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Does Curvature Enhance Forecasting? *

Caio Almeida[†] Romeu Gomes [‡] André Leite [§]
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Abstract

In this paper, we analyze the importance of curvature term struture movements on forecasts of interest rate means. An extension of the exponential three-factor Diebold and Li (2006) model is proposed, where a fourth factor captures a second type of curvature. The new factor increases model ability to generate more volatile and non-linear yield curves, leading to a significant improvement of forecasting ability, in special for short-term maturities. A forecasting experiment adopting Brazilian term structure data on Interbank Deposits (IDs) generates statistically significant lower bias and Root Mean Square Errors (RMSE) for the double curvature model, for most examined maturities, under three different forecasting horizons. Consistent with recent empirical analysis of bond risk premium, when a second curvature is included, despite explaining only a small portion of interest rate variability, it changes the structure of model risk premium leading to better predictions of bond excess returns.

Keywords: Parametric Term Structure Models, Principal Components, Vector Autoregressive Models, Interest Rate Mean Forecasting.

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1 Introduction

Understanding the evolution of the term structure of interest rates is important for a variety of reasons. Portfolio managers will adopt yield curve models for allocation purposes, while risk managers and macroeconomists will extract term structure movements to mimick their behavior or use them for monetary policy purposes. For each specific application, innumerous statistical procedures, parametric models, and dynamic arbitrage-free models are available[1], and one has to be criative, and sometimes pragmatic to identify the perfect matching between model and application.

In particular, forecasting interest rate means is an issue that has recently attracted the attention of researchers. Predictability questions raised by Fama and Bliss (1987) have recently been revisited through the lens of dynamic term structure models in Duffee (2002) and Dai and Singleton (2002). In a different strand, Ang and Piazzesi (2003) analyzed a Gaussian affine model with macroeconomic variables and showed that macro variables contribute to a better forecasting of the yield curve dynamics. In a simple parametric latent factor setting, Diebold and Li (2006) (DL, hereafter) proposed to forecast interest rate means with a variation of the Nelson and Siegel (1987) model, parameterizing the term structure as a sum of three basic movements: level, slope and curvature. They extract time-series for those movements to forecast the future evolution of the whole term structure, and their model quickly became a benchmark on forecasting exercises[2]. Although parameterizing the yield curve evolution by a sum of movements is not a particularly new topic[3], their model outperforms a variety of reliable candidates including principal components, and the random walk, indicating that the specific choice of parametric functions matters on forecasting problems.

Despite the fact that based on the seminal work by Litterman and Scheinkman (1991) most authors adopt three-factor term structure models[4], Cochra-

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[1] To cite a few, McCulloch (1971) presented a cubic splines model to estimate a cross-sectional term structure; Vasicek (1977) proposed one of the first affine dynamic term structure models, a one-factor gaussian model; Litterman and Scheinkman (1991) adopted principal component analysis to extract term structure movements; Heath, Jarrow and Morton (1992) proposed a general theory for arbitrage-free dynamic models; Duffie and Kan (1996) proposed affine multi-factor models; Ahn et al. (2002) proposed quadratic term structure models, one of the most recently developed multi-factor dynamic models.

[2] See, for instance, Almeida and Vicente (2007), Bowsher and Meeks (2006), Huse (2007), and Kargin and Onatski (2007) for comparisons of the DL method to other forecasting methods.

[3] See, for instance Litterman and Scheinkman (1991) and Almeida et al. (2003).

[4] Some exceptions include Svenson (1994), Fan et al. (2003), Bester (2004), Collin Dufresne et al. (2006), and Han (2007).

ne and Piazzesi (2005) show that the fourth principal component of the U.S. zero coupon curve is responsible for explaining a large portion of bond return predictability. By performing regressions of bond excess returns on forward rates, they obtain a tent-shaped factor common to bonds of different maturities, which predicts bond returns with high R2's, of the order of 40%. They further show that the fourth principal component is responsible for explaining more than 20% of return predictability captured by this specific tent-shaped factor. Moreover, they stress the fact that this fourth factor is usually neglected by the literature in dynamic term structure models because it usually explains only a tiny portion of the variability of in-sample (contemporaneous) interest rate movements.

In this paper, motivated by those results provided by Cochrane and Piazzesi (2005), we extend the DL model to incorporate a fourth factor driving a second type of curvature [5]. The role of this new factor is to improve model ability in capturing more volatile and non-linearly changing yield curves, and from a principal component perspective, to capture the dynamics of the usually neglected fourth principal component[6]. This second curvature factor alters the dynamics of the original slope and curvature factors and therefore alters the bond risk premia structure of the model, a fundamental component to improve forecasting ability.

A forecasting exercise adopting high frequency (daily) Brazilian fixed income data indicates that the new model outperforms the DL model on both bias and Root Mean Square Errors (RMSE) criteria, for most maturities, and on three different forecasting horizons (1-day, 1- and 3- month). Results are confirmed to be statistically significant with Diebold and Mariano (1995) tests under a quadratic loss function. Although it might appear to be natural that a more complex model will obtain better results than a simpler model, this is usually true only when fitting in sample data. When dealing with out-of-sample data, due to possible in-sample overfitting problems, more complex models might not be able to capture the correct dynamics of the observed phenomenon, and might end up achieving worse forecasting results[7]. In this sense, the results obtained in this work indicate that curvature, or more generaly, more complex movements of the term structure

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[5] Other extensions of the DL model include Fontaine and Garcia (2007), and Huse (2007). Fontaine and Garcia (2007) include an extra liquidity factor on the model. Huse (2007) maps the three DL extracted term structure movements into observable macroeconomic variables.

[6] In a static setting, the proposed model is equivalent to Svenson's (1994) model, which is an extention of the Nelson and Siegel (1987) model, containing two curvature factors.

[7] In addition, the larger the number of the parameters in a model, the higher the chances of having identification problems.

should be seriously considered as important elements for a better identification of bond risk premia, a point that reinforces the results advocated by Cochrane and Piazzesi (2005)[8]. In fact, by looking at Figure 3 and Table 1, which present respectively principal component loadings and eingenvalues decomposition for U.S. and Brazilian zero-coupon data, one can observe that despite shorter in maturities, the Brazilian term structure presents curvature factors with much higher importance than the corresponding U.S. ones[9].

The paper is organized as follows. Sections 2 and 3 respectively present the DL and the Exponential Double Curvature (EDC) models, explaining how to estimate and forecast with those models. Section 4 presents empirical results: the dataset is explained, model estimation results are presented, and a forecasting exercise is performed. Section 5 offers some concluding comments and possible topics for future research.

2 The Diebold and Li Model (DL Model)

DL modified the exponential model proposed by Nelson and Siegel (1987), considering the following parametric form for the term structure evolving through time:

$$R_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) \tag{1}$$

Despite proposing a time-varying decay parameter (λ_t), DL fixed its value at $\lambda_t = \lambda = 0.0609\ \forall t$, to maximize the curvature of the term structure at 30 months (a medium term factor for the U.S. term structure). On the current work λ is set equal to 3.58, maximizing the curvature loadings at a maturity of 6.8 months[10]. Figure 1 presents the loadings of the three movements captured by the model. The dashed line represents the loadings of the level factor. A shock on variable β_1 changes yields for all maturities τ in the same direction. The solid line represents the loadings of the slope factor. A positive shock to β_2 increases short-term yields approximately preserving long term yields the same. The dotted line captures the loadings of the curvature factor. A

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[8] For a deeper analysis of bond risk premium in dynamic models, see Cochrane and Piazzesi (2006), who construct an affine model consistent with the stylized facts observed in Cochrane and Piazzesi (2005).

[9] U.S. zero coupon data appears in monthly frequency ranging from 1985 to 2000. Brazilian zero coupon data appears in daily frequency ranging from 2004 to 2006.

[10] We express time to maturity in years while DL express time to maturity in months. Then to compare our lambda to DL lambda is necessary to multiply the DL lambda by twelve.

positive shock to β_3 primarily makes medium-term yields to go up, preserving the two extremes of the curve approximately the same.

2.1 Model Estimation Procedure

The model is estimated in a two-step procedure, which clearly depends on specific assumptions about the process λ. Choosing the structure of the time series of λ is a difficult problem. Section 2.3 provides a discussion on the choice of the parameter λ.

In principle, if λ is not fixed, the first step consists of running cross-section non-linear regressions were observed yields are linear combination of the three proposed movements plus an error term

$$R_t(\tau)_{observed} = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) + \epsilon_t(\tau). \quad (2)$$

On the other hand, in their work, DL suggest keeping λ as a fixed value. In this case, instead of running a sequency of cross-section non-linear regressions, there will be linear regressions relating yields to β parameters, and for each fixed date the parameters (movements) can be obtained by minimizing the sum of squared residuals of these cross-section linear regressions

$$\hat{\beta}_t = argmin\left(\sum_{j=1}^{N_t}\epsilon_t(\tau_j,\lambda)^2\right), \quad (3)$$

where $\hat{\beta}_t$ represents a vector with stacked betas, N_t represents the number of observed yields for date t, τ_j is the time to maturity of the j_{th} yield on that same date, and λ is fixed at a constant value.

This first step generates a time series for each of the term structure movements that were implied by the observed term structure data, and minimize the sum of squared residuals, for each independent subset of cross section yields.

In a second step, univariate autoregressive time series models are fitted to those three term structure movements. For $i = 1, 2, 3$, one fit

$$\hat{\beta}_{it} = c_i + \phi_i\hat{\beta}_{it-1} + \eta_{it}, \quad (4)$$

where c_i is a constant, ϕ_i is a number, and η_i is a univariate zero mean gaussian error.

In their work, DL also experiment with vector autoregressive (VAR) models but identify that, for the particular period of the U.S. term structure analized in their paper, the independent univariate autoregressive processes are better forecasters than the vector autoregressive model. In contrast with their results, considering the Brazilian term structure analyzed in this work, the VAR version of the model presents superior forecasting results when compared to the univariate version.

The VAR is fitted by

$$\hat{\beta}_t = c_{DL} + \phi_{DL}\hat{\beta}_{t-1} + \bar{\eta}_t, \tag{5}$$

where c is a 3×1 vector of constants, ϕ_{DL} is a 3×3 matrix, and $\bar{\eta}$ is a multivariate zero mean gaussian error, with a free correlation structure, not necessarily the identity matrix.

2.2 Forecasting

Under the univariate model, for each specific movement ($i = 1, 2, 3$), forecasts for its conditional mean is produced by

$$\hat{\hat{\beta}}_{it} = E_{t-1}\left[\hat{\beta}_{it}\right] = \hat{c_i} + \hat{\phi_i}\hat{\beta}_{it-1}, \tag{6}$$

where $E_{t-1}[\cdot]$ denotes conditional expectation with information set at time $t-1$. Once knowing the conditional forecasts for each movement, for any fixed maturity, model implied yield forecasts can be easily produced with the use of

$$E_{t-1}\left[R_t(\tau)\right] = \hat{\hat{\beta}}_{1t} + \hat{\hat{\beta}}_{2t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \hat{\hat{\beta}}_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right). \tag{7}$$

Similarly, for the VAR multivariate model, forecasts for the conditional means of all movements are jointly produced by

$$\hat{\hat{\beta}}_t = E_{t-1}\left[\hat{\beta}_t\right] = \hat{c_{DL}} + \hat{\phi_{DL}}\hat{\beta}_{t-1}. \tag{8}$$

For longer horizon forecasts or multi-step forecasts, there are two alternatives that might be adopted: Estimate the model with the original data frequency and produce multi-step forecasts, or estimate the model by regressing movements at time t (β_{it}'s) on movements at time $t-h$ ($\beta_{i(t-h)}$'s), were h is the number of time slots within each particular forecasting horizon. For instance, if one is interested in one-month horizon forecasts, and is using daily data to estimate the model, a regression of factors on their 21-day lagged values should be performed. DL suggest this last method as the optimal one when the purpose is to minimize the RMSE. Following their suggestion, the lagged-values method is adopted, but forecasts with the multi-step method were also produced, are available upon request, and do not change the qualitative results of this paper.

2.3 Choice of λ

This is a very important and difficult issue to solve. How should one choose the λ process? Should it be a stochastic process like the betas, a deterministic process, or simply a constant value for all dates? In their work, DL decided for the last and simpler solution, to fix it to a constant value, advocating in favor of simplicity and parcimoniousness[11]. In this work, the value of λ is also kept fixed, but there is a difference in the procedure that defines how the value of λ is chosen.

DL argue that historically the curvature has been linked to changes of medium term yields, and that usually 2- and 3- year yields were used to represent medium term yields. For this reason, they decided to choose λ to maximize the curvature loadings at the average of these two maturities, that is, at 30 months. In this work, a more interesting and less arbitrary way to choose the fixed value for λ is adopted. The idea is to search for a value under which the DL model generate its best forecasting results. In this sense, if the new proposed model generate better results, it will happen under the best possible scenario for the DL model.

Following this idea, the initial time series of observed yields was divided in two sets, the “in-sample” one, composed by 300 daily observations ranging from November of 2004 to December of 2005, and the “out-of-sample” set, composed by 234 observations ranging from January of 2006 to December of 2006. A large grid of values for λ was produced, and for each λ, time series of term structure movements were estimated, and vector autoregressive models were estimated based on in-sample data. For each fixed out-of-sample date, the vector autoregressive models generated one-day ahead forecasts for yields for all maturities, and those forecasts were compared to the true observed ID yields. The value for λ that minimized the RMSE for 1-step ahead forecasts on the Brazilian term structure of IDs was $\lambda = 3.58$. As explained before, this value maximizes the curvature loadings at a maturity of 6.8 months.

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[11] Note that if λ varies along time this will imply a change on the loadings of the slope and curvature factors and the procedure will not be exactly consistent with a raw application of principal component analysis as done by Litterman and Scheinkman (1991). However, there might be cases were indeed dynamic loadings will better capture the dynamics of certain term structures.

3 The Exponential Double Curvature Model (EDC Model)

In this section we show how to extend the DL model to incorporate a fourth factor, which represents a second type of curvature. In this case, the term structure will evolve along time according to the following equation:

$$\begin{aligned} R_t(\tau) = \beta_{1t} + \beta_{2t}\left(\tfrac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3t}\left(\tfrac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda\tau}\right) + \\ +\beta_{4t}\left(\tfrac{1-e^{-\tilde{\lambda}_t\tau}}{\tilde{\lambda}_t\tau} - e^{-\tilde{\lambda}_t\tau}\right). \end{aligned} \tag{9}$$

Note that the first three movements are exactly the ones that appear in the DL model. The fourth term is a copy of the third one, with a different λ, though. We argue that this subtle change will be very important to model term structures of interest rates that are more volatile than the U.S. curve. This will generally be the case for emerging markets curves, corporate bonds curves, and credit derivatives markets, indicating that this small extension might potentially produce a huge gain in forecasting abilities. Figure 2 presents the loadings of those four movements when the fixed value for $\lambda = 3.58$ maximizes the first curvature loadings at 6.8 months, and the fixed value for $\tilde{\lambda} = 7.16$ maximizes the second curvature loadings at 3.4 months. It will be observed in the empirical section that having an even shorter-term curvature than the first one (at 6.8 months) will be very important to improve short-term in-sample fitting and also out-of-sample forecasting ability.

3.1 Model Estimation Procedure

Similarly to DL, the model is estimated in a two-step procedure. The cross-section regressions are implemented writing the observed yields as a linear combination of the four proposed movements plus an error term:

$$\begin{aligned} R_t(\tau)_{observed} = \beta_{1t} + \beta_{2t}\left(\tfrac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\tfrac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \\ +\beta_{4t}\left(\tfrac{1-e^{-\tilde{\lambda}\tau}}{\tilde{\lambda}\tau} - e^{-\tilde{\lambda}\tau}\right) + \tilde{\epsilon}_t(\tau) \end{aligned} \quad . \tag{10}$$

Note that λ, the parameter that defines the slope and first curvature decaying factor, is fixed to the same constant used on the version of the DL model adopted for the Brazilian term structure, $\lambda = 3.58$. To choose the parameter that defines the decaying factor of the second curvature $\tilde{\lambda}$ we run non-linear regression according to Equation 9 with $\lambda = 3.58$ for each day in the "in-sample" set obtaining a times series of $\tilde{\lambda}_t$. Then we fix $\tilde{\lambda}$ equals to the mean value of the $\tilde{\lambda}_t$'s. Figure 4 shows that the short-end of the Brazilian yield curve presents a large number of instruments and also a strong curvature effect. The specific value of $\tilde{\lambda} = 7.16$ captures this strong curvature effect, maximizing the second curvature at 3.4 months.

The β parameters are obtained, for each fixed date, through a minimization of the sum of squared residuals of the above cross-section regressions:

$$\hat{\beta}_t = argmin \left(\sum_{i=1}^{N_t} \tilde{\epsilon}_t(\tau_i, \lambda, \tilde{\lambda})^2 \right), \tag{11}$$

where $\hat{\beta}_t$ represents a vector with stacked betas, N_t represents the number of observed yields for date t, τ_i the time to maturity of the i_{th} yield on that same date, $\lambda = 3.58$, and $\tilde{\lambda} = 7.16$.

The first step produces time series for each of the term structure movements. In a second step, a Vector autoregressive time series model is fitted to those four term structure movements:

$$\hat{\beta}_t = c_{EDC} + \phi_{EDC}\hat{\beta}_{t-1} + \tilde{\eta}_t \tag{12}$$

As discussed before, while DL obtained better results with the use of univariate models instead of a VAR, their results were not confirmed for the Brazilian term structure data. Similarly, for the EDC model the VAR forecasting ability is higher than that of univariate autoregressive models.

3.2 Forecasting

Under the adopted VAR multivariate model, forecasts for the conditional means of all movements are jointly produced by:

$$\hat{\hat{\beta}}_t = E_{t-1}\left[\hat{\beta}_t\right] = \widehat{c_{EDC}} + \widehat{\phi_{EDC}}\hat{\beta}_{t-1}, \tag{13}$$

where $E_{t-1}[\cdot]$ denotes conditional expectation with information set at time $t-1$. Once knowing the conditional forecasts for each movement, for any fixed maturity, model implied yield forecasts can be easily produced with the use of:

$$\begin{gathered} E_{t-1}\left[R_t(\tau)\right] = \hat{\hat{\beta}}_{1t} + \hat{\hat{\beta}}_{2t}\left(\tfrac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \hat{\hat{\beta}}_{3t}\left(\tfrac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + \\ \hat{\hat{\beta}}_{4t}\left(\tfrac{1-e^{-\tilde{\lambda}\tau}}{\tilde{\lambda}\tau} - e^{-\tilde{\lambda}\tau}\right). \end{gathered} \tag{14}$$

For longer horizon forecasts, the same method proposed to DL and described in Section 2.2 of re-estimating the VAR for each forecasting horizon is adopted.

4 Empirical Results

4.1 Data

In this section, the Brazilian market of ID Futures and the dataset adopted are briefly described. For more detailed information on the products and available datasets see www.bmf.com.br/portal/portal_english.asp.

4.1.1 ID Futures

The one-day interbank deposit future contract (ID Future) with maturity T is a future contract whose underlying asset is the accumulated daily ID rates[12] capitalized between the trading time t ($t \leq T$) and T. The contract size corresponds to R$ 100,000.00 (one hundred thousand Brazilian Reals) discounted by the accumulated short-rate negociated between the buyer and the seller of the contract. If you buy an ID Future at a price $\overline{ID}$[13] at time t and hold it until the maturity T, your gain/loss is

$$100000 \cdot \left(\frac{\prod_{i=1}^{\zeta(t,T)} (1 + ID_i)^{(1/252)}}{\left(1 + \overline{ID}\right)^{\zeta(t,T)/252}} - 1 \right),$$

where ID_i denotes the ID rate $i - 1$ days after the trading time t. The function $\zeta(t, T)$ represents the numbers of days between times t and T.

This contract is very similar to a zero coupon bond, except that it pays margin adjustments every day. Each daily cash flow is the difference between the settlement price[14] on the current day and the settlement price on the day before corrected by the ID rate of the day before.

BM&F is the entity that offers the ID Future. The number of authorized contract-maturity months is fixed by BM&F (on average, there are about twenty authorized contract-maturity months for each day but only around ten are liquid). Contract-maturity months are the first four months subsequent

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[12] The ID rate is the average one-day interbank borrowing/lending rate, calculated by CETIP (Central of Custody and Financial Settlement of Securities) every workday. The ID rate is expressed in effective rate per annum, based on 252 business-days.

[13] The ID-Future is quoted in interest rate per annum based on 252 business days.

[14] The settlement price at time t of a ID Future with maturity T is equal to R$ 100,000.00 discounted by its closing price quotation.

to the month in which a trade has been made and, after that, the months that initiate each following quarter. Expiration date is the first business day of the contract-maturity month.

4.1.2 Database Adopted

Data consisted of 534 daily observations of ID Futures yields with average maturities of 0.05, 0.13, 0.22, 0.32, 0.51, 0.76, 1.01, 1.27, 1.55, 1.89, 2.18, 2.42, 2.59, 2.70 years. Those yields were observed between November of 2004 and December of 2006, and represent the most liquid IDs traded during those two years.

4.2 Model Fitting

Figure 4 presents four examples of in-sample fitting of the EDC model, for arbitrarely chosen moments. Note that the yield curve is inverted in two of them and twisted in the other two, demanding a very flexible curvature factor. Largest fitting errors are of the order of 10 basis points indicating that the model fits well the 14 observed points with the four parameters (that is, factors) that represent term structure movements.

Figures 5 and 6 present the time series of the term structure movements extracted adopting respectively the DL and the EDC models. Under both models, the level is the most stable movement oscillating around a long term mean of 0.17, while slope and curvature switch signs along time, both being primarily positive in the first half of the sample and negative in the second half. For the EDC model, the second curvature appears to be a mirror of the first curvature and indeed Table 3 confirms this fact. This table presents the coefficient of correlation between the time-series of any two movements extracted under the EDC model, and indicates a negative correlation of -0.92 between the two curvature factors. In addition, note that all the movements are highly correlated (except for level and slope) indicating that a VAR structure is more suited to capture the time series behavior of the four movements. Similarly Table 2 indicates that curvature is correlated to both level and slope under the DL model, also indicating that a VAR would be a process more adequate than univariate autoregressions to fit to the time series of those three movements together.

Figure 7 presents for the level, slope and first curvature, the distance between the time series obtained under the two models. It shows that once a second curvature is included in the model it changes the behavior of the previously extracted level, slope, and curvature movements. This effect is stronger for the curvature factor, but it also appears significant on the slope factor. When the second curvature factor is included it produces potentially two effects on the dynamics of the term structure: the first, an inclusion of a new movement, and the second, a change on all existing movements. Note that this effect is more complex than simply including a fourth principal component to capture a second type of curvature because by the orthogonality of the principal components there wouldn't be any change to the time series of the previous three principal components already used. In fact, the effect of including a second curvature factor has a disciplinary effect on the previously extracted term structure movements, providing higher ability to capture bond risk premia and consequently interest rate conditional means. Almeida and Vicente (2007) find that imposing no-arbitrage restrictions to a polynomial term structure model, induces the existence of conditionally deterministic movements for the term structure that increase model forecasting ability, when compared to a corresponding version that allows for arbitrages. They associate this improvement in model forecasting to a better ability of capturing bond risk premia. One shall observe with the proposed forecasting exercise that extending the DL model to include a second curvature term has a similar effect here.

4.3 Forecasting Exercise

A forecasting exercise is formulated by separating the sample in two parts, the estimation part, composed of the first 300 daily observations ranging from November of 2004 to December of 2005, and a second part used to access model ability to produce out-of-sample forecasts, ranging from January of 2006 to December of 2006. Forecasts are performed for three different forecasting horizons: one day, one month, and three months. For each horizon and for each out-of-sample observation, the two models were re-estimated. Bellow we present, for each model, the three ϕ-matrices representing the estimated VARs using only the in-sample set.

$$\phi_{DL}^{1} = \begin{bmatrix} 0.0101 & -0.0062 & -0.0250 \\ 0.9348 & 0.0411 & 0.1595 \\ -0.0159 & 0.9918 & 0.0378 \end{bmatrix}$$

$$\phi_{DL}^{21} = \begin{bmatrix} 0.1379 & -0.1379 & -0.0848 \\ 0.1297 & 0.8644 & 0.6524 \\ -0.1685 & 0.9502 & -0.5115 \end{bmatrix}$$

$$\phi_{DL}^{63} = \begin{bmatrix} 0.1751 & -0.1348 & 0.2745 \\ -0.0779 & 0.8544 & -1.2463 \\ -0.2264 & 0.4387 & -1.9761 \end{bmatrix}$$

$$\phi^{1}_{EDC} = \begin{bmatrix} 0.0103 & -0.0119 & -0.0423 & 0.0200 \\ 0.9335 & 0.0753 & 0.2646 & -0.1247 \\ -0.0173 & 1.0073 & 0.0760 & -0.0354 \\ 0.0096 & 0.0300 & 1.0366 & -0.1202 \end{bmatrix}$$

$$\phi^{21}_{EDC} = \begin{bmatrix} 0.1263 & -0.1152 & -0.0145 & -0.0956 \\ 0.1905 & 0.7499 & 0.3005 & 0.4665 \\ -0.1086 & 0.8320 & -0.8376 & 0.5164 \\ 0.1117 & 0.1043 & 1.1167 & -0.3345 \end{bmatrix}$$

$$\phi^{63}_{EDC} = \begin{bmatrix} 0.0847 & -0.1082 & 0.4651 & -0.3401 \\ 0.4030 & 0.7617 & -2.1702 & 1.6654 \\ -0.0221 & 0.2786 & -2.6404 & 1.2397 \\ 0.2303 & 0.2533 & 0.6727 & -0.1333 \end{bmatrix}$$

Note that those matrices change substantially across forecasting horizons, but for all matrices, there is an intense interaction among movements indicating that at least from an in-sample viewpoint the VAR is more appropriate to capture term structure dynamics than separate autoregressive equations for each movement. In addition, for all the presented VARs, all the roots from the characteristic polynomials lie within the unit circle, indicating that they satisfy conditions that guarantee achievement of stable forecasts.

Table 4 presents the bias for both models, for eight different chosen maturities and the three forecasting horizons[15]. Note that on 19 out of 24 values on the Table, the absolute value of bias is smaller under the EDC model. Models have smaller bias discrepancy for the 1-month forecasting horizon. On the other hand, for the 1-day and 3-month horizons they present very distinct bias behavior. For instance, for the 1-day forecasting horizon, for some particular maturities, the DL model presents a positive bias while the EDC model presents a negative bias, and for other maturities their biases switch signs. In particular, it is worth noting that for the shortest maturity (of 13 days) the DL presents higher bias and RMSE (see Table 5), indicating that this fourth curvature factor simultaneously decreases variability and slightly deslocates predictions on the short-end of the curve yields to the right direction. For longer forecasting horizons (1- and 3- month), both models underestimate yield movements presenting negative bias. In particular, for the 3-month horizon, the EDC model presents bias that are around 25%-30% smaller than those corresponding to the DL model.

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[15] Results for all other initially adopted maturities present similar interpretation and are available upon request.

Similarly, Table 5 presents the RMSE for both models. Note that for all table entries but one, the RMSEs of the EDC model are lower than those of the DL model. Moreover, for shorter maturities the differences are higher. Consider, for instance, the 0.32 maturity (around 4-month maturity): For all forecasting horizons the RMSEs of the DL model are more than 20% higher than the corresponding EDC RMSEs. For longer maturities, the differences are smaller but some are still significant, like the 145.4 basis points RMSE for the EDC model under the one year maturity against 160.2 for the DL model under the same maturity.

Table 6 presents Diebold and Mariano (1995) S1 and S2 (size corrected) statistics using a quadratic loss function. Positive values are in favor of the EDC model, where values higher than 1.96 indicate significance at a 95% confidence level and values higher than 2.57 indicate significance at a 99% confidence level. Note that 19 out of 24 values of the S1 statistics are higher than 1.96, indicating that most of the difference in bias and RMSE across models are significant at a 95% confidence interval. This is confirmed by the values of the S2 statistics which is robust to small samples. Similarly, 18 out of 24 values are significant under this statistics, in favor of the EDC model. On the other hand, for the longest maturity (2.7 years) the S2 statistics indicate that the DL model would be doing a better forecasting job under the 1-month forecasting horizon. This might be an effect of having the second curvature loadings centered on the short-end of the yield curve.

The three mentioned tables clearly evidence the superior performance of the model that considers a fourth term structure movement, consistent with the findings that higher order principal components of the yield curve might have an important job on capturing bond risk premium. In fact, other studies also indicate the importance of residual term structure terms in explaining derivatives movements (Heidari and Wu (2003)) and also interest rate movements themselves (Bali et al. (2007)).

5 Conclusion

In a recent paper, Cochrane and Piazzesi (2005) show the importance of the fourth principal component of zero coupon yields on predicting bond excess returns. They first find a certain robust tent-shaped return forecasting factor by running regressions of bond excess returns on forward rates, and show that the fourth principal component factor which explains only 0.02% of the variability of yields, explains more than 20% of bond risk premia captured by this return forecasting factor. Consistently with these results, the parametric three-factor exponential model of Diebold and Li (2006) is extended in this paper to contain a fourth factor, related to a second type of curvature, that significantly improves model forecasting ability by lowering both bias and RMSE on out-of-sample forecasts. An empirical exercise adopting high frequency (daily) fixed income data from the most liquid Brazilian fixed income market documents the superior performance of the new model. The proposed model, named Exponential Double Curvature Model, is equivalent to the Svenson (1994) model in a dynamic setting. It outperforms the Diebold and Li (2006) model for most maturities under three different forecasting horizons: a very short 1-day horizon, and 1- and 3- month horizons. The results presented in this paper confirm in a dynamic econometric setting the results provided by Cochrane and Piazzesi (2005), and suggest that this extended model should be adopted on forecasting exercises, specially on markets with more volatile yield curves, like emerging markets, corporate bond markets, and credit derivative markets. One possible suggestion for future work include testing more complex econometric systems, like substituting the two step estimation procedure by a one-step estimation adopting a kalman filter[16]. Another possibility would be to follow Almeida and Vicente (2007) considering a detailed analysis of bond risk premia structure implied across models.

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[16] See Diebold et al. (2006) for an example of Kalman filtering estimation of a term structure model in a macro-finance setting.

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Movement/ Yield Curve	Level	Slope	Curvature	Sec. Curv.	Total
Brazilian	91.08%	6.21%	1.57%	0.84%	99.70%
U.S.	92.08%	7.47%	0.32%	0.06%	99.92%

Table 1: Variation of the Term Structure Explained by the First Four Principal Components
This table presents the percent of variation explained by the first four principal components extracted from the U.S. and the Brazilian term structures of interest rates. The U.S. term structure is represented by yields of zero coupon treasury bonds with maturities up to 10 years, with monthly observations from 1985 to 2000. The Brazilian term structure is represented by ID Futures with average maturities up to 2.7 years, with daily observations from November of 2004 to December of 2006. The "total" column indicates the sum of the variance explained by the first four components, for each curve.

Movement	Level	Slope	Curvature
Level	1.00	-0.01	0.47
Slope	-0.01	1.00	0.67
Curvature	0.47	0.67	1.00

Table 2: Correlation Structure: Term Structure Movements from the DL Model
This table presents the coefficient of correlation between any two movements extracted using the DL three factor model. Movements come from daily Brazilian IDs term structure data ranging from November of 2004 to December of 2006.

Movement	Level	Slope	Curvature 1	Curvature 2
Level	1.00	0.04	0.39	-0.33
Slope	0.04	1.00	0.80	-0.77
Curvature 1	0.39	0.80	1.00	-0.92
Curvature 2	-0.33	-0.77	-0.92	1.00

Table 3: Correlation Structure: Term Structure Movements from the EDC Model
This table presents the coefficient of correlation between any two movements extracted using the EDC four factor model proposed in this paper. Movements come from daily Brazilian IDs term structure data ranging from November of 2004 to December of 2006.

Maturity	0.05	0.13	0.32	0.51	1.01	1.55	2.18	2.70
Model	**1-Day Ahead Forecast**							
DL	-5.6	1.2	3.5	-0.1	-3.8	-1.2	-1.3	-0.7
EDC	-2.2	-0.3	1.5	0.2	-2.2	-0.1	-0.8	-1.3
Model	**1-Month Ahead Forecast**							
DL	-6.5	-4.1	-7.5	-13.9	-21.3	-18.7	-17.4	-14.2
EDC	-5.9	-4.1	-7.9	-14.4	-20.8	-17.4	-15.6	-12.3
Model	**3-Month Ahead Forecast**							
DL	-57.0	-81.1	-110.5	-120.8	-114.1	-91.9	-74.0	-50.7
EDC	-48.3	-49.4	-75.1	-92.9	-94.7	-71.1	-50.6	-24.7

Table 4: Bias on Out-of-Sample Forecasts (in bps)
This table presents the bias for 1-day, 1-month and 6-month ahead out-of-sample forecasts obtained for the DL and the EDC models. Models were estimated in a two-step procedure, with cross-sectional independent regressions in the first step, and in the second step, VAR models to fit term structure movements in both models. Out-of-sample data ranges from January 2006 to December 2006, with a total of 234 observations.

Maturity	0.05	0.13	0.32	0.51	1.01	1.55	2.18	2.70
Model	**1-Day Ahead Forecast**							
DL	13.1	6.1	9.7	10.0	11.4	15.5	15.2	16.6
EDC	8.0	5.6	5.8	8.7	10.4	14.1	14.9	15.8
Model	**1-Month Ahead Forecast**							
DL	21.2	19.1	33.2	41.6	57.7	73.0	82.6	86.3
EDC	16.7	15.9	28.0	37.8	56.5	72.4	82.2	86.2
Model	**3-Month Ahead Forecast**							
DL	75.1	100.6	135.1	149.4	160.2	163.2	166.1	159.4
EDC	70.2	77.2	104.3	124.5	145.4	153.9	161.6	162.5

Table 5: RMSE for Out-of-Sample Forecasts (in bps)

This table presents RMSE for 1-day, 1-month and 6-month ahead out-of-sample forecasts obtained for the DL and the EDC models. Models were estimated in a two-step procedure, with cross-sectional independent regressions in the first step, and in the second step, VAR models to fit term structure movements in both models. Out-of-sample data ranges from January 2006 to December 2006, with a total of 234 observations.

Maturity	0.05	0.13	0.32	0.51	1.01	1.55	2.18	2.70
Model	**1-Day Ahead Forecast**							
S1	7.6	1.69	10.6	4.8	4.2	6.6	4.5	3.7
S2	7.1	2.7	9.2	3.4	5.1	7.6	4.8	0.7
Model	**1-Month Ahead Forecast**							
S1	6.6	4.7	7.2	5.6	2.4	1.8	1.4	0.5
S2	5.5	4.2	5.5	3.4	2.2	2.9	1.8	-4.0
Model	**3-Month Ahead Forecast**							
S1	2.7	7.5	7.7	6.8	5.1	3.8	2.0	-1.4
S2	-1.5	5.5	5.8	5.6	3.8	3.2	1.8	-0.8

Table 6: Out-of-Sample Statistical Significance of Difference in Forecasting Ability: Diebold and Mariano (1995) Tests
This table presents the Diebold and Mariano (1995) S1 and S2 size statistics for 1-day, 1-month and 3-month ahead out-of-sample forecasts. Comparisons are done as functions of Mean Squared Errors (MSE). Out-of-sample data ranges from January 2006 to December 2006, with a total of 234 observations. Positive values are in favor of the EDC model. Large values for $S1$ and $S2$ indicate high probability of rejecting the null hypothesis that the difference in Mean Square Errors is negligible. Absolute values larger than 1.96 indicate significance at a 95% confidence level, and larger than 2.57, indicate significance at a 99% level.

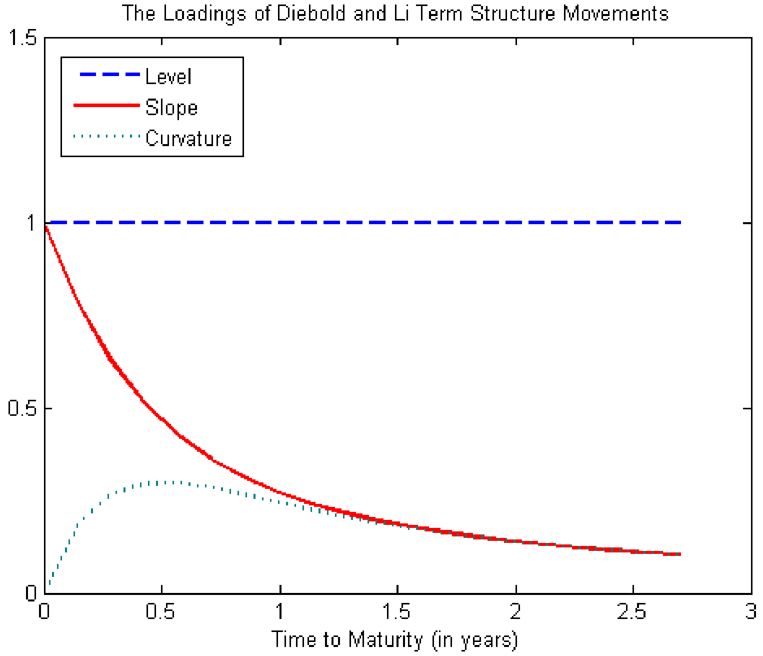

Figure 1: A Cross Section View of Term Structure Movements Under the DL Model
This picture presents the loadings of the level, slope and curvature movements under the DL model.

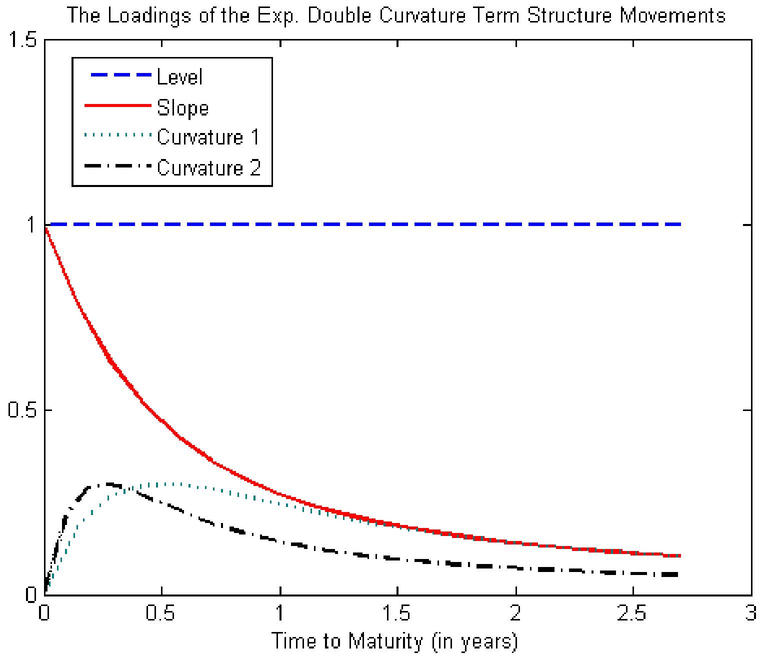

Figure 2: A Cross Section View of Term Structure Movements Under the EDC Model
This picture presents the loadings of the level, slope and curvature movements under the EDC model.

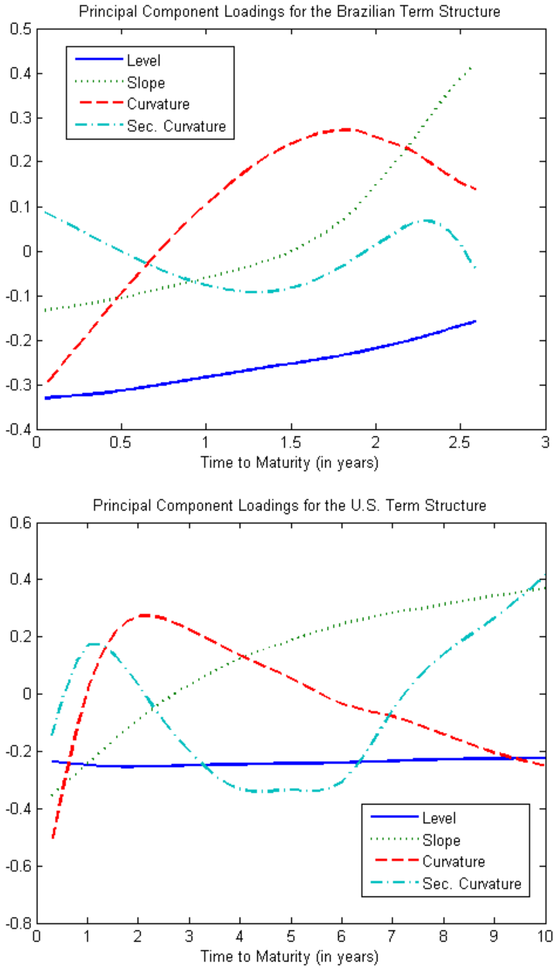

Figure 3: Principal Components Loadings
This picture presents the loadings of the level, slope and curvature movements obtained with an application of Principal Component Analysis to the Brazilian and the U.S. term structures of interest rates.

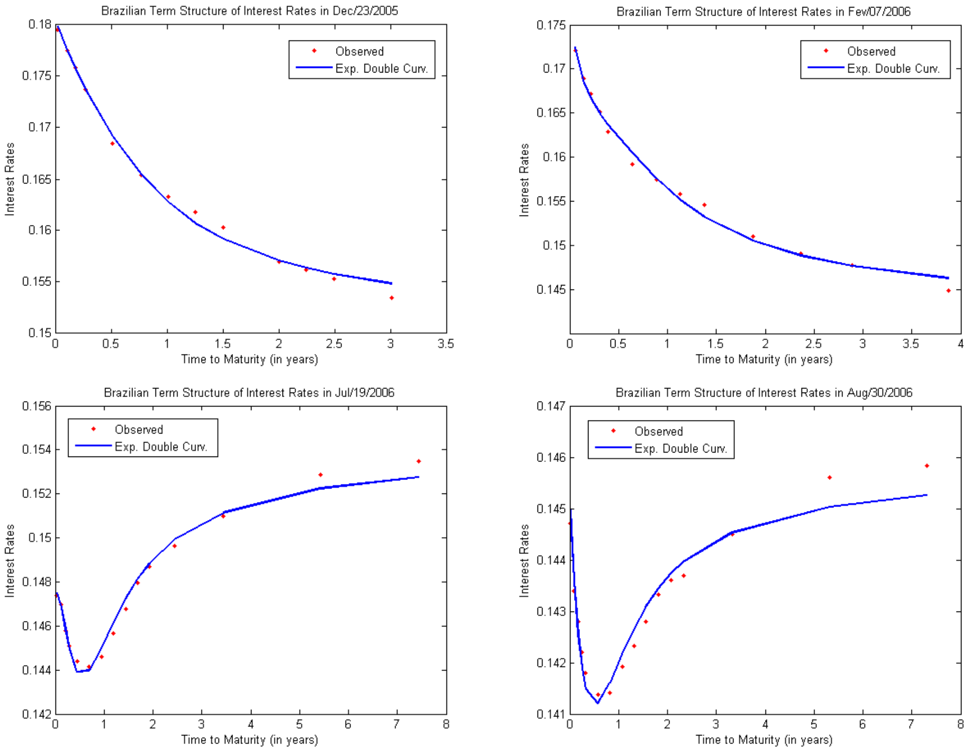

Figure 4: Some Pictures of Term Structure Cross Section Estimation

This picture presents observed and EDC model implied term structures for four different moments of the database.

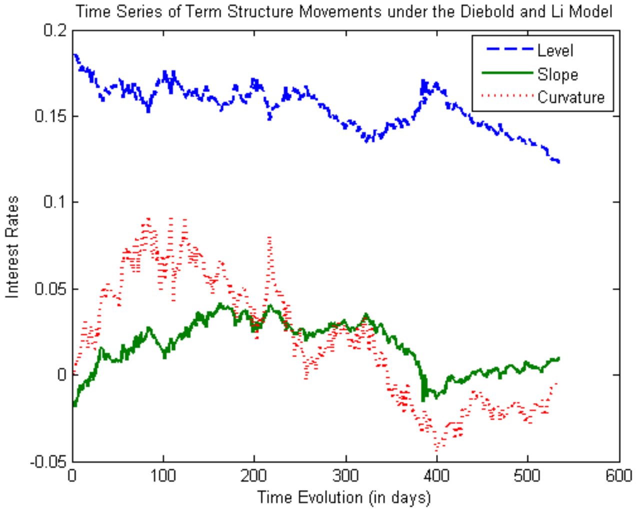

Figure 5: Term Structure Movements Under the DL Model
This picture presents the time series of the level, slope and curvature captured under the DL model.

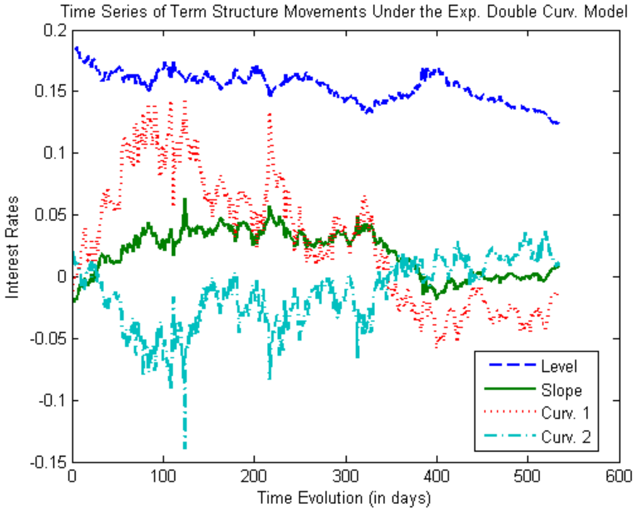

Figure 6: Term Structure Movements Under the EDC Model
This picture presents the time series of the level, slope, first and second curvatures captured by the EDC Model.

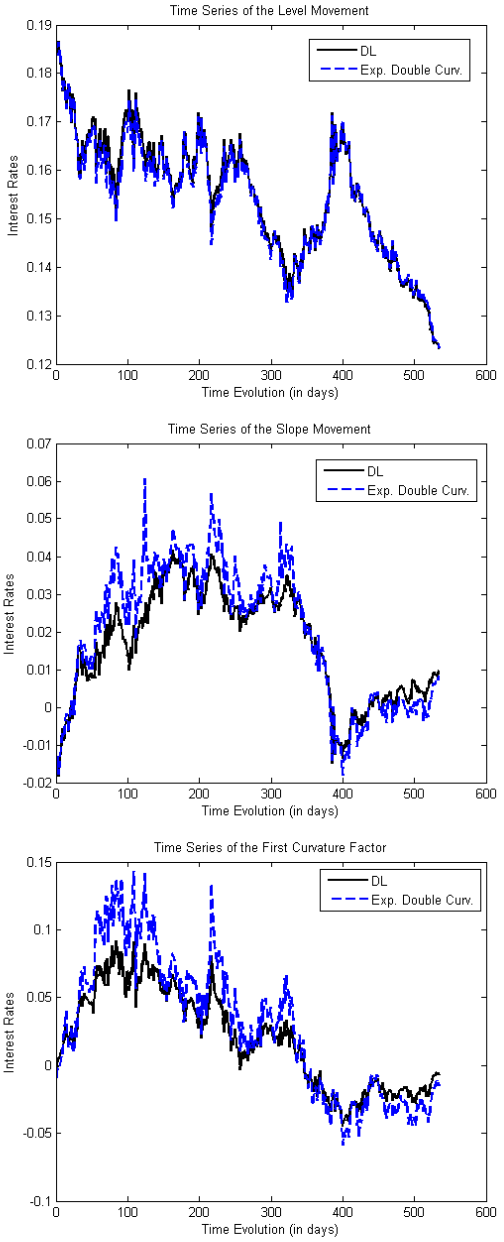

Figure 7: The Distance Between Movements Across Models
This picture presents the time series of the three main term structure movements extracted under the two adopted models.

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