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Caixa Postal 8.670

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Phones: (5561) 3414-3710 and 3414-3567

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E-mail: editor@bcb.gov.br

Evaluating Value-at-Risk models via quantile regressions

Wagner P. Gaglianone* Luiz Renato Lima† Oliver Linton‡

Abstract

We propose an alternative backtest to evaluate the performance of Value-at-Risk (VaR) models. The presented methodology allows us to directly test the performance of many competing VaR models, as well as identify periods of an increased risk exposure based on a quantile regression model (Koenker & Xiao, 2002). Quantile regressions provide us an appropriate environment to investigate VaR models, since they can naturally be viewed as a conditional quantile function of a given return series. A Monte Carlo simulation is presented, revealing that our proposed test might exhibit more power in comparison to other backtests presented in the literature. Finally, an empirical exercise is conducted for daily S&P500 series in order to explore the practical relevance of our methodology by evaluating five competing VaRs through four different backtests.

Keywords: Value-at-Risk, Backtesting, Quantile Regression
JEL Classification: C12, C14, C52, G11

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†Corresponding author. Department of Economics, University of Illinois and Graduate School of Economics, Getulio Vargas Foundation, Praia de Botafogo 190, s.1104, Rio de Janeiro, RJ 22.253-900, Brazil (e-mail: luizr@fgv.br).

‡Department of Economics, London School of Economics, Houghton Street, London WC2A 2AE, United Kingdom (e-mail: o.linton@lse.ac.uk).

1 Introduction

Several large scale crashes and financial losses in the previous decades, such as the "Black Monday" in 1987 (with a 23% drop in value of U.S. stocks, equivalent to $1trillion lost in one day), or the Asian turmoil of 1997 and Russian default (leading to a near failure of LTCM) in 1998 and, more recently, the World Trade Center attack in 2001, freezing the financial market for six days, with U.S. stock market losses of $1.7 trillion, have brought risk management of financial institutions to the forefront of internal control and regulatory debate. Value-at-Risk (hereafter, VaR) models arose as a subject for both regulators and investors concerned with large crashes and the respective adequacy of capital to meet such risk.

In fact, VaR is a statistical risk measure of potential losses, and summarizes in a single number the worst loss over a target horizon that will not be exceeded with a given level of confidence. Despite several other competing risk measures proposed in the literature, VaR has effectively become a cornerstone of internal risk management systems in financial institutions, following the success of the J.P. Morgan RiskMetrics system, and nowadays form the basis of the determination of market risk capital, since the 1996 Amendment of the Basel Accord.

A crucial question that arises in this context is how to evaluate the performance of a VaR model? When several risk forecasts are available, it is desirable to have formal testing procedures for comparison, which do not necessarily require knowledge of the underlying model, or, if the model is known, which do no restrict attention to a specific estimation procedure. The literature has proposed several tests (also known as "backtests"), such as Kupiec (1995), Christoffersen (1998) and Engle and Manganelli (2004), mainly focused on a hit sequence from which statistical properties are derived and further tested.

In this article, we go a step further by arguing that these backtests may provide only necessary but not sufficient conditions to test whether or not a given VaR measure is properly specified. In fact, by investigating solely a violation sequence, one might ignore an important piece of information contained in the magnitude of violations. In this sense, we propose an alternative backtest based on a quantile regression framework that can properly account for it. It is natural to evaluate a VaR model by a quantile regression method due to its capability of conditional distribution exploration with distribution-free assumption, also allowing for serial correlation and conditional heteroskedasticity. Furthermore, Value-at-Risk models are nothing else than conditional quantiles functions, as will be further explored throughout this paper.

There are a variety of approaches to estimate conditional quantiles in general and Value-at-Risk in particular. A short list includes Koenker and Zhao (1996), Danielsson and de Vries (1997), Embrechts et al. (1997, 1999), Chernozhukov and Umantsev (2001), Christoffersen et al. (2001). For instance, Koenker and Zhao (1996) provide a discussion about conditional quantile estimation and inference under Engle's (1982) ARCH models, whereas Hafner and Linton (2006) show that a QAR(p) process can be represented by a semi-strong ARCH(p) process, and the GARCH(1,1) can be nested by a QAR process extended to infinite order.

Quantile regressions can also be used to construct VaR measures without imposing a parametric distribution or the iid assumption: Chen (2001) discusses a multiperiod VaR model based on quantile regressions, and Wu and Xiao (2002) present an ARCH quantile regression approach to estimate VaR and left-tail measures (see also Chen & Chen, 2002). Surprisingly, however, little empirical work has been done by using quantile regressions to evaluate competing VaR models (e.g., Engle and Manganelli (2004) and Giacomini and Komunjer (2005)).

This way, the main objective of this paper is to provide a backtest based on quantile regressions that allows us to formally evaluate (through a standard Wald statistic) the performance of a VaR model, and also permits one to identify periods of an increased risk exposure, which we believe to be a novelty in the literature. The test statistic is derived from a Mincer-Zarnowitz (1969) type-regression considered in a quantile environment.

The proposed test is quite simple to be computed and can be carried out using software available for conventional quantile regression, and also presents the advantage of making "full use of information", in the sense that takes into account the magnitudes of model violations, rather than simply checking whether the violation series follows an iid sequence. In addition, our methodology is applicable even when the VaR does not come from a conditional volatility model.

The practical relevance of our theoretical results are documented by a small Monte Carlo simulation, in which the quantile regression test seems to have more power in comparison to other backtests, previously documented in the literature as exhibiting low power to detect misspecified VaRs in finite samples (see Kupiec (1995), Pritsker (2001) and Campbell (2005)). The increased power might be due to the quantile framework, which provides an adequate null hypothesis, in comparison to other backtests, which is also an issue addressed in this paper.

This study is organized as follows: Section 2 defines Value-at-Risk, presents a quantile regression-based hypothesis test to evaluate VaRs and describes other backtests suggested in the literature. Section 3 shows the Monte Carlo simulation comparing the size and power of the competing backtests. Section 4 provides an empirical exercise based on daily S&P500 series, and Section 5 summarizes our conclusions.

2 The econometric model and assumptions

Value-at-Risk models were developed in response to the financial disasters of the early 90s, and have become a standard measure of market risk, which is increasingly used by financial and non-financial firms as well. VaR models have also been sanctioned for determining market risk capital requirements for financial institutions through the 1996 Market Risk Amendment to the Basle Accord.[1]

According to Jorion (2007), Mr. Till Guldimann is the creator of the term "Value-at-Risk", while head of global research at J.P. Morgan in the late 80s. The introduction of the VaR concept through the RiskMetrics methodology has collapsed the entire distribution of the portfolio returns into a single number, which investors have found very useful and easily interpreted as a measure of market risk. Generally speaking, Value-at-Risk can be interpreted as the amount lost on a portfolio, with a given small probability, over a fixed time period.

Jorion (2007) also argues that a VaR summarizes the worst loss (or the highest gain) of a portfolio over a target horizon that will not be exceeded with a given level of confidence. The author formally defines VaR as the quantile of the projected distribution of gains and losses over the target horizon. If $\tau^* \in (0;1)$ is the selected tail level of the mentioned distribution, the respective VaR is implicitly defined by the following expression:

$$\Pr\left[R_t \leq V_t | \mathcal{F}_{t-1}\right] = \tau^*, \tag{1}$$

where $\mathcal{F}_{t-1}$ is the information set available at time $t-1$, R_t is the return series and V_t is the respective VaR. From this definition, it is clear that finding a VaR is essentially the same as finding a $(100 * \tau)\%$ conditional quantile. Note that, for convention, the VaR is defined for the right tail of the distribution, which is assumed without loss of generality, since our methodology can easily be adapted to investigate the left tail.[2] In this case, the VaR would be defined by

[1] See Appendix B for further details.

[2] According to Nankervis et al. (2006), it is usual that VaR is separately computed for the left and right tails of the distribution depending on the position of the risk managers or traders. For traders with a long position (when they buy and hold a traded asset), the risk comes from a drop in the price of the asset, while traders with a short position (who first borrow the asset and subsequently sell it in the market) lose money when the price increases. Due to the existence of leverage effects, a well-known stylized fact in financial asset returns, models that allow positive and negative returns to have different impacts on volatility are required to compute and distinguish the VaR for the long and short positions.

$\Pr\left[R_t \leq -V_t|\mathcal{F}_{t-1}\right] = \tau^*$. Note that the sign is changed to avoid a negative number in the V_t time series, since the VaR is usually reported by risk managers as a positive number.

Following the idea of Christoffersen et al. (2001), one can think of generating a VaR measure as the outcome of a quantile regression, treating volatility as a regressor. For instance, from a regression of the form: $y_t = \alpha_0(U_t) + \alpha_1(U_t)\sigma_t^2$, where σ_t^2 is the conditional volatility of y_t, it follows that $Q_{y_t}(\tau \mid \mathcal{F}_{t-1}) = \alpha_0(\tau) + \alpha_1(\tau)\sigma_t^2$, which implies that the conditional quantile[3] is some linear function of volatility. In this sense, Engle and Patton (2001) argue that a volatility model is typically used to forecast the absolute magnitude of returns, but it may also be used to predict quantiles.

In this paper, we adapt the model suggested by Christoffersen et al. (2001) to investigate the accuracy of a given VaR model. In other words, instead of using the conditional volatility as a regressor, we simply use V_t in place of σ_t^2 in the above model, where V_t is the VaR measure of interest. That is exactly the idea we next explore, where a convenient hypothesis test is formally derived to evaluate VaR models. In sum, we consider the following model

$$\begin{aligned} R_t &= \alpha_0(U_t) + \alpha_1(U_t)V_t && (2)\\ &= x_t'\beta(U_t), && (3)\end{aligned}$$

where U_t is an iid standard uniform random variable, $U_t \sim U(0,1)$, the functions $\alpha_i(U_t)$, $i = 0, 1$ are assumed to be comonotonic, $\beta(U_t) = [\alpha_0(U_t); \alpha_1(U_t)]'$ and $x_t' = [1, V_t]$. Notice that Eq. (2) can be re-written as

$$R_t = \varphi_t + \epsilon_t, \tag{4}$$

where $\varphi_t = \alpha_0 + \alpha_1(U_t)V_t$, $\epsilon_t = \alpha_0(U_t) - \alpha_0$ is an iid random variable and $\alpha_0 = E[\alpha_0(U_t)]$. An important feature of (4) is that the conditional mean is affected by the VaR, which was computed using information available up to period $t-1$. Since the value at risk V_t is nothing else than the conditional quantile of R_t, then the above model can be seen as a quantile-in-mean model. Indeed, if we allow V_t to be equal to the conditional variance of R_t, then the above model becomes a particular case of the so-called ARCH-in-mean model introduced by Engle (1987).

Following Koenker and Xiao (2002), we will assume that the returns $\{R_t\}$ are, conditional on $\mathcal{F}_{t-1}$, independent with linear conditional quantile functions given by (5). Since V_t is already

[3] Where the quantile function of a given random variable z_t is defined as the reciprocal of its cumulative distributive function F_z, i.e., $Q_{z_t}(\tau) = F_z^{-1}(\tau) = \inf\{z : F(z) \geq \tau\}$.

available at the end of period $t-1$, before the realization of R_t at time t, then we can compute the conditional quantile of R_t as follows:

$$Q_{R_t}(\tau \mid \mathcal{F}_{t-1}) = \alpha_0(\tau) + \alpha_1(\tau)\, V_t. \tag{5}$$

Now, recall the definition of Value-at-Risk (V_t), in which the conditional probability of a return R_t to be lower than V_t, over the target horizon, is equal to $\tau^* \in (0,1)$, i.e., $\Pr(R_t \leq V_t \mid \mathcal{F}_{t-1}) = \tau^*$. From this definition, it is clear that finding a VaR is exactly the same as finding a conditional quantile function. In fact, from our quantile regression methodology, we also have that $\Pr(R_t \leq Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1}) \mid \mathcal{F}_{t-1}) = \tau^*$. Thus, by considering that the VaR model's true level of coverage is τ^*, it follows that V_t must coincide with the related conditional quantile function of R_t at the same level τ^*. Therefore, a natural way to test for the overall performance of a VaR model is to test the null hypothesis

$$Ho : \begin{cases} \alpha_0(\tau^*) = 0 \\ \alpha_1(\tau^*) = 1 \end{cases}. \tag{6}$$

This hypothesis can be presented in a classical formulation as $Ho : R\theta(\tau^*) = r$, for the fixed quantile $\tau = \tau^*$, where R is a 2×2 identity matrix; $\beta(\tau^*) = [\alpha_0(\tau^*); \alpha_1(\tau^*)]'$ and $r = [0; 1]$. Note that, due to the simplicity of our restrictions, the later null hypothesis can still be reformulated as $Ho : \theta(\tau^*) = 0$, where $\theta(\tau^*) = [\alpha_0(\tau^*); (\alpha_1(\tau^*) - 1)]'$.[4]

The first issue to implement such a hypothesis test is to construct the confidence intervals for the estimated coefficients $\widehat{\theta}(\tau^*)$. Following Koenker (2005, p.74) the method used in this paper to compute the covariance matrix of the estimated coefficients takes the form of a Huber (1967) sandwich:[5]

$$\sqrt{T}(\widehat{\theta}(\tau^*) - \theta(\tau^*)) \xrightarrow{d} N(0, \tau^*(1-\tau^*)H_{\tau^*}^{-1}JH_{\tau^*}^{-1}) = N(0, \Lambda_{\tau^*}), \tag{7}$$

[4] Recall that our focus is to test a VaR on the right tail of the distribution of returns. In order to investigate a VaR for the left tail, one must consider the modified null hypothesis: $\widetilde{\theta}(\tau^*) = 0$, in which $\widetilde{\theta}(\tau^*) \equiv [\alpha_0(\tau^*); (\alpha_1(\tau^*) + 1)]'$.

[5] A technical issue on the estimation process emerges from the fact that the objective function is not differentiable with respect to parameters at interested quantiles. The discontinuity in first order condition of the corresponding objective function makes the derivation of asymptotics of quantile regression estimators quite difficult, since conventional techniques (based on Taylor expansion) are no more applicable. The argument of stochastic uniform continuity, called stochastic equicontinuity, is one of the solutions for deriving the asymptotics from the non-differentiable objective function, revalidating the conventional techniques under nonstandard conditions. This idea was pioneering illustrated by Huber (1967) in discussion of deriving the asymptotics of maximum likelihood estimators with iid random variables under nonstandard conditions. The main idea is to make the discontinuous first order conditions asymptotically and uniformly continuous by stochastic equicontinuity argument, i.e., by approximating it through a uniformly continuous function. After justifying stochastic equicontinuity, all conventional techniques for deriving asymptotics are again applicable. See Chen (2001) for further details.

where $J = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} x_t x_t'$ and $H_{\tau^*} = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} x_t x_t' [f_t(Q_{R_t}(\tau^*|x_t)]$ under the quantile regression model $Q_{R_t}(\tau \mid x_t) = x_t'\theta(\tau)$. The term $f_t(Q_{R_t}(\tau^*|x_t)$ represents the conditional density of R_t evaluated at the quantile τ^*. Given that we are able to compute the covariance matrix of the estimated $\widehat{\theta}(\tau)$ coefficients, we can now construct our hypothesis test to verify the performance of the Value-at-Risk model based on quantile regressions (hereafter, VQR test).

Definition 1: Let our test statistic be defined under the null by $\zeta_{VQR} = T[\widehat{\theta}(\tau^*)'(\tau^*(1-\tau^*)H_\tau^{-1}JH_\tau^{-1})^{-1}\widehat{\theta}(\tau^*)$

In addition, consider the following assumptions:

Assumption 1: Let $x_t \geq 0$ be measurable with respect to $\mathcal{F}_{t-1}$ and $z_t \equiv \{R_t; x_t\}$ be a strictly stationary process;

Assumption 2: (Density) Let $\{R_t\}$ have distribution functions F_t, with continuous Lebesgue densities f_t uniformly bounded away from 0 and ∞ at the points $Q_{R_t}(\tau \mid x_t) = F_{R_t}^{-1}(\tau \mid x_t)$;

Assumption 3: (Design) There exist positive definite matrices J and H_τ, such that $J = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} x_t x_t'$ and $H_\tau = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} x_t x_t' [f_t(Q_{R_t}(\tau \mid x_t))]$;

Assumption 4: $\max_{i=1,\dots,T} \|x_i\| / \sqrt{T} \to 0$.

The following Proposition, which is merely an application of Hendricks and Koenker (1992) and Koenker (2005, Theorem 4.1), by considering a fixed quantile τ^*, summarizes our VQR test, designed to check whether the Value-at-Risk model (V_t) equals the respective conditional quantile function of R_t (at quantile τ^*), obtained from (2), i.e., $Ho : V_t = Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$.

Proposition 1 *(VQR test) Consider the quantile regression (2). Under the null hypothesis (6), if assumptions (1)-(4) hold, then, the test statistic ζ_{VQR} is asymptotically chi-squared distributed with two degrees of freedom.*

Proof. See Appendix. ■

Remark 1: The Wald statistic is often adopted in joint tests for quantile regressions, such as Machado and Mata (2004), and Proposition 1 is a special case of a general linear hypothesis test, which was properly adapted to our setup. According to Schulze (2004), a more general Wald statistic is given by $W = T(R\widehat{\varphi} - r)'(R\widehat{\Omega}R')^{-1}(R\widehat{\varphi} - r)$, where $\widehat{\varphi} = [\widehat{\alpha}(\tau_1); \dots; \widehat{\alpha}(\tau_m)]'$ and $\widehat{\Omega}$ is the estimated asymptotic joint matrix of the estimated coefficients considering a full range of quantiles $\tau \in [\tau_1; ...; \tau_m]$.[6] Note that this formulation includes a wide variety of testing situations. However, since we are only focused on testing the estimated coefficients $\widehat{\alpha}(\tau)$ for the specific quantile $\tau = \tau^*$, we adopted the simplified version of the Wald statistic presented in Proposition 1.

Remark 2: Assumption (1) together with comonotonicity of $\alpha_i(U_t)$, $i = 0, 1$ guarantee the monotonic property of the conditional quantiles. We recall the comment of Robinson (2006), in which the author argues that comonotonicity may not be sufficient to ensure monotonic conditional quantiles, in cases where x_t can assume negative values. Assumption (2) relaxes iid in the sense that allows for non-identical distributions. Bounding the quantile function estimator away from 0 and ∞ is necessary to avoid technical complications. Assumptions (2)-(4) are quite standard in the quantile regression literature (e.g., Koenker and Machado (1999) and Koenker and Xiao (2002)) and familiar throughout the literature on M-estimators for regression models, and are crucial to claim the CLT of Koenker (2005, Theorem 4.1).

Remark 3: Under the null hypothesis it follows that $V_t = Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$, but under the alternative hypothesis the randomness nature of V_t, captured in our model by the estimated coefficients $\widehat{\theta}(\tau^*) \neq 0$, can be represented by $V_t = Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1}) + \eta_t$, where η_t represents the measurement error of the VaR on estimating the latent variable $Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$. Note that assumptions (1)-(4) are easily satisfied under the null and the alternative hypotheses. In particular, note that assumption (4) under H_1 implies that also η_t is bounded.

Remark 4: According to Giacomini and Komunjer (2005), when several forecasts of the same variable are available, it is desirable to have formal testing procedures, which do not necessarily require knowledge of the underlying model, or, if the model is known, which do no restrict attention to a specific estimation procedure. Note that assumptions (1)-(4) do not restrict our methodology to those cases in which V_t is constructed from a conditional volatility model, but instead allow for several cases, such as a Pareto or a Cauchy distribution of returns (in which the mean and variance do not even exist), frequently used in the EVT literature (see McNeil & Frey (2000) and Huisman et al. (2001)). In fact, our proposed methodology can be applied to a broad number of situations, such as:

(i) The model used to construct V_t is known. For instance, a risk manager trying to construct a reliable VaR measure. In such a case, it is possible that: (ia) V_t is generated from a conditional volatility model, e.g., $V_t = g(\widehat{\sigma}_t^2)$, where $g()$ is some function of the estimated conditional variance

[6] See Koenker & Basset (1978, 1982 a, b); Koenker & Portnoy (1999) and Koenker (2005, p. 76) for further details.

$\widehat{\sigma}_t^2$, say from a GARCH model; or (ib) V_t is directly generated, for instance, from a CAViaR model[7] or an ARCH-quantile method;[8]

(ii) V_t is generated from an unknown model, and the only information available is $\{R_t; V_t\}$. In this case, we are still able to apply Proposition 1 as long as assumptions (1)-(4) hold. This might be the case described in Berkowitz and O'Brien (2002), in which a regulator investigates the VaR measure reported by a supervised financial institution (see appendix B for further details);

(iii) V_t is generated from an unknown model, but besides $\{R_t; V_t\}$ a confidence interval of V_t is also reported. Suppose that a sequence $\{R_t; V_t; \underline{V}_t; \overline{V}_t\}$ is known, in which $\Pr\left[\underline{V}_t < V_t < \overline{V}_t \mid \mathcal{F}_{t-1}\right] = \delta$, where $[\underline{V}_t; \overline{V}_t]$ are respectively lower and upper bounds of V_t, generated (for instance) from a bootstrap procedure, with a confidence level δ. One could use this additional information to investigate the considered VaR by making a connection between the confidence interval of V_t and the previously mentioned measurement error η_t. The details of this route remain an issue to be further explored.

In next section, we provide an additional framework that might be useful for those interested in improving the performance of a rejected VaR as well as choosing the best model among competing measures.

2.1 Periods of risk exposure

The conditional coverage literature (e.g., Christoffersen (1998)) is concerned with the adequacy of the VaR model, in respect to the existence of clustered violations. In this section, we will take a different route to analyze the conditional behavior of a VaR measure. According to Engle and Manganelli (2004), a good Value-at-Risk model should produce a sequence of unbiased and uncorrelated hits H_t, and any noise introduced into the Value-at-Risk measure would change the conditional probability of a hit, vis-à-vis the related VaR. Given that our study is entirely based on a quantile framework, besides the VQR test, we are also able to identify the exact periods in which the VaR produces an increased risk exposure in respect to its nominal level τ^*, which is quite a novelty in the literature. To do so, let us first introduce some notation:

Definition 2: $W_t \equiv \{\widetilde{\tau} \in [0;1] \mid V_t = \widehat{Q}_{R_t}(\widetilde{\tau} \mid \mathcal{F}_{t-1})\}$, representing the "fitted quantile" of the VaR measure at period t given the regression model (2).

[7] See section 2.2 for more details regarding the CAViaR model.

[8] A quantile regression model that allows for ARCH effect. See Koenker & Zhao (1996) and Wu & Xiao (2002) for further details.

In other words, W_t is obtained by comparing V_t with a full range of estimated conditional quantiles evaluated at $\tau \in [0;1]$. Note that W_t enables us to conduct a local analysis, whereas the proposed VQR test is designed for a global evaluation based on the whole sample. It is worth mentioning that, based on our assumptions, $Q_{R_t}(\tau \mid \mathcal{F}_{t-1})$ is monotone increasing in τ, and W_t by definition is equivalent to a quantile level, i.e., $W_t > \tau^* \Leftrightarrow Q_{R_t}(W_t \mid \mathcal{F}_{t-1}) > Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$. Also note that W_t should (ideally) be as close as possible to τ^* for all t given that the VaR is computed for the fixed level τ^*. However, due to modeling procedures (i.e., in practice), it might be different from τ^*, suggesting that V_t could not belong to the proper conditional quantile of interest.

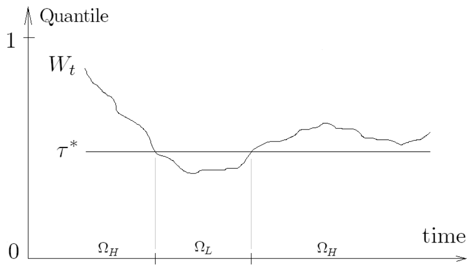
Now consider the set of all observations $\Omega = 1, ..., T$, in which T is the sample size, and define the following partitions of Ω:

Definition 3: $\Omega_H \equiv \{t \in \Omega \mid W_t \geqslant \tau^*\}$, representing the periods in which the VaR belongs to a quantile above the level of interest τ^* (indicating a conservative model);

Definition 4: $\Omega_L \equiv \{t \in \Omega \mid W_t < \tau^*\}$, representing the periods in which the VaR is below the nominal τ^* level and, thus, underestimate the risk in comparison to τ^*.

Since we partitioned the set of periods into two categories, i.e. $\Omega = \Omega_H + \Omega_L$, we can now properly identify the so-called periods of "risk exposure" Ω_L. Let us summarize the previous concepts through the following schematic graph:

Figure 1 - Periods of risk exposure



It should be mentioned that a VaR model that exhibits a good performance in the VQR test (i.e., in which Ho is not rejected) is expected to exhibit W_t as close as possible to τ^*, fluctuating around τ^*, in which periods of W_t below τ^* are balanced by periods above this threshold. On the other hand, a VaR model rejected by the VQR test should present a W_t series detached from τ^*, revealing the periods in which the model is conservative or underestimate risk. This additional information can be extremely useful to improve the performance of the underlying Value-at-Risk model, since the periods of risk exposure are now easily revealed.

Another important issue regarding model analysis is the choice of competing VaRs. Instead of only checking the performance of a single model, one might be interested in ranking several VaR measures (see Giacomini and Komunjer, 2005). Although this is not the main objective of this paper, we outline a simple nonparametric procedure, inspired by Lopez (1999), in which a loss function is used to measure the "conditional coverage distance" of a VaR from its nominal benchmark τ^*. According to the author, a numerical score could reflect regulatory concerns and provide a measure of relative performance to compare competing VaR models across time and institutions.

The generic loss function suggested by Lopez (1999) is given by $C(R_t; V_t) = \sum_{t=1}^{T} C_t(R_t; V_t)$, where $C_t(.) = \begin{cases} f(R_t; V_t) & ; \text{ if } R_t > V_t \\ g(R_t; V_t) & ; \text{ if } R_t \leq V_t \end{cases}$. Accurate VaR estimates are expected to generate lower numerical scores. Once the f and g functions are defined, the loss function can be constructed and used to evaluate the performance of a set of VaR models. Among several different specifications, Lopez (1999) suggests adopting $f(R_t; V_t) = 1 + (R_t - V_t)^2$ and $g(R_t; V_t) = 0$. An interesting advantage of this specification is to consider the magnitude of violations, since the magnitude as well as the number of violations is a serious matter of concern to regulators and risk managers. In addition, loss functions may be more suited to discriminate among competing VaR models than deciding for the accuracy of a single VaR model.

In this paper, we adapt the previous approach to our setup in order to rank competing VaRs. Let's first define C_t as the "empirical distance" of V_t in respect to the conditional quantile function $Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$, based on the W_t series:

$$C_t \equiv |W_t - \tau^*| . \tag{8}$$

Now, define the loss function $L(V_t)$ that summarizes the distances C_t, and assigns weights $[\gamma_1; \gamma_2]$ to each distance, according to the indicator function $I_t = \begin{cases} 1 & ; \text{ if } W_t > \tau^* \\ 0 & ; \text{ if } W_t \leq \tau^* \end{cases}$:

$$L(V_t) \equiv \frac{1}{T} \sum_{t=1}^{T} C_t * (\gamma_1 I_t + \gamma_2 (1 - I_t)). \tag{9}$$

This loss function is very convenient, since it is very easy to be computed and non-parametrically provides us an empirical way to rank a set of VaRs. Thus, the choice among $i = 1, ..., n$ competing models could be based on the minimization of the proposed loss function, by simply choosing $i = \arg\min_{i=1,...,n}[L(V_t^i)]$. In addition, note that by setting $\gamma_1 < \gamma_2$ one could penalize more the periods of risk exposure than those periods in which the "fitted quantile" W_t is above τ^*. Therefore, an asymmetric evaluation is allowed by this framework, in which the choice of weights $[\gamma_1; \gamma_2]$ could also be driven by the risk aversion degree of the regulator (or the risk manager).[9] Despite its simplicity, the descriptive statistic $L(.)$ might be useful to illustrate model comparison in our empirical exercise. It is worth mentioning that the backtest literature is mainly focused just on the signal I_t (as detailed in next section), whereas in this paper we try to go a step further by also considering the valuable information contained in the magnitude of the VaR violations.

2.2 Other Backtests

Generally speaking, backtesting a VaR model means checking whether the realized daily returns are consistent with the corresponding VaR produced by an internal model of a financial institution, over an extended period of time. Crouhy et al. (2001) argue that backtests provide a key check of how accurate and robust models are, by considering ex-ante risk measure forecasts and comparing it to ex-post realized returns. The authors also state: *"Backtesting is a powerful process with which to validate the predictive power of a VaR model ... and in effect, a self-assessment mechanism that offers a framework for continuously improving and refining risk modeling techniques"*.

According to Hull (2005), it represents a way to test how well VaR estimates would have performed in the past, i.e., how often was the actual 1-day (or 10-day) loss greater than the 95% (or 99%) VaR measure. Before presenting some commonly discussed backtests, let's initially recall that R_t is the observed returns and that V_t is the respective 1-day VaR defined for a quantile level τ^*. Now define a violation sequence[10] by the following indicator function:

$$H_t = \begin{cases} 1 \ ; \ \text{if } R_t > V_t \\ 0 \ ; \ \text{if } R_t \leq V_t \end{cases}, \tag{10}$$

and compute the number of violations $N = \sum_{t=1}^{T} H_t$. Based on these definitions, we now present some backtests usually mentioned in the literature to identify misspecified VaR models (see Dowd (2005) and Jorion (2007) for a detailed description):

(i) Kupiec (1995): Some of the earliest proposed VaR backtests is due to Kupiec (1995), which proposes a nonparametric test based on the proportion of exceptions. Assume a sample size of T observations and a number of violations of N. The objective of the test is to know whether or not $\widehat{p} \equiv N/T$ is statistically equal to τ^* (the VaR confidence level).

$$Ho : p = E(H_t) = \tau^*. \tag{11}$$

[9] A more general setup could consider the weights as functions of the distance C_t, i.e., $\gamma_i = \gamma_i(C_t)$.

[10] Also called in the literature by "exception" or "hit sequence".

The probability of observing N violations over a sample size of T is driven by a Binomial distribution and the null hypothesis Ho: $p = \tau^*$ can be verified through a LR test of the form (also known as the unconditional coverage test):

$$LR_{uc} = 2\ln\left(\frac{\widehat{p}^N(1-\widehat{p})^{T-N}}{\tau^{*N}(1-\tau^*)^{T-N}}\right), \tag{12}$$

which follows (under the null) a chi-squared distribution with one degree of freedom. It also should be mentioned that this test is uniformly most powerful (UMP) test for a given T. However, Kupiec (1995) finds that the power of his test is generally poor in finite samples, and the test becomes more powerful only when the number of observations is very large.[11]

(ii) Christoffersen (1998): The unconditional coverage property does not give any information about the temporal dependence of violations, and the Kupiec (1995) test ignores conditioning coverage, since violations could cluster over time, which should also invalidate a VaR model. In this sense, Christoffersen (1998) extends the previous LR statistic to specify that the hit sequence should also be independent over time. The author argues that we should not be able to predict whether the VaR will be violated, since if we could predict it, then, that information could be used to construct a better risk model. The proposed test statistic is based on the mentioned hit sequence H_t, and on T_{ij} that is defined as the number of days in which a state j occurred in one day, while it was at state i the previous day. The test statistic also depends on π_i, which is defined as the probability of observing a violation, conditional on state i the previous day. It is also assumed that the hit sequence follows a first order Markov sequence with transition matrix given by

$$\Pi = \begin{array}{c} \text{Previous day} \\ \begin{bmatrix} 1-\pi_0 & 1-\pi_1 \\ \pi_0 & \pi_1 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{current day (violation)} \\ \text{no violation} \end{array} \tag{13}$$

Note that under the null hypothesis of independence, we have that $\pi = \pi_0 = \pi_1 = (T_{01}+T_{11})/T$, and the following LR statistic can, thus, be constructed:

$$LR_{ind.} = 2\ln\left(\frac{(1-\pi_0)^{T_{00}}\pi_0^{T_{01}}(1-\pi_1)^{T_{10}}\pi_1^{T_{11}}}{(1-\pi)^{(T_{00}+T_{10})}\pi^{(T_{01}+T_{11})}}\right). \tag{14}$$

The joint test, also known as "conditional coverage test", includes unconditional coverage and independence properties, and is simply given by $LR_{cc} = LR_{uc} + LR_{ind.}$; where each component follows a chi-squared distribution with one degree of freedom, and the joint statistic LR_{cc} is asymptotically distributed as $\chi^2_{(2)}$. An interesting feature of this test is that a rejection of the conditional

[11] According to Kupiec (1995), it would require more than six violations during a one-year period (250 trading days) to conclude that the model is misspecified. See Crouhy et al. (2001) for further details.

coverage may suggest the need for improvements on the VaR model, in order to eliminate the clustering behavior. On the other hand, the proposed test has a restrictive feature, since it only takes into account the autocorrelation of order 1 in the hit sequence.

(iii) Engle and Manganelli (2004): The Conditional Autoregressive Value-at-Risk by Regression Quantiles (CAViaR) model is proposed by the authors, which define $V_t(\tau)$ as the solution to $\Pr[R_t < -V_t(\tau) \mid \mathcal{F}_{t-1}) = \tau$; and describe the (generic) specification: $V_t(\tau) = \beta_0 + \sum_{i=1}^{q} \beta_i V_{t-i}(\tau) + \sum_{j=1}^{r} \gamma_j x_{t-j}$; where $[\beta_i; \gamma_j]$ are unknown parameters to be estimated and x_t is a generic vector of time t observable variables. The CAViaR approach directly models the return quantile rather than specifying a complete data generating process. The authors define various dynamic models for V_t itself, including the adaptative model: $V_t(\tau) = V_{t-1}(\tau) + \beta\left[\mathbf{1}(R_{t-1} \leq -V_{t-1}) - \tau\right]$; symmetric absolute value: $V_t(\tau) = \beta_0 + \beta_1 V_{t-1}(\tau) + \beta_2 \left|R_{t-1}\right|$; asymmetric slope: $V_t(\tau) = \beta_0 + \beta_1 V_{t-1}(\tau) + \beta_2 R_{t-1}^{+} + \beta_3 R_{t-1}^{-}$; in which R_t is the return series. The estimation of the CAViaR model uses standard quantile regression techniques, but the model is only possible in GARCH special cases. How to proper simulate the model is another issue to be further explored.

Besides proposing the CAViaR class of models to directly estimate the conditional quantile process, Engle and Manganelli (2004) also suggested a specification test, also known as the dynamic conditional quantile (DQ) test, which involves running the following regression

$$DQ_{oos} = (Hit_t' X_t [X_t' X_t]^{-1} X_t' Hit_t)/(\tau(1-\tau)), \tag{15}$$

where $X_t = [c, V_t(\tau), Z_t]$; Z_t denotes lagged Hit_t; $Hit_t = I_t(\tau) - \tau$; and $I_t(\tau) = \begin{cases} 1 & ; \text{ if } R_t < V_t(\tau) \\ 0 & ; \text{ if } R_t \geq V_t(\tau) \end{cases}$. The null hypothesis is the independence between Hit_t and X_t. Under the null, the proposed metric to evaluate one-step-ahead forecasts (DQ_{out}) follows a χ_q^2, in which $q = rank(X_t)$. Note that the DQ test can be used to evaluate the performance of any type of VaR methodology (and not only the CAViaR family).

(iv) Berkowitz et al. (2006): These authors recently proposed a unified approach to a VaR assessment, based on the fact that the unconditional coverage and independence hypotheses are nothing but consequences of the martingale difference hypothesis of the Hit_t process, i.e., $E(I_t(\tau) - \tau \mid \mathcal{F}_{t-1}) = 0$, where $I(\tau)$ is defined as above. Based on a Ljung-Box type-test, they consider the nullity of the first K autocorrelations of the Hit_t process, instead of only considering the autocorrelation of order one, as done by Christoffersen (1998).

Several other related procedures are also well documented in the literature, such as the non-parametric test of Crnkovic and Drachman (1997),[12] the duration approach of Christoffersen and Pelletier (2004),[13] and the encompassing test of Giacomini and Komunjer (2005).[14] However, as shown in several simulation exercises (e.g., Kupiec (1995), Pritsker (2001) and Campbell (2005)), backtests generally have low power and are, thus, prone to misclassifying inaccurate VaR estimates as "acceptably accurate". The standard backtests often lack power, especially when the VaR confidence level is high and the number of observations is low, which has lead to a search for improved tests. Since the original Kupiec (1995) test is the most powerful among its class, more effective backtests would have to focus on different hypothesis or use more information, according to Jorion (2007). In next section, we exactly address this topic by comparing our proposed quantile regression-based backtest with some previously mentioned procedures.

2.3 Nested null hypotheses

In this section, we construct a parallel of our setup with some backtests. Since the main concern of the backtest literature is to evaluate the VaR accuracy, we pose a relevant question in this context: What do we really want to test? Given that a VaR measure is implicitly defined by Property 1 (hereafter, P1, reproduced below), the core issue of a backtest should be to verify whether it (in fact) is true. As we next show, the quantile regression framework provides a natural way to investigate the performance of a VaR model, and the proposed VQR test consists on a sufficient condition for P1. We also show that our considered null hypothesis implies some null hypotheses used in the literature to construct backtests, which are only necessary conditions for P1. To do so, firstly recall that a "violation" sequence is here defined by the following indicator (hit) function: $H_t = \begin{cases} 1 \ ; \ \text{if } R_t > V_t \\ 0 \ ; \ \text{if } R_t \leq V_t \end{cases}$, and secondly consider $\tau^{**} = (1 - \tau^*)$, in order to properly compare our VQR test, originally constructed for the right tail of the returns distribution, with the other considered backtests based on the left tail.

[12] The authors use a Kuiper's statistic, based not only on a selected quantile but focused on the entire forecasted distribution.

[13] The key idea is that if the VaR model is correctly specified for a given coverage rate p, then, the conditional expected duration between violations should be a constant $1/p$ days. The independence test based on duration allows one to consider wider dependences than those chosen under the Markov chain hypothesis. However, the core idea remains unchanged, and consists in putting the conditional coverage hypothesis to the test, still ignoring the magnitude of violations.

[14] A conditional quantile encompassing test is provided based on GMM estimation, with the focus on relative model evaluation, which involves comparing the performance of competing VaR models, and choosing the one that performs the best. The encompassing approach also gives a theoretical basis for quantile forecast combination, in cases when neither forecast encompasses its competitor. As a by-product, their framework also provides a link to the conditional coverage test of Christoffersen (1998).

Property 1: $\Pr[R_t \leq V_t|\mathcal{F}_{t-1}] = \tau^*$;

Statement 1: (Null hypothesis of the VQR test) $V_t = Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$;

Statement 2: Berkowitz et al. (2006): $E(H_t - \tau^{**} \mid \mathcal{F}_{t-1}) = 0$;

Statement 3: Christoffersen (1998): $E[(H_t - \tau^{**})(H_{t-1} - \tau^{**})] = 0$;

Statement 4: Kupiec (1995): $E(H_t) = \tau^{**}$.

Proposition 2 *(Nested Hypotheses) Consider Property 1 and statements S1-S4. If assumptions (1)–(4) hold for the regression (2), then, it follows that:*

(i) $P1 \Leftrightarrow S1$

(ii) $S1 \Rightarrow S2, S3, S4$

(iii) $S2, S3, S4 \nRightarrow S1$

Proof. See Appendix. ■

Our proposed test aims to break down the paradigm of the hit sequence in the backtest literature, which investigates the accuracy of a VaR measure basically through the behavior of its hit sequence. Proposition 2 states an important result that the VQR test is a necessary and sufficient condition to verify Property 1. In addition, it also shows that our null hypothesis implies some null hypothesis of the backtest literature, but the reverse does not hold. In other words, assumption S1 is a sufficient condition for Property 1, whereas statements S2-S4 are only necessary conditions.

A small Monte Carlo simulation is conducted in next section, in order to verify the size and power of the VQR test in finite samples. Overall, the quantile regression test seems to have relatively more power in comparison to other backtests, previously documented in the literature as exhibiting low power to detect poor VaR models. These results are consistent with previous findings in Kupiec (1995), Pritsker (2001) and Campbell (2005). The increased power might be due to the quantile framework, which can provide an adequate null hypothesis as stated in Proposition 2.

3 Monte Carlo simulation

A small simulation experiment is conducted in order to investigate the finite sample properties of the VQR test, in comparison to other tests presented in the literature, such as the unconditional coverage test of Kupiec (1995), the conditional coverage test of Christoffersen (1998), and the out-of-sample DQ test of Engle and Manganelli (2004). To do so, we use two Data-Generating Processes: DGP1 is the RiskMetrics[15], and DGP2 is the GARCH(1,1) model $\sigma_t^2 = 0.02 + 0.05y_{t-1}^2 + 0.93\sigma_{t-1}^2$. In addition, we assume (in both DGPs) that $y_t = \sigma_t\varepsilon_t$, in which $\varepsilon_t \sim N(0,1)$. For each DGP, we generate $T + 2,000$ observations, discarding the first $2,000$ observations. Then, a total amount of $i = 5,000$ replications of the $\{y_t\}_{t=1}^{T}$ process is considered for each DGP.

We follow here the same computational strategy of Lima & Neri (2006), in which a hybrid solution using R and Ox environments is adopted, since the proposed simulation is extremely computational intensive. Ox is much faster than R in large computations, and also makes use of the package G@RCH 4.2 (see Laurent & Peters, 2006), which easily allows us to generate GARCH specifications. On the other hand, the R language is more interactive and user-friendly than Ox and the VQR test must in fact be conducted in R, since its package for quantile regressions (quantreg) is more complete and updated than the Ox package. Therefore, we proceed as follows: an Ox code initially generates the time series y_t for each DGP, and save all the replications in the hard disk. Next, an R code computes the four considered backtests for all replications and saves the final results in a text file.

For the size investigation, in order to generate data that supports the null hypothesis, we compute the respective VaR at the (standard normal) quantile τ^*. In other words, the VaR for $\tau^* = 95\%$ is given by $V_t = 1.64 * \sigma_t$, and for $\tau^* = 99\%$ is computed by $V_t = 2.33{*}\sigma_t$. The empirical sizes for $T = \{250; 500; 1,000\}$ and the quantile levels $\tau^* = 95\%$ or 99% are presented in next table (for a nominal size of 5%):

[15] Recall that RiskMetrics is just an integrated GARCH(1,1) model with the autoregressive parameter set to 0.94, i.e., $\sigma_t^2 = c + 0.06y_{t-1}^2 + 0.94\sigma_{t-1}^2$. In our simulation, we set $c = 0.02$.

Table 1 - Size investigation (T=250)

	$\tau^* = 95\%$		$\tau^* = 99\%$	
	DGP1	DGP2	DGP1	DGP2
ζ_{VQR}	0.0705	0.0591	0.1801	0.1851
ζ_{Kupiec}	0.0101	0.0069	0.0079	0.0007
$\zeta_{\text{Christ.}}$	0.1073	0.1053	0.0215	0.0175
ζ_{DQ}	0.0739	0.0429	0.0806	0.0931
T=500				
ζ_{VQR}	0.0632	0.0545	0.1114	0.1299
ζ_{Kupiec}	0.0088	0.0084	0.0242	0.0198
$\zeta_{\text{Christ.}}$	0.0960	0.1089	0.0325	0.0267
ζ_{DQ}	0.0592	0.0577	0.0762	0.0781
T=1,000				
ζ_{VQR}	0.0541	0.0513	0.0950	0.0991
ζ_{Kupiec}	0.0247	0.0192	0.0368	0.0254
$\zeta_{\text{Christ.}}$	0.0986	0.0920	0.0374	0.0332
ζ_{DQ}	0.0562	0.0517	0.0801	0.0855

Note: The values above represent the percentage of p-values below the nominal level of significance $\alpha = 5\%$.

Firstly, note that for $T = 250$, the VQR and DQ backtests exhibit relatively good sizes for $\tau^* = 95\%$. On the other hand, for $\tau^* = 99\%$ the results are slightly distorted: the DQ and VQR tests tend to over-reject the VaR model, whereas the Kupiec and Christoffersen backtests tend to under-reject it. The main reason is that, for $T = 250$ only a small number of observations is expected at the extreme quantiles, which is a serious problem for all backtests, and might also affect the QR estimation.

The increase of the sample size T can give us some flavor of the asymptotic behavior in the size investigation. Recall that each backtest is constructed to investigate different null hypotheses, which might partially explain the results presented in Table 1. In addition, note that an increase of the sample size produces the following effect in our simulation: As long as T increases, the estimation of the extreme quantiles becomes more precise, leading to a better estimation of the quantile density function evaluated at those quantiles. As a result, the empirical size of the proposed test tends to approach its nominal size (5%) as T goes to infinity.

Despite the relatively large sample size when $T = 1,000$, note that for $\tau^* = 99\%$ one should expect only 10 observations of y_t above the VaR measure, which could seriously influence the performance of any backtest. However, the small sample size should not be viewed anymore as a restriction, given that nowadays it is common to deal with intra-day data, and even for daily frequency, a sample size of $T = 1,000$ only requires four years of database.

Moreover, backtesting involves balancing two types of errors and dealing with the tradeoff between rejecting a correct model versus accepting a misspecified one. According to Christoffersen (2003, p.186), in risk management, may be very costly if the test fail to reject an incorrect model. Jorion (2007), in the same line, says that one would want a framework that has high power of rejecting an incorrect model. Therefore, if one is more concerned with discarding a poor VaR model (and, thus, the power of the tests), instead of validating a good VaR specification, the numerical results might be favorable to the VQR approach, as we shall next see.

In the power analysis, we conduct the investigation along three main directions: the sample size $T = \{250; 500; 1,000\}$, the quantile level $\tau^* = 95\%$ or 99%; and finally the set of alternative hypothesis. The first set of H_1 (here, so-called method 1) considers a sequence of DGPs based on a GARCH(1,1), with coefficients: $c = 0.02$; $\alpha = 0.06 - \phi/20$; $\beta = 0.94 - \phi/2$, and a Gamma (a, b) distribution with parameters $a = 200e^{-5\phi}; b = 5$. We control the "degree of misspecification" through the parameter $\phi \in [0, 1]$, which ranges from 0 to 1 with increments of 0.1. Then, in order to replicate a realistic situation, a VaR is estimated for each DGP via a RiskMetrics model with normal distribution. Note that when $\phi = 0$ we are under the null hypothesis,[16] but as long as we increase ϕ the alternative hypothesis is simulated.

The second approach for H_1 (method 2) is constructed as a complementary exercise, in which we now fix the DGP and then generate a sequence of VaRs. The idea is based on Engle and Manganelli (2004), which argue that: *"any noise introduced into the quantile estimate will change the conditional probability of a hit given the estimate itself"*. To do so, we initially generate y_t and σ_t^2 according to DGP2. Then, we construct a sequence of VaRs in the following way: $V_t(\phi) \equiv Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1}) + \phi\eta_t$, where $Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$ comes from the DGP2; $\eta_t \sim iid\ N(0, 1)$; $\phi \in [0, 1]$ ranges from 0 to 1. Note that the "degree of misspecification" is (again) given by ϕ, in which $V_t(\phi = 0)$ satisfies H_0, but as long as the ϕ parameter is augmented we expect to generate quite poor VaR measures due to the additional white noise η_t.

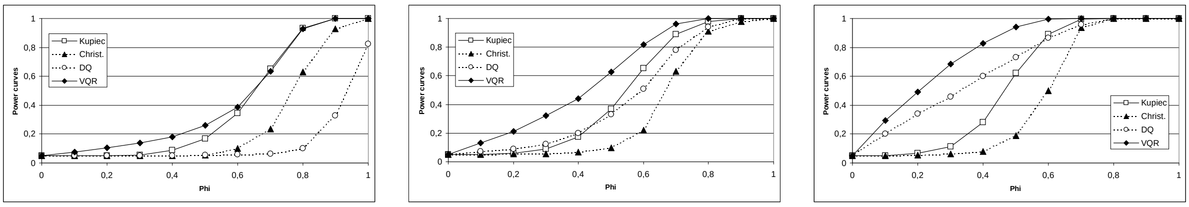
A final simulation for the power analysis is given by method 3, in which the DGP2 is used to generate y_t and σ_t^2, but the sequence of VaRs is now constructed from a normal-GARCH (1,1)

[16] Recall that a Gamma (a, b) distribution tends to a normal distribution as long as $a \to \infty$.

specification with the following coefficients: $c = 0.02$; $\alpha = 0.05 + \phi/5$; $\beta = 0.93 - \phi/5$. This way, when $\phi = 0$, we are under the null hypothesis, and the model used for the VaR is compatible with the adopted DGP. However, as long as the ϕ parameter increases, the constructed VaR measure will come from an increasingly misspecified volatility model. We next present the results for the power investigation, which are already corrected[17] for the size distortions shown in Table 1, i.e., size-adjusted power results (for a nominal size of 5%).

Figure 2a - Size-corrected Power Curves - Method 1 ($\tau^* = 95\%$)

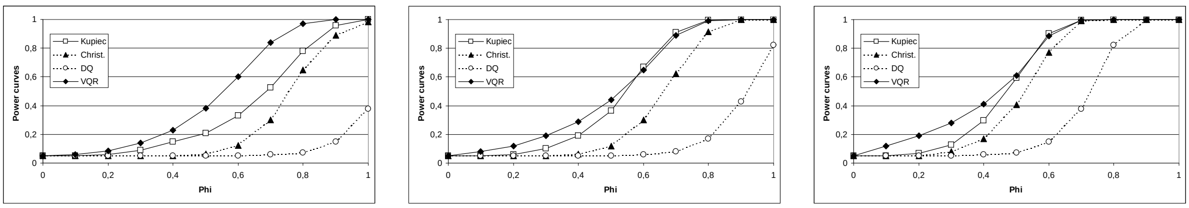
$T = 250$ $T = 500$ $T = 1,000$



Notes: Nominal level of significance is $\alpha = 5\%$. The results for methods 2 and 3 are presented in appendix.

Figure 2b - Size-corrected Power Curves - Method 1 ($\tau^* = 99\%$)

$T = 250$ $T = 500$ $T = 1,000$



Notes: Nominal level of significance is $\alpha = 5\%$. The results for methods 2 and 3 are presented in appendix.

The previous plots reveal meaningful differences among the considered tests. A very nice result is that in the smallest sample size case ($T = 250$, with $\tau^* = 99\%$), our test is indeed the most powerful among the considered backtests. Note that as long as T increases, all curves becomes closer to the origin, increasing the chance of detecting a misspecified model, which is a natural response since greater sample sizes lead to smaller variances. An important remark is that, in the same line of Engle and Manganelli (2004), one could include other (exogenous or lagged) variables in $\mathcal{F}_{t-1}$ and, thus, in the quantile regression (2), in order to still further increase the power of the VQR test in different directions.

The results for methods 2 and 3 are presented in appendix. The DQ curve exhibits the best shape in method 2, whereas the Christoffersen (1998) and Kupiec (1995) tests are relatively better

[17] "Size-corrected power" is just power using the critical values that would have yielded correct size under the null hypothesis.

than the VQR test for $\tau^* = 99\%$, but the VQR shows again a superior behavior for $\tau^* = 95\%$. Regarding method 3, the VQR test exhibits a good performance, beating the other backtests in almost all situations.

Previously results in the literature have already suggested that the Kupiec (1995) test might exhibit low power against poor VaR methodologies: Kupiec (1995) itself describes how his test has a limited ability to distinguish among alternative hypotheses and thus has low power in samples of size $T = 250$. See also Pritsker (2001), Campbell (2005) and Giacomini & Komunjer (2005).[18] In fact, our results reconfirm these earlier findings, and also suggest that the VQR test might be more powerful under some directions of the alternative hypothesis.

Besides the sample size, another reason to support the simulation results is given by Proposition 2, in which the null hypothesis of the VQR test seems to be a sufficient condition for the validity of Property 1, whereas the Kupiec (1995) test is only a necessary condition. Note that the unconditional coverage test of Kupiec (1995) is a LR test, which is uniformly most powerful for a given sample size. However, the related low power of this test in small samples is due to its inappropriate null hypothesis regarding Property 1. What we really want to test? Recall that an ideal VaR model should be well represented by Property 1. Yet another reason for the reported lack of power is the choice of a high confidence level (99%) that generates too few exceptions for a reliable backtest. Thus, simply changing the VaR quantile level from 99% to 95% sharply reduces the probability of accepting a misspecified model.[19]

4 Empirical exercise

4.1 Data

In this section, we explore the empirical relevance of the theoretical results previously derived. This is done by evaluating and comparing five different VaR models, based on the VQR test and other competing procedures commonly presented in the backtest literature. To do so, we investigate the daily returns of S&P500 over the last 4 years,[20] with an amount of $T = 1,000$ observations, depicted in the following figure:[21]

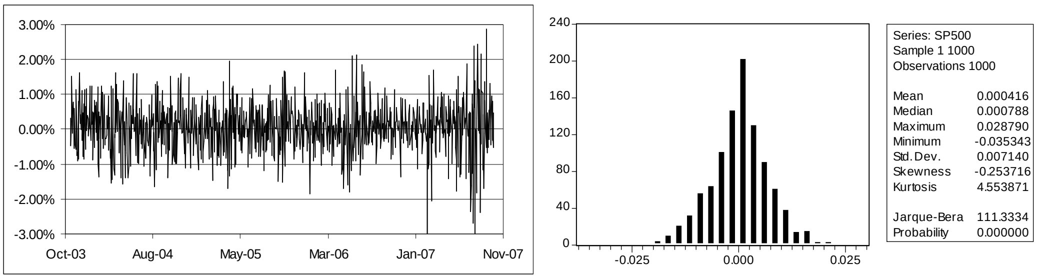
[18] According to the authors, the unconditional coverage test of Kupiec (1995) assumes away parameter estimation uncertainty and, as we already discussed, only investigates the hit sequence instead of the magnitude of the violations.

[19] This could explain why some banks prefer to choose $\tau^* = 0.95$, in order to be able to observe sufficient number of observations to validate the internal model. See Jorion (2007, p. 147) for further details.

[20] In appendix D, we conduct the empirical exercise for two additional datasets: (i) The FTSE100 index from the United Kingdom; and (ii) the IBOVESPA index from Brazil.

[21] We take the log-difference of the value of the S&P500 index in order to convert the data into returns.

Figure 3 - S&P500 daily returns (%)



Notes: a) The sample covers the period from 23/10/2003 until 12/10/2007;

b) Source: Yahoo!Finance.

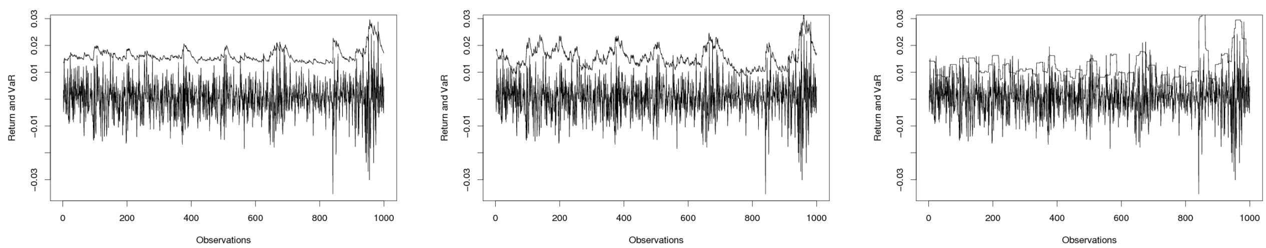
Note from the graph and the summary statistics the presence of common stylized facts about financial data (e.g., volatility clustering; mean reverting; skewed distribution; kurtosis > 3; and non-normality. See Engle and Patton (2001) for further details). In addition, an analysis of the correlogram of the returns (not reported) indicates only weak dependence in the mean. In this sense, a detailed analysis over a full range of quantiles could still be conducted based on the "quantilograms" of Linton and Whang (2007), which propose a diagnostic tool for directional predictability, by measuring nonlinear dependence based on the correlogram of the quantile hits. The authors provide a method to compute the correlogram of the quantile hits, so-called the "quantilogram", and to display this along with pointwise confidence bands, resulting in additional information in respect to the standard correlogram.

It is worth mentioning that Linton and Whang (2007) apply their methods to S&P500 stock index return data, from 1955 to 2002, and the empirical results suggest some directional predictability in daily returns, especially at the extreme lower quantiles. In addition, there is not much individual evidence of predictability in the median, which is similar to evidence at the mean using standard correlogram. In other words, extreme losses in one period are likely to be succeeded by large losses in the next period. This way, a good VaR measure should be able to capture this kind of dynamics. Note that our proposed framework is related to the quantilogram approach, in the sense that we also make use of additional information by investigating the magnitude of hits, and not only the hit sequence, but here we solely focus on model evaluation.

The five Value-at-Risk models adopted in our evaluation procedure are the following: Rolling Window (1 and 3 months), GARCH (1,1), RiskMetrics (hereafter, RM) and CAViaR. In the first two approaches, the last 30 (and 90) days of data are used to calculate the conditional variance (σ_t^2), based on a moving average of past observations. The third and fourth approaches are nothing else than conditional volatility models based on a GARCH (1,1) model,[22] since RiskMetrics is just an integrated GARCH(1,1) model with the autoregressive parameter set to 0.94. The respective VaR measures of these first four volatility models are, then, constructed by a linear function of σ_t (assuming normality). For instance, the Value-at-Risk for $\tau^* = 99\%$ is given by $V_t = 2.33 * \sigma_t$. Regarding the CAViaR model, we considered the asymmetric slope model: $V_t(\tau) = \beta_0 + \beta_1 V_{t-1}(\tau) + \beta_2 R_{t-1}^{+} + \beta_3 R_{t-1}^{-}$; in which R_t is the return series.

Practice generally shows that these various models lead to widely different VaR levels for the same considered return series, leading us to the crucial issue of model comparison and hypothesis testing. The Rolling Window method (also called Historical Simulation, hereafter, HS) has serious drawbacks and is expected to generate poor VaR measures, since it ignores the dynamic ordering of observations, and volatility measures look like "plateaus", due to the so-called "ghost effect". On the other hand, as shown by Christoffersen et al. (2001), apud Giacomini and Komunjer (2005), the GARCH-VaR model is the only VaR measure, among several alternatives considered by the authors, which passes the Christoffersen's (1998) conditional coverage test. The JP Morgan's RiskMetrics-VaR model is chosen as a benchmark model commonly used by practitioners. Finally, Engle and Manganelli (2004) show that the "asymmetric absolute value" and "asymmetric slope" models are the best CAViaR specifications for the S&P500 data.

Figure 4 - S&P500 daily returns (R_t) and VaR (99%) V_t
GARCH(1,1), CAViaR and Rolling Window (1 month)



[22] The following GARCH (1,1) model was estimated through EViews: $\sigma_t^2 = 2.44E-06 + 0.049535 y_{t-1}^2 + 0.901294 \sigma_{t-1}^2$.

4.2 Results

Based on the quantile regression framework, we are now able to construct the VQR test for the five considered VaRs. The main results are summarized in the following table:

Table 2 - Results of the VQR test ($\tau^* = 99\%$)

$Ho : V_t = Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$

	CAViaR	GARCH	RM	HS1m	HS3m
$\widehat{\alpha}_0(\tau^*)$	0.00205 (0.00232)	-0.00594 (0.00976)	-0.00267 (0.00202)	0.00677 (0.00252)	0.00298 (0.00255)
$\widehat{\alpha}_1(\tau^*)$	0.83323 (0.19955)	1.39269 (0.63097)	1.16941 (0.08170)	0.80103 (0.22783)	0.91397 (0.19632)
ζ_{VQR}	0.81351	0.39766	15.23240	31.94366	11.87233
p-value	0.66581	0.81968	0.00049	1.15e-07	0.00264

Note: a) Standard error in parentheses.

As already expected, the rolling window models are all rejected, whereas the GARCH(1,1) and CAViaR models do not fail at the VQR test, which is a result perfectly in line with the literature (e.g., Christoffersen et al. (2001) and Giacomini and Komunjer (2005)). In addition, the RiskMetrics-VaR is rejected for $\tau^* = 99\%$. It should be mentioned that violations that are clustered in time are more likely to occur in a VaR model obtained from a rolling window procedure, which increases the number of scenarios for our backtest evaluation. We now present the results of other backtests often used in the literature for VaR evaluation:

Table 3 - Backtests comparison ($\tau^* = 99\%$)

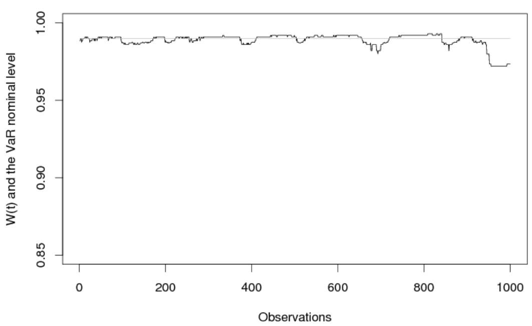
	CAViaR	GARCH	RM	HS1m	HS3m
% of hits	0.9	1.2	1.1	5.6	2.5
ζ_{Kupiec}	0.74884	0.53556	0.75198	0.00000 (**)	0.00000 (**)
$\zeta_{\text{Christ.}}$	0.87539	0.71333	0.84163	0.00000 (**)	0.00017 (**)
ζ_{DQ}	0.97173	0.94656	0.13848	0.00000 (**)	0.00000 (**)
ζ_{VQR}	0.66581	0.81968	0.00049 (**)	0.00000 (**)	0.00264 (**)

Notes: P-values are shown in the ζ's rows; (**) means rejection at 1%.

Note that the GARCH(1,1)-VaR model provides a quite good VaR measure, according to all considered backtests, despite its simplicity and the assumption of normality. Overall, the results are similar to those obtained from the VQR test, excepting the RiskMetrics model. The results of Table 3 indicate that RiskMetrics is only rejected by the VQR test, which is compatible[23] with the previous results of the Monte Carlo simulation (see Figure 2b, $T = 1,000$). In other words, our methodology is able to reject more VaR models in comparison to other backtests, which might be a major advantage of our approach. In fact, recall that the VQR test has more power in some directions of the alternative hypothesis, as described in the power investigation of section 3. The main reason could be that the other backtests are all based on a hit sequence, ignoring the respective magnitude of violations, which is properly considered in the quantile regression setup.

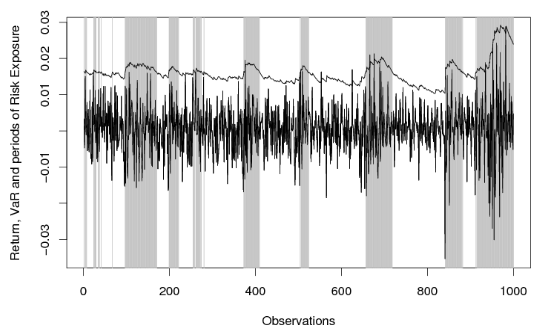
As a result of our proposed methodology, we are also able to construct the W_t series, described in section 2.1, in order to reveal the periods of risk exposure. Recall that whenever W_t is below the benchmark level τ^*, the VaR model increases the risk exposure by underestimating the related conditional quantile of returns, since (ideally) W_t should be as close as possible to τ^*. To illustrate the methodology, the estimated W_t series as well as the periods of risk exposure for the RiskMetrics-VaR(99%) model are depicted in Figures 5 and 6, where the gray bars indicate periods in which $W_t < \tau^*$.

Figure 5 - W_t (RiskMetrics-VaR 99%)



Notes: a) The black series is the computed W_t;
$W_t \equiv \{\widetilde{\tau} \in [0;1] \mid V_t = \widehat{Q}_{R_t}(\widetilde{\tau} \mid \mathcal{F}_{t-1})\}$

Figure 6 - R_t and V_t (RiskMetrics-VaR 99%)



Note: Gray bars indicate $W_t < \tau^*$;

In other words, gray bars suggest periods in which the VaR measure underestimates the risk exposure. Since the RiskMetrics-VaR(99%) model is rejected by the VQR test, the risk exposure periods could be very useful for risk managers interested in improving the accuracy of the underlying model. For instance, a visual inspection on figure 6 indicates that the RiskMetrics model usually

[23] Also note that the rank of the DQ test in the power curves of Figure 2b ($T = 1,000$) is not exactly the same as the rank of p-values in Table 3 (RM column). One possible explanation is that the power curves are size-adjusted, and the DQ test (see Table 1, $\tau^* = 99\%$, $T = 1,000$) is oversized whereas the Kupiec and Christoffersen backtests are undersized.

underestimates (gray bars) the degree of risk for high volatility periods. Therefore, we are able to unmask the bad performance of the RiskMetrics model in our empirical exercise based on a local behavior analysis, which brings some additional (and important) information to the backtest investigation by exposing some "reasons of rejection". Note that this local behavior investigation could only be conducted through our proposed quantile regression methodology, which we believe to be a novelty in the backtest literature.

Other relevant issue regarding VaR evaluation is the comparison among several competing models. Although it is not the main objective of this paper, we outline (for the sake of completion of our empirical exercise) a simple nonparametric decision rule for model selection and apply it to our empirical exercise (see Giacomini and Komunjer (2005) for a detailed discussion of model comparison). We are, thus, concerned with relative evaluation, which involves comparing the performance of competing models and choosing the one that performs the best according to our suggested criterion of section 2.1. The main results are next summarized:

Table 4 - Loss Function $L(V_t)$ for $\tau^* = 99\%$

CAViaR	GARCH	RM	HS1m	HS3m
0.00320	0.00497	0.00560	0.06843	0.02099

Notes: a) Recall that $C_t \equiv |W_t - \tau^*|$; $I_t = \begin{cases} 1 \ ; \text{ if } W_t > \tau^* \\ 0 \ ; \text{ if } W_t \leq \tau^* \end{cases}$;

and $L(V_t) \equiv \frac{1}{T}\sum_{t=1}^{T} C_t * (\gamma_1 I_t + \gamma_2(1 - I_t))$;

b) We adopted $\gamma_1 = 1.0$ and $\gamma_2 = 1.5$.

Based on this procedure, one should choose the model in which W_t best tracks the desired τ^* level, according to the asymmetric weights γ_1 and γ_2.In our exercise, the CAViaR model exhibits the best performance (i.e., lowest value of $L(V_t)$), which is a natural result, given that it is exactly designed to produce Value-at-Risk measures, whereas the other discussed VaRs are only obtained from conditional volatility models together with the assumption of normality. Therefore, the proposed methodology to identify periods of risk exposure could be used to increase the performance of a poor VaR model, whereas, the suggested $L(V_t)$ distance could be applied to rank and select among competing models.

5 Conclusions

Backtesting could prove very helpful in assessing Value-at-Risk models and is nowadays a key component for both regulators and risk managers. Since the first procedures suggested by Kupiec (1995) and Christoffersen (1998), a lot of research has been done in the search for adequate methodologies to assess and help improve the performance of VaRs, which (preferable) do not require the knowledge of the underlying model.

As noted by the Basle Committee (1996), the magnitude as well as the number of exceptions of a VaR model is a matter of concern. The so-called "conditional coverage" tests indirectly investigate the VaR accuracy, based on a "filtering" of a serially correlated and heteroskedastic time series (V_t) into a serially independent sequence of indicator functions (hit sequence H_t). Thus, the standard procedure in the literature is to verify whether the hit sequence is iid. However, an important piece of information might be lost in that process: not only is the sequence of past hits that matters, but also the magnitude of H_t is of vital importance, since the conditional distribution of returns is dynamically updated. This issue is also discussed by Campbell (2005), which states that the reported quantile provides a quantitative and continuous measure of the magnitude of realized profits and losses, while the hit indicator only signals whether a particular threshold was exceeded. In this sense, the author suggests that quantile tests can provide additional power to detect an inaccurate risk model.

That is exactly the objective of this paper: to provide a VaR-backtest fully based on a quantile regression framework. Our proposed methodology enables us to: (i) formally conduct a Wald-type hypothesis test to evaluate the performance of VaR; and (ii) identify periods of an increased risk exposure. We illustrate the usefulness of our setup through an empirical exercise with daily S&P500 returns, in which we constructed five competing VaR models and evaluate them through our proposed test (and through other three backtests). In addition, we also suggest a simple nonparametric procedure to rank the competing models.

Since a Value-at-Risk model is implicitly defined as a conditional quantile function, the quantile approach provides a natural environment to study and investigate VaRs. One of the advantages of our approach is the increased power of the suggested quantile regression-backtest in comparison to some established backtests in the literature, as suggested by a small Monte Carlo simulation. Perhaps most importantly, our backtest is applicable under a wide variety of structures, since it does not depend on the underlying VaR model, covering either cases where the VaR comes from a conditional volatility model, or it is directly constructed (e.g., CAViaR or ARCH-quantile methods) without relying on a conditional volatility model. We also introduce a main innovation: based on the quantile estimation, one can also identify periods in which the VaR model might increase the risk exposure, which is a key issue to improve the risk model, and probably a novelty in the literature. A final advantage is that our approach can easily be computed through standard quantile regression softwares.

Although the proposed methodology have several appealing properties, it should be viewed as complementary rather than competing with the existing approaches, due to the limitations of the quantile regression technique discussed along this paper. Furthermore, several important topics remain for future research, such as: (i) time aggregation: how to compute and properly evaluate a 10-day regulatory VaR? Risk models constructed through QAR (Quantile Autoregressive) technique can be quite promising due to the possibility of recursively generation of multiperiod density forecast (see Koenker and Xiao (2006b)); (ii) Our randomness approach of VaR also deserves an extended treatment and leaves room for weaker conditions; (iii) multivariate VaR: although the extension of the analysis for the multivariate quantile regression is not straightforward, several proposals have already been suggested in the literature (e.g., Chaudhuri (1996) and Laine (2001)); (iv) inclusion of other variables to increase the power of VQR test in other directions; (v) improvement of the BIS formula for market required capital; among many others.

According to the Basel Committee (2006), new approaches to backtesting are still being developed and discussed within the broader risk management community. At present, different banks perform different types of backtesting comparisons, and the standards of interpretation also differ somewhat across banks. Active efforts to improve and refine the methods currently in use are underway, with the goal of distinguishing more sharply between accurate and inaccurate risk models. We aim to contribute to the current debate by providing a quantile technique that can be useful as a valuable diagnostic tool, as well as a mean to search for possible model improvements.

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Appendix A. Proofs of Propositions

Proof of Proposition 1. By Assumption (1), $\alpha_i(U_t)$ are increasing functions of the iid standard uniform random variable U_t and, thus, $Q_{\alpha_i(U_t)} = \alpha_i(Q_{U_t}) = \alpha_i(\tau)$, since for any monotone increasing function g and a standard uniform random variable, U, we have $Q_{g(U)}(\tau) = g(Q_U(\tau)) = g(\tau)$, where $Q_U(\tau) = \tau$ is the quantile function of U_t. By comonotonicity, we have that $Q_{\sum_{i=1}^{p}\alpha_i(U_t)} = \sum_{i=1}^{p} Q_{\alpha_i(U_t)}$. This way, by also considering assumption (1), we guarantee that the conditional quantile function is monotone increasing in τ, which is a crucial property of Value-at-Risk models. In other words, we have that $Q_{R_t}(\tau_1 \mid \mathcal{F}_{t-1}) < Q_{R_t}(\tau_2 \mid \mathcal{F}_{t-1})$ for all $\tau_1 < \tau_2 \in (0;1)$. Assumptions (2)-(4) are regularity conditions necessary to define the asymptotic covariance matrix, and a continuous conditional quantile function, needed for the CLT (7) of Koenker (2005, Theorem 4.1). A sketch of the proof of this CLT, via a Bahadur representation, is also presented in Hendricks and Koenker (1992, Appendix). Given that we established the conditions for the CLT (7), our proof is concluded by using standard results on quadratic forms: For a given random variable $z \sim N(\mu, \Sigma)$ it follows that $(z-\mu)'\Sigma^{-1}(z-\mu) \sim \chi^2_r$ where $r = rank(\Sigma)$. See Johnson and Kotz (1970, p. 150) and White (1984, Theorem 4.31) for further details. ■

Lemma 1 *Consider two independent random variables X and Y. If X has a continuous pdf and $Y \sim N(0,1)$, then,* $\Pr(X > y \cap Y = y) = \Pr(X > 0)$*.*

Proof. Initially define the following events $(A): X > 0$; $(B): Y > 0$; $(C): X > Y$. Thus, our objective is to show that $\Pr(C) = \Pr(A)$. Firstly, note that $\Pr(C) = \Pr(A \cap C) + \Pr(A^c \cap C)$ and $\Pr(A) = \Pr(A \cap B) + \Pr(A \cap B^c)$. Moreover, $\Pr(C) = [\Pr(A \cap B \cap C) + \Pr(A \cap B^c \cap C)] + [\Pr(A^c \cap B \cap C) + \Pr(A^c \cap B^c \cap C)]$ and $\Pr(A) = [\Pr(A \cap B \cap C) + \Pr(A \cap B \cap C^c)] + [\Pr(A \cap B^c \cap C) + \Pr(A \cap B^c \cap C^c)]$. This way, $\Pr(C) - \Pr(A) = \Pr(A^c \cap B \cap C) + \Pr(A^c \cap B^c \cap C) - \Pr(A \cap B \cap C^c) - \Pr(A \cap B^c \cap C^c)$. Since $\Pr(A^c \cap B \cap C) = \Pr(A \cap B^c \cap C^c) = 0$, by construction, it follows that $\Pr(C) - \Pr(A) = \Pr(A^c \cap B^c \cap C) - \Pr(A \cap B \cap C^c)$. However, since Y has zero mean with a symmetric pdf, it follows that $\Pr(Y > x) = \Pr(Y < -x)$, where $x \in \mathbb{R}^+$. In other words, for any $X = x \in \mathbb{R}$ we have that $\Pr(Y > X \cap X, Y > 0) = \Pr(Y < X \cap X, Y < 0)$. Therefore, $\Pr(C) - \Pr(A) = 0$. ■

Proof of Proposition 2. (i) $P1 \Leftrightarrow S1$ Assume that the nominal quantile level of the VaR model is τ^*, i.e., $\Pr[R_t \le V_t | \mathcal{F}_{t-1}] = \tau^*$. If assumptions (1)-(4) hold, then, it follows that

$Q_{R_t}(\tau \mid \mathcal{F}_{t-1}) = \inf\{R_t : F(R_t \mid \mathcal{F}_{t-1}) \geq \tau\}$ and, thus, $\Pr(R_t \leq Q_{R_t}(\tau \mid \mathcal{F}_{t-1}) \mid \mathcal{F}_{t-1}) = \tau$. In particular, for $\tau = \tau^*$, we have that $\Pr(R_t \leq Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1}) \mid \mathcal{F}_{t-1}) = \tau^*$. Therefore, it follows that $\tau^* = \Pr(R_t \leq V_t \mid \mathcal{F}_{t-1}) = \Pr(R_t \leq Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1}) \mid \mathcal{F}_{t-1}) \Leftrightarrow V_t = Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$.

(iia) $S1 \Rightarrow S2$ From the definition of H_t, it follows that $E(H_t \mid \mathcal{F}_{t-1}) = 1 * \Pr(R_t > V_t \mid \mathcal{F}_{t-1}) + 0 * \Pr(R_t \leq V_t \mid \mathcal{F}_{t-1}) = \Pr(R_t > V_t \mid \mathcal{F}_{t-1}) = \Pr(R_t > Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1}) \mid \mathcal{F}_{t-1})$, where the last equality is due to S1. This way, $E(H_t \mid \mathcal{F}_{t-1}) = 1 - \Pr(R_t \leq Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1}) \mid \mathcal{F}_{t-1}) = 1 - \tau^* = \tau^{**}$ based on the definition of the conditional quantile function. Therefore, $E(H_t - \tau^{**} \mid \mathcal{F}_{t-1}) = 0$.

(iib) $S1 \Rightarrow S3$ From the previous item, it follows that $S1 \Rightarrow S2$. Following Berkowitz et al. (2006), the martingale difference hypothesis ($S2$) naturally implies that the demeaned violation sequence is uncorrelated at all leads and lags. More specifically, the violation sequence has a first-order autocorrelation of zero, which is exploited by the Markov test of Christoffersen (1998). In other words, $S2 \Rightarrow S3$ and, therefore, $S1 \Rightarrow S3$. In addition, note that $E(H_t^2 \mid \mathcal{F}_{t-1}) = \Pr(R_t > V_t \mid \mathcal{F}_{t-1}) = \tau^{**}$ and $Var(H_t \mid \mathcal{F}_{t-1}) = E(H_t^2 \mid \mathcal{F}_{t-1}) - [E(H_t \mid \mathcal{F}_{t-1})]^2 = \tau^{**} - (\tau^{**})^2 = \tau^{**}(1-\tau^{**})$. Therefore, the random variable H_t follows a Bernoulli (τ^{**}) distribution.

(iic) $S1 \Rightarrow S4$ From item (iia), it follows that $S1 \Rightarrow S2$. Applying the law of iterated expectations on $S2$, it follows that $E(H_t) = \tau^{**}$.

(iiia) $S2 \nRightarrow S1$ Consider the following VaR model $V_t = Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1}) + \eta_t$, where $\eta_t \sim iid$ $N(0,1)$, inspired by Engle & Manganelli (2004), which argue that: *"any noise introduced into the quantile estimate will change the conditional probability of a hit given the estimate itself"*. Firstly, note that $E(H_t \mid \mathcal{F}_{t-1}) = 1 * \Pr(R_t > V_t \mid \mathcal{F}_{t-1}) + 0 * \Pr(R_t \leq V_t \mid \mathcal{F}_{t-1}) = \Pr(R_t > V_t \mid \mathcal{F}_{t-1}) = \Pr(R_t > (Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1}) + \eta_t) \mid \mathcal{F}_{t-1})$. Now, apply Lemma 1 by defining $X = R_t - Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$ and $Y = \eta_t$. Thus, $\Pr(R_t > V_t \mid \mathcal{F}_{t-1}) = \Pr(R_t > Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1}) \mid \mathcal{F}_{t-1}) = 1 - \tau^* = \tau^{**}$, based on the definition of the conditional quantile function. This way, $E(H_t \mid \mathcal{F}_{t-1}) = \tau^{**}$ and $Var(H_t \mid \mathcal{F}_{t-1}) = \tau^{**}(1 - \tau^{**})$. Therefore, the considered VaR model V_t satisfies $S2$. On the other hand, by definition, V_t clearly does not satisfy $S1$, since $V_t \neq Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$.

(iiib) $S3 \nRightarrow S1$ Based on the same example of item (iiia), it follows that $E(H_t - \tau^{**} \mid \mathcal{F}_{t-1}) = 0$ and, thus, $E(H_t - \tau^{**})(H_{t-1} - \tau^{**}) = 0$, i.e., $S3$ holds, whereas, $S1$ does not hold by construction.

(iiic) $S4 \nRightarrow S1$ From $S4$, we have that $E(H_t) = \tau^{**}$, which is not sufficient to guarantee that $E(H_t \mid \mathcal{F}_{t-1}) = \tau^{**}$ neither $\Pr(R_t > V_t \mid \mathcal{F}_{t-1}) = \tau^{**}$, i.e. $\Pr(R_t \leq V_t \mid \mathcal{F}_{t-1}) = \tau^*$, or $V_t = Q_{R_t}(\tau^* \mid \mathcal{F}_{t-1})$. ■

Appendix B. Regulatory Framework

The Basle Accord, also known as the 1988 Bank of International Settlements (BIS) Accord, established international guidelines that linked bank's capital requirements to their credit exposures. The "1996 Amendment" extended the initial Accord to include risk-based capital requirements for the market risks that banks incur in their trading accounts, officially consecrating the use of internal models based on Value-at-Risk methodologies to assess market risk exposure. The fact that banks were required to hold capital to face market risk associated with their trading positions intends to create incentives for them to develop their own internal VaR models. The advantage for the banks using an internal model should be a substantial reduction in regulatory capital. The current regulatory framework uses a so-called "traffic-light" approach for the daily market required capital (MRC_t), which is calculated in the following way:

$$MRC_t = \max(V_t; \frac{k}{60}\sum_{i=0}^{59} V_{t-i}) + SRC_t, \tag{16}$$

where V_t is the daily global VaR calculated for the 99% one sided significance level, over a 10-day forecast horizon, SRC_t is a specific risk charge (for the portfolio's idiosyncratic risk), and k represents a multiplicative factor applied to the average VaR and depends on the backtesting results, as it follows:

Table 5 - Multiplier (k) based on the number of exceptions (N)

"Traffic-light"	N	k
Green Zone	4 or fewer	3.00
Yellow Zone	5	3.40
Yellow Zone	6	3.50
Yellow Zone	7	3.65
Yellow Zone	8	3.75
Yellow Zone	9	3.85
Red Zone	10 or more	4.00

where N is the number of violations of V_t in the previous one year of historical data (250 trading days).[24] The k factor can be set by individual supervisory authorities on the basis of

[24] According to Crouhy et al. (2001), when being employed in relation to regulatory requirements, backtests must compare daily VaR forecasts against two measures of the profit & loss (P&L) results: (i) The actual net trading P&L for the next day; (ii) The theoretical P&L, also called "static P&L", that would have occurred if the position at the close of the previous day had been carried forward to the next day, i.e., the revenue that would have been realized had the bank's positions remained the same throughout the next day. The main reason is that VaR measures should not be compared against actual trading outcomes,

their assessment of the quality of the bank's risk management system, directly related to the ex-post performance of the model, thereby introducing a built-in positive incentive to maintain the predictive quality of the model.

According to the Basle Committee (2006), it is with the statistical limitations of "backtesting" in mind that the Basle Committee introduced a framework for the supervisory interpretation of backtesting results that encompasses a range of possible responses, depending on the strength of the signal generated from the backtest. These responses are classified into three zones, distinguished by colours into a hierarchy of responses. The green zone corresponds to backtesting results that do not themselves suggest a problem with the quality or accuracy of a bank's model. The yellow zone encompasses results that do raise questions in this regard, but where such a conclusion is not definitive. In this case, the penalty is up to the supervisor, depending to the reason for the violation. The red zone indicates a backtesting result that almost certainly indicates a problem with a bank's risk model. Mr. Tommaso Padoa-Schioppa, former chairman of the Basle Committee (apud Jorion, 2007), argues that this system is *"designed to reward truthful internal monitoring, as well as developing sound risk management systems."*

Furthermore, regulators accept that it is the nature of the modern banking world that institutions will use different assumptions and modeling techniques (see Crouhy et al. (2001)). The regulators take account of this for their own purposes by requiring institutions to scale up the VaR number derived from the internal model by a k factor, which can be viewed as "insurance" against model misspecification or can also be regarded as a safety factor against "non-normal market moves". In the same line, Jorion (2007) argues that k also accounts for additional risks not modeled by the usual applications of VaR. According to the author, studies of portfolios based on historical data, reporting the performance of the MRC_t during the turbulence of 1998, have shown that while 99% VaR is often exceeded, a multiplier of 3 provides adequate protection against extreme losses. Jorion (2007) also provides a very interesting (possible) rationale for the multiplicative

since the actual outcomes would inevitably be "contaminated" by changes in portfolio composition during the holding period. In addition, the inclusion of fee income together with trading gains and losses resulting from changes in the composition of the portfolio should also not be included in the definition of the trading outcome because they do not relate to the risk inherent in the static portfolio that was assumed in constructing the value-at-risk measure. Since this fee income is not typically included in the calculation of the risk measure, problems with the risk measurement model could be masked by including fee income in the definition of the trading outcome used for backtesting purposes. For these reasons, Supervisors will have national discretion to require banks to perform backtesting on either hypothetical (i.e. using changes in portfolio value that would occur were end-of-day positions to remain unchanged), or actual trading (i.e. excluding fees, commissions, and net interest income) outcomes, or both.

k factor, due to Stahl (1997), based on the Chebyshev's inequality.[25]

However, there are several critiques to the k multiplier in the literature. For instance, Danielsson et al. (1998) argue that current VaR regulation may, perversely, provide incentives for banks to underestimate VaR as much as possible. The ISDA/LIBA 1996 Joint Models task force (apud Crouhy et al., 2001) considers that a multiplier of any size is an unfair penalty on banks that are already sophisticated in the design of their own risk management system. In addition, ISDA also argues that an arbitrarily high scaling factor may even provide perverse incentives to abandon initiatives to implement prudent modifications of the internal model.

In this sense, Berkowitz and O'Brien (2002) report too few violations of actual VaRs in the U.S., indicating overly conservative models for six large commercial banks. These results are quite surprising because they imply that the market risk charges are too high. Recall that a poor VaR specification might lead to a higher capital requirement, which provides an incentive for the banks to improve their internal risk models. However, the capital requirement might not be a binding condition, since the capital that U.S. banks currently hold is above the regulatory capital. Another potential explanation is the existence of incentives for no violations: banks could prefer to report higher VaR numbers to avoid the possibility of regulatory intrusion.

Another important issue regarding the regulatory framework is the "square-root-of-time rule". The current regulatory framework requires that financial institutions use their own internal risk models to calculate and report their 99% VaR over a 10-day horizon.[26] In practice, however, banks are allowed (during an initial phase of the implementation of the internal model) to compute their 10-day ahead VaR by scaling up their 1-day VaR by $\sqrt{10}$, i.e., banks may use VaR numbers calculated according to shorter holding periods scaled up to ten days by the square root of time.

If we assume that returns are $iid \sim N(\mu, \sigma^2)$, then, the 10-day return is also normally distributed with mean 10μ and variance $10\sigma^2$. Thus, it follows that $V_t^{(10-day)} = \sqrt{10}V_t^{(1-day)}$. It is well known

[25] The main idea is to generate a robust upper limit to the VaR when the model is misspecified. Let x be a random variable with expected value μ and finite variance σ^2. Then for any real number $r > 0$ it follows that $\Pr(\,|x-\mu| > r\sigma) \leq \frac{1}{r^2}$. By assuming a symmetric distribution, we have that $\Pr((x-\mu) < -r\sigma) \leq \frac{1}{2r^2}$. Now, set the desired confidence level $\tau^* = 1\%$ on the right side of the previous expression in order to obtain the respective value of r, i.e., provided that $1/2r^2 = 0.01 \therefore r = 7.071$. Thus, last expression becomes $\Pr((x-\mu) < -7.071\sigma) \leq 0.01$, where the maximum VaR measure is $V_t^{\max} = 7.071\sigma$. Say that the bank report its VaR model using a normal distribution, we have that $V_t = 2.326\sigma$. If the true distribution is misspecified, the correction factor is then $k = \frac{V_t^{\max}}{V_t} = \frac{7.071\sigma}{2.326\sigma} = 3.03$, which is an attempt to justify the correction factor adopted by the Basel Committee.

[26] The 10-day holding period means that regulators are asking banks to consider that they might not be able to liquidate their positions for a 2-week period.

that the self-additivity of normal distributions implies the $\sqrt{T}$ scaling factor for multiperiod VaR. However, for heavy-tailed distributions this factor can be different for the largest risks. Danielsson and de Vries (2000) argue that the appropriate method for scaling up a single day VaR to a multiperiod VaR is the "alpha-root rule", where alpha is the number of finite bounded moments, also known as the tail index. According to the authors, heavy tailed distributions are self–additive in the tails, implying a scaling factor $T^{1/\alpha}$. Danielsson and Zigrand (2005) argue that the "square-root-of-time rule" could lead to a systematic underestimation of risk. See also Taylor (1999), which proposed a procedure to estimate a conditional quantile model over the next n periods, and Chen (2001) for a forecasting multiperiod VaR based on a quantile regressions.

A final remark is about the backtest implicitly incorporated into the BIS formulation. Campbell (2005) notes that the k multiplier is solely determined by the number of hits in the past 250 trading days in the same manner as the Kupiec (1995) test. This way, the market capital requirement can be interpreted as an unconditional coverage test that mandates a larger market risk capital set-aside as the evidence that the VaR model under consideration is misspecified. Jorion (2007) argues that regulators operate under different constraints from financial institutions and, since they do not have access to every component of the models, the approach is at a broader level. However, a serious caveat of the Kupiec (1995) test is the difficulty to detect VaR models that systematically under report risk (low power) in sample sizes consistent with the regulatory framework (i.e., $T = 250$). According to Jorion (2007), the lack of power of this framework is due to the choice of the high VaR confidence level (99%) that generates too few exceptions for a reliable test.[27]

A second drawback is that unconditional coverage tests may fail to detect VaR models with adequate unconditional coverage, but with dependent VaR violations, in which case the independence test is recommended. According to Campbell (2005), the unconditional coverage and independence (no clustering) properties are separate and distinct, and must (both) be satisfied by an accurate VaR model. In this paper, we also showed that these two conditions are necessary but not sufficient conditions for a desirable VaR measure (Proposition 2), due to the limited information contained in the hit sequence that ignores the respective magnitude of violations. This issued is also discussed by Campbell (2005), which states that the reported quantile provides a quantitative and continuous measure of the magnitude of realized profits and losses while the hit indicator only signals whether a particular threshold was exceeded. In this sense, the author suggests that quantile tests can

[27] Note that there are many combinations of confidence level, the horizon and the multiplicative factor that would yield the same capital charge MRC_t. This way, some suggestions to increase the power of the backtest, already pointed out by Jorion (2007, p.150-151), are to increase the number of observations T from 250 to 1,000 or decrease the confidence level from 99% to 95%.

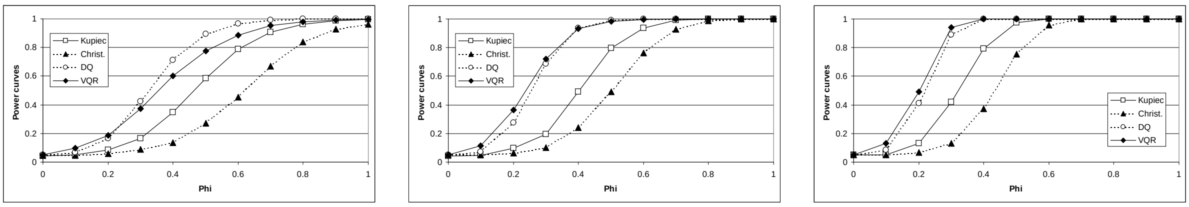
provide additional power to detect an inaccurate risk model, which is exactly the idea we discussed throughout this paper.

Jorion (2007) says that capital requirements will evolve automatically at the same speed as risk measurement techniques. According to the Basle Committee (2006), the essence of all backtesting efforts is the comparison of actual trading results with model-generated risk measures. If this comparison is close enough, the backtest raises no issues regarding the quality of the risk measurement model. In some cases, however, the comparison uncovers sufficient differences that problems almost certainly must exist, either with the model or with the assumptions of the backtest. In between these two cases there is a gray area where the test results are, on their own, inconclusive. Based on a quantile regression framework, we try to contribute to the debate inside the "gray area".

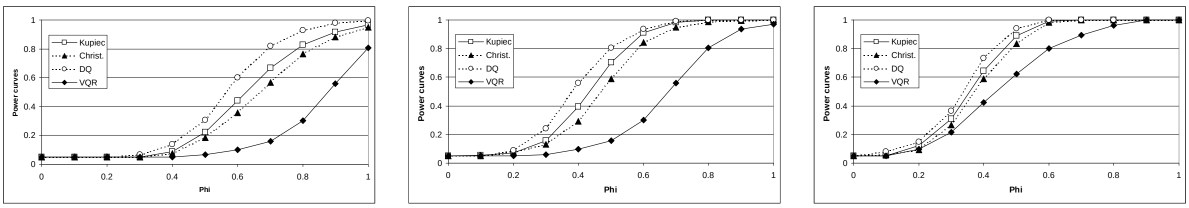
Appendix C. Monte Carlo simulation

Figure 7 - Size-corrected Power Curves - Method 2

$T = 250$; $\tau^* = 95\%$ | $T = 500$; $\tau^* = 95\%$ | $T = 1,000$; $\tau^* = 95\%$



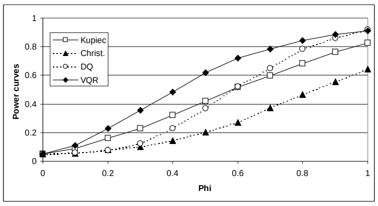
$T = 250$; $\tau^* = 99\%$ | $T = 500$; $\tau^* = 99\%$ | $T = 1,000$; $\tau^* = 99\%$



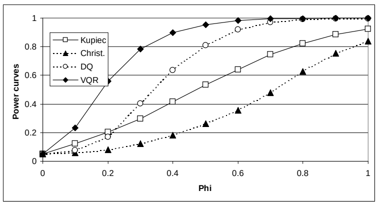
Note: Nominal level of significance is $\alpha = 5\%$.

Figure 8 - Size-corrected Power Curves - Method 3

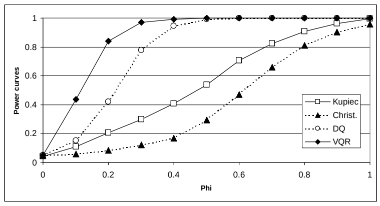
$T = 250$; $\tau^* = 95\%$



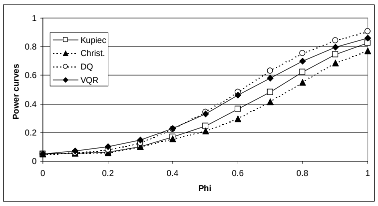
$T = 500$; $\tau^* = 95\%$



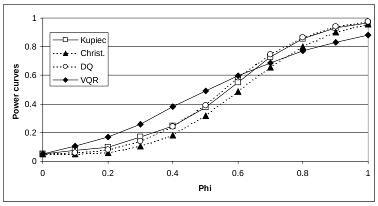
$T = 1,000$; $\tau^* = 95\%$



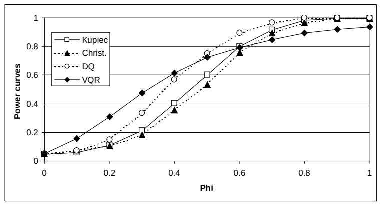
$T = 250$; $\tau^* = 99\%$



$T = 500$; $\tau^* = 99\%$



$T = 1,000$; $\tau^* = 99\%$

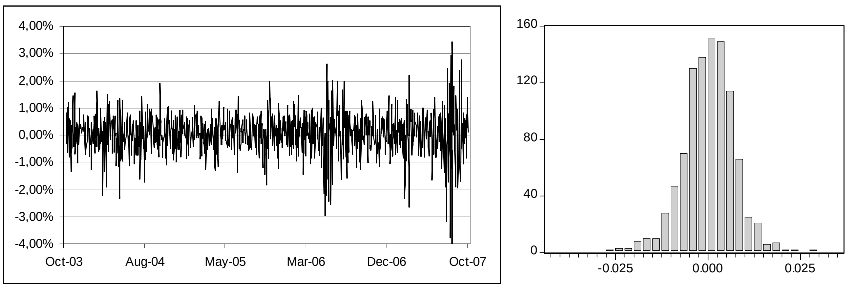


Note: Nominal level of significance is $\alpha = 5\%$.

Appendix D. Empirical exercise - other datasets

In this section, we present further results of our empirical exercise for two additional datasets: (i) The FTSE100 index from the United Kingdom; and (ii) the IBOVESPA index from Brazil. We investigate daily returns over the last 4 years, with an amount of $T = 1,000$ observations, following the same procedure[28] presented in section 4.

Figure 9 - FTSE 100 (UK) daily returns (%)



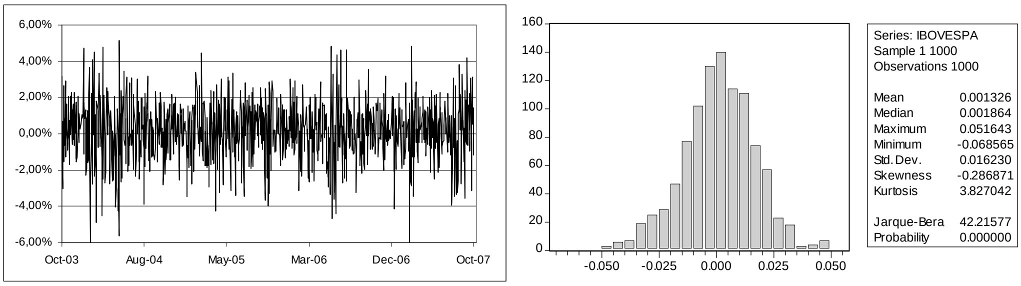
Series: FTSE100	
Sample 1 1000	
Observations 1000	
Mean	0.000456
Median	0.000841
Maximum	0.034441
Minimum	-0.041850
Std. Dev.	0.007587
Skewness	-0.442815
Kurtosis	6.116434
Jarque-Bera	437.3542
Probability	0.000000

Notes: a) The sample covers the period from 30/10/2003 until 12/10/2007;

b) Source: Yahoo!Finance.

[28] The following GARCH (1,1) models were estimated (EViews) for the FTSE index: $\sigma_t^2 = 2.72E-06 + 0.100860 y_{t-1}^2 + 0.846919 \sigma_{t-1}^2$, and for the Ibovespa index: $\sigma_t^2 = 1.21E-05 + 0.051264 y_{t-1}^2 + 0.902895 \sigma_{t-1}^2$.

Figure 10 - Ibovespa (Brazil) daily returns (%)



Notes: a) The sample covers the period from 02/10/2003 until 11/10/2007;

b) Source: Yahoo!Finance.

Table 6 - Backtests comparison - **FTSE - UK** ($\tau^* = 99\%$)

	CAViaR	GARCH	RM	HS1m	HS3m
% of hits	1.2	0.8	0.9	5.6	2.0
ζ_{Kupiec}	0.53557	0.51213	0.74883	0.00000 (**)	0.00509 (**)
$\zeta_{\text{Christ.}}$	0.71332	0.75621	0.87539	0.00000 (**)	0.01415 (*)
ζ_{DQ}	0.96288	0.96306	0.77637	0.00000 (**)	7.67e-06 (**)
ζ_{VQR}	0.90754	0.03584 (*)	0.22325	1.72e-06 (**)	0.02276 (*)

Notes: P-values are shown in the ζ's rows; (**) means rejection at 1%;

and (*) means rejection at 5%.

Table 7 - Backtests comparison - **Ibovespa - Brazil** ($\tau^* = 99\%$)

	CAViaR	GARCH	RM	HS1m	HS3m
% of hits	0.9	0.9	1.1	6.9	2.5
ζ_{Kupiec}	0.74883	0.74883	0.75198	0.00000 (**)	0.00006 (**)
$\zeta_{\text{Christ.}}$	0.87540	0.87538	0.84163	0.00000 (**)	0.00012 (**)
ζ_{DQ}	0.99867	0.99852	0.49423	0.00000 (**)	0.00000 (**)
ζ_{VQR}	0.98365	0.94018	0.10785	0.00095 (**)	0.00132 (**)

Notes: P-values are shown in the ζ's rows; (**) means rejection at 1%.

Table 8 - Loss Function $L(V_t)$ for $\tau^* = 99\%$

	CAViaR	GARCH	RM	HS1m	HS3m
FTSE (UK)	0.00236	0.00241	0.00413	0.06491	0.01285
Ibovespa (Brazil)	0.00210	0.00174	0.00492	0.09220	0.02059

Notes: a) Recall that $C_t \equiv |W_t - \tau^*|$; $I_t = \begin{cases} 1 & ; \text{ if } W_t > \tau^* \\ 0 & ; \text{ if } W_t \leq \tau^* \end{cases}$;

and $L(V_t) \equiv \frac{1}{T}\sum_{t=1}^{T} C_t * (\gamma_1 I_t + \gamma_2(1 - I_t))$;

b) We adopted $\gamma_1 = 1.0$ and $\gamma_2 = 1.5$.

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