the *Capital Asset Pricing Model*, CAPM. Damodaran (2002, chap. 21) suggests that, in the case of banks, there should not be a leverage correction in the beta of the CAPM, since debt works for the bank as accounts payable would for a non-financial firm. Table 2.4 shows CAPM betas of Brazilian banks listed at the São Paulo's Stock Exchange (Bolsa de Valores de São Paulo, BOVESPA).

Using the average of these betas weighted by market capitalization of each stock, the obtained banking sector beta is about 0.9.

Copeland et al. (2000, chap. 10) suggest that a reasonable value for the market premium risk, the difference between the average return of a theoretical market portfolio and the risk free rate, is around 5%, based on estimates that used data from Unites States' companies. Cysne (2006) compares his results with four others that used Brazilian data, and shows values that range from 10% to 29%. Nevertheless, all estimates uses data from a high inflation period, what may distort the results, and because of this, the lower value of the range will be used. On the other hand, by the second half of 2006,

| Name | Type | Beta | Market Cap. (R$ millions) | Weighted beta |
|---|---|---|---|---|
| ALFA HOLDING S.A. | Preferred | 0.16 | 56.75 | 0.00004 |
| BCO ALFA DE INVESTIMENTO S.A. | Common | 0.27 | 317.76 | 0.00036 |
| BCO ALFA DE INVESTIMENTO S.A. | Preferred | 0.31 | 210.40 | 0.00027 |
| BCO AMAZONIA S.A. | Common | 0.63 | 847.87 | 0.00221 |
| BCO ESTADO DE SERGIPE S.A. - BANESE | Preferred | 0.47 | 158.12 | 0.00031 |
| BCO ESTADO DE SAO PAULO S.A. - BANESPA | Common | 0.46 | 6,238.40 | 0.01188 |
| BCO ESTADO DE SAO PAULO S.A. - BANESPA | Preferred | 0.53 | 6,587.13 | 0.01446 |
| BCO ESTADO DO RIO GRANDE DO SUL S.A. | Preferred | 0.42 | 1,271.33 | 0.00221 |
| BCO BRADESCO S.A. | Common | 1.09 | 35,420.78 | 0.15988 |
| BCO BRADESCO S.A. | Preferred | 1.05 | 37,478.04 | 0.16295 |
| BCO BRASIL S.A. | Common | 0.95 | 42.214,91 | 0.16607 |
| BCO ITAU HOLDING FINANCEIRA S.A. | Common | 0.77 | 36,478.97 | 0.11631 |
| BCO ITAU HOLDING FINANCEIRA S.A. | Preferred | 1.01 | 42,935.26 | 0.17957 |
| BCO SUDAMENRIS BRASIL S.A. | Common | 0.69 | 3,037.20 | 0.00868 |
| UNIBANCO UNIAO DE BCOS BRASILEIROS S.A. | Common | 0.64 | 18,891.45 | 0.05007 |
| UNIBANCO UNIAO DE BCOS BRASILEIROS S.A. | Preferred | 0.82 | 9,347.33 | 0.03174 |
| TOTAL | | | 241,491.71 | 0.90700 |

Source: Bloomberg, 10/30/2006.

Table 1: Brazilian bank's betas with respect to São Paulo's Stock Exchange Index, IBOVESPA.

Brazilian government securities were paying interests of about 13.5% a year. The CAPM will predict, therefore, a $ROE$ of 22.5% a year.

## 2.5 Optimization

The assumption of RAROC and EVA models is that shareholders will seek the highest expected economic profit $E(\widetilde{\Pi})$ and any positive economic profit will enhance the shareholders value. Nevertheless, as previously discussed, it is necessary to account for the possibility of the failure loss $\widetilde{L}_f^i$. Thus, while deciding the best allocation between liquid and illiquid assets, a risk neutral agent should maximize the expected economic profit, given the possibility of loss due to the lack of liquidity: $(1 - P_f^i) \times E(\widetilde{\Pi}^i) + P_f^i \times [E(\widetilde{\Pi}^i) - E(\widetilde{L}_f^i)] = E(\widetilde{\Pi}^i) - P_f^i E(\widetilde{L}_f^i)$. The complete problem, given the restrictions discussed in the description of the model, may be given by:

$$
\begin{aligned}
&\max_{\mathbf{w}^i} E\left(\widetilde{\Pi}^i\right) - P_f^i E\left(\widetilde{L}_f^i\right) \\
&\text{subject to} \\
&i) \quad \mathbf{O}^i \geqq A_{L,D}\mathbf{w}^i; \\
&ii) \;\; \mathbf{w}^i \mathbf{1}^\top = 1; \\
&iii) \; \mathbf{w}^i \geqq \mathbf{0}.
\end{aligned}
\tag{67}
$$

From restriction (ii) in (67), it is possible to write $w_I = 1 - w_L$, and the problem can be treated as the optimization of a function of one real variable, with the domain in an closed interval. This interval is given by restrictions (i) and (iii) in (67), which can be summarized by:

$$\max\left\{0, 1 - \frac{O_I^i}{A_{L,D}^i}\right\} \leq w_L \leq \min\left\{1, \frac{O_L^i}{A_{L,D}}\right\}. \tag{68}$$

The optimization problem can be, therefore, completely described by:

$$\begin{aligned}
&\max_{w_L^i} A_{L,D}\mathbf{w}^i \left[E\left(\widetilde{\mathbf{r}}^i\right) - \mathbf{R}^i\right]^\top - P_f^i E(\widetilde{L}_f^i) \\
&\text{subject to} \\
&\max\left\{0, 1 - \frac{O_I^i}{A_{L,D}^i}\right\} \leq w_L \leq \min\left\{1, \frac{O_L^i}{A_{L,D}}\right\}.
\end{aligned} \tag{69}$$

If (56) is used, and $E(\widetilde{L}_f^i)$ is not treated as a function of $\mathbf{w}^i$, then (69) becomes:

$$\begin{aligned}
&\max_{w_L^i} A_{L,D}\mathbf{w}^i \left[E\left(\widetilde{\mathbf{r}}^i\right) - \mathbf{R}^i\right]^\top - P_f^i \frac{\beta^i}{1 - \xi^i} \\
&\text{subject to} \\
&\max\left\{0, 1 - \frac{O_I^i}{A_{L,D}^i}\right\} \leq w_L \leq \min\left\{1, \frac{O_L^i}{A_{L,D}}\right\},
\end{aligned} \tag{70}$$

but in either case, $P_f^i = P_f^i(\mathbf{w}^i) = P_f^i(w_L^i)$.

If the failure probability has been estimated through simulation, it is only necessary to find the greatest value of (69) or (70) to the simulated values of $w_L^i$ in the interval (68).

Though, as neither data nor parameters to a simulation are widely available outside the financial institution, it will be useful to use expression (52) in order to quantitatively assess the results of the model. While it is very simple to numerically obtain the proposed optimization, the fact that $\mu_C^i$ and specially $\sigma_C^i$ are functions of $w_L^i$ makes the derivatives of $P_f^i(w_L^i)$ too big and clumsy to allow an algebraic analysis from which one could get any intuition of the optimization process. On the other hand, the result of a change in $w_L^i$ on $\mu_C^i$ has a secondary effect on the availability of resources, when compared to the change on the investment in liquid assets. In fact, by using (15) it is possible to obtain:

$$\frac{d\mu_C^i}{dw_L^i} = \frac{A_{L,D}^i(\mu_{C,I}^i - \mu_L^i)}{A_C^i}, \tag{71}$$

and, therefore, a small change $\Delta w_L^i$ in $w_L^i$ results in a change in the cash generated in the period $\delta t$ of about:

$$A_C^i \Delta\mu_C^i \delta t \approx \frac{A_{L,D}^i(\mu_{C,I}^i - \mu_L^i)}{A_C^i} \Delta w_L^i A_C^i \delta t = (\mu_{C,I}^i - \mu_L^i)\Delta w_L^i A_{L,D}^i \delta t. \quad (72)$$

As $\delta t$ is ideally small, and $(\mu_{C,I}^i - \mu_L^i)$ is most likely a number smaller than one,[14] then $(\mu_{C,I} - \mu_L^i)\Delta w_L^i A_{L,D}^i \delta t$ is much smaller than the change in the investment in liquid assets given by $\Delta w_L^i A_{L,D}^i$. For example, even if $\delta t$ represents a month, i.e. $\delta t = \frac{1}{12}$, and $(\mu_{C,I} - \mu_L^i) = 25\%$, then the change in the cash generated in the period $\delta t$ due to a change in $w_L^i$ is 48 times smaller than the change in the investment in liquid assets given by $\Delta w_L^i A_{L,D}^i$. A similar analysis may be done to $\sigma_C^i$, in such a way that in the following discussion it is assumed that $\delta t$ and $(\mu_{C,I}^i - \mu_L^i)$ are small enough so that $\mu_C^i$ and $\sigma_C^i$ may be treated as constants with respect to $w_L^i$.

By the theorem of Weierstrass, if an objective function is continuous in a compact interval, then the function has a maximum and a minimum in this interval. The failure probability as defined in (52) is a continuous function of $w_L^i$, and the interval (68) is a compact set, such that there is a maximum and a minimum.

The cumulative probability function of the normal distribution is twice differentiable, such that the optimal choice will be either a corner solution, given by one of the extremes in the interval (68), or a value that causes the derivative of (70) to equal zero and the second derivative to be negative, what results, for $\beta^i > 0$ and $\xi^i < 1$, in the conditions:

$$\frac{\mathrm{d}P_f^i(w_L^i)}{\mathrm{d}w_L^i} = \frac{1-\xi^i}{\beta^i} A_{L,D} \left\{ \left[E(\widetilde{\mathbf{r}}^i) - \mathbf{R}^i\right] \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\} \quad (73)$$

and

$$\begin{aligned} -\frac{\beta^i}{1-\xi^i} \frac{\mathrm{d}^2 P_f^i(w_L^i)}{(\mathrm{d}w_L^i)^2} &> 0 \\ \Rightarrow \frac{\mathrm{d}^2 P_f^i(w_L^i)}{(\mathrm{d}w_L^i)^2} &> 0. \end{aligned} \quad (74)$$

In order to exist an interior solution, it is necessary, even though not sufficient, that both conditions are satisfied. With the objective to make the

[14] The difference between median values of $\mu_{C,I}^i$ and of $\mu_L^i$ found in the empirical section is 25%.

manipulation of the derivatives easier, the failure probability in (52) may be written as:

$$P_f^i(w_L^i) = e^{-\lambda\delta t}\Phi(-w_L^i a - b) + (1 - e^{-\lambda\delta t})\Phi(-w_L^i a - c), \tag{75}$$

with:

$$a = \frac{A_{L,D}^i}{\sqrt{(A_C^i\sigma_C^i)^2\delta t + (\sigma_F^i)^2\delta t}}; \tag{76}$$

$$b = \frac{A_C^i\mu_C^i\delta t + \mu_F^i\delta t}{\sqrt{(A_C^i\sigma_C^i)^2\delta t + (\sigma_F^i)^2\delta t}}; \tag{77}$$

$$c = \frac{A_C^i\mu_C^i\delta t + \mu_F^i\delta t - J^i}{\sqrt{(A_C^i\sigma_C^i)^2\delta t + (\sigma_F^i)^2\delta t}}. \tag{78}$$

Therefore,

$$\frac{\mathrm{d}P_f^i(w_L^i)}{\mathrm{d}w_L^i} = -a \times \left[e^{-\lambda\delta t}\phi(-w_L^i a - b) + (1 - e^{-\lambda\delta t})\phi(-w_L^i a - c)\right], \tag{79}$$

and

$$\frac{\mathrm{d}^2P_f^i(w_L^i)}{(\mathrm{d}w_L^i)^2} = a^2 \times \left[e^{-\lambda\delta t}\phi'(-w_L^i a - b) + (1 - e^{-\lambda\delta t})\phi'(-w_L^i a - c)\right] \tag{80}$$

where $\phi(x)$ is the standard normal distribution probability density function and $\phi'(x)$ is its first derivative.

From (73) and (79), there will be an interior critical point if it is possible to obtain

$$\begin{aligned} &\frac{\xi^i - 1}{a\beta^i}A_{L,D}\left\{\left[E(\widetilde{\mathbf{r}}^i) - \mathbf{R}^i\right]\left[\begin{array}{c} -1 \\ 1 \end{array}\right]\right\} = \\ &= e^{-\lambda\delta t}\phi(-w_L^i a - b) + (1 - e^{-\lambda\delta t})\phi(-w_L^i a - c) \end{aligned} \tag{81}$$

for some $w_L^i$ that satisfies (68). If there is a solution, equation (81) may be trivially solved through numerical methods. If there is not a solution, the maximum will occur at one of the interval extremes.

The right hand side of the equation is clearly limited below by zero, since it results from the sum of products of functions whose images in the real domain are always positive. As it is assumed that $\xi^i < 1$ so that the expected value of the distribution of losses is finite, there can only be a solution for equation (81) if the expected return in excess to the opportunity cost is bigger for the illiquid assets. Moreover, the higher possible value for the expression is $\phi(0) = 1/\sqrt{2\pi}$. This is an intuitive result and would remain the same if $\phi(x)$ was any symmetric function, differentiable in every point, with a global maximum at zero and monotonic to the left and to the right of its maximum. Anyway, this statement can be very easily verified through calculus by simply equating the partial derivatives of the function $f(x, y, z) = z\phi(x)+(1-z)\phi(x+y)$ to zero. The resulting system of equations has a solution for $x = y = 0$, regardless of the value of $z$, such that the function has a maximum value of $f(0, 0, z) = z\phi(0) + (1 - z)\phi(0) = \phi(0)$ in any point of the line $(0, 0, z) \in \mathbb{R}^3$.

In summary, in order to interior critical points exist it is necessary, but not sufficient, that:

$$0 < \frac{(\xi^i - 1)A^i_{L,D}}{a\beta^i} \left\{ \left[E(\widetilde{\mathbf{r}}^i) - \mathbf{R}^i\right] \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\} < \frac{1}{\sqrt{2\pi}}. \tag{82}$$

Alternatively, by using (50), (56), and (76), condition (82) may be rewritten as:

$$0 < \frac{\sigma(\delta\widetilde{A}^i_C + \delta\widetilde{F}^i_{S,NJ})}{E(\widetilde{L}^i_f)} \left\{ \left[E(\widetilde{\mathbf{r}}^i) - \mathbf{R}^i\right] \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} < \frac{1}{\sqrt{2\pi}}, \tag{83}$$

which is easier to be interpreted: there will be no interior solution if there is a large advantage in investing in illiquid assets or if the expected loss is too small when related to the volatility in cash generation and funding availability.

Regarding the number of critical points, if $c$ is a positive value then the right hand side of equation (81) is monotonically decreasing with respect to the rises in the proportion of liquid assets and there could only be a single critical point. If $J^i$ is large enough so that $c$ is negative, then, starting from $w^i_L = 0$, there will be at least one non-empty interval in which $-w^i_L a - c > 0$ and $\phi(-w^i_L a - c)$ will grow with $w^i_L$. If, in addition, the parameters $\lambda$ and $\delta t$ are large enough so that $1 - e^{-\lambda\delta t}$ has a value of the same magnitude of $e^{-\lambda\delta t}$, allowing that the effect of an increase in $(1 - e^{-\lambda\delta t})\phi(-w^i_L a - c)$ be significant with respect to a decrease in $e^{-\lambda\delta t}\phi(-w^i_L a - b)$, then there could be two or even three critical points. Figure 2 depicts these effects on $-\frac{1}{a}\frac{dP^i_f}{dw^i_L}$, which is equal to $e^{-\lambda\delta t}\phi(-w^i_L a - b) + (1 - e^{-\lambda\delta t})\phi(-w^i_L a - c)$.

The surfaces show the behavior of the values of $-(1/a)\left(dP^i_f/dw^i_L\right)$ for $0 \leq w^i_L \leq 1$, $0.0004 \leq \lambda\delta t \leq 0.42$, or $0.0004 \leq \lambda\delta t \leq 2$ to make the visualization easier. The parameters $b$ and specially $c$ were altered so that it is possible to realize their influence in the shape of the surface.

In figures 2i and 2ii, where $c \geq 0$, it is possible to see that $-(1/a)\left(dP^i_f/dw^i_L\right)$ is monotonically decreasing for any $\lambda\delta t$. In the remaining figures, with $c < 0$,

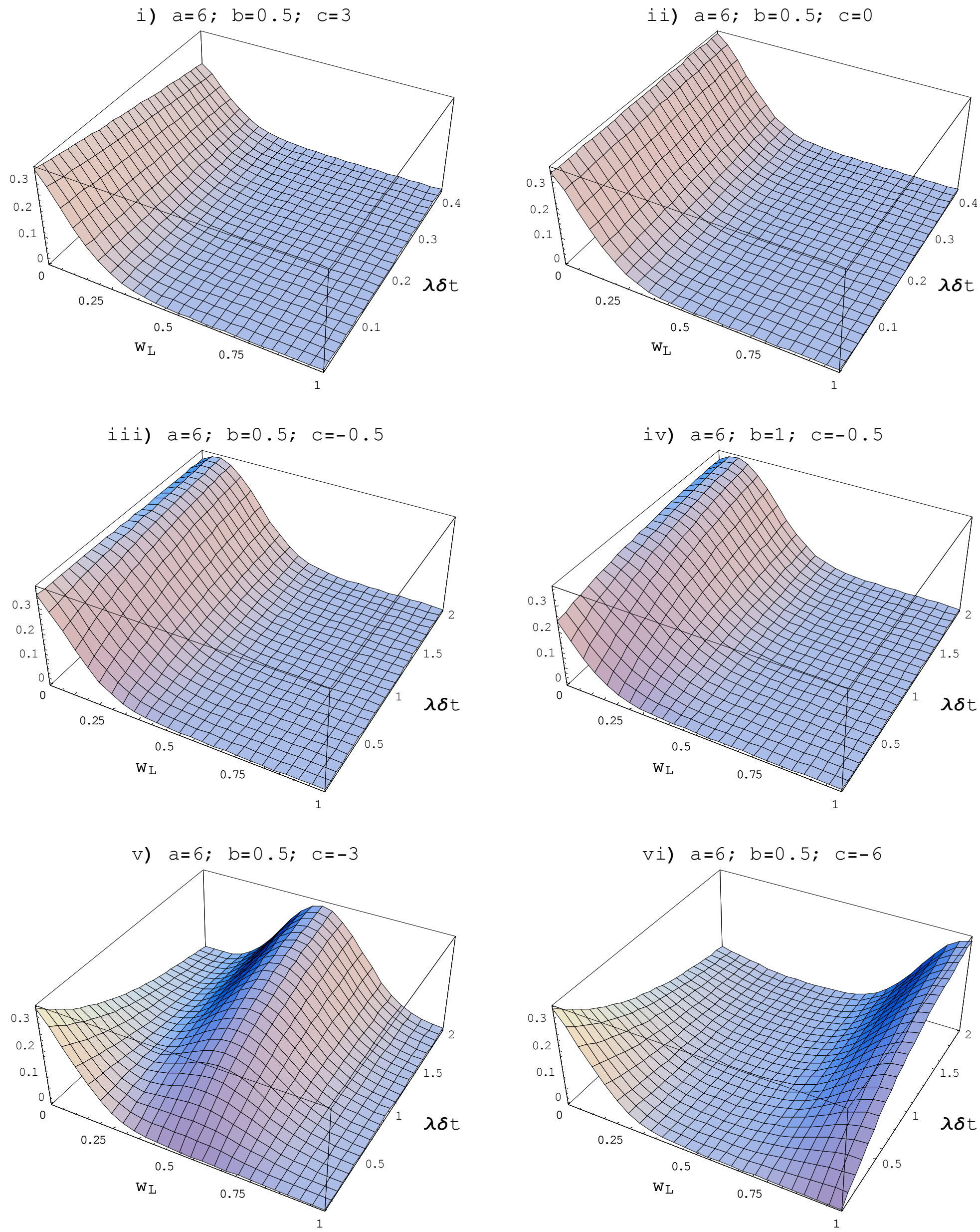



Figure 2: Behavior of $e^{-\lambda\delta t}\phi(-w_L^i a - b) + (1 - e^{-\lambda\delta t})\phi(-w_L^i a - c)$ given $0 \leq w_L^i \leq 1$, $0.0004 \leq \lambda\delta t \leq 0.42$ or $0.0004 \leq \lambda\delta t \leq 2$, $a = 6$, and several values for the parameters $b$ and $c$.

one can see that for sufficiently high values of $\lambda\delta t$, there are intervals where $-(1/a)\left(dP_f^i/dw_L^i\right)$ grows before it starts to decrease. For very small values of $\lambda\delta t$, on the other hand, $-(1/a)\left(dP_f^i/dw_L^i\right)$ is monotonically decreasing.

It is necessary for a critical point to be a local maximum, by (74) and (81), that:

$$e^{\lambda\delta t}\phi'(-w_L^i a - b) + (1 - e^{\lambda\delta t})\phi\prime(-w_L^i a - c) > 0, \tag{84}$$

given $a > 0$.[15]

It is known that $\phi'(x) > 0$ for every negative value of $x$, and one can assume that $b > 0$, otherwise the average cash generation and the funding opportunities would be null or negative. Moreover, the chances are that the probability of no jumps, $e^{-\lambda\delta t}$, is bigger than the probability of at least one jump, $1 - e^{-\lambda\delta t}$. For example, with $\lambda = 4.8$ and $\delta t = \frac{1}{12}$ the probability of no jumps is at least twice as big as the probability of the occurrence of jumps, and this proportion scales up to around $2,500$ to 1 when $\lambda = 0.1$ and $\delta t = \frac{1}{252}$. As a result, the inequality (84) would only be violated if $J^i$ is large enough so that the value of $c$ is sufficiently negative. Nevertheless, $\phi'(x)$ has a minimum for $x > 0$, or a maximum in absolute values, after which the value of $\phi'(x)$ starts to grow again, or diminish in absolute values, towards zero as $x$ tends to infinity. This way, $\phi'(-w_L^i a - c)$ can only be sufficiently negative in a limited range of positive values of $-w_L^i a - c$, depending on the values of $\lambda$ and $\delta t$, so that $(1 - e^{-\lambda\delta t})\phi'(-w_L^i a - c)$ can be larger, in absolute values, than $e^{-\lambda\delta t}\phi'(-w_L^i a - b)$. For sufficiently small values of $\lambda$ and $\delta t$, $e^{-\lambda\delta t}$ will be much bigger than $1 - e^{-\lambda\delta t}$ and the inequality cannot be violated.

Figure 3 shows three situations that could typically occur[16] when $c < 0$, since if $c > 0$ there is a single critical point, which corresponds to the value of $w_L^i$ that maximizes the objective function. This figure corresponds to figure 2v where three transversal sections are analyzed. The intersection between these sections and the surface is highlighted and is the graphic of $e^{-\lambda\delta t}\phi(-w_L^i a - b) + (1 - e^{-\lambda\delta t})\phi(-w_L^i a - c)$ given a value of $\lambda\delta t$, with $w_L^i$ changing from 0 to 1.

The value of $\left[(\xi^i - 1)A_{L,D}^i/(a\beta^i)\right]\{[E(\widetilde{r}_L^i) - R_L^i] - [E(\widetilde{r}_I^i) - R_I^i]\}$, which should be equal to $e^{-\lambda\delta t}\phi(-w_L^i a - b) + (1 - e^{-\lambda\delta t})\phi(-w_L^i a - c)$ so that there is a critical point, according to 81, is represented by the horizontal lines also highlighted in each of the transversal section. The critical points correspond to the intersections between the horizontal lines and the graphic of

[15]If $a$ is negative or zero, then the bank is already in a liquidity crises or there are not available resources for allocation.

[16]There are two other possibilities that involve the existence of saddle points: one saddle point as the only critical point, or a local maximum and a saddle point.

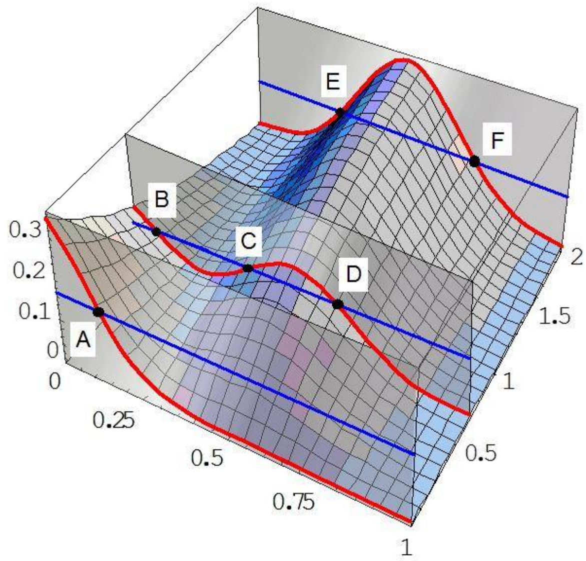



Figure 3: Typical situations for the maximization problem when $c < 0$, where there is a local maximum (point A), two maxima and one minimum (maxima at B and D, and minimum at C), or a maximum (point F), and a minimum (point E).

$e^{-\lambda\delta t}\phi(-w_L^i a - b) + (1 - e^{-\lambda\delta t})\phi(-w_L^i a - c)$, and if the slope of this graphic with respect to changes in $w_L^i$ is negative in the intersection, this means that:

$$-a \times [e^{-\lambda\delta t}\phi'(-w_L^i a - b) + (1 - e^{-\lambda\delta t})\phi'(-w_L^i a - c)] < 0$$
$$\Rightarrow e^{-\lambda\delta t}\phi'(-w_L^i a - b) + (1 - e^{-\lambda\delta t})\phi'(-w_L^i a - c) > 0$$

and that critical point is, by 84, a local maximum. On the other hand, if the slope of the graphic is positive with respect to changes in $w_L^i$, then the intersection corresponds to a local minimum.

At point A $\lambda\delta t$ is too small for $(1 - e^{-\lambda\delta t})\phi'(-w_L^i a - c)$ to have any significant influence, and there is only one critical point. The slope of the curve at this point is negative and, therefore, it is a local maximum, which, in this case, is also the maximum in all the interval $0 \leq w_L^i \leq 1$. Points B, C and D represent a situation where there can be three critical points: two local maxima at points B and D (one of them will be the maximum in the whole interval), since the curve shows negative slope at these points, and one local minimum at C. Finally, points E and F represent a situation where there is one local minimum (point E) and one maximum (point F).

Corner solutions mean that at the present volume of resources, $A_{L,D}^i$, it is not possible to reach the optimal liquidity level in a single period, and the model have to be applied dynamically throughout the time. This occurs either because the cash generation plus liquid strategic positions are already enough to cover for liquidity losses, in the case where $w_L^i = 0$ is the solution, or because it is necessary to raise the liquidity and to lower the inventory of illiquid assets[17], in the case where $w_L^i = 1$ is the solution.

# 3 Empirical Test of the Model

This section describes an empirical test of the model that uses Brazilian banks data available to the Banco Central do Brasil (Central Bank of Brazil), BCB. This means that the information used is more detailed than public information, but it is not as complete as managerial information, so that it will be necessary to make some assumptions to provide estimates of unobservable values. As it is not an effective implementation of the model, but a general assessment of its viability, the lack of precision is not a problem. Additionally, as part of the information used is protected by law, all data is presented in aggregate form and without identification.

There is, though, an important problem: the lack of data about the failure losses. As in the case of operational risks, this kind of information is not easily available, and specific initiatives need to be taken so that the data starts to be collected. Thus, the results presented here are conditional to arbitrary loss values, assumed to be reasonable choices.

Whenever accounting data is used, the Brazilian standard COSIF code[18] for the accounts used will be cited for reference.

## 3.1 Methodology

In order to calibrate the model and solve the maximization problem, it is necessary to obtain the following parameters:

a. Total cash generating assets: $A_C^i$;

b. Amount of liquid resources available to discretionary allocation: $A_{L,D}^i$;

[17] This may be accomplished simply by not making new investments and letting the current assets to mature, or by engaging loan sales or securitization. The best course of action depends on how low is the liquidity, and on strategic issues, for example, the preservation of market share.

[18] COSIF stands for *Plano Contábil das Instituições do Sistema Financeiro Nacional*, or Accounting Plan for the Institutions of the National Financial System. It is a standardized accounting plan for financial institutions established in Brazil through the Circular 1.272 of the Central Bank of Brazil in December 29th, 1987.

c. Mean continuous cash yield on $A_C^i$: $\mu_C^i$;

d. Standard deviation of the cash yield on $A_C^i$ in annualized continuous rates: $\sigma_C^i$;

e. Drift component of the funding opportunities distribution: $\mu_F^i$;

f. Diffusion component of the funding opportunities distribution: $\sigma_F^i$;

g. Jump component of the funding opportunities distribution: $J^i$;

h. Poisson process intensity: $\lambda$;

i. Expected value of the loss due to the lack of liquidity: $E(\widetilde{L}_f^i)$;

j. Expected return of the investments in liquid assets:$E(\widetilde{r}_L^i)$;

k. Expected return of the investments in illiquid assets: $E(\widetilde{r}_I^i)$;

l. Opportunity cost of equity: $ROE$;

m. Required capital to invest in liquid assets: $K(A_{L,D}^i, f_{\widetilde{r}_L})$;

n. Required capital to invest in illiquid assets: $K(A_{L,D}^i, f_{\widetilde{r}_I})$.

The available information is:

- On a daily basis: data about the deposits and total interbank credit operations from September 30, 2004, to September 29, 2006;
- On a monthly basis: accounting information about financial institutions from July 2003 to June 2006.

As most of the available information is on monthly basis, it will be used $\delta t = 1/12$. The total volume of loans plus the total volume of securities and derivative operations was used as the information in (a). Operations with securities and derivatives will be generically called treasury operations, henceforth. The COSIF account 3.1.0.00.00-0 was used for the value of credit operations. Accounts 1.3.0.00.00-4 and 4.7.0.00.00-1 where used for the value of treasury operations.

In order to obtain item (b), it was used the average variation on the inventory of credit and treasury operations. As the treasury operations are marked to market, part of the variation does not come from new investment, but from capital gains of existing assets, and in this case, the total invested amount is overestimated. On the other hand, debt payments by bank's clients decrease the loan portfolio, and in this case, the variation underestimates the invested amount, and both effects help to reduce the final distortion.

As there is not available information about the exact structure of liquid and illiquid assets, the credit portfolio was used as a proxy of the cash generating illiquid assets, and treasury operations were used as proxy of cash generating liquid assets in order to estimate items (c) and (d). Equation (8) and information about revenues from credit operations in COSIF accounts 7.1.1.00.00-1, 7.1.2.00.00-4, 7.1.3.00.00-7 and 7.1.4.00.00-0[19] were used to estimate the cash yields from illiquid assets. It is not possible to use (10) for treasury operations without information that allows to tell price changes from new investments. Though, as treasury operations are marked to market, it was used the ratio between the profit of treasury operations and the total amount of treasury operations as the cash yield from treasury operations. The time series of yields were used to estimate variances and covariances: $(\sigma^i_{C,I})^2, (\sigma^i_L)^2$, and $\sigma^i_{L,I}$, allowing for the obtention of estimates of $\mu^i_C$ and $\sigma^i_C$ for any portfolio of credit and treasury operations.

The parameters corresponding to items (e) to (h) were estimated from the data on daily basis, according to the methodology consolidated in equations (37) to (48). $\delta\widetilde{F}^i_S$ corresponds to the change on deposits and on net interbank credit operations.

The greatest challenge to the practical implementation of the model is the absence of historical data about losses stemming from the lack of liquidity. In section 2.3 I discuss an Extreme Value Theory approach to the estimation of the loss given a failure, but as the data required to proceed the estimation is not available, I will follow, in this empirical test, the more traditional approach of assuming that the loss given a failure and the amount by which the liquidity constraint is violated are proportional, where the proportionality coefficient represents a punitive funding cost. For now, I will assume an unitary coefficient for the punitive funding cost, which, *a priori*, is as arbitrary as any other choice. Later on, this assumption will be replaced by another estimate based on the findings of the empirical tests. Thus, according to the liquidity restrictions in (49), this means that the value of the expected loss given a failure will be proportional to the mathematical expectation of by how much $\delta\widetilde{A}^i_C + \delta\widetilde{F}^i_{S,NJ}$ or $\delta\widetilde{A}^i_C + \delta\widetilde{F}^i_{S,J}$ will result less than $-w^i_L A^i_{L,D}$, conditional to the violation of the liquidity restriction:

[19] Revenues, started by digit 7, and expenses, started by digit 8, are accumulated monthly and transferred to shareholders' equity every semester so that, except for January and July, monthly changes and not absolute values were actually used.

$$
\begin{gathered}
E(\widetilde{L}_f^i) = r_P E\left[-w_L^i A_{L,D}^i - \left(\delta\widetilde{A}_C^i + \delta\widetilde{F}_S^i\right) \middle| \delta\widetilde{A}_C^i + \delta\widetilde{F}_S^i < -w_L^i A_{L,D}^i\right] \\
\Rightarrow E(\widetilde{L}_f^i) = r_P \left[\begin{array}{cc} e^{-\lambda\delta t} & (1-e^{-\lambda\delta t}) \end{array}\right] \\
\left[\begin{array}{c} E\left[-w_L^i A_{L,D}^i - \left(\delta\widetilde{A}_C^i + \delta\widetilde{F}_{S,NJ}^i\right) \mid \delta\widetilde{A}_C^i + \delta\widetilde{F}_{S,NJ}^i < -w_L^i A_{L,D}^i\right] \\ E\left[-w_L^i A_{L,D}^i - \left(\delta\widetilde{A}_C^i + \delta\widetilde{F}_{S,J}^i\right) \mid \delta\widetilde{A}_C^i + \delta\widetilde{F}_{S,J}^i < -w_L^i A_{L,D}^i\right] \end{array}\right],
\end{gathered} \tag{85}
$$

where $r_P$ is the punitive funding cost. As discussed above, $r_P$ will be assumed to be one for the moment.

The mean excess function of a random variable $\widetilde{X}$ is defined as:[20]

$$
e_{\widetilde{X}}(u) = E(\widetilde{X} - u \mid \widetilde{X} > u). \tag{86}
$$

In the case of normal random variables with mean $\mu$ and standard deviation $\sigma$, the mean excess function is known and given by:[21]

$$
e_{N(\mu,\sigma)}(u) = (\mu - u) + \sigma \frac{\phi\left(\frac{u-\mu}{\sigma}\right)}{1 - \Phi\left(\frac{u-\mu}{\sigma}\right)}. \tag{87}
$$

The expectations in (85) correspond to mean excess functions, but applied to the left tail, in such a way that, by the asymmetry of the normal distribution, it is possible to rewrite (85) with $r_P = 1$ as:[22]

$$
\begin{aligned}
E(\widetilde{L}_f^i) = e^{-\lambda\delta t} e_{\delta\widetilde{A}_C^i + \delta\widetilde{F}_{S,NJ}^i}\left[2E(\delta\widetilde{A}_C^i + \delta\widetilde{F}_{S,NJ}^i) + w_L^i A_{L,D}^i\right] + \\
+ (1 - e^{-\lambda\delta t}) e_{\delta\widetilde{A}_C^i + \delta\widetilde{F}_{S,J}^i}\left[2E(\delta\widetilde{A}_C^i + \delta\widetilde{F}_{S,J}^i) + w_L^i A_{L,D}\right].
\end{aligned} \tag{88}
$$

As $E(\widetilde{L}_f^i)$ is a function of $w_L^i$, the optimization was done numerically.

This result is only a conjecture, assumed to be a reasonable one, that is used to allow for the test of the model with available information. A test of how both representations of $E(\widetilde{L}_f^i)$ shown in this work fits to data would only be possible with the availability of historical data of losses caused by the lack of liquidity.

---

[20] See, for example, Embrechts et al. (1997).

[21] In order to obtain the mean excess function for a particular distribution one should remember that $E(\widetilde{X} - u \mid \widetilde{X} > u) = \int_u^\infty (x-u) f_{\widetilde{X}}(x)dx / \int_u^\infty f_{\widetilde{X}}(x)dx$. In the case of a normal distribution, result (87) follows rather straightforwardly from the substitution $z = (x-\mu)/\sigma \Rightarrow dz = dx/\sigma$.

[22] Alternatively, by using $E(-u - \widetilde{X} \mid \widetilde{X} < -u) = \int_{-\infty}^{-u} (-u-x) f_{\widetilde{X}}(x)dx / \int_{-\infty}^{-u} f_{\widetilde{X}}(x)dx$ it is possible to obtain an equivalent expression without the use of the mean excess function. The mean excess function representation was chosen because it is more condensed.

The historical average returns on treasury and credit portfolios, net of funding costs, were used to estimate the expected returns on liquid and illiquid assets, respectively. Marginal operational expenses were assumed to be zero. In the case of the treasury portfolio, the profits and losses used were in COSIF accounts 7.1.5.00.00-3 and 8.1.5.00.00-0, and each return was calculated as the profits and losses over the total treasury portfolio, net of the monthly accumulated CDI rate, which was used as a transfer price. CDI stands for *Certificado de Depósito Interbancário*, or Certificate of Interbank Deposit, and the CDI rate is an average of interest rates on unsecured interbank short term funding, but its value is usually very close to the rates of repurchase agreements of securities issued by the federal government.

With respect to the credit portfolio, profits and losses were estimated as the revenues from several types of credit operations less the corresponding expenses, which include funding costs by COSIF accounting standards. Revenues were obtained from accounts 7.1.1.00.00-1, 7.1.2.00.00-4, 7.1.3.00.00-7, and 7.1.4.00.00-0, and expenses from accounts 8.1.1.00.00-8, 8.1.2.00.00-1, 8.1.3.00.00-4, and 8.1.4.00.00-7.

The value suggested in section 2.4 was used as the opportunity cost of equity, $ROE$, item (l). The initial value of 22.5% a year was adjusted, though, to include the mean proportion of taxes and profit-sharing, which were not deducted from the returns above described. The proportion of taxes and profit-sharing was obtained from the monthly values of taxes plus profit-sharing, registered on accounts 8.9.00.00-9 and 8.9.7.00.00-8 respectively, over the final result of the period, given by the difference between total revenues, COSIF account 7.0.0.00.00-9, and total expenses, COSIF account 8.0.0.00.00-6. The value used to estimate de adjusted $ROE$ was the median of the proportions from July 2003 and June 2006. The median was used instead of the mean because in some cases there were extreme variations so that the proportion became greater than 1, what is obviously unsustainable through time. The annual rate obtained was then converted to a monthly rate:

$$ROE = \left( 1 + \frac{22.5\%}{1 - \begin{array}{c}\text{median of the proportion}\\ \text{of taxes and profit-sharing}\end{array}} \right)^{\frac{1}{12}} - 1. \tag{89}$$

The accounting historical $ROE$ was also estimated for comparison. It was obtained from the division of the final result of the period by the shareholer's equity, COSIF account 6.0.0.00.00-2.

The economic capital required by the investment in liquid assets, item (m), was obtained from a simplified version of parametric VaR, which were estimated from the mean and standard deviation of returns on treasury operations, under the hypothesis of normal distribution, at the confidence level of 99.9%:[23]

$$w_L^i K(A_{L,D}^i, f_{\tilde{r}_L}) = VaR_{99.9\%,L} = -w_L^i A_{L,D} \times \left[ E(\tilde{r}_L^i) - 3.09 \times \sigma(\tilde{r}_L^i) \right]. \quad (90)$$

In the case of the economic capital required by the investment in illiquid assets, item (n), as the portfolio is not marked to market, it would be necessary to use more sophisticated methodologies such as RiskMetrics™ or CreditRisk+.[24] There are not, though, available information to apply these models. Additionally, as there are only 36 months in the data base and a much smaller number of sample points representing losses, the estimation of an adequate percentile or even the parametrization of the credit loss distribution is hindered. Given such restrictions, it was chosen the simpler path of using the legal requirement of 11% of the APR (*Ativo Ponderado ao Risco*, or Risk Weighted Assets, which is the Brazilian implementation of the Basel Accord's credit risk-adjusted assets). The proportion of 11%, greater than the original Basel Accord's minimum requirement of 8%, was defined by the Resolução 2.606, issued May 27th, 1999, by the CMN (*Conselho Monetário Nacional*, or National Monetary Council). The definition of APR was introduced by the Circular 2.099, issued August 17th, 1994, by the BCB (*Banco Central do Brasil*, Central Bank of Brazil), effectively implementing the Brazilian version of the first Basel Accord. The weights applied to the various assets in the composition of the APR have changed through time, but in general, credit operations are integrally[25] added to the APR (that is, 100% weight). Therefore:

$$K(A_{L,D}^i, f_{\tilde{r}_I}) = 0.11 \times A_{L,D}^i. \quad (91)$$

Even though the use of a constant factor ignores the quality of credit portfolios, one of the problems that lead to the proposal of a new capital accord, it is a *de facto* requirement until the implementation of Basel II.

[23]The economic profit model that is optimized already uses returns that have $\delta t$ periodicity.

[24]The reference documentation for RiskMetrics and CreditRisk+ models are Gupton et al. (1997) and CSFB (1997), respectively. There is a brief description of both in Saunders and Cornett (2006, chap. 11) and Saunders (1999, chap. 4 and 7).

[25]Most notable exceptions are habitational loans, with 50% weighting, (Circular 2.568, issued May 4th, 1995) and tax credits, with 300% weighting (Circular 2.916, issued August 6th, 1999). The accounts with the needed details to make a precise assessment of the APR (1.6.4.30.00-4 and 1.6.4.60.30-4 for habitational loans and 1.8.8.25.00-2 for tax credits), are protected by law, and it was chosen to use public information whenever possible.

Finally, it is important to note that, with exception of items (e) to (h), for which the daily data base was used, every other items were estimated with public information.

## 3.2 Results

The monthly database contains accounting information organized as described by the Top 50 methodology of the BCB[26]. The used information corresponds to all Brazilian depositary financial institutions, and the accounting data is consolidated, when it is the case, by financial conglomerates. This subset of the available accounting information is called "Banking - Consolidated I" in the Top 50 methodology. All estimates were done using data from July 2003 to June 2006.

From the initial 257 conglomerates and financial institutions, 82 without credit or treasury operations were filtered, and 127 more, which presented negative historical average return on either portfolio, were also excluded. Table 2 presents a summary of the estimated values after the exclusion of institutions without credit or treasury portfolios.

| Estimate of | Average | Median | Maximum | Minimum | Standard deviation |
|---|---|---|---|---|---|
| $A_C(\times 10^6)$ | 10,341.79 | 1,299.74 | 184,778.84 | 5.15 | 27,045.74 |
| $A_{L,D}(\times 10^6)$ | 106.87 | 7.54 | 1,219.92 | -65.79 | 258.64 |
| $\mu_{C,I}$ | 0.860262 | 0.459656 | 9.076639 | 0.114010 | 1.202106 |
| $\sigma_{C,I}$ | 0.170609 | 0.038728 | 2.102526 | 0.002812 | 0.383777 |
| $\mu_L$ | 0.108560 | 0.151802 | 0.700159 | -2.414947 | 0.245505 |
| $\sigma_L$ | 0.091945 | 0.049350 | 1.806865 | 0.006345 | 0.190377 |
| $\sigma_{L,I}$ | 0.000018 | 0.000011 | 0.028413 | -0.036656 | 0.005963 |
| $w_{C,I}$ | 0.581105 | 0.621699 | 0.998942 | 0.001160 | 0.281870 |
| $w_{C,L}$ | 0.418895 | 0.378301 | 0.998840 | 0.001058 | 0.281870 |
| $\mu_C$ | 0.381063 | 0.336335 | 2.001438 | -0.016119 | 0.242569 |
| $\sigma_C$ | 0.062352 | 0.030106 | 1.236773 | 0.004200 | 0.135129 |
| $E(\tilde{r}_I)$ | 0.011478 | 0.008840 | 1.459131 | -0.454512 | 0.159028 |
| $\sigma(\tilde{r}_I)$ | 0.068652 | 0.009200 | 1.474703 | 0.000755 | 0.196751 |
| $E(\tilde{r}_L)$ | -0.003012 | -0.001062 | 0.093066 | -0.113950 | 0.016213 |
| $\sigma(\tilde{r}_L)$ | 0.026927 | 0.014502 | 0.521380 | 0.000609 | 0.060595 |
| $ROA^*$ | 0.001490 | 0.001163 | 0.019791 | -0.004143 | 0.002779 |
| $ROE^*$ | 0.007867 | 0.007687 | 0.047363 | -0.032361 | 0.011791 |
| $ROE$ | 0.027097 | 0.027018 | 0.061080 | 0.014540 | 0.008150 |

* Historical accounting average of the monthly return on assets (ROA) and on shareholder's equity (ROE).

Table 2: Parameters estimates for depositary institutions with non zero value of credit and treasury operations.

The estimate of parameters after excluding the cases where historical negative average returns occurred is presented in table 3.

[26]A complete description of the methodology is available at http://www.bcb.gov.br/Fis/Top50/Port/default-i.asp?idioma=I&id=50top.

| Estimate of | Average | Median | Maximum | Minimum | Standard deviation |
|---|---|---|---|---|---|
| $A_C(\times 10^6)$ | 14,360.72 | 311.41 | 105,090.17 | 20.52 | 28,575.46 |
| $A_{L.D}(\times 10^6)$ | 157.70 | 2.78 | 1,191.94 | -40.45 | 310.91 |
| $\mu_{C,I}$ | 0.688399 | 0.460295 | 9.076639 | 0.153307 | 1.255961 |
| $\sigma_{C,I}$ | 0.083614 | 0.030826 | 1.867301 | 0.004902 | 0.266705 |
| $\mu_L$ | 0.199823 | 0.187244 | 0.394792 | 0.136192 | 0.043624 |
| $\sigma_L$ | 0.059932 | 0.046709 | 0.447019 | 0.007408 | 0.072994 |
| $\sigma_{L,I}$ | -0.000199 | 0.000023 | 0.001717 | -0.012600 | 0.001904 |
| $w_{C,I}$ | 0.711730 | 0.697291 | 0.997429 | 0.035632 | 0.200094 |
| $w_{C,L}$ | 0.288270 | 0.302709 | 0.964368 | 0.002571 | 0.200094 |
| $\mu_C$ | 0.419358 | 0.388919 | 0.743883 | 0.161078 | 0.146941 |
| $\sigma_C$ | 0.036183 | 0.027623 | 0.107942 | 0.004200 | 0.030456 |
| $E(\widetilde{r}_I)$ | 0.052345 | 0.015188 | 1.459131 | 0.000466 | 0.208284 |
| $\sigma(\widetilde{r}_I)$ | 0.041090 | 0.005015 | 1.474703 | 0.000755 | 0.211747 |
| $E(\widetilde{r}_L)$ | 0.003280 | 0.002150 | 0.019973 | 0.000016 | 0.003903 |
| $\sigma(\widetilde{r}_L)$ | 0.017572 | 0.013357 | 0.133008 | 0.000616 | 0.021646 |
| $ROA^*$ | 0.002039 | 0.001541 | 0.010060 | -0.004078 | 0.002580 |
| $ROE^*$ | 0.009916 | 0.008295 | 0.047363 | -0.015611 | 0.010429 |
| $ROE$ | 0.027063 | 0.028182 | 0.047146 | 0.014540 | 0.007372 |
| $K(A_{L,D}, f_{\widetilde{r}_I}/A_{L,D})$ | 0.110000 | 0.110000 | 0.110000 | 0.110000 | 0.000000 |
| $K(A_{L,D}, f_{\widetilde{r}_L}/A_{L,D})$ | 0.051018 | 0.037273 | 0.405203 | 0.001635 | 0.064976 |
| $R_I$ | 0.002977 | 0.003100 | 0.005186 | 0.001599 | 0.000811 |
| $R_L$ | 0.001347 | 0.000900 | 0.013525 | 0.000064 | 0.001985 |

* Historical accounting average of the monthly return on assets (ROA) and on shareholder's equity (ROE).

Table 3: Parameters estimates for depositary institutions with non zero value of credit and treasury operations and positive average historical returns.

In 12 of the 48 cases summarized in table 3, the estimate of $A_{L,D}$ resulted in a negative value. The average proportion between the estimates of $A_{L,D}$ and $A_C$ in the cases where $A_{L,D} \geq 0$ is 1.17%, and this proportion was used to estimate positive values for $A_{L,D}$ when $A_{L,D} < 0$.

The information from the "Banking - Consolidated I" of the Top 50 methodology are consolidated for institutions that are in the same conglomerate, but information of individual institutions are also available. Therefore, the last filter has excluded the institutions that have been listed individually, but at June 2006 were part of a financial conglomerate, in order to avoid double counting, resulting in a sample of 28 institutions or conglomerates. The estimates of $\mu_f$, $\sigma_F$, $\lambda$ and $J$ were done for these 28 institutions and 3 more were removed, two of them because they presented zero value of deposits and one more because a negative estimate of $\sigma_F^2$. By equation (47), this occurs if $\widehat{\sigma}^2(\delta\widetilde{F}_S) < (\widehat{J})^2 \times \widehat{\lambda}\delta t$, in what case the model (30) obviously does not fit the data. Figure 4 shows the behavior of deposits plus net interbank credit operations (active less passive operations) of the institution.

Only two other institutions showed a pattern of deposits plus net interbank credit operations that resembles the one in figure 4, but they did not generated negative estimates of $\sigma_F^2$. Table 4 shows the summary of estimates of all parameters for the final set of 25 financial institutions and conglomer-

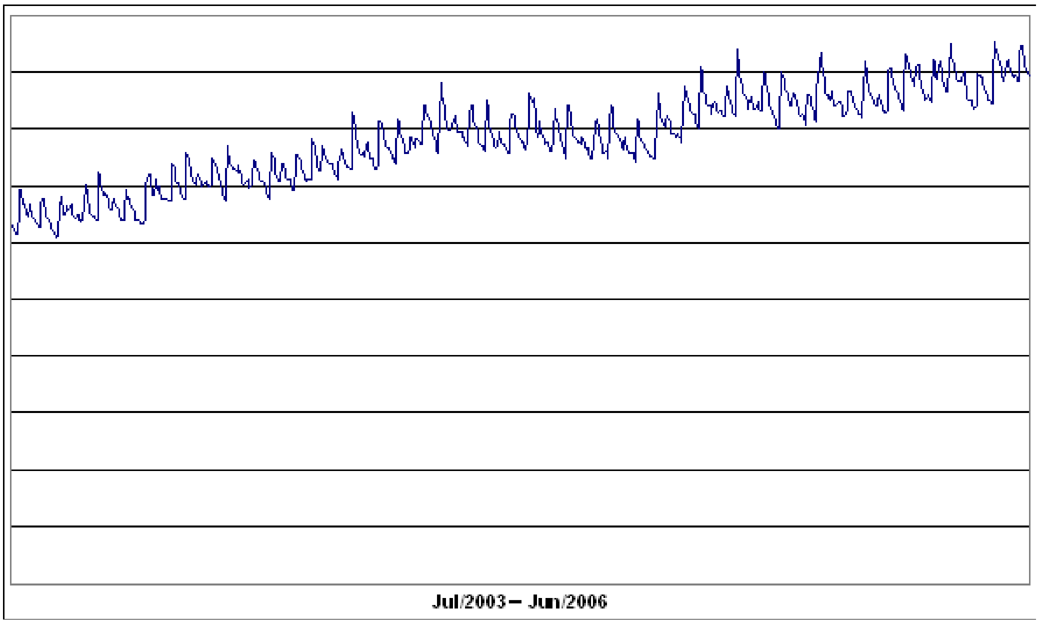



Figure 4: Daily behavior of deposits plus net interbank credit operations from the institution with negative estimate of $\sigma_F^2$.

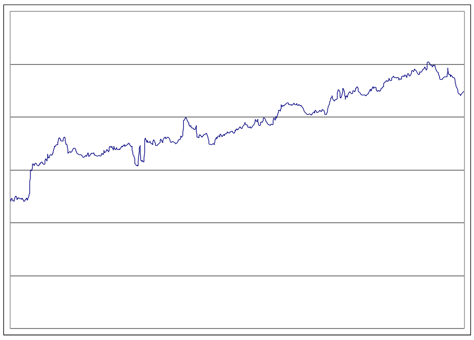
ates.

For 12 from the 25 institutions and conglomerates, the estimated value of $J$ was negative, indicating positive jumps and not sudden drops on the amount of funding opportunities. In these cases, and even in other cases where the estimate of $J$ was positive, there were not drastic reductions on the funding opportunities, what leads to downward biased estimates of the jump, a common problem while estimating rare events. Nevertheless, the purpose of the test is to show that the proposed model may have an important effect on value creation, thus, given that a higher value of $J$ only raises the failure probability, if even with downward biased estimates the use of the model still leads to signs of value creation, then its importance tends to increase with better estimates of $J$. In order to illustrate these effects, figure 5 shows some patterns of deposits plus net interbank credit operations for institutions whose estimates of $J$ were negative, and figure 6 shows some cases where $J$ estimates were positive.

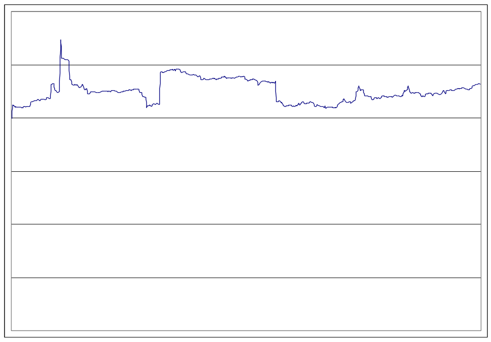
| Estimate of | Average | Median | Maximum | Minimum | Standard deviation |
|---|---|---|---|---|---|
| $A_C(\times 10^6)$ | 16,062.53 | 292.47 | 105,090.17 | 21.21 | 31,005.96 |
| $A_{L.D}(\times 10^6)$ | 175.43 | 2.76 | 902.17 | -19.92 | 323.83 |
| $\mu_{C,I}$ | 0.473655 | 0.441006 | 1.174521 | 0.208630 | 0.198558 |
| $\sigma_{C,I}$ | 0.038726 | 0.024972 | 0.169484 | 0.004902 | 0.043225 |
| $\mu_L$ | 0.199783 | 0.187300 | 0.394792 | 0.168443 | 0.045790 |
| $\sigma_L$ | 0.044745 | 0.032678 | 0.122689 | 0.007444 | 0.031575 |
| $\sigma_{L,I}$ | -0.000040 | -0.000008 | 0.001361 | -0.001262 | 0.000437 |
| $w_{C,I}$ | 0.735366 | 0.722447 | 0.997429 | 0.285185 | 0.174000 |
| $w_{C,L}$ | 0.264634 | 0.277553 | 0.714815 | 0.002571 | 0.174000 |
| $\mu_C$ | 0.389769 | 0.381001 | 0.648272 | 0.194777 | 0.118437 |
| $\sigma_C$ | 0.029496 | 0.023820 | 0.107942 | 0.004200 | 0.027593 |
| $E(\widetilde{r}_I)$ | 0.019043 | 0.013219 | 0.083480 | 0.000667 | 0.017953 |
| $\sigma(\widetilde{r}_I)$ | 0.009426 | 0.003986 | 0.054114 | 0.000755 | 0.013111 |
| $E(\widetilde{r}_L)$ | 0.003040 | 0.002069 | 0.019973 | 0.000322 | 0.003995 |
| $\sigma(\widetilde{r}_L)$ | 0.013241 | 0.009733 | 0.039723 | 0.000838 | 0.010244 |
| $ROA^*$ | 0.001890 | 0.001558 | 0.009468 | -0.000733 | 0.002064 |
| $ROE^*$ | 0.010039 | 0.007530 | 0.047363 | -0.001883 | 0.009400 |
| $ROE$ | 0.028102 | 0.028951 | 0.037423 | 0.017056 | 0.006512 |
| $K(A_{L,D}, f_{\widetilde{r}_I}/A_{L,D})$ | 0.110000 | 0.110000 | 0.110000 | 0.110000 | 0.000000 |
| $K(A_{L,D}, f_{\widetilde{r}_L}/A_{L,D})$ | 0.037874 | 0.027850 | 0.117140 | 0.002266 | 0.029587 |
| $R_I$ | 0.003091 | 0.003185 | 0.004117 | 0.001876 | 0.000716 |
| $R_L$ | 0.001048 | 0.000872 | 0.003182 | 0.000064 | 0.000810 |
| $E(\widetilde{r}_I) - R_I$ | 0.015951 | 0.010122 | 0.079921 | -0.002613 | 0.018031 |
| $E(\widetilde{r}_L) - R_L$ | 0.001992 | 0.000695 | 0.017397 | -0.001113 | 0.003604 |
| $\mu_F(\times 10^6)$ | 1,334.89 | 28.13 | 7,704.08 | -1,125.74 | 2,682.64 |
| $\sigma_F(\times 10^6)$ | 1,416.79 | 52.89 | 7,803.93 | 6.47 | 2,568.75 |
| $\lambda$ | 1.770721 | 0.020517 | 37.936676 | 0.000002 | 7.554380 |
| $J(\times 10^6)$ | 1,465.18 | 7.36 | 48,772.95 | -31,983.82 | 13,027.97 |

* Historical accounting average of the monthly return on assets (ROA) and on shareholder's equity (ROE).

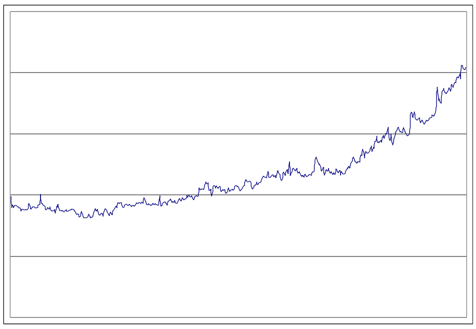
Table 4: Estimates of all parameters for the final set of 25 financial institutions and conglomerates.



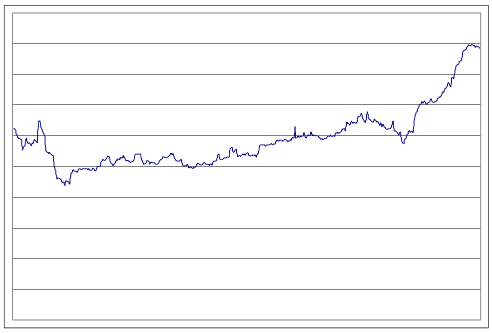
$\lambda = 0.57$



$\lambda = 0.42$

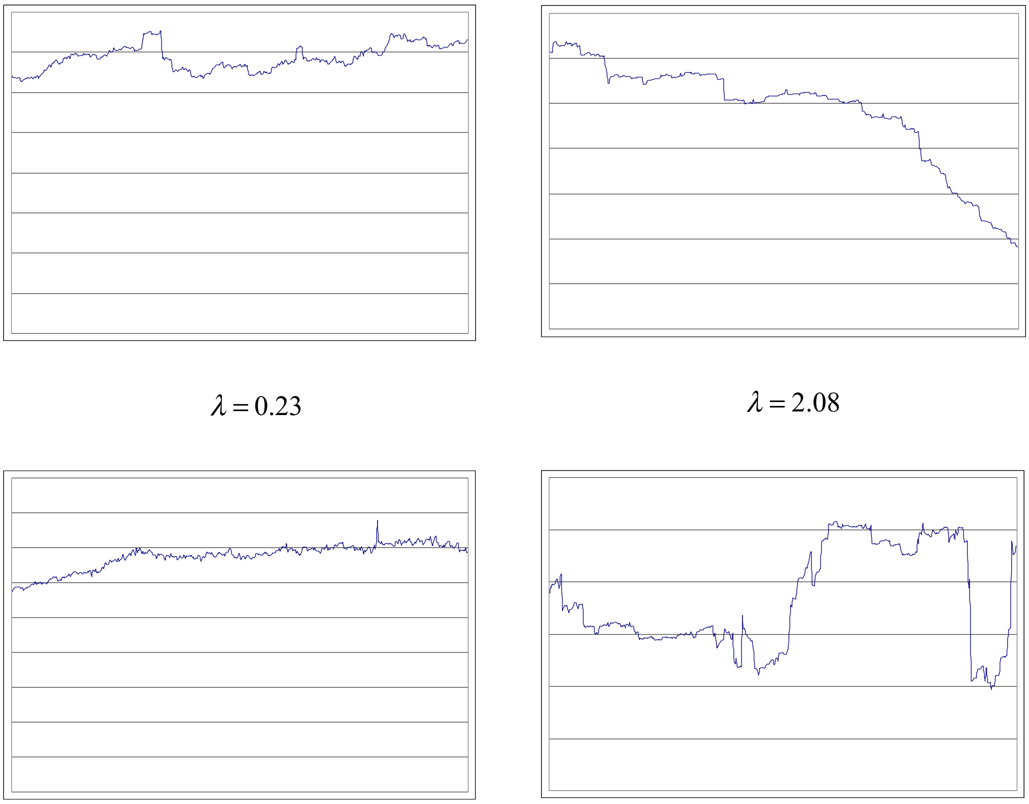


$\lambda = 0.02$



$\lambda = 0.005$

Figure 5: Daily behavior of deposits plus net interbank credit operations for institutions and conglomerates whose estimates of $J$ were negative.



$\lambda = 0.23$ $\lambda = 2.08$

$\lambda = 0.0004$ $\lambda = 0.002$

Figure 6: Daily behavior of deposits plus net interbank credit operations for institutions and conglomerates whose estimates of $J$ were positive.

The difference between the expected economic profit and the expected failure loss was optimized with the estimated parameters. The results are summarized in table 5.

| Result of | Average | Median | Maximum | Minimum | Standard deviation |
|---|---|---|---|---|---|
| $E(\widetilde{L}_F)/A_C$ | 0.054154 | 0.019235 | 0.472060 | 0.004454 | 0.099808 |
| $P_f$ | 0.212895 | 0.117691 | 0.986309 | 0.001605 | 0.223849 |
| $E(\widetilde{\Pi})/SE$ | 0.000366 | 0.000053 | 0.003513 | -0.000031 | 0.000926 |
| $[E(\widetilde{\Pi}) - P_f E(\widetilde{L}_f)]/SE$ | -0.092487 | -0.008128 | 0.002569 | -1.121093 | 0.242609 |
| $P_f E(\widetilde{L}_f)/SE$ | 0.096506 | 0.005918 | 1.121101 | 0.000029 | 0.247831 |
| $P_f E(\widetilde{L}_f)/E(\widetilde{\Pi})$ | 6,216.33 | 28.11 | 149,631.25 | -24,004.14 | 32,647.69 |

SE = Shareholder's equity.

Table 5: Summary of the optimization results, given the possibility of loss due to the lack of liquidity.

There were five interior solutions, but three of them were very close to the extreme, with $w_L \approx 1$. The two other interior solutions occurred for $w_L \approx 0.2$ and $w_L \approx 0.9$. The corner solutions comprehended fifteen solutions with $w_L = 1$ and only one solution $w_L = 0$. In three cases the value of $\Phi(x)$ was too close to 1, and it was not possible to proceed the optimization due to the near singular value of the mean excess function argument.

The optimization was repeated without considering the risk of losses due to the lack of liquidity, that is, only the economic profit was optimized. The results are in table 6.

| Result of | Average | Median | Maximum | Minimum | Standard deviation |
|---|---|---|---|---|---|
| $E(\widetilde{L}_F)/A_C$ | 0.056773 | 0.021699 | 0.482926 | 0.005647 | 0.101611 |
| $P_f$ | 0.243920 | 0.179412 | 0.988160 | 0.000532 | 0.223953 |
| $E(\widetilde{\Pi})/SE$ | 0.000690 | 0.000349 | 0.004321 | 0.000038 | 0.001002 |
| $[E(\widetilde{\Pi}) - P_f E(\widetilde{L}_f)]/SE$ | -0.099340 | -0.015155 | 0.001061 | -1.148484 | 0.248770 |
| $P_f E(\widetilde{L}_f)/SE$ | 0.10379796 | 0.0143496 | 1.149058842 | 2.894E-05 | 0.25420239 |
| $P_f E(\widetilde{L}_f)/E(\widetilde{\Pi})$ | 423.815507 | 29.400467 | 3,628.703639 | 0.100477 | 879.850675 |

SE = Shareholder's equity.

Table 6: Summary of the optimization neglecting the risk of losses due to the lack of liquidity.

In this case, there were twenty corner solutions with $w_L = 0$ and only two corner solutions with $w_L = 1$. Again, in three cases there were near singular values for the arguments of the mean excess function.

There are some important conclusions to be drawn from the results of the optimizations in tables 5 and 6. Initially, as expected, if the losses from the lack of liquidity are not considered, the optimization favors the investment in illiquid assets. When the loss possibility is taken into account, the reverse scenario occurs. Additionally, at least as modeled here, the failure probability values are considerable. The failure probability was, on average, 21.3% when the optimization took into account the possibility of loss due to the lack of liquidity, and 24.4% when the liquidity restriction was ignored. The difference between the medians, least influenced by extreme values, was even bigger: the median failure probability considering losses because of the lack of liquidity was 11.8%, while ignoring liquidity risk it reached 18.0%. The expected loss also had relevant values, representing, on average, 5.5% of all inventory of cash generating assets. The combination of both, failure probability multiplied by the expected loss, has a median value of 0.6% of shareholder's equity, and more than the double, approximately 1.4%, when losses because the lack of liquidity are ignored. Thus, even if $E(\widetilde{\Pi})/SE$ is higher when the optimization does not account for the liquidity restriction, the effect of the losses because of the lack of liquidity is enough to provide an excess return on shareholder's equity of 0.7% a month, or 8.8% a year, on average, and 8.5% a year when medians are compared.

This result presents some robustness to changes in the parameters. The optimization was repeated for 20% increases and 20% decreases in all parameters, except for $E(\widetilde{r}_L)$ and $\sigma(\widetilde{r}_L)$ that remained the same so that there would be some change in the difference between $E(\widetilde{r}_I) - R_I$ and $E(\widetilde{r}_L) - R_L$. The results are shown in table 7.

$\frac{E(\tilde{\Pi})-P_f E(\tilde{L}_f)}{SE}$ for optimizations with and without the liquidity restriction

| | Parameters +20% | Original parameters | Parameters -20% |
|---|---|---|---|
| Difference between medians | 0.513% (6.3% a year) | 0.703% (8.8% a year) | 0.601% (7.5% a year) |
| Difference between averages | 0.694% (8.7% a year) | 0.685% (8.5% a year) | 0.654% (8.1% a year) |

Table 7: Difference between the values of $[E(\widetilde{\Pi}) - P_f E(\widetilde{L}_f)]/SE$ estimated by optimizations with and without the liquidity constraint.

The optimizations were repeated with a larger amount of available resources, in order to allow for more interior solutions. By using an $A_{L,D}$ four times bigger, the number of interior solutions grew from five to six, with only one resulting in $w_L \approx 1$. Additionally, the number of corner solutions with $w_L = 0$ went from two to three and the number of corner solutions with $w_L = 1$ felt from fifteen to thirteen. The summary of the results is on table 8.

Comparing the results on table 8 with the ones in the optimization of the expected economic profit without the liquidity constraint, as summarized on table 9, it is possible to observe a gain between the medians of the returns on shareholder's equity of about 11.1% a year, since the greater availability of liquid resources to allocation allows for a more pronounced reduction of the losses because of the lack of liquidity.

| Result of | Average | Median | Maximum | Minimum | Standard deviation |
|---|---|---|---|---|---|
| $E(\widetilde{L}_F)/A_C$ | 0.047934 | 0.015259 | 0.440059 | -0.004371 | 0.093783 |
| $P_f$ | 0.149206 | 0.075162 | 0.979191 | 0.001605 | 0.211223 |
| $E(\widetilde{\Pi})/SE$ | 0.001763 | 0.000213 | 0.017282 | -0.000124 | 0.004187 |
| $[E(\widetilde{\Pi}) - P_f E(\widetilde{L}_f)]/SE$ | -0.070300 | -0.002208 | 0.014023 | -1.037530 | 0.224721 |
| $P_f E(\widetilde{L}_f)/SE$ | 0.075369 | 0.002409 | 1.037560 | -0.000050 | 0.229088 |
| $P_f E(\widetilde{L}_f)/E(\widetilde{\Pi})$ | 1,414.28 | 0.79 | 34,620.30 | -3,627.48 | 7,481.72 |

SE = Shareholder's equity.

Table 8: Summary of the results after optimizing the expected economic profit given the possibility of loss due to the lack of liquidity for a volume of available resources, $A_{L,D}$, four times bigger.

| Result of | Average | Median | Maximum | Minimum | Standard deviation |
|---|---|---|---|---|---|
| $E(\widetilde{L}_F)/A_C$ | 0.055896 | 0.021699 | 0.482926 | 0.005647 | 0.101778 |
| $P_f$ | 0.226662 | 0.151160 | 0.988160 | 0.000000 | 0.222082 |
| $E(\widetilde{\Pi})/SE$ | 0.002761 | 0.001395 | 0.017282 | 0.000153 | 0.004006 |
| $[E(\widetilde{\Pi}) - P_f E(\widetilde{L}_f)]/SE$ | -0.090973 | -0.011487 | 0.014023 | -1.146760 | 0.249564 |
| $P_f E(\widetilde{L}_f)/SE$ | 0.097414 | 0.011046 | 1.149059 | 0.000029 | 0.254890 |
| $P_f E(\widetilde{L}_f)/E(\widetilde{\Pi})$ | 70.95 | 7.35 | 499.74 | 0.03 | 137.21 |

SE = Shareholder's equity.

Table 9: Summary of the results after optimizing the expected economic profit, without the liquidity constraint, for a volume of available resources, $A_{L,D}$, four times bigger.

Finally, the existence of several corner solutions with $w_L = 1$ may indicate that the punitive rate $r_p = 1$ is not adequate, and the expected losses are smaller. Thus, a final test was performed where $E(\widetilde{L}_f)$ was divided by a factor (the inverse of $r_P$) so that the average of the optimized $w_L$, weighted by $A_{L,D}$, was next to 50%. By dividing the original expected loss by 8.6, what corresponds to $r_P \approx 11.6\%$, it was possible to obtain an weighted average $w_L$ of 49.9%. On this condition, there were five interior solutions, two of them very close to $w_L = 0$, plus three solutions $w_L = 0$. The corner solutions with $w_L = 1$ amounted to 14. The summary of the optimization results is on table 10.

| Result of | Average | Median | Maximum | Minimum | Standard deviation |
|---|---|---|---|---|---|
| $E(\widetilde{L}_F)/A_C$ | 0.05484099 | 0.01923523 | 0.47205997 | 0.00564704 | 0.09949854 |
| $P_f$ | 0.21917888 | 0.17941218 | 0.98630901 | 0.00053174 | 0.22120093 |
| $E(\widetilde{\Pi})/SE$ | 0.00048934 | 0.00012914 | 0.00432062 | -0.00003094 | 0.00104585 |
| $[E(\widetilde{\Pi}) - P_f E(\widetilde{L}_f)]/SE$ | -0.01033421 | -0.00092684 | 0.00394163 | -0.13035304 | 0.02839224 |
| $P_f E(\widetilde{L}_f)/SE$ | 0.01128729 | 0.00091178 | 0.13036053 | 0.00000336 | 0.02879175 |
| $P_f E(\widetilde{L}_f)/E(\widetilde{\Pi})$ | 718.89 | 2.21 | 17,398.98 | -2,791.18 | 3,797.00 |

SE = Shareholder's equity.

Table 10: Summary of the optimization results, given the possibility of loss due to the lack of liquidity and an expected loss 8.6 times smaller.

When the optimization is performed without considering losses because of the lack of liquidity, the weighted average of $w_L$ drops to 0.81%. The results are summarized on table 11.

| Result of | Average | Median | Maximum | Minimum | Standard deviation |
|---|---|---|---|---|---|
| $E(\widetilde{L}_F)/A_C$ | 0.056773 | 0.021699 | 0.482926 | 0.005647 | 0.101611 |
| $P_f$ | 0.243920 | 0.179412 | 0.988160 | 0.000532 | 0.223953 |
| $E(\widetilde{\Pi})/SE$ | 0.000690 | 0.000349 | 0.004321 | 0.000038 | 0.001002 |
| $[E(\widetilde{\Pi}) - P_f E(\widetilde{L}_f)]/SE$ | -0.010882 | -0.001263 | 0.003942 | -0.133037 | 0.029021 |
| $P_f E(\widetilde{L}_f)/SE$ | 0.012070 | 0.001669 | 0.133611 | 0.000003 | 0.029558 |
| $P_f E(\widetilde{L}_f)/E(\widetilde{\Pi})$ | 49.28 | 3.42 | 421.94 | 0.01 | 102.31 |

SE = Shareholder's equity.

Table 11: Summary of the optimization results, with an expected loss 8.6 times smaller, and without considering the liquidity restriction.

From tables 10 and 11, the proposed model produces an annual gain on the return on shareholder's equity of 0.7%, considering averages, and 0.4%, considering medians. These results were obtained for $r_P \approx 11.6\%$, and it is fair to assume that a better assessment of the punitive rate should be a value higher than the interbank credit rate of about 13.5% a year, what would lead to yearly gains in the $ROE$ higher than 0.7%.

# 4 Conclusions

This work introduces a model of liquidity management that uses the consolidated and wide spread RAROC and EVA models, resulting in a shot-term optimal allocation strategy that seamlessly integrates bank wide risks with little additional effort for those institutions that already use economic profit models. By maintaining the strategic allocation decision exogenous and the liquidity cushion decision risk neutral, it was possible to eliminate the need for arbitrary utility functions and the complexities of properly accounting for risk throughout all bank operations, and focus on the liquidity management decision in order to avoid losses and maximize value to the shareholder. Risk neutral pricing alternatives were also avoided, despite their theoretical appeal, because they often assume complete markets with the possibility of perfect hedge of the liquidity risk, what is not ever the case, specially in less developed markets, as the Brazilian one, or even in the case of generalized confidence crisis as the one that is presently occurring due to the problems with subprime mortgages in United States. At the end, the resulting model was kept quite simple, with the additional benefit of incorporating the liquidity management to largely used economic profit models. The model proposed here presents an evolutive feature that allows for its implementation as a complement of well known practices, consequently, at lower costs and reduced impacts over current activities and routines.

The suggested liquidity restriction can be applied with any desired level of complexity, either through parametric or historical simulations. Nevertheless, a closed form solution, given some simplifying assumptions, was presented and analyzed thoroughly. The availability of such solution allows for, among other things, the use of the failure probability by the regulatory authority as an indicator that helps to monitor a big number of banks at once.[27] The closed form to the failure probability also may be used as a proxy of the real, but opaque value, allowing for empirical tests related to the funding liquidity risk. Given the relative scarcity of academic production about optimal liquidity and the relevance of the theme to the stability of the financial system (see Cifuentes et al., 2005), this possibility is another important contribution of this work. The alternative estimation procedure for the loss given a failure based on the Extreme Value Theory is also a contribution, even though it deserves further research.

Finally, it was also presented an empirical assessment of the model using data from Brazilian banks corresponding to the period from July 2003 to June 2006. Relevant values were found to the failure probability, to the expected loss and to $ROE$ gains over an optimization that makes no regards to liquidity risks. The unavailability of more accurate managerial data made it necessary to use proxies and estimates, what introduced imprecisions that are difficult to quantify. As a result, it is not possible to assess the significance of the values found. Nevertheless, the proposed model was able to show gains under several assumptions in robustness tests. There are, therefore, indications that the model may have relevant impact in liquidity management and value creation in banks.

[27] Clearly, given its underling simplifying assumptions, this indicator should be used in conjunction with others.

# References


Albanese, C. and Campolieti, G. (2006). *Advanced derivatives pricing and risk management: theory, tools, and hands-on programming applications.* Elsevier Academic Press.

Artzner, P. F., Delbaen, F., Eber, J. M., and Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, (9):203–228.

Banks, E. (2004). *Liquidity risk: managing asset and funding risks.* Palgrave Macmillan.

Bessis, J. (2002). *Risk management in banking.* John Wiley & Sons, 2nd edition.

Bhattacharya, S. and Fulghieri, P. (1994). Uncertain liquidity and interbank contracting. *Economic Letters*, 44:287–294.

Bingham, N. H. and Kiesel, R. (2004). *Risk-neutral valuation: pricing and hedging of financial derivatives.* Springer, 2nd edition.

BIS (2000). Sound practices for managing liquidity in banking organizations. Basel Committee on Banking Supervision 69, Bank for International Settlement (BIS). Available at http://www.bis.org/publ/bcbs69.pdf.

BIS (2006a). International convergence of capital measurement and capital standards: a revised framework - comprehensive version. Basel Committee on Banking Supervision 128, Bank for International Settlement (BIS). Available at http://www.bis.org/publ/bcbs128.htm.

BIS (2006b). The management of liquidity risk in financial groups. Joint forum issues paper on funding liquidity risk management, Bank for International Settlement (BIS). Available at http://http://www.bis.org/press/p060503.htm.

Cai, Z. and Hong, Y. (2003). *Recent advances and trends in nonparametric statistics*, chapter Nonparametric methods in continuous-time finance: a selective review. Elsevier.

Campbell, J., Lo, A. W., and MacKinlay, A. C. (1997). *The econometrics of financial markets.* Princeton University Press.

Cifuentes, R., Shin, H. S., and Ferrucci, G. (2005). Liquidity risk and contagion. *Journal of the European Economic Association*, 3(2-3):556–566.

Copeland, T., Koller, T., and Murrin, J. (2000). *Valuation: measuring and managing the value of companies.* John Wiley & Sons, 3rd edition.

Crouhy, M., Galai, D., and Mark, R. (2001). *Risk Management.* McGraw-Hill.

CSFB (1997). Creditrisk+: a credit risk management framework. Technical report, Credit Suisse First Boston International. Available at http://www.csfb.com/institutional/ research/assets/creditrisk.pdf.

Cysne, R. P. (2006). Equity premium puzzle: evidence from brazilian data. *Economia Aplicada*, 10(2):161–180.

Damodaran, A. (2002). *Investment valuation: tools and techniques for determining the value of any asset.* John Wiley & Sons, 2nd edition.

Embrechts, P., Klüppelberg, C., and Mikosch, T. (1997). *Modeling extremal events for insurance and finance.* Springer.

Fama, E. (1991). Efficient capital markets ii. *The Journal of Finance*, 46(5):1575–1617.

Fisher, R. A. (1928). Moments and product moments of sampling distributions. *Proceedings of the London Mathematical Society*, 30(199-238).

Gourieroux, C. and Jasiak, J. (2001). *Financial econometrics: problems, models, and methods.* Princeton University Press.

Gup, B. E. and Kolari, J. W. (2005). *Commercial banking: the management of risk.* John Wiley & Sons, 3rd edition.

Gupton, G. M., Finger, C. C., and Bhatia, M. (1997). Creditmetrics™ - technical document. Technical report, RiskMetrics Group. Available at http://www.riskmetrics.com/cmtdovv.html.

Hamilton, J. (1994). *Times series analysis.* Princeton University Press.

Hastings, D. F. (2006). *Banking: gestão de ativos, passivos e resultados em instituições financeiras.* Saraiva.

Hempel, G. H., Simonson, D. G., and Coleman, A. B. (1994). *Bank management: text and cases.* John Wiley & Sons, 4th edition.

Ho, T. and Saunders, A. (1981). The determinants of bank interest margins: theory and empirical evidence. *Journal of Financial and Quantitative Analysis*, 16(4):581–600.

Ho, T. and Stoll, H. (1980). On dealer markets under competition. *The Journal of Finance*, 35(2):259–267.

Hull, J. C. (2003). *Options, Futures, and Other Derivatives.* Prentice Hall.

Jorion, P. (2001). *Value at risk: the new benchmark for managing financial risk.* McGraw-Hill, 2nd edition.

Kidwell, D. S., Peterson, R. L., and Blackwell, D. W. (1997). *Financial institutions, markets, and money.* Dryden Press, 6th edition.

Klebaner, F. C. (2004). *Introduction to stochastic calculus with applications.* Imperial College Press.

Koch, T. W. and MacDonald, S. S. (2000). *Bank Management.* Dryden Press, 4th edition.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1):77–91.

Marshall, C. (2001). *Measuring and managing operational risks in financial institutioins.* John Wiley & Sons.

McNeil, A. J., Frey, R., and Embrechts, P. (2005). *Quantitative risk management: concepts, techniques, and tools.* Princeton University Press.

Rose, P. S. (1996). *Commercial bank management.* Irwin, 3rd edition.

Saita, F. (2004). Risk capital aggregation: the risk manager's perspective. EFMA 2004 Basel Meetings Paper. Available at <http://ssrn.com/abstract=496684>.

Saunders, A. (1999). *Credit risk measurement: new approaches to value-at-risk and other paradigmsCredit risk measurement: new approaches to value-at-risk and other paradigms.* John Wiley & Sons.

Saunders, A. and Cornett, M. M. (2006). *Financial institutions management: a risk management approach.* McGraw-Hill/Irwin, 5th edition.

Schroeck, G. (2002). *Risk management and value creation in financial institutions.* Jonh Wiley & Sons.

Van Greuning, H. and Bratanovic, S. B. (2003). *Analyzing and managing banking risk: a framework for assessing corporate governance and financial risk.* World Bank, 2nd edition.

Xavier, F. and Rochet, J.-C. (1999). *Microeconomics of banking*. The MIT Press.

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