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Term Structure Movements Implicit in Option Prices *

Caio Ibsen R. Almeida† José Valentim M. Vicente ‡

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Abstract

This paper analyzes how including options in the estimation of a dynamic term structure model impacts the way it captures term structure movements. Two versions of a multi-factor Gaussian model are compared: One adopting only bonds data, and the other adopting a joint dataset of bonds and options. Term structure movements extracted under each version behave distinctly, with slope and curvature presenting higher mean reversion rates when options are adopted. The composition of bond risk premium is also affected, with considerably more weight attributed to the level factor when options are included. The inclusion of options in the estimation of the dynamic model also improves the pricing of out-of-sample options.

Keywords: Dynamic Term Structure Models, Latent Factors, Bond Risk Premium, Interest Rates Option Pricing.
JEL Classification: C51, G12.

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1 Introduction

It is a established fact that options embed investor's expectations on different economic variables impacting prices of underlying securities[1]. In particular, fixed income options should be expected to affect market participants perception for the importance of each movement driving the term structure of interest rates[2]. Adopting a dynamic term structure model with multiple sources of uncertainty and a time varying market price of risk, this research addresses the question of how options affect the shape of those movements, as well as the importance of each movement on the pricing of bonds.

Based on closed-form formulas for bonds and asian option prices[3] (liquid options within the Brazilian fixed income market), two versions of a three-factor gaussian model are estimated by Maximum Likelihood: The first adopting only bonds data (bond version), and the other combining bonds and at-the-money fixed-maturity options data (option version). The main findings are that options affect basically three dimensions of the dynamic model: Types of term structure movements, bond risk premia decomposition, and dynamic first order hedging weights when hedging options.

Adopting options to estimate dynamic term structure models might be useful in different contexts, as shown by the following examples. Bikbov and Chernov (2004) use eurodollar options to economically discriminate among different affine models with stochastic volatility. Almeida et al. (2006) show that options are important to predict excess returns of long term U.S swaps. Graveline (2006) identifies that exchange rate options are useful to explain the forward premium anomaly, and Joslin (2006) statistically tests the existence of unspanned stochastic volatility

[1] See, for instance, Bakshi et al. (1997), Dumas et al. (1998), Bates (2000), Pan (2002), and Garcia et al. (2003), among others.

[2] See Litterman and Scheinkman (1991) for an application of Principal Component Analysis to the U.S Treasury term structure.

[3] For the pricing of fixed income asian options under one-dimensional affine models see Leblanc and Scaillet (1998), Cheuk and Vorst (1999) or Dassios and Nagaradjasarma (2003). Vicente and Almeida (2006) provide a methodology to efficiently price those options under general affine models.

(Collin Dufresne and Goldstein (2002)) adopting caps and swaptions on the estimation of affine models[4]. In contrast, this work is focused in the transformations that happen to the dynamic factors, and consequently to the stochastic discount factor and risk premium structures, once options are adopted.

Results in this paper show, for the particular database adopted, that the level is a robust factor common to both versions of the estimated model, while slope and curvature are less persistent under the option version of the model (see Figure 3). These movements present much higher mean reversion rates under the option version, indicating that while information contained in bonds and at-the-money options agree on the main factor driving term structure movements, the information implicit in those option prices suggest faster variations for the secondary movements of the term structure.

Bond risk premia is slightly less volatile on the option version, and is more concentrated on the level factor. For instance, while around 80% of the one-year premium is concentrated on the level factor under the option version, only 12% is due to the level factor under the bond version[5].

A comparison of the two estimated versions is also performed with respect to: Pricing of in-sample bonds, pricing of out-of-sample options, and delta-hedging of an at-the-money option[6]. Results indicate that the bond version better captures the term structure of bond yields, but is out-performed by the option version in the option pricing and hedging exercises. In general, whenever larger option mispricings occur, the bond version underestimates prices, while the option version overestimates them, as can be observed in Figures 8 and 9. From a hedging perspective, the bond version is only able to capture 5.10% price movements of the at-the-money option adopted, contrasted to a 94.74% fraction for the option version[7]. When analyzing

[4]For examples of other research works adopting joint datasets of underlying and option prices to estimate dynamic term structure models, see Longstaff et al. (2001), Umantsev (2002), and Han (2004).

[5]Note that although the loadings of the level factor coincide under the two versions, the time series of this factor are distinct, being slightly less volatile under the option-version (see Figure 4).

[6]Similar questions are addressed by Driessen et al. (2003), with the use of Heath et al. (1992) term structure models.

[7]Note that this was expected since the option version is perfectly pricing this option, and the

the dynamic hedging weights attributed to each factor under each version, it is clear that both versions give no importance to the curvature dynamic factor when hedging the at-the-money option, while level and slope weights are much more volatile under the option version of the model.

The paper is organized as follows. Section 2 describes the market of ID-futures (bonds), and IDI options. Section 3 presents the model, the pricing of zero-coupon bonds and IDI options, and first order dynamic hedging properties of such options. Section 4 describes and implements the estimation process under each version. Section 5 compares the two dynamic versions of the model considering the empirical dimensions described above. Section 6 concludes. Appendix A contains theoretical results on the pricing of fixed income instruments under the model. Appendix B presents a detailed description of the Maximum Likelihood estimation procedure adopted.

2 Data and Market Description

The following two subsections explain how ID-futures and IDI options work. For more details on these contracts see the Brazilian Mercantile & Future Exchange (BM&F) webpage[8]. Subsection 2.3 describes the data adopted in this work.

2.1 ID-Futures

The One-Day Interbank Deposit Future Contract (ID-future) with maturity T is a future contract whose underlying asset is the accumulated daily ID rates[9] capitalized between the trading time t ($t \leq T$) and T. The contract size corresponds to R$ 100,000.00 (one hundred thousand Brazilian Real) discounted by the accumulated rate negotiated between the buyer and the seller of the contract. Then, if one buys

4.79% variability of prices not captured in the delta-hedge is due to second order effects. The only reason to provide hedging results under the option version is to allow comparison of dynamic hedging weights across versions.

[8]http://www.bmf.com.br/indexenglish.asp

[9]The ID rate is the average one-day interbank borrowing/lending rate, calculated by CETIP (Central of Custody and Financial Settlement of Securities) every workday. The ID rate is expressed in effective rate per annum, based on 252 business days.

an ID-future at a price $\overline{ID}$ at time t and holds it until the maturity T, his gain/loss is

$$100000 \cdot \left(\frac{\prod_{i=1}^{\zeta(t,T)} (1 + ID_i)^{(1/252)}}{\left(1 + \overline{ID}\right)^{\zeta(t,T)/252}} - 1 \right),$$

where ID_i denotes the ID rate $i - 1$ days after the trading time t, and function $\zeta(t, T)$ represents the numbers of days between times t and T[10].

Apart from daily cash-flows exchanged between margin accounts, this contract behaves like a zero coupon bond, and a no-arbitrage argument combined with a swap fixed-floating rate makes it equivalent to a zero coupon for pricing purposes. Each daily cash flow is the difference between the settlement price[11] on the current day and the settlement price on the day before, corrected by the ID rate of the day before.

BM&F is the entity that offers ID-futures. The number of authorized contract-maturity months is fixed by BM&F (on average, there are about twenty authorized contract-maturity months within each day but only about ten are liquid). Contract-maturity months are the first four months subsequent to the month in which a trade has been made and, after that, the months that initiate each following quarter. Expiration date is the first business day of the contract-maturity month.

2.2 IDI and its Option Market

The IDI index is defined as the accumulated ID rate. Using the association between the short term rate r_t and the continuously-compounded ID rate, the IDI index can be written as the exponential of the accumulated short term interest rate

$$IDI_t = IDI_0 \cdot \mathrm{e}^{\int_0^t r_u \mathrm{d}u}. \tag{1}$$

This index has been fixed to the value of 100000 points in January 2, 1997. It has actually been resettled to its initial value most recently in January 2, 2003.

[10]Without any loss of generality, in this paper, the continuously-compounded ID rate is directly associated to the short term rate r_t. Then the gain/loss can be written as $100000 \cdot \left(\mathrm{e}^{\int_t^T (r_u - \overline{r})\mathrm{d}u} - 1\right)$, where $\overline{r} = \ln(1 + \overline{ID})$.

[11]The settlement price at time t of a ID-future with maturity T is equal to R$ 100,000.00 discounted by its closing price quotation.

An IDI option with time of maturity T is an European option whose underlying asset is the IDI and whose payoff depends on IDI_T. When the strike is K, the payoff of an IDI option is $L_c(T) = (IDI_T - K)^+$ for a call and $L_p(T) = (K - IDI_T)^+$ for a put.

BM&F is also the entity that offers the IDI option[12]. Strike prices (expressed in index points) and the number of authorized contract-maturity months are established by BM&F. Contract-maturity months can be any month, and the expiration date is the first business day of the maturity month. On average, there are about 30 authorized series[13] within each day for call options, but no more than ten call options series are liquid.

2.3 Data

Data consists on time series of ID-futures yields for all different liquid maturities, and prices of IDI options for different strikes and maturities, covering the period from January, 2003 to December, 2005.

BM&F maintains a daily historical database with prices and number of trades for all ID-futures and IDI options that have been traded within a day. Interest rates for zero coupon bonds with fixed maturities are estimated with a cubic interpolation scheme applied to the ID-futures dataset. On the estimation process of the Gaussian model, yields from bonds with fixed maturities of 1, 21, 63, 126, 189, 252 and 378 days are adopted[14].

Regarding options, two different databases are selected. The first, used on the estimation of the option version of the dynamic model, is composed by an at-the-money fixed-maturity IDI call[15], with time to maturity equal to 95 days[16]. The

[12]There is also considerable trading over-the-counter.

[13]A series is just a set of characteristics of the option contract, which determine its expiration date and strike price.

[14]There exist deals within this market with longer maturities (up to ten years) but the liquidity is considerably lower.

[15]Moneyness is defined by the ratio present value of strike over current IDI value.

[16]The at-the-money IDI call prices are obtained by an interpolation of Black implied volatilities in a similar procedure to that adopted to construct original VIX volatilities.

second is composed by picking up within each day the most liquid IDI call[17].

The first database containing options is used to estimate the dynamic model (option version), and the second is used to test the pricing performance of the two versions. As hedging can not be tested with the database on the most liquid IDI options because moneyness and/or maturity change through time, the hedging is performed using the at-the-money options of the first database[18].

After excluding weekends, holidays, and no-trade workdays, there exists a total of 748 daily observations of yields from zero coupon bonds, and option prices.

3 The Model

The uncertainty in the economy is characterized by a filtered probability space $(\Omega, (\mathcal{F}_t)_{t\geq 0}, \mathcal{F}, \mathbb{P})$. The existence of a pricing measure $\mathbb{Q}$ under which discounted bond prices are martingales is assumed, and the model is specified through the definition of the short term rate r_t as a sum of N Gaussian random variables:

$$r_t = \phi_0 + \sum_{i=1}^{N} X_t^i, \tag{2}$$

where the dynamics of process X is given by

$$\mathrm{d}X_t = -\kappa X_t \mathrm{d}t + \rho \mathrm{d}W_t^{\mathbb{Q}}, \tag{3}$$

with $W^{\mathbb{Q}}$ being an N-dimensional brownian motion under $\mathbb{Q}$, κ a diagonal matrix with κ_i in the i_{th} diagonal position, and ρ is a matrix responsible for correlation among the X factors. The connection between martingale probability measure $\mathbb{Q}$ and objective probability measure $\mathbb{P}$ is given by Girsanov's Theorem with an essentially affine (Duffee (2002))[19] market price of risk

$$\mathrm{d}W_t^{\mathbb{P}} = \mathrm{d}W_t^{\mathbb{Q}} - \lambda_X X_t \mathrm{d}t, \tag{4}$$

[17] Moneyness and time-to-maturity of liquid options are readily available upon request.

[18] In this case it should be clear that the option version will outperform the bond version since the first perfectly prices the at-the-money option. However, as explained in the empirical section the most interesting aspect of this hedging exercise is to compare the dynamic allocations provided to each term structure movement by each model.

[19] Constrained for admissibility purposes (see Dai and Singleton (2000)).

where λ_X is an $N \times N$ matrix and $W^{\mathbb{P}}$ is a brownian motion under $\mathbb{P}$.

Lemma 1 *Let $y(t,T) = \int_t^T r_u du$. Then, under measure $\mathbb{Q}$ and conditional on the sigma field $\mathcal{F}_t$, y is normally distributed with mean $M(t,T)$ and variance $V(t,T)$, where*

$$M(t,T) = \phi_0 \tau + \sum_{i=1}^{N} \frac{1 - \mathrm{e}^{-\kappa_i \tau}}{\kappa_i} X_t^i \tag{5}$$

and

$$\begin{aligned} V(t,T) = {} & \textstyle\sum_{i=1}^{N} \frac{1}{\kappa_i^2} \left(\tau + \frac{2}{\kappa_i} \mathrm{e}^{-\kappa_i \tau} - \frac{1}{2\kappa_i} \mathrm{e}^{-2\kappa_i \tau} - \frac{3}{2\kappa_i} \right) \sum_{j=1}^{N} \rho_{ij}^2 + \\ & + 2 \textstyle\sum_{i=1}^{N} \sum_{k>i} \frac{1}{\kappa_i \kappa_k} \left(\tau + \frac{\mathrm{e}^{-\kappa_i \tau} - 1}{\kappa_i} + \frac{\mathrm{e}^{-\kappa_k \tau} - 1}{\kappa_k} - \frac{\mathrm{e}^{-(\kappa_i + \kappa_k)\tau} - 1}{\kappa_i + \kappa_k} \right) \sum_{j=1}^{N} \rho_{ij} \rho_{kj}, \end{aligned} \tag{6}$$

where $\tau = T - t$.

Proof. See Appendix A. ■

3.1 Pricing Zero Coupon Bonds

Let $P(t,T)$ denote the time t price of a zero coupon bond maturing at time T, paying one monetary unit. It is known that Multi-factor Gaussian models offer closed-form formulas for zero coupon bond prices. The next lemma presents a simple proof of this fact for the particular model in hand.

Lemma 2 *The price at time t of a zero coupon bond maturing at time T is*

$$P(t,T) = \mathrm{e}^{A(\tau) + B(\tau)' X_t}, \tag{7}$$

where $A(\tau) = -\phi_0 \tau + \frac{1}{2} V(t,T)$ and $B(\tau)$ is a column vector with $-\frac{1 - \mathrm{e}^{-\kappa_i \tau}}{\kappa_i}$ as the i_{th} element.

Proof. See Appendix A. ■

Using Equation (7) and Itô's lemma one can obtain the dynamics of a bond price under the martingale measure $\mathbb{Q}$

$$\frac{\mathrm{d}P(t,T)}{P(t,T)} = r_t \mathrm{d}t + B(\tau)' \rho \mathrm{d}W_t^{\mathbb{Q}}. \tag{8}$$

To hold this bond, the investors will ask for an instantaneous expected excess return. Then, under the objective measure, the bond price dynamics is

$$\frac{\mathrm{d}P(t,T)}{P(t,T)} = (r_t + z^i(t,T))\mathrm{d}t + B(\tau)'\rho \mathrm{d}W_t^{\mathbb{P}}. \tag{9}$$

Applying Girsanov's Theorem to change measures the instantaneous premium is obtained as

$$z^i(t,T) = B(\tau)'\rho\lambda_X X_t. \tag{10}$$

3.2 Pricing IDI Options

IDI options, are continuous-time asian options, which have been priced before with the use of single factor term structure models[20]. This research generalizes those models by adopting multiple factors to drive the uncertainty of the yield curve, a usual practice since the work of Duffie and Kan (1996) and Dai and Singleton (2000)[21]. Option pricing is provided in what follows.

Denote by $c(t,T)$ the time t price of a call option on the IDI index, with time of maturity T, and strike price K, then

$$\begin{aligned} c(t,T) &= \mathrm{E}^{\mathbb{Q}}\left[\mathrm{e}^{-\int_t^T r_u \mathrm{d}u} \max(IDI_T - K, 0)|\mathcal{F}_t\right] = \\ &= \mathrm{E}^{\mathbb{Q}}\left[\max(IDI_t - K\mathrm{e}^{-y(t,T)}, 0)|\mathcal{F}_t\right]. \end{aligned} \tag{11}$$

Lemma 3 *The price at time t of the above mentioned option is*

$$c(t,T) = IDI_t\Phi(d) - KP(t,T)\Phi(d - \sqrt{V(t,T)}), \tag{12}$$

where Φ denotes the cumulative normal distribution function, and d is given by

$$d = \frac{\log\frac{IDI_t}{K} - \log P(t,T) + V(t,T)/2}{\sqrt{V(t,T)}}. \tag{13}$$

[20] Vieira Neto & Valls (1999) adopted the Vasicek (1977) model, and Fajardo & Ornelas (2003) adopted the Cox et al. (CIR, 1985) model.

[21] which respectively provided theoretical and empirical support for multi-factor affine models. Multiple factors driving term structure movements have been advocated since the work of Litterman and Scheinkman (1991). For examples of empirical applications with multi-factor versions of affine models see Dai and Singleton (2002), Sangvinatsos and Watcher (2005), and Collin Dufresne et al. (2006), among others.

Proof. See Appendix A. ■

If $p(t,T)$ is the price at time t of the IDI put with strike K and maturity T then, by the put-call parity

$$p(t,T) = KP(t,T)\Phi(\sqrt{V(t,T)} - d) - IDI_t\Phi(-d). \tag{14}$$

3.3 Hedging IDI Options

Whenever hedging a certain instrument, one is interested in the composition of a portfolio which approximately neutralizes variations on the price of this instrument. To that end, usually one should make use of a set of additional instruments which present dynamics related to the dynamics of the targeted instrument. Alternatively, it is known that each state variable driving uncertainty on the term structure is responsible for one type of movement. These movements are represented by the state variables loadings as a function of time to maturity (see Section 5 for a concrete example). Similarly to Li and Zhao (2005), this research assumes that those state variables are tradable assets which can be used as instruments to compose the hedging portfolio. The main advantage of this approach is to avoid introduction of additional sources of error due to approximate relations between the hedging instruments and the state variables.

The goal of this hedging analysis is to identify if the bond version of model captures the dynamics of IDI options. A delta hedging procedure is performed by equating the first derivatives (with respect to state variables) of the hedging portfolio to the first derivatives (with respect to state variables) of the instrument being hedged, which was chosen, for illustration purposes, to be one contract of a call on the IDI index with strike K, and time of maturity T. Letting Π_t denote the time t value of the hedging portfolio, by assumption it must satisfy

$$\Pi_t = q_t^1 X_t^1 + q_t^2 X_t^2 + ... + q_t^N X_t^N, \tag{15}$$

where q_t^i is the number of units of X_t^i in the hedging portfolio, and X_t^i is the i^{th} term structure dynamic factor. By simply equating the first order variation of Π_t to the first order variation of the IDI option price $c(t,T)$, it is obtained that $q_t^i = \frac{\partial c(t,T)}{\partial X_t^i}$. Calculating the partial derivatives using Equation (12) it follows that

$$q_t^i = \frac{1-\mathrm{e}^{-\kappa_i \tau}}{\kappa_i \sqrt{V(t,T)}} \Big[IDI_t \Phi'(d) + \\ KP(t,T)\sqrt{V(t,T)}\Phi(d-\sqrt{V(t,T)}) - KP(t,T)\Phi'(d-\sqrt{V(t,T)}) \Big]. \qquad (16)$$

On the empirical exercise presented bellow, Equation (16) is used to readjust the hedging on a daily basis.

4 Parameters Estimation

In this section, two versions of a three factor Gaussian model[22] are estimated. Model parameters are obtained based on a maximum likelihood procedure adopted by Chen and Scott (1993) and exposed in Appendix B, in an extended form considering options in the estimation process:

- On the bond version, only ID-futures data, in form of fixed maturity zero coupon bond implied yields, is used in the estimation process. Bonds with maturities of 1, 126, and 252 days are observed without error[23]. For each fixed t, the state vector is obtained through the solution of the following linear system:

$$\begin{aligned} rb_t(0.00397) &= -\tfrac{A(0.00397,\phi)}{0.00397} - \tfrac{B(0.00397,\phi)'}{0.00397} X_t \\ rb_t(0.5) &= -\tfrac{A(0.5,\phi)}{0.5} - \tfrac{B(0.5,\phi)'}{0.5} X_t \qquad (17) \\ rb_t(1) &= -\tfrac{A(1,\phi)}{1} - \tfrac{B(1,\phi)'}{1} X_t. \end{aligned}$$

  Bonds with time to maturity of 21, 63, 189 and 378 days, are assumed to be

[22] According to a principal component analysis applied to the covariance matrix of observed yields, three factors are sufficient to describe 99.5% of the variability of the term structure of ID bonds.

[23] Inversions of the state vector considering other combinations of bonds were also tested offering similar qualitative results in what regards parameter estimation and bond pricing errors.

observed with gaussian errors u_t uncorrelated in the time dimension:

$$
\begin{aligned}
rb_t(0.0833) &= -\tfrac{A(0.0833,\phi)}{0.0833} - \tfrac{B(0.0833,\phi)'}{0.0833}X_t + u_t(0.0833) \\
rb_t(0.25) &= -\tfrac{A(0.25,\phi)}{0.25} - \tfrac{B(0.25,\phi)'}{0.25}X_t + u_t(0.25) \\
rb_t(0.75) &= -\tfrac{A(0.75,\phi)}{0.75} - \tfrac{B(0.75,\phi)'}{0.75}X_t + u_t(0.75) \\
rb_t(1.5) &= -\tfrac{A(1.5,\phi)}{1.5} - \tfrac{B(1.5,\phi)'}{1.5}X_t + u_t(1.5).
\end{aligned}
\tag{18}
$$

The Jacobian matrix is

$$
Jac_t = \begin{bmatrix} -\tfrac{B(0.00397,\phi)'}{0.00397} \\ -\tfrac{B(0.5,\phi)'}{0.5} \\ -\tfrac{B(1,\phi)'}{1} \end{bmatrix}; \tag{19}
$$

- On the option version, options are included in the estimation procedure. This is done by assuming that the instruments observed without error are bonds with maturities of 1 and 189 days, and the at-the-money IDI call option with time to maturity of 95 days. The state vector is obtained through the solution of the following non-linear system

$$
\begin{aligned}
rb_t(0.00397) &= -\tfrac{A(0.00397,\phi)}{0.00397} - \tfrac{B(0.00397,\phi)'}{0.00397}X_t \\
rb_t(0.75) &= -\tfrac{A(0.75,\phi)}{0.75} - \tfrac{B(0.75,\phi)'}{0.75}X_t \\
cs_t &= c(t, t+0.377),
\end{aligned}
\tag{20}
$$

where $c(t,T)$ is given by Equation (11).

Bonds with time to maturity equal to 21, 63, 252, and 378 days, are priced with uncorrelated gaussian errors u_t:

$$
\begin{aligned}
rb_t(0.0833) &= -\tfrac{A(0.0833,\phi)}{0.0833} - \tfrac{B(0.0833,\phi)'}{0.0833}X_t + u_t(0.0833) \\
rb_t(0.25) &= -\tfrac{A(0.25,\phi)}{0.25} - \tfrac{B(0.25,\phi)'}{0.25}X_t + u_t(0.25) \\
rb_t(1) &= -\tfrac{A(1,\phi)}{1} - \tfrac{B(1,\phi)'}{1}X_t + u_t(1) \\
rb_t(1.5) &= -\tfrac{A(1.5,\phi)}{1.5} - \tfrac{B(1.5,\phi)'}{1.5}X_t + u_t(1.5).
\end{aligned}
\tag{21}
$$

The Jacobian matrix is

$$Jac_t = \begin{bmatrix} -\frac{B(0.00397,\phi)'}{0.00397} \\ \\ -\frac{B(0.75,\phi)'}{0.75} \\ \\ q_t \end{bmatrix},$$

where $q_t = \left[q_t^1, \ldots, q_t^N\right]$ with q_t^i calculated for $T = t + 0.377$.

Under both versions of the model, the transition probability $p(X_t|X_{t-1};\phi)$ is a three-dimensional gaussian distribution with known mean and variance as functions of parameters appearing in ϕ.

Tables 1 and 2 present respectively the values of the parameters estimated for each version of the model. Standard deviations are obtained by the BHHH method (see Davidson & MacKinnon (1993)). Under both versions most of the parameters are significant at a 95% confidence interval, except for a few risk premia parameters, and one parameter which comes from the correlation matrix of the brownian motions. The long term short rate mean ϕ_0 was fixed equal to 0.18, compatible with the ID short-rate sample mean of 0.1778[24].

5 Empirical Results

Figure 1 presents the evolution of some bond yields extracted from ID-futures data, from January, 2003 to December, 2005. Yields range from a maximum of 25% observed in the beginning of the sample period to a minimum of 15% in February, 2004. This high variability of yields anticipates that it is not simple to capture all cross section variation with a time homogeneous dynamic model.

Figure 2 presents the average observed and model implied term structures of interest rates for zero coupon bonds, under each estimated version. Its clear from the picture that on the pricing of bonds, the bond version outperforms the option

[24]Optimization including this parameter was also experimented, but generated results with higher standard errors for a considerable fraction of the parameter vector.

version [25]. Under the bond version, the mean absolute error for yields of zero coupon bonds with time to maturity 21, 63, 189 and 378 days are respectively 18.10 bps[26], 6.93 bps, 1.76 bps and 11.52 bps. The errors standard deviations, which provide a metric for their time series variability, are 24.52 bps, 9.52 bps, 2.26 bps and 14.07 bps. Under the option version, the mean absolute error for yields of bonds with time to maturity 21, 63, 126 and 378 days are respectively 29.72 bps, 14.89 bps, 12.93 bps and 39.03 bps, with standard deviations of 35.37 bps, 17.70 bps, 15.92 bps and 46.54 bps.

5.1 Term Structure Movements and Bond Risk Premium

Figure 3 presents the loadings of the three dynamic factors under each version of the model (solid lines correspond to the bond version, dotted lines to the option version). The level factor[27] presents loadings indistinguishable across versions. However, slope and curvature factors are clearly different. They both present higher curvatures under the option version, suggesting that option investors tend to react faster (than bond investors) to news that affect the term structure of bond risk premiums in an asymmetric way[28]. Figure 4 presents the state variables driving each term structure movement, for the two versions of the model[29]. Note that the time series of the slope and curvature factors, under the option version, present spikes that are consistent with fast mean reverting variables.

An important point related to the modification of term structure movements is to understand what are the implications on investor's interpretation of risks when options are or not included in the estimation process. This might be addressed

[25]Under the bond version, the three dimensional latent vector X, characterizing uncertainty in the economy, is fully inverted from bonds data. In contrast, the option version only captures the yields of two bonds without errors, because the third instrument priced without error is an at-the-money option.

[26]Bps stands for basis points. One basis point is equivalent to 0.01%.

[27]It is the one with slowest mean reversion speed and responsible for explaining most of the variation on yields.

[28]Note that a shock on the level factor affects the risk premium term structure in a symmetric way.

[29]The average value of the short-rate (ϕ_0) should be added to the level state variable, in order to obtain the level factor.

in at least two ways: By observing the time series of model implied bond risk premiums and contrasting across versions, or directly observing bond risk premium decomposition as a combination of term structure movements, under each version.

Figure 5 presents pictures of the term structures of bond instantaneous risk premium (measured by Equation (10)) in different instants of time. Note that the cross section of premiums is very distinct across versions, and in particular, the longer the maturity the larger the difference between the risk premium implied by each version. In addition, under the option version, the term structure of risk premiums is better approximated by a linear function, and risk premiums are in general lower. The time series behavior of the premiums might be better observed in Figure 6, which presents the evolution of the instantaneous risk premium for the 1-year bond, under the two versions. During the period from September of 2003 to December of 2004, the premium is significantly higher under the bond version. That was a period where interest rates were consistently being lowered by the Central Bank of Brazil, and in this context, the smaller premium (under the option version) indicates the possibility of an inertia of bond investors in reestimating their expectations for long term behavior of interest rates, as opposed to a fast reaction of option market players.

The risk premium decomposition across movements of the term structure provides a direct way of identifying the shifts in importance of factors once options are adopted in the estimation process. From Equation (10), it is clear that risk premium is a linear combination of the state variables: $z(t, t+\tau) = a_1(\tau)X_t^1 + a_2(\tau)X_t^2 + a_3(\tau)X_t^3$. Figure 7 presents the term structure of risk premiums decomposed for each maturity among the three movements: Level, slope and curvature. Solid lines represent the bond version and dashed lines the option version. For each fixed maturity, the sum of the absolute weights on the three movements gives 100%. The decomposition presents a clearly distinct pattern for maturities bellow and above 0.5 years, under both versions. For instance, under the bond version, the curvature factor explains more than 70% of the premium for short maturities while curvature and slope together explain the premium for longer maturities. Under the option version the level factor explains most of the premium for longer maturities while it splits this role with the curvature factor for shorter maturities. Under both versions the slope contributes negatively to the risk premium decomposition. In general, risk premium is more sensitive to the curvature and slope factors under the bond version, and to the level and curvature factors under the option version. Contrasting factor loadings and risk premiums, it is possible to identify that the use of options data provides less persistent slope and curvature movements, but prices the most persistent factor (level). On the other hand, when only bonds are adopted in the estimation process, secondary movements (slope and curvature) are more persistent, but are priced in stead of the level movement (still the most persistent factor). Results tend to suggest that within the Brazilian fixed income market, option investors are more concerned with monetary policy through the level of interest rates, while bond investors are more concerned with the volatility of interest rates through curvature and slope (see Litterman et al. (1991)).

5.2 Pricing and Hedging Options

The goal of the next exercise is to understand how useful could be the inclusion of options on the estimation process of the dynamic model when pricing and hedging options. Since under the option version, an at-the-money option is used to invert the state vector, this exercise is only interesting if out-of-sample options are adopted. The database of most liquid IDI call options is adopted, when comparing pricing performances across versions.

Figure 8 presents observed option prices versus model implied prices. Points represent the bond version and x's the option version. For modeling purposes, an ideal relation would be a 45 degree line passing through the origin with angular coefficient equal to 1 (solid line in Figure 8). Under the bond version, a linear regression of observed prices depending on model prices, presents a $R^2 = 97.5\%$, an angular coefficient equal to 1.0423 (p-value < 0.01) and a linear coefficient of 86.83

(p-value < 0.01). The high R^2 indicates that the option prices obtained under the bond version correctly captures the time series variability of observed option prices (high correlation). However, the high value for the linear coefficient implies that the bond version consistently underestimates option prices. The underestimation of option prices is confirmed by Figure 9, which presents the relative error defined by model price minus observed price, divided by observed price. Note how under the bond version it is smaller than zero during most of the time. The absolute relative pricing error presents an average of 17.53%[30].

When the same regression is provided for the option version, the R^2 is slightly bellow, achieving 97.2%, probably due to some mispricing of options with prices in the range $[1500, 3000]$ (see Figure 8). On the other hand, both the angular coefficient of 1.0121 (p-value < 0.01) and the linear coefficient of 11.67 (p-value $= 0.14$) are closer to ideal values. The smaller linear coefficient indicates that once options are adopted in the estimation process they help the dynamic model to better capture the level of option prices. The dotted line in Figure 9 presents the relative pricing error for the option version. Note that it clearly outperforms the bond version, except for the end of the sample period when it overestimates option prices. It achieves an average absolute value of 10.75%, a 40% improvement with respect to the bond version.

The next step implements a dynamic delta-hedging strategy on the fixed-maturity at-the-money IDI call option[31]. Note that if the hedging is effective, variations on the hedging portfolio should approximately offset variations on the option price. The correlation coefficients between these variations are 5.10% and 94.74% for the bond and option versions respectively, directly suggesting that the option based version is much more efficient when hedging. In fact, one could expect with no surprises that the option version would be able to perform an excellent hedging since the at-the-money option is inverted to extract the state vector. In this sense, the

[30] For comparison purposes, see Jagannathan et al. (2003) who price U.S. caps adopting a three-factor CIR model estimated with U.S Libor and swaps data.

[31] On the hedging analysis a fixed-maturity, fixed-moneyness option is adopted, otherwise changes in prices would reflect not only the price dynamics but also changes on the type of the option.

hedging error for the option model is essentially a second order error not captured by the delta-hedging procedure. However, the result of interest is the comparison of dynamic hedging weights across versions. Figure 10 displays the number of units in the hedging portfolio invested on each state variable. Observe that in both versions of the model the option is more sensitive to the level factor and less sensitive to the curvature factor, and in particular under the option version, the allocations to both level and slope factors are much more volatile. This high volatility of the allocations reflects the fact that at-the-money options are highly sensitive to changes in their underlying assets, which in the particular case are interest rates.

6 Conclusion

A dynamic multi-factor Gaussian model is estimated based on two different sets of Brazilian fixed income instruments, one adopting only bonds data, and the other combining bonds and options data. The main interest is to verify if (and how) options change the loadings and dynamic time series of the main movements that drive the term structure of interest rates. It is identified that option prices bring information that primarily affect the speeds of mean reversion of the slope and curvature of the yield curve, and also affect the decomposition of bond risk premia. In fact, considerably more weight is given to the level factor, which ends up explaining around 80% of the premium for longer maturities, when options are adopted in the estimation process.

In addition, when delta-hedging an at-the-money option, both implemented versions give little importance to the curvature factor, while the option version presents much more volatile weights on slope and level factors, which seem to be necessary to capture the dynamics of option prices.

These results lead to the conclusion that whenever analyzing risk premium through the lens of a dynamic term structure model, or performing hedging of fixed income options, options should be incorporated to the estimation process of the dynamic model, and the effect of including it should be compared to a model estimated based on only bonds data.

Appendix A

Proof. Lemma 1

By Ito's rule, for each $t<T$ the unique strong solution of (3) is[32]

$$X_T^i = X_t^i \mathrm{e}^{-\kappa_i (T-t)} + \sum_{j=1}^N \rho_{ij} \int_t^T \mathrm{e}^{-\kappa_i (T-s)} \mathrm{d}W_s^j, \quad i=1,\ldots,N.$$

Then

$$r_T = \phi_0 + \sum_{i=1}^N \left(X_t^i \mathrm{e}^{-\kappa_i (T-t)} + \sum_{j=1}^N \rho_{ij} \int_t^T \mathrm{e}^{-\kappa_i (T-s)} \mathrm{d}W_s^j \right).$$

Stochastic integration by parts implies that

$$\int_t^T X_u^i \mathrm{d}u = \int_t^T (T-u)\, \mathrm{d}X_u^i + (T-t)\, X_t^i. \tag{22}$$

By definition of X, the integral in the right-hand side can be written as

$$\int_t^T (T-u)\, \mathrm{d}X_u^i = -\kappa_i \int_t^T (T-u)\, X_u^i \mathrm{d}u + \sum_{j=1}^N \rho_{ij} \int_t^T (T-u)\, \mathrm{d}W_u^j.$$

Note also that

$$\int_t^T (T-u)\, X_u^i \mathrm{d}u =$$

$$= X_t^i \int_t^T (T-u)\, \mathrm{e}^{-\kappa_i (u-t)} \mathrm{d}u + \sum_{j=1}^N \rho_{ij} \int_t^T (T-u) \int_t^u \mathrm{e}^{-\kappa_i (u-s)} \mathrm{d}W_s^j \mathrm{d}u.$$

Calculating separately the last two integrals, the following result holds

$$\int_t^T (T-u)\, \mathrm{e}^{-\kappa_i (u-t)} \mathrm{d}u = \left(\frac{T-t}{\kappa_i} + \frac{\mathrm{e}^{-\kappa_i (u-t)} - 1}{\kappa_i^2} \right)$$

and, again by integration by parts,

$$\int_t^T (T-u) \int_t^u \mathrm{e}^{-\kappa_i (u-s)} \mathrm{d}W_s^j \mathrm{d}u =$$

$$= \int_t^T \left(\int_t^u \mathrm{e}^{\kappa_i s} \mathrm{d}W_s^j \right) \mathrm{d}_u \left(\int_t^u (T-v)\, \mathrm{e}^{-\kappa_i v} \mathrm{d}v \right) =$$

$$= \left(\int_t^T \mathrm{e}^{\kappa_i u} \mathrm{d}W_u^j \right) \left(\int_t^T (T-v)\, \mathrm{e}^{-\kappa_i v} \mathrm{d}v \right) -$$

$$- \int_t^T \left(\int_t^u (T-v)\, \mathrm{e}^{-\kappa_i v} \mathrm{d}v \right) \mathrm{e}^{\kappa_i u} \mathrm{d}W_u^j =$$

$$= \int_t^T \left(\int_u^T (T-v)\, \mathrm{e}^{-\kappa_i v} \mathrm{d}v \right) \mathrm{e}^{\kappa_i u} \mathrm{d}W_u^j =$$

$$\frac{1}{\kappa_i} \int_t^T \left(T-u + \frac{\mathrm{e}^{-\kappa_i (T-u)} - 1}{\kappa_i} \right) \mathrm{d}W_u^j.$$

[32]In this appendix we drop the superscript $\mathbb{Q}$ and denote the N-dimensional brownian motion $W^{\mathbb{Q}}$ simply by W.

Substituting the previous terms in Equation (22), the following result holds

$$\int_t^T X_u^i \mathrm{d}u = (T-t)\, X_t^i -$$

$$-\kappa_i \left[X_t^i \left(\frac{T-t}{\kappa_i} + \frac{\mathrm{e}^{-\kappa_i(T-t)}-1}{\kappa_i^2} \right) + \sum_{j=1}^N \frac{\rho_{ij}}{\kappa_i} \int_t^T \left(T-u+\frac{\mathrm{e}^{-\kappa_i(T-u)}-1}{\kappa_i} \right) \mathrm{d}W_u^j \right] +$$

$$+\sum_{j=1}^N \rho_{ij} \int_t^T (T-u)\, \mathrm{d}W_u^j =$$

$$= -\frac{\mathrm{e}^{-\kappa_i(T-u)}-1}{\kappa_i} X_t^i + \sum_{j=1}^N \rho_{ij} \int_t^T -\frac{\mathrm{e}^{-\kappa_i(T-u)}-1}{\kappa_i} \mathrm{d}W_u^j =$$

$$= \frac{1-\mathrm{e}^{-\kappa_i(T-u)}}{\kappa_i} X_t^i + \frac{1}{\kappa_i} \sum_{j=1}^N \rho_{ij} \int_t^T \left(1-\mathrm{e}^{-\kappa_i(T-u)}\right) \mathrm{d}W_u^j,$$

that is,

$$\int_t^T X_u^i \mathrm{d}u = \frac{1-\mathrm{e}^{-\kappa_i(T-u)}}{\kappa_i} X_t^i + \frac{1}{\kappa_i} \sum_{j=1}^N \rho_{ij} \int_t^T \left(1-\mathrm{e}^{-\kappa_i(T-u)}\right) \mathrm{d}W_u^j. \tag{23}$$

Then $y(t,T) = \phi_0 (T-t) + \sum_{i=1}^N \int_t^T X_u^i \mathrm{d}u$ conditional on $\mathcal{F}_t$ is normally distributed (see Duffie (2001)) with mean

$$M(t,T) = \phi_0 (T-t) + \sum_{i=1}^N \frac{1-\mathrm{e}^{-\kappa_i(T-t)}}{\kappa_i} X_t^i, \tag{24}$$

where the fact that the stochastic integral in (23) is a martingale was used. The variance of $y(t,T)|\mathcal{F}_t$ is

$$V(t,T) = \mathrm{var}^{\mathbb{Q}} \left[\sum_{i=1}^N \frac{Y_i}{\kappa_i} | \mathcal{F}_t \right], \tag{25}$$

where $Y_i = \sum_{j=1}^N \rho_{ij} \int_t^T \left(1-\mathrm{e}^{-\kappa_i(T-u)}\right) \mathrm{d}W_u^j$. Then

$$V(t,T) = \sum_{i=1}^N \frac{\mathrm{var}^{\mathbb{Q}}(Y_i|\mathcal{F}_t)}{\kappa_i^2} + 2\sum_{i=1}^N \sum_{k>i} \frac{\mathrm{cov}^{\mathbb{Q}}(Y_i, Y_k|\mathcal{F}_t)}{\kappa_i \kappa_k}.$$

Using Ito's isometry

$$\begin{aligned} V(t,T) &= \sum_{i=1}^N \frac{1}{\kappa_i^2} \sum_{j=1}^N \rho_{ij}^2 \int_t^T \left(1-\mathrm{e}^{-\kappa_i(T-u)}\right)^2 \mathrm{d}u + \\ &+2\sum_{i=1}^N \sum_{k>i} \frac{1}{\kappa_i\kappa_k} \sum_{j=1}^N \rho_{ij}\rho_{kj} \int_t^T \left(1-\mathrm{e}^{-\kappa_i(T-u)}\right)\left(1-\mathrm{e}^{-\kappa_k(T-u)}\right) \mathrm{d}u. \end{aligned} \tag{26}$$

At this point, simple integration produces

$$\begin{aligned} V(t,T) &= \sum_{i=1}^N \frac{1}{\kappa_i^2} \left(\tau + \frac{2}{\kappa_i}\mathrm{e}^{-\kappa_i\tau} - \frac{1}{2\kappa_i}\mathrm{e}^{-2\kappa_i\tau} - \frac{3}{2\kappa_i} \right) \sum_{j=1}^N \rho_{ij}^2 + \\ &+2\sum_{i=1}^N \sum_{k>i} \frac{1}{\kappa_i\kappa_k} \left(\tau + \frac{\mathrm{e}^{-\kappa_i\tau}-1}{\kappa_i} + \frac{\mathrm{e}^{-\kappa_k\tau}-1}{\kappa_k} - \frac{\mathrm{e}^{-(\kappa_i+\kappa_k)\tau}-1}{\kappa_i+\kappa_k} \right) \sum_{j=1}^N \rho_{ij}\rho_{kj}, \end{aligned} \tag{27}$$

where $\tau = T - t$. ■

Proof. Lemma 2

The martingale condition for bond prices (Duffie (2001)) gives:

$$P(t,T) = \mathrm{E}^{\mathbb{Q}}\left[\mathrm{e}^{-\int_t^T r_u \mathrm{d}u}|\mathcal{F}_t\right] = \mathrm{E}^{\mathbb{Q}}\left[\mathrm{e}^{-y(t,T)}|\mathcal{F}_t\right]. \tag{28}$$

Now the normality of variable $y(t,T)|\mathcal{F}_t$ (Lemma 1), and a simple property of the mean of log-normal distributions complete the proof. ■

Proof. Lemma 3

By Equation (11) the proof consists of a simple calculation of the expectation $\mathrm{E}^{\mathbb{Q}}\left[\max\left(IDI_t - K\mathrm{e}^{-y}, 0\right)|\mathcal{F}_t\right]$.

$$\begin{aligned} c\left(t,T\right) &= \mathrm{E}^{\mathbb{Q}}\left[\max\left(IDI_t - K\mathrm{e}^{-y}, 0\right)|\mathcal{F}_t\right] = \\ &= \int_{-\infty}^{\infty} \tfrac{1}{\sqrt{2\pi V(t,T)}} \max\left(IDI_t - K\mathrm{e}^{-y}, 0\right) \mathrm{e}^{-\frac{(y-M(t,T))^2}{2V(t,T)}} \mathrm{d}y = \\ &= \int_{\log(K/IDI_t)}^{\infty} \tfrac{1}{\sqrt{2\pi V(t,T)}} \left(IDI_t - K\mathrm{e}^{-y}\right) \mathrm{e}^{-\frac{(y-M(t,T))^2}{2V(t,T)}} \mathrm{d}y. \end{aligned} \tag{29}$$

Making the substitution $z = \frac{y-M(t,T)}{\sqrt{V(t,T)}}$ the following result holds:

$$\begin{aligned} c\left(t,T\right) &= \int_{-d}^{\infty} \tfrac{1}{\sqrt{2\pi}} \left(IDI_t - K\mathrm{e}^{-z\sqrt{V(t,T)}-M(t,T)}\right) \mathrm{e}^{-\frac{1}{2}z^2}\mathrm{d}z = \\ &= IDT_t \int_{-\infty}^{d} \tfrac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{1}{2}z^2}\mathrm{d}z - K\int_{-d}^{\infty}\tfrac{1}{\sqrt{2\pi}}\mathrm{e}^{-z\sqrt{V(t,T)}-M(t,T)-\frac{1}{2}z^2}\mathrm{d}z = \\ &= IDI_t\Phi\left(d\right) - K\mathrm{e}^{-M(t,T)+\frac{V(t,T)}{2}}\int_{-d}^{\infty}\tfrac{1}{\sqrt{2\pi}}\mathrm{e}^{-\frac{1}{2}\left(z+\sqrt{V(t,T)}\right)^2}\mathrm{d}z. \end{aligned} \tag{30}$$

where d is given by Equation (13). Making a new substitution $v = z + \sqrt{V\left(t,T\right)}$ and using Lemma 2 results in Equation (12). ■

Appendix B

In this work, the maximum likelihood estimation procedure described in Chen and Scott (1993), is extended to deal with options[33]. The following bond yields are observed along H different days: $rb_t(1/252)$, $rb_t(21/252)$, $rb_t(63/252)$, $rb_t(126/252)$, $rb_t(189/252)$, $rb_t(1)$ and $rb_t(1.5)$[34]. Let rb represent the $H \times 7$ matrix containing the yields for all H days. In addition, the price cs_t for an at-the-money call with time to maturity 95/252 years is observed during the same H days. Let cs be the vector of length H that represents these call prices. The ID bonds and the at-the-money IDI call are called reference market instruments. Denote by $rmi = [rb, cs]$ the $H \times 8$ matrix containing the yields and the price of these reference market instruments. Assume that model parameters are represented by vector ϕ and a time unit equal to Δt. Finally, let $g_i(X_t; t, \phi)$ be the function that maps reference market instrument i into state variables.

As three factors are adopted to estimate the model, it is assumed that reference market instruments, say i_1, i_2 and i_3, are observed without error. For each fixed t, the state vector is obtained through the solution of the following system:

$$\begin{aligned} g_{i_1}(X_t; t, \phi) &= rmi(t, i_1) \\ g_{i_2}(X_t; t, \phi) &= rmi(t, i_2) \\ g_{i_3}(X_t; t, \phi) &= rmi(t, i_3). \end{aligned} \tag{31}$$

Reference market instruments i_4, i_5, i_6, i_7 and i_8, are assumed to be observed with gaussian uncorrelated errors u_t:

$$\begin{aligned} & rmi(t, \begin{bmatrix} i_4 & i_5 & i_6 & i_7 & i_8 \end{bmatrix}) - u_t = \\ & \begin{bmatrix} g_{i_4}(X_t; t, \phi) & g_{i_5}(X_t; t, \phi) & g_{i_6}(X_t; t, \phi) & g_{i_7}(X_t; t, \phi) & g_{i_8}(X_t; t, \phi) \end{bmatrix} \end{aligned} \tag{32}$$

The log-likelihood function can be written as

$$\begin{aligned} & L(\phi, rb) = \sum_{t=2}^{H} \log p(X_t | X_{t-1}; \phi) - \\ & - \sum_{t=2}^{H} \log |Jac_t| - \frac{H-1}{2} \log |\Omega| - \frac{1}{2} \sum_{t=2}^{H} u_t' \Omega^{-1} u_t, \end{aligned} \tag{33}$$

[33] For the estimation of more general dynamic term structure models on joint U.S swaps and caps, see for instance, Han (2004), Almeida et al. (2006), Joslin (2006), or Graveline (2006), among others.

[34] $rb_t(\tau)$ stands for the time t yield of a bond with time to maturity τ.

where:

1. $Jac_t = \begin{bmatrix} \frac{\partial g_{i_1}(X_t;t,\phi)}{\partial X_t} \\ \\ \frac{\partial g_{i_2}(X_t;t,\phi)}{\partial X_t} \\ \\ \frac{\partial g_{i_3}(X_t;t,\phi)}{\partial X_t} \end{bmatrix}$ is the Jacobian matrix of the transformation defined by Equation (31);

2. Ω represents the covariance matrix for u_t, estimated using the sample covariance matrix of the u_t's implied by the extracted state vector;

3. $p(X_t|X_{t-1};\phi)$ is the transition probability from X_{t-1} to X_t under the objective probability measure $\mathbb{P}$.

The final objective within this procedure is to estimate vector ϕ which maximizes function $L(\phi, rb)$. In order to (try to) avoid possible local minima, several different starting parameter vectors are tested and, for each one, a search for the optimal point is performed with alternating use of Nelder-Mead Simplex algorithm for non-linear optimization and gradient-based optimization methods.

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Parameter	Value	Standard Error	ratio $\frac{\text{abs(Value)}}{\text{Std Error}}$
κ_1	6.3435	0.0889	**71.34**
κ_2	1.6082	0.0174	**92.47**
κ_3	0.0003	0.00001	**12.65**
ρ_{11}	0.0919	0.0021	**43.07**
ρ_{21}	-0.0216	0.0034	**6.30**
ρ_{22}	0.0400	0.0010	**40.22**
ρ_{31}	-0.0008	0.0016	0.47
ρ_{32}	-0.0192	0.0004	**50.85**
ρ_{33}	0.0112	0.0001	**108.49**
$\lambda_X(11)$	-329.7170	109.0627	**3.02**
$\lambda_X(21)$	42.9899	68.3982	0.62
$\lambda_X(22)$	0.5462	12.0799	0.05
$\lambda_X(31)$	-200.4261	39.4736	**5.07**
$\lambda_X(32)$	258.7188	10.6457	**24.30**
$\lambda_X(33)$	-75.3815	7.9478	**9.48**
ϕ_0	0.18	-	-

Table 1: Parameters and Standard Errors Obtained Under the Bond Version.

Parameter	Value	Standard Error	ratio $\frac{\text{abs(Value)}}{\text{Std Error}}$
κ_1	37.6296	10.8910	**3.46**
κ_2	3.4565	0.1858	**18.60**
κ_3	0.0003	0.00002	**16.96**
ρ_{11}	0.0919	0.0040	**23.96**
ρ_{21}	-0.0415	0.0044	**9.41**
ρ_{22}	0.0729	0.0016	**45.45**
ρ_{31}	-0.0006	0.0017	0.39
ρ_{32}	-0.0332	0.0016	**20.72**
ρ_{33}	0.0194	0.0003	**69.66**
$\lambda_X(11)$	-240.0116	129.1894	1.86
$\lambda_X(21)$	-137.1462	63.9335	**2.15**
$\lambda_X(22)$	0.0376	12.4838	0.00
$\lambda_X(31)$	-260.0849	84.7153	**3.07**
$\lambda_X(32)$	16.917	26.6624	0.63
$\lambda_X(33)$	-278.9916	13.1735	**21.17**
ϕ_0	0.18	-	-

Table 2: Parameters and Standard Errors Obtained Under the Option Version.

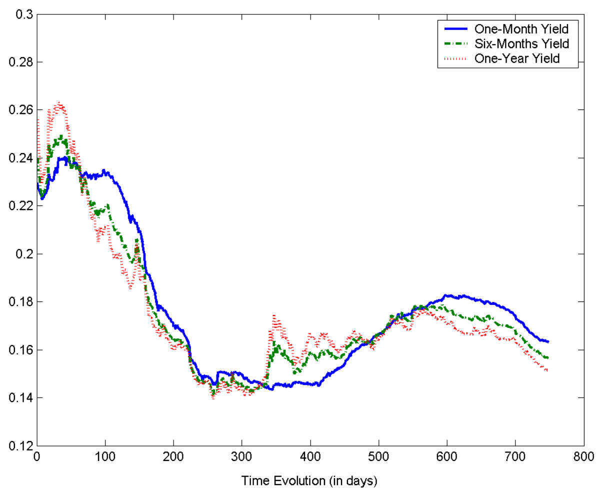

Figure 1: Time Series of Brazilian Bonds Yields: From January, 2003 to December, 2005.

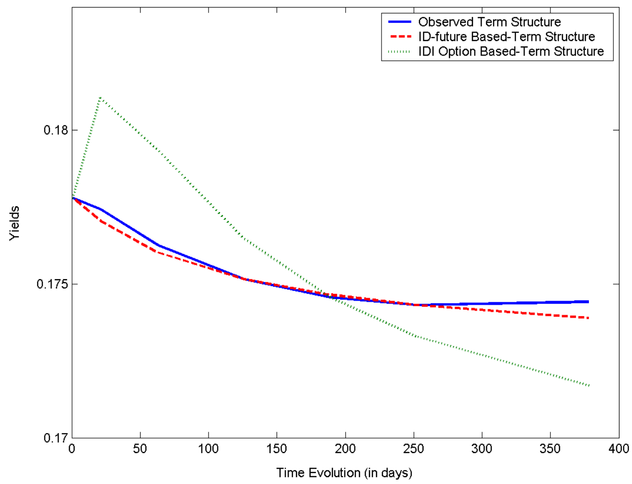

Figure 2: Average Observed and Model-Implied Cross Section of Yields.

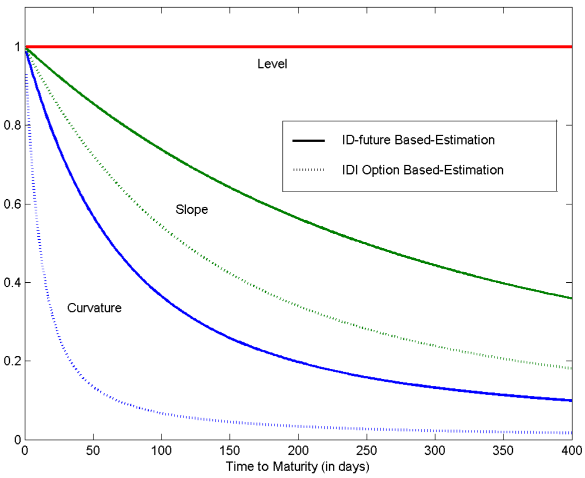

Figure 3: Loadings of the Three Dynamic Factors.

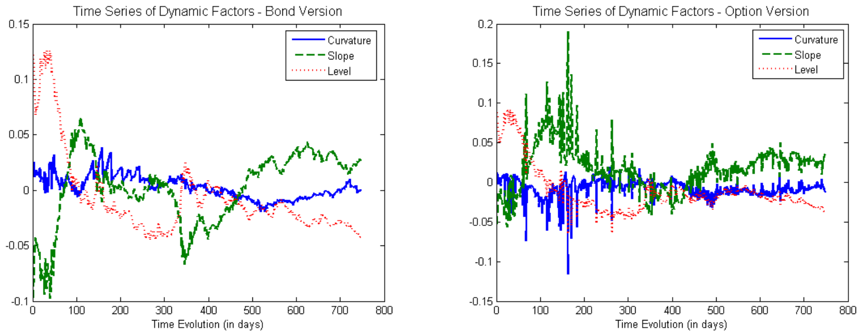

Figure 4: Time Series of the State Variables.

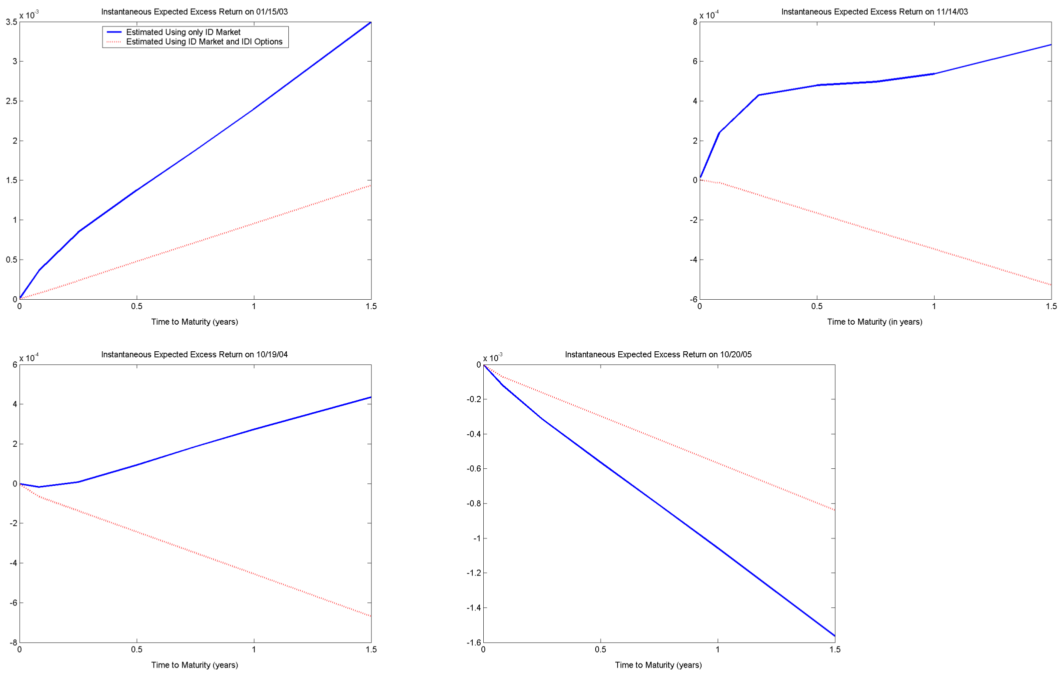

Figure 5: Examples of Cross-Section Instantaneous Expected Excess Returns.

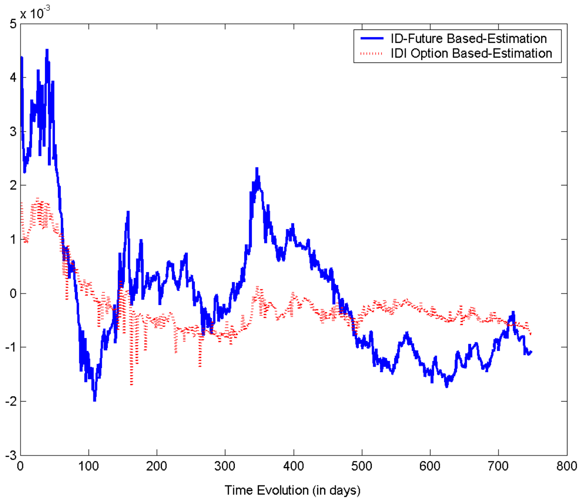

Figure 6: Time Series of Instantaneous Expected Excess Return for the 1-year Bond.

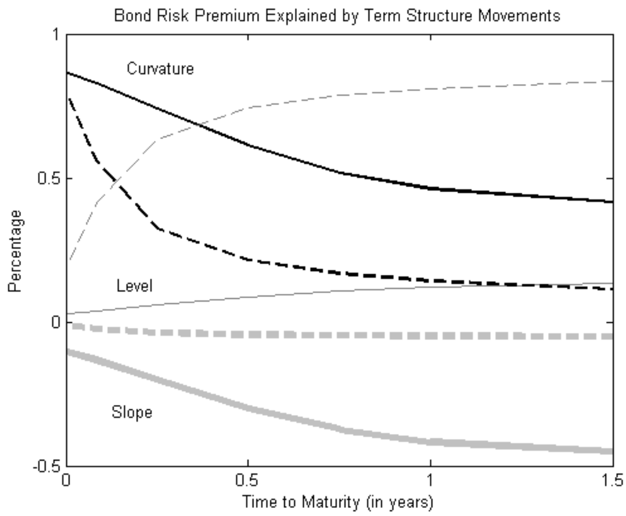

Figure 7: The Bond Risk Premium Decomposition for the Bond Version (Solid Line) and Option Version (Dashed Line).

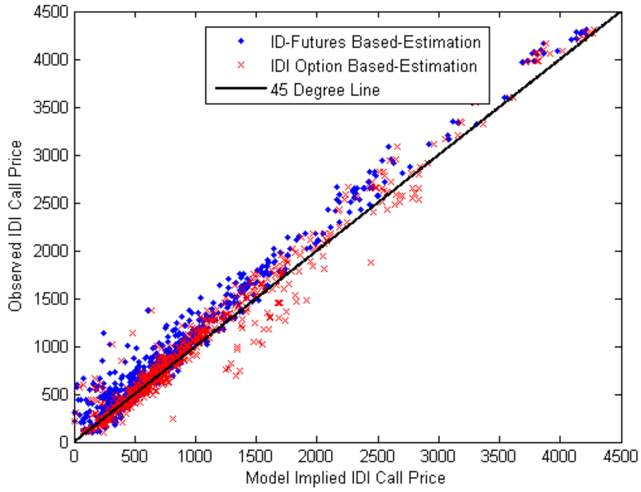

Figure 8: Observed IDI Call Price as a Linear Approximation of the Model-Implied Price

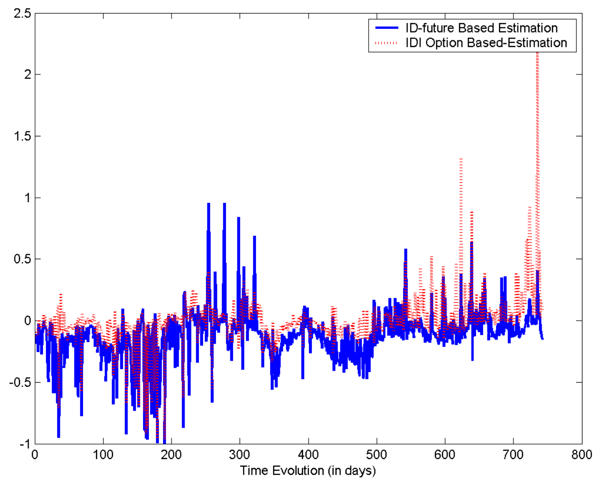

Figure 9: Model Relative Error when Pricing an IDI Call Based on Parameters Estimated Under the Bond Version (Solid Line) and Option Version (Dotted Line).

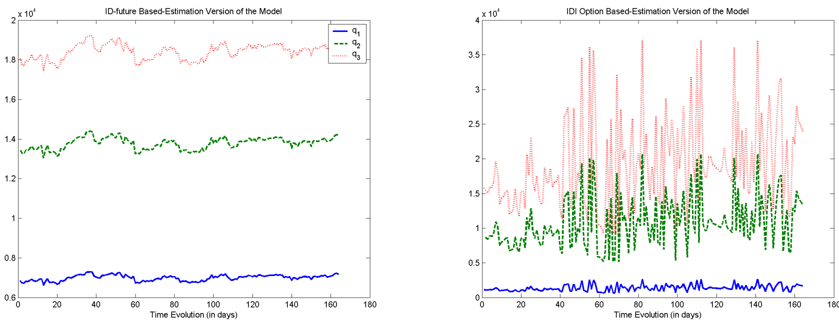

Figure 10: Units of State Variables in the Hedging Portfolio Under Both Versions of the Model.

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