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Contagion, Bankruptcy and Social Welfare Analysis in a Financial Economy with Risk Regulation Constraint*

Aloísio P. Araújo[†] José Valentim M. Vicente[‡]

Abstract

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In the last years, regulatory agencies of many countries in the world, following recomendations of Basel Committee, have compeled financial institutions to maintain minimum capital requirements to cover market and credit risks. This paper investigates the consequences about social welfare, contagion and the bankruptcy probability of such practice. We show that for each financial institution there is a level of regulation that maximizes its utility. Another important result asserts that risk regulation decreases contagion and under certain conditions can reduce the bankruptcy probability. We also analyze the trade-off faced by regulators involving the financial institutions welfare versus bankruptcy and contagion probabilities.

JEL classification: G18; G20; D60; C68.
Keywords: Basel Capital Accord, VaR, Banking Regulation and Social Welfare.

*The opinions expressed herein are those of the authors and not necessarily those of the Central Bank of Brazil.

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1 Introduction

In the last two decades, many regulatory agencies around the world have introduced formal capital requirements to control banks risks based on the recommendations of the 1988 Basel Accord on capital standards and its following amendments.

This Accord was the first successful attempt to harmonize international rules of bank capital[1] and resulted from a process under the heading of the Basel Committee on Banking Supervision[2]. The 1988 Basel Accord is a document approved in July 1988 by the member countries of the Committee establishing minimum capital requirements for credit risk. Basically, it imposes a capital requirement of at least 8% of the Risk-Adjusted Asset (RAA), defined as the sum of asset positions multiplied by asset-specific risk weights.

In January 1996, the Committee released a new document named Amendment to the Capital Accord to Incorporate Market Risks (Basel Committee on Banking Supervision, 1996a)[3] defining criteria for capital requirements to cover market risk. Since then the minimum regulatory capital of a financial institution is the sum of an amounts to cover credit and market risks[4]. In order to gauge market risk the Basel Committee adopted the well known Value-at-Risk (VaR) metric[5].

Regardless of legal requirements, several financial institutions have recently adopted internal VaR-based models for market risk management. Most of this self-discipline process stemmed from demand of stockholders and investors who were concerned with the increase of volatility in a globalized world and wanted transparency in the management of their resources.

Many recent studies have addressed the economic implications of the

[1]See Freixas and Santomero (2002) or Santos (2002) for a review of the theoretical justifications for bank capital requirements.

[2]The Basel Committee was set up in 1974 under the auspices of the Bank for International Settlements (BIS) by the central banks of the G10 members.

[3]For an overview on the Amendment to the Capital Accord to Incorporate Market Risk, see the Basel Committee on Banking Supervision (1996b).

[4]Recently, the Basel Committee released another document, commonly known as Basel II, that revises the original framework for setting capital charges for credit risk and introduces capital charge to cover operational risk.

[5]VaR represents the maximum loss to which a portfolio is subject for a given confidence interval and time horizon. For instance, a one-day 99% VaR of R$ 10 million means that there is only 1 in 100 chance of the portfolio loss to exceed R$ 10 million at the end of the next business day. For an overview of VaR, see Duffie & Pan (1997).

adoption of capital requirements based on the Basel Accord proposals. Rochet (1992) analyzes the consequences of capital requirements on the portfolio choices of banks and showed that the optimal risk weigth must be proportional to the systemic risk of the assets (their betas). Jackson et. al (1999) review the empirical evidence on the impact of the 1988 Basel Accord. Blum (1999) point out that, in a dynamic framework, a capital intertemporal effect can arise which leads to an increase in bank's risk. Marshall & Venkataraman (1999) use a simple model to evaluate alternative bank capital regulatory proposals for market risk. Basak & Shapiro (2001) investigate the implications of the investment decision problem when the trader is subject to an exogenous VaR limit. Daníelsson & Zigrand (2003) use an equilibrium model to study the implications on asset prices and variances due to the introduction of a VaR-based risk regulation. Daníelsson et al. (2004) extend the model proposed by Daníelsson & Zigrand (2003) to a multiperiod environment and estimate the intensity of adverse impacts of VaR-based risk constraint. Cuoco & Liu (2006) study the behavior of a financial institution subject to capital requirements based on self-reported VaR measures. Leippold et al. (2006) consider the asset-pricing implications of VaR regulation in incomplete continuous-time economies.

The aim of the present study is to investigate the welfare properties, the bankruptcy probability and the contagion among financial institutions in an economy with capital requirements to cover risks using an equilibrium model similar to one proposed by Daníelsson et al. (2003)[6].

We start by analyzing the welfare effects of the introduction of VaR-based capital requirements. Surprisingly, we show that some institutions can be better in a regulated economy (i.e., an economy where all financial institutions must satisfy the risk regulation constraint) than an unregulated economy (i.e., an economy where there are no risk limits). Another important result states that a VaR-based risk regulation can reduce the financial fragility of the market, defined as the sum of the bankruptcy probabilities of all financial agents. We also show that the tighter is the regulation, the smaller is the probability of contagion. Next we consider an economy with RAA-based risk constraint. In this context we show that if the weights are misadjusted, besides to the decrease in the prices of risky assets, the regula-

[6] In the same spirit of Danielsson et al. (2003) we don't model reasons to the presence of risk regulation. Simply we suppose that it exists (probably due to a market failure) and assess the economic consequence of it.

tion can increase the bankruptcy probability of financial institutions.

The structure of the paper is as follows. In Section 2 we present the ingredients of the model. Section 3 describes the VaR-based risk constraint and establishes conditions for the existence of equilibrium. In Section 4 we study the welfare of financial institution in a regulated economy. Section 5 analyzes the total bankruptcy probability before and after the introduction of a VaR-based risk regulation. In Section 6 we present a new approach to evaluate the contagion in an economy with risk constraint. In Section 7 we study through a simple example the problem of RAA-based risk regulation. Section 8 concludes. Proofs are contained in the Appendix.

2 The Model

Consider a two period economy ($t = 0, 1$) according to proposed by Daníelsson & Zigrand (2003). At $t = 0$ agents (financial institutions) invest in $N + 1$ assets that mature at $t = 1$. The asset 0 is risk-free and yields payoff d_0. The risky assets are nonredundant and promise at $t = 1$ a payoff

$$\boldsymbol{d} = \begin{pmatrix} d_1 \\ \vdots \\ d_N \end{pmatrix},$$

that follow a Gaussian distribution with mean μ and covariance matrix Σ.

The price of asset i is denoted by q_i. The return on asset i is defined by

$$R_i \equiv \frac{d_i}{q_i}.$$

We follow common modelling practice by endowing financial institutions with their own utility functions (such as in Basak and Shapiro, 2001, for instance). There is a continuum of small agents characterized by a constant coefficient of absolute risk aversion (CARA) h. The population of agents is such that h is uniformly distributed on the interval $[\ell, 1]$. To guarantee that all agents are risk-averse, let us suppose that $\ell > 0$.

Let x^h and y_i^h be the number of units of the risk-free asset and of the risky asset i, respectively, held by financial institution h at $t = 0$. Then the wealth of agent h at time $t = 1$ is

$$W_1^h = d_0 x^h + \sum_i d_i y_i^h.$$

The agents choose the portfolio that maximizes the expected value of their wealth utility $u^h\left(W_1^h\right)$ subject to budget and risk constraints.

The time-zero wealth of an agent of type h comprises initial endowments in the risk-free asset, θ_0^h, as well in risky assets, $\boldsymbol{\theta}^h = \left(\theta_1^h, \ldots, \theta_N^h\right)'$.

The budget constraint of institution h at $t = 0$ is

$$q_0 x^h + \sum_i q_i y_i^h \leq W_0^h,$$

where $W_0^h = q_0\theta_0^h + \sum_i q_i\theta_i^h$ is the initial wealth of agent h.

The role of the regulatory agency consists to limiting the set of investments opportunities in the risky assets. That is, the regulatory agency introduces a new constraint (hereafter, denominated risk constraint) that can be written as

$$\boldsymbol{y}^h \in \Upsilon, \quad \forall h \in [\ell, 1], \tag{1}$$

for some $\Upsilon \subseteq \mathbb{R}^N$. Of course, the regulatory agency's aim is to choose Υ so as to minimize the financial fragility of the market, damaging as little as possible the economy. Different choices for Υ correspond to different bank capital regulatory proposals.

Therefore, the investment problem of financial institution h is[7]

$$\begin{array}{cc} \underset{\left(x^h, \boldsymbol{y}^h\right)}{\text{Max}} & \mathcal{E}\left(u^h\left(W_1^h\right)\right) \\ \text{s.a.} & q_0 x^h + \sum_{i=1}^N q_i y_i^h \leq q_0\theta_0^h + \sum_{i=1}^N q_i\theta_i^h, \\ & \boldsymbol{y}^h \in \Upsilon \end{array}$$

As the budget constraint is homogeneous of degree zero in prices, we can normalize, without loss of generality, the price of risk-free asset to $q_0 = 1$. Moreover, since u^h is strictly increasing, the budget constraint must be bind.

[7]Hereafter, when there isn't any doubt about the notation, for $x \in \mathbb{R}$ and $\boldsymbol{y} \in \mathbb{R}^N$ we write simply $(x, \boldsymbol{y})$ instead of $(x, \boldsymbol{y}')$.

The next lemma is a direct consequence of the properties of a continuous function defined on a compact set[8].

Lemma 1 *If Υ is compact and convex then the problem of financial institution has only one solution.*

A competitive equilibrium for the economy in question is an asset price vector $(q_0, \boldsymbol{q}) = (q_0, q_1, \ldots, q_N)$ and a mapping $h \in [\ell, 1] \mapsto \left(x^h, \boldsymbol{y}^h\right)$, such that

1. $\left(x^h, \boldsymbol{y}^h\right)$ solves the problem of financial institution h when assets prices are equal to $(q_0, \boldsymbol{q}')$.

2. Market clears, that is, $\int_\ell^1 \boldsymbol{y}^h \mathrm{d}h = \boldsymbol{\theta}$ and $\int_\ell^1 x^h \mathrm{d}h = \theta_0$, where $\boldsymbol{\theta} = \int_\ell^1 \boldsymbol{\theta}^h \mathrm{d}h$ is the aggregate amount of risky assets and $\theta_0 = \int_\ell^1 \theta_0^h \mathrm{d}h$ is the aggregate amount of risk-free asset.

3 VaR-Based Risk Constraint

Market risk is the risk that the value of an investment decreases due to changes in market factors (like equity and commodities prices, interest rates and rate of exchange). To asses the soundness of a financial institution it's fundamental to measure its exposure to market risk. In recent years, the risk metric knonw as VaR has become the major market risk metric for regulatory purposes as well as a standard industry tool. Following this trend we suppose that the regulatory agency make use of VaR to limit market risk of financial institutions. VaR is usually defined as

$$VaR^\alpha \equiv -\inf\left\{x \in \mathbb{R}; \mathcal{P}\left[W_1^h - \mathcal{E}\left(W_1^h\right) \leq x\right] > \alpha\right\}, \qquad (2)$$

where $\mathcal{P}$ is the probability measure corresponding to risky assets payoff distribution, $\mathcal{E}$ is the expected value relative to this measure and α is the significance level adopted (the probability of losses exceeding the VaR)[9].

[8] For an analysis of optimal portfolio choice with compact and convex constraints see Elsinger & Summer (1999).

[9] VaR when defined by Equation 2 is known as relative VaR, while the absolute VaR is the VaR defined without reference to the expected value (see Jorion, 2001).

In a simple way, VaR is the loss, which is exceeded with some given probability, α, over a given horizon. This easy interpretation is one of the reasons that justify the large use of VaR as standard market risk metric[10].

The risk constraint is fixed as an uniform upper bound to VaR, that is,

$$VaR^{\alpha} \leq \overline{VaR}, \tag{3}$$

where $\overline{VaR}$ is a VaR exogenous bound set by the regulatory agency. By using normal distribution properties, the risk constraint can be rewritten as an exogenous upper limit for the portfolio variance

$$\Upsilon = \left\{ \boldsymbol{y} \in \mathbb{R}^N ; \boldsymbol{y}' \boldsymbol{\Sigma} \boldsymbol{y} \leq \nu \right\}, \tag{4}$$

where the paramater ν, called nonseverity of the risk constraint, depends on α and $\overline{VaR}$.

The next proposition characterizes the solution of the problem of financial institutions. The demonstration of this proposition can be found in Daníelsson & Zigrand (2003).

Proposition 1 *Let $\left(x^h, \boldsymbol{y}^h\right)$ be the solution of the problem of financial institution h when the price vector of risky assets is $\boldsymbol{q}$. We have:*

1. *If $h \geq \sqrt{\frac{\rho}{\nu}}$ then*
$$\boldsymbol{y}^h = \frac{1}{h} \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{\mu} - r_0 \boldsymbol{q} \right), \tag{5}$$
where $\rho = \left(\boldsymbol{\mu} - r_0 \boldsymbol{q} \right)' \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{\mu} - r_0 \boldsymbol{q} \right)$ and r_0 is the risk-free rate.

2. *If $h < \sqrt{\frac{\rho}{\nu}}$ then*
$$\boldsymbol{y}^h = \sqrt{\frac{\nu}{\rho}} \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{\mu} - r_0 \boldsymbol{q} \right). \tag{6}$$

In any case $x^h = \theta_0^h + \sum_i q_i \theta_i^h - \sum_i q_i y_i^h$.

[10] In spite of its widespread adoption, VaR is not without controversy. Its major problem relies on the fact that it is not a coherent measure of risk (VaR fails the sub-additive property, see Artzner et al., 1999). Besides, Kerkhof & Melenberg (2004) use a backtesting procedure to show that expected shortfall, a coherent measure of risk, produces better results than VaR.

Note that the introduction of the risk constraint prevents optimal risk sharing since all institutions with CARA less than or equal to $\sqrt{\frac{\rho}{\nu}}$ choose the same portfolio.

After solving the problem of the financial institutions, the market clears condition automatically provides the equilibrium prices, as presented in the following proposition (again, the demonstration is in Daníelsson & Zigrand, 2003).

Proposition 2 *Suppose that $R_i > r_0$ for all $i = 1, \ldots, N$. Then, the equilibrium price vector of risky assets is*

$$\boldsymbol{q} = \frac{1}{r_0}\left(\boldsymbol{\mu} - \Psi \boldsymbol{\Sigma}\boldsymbol{\theta}\right), \tag{7}$$

where Ψ is the market price of risk scalar (see Daníelsson & Zigrand, 2003). Denoting by $F(\cdot)$ the non-principal branch of the Lambert correspondence[11]*, we have*

$$\Psi = \begin{cases} \frac{1}{\ln \ell^{-1}} & \text{if} \quad 0 \leq \kappa \leq \ell \ln \ell^{-1} \\ -\frac{\kappa+\ell}{\kappa F(-(\kappa+\ell)e^{-1})} & \text{if} \quad \ell \ln \ell^{-1} < \kappa < 1-\ell \\ \text{any number} \geq \frac{1}{1-\ell} & \text{if} \quad \kappa = 1-\ell, \end{cases} \tag{8}$$

where

$$\kappa = \sqrt{\frac{\boldsymbol{\theta}'\boldsymbol{\Sigma}\boldsymbol{\theta}}{\nu}}.$$

An equilibrium fails to exist if $\kappa > 1 - \ell$.

Figure 1 illustrates Ψ as a function of κ. When $\kappa = 1-\ell$ the equilibrium is undetermined. If equilibrium exists and at least one institution hits the risk constraint then $\ell \ln \ell^{-1} < \kappa < 1 - \ell$, hence Ψ is a strictly increasing function of κ and consequently a strictly decreasing function of ν. This implies that the tighter is the regulation (that is, the smaller is ν) the less will be the risky assets equilibrium prices as highlighted in Daníelsson & Zigrand (2003).

[11] The non-principal branch of the Lambert correspondence is the inverse of the function $f : (-\infty, -1] \mapsto \left[-e^{-1}, 0\right)$ defined by $f(x) = xe^x$. For more details and properties of the Lambert correspondence see Corless et al (1996).

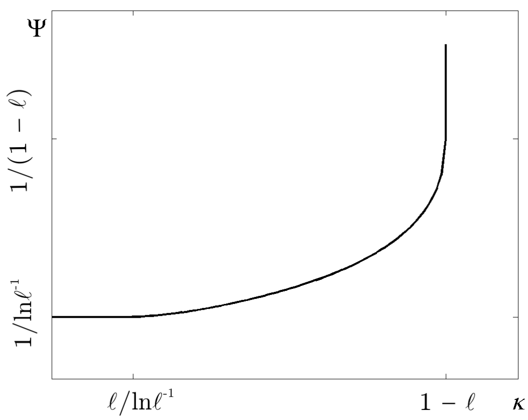

Figure 1: Illustration of Ψ.

4 Welfare of Financial Institutions

To measure the financial institutions welfare we suppose that we have a linear-in-utilitily social welfare function, also called Bergson welfare function (see Varian, 1992), in which the weigth of each agent is equal to the inverse of its CARA. That is, we suppose that the regulatory agency consider more important the financial institutions less risk averse. Of course, other schemes can be considered such as assigning the same weights for all institutions or assigning higher weights to the more risk averse institutions.

Definition 1 *Let $\left\{\left(x^h, \boldsymbol{y}^h\right)\right\}_{h\in[\ell,1]}$ be an equilibrium allocation for the economy under analysis. We define the financial institutions welfare function by:*

$$\Lambda_f(\nu) \equiv -\int_\ell^1 \frac{\ln\left\{-\mathcal{E}\left[u^h\left(W_1^h\right)\right]\right\}}{h}\mathrm{d}h.$$

Proposition 3 *Suppose that for the economy considered here equilibrium exists and at least one financial institution hits the risk constraint. Then the financial institutions welfare function is given by:*

$$\Lambda_f(\nu) = r_0\theta_0 + \mu\theta + \frac{\theta'\Sigma\theta}{4\kappa^2}\left[(\kappa\Psi)^2 - (\ell+\kappa)\kappa\Psi + \ell^2\right]$$

Proposition 4 *If equilibrium exists and at least one financial institution hits the risk constraint, the financial institutions welfare function is increasing in* ν*.*

Proposition 4 tells us that the tighter is the risk regulation the lower is the welfare of financial institutions as a whole. But, what happens at individual level? Would it be possible for a financial institution to increase its welfare in a regulated economy? Proposition 5 (below) states that, under certain conditions, the answer to the last question is positive. The intuition is immediate: At a regulated economy, agents little risk averse decrease their positions in riskier assets, then prices of these assets fall, which makes interesting for other agents to buy them, thus increasing these agents' utility. Therefore, each financial institution maximizes its utility for a certain value of the nonseverity parameter that doesn't correspond necessarily to the situation of an unregulated economy ($\nu = \infty$). Before presenting Proposition 5 we are going to estabilish some preliminary calculations and notations.

Denote by $\overline{\nu}$ the maximum value of ν such as at least one institution hits the risk constraint and by $\underline{\nu}$ the lower value of ν that equilibrium exists. In other words,

$$\overline{\nu} = \frac{\boldsymbol{\theta}'\boldsymbol{\Sigma}\boldsymbol{\theta}}{(\ell \ln \ell^{-1})^2} \quad \text{and} \quad \underline{\nu} = \frac{\boldsymbol{\theta}'\boldsymbol{\Sigma}\boldsymbol{\theta}}{(1-\ell)^2}.$$

Consider the following functions:

1. $g_1(\nu) : [\underline{\nu}, \overline{\nu}] \mapsto [\ell, 1]$, defined by $g_1(\nu) = \kappa\Psi + \kappa^3\Psi'(\kappa)\left(\frac{1}{1-\ell} - \frac{1}{\kappa}\right)$,
2. $g_2(\nu) : [\underline{\nu}, \overline{\nu}] \mapsto [\ell, 1]$, defined by $g_2(\nu) = \kappa\Psi$ and
3. $g_3(\nu) : [\underline{\nu}, \overline{\nu}] \mapsto \left[\frac{1-\ell}{\ln \ell^{-1}}, 1\right]$, defined by $g_3(\nu) = \Psi(1-\ell)$;

where $\Psi'(\kappa)$ is the derivative of Ψ, that is

$$\Psi'(\kappa) = \frac{1}{\kappa F\left(-(\kappa+\ell)\,\mathrm{e}^{-1}\right)}\left[\frac{\ell}{\kappa} + \frac{1}{F\left(-(\kappa+\ell)\,\mathrm{e}^{-1}\right)+1}\right].$$

It is easy to see that $g_1(\underline{\nu}) = g_2(\underline{\nu}) = g_3(\underline{\nu}) = 1$. Since κ, Ψ and Ψ' are strictly decreasing functions of ν[12] we have that g_1, g_2 and g_3 are strictly decreasing function of ν too. Figure 2 shows graphs of these three functions.

[12] Ψ' is a decreasing function of ν because $\Psi(\kappa)$ is a convex function, thus $\Psi''(\kappa) > 0$. Hence $\Psi'(\kappa)$ is increasing in κ and therefore decreasing in ν.

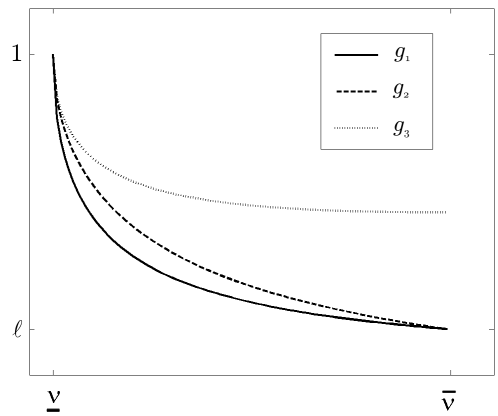

Figure 2: Graphs of functions g_1, g_2 and g_3.

If we fix the market parameters ($\boldsymbol{\Sigma}$ and $\boldsymbol{\mu}$) then the welfare of financial institution h is given by its expected utility at $t = 1$:

$$\mathcal{E}\left(u^h\left(W_1^h\right)\right) = r_0\left(\theta_0^h + \boldsymbol{q}\boldsymbol{\theta}^h - \boldsymbol{q}\boldsymbol{y}^h\right) + \boldsymbol{\mu}\boldsymbol{y}^h - h\frac{\boldsymbol{y}^{h\prime}\Sigma\boldsymbol{y}^h}{2}.$$

Therefore, in equilbrium, the welfare of institution h depends on the nonseverity parameter ν. If the aggregate endowment of the risky assets is uniformly distributed between all agents (that is, $\boldsymbol{\theta}^h = \frac{\boldsymbol{\theta}}{1-\ell}$) then, after some algebraic manipulations, it is possible to show that analyzing the welfare of institution h as function of ν is equivalent to studying the function $f^h(\nu)$: $[\underline{\nu}, \overline{\nu}] \mapsto \mathbb{R}$ defined by:

$$f^h(\nu) = \begin{cases} \frac{\Psi^2}{2h} - \frac{\Psi}{1-\ell} & \text{if } \nu \geq g_2^{-1}(h) \\ \\ \frac{\Psi}{\kappa} - \frac{h}{2\kappa^2} - \frac{\Psi}{1-\ell} & \text{if } \nu < g_2^{-1}(h) \end{cases} \tag{9}$$

The higher is $f^h(\nu)$, the greater is the welfare of institution h.

Now we are able to present the main result of this Section.

Proposition 5 *Let $f^h(\nu)$ defined by Equation 9, then:*

1. *For $\frac{1-\ell}{\ln \ell^{-1}} < h \leq 1$ we have*
   - *If $g_3^{-1}(h) < \nu \leq \overline{\nu}$ then $f^h(\nu)$ is strictly increasing.*

- *If* $g_2^{-1}(h) < \nu \leq g_3^{-1}(h)$ *then* $f^h(\nu)$ *is strictly decreasing.*
- *If* $g_1^{-1}(h) < \nu \leq g_2^{-1}(h)$ *then* $f^h(\nu)$ *is strictly decreasing.*
- *If* $\underline{\nu} < \nu \leq g_1^{-1}(h)$ *then* $f^h(\nu)$ *is strictly increasing.*

2. *For* $\ell \leq h \leq \frac{1-\ell}{\ln \ell^{-1}}$ *we have*
   - *If* $g_2^{-1}(h) < \nu \leq \overline{\nu}$ *then* $f^h(\nu)$ *is strictly decreasing.*
   - *If* $g_1^{-1}(h) < \nu \leq g_2^{-1}(h)$ *then* $f^h(\nu)$ *is strictly decreasing.*
   - *If* $\underline{\nu} < \nu \leq g_1^{-1}(h)$ *then* $f^h(\nu)$ *is strictly increasing.*

In any case $f^h(\overline{\nu}) = \frac{1}{\ln \ell^{-1}}\left(\frac{1}{2h \ln \ell^{-1}} - \frac{1}{1-\ell}\right)$ *and* $f^h(\underline{\nu}) = -\frac{h}{2(1-\ell)^2}$

The next proposition shows that between the tightest level ($\nu = \underline{\nu}$) and the softest level ($\nu = \overline{\nu}$) of regulation, all financial institutions prefer the last one.

Proposition 6 *For all* h *we have* $f^h(\overline{\nu}) \geq f^h(\underline{\nu})$.

By Proposition 6 we have that if $\ell \leq h \leq \frac{1-\ell}{\ln \ell^{-1}}$ then the maximum of $f^h(\nu)$ occurs when $\nu = g_1^{-1}(h)$. However, if $\frac{1-\ell}{\ln \ell^{-1}} < h \leq 1$ there are two possible candidates for the maximum of $f^h(\nu)$: the same $g_1^{-1}(h)$ or $\overline{\nu}$. The next proposition gives conditions that allow us to decide in which of these points the function $f^h(\nu)$ assumes its maximum.

Proposition 7 *Let* $t(h) : \left[\frac{1-\ell}{\ln \ell^{-1}}, 1\right] \mapsto \mathbb{R}$ *defined by*

$$t(h) = \frac{\Psi}{\kappa} - \frac{h}{2\kappa^2} - \frac{\Psi}{1-\ell} - \frac{1}{\ln \ell^{-1}}\left(\frac{1}{2h \ln \ell^{-1}} - \frac{1}{1-\ell}\right),$$

where κ *and* Ψ *are calculated at* $\nu = g_1^{-1}(h)$. *The function* $t(h)$ *is strictly decreasing and has only one root. Denoting by* h^* *this root we have*

1. *If* $\frac{1-\ell}{\ln \ell^{-1}} \leq h \leq h^*$ *then the maximum of* $f^h(\nu)$ *occurs when* $\nu = g_1^{-1}(h)$.
2. *If* $h^* \leq h \leq 1$ *then the maximum of* $f^h(\nu)$ *occurs when* $\nu = \overline{\nu}$.

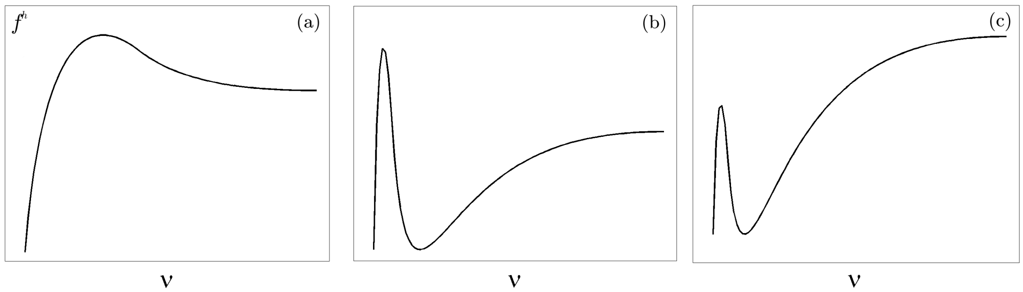

Figure 3: Function f^h. At (a) $h \in \left[\ell, \frac{1-\ell}{\ln \ell^{-1}}\right]$, at (b) $h \in \left[\frac{1-\ell}{\ln \ell^{-1}}, h^*\right]$ and at (c) $h \in [h^*, 1]$.

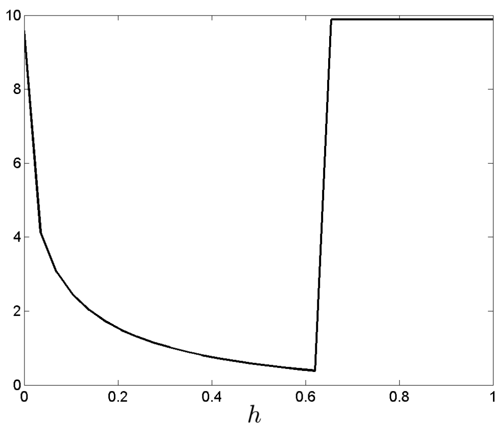

Figure 4: Optimum level of regulation (ν) as a function of h.

Figure 3 illustrates the graphs of $f^h(\nu)$ for $h \in \left[\ell, \frac{1-\ell}{\ln \ell^{-1}}\right]$, $h \in \left[\frac{1-\ell}{\ln \ell^{-1}}, h^*\right]$ and $h \in [h^*, 1]$.

Observe that if $h < h^*$ the financial institution h prefers the regulation to be fixed in a specific level $\nu < \overline{\nu}$. If $h > h^*$ then financial institutions h prefers no regulation (that is, $\nu \geq \overline{\nu}$). The reasoning behind it is very simple: to get benefit with the regulation these financial institutions would prefer a level of regulation tighter than $\underline{\nu}$, but in this case there isn't equilibrium. Since it is impossible, they have no gain with regulation hence they prefer $\nu = \overline{\nu}$. Figure 4 shows the optimum ν as a function of h.

5 Bankruptcy Probability

The financial institution h goes to bankrupty if its wealth at $t = 1$ is less than or equal to zero. If equilibrium exists and at least one institution reaches the risk constraint the probability of this to occur is

$$pb^h \equiv \mathcal{P}\left[W_1^h < 0\right] = \Phi\left(-\frac{m^h}{s^h}\right),$$

where $m^h = r_0 W_0^h + \Psi \boldsymbol{\theta}' \boldsymbol{\Sigma} y^h$ and $s^h = \sqrt{y^{h\prime} \boldsymbol{\Sigma} y^h}$ are, respectively, the mean and the standard deviation of W_1^h, and Φ represents the cumulative standard normal distribution function. Since Φ is strictly increasing, to analyze the behavior of pb^h as a function of the nonseverity parameter ν, it is enough to study how $\frac{m^h}{s^h}$ varies when the regulatory agency modifies ν. The greater is this quotient, the less is the default probability of institution h. Using Propositions 1 and 2 it is easy to see that in equilibrium we have

1. If $h < g_2(\nu)$ then
$$\frac{m^h}{s^h} = \frac{\kappa r_0 W_0^h}{\sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}}} + \Psi \sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}}.$$

2. If $h \geq g_2(\nu)$ then
$$\frac{m^h}{s^h} = \frac{r_0 W_0^h h}{\Psi \sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}}} + \Psi \sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}}.$$

For the purpose of comparison, the value of this quotient in an unregulated economy is

$$\frac{m^h}{s^h} = \frac{r_0 W_0^h h \ln \ell^{-1}}{\sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}}} + \frac{\sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}}}{\ln \ell^{-1}} \quad \forall h.$$

Proposition 8 *Assume that equilibrium exists and at least one institution hits the risk constraint. Let $\tilde{\nu}$ be the nonseverity parameter value such as $\Psi = \tilde{\Psi}$, where $\tilde{\Psi} \equiv \sqrt{\frac{h r_0 W_0^h}{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}}}$. That is, considering Ψ as function of ν we have $\tilde{\nu} = \Psi^{-1}\left(\tilde{\Psi}\right)$ (if $\tilde{\Psi} \leq \frac{1}{\ln \ell^{-1}}$ set $\tilde{\nu} = \overline{\nu}$ and if $\tilde{\Psi} \geq \frac{1}{1-\ell}$ set $\tilde{\nu} = \underline{\nu}$).*

1. *If* $\tilde{\nu} \leq g_2^{-1}(h)$ *then* $\frac{m^h}{s^h}$ *is a decreasing function of* ν *on the interval* $\left[\underline{\nu}, g_2^{-1}(h)\right]$ *and is an increasing function of* ν *on the interval* $\left[g_2^{-1}(h), \overline{\nu}\right]$.

2. *If* $g_2^{-1}(h) < \tilde{\nu} \leq \overline{\nu}$ *then* $\frac{m^h}{s^h}$ *is a decreasing function of* ν *on the interval* $[\underline{\nu}, \tilde{\nu}]$ *and is an increasing function of* ν *on the interval* $[\tilde{\nu}, \overline{\nu}]$.

3. *If* $\tilde{\nu} > \overline{\nu}$ *then* $\frac{m^h}{s^h}$ *is a decreasing function of* ν.

Proposition 8 gives interesting conclusions on the effectiveness of the risk regulation (effectiveness is understood here as the reduction of the bankruptcy probability):

1. The greater is W_0^h, the less is $\tilde{\nu}$. Then if the institution is highly capitalized, the regulation can increase its bankruptcy probability. On the other hand, if the net worth of an institution is small, then, from the regulatory agency point of view, the regulation is always beneficial, since the more severe it is, the less is the default probability of the institution.

2. The more nervous is the market, the more effective will be the regulation.

3. The regulation is more effective for the institutions little risk averse (small h). If the institution will be super conservative then the regulation can increase its bankruptcy probability.

Figure 5 presents the graphs of $\frac{m^h}{s^h}$ (solid line) for cases 1 and 3 of Proposition 8. The horizontal dash-dot line represents the same relation in an unregulated economy.

Evidently, the regulatory agency must consider the system as a whole and not an institution in particular. Therefore, it is interesting to analyze the total bankruptcy probability, defined as the sum (integral) of the default probability of all institutions,

$$pgb \equiv \int_{\ell}^{1} pb^h \mathrm{d}h. \tag{10}$$

Directly related (and more treatable from the algebraic point of view) with the metric defined by Equation 10 is the integral in h of the quotient $\frac{m^h}{s^h}$,

$$\Lambda_s(\nu) \equiv \int_{\ell}^{1} \frac{m^h}{s^h} \mathrm{d}h. \tag{11}$$

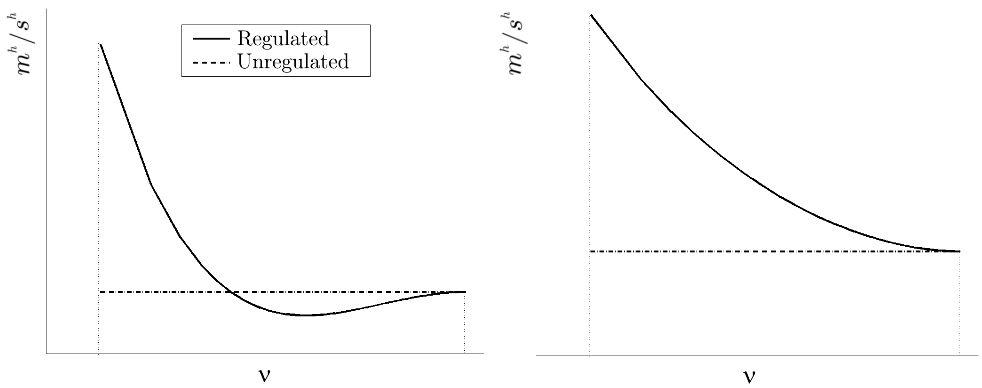

Figure 5: Graphs of the function $\frac{m^h}{s^h}$. In (a) $\tilde{\nu} \leq g_2^{-1}(h)$ and in (b) $\tilde{\nu} > \overline{\nu}$.

If the initial endowment of the assets is uniformly distributed between the agents, then $W_0^h = W_0$ for all h. In this case

$$\Lambda_s(\nu) = \frac{r_0 W_0}{\sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}}} \left(\frac{\kappa^2 \Psi}{2} + \frac{1}{2\Psi} - \kappa \ell \right) + \Psi (1 - \ell) \sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}}. \qquad (12)$$

The first and the second terms of the left side of Equation 12 are, respectively, increasing and decreasing functions of ν. Then the phenomenon already observed individually happens again in global level: If the level of capitalization of the financial institutions is high or the degree of market nervousness is low, then the regulation can have contrary effect to the planned (that is, to increase the financial fragility of the institutions). On the other hand, if the institutions have a small initial wealth or the market is nervous then the risk regulation presents the benefit to diminish the number of bankruptcies. Figure 6 shows these two situations.

6 Financial Market Contagion

In this section we analyze the problem of financial market contagion. Contagion is the transmission of shocks to other financial institutions, beyond any fundamental link among the institutions and beyond common shocks. Contagion can take place both during “good times” and “bad times”. Then, contagion does not need to be related to crises. However, contagion has been emphasized during crisis times. Examples of recent contagious episodes are

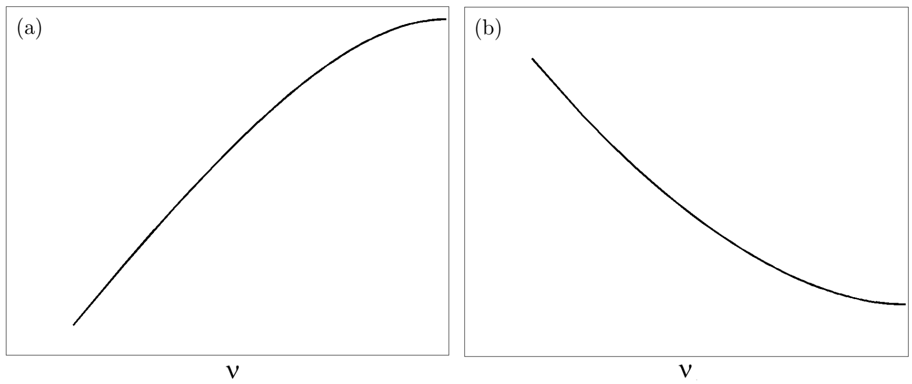

Figure 6: Graphs of the function Λ_s. In letter (a) the level of capitalization of the financial institutions is high and in letter (b) the opposite occurs.

the Tequila crisis of 1994-95, the East Asian crisis of 1997 and the Russian crisis of 1998 (for details about these episodes see Kaminsky & Reinhart, 1998).

Based on the model presented in Section 2 we develop a new approach to evaluate the contagion in an economy in which financial institutions are subject to VaR-based risk constraint. We are not aware of any work that studies this question from an equilibrium point of view[13]. To introduce the possibility of contagion we increase the portfolios space of each financial institution allowing investments among them. To avoid an infinite dimensional optimization problem, instead of a continuum of financial institutions considered in the basic model we suppose that there is a finite number of them. Let's describe in more details a simple version of the contagion model where there is only three financial institutions. Generalizations of this particular case are immediate.

Consider a two period economy with three financial institutions A, B and C. There is two risky assets with payoff $\boldsymbol{d}$ normaly distributed. To make investments of one financial institution in another interesting we have to introduce a friction on the market. There are many ways to do this. Here we opt to prevent that some financial institutions have access to the whole financial market. More specifically, financial institution C can invest in both

[13] Tsomocos (2003) characterizes contagion and financial fragility as an equilibrium phenomenon but he doesn't analyze the properties of this equilibrium.

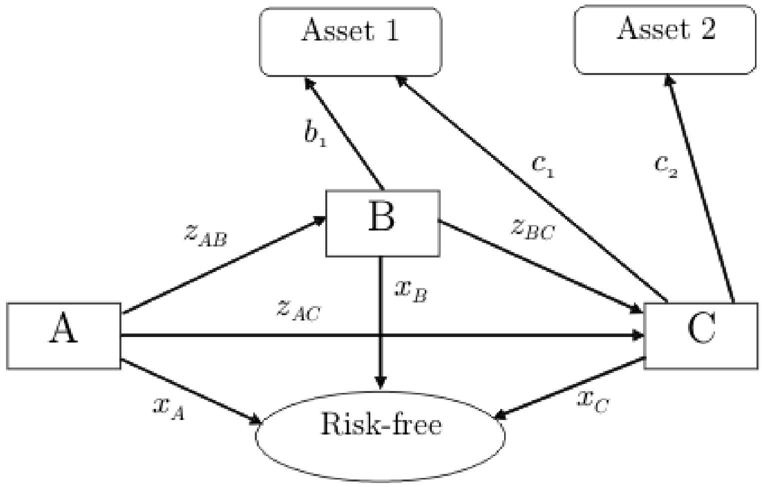

Figure 7: Financial market contagion model.

risky assets and in the risk-free asset. Its portfolio is (x_c, c_1, c_2), where x_c is number of units of the risk-free asset held by C and $\boldsymbol{c} = (c_1, c_2)'$ is the risky asset portfolio of C. The initial endowment of C is $(\theta_0^C, \theta_1^C, \theta_2^C)$. Financial institution B can invest in the risky asset 1, in the risk-free asset and in financial institution C. Its portfolio is (x_B, b_1, z_{BC}) where z_{BC} is the sharing of B in C and its initial endowment is (θ_0^B, θ_1^B). Finally, financial institution A can invest only in B and C and in the risk-free asset. Its portfolio is (x_A, z_{AB}, z_{AC}) where z_{AB} is the sharing of A in B and z_{AC} is the sharing of A in C and its initial endowment is θ_0^A. The CARA of these financial institutions are h_C, h_B and h_A, respectively. To avoid situations where a financial institution fully buy another institution we suppose $h_A \geq h_B \geq h_C$. Figure 7 illustrate the model.

The wealths of institutions at $t = 1$ are

$$W_C = r_0 x_C + \boldsymbol{d} \cdot \boldsymbol{c},$$

$$\begin{aligned} W_B &= r_0 x_B + d_1 b_1 + z_{BC} W_C \\ &= r_0 x_B + z_{BC} r_0 x_C + \boldsymbol{d} \cdot \boldsymbol{\beta} \quad \text{and} \end{aligned}$$

$$\begin{aligned} W_A &= r_0 x_A + z_{AB} W_B + z_{AC} W_C \\ &= r_0 x_A + z_{AB} r_0 x_B + z_{AB} z_{BC} r_0 x_C + z_{AC} r_0 x_C + \boldsymbol{d} \cdot \boldsymbol{\alpha}, \end{aligned}$$

where

$$\boldsymbol{\beta} = \begin{pmatrix} b_1 + z_{BC}c_1 \\ z_{BC}c_2 \end{pmatrix}$$

and

$$\boldsymbol{\alpha} = \begin{pmatrix} z_{AB}\beta_1 + z_{AC}c_1 \\ z_{AB}\beta_2 + z_{AC}c_2 \end{pmatrix}$$

The budget constraints for A, B and C are respectively:

$$x_A + z_{AC}\tfrac{K_P^C}{1-z_{AB}-z_{BC}} + z_{AB}\tfrac{K_P^C}{1-z_{AB}} = \theta_0^A,$$

$$x_B + q_1 b_1 + z_{BC}\tfrac{K_P^C}{1-z_{AC}-z_{BC}} = \tfrac{K_P^B}{1-z_{AB}} \quad \text{and}$$

$$x_C + \boldsymbol{q}\cdot\boldsymbol{c} = \tfrac{K_P^C}{1-z_{AC}-z_{BC}},$$

where $K_P^C = \theta_0^C + \boldsymbol{q}\cdot\boldsymbol{\theta}$ and $K_P^B = \theta_0^B + q_1\theta_1^B$ are the equity capital of C and B, respectively.

The risk constraints for A, B and C are respectively:

$$\boldsymbol{c}'\boldsymbol{\Sigma}\boldsymbol{c} \leq \nu,$$

$$\boldsymbol{\beta}'\boldsymbol{\Sigma}\boldsymbol{\beta} \leq \nu \quad \text{and}$$

$$\boldsymbol{\alpha}'\boldsymbol{\Sigma}\boldsymbol{\alpha} \leq \nu.$$

Each institution maximizes the expected value of its wealth utility subject to the budget and risk constraints. To solve the problem of financial institutions we proceed in the same way that was done in Section 2. If the risky asset price is $\boldsymbol{q}$, then the optimum portfolios are:

1. For financial institution C:

   (a) If $h_C \geq \sqrt{\frac{\rho_C}{\nu}}$ then

   $$\boldsymbol{c} = \frac{1}{h_C}\boldsymbol{\Sigma}^{-1}\boldsymbol{e}_C, \tag{13}$$

   where $\rho_C = \boldsymbol{e}_C'\boldsymbol{\Sigma}^{-1}\boldsymbol{e}_C$ and $\boldsymbol{e}_C = (\boldsymbol{\mu} - r_0\boldsymbol{q})$.

   (b) If $h_C < \sqrt{\frac{\rho_C}{\nu}}$ then

   $$\boldsymbol{c} = \sqrt{\frac{\nu}{\rho_C}}\boldsymbol{\Sigma}^{-1}\boldsymbol{e}_C, \tag{14}$$

2. For financial institution B:

   (a) If $h_B \geq \sqrt{\frac{\rho_B}{\nu}}$ then

$$\boldsymbol{\beta} = \frac{1}{h_B}\boldsymbol{\Sigma}_B^{-1}\boldsymbol{e}_B, \tag{15}$$

   where $\rho_B = \boldsymbol{e}'_B\left(\boldsymbol{B\Sigma B}'\right)^{-1}\boldsymbol{e}_B$, $\boldsymbol{e}_B = (\mu_1 - r_0 q_1, (\boldsymbol{\mu} - r_0\boldsymbol{q})\cdot\boldsymbol{c})'$, $\boldsymbol{\Sigma}_B = \boldsymbol{B\Sigma}$ and

$$\boldsymbol{B} = \left[\begin{array}{cc} 1 & 0 \\ \\ c_1 & c_2 \end{array}\right].$$

   (b) If $h_B < \sqrt{\frac{\rho_B}{\nu}}$ then

$$\boldsymbol{\beta} = \sqrt{\frac{\nu}{\rho_B}}\boldsymbol{\Sigma}_B^{-1}\boldsymbol{e}_B, \tag{16}$$

3. For financial institution A:

   (a) If $h_A \geq \sqrt{\frac{\rho_A}{\nu}}$ then

$$\boldsymbol{\beta} = \frac{1}{h_A}\boldsymbol{\Sigma}_A^{-1}\boldsymbol{e}_A, \tag{17}$$

   where $\rho_A = \boldsymbol{e}'_A\left(\boldsymbol{A\Sigma A}'\right)^{-1}\boldsymbol{e}_A$,
   $\boldsymbol{e}_A = ((\mu_1 - r_0 q_1)b_1 + z_{BC}(\boldsymbol{\mu} - r_0\boldsymbol{q})\cdot\boldsymbol{c}, (\boldsymbol{\mu} - r_0\boldsymbol{q})\cdot\boldsymbol{c})'$, $\boldsymbol{\Sigma}_A = \boldsymbol{A\Sigma}$ and

$$\boldsymbol{A} = \left[\begin{array}{cc} \beta_1 & \beta_2 \\ \\ c_1 & c_2 \end{array}\right].$$

   (b) If $h_A < \sqrt{\frac{\rho_A}{\nu}}$ then

$$\boldsymbol{\beta} = \sqrt{\frac{\nu}{\rho_A}}\boldsymbol{\Sigma}_A^{-1}\boldsymbol{e}_A, \tag{18}$$

To find the equilibrium prices we have to use the market clears condition:

$$\begin{aligned} c_1 + b_1 &= \theta_1 = \theta_1^B + \theta_1^C \\ c_2 &= \theta_2 = \theta_2^C \end{aligned}$$

To solve this system we have to use the MatLab *fsolve* function since the system is non-linear and there isn't a closed form solution.

Now we are ready to define metrics of contagion that allow us to evaluate the impact of risk regulation on the financial institutions contagion. Since W_A, W_B and W_C are normal with mean and variance known it is easy to compute the following probabilities:

$$p_A \equiv \mathcal{P}\left[W_A \leq 0\right],$$

$$p_B \equiv \mathcal{P}\left[W_B \leq 0\right] \quad \text{and}$$

$$p_C \equiv \mathcal{P}\left[W_C \leq 0\right].$$

Besides the probabilities above, to measure the contagion we need to compute conditional probabilities like $\mathcal{P}\left[W_i \leq 0 \cap W_j \leq 0\right]$ where $i, j = A, B, C$. Let D_{BC} be the region of the payoff plane such as

$$\boldsymbol{c} \cdot \boldsymbol{d} \leq -r_0 x_C$$

$$\boldsymbol{\beta} \cdot \boldsymbol{d} \leq -r_0 x_B - r_0 z_{BC} x_C$$

then

$$p_{BC} \equiv \mathcal{P}\left[W_B \leq 0 \cap W_C \leq 0\right] = \int_{D_{BC}} N\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right),$$

where $N\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right)$ is the density probability function of a bidimensional normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$.

Let D_{AC} be the region of the payoff plane such as

$$\boldsymbol{c} \cdot \boldsymbol{d} \leq -r_0 x_C$$

$$\boldsymbol{\alpha} \cdot \boldsymbol{d} \leq -r_0 x_A - r_0 z_{AB} x_B - r_0 (z_{AC} + z_{AB} z_{BC})$$

then

$$p_{AC} \equiv \mathcal{P}\left[W_A \leq 0 \cap W_C \leq 0\right] = \int_{D_{AC}} N\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right).$$

Finally, let D_{AB} be the region of the payoff plane such as

$$\boldsymbol{\beta} \cdot \boldsymbol{d} \leq -r_0 x_B - r_0 z_{BC} x_C$$

$$\boldsymbol{\alpha} \cdot \boldsymbol{d} \leq -r_0 x_A - r_0 z_{AB} x_B - r_0 (z_{AC} + z_{AB} z_{BC})$$

then

$$p_{AB} \equiv \mathcal{P}\left[W_A \leq 0 \cap W_B \leq 0\right] = \int_{D_{AB}} N\left(\boldsymbol{\mu}, \boldsymbol{\Sigma}\right).$$

We define the contagion metric of institution i on institution j ($i > j$ in a lexicographic order $i, j = A, B, C$) by the bankruptcy probability of j conditional on the bankruptcy probability of i, that is

$$\mathcal{C}_{CB} = \mathcal{P}\left[W_B \leq 0 |\, W_C \leq 0\right] = \frac{p_{BC}}{p_C} \tag{19}$$

$$\mathcal{C}_{CA} = \mathcal{P}\left[W_A \leq 0 |\, W_C \leq 0\right] = \frac{p_{AC}}{p_C} \tag{20}$$

$$\mathcal{C}_{BA} = \mathcal{P}\left[W_A \leq 0 |\, W_B \leq 0\right] = \frac{p_{AB}}{p_B} \tag{21}$$

The contagion metrics $\mathcal{C}_{CB}$, $\mathcal{C}_{AC}$ and $\mathcal{C}_{AB}$ are increasing functions of the nonseverity parameter ν. In other words the tighter is the regulation the smaller is the contagion. Figure 8 illutrates $\mathcal{C}_{CB}$ for $\ell = 0.0011$, $\boldsymbol{\theta} = (1.5, 0.9)'$, $\boldsymbol{\mu} = (1.5, 1.2)'$, $r_0 = 1.00013$, $h_A = 0.5$, $h_B = 0.4$, $h_C = 0.1$ and

$$\boldsymbol{\Sigma} = \begin{bmatrix} 0.6 & 0.25 \\ 0.25 & 0.4 \end{bmatrix}.$$

In Section 4 we show that for each institution there is a value of ν that maximizes its utility. The same question about the optimum level of regulation for each financial institution can be done to an economy with possibility of contagion. Let ν_h be the value of the nonseverity parameter that maximizes the utility function of financial institution h. That is,

$$\nu_h \quad \in \quad \text{Arg Máx}\, \mathcal{E}\left[u^h(W_h)\right], \tag{22}$$

where W_h is the wealth of financial institution h at $t = 1$ in an equilibrium allocation.

Of course, since financial institution C is the less risk averse of all, it has no benefit with regulation, that is, $\nu_{h_C} = \infty$. For institutions B and C ν_h depends on factors like market conditions and the difference among the coefficients of risk aversion. Let's analyze in more details ν_{h_B} when these factors varies. The analysis of ν_{h_A} is very similar and we don't show it here.

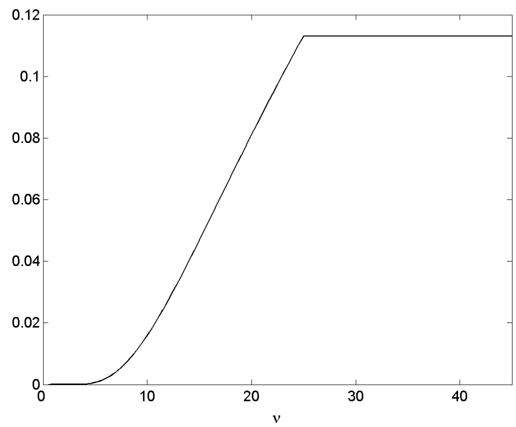

Figure 8: Contagion probability of C on B.

On the one hand financial institution B prefer a tight level of regulation. For example, since B invests on C, B would like that C doesn't take excessive risk to prevent that C go to bankrupt. Also, if asset 1 volatility is greater than asset 2 volatility then regulation is beneficial to B since the only way that B has to invest on asset 2 is investing in C and the regulation can lead C to concentrate its portfolio on asset 2. On the other hand, it is possible, for example, that institution B wishes to invest a large amount on asset 2 and the preferences of B and C are very similar (that is, $h_B \approx h_C$). But asset 2 is accessible only to institution C which has an upper limit on its investments in asset 2 and institution B has an upper limit on its investments in institution C. Then, in equilibrium, the number of units of asset 2 effectively hold by institution B can be smaller than in unregulated economy. In this case institution B prefers a soft level of regulation.

Table 1 shows values of ν_B as a function of h_B, h_C, asset 1 variance and asset 2 variance[14]. The other model parameters are fixed and equal to the values used in the exercise described in Figure 8.

The tightest level of regulation that B prefers occurs when the difference between h_B and h_C is great and asset 1 is much more volatile than asset 2. In this case financial institution B strongly prefer regulation because it makes the portfolio of C concentrated on asset 2. But if the preferences

[14]Since there isn't a closed form solution to the problem defined by Equation 22, in this example we use numerical methods to solve it.

h_B	h_C	σ_1^2	σ_2^2	ν_B
0.4	0.1	0.8	0.2	1.12
0.4	0.1	0.6	0.4	1.88
0.4	0.1	0.2	0.6	2.43
0.3	0.26	0.2	0.25	4.57
0.27	0.26	0.01	0.25	7.93

Table 1: Different values of ν_B as a function of the model's parameters.

of B and C are very similar and the asset 1 variance is very small then regulation is undesirable for financial institution B since it adversely affects C and consequently B too.

Observe that in an economy with contagion and an incomplete asset structure, institution B wants that institution C has preferences very similar to its own. But we showed in Section 2 that regulation can affect the effective degree of risk aversion. Then what B wishes is that the regulation makes the C effective degree of risk aversion equal to its own.

7 RAA-Based Risk Constraint

In this Section we study the economic effects of the capital requirement for covering risk based on the RAA scheme. The model is the same presented in Section 2. This model, despite its simplicity, is sufficiently flexible to cover a series of interesting situations. In contrast to the deep analysis of VaR-based risk regulation, in the study of RAA-based regulation we will work in a more informal way. The main conclusions will be extracted from simple numerical examples.

The Basel proposal for covering credit risk consists in using what is known by Risk-Adjusted Assets. Basically, the idea is to separate the assets of the financial institutions in I groups and to apply in each group an asset-specific risk weight. The positions bought and sold in different assets must be added in absolute value. The result of this account is the RAA. The RAA must be less than or equal to a fraction of the institution net worth.

In this case, the risk constraint assumes the following form:

$$\Upsilon = \left\{ \boldsymbol{y} \in \mathbb{R}^N ; \beta_1 \left| q_1 y_1 \right| + \ldots + \beta_N \left| q_N y_N \right| \leq W_0^h \right\},$$

where $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_N) \in \mathbb{R}^N_{++}$ are weight factors. Of course, if $N > I$ then

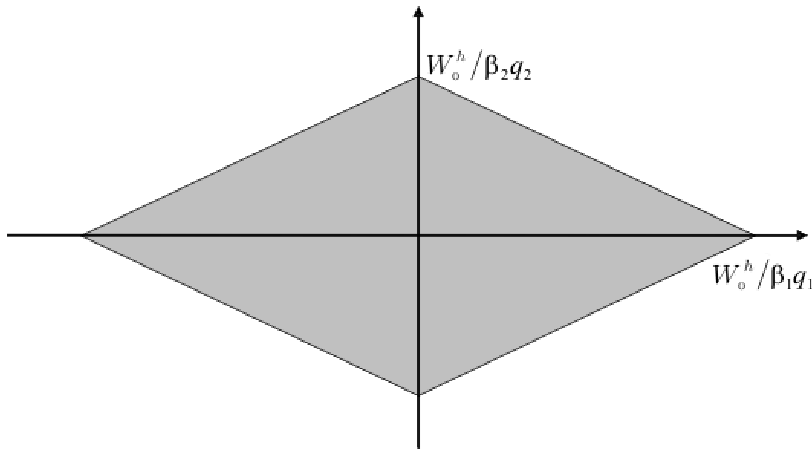

Figure 9: RAA-based risk constraint ($N = 2$).

at least two β's are equal, that is, if there is more assets than groups then at least two assets have the same weight factor.

When we have only two risky assets and the prices of these assets are positive then the risk constraint is a lozenge as illustrated in Figure 9. The institution h problem can be written as

$$\begin{array}{ll} \underset{\boldsymbol{y}^h}{\text{Min}} & (r_0\boldsymbol{q} - \boldsymbol{\mu})' \boldsymbol{y}^h + h\frac{\boldsymbol{y}^{h\prime}\boldsymbol{\Sigma}\boldsymbol{y}^h}{2} \\ \text{s.a.} & \beta_1 q_1 y_1^h + \beta_2 q_2 y_2^h \leq W_0^h \\ & \beta_1 q_1 y_1^h - \beta_2 q_2 y_2^h \leq W_0^h \\ & -\beta_1 q_1 y_1^h + \beta_2 q_2 y_2^h \leq W_0^h \\ & -\beta_1 q_1 y_1^h - \beta_2 q_2 y_2^h \leq W_0^h \end{array}$$

Hence, to solve the previous problem we have to consider nine different cases (depending on which restrictions are active in the optimum). For example, $\boldsymbol{y}^h = \frac{1}{h}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - r_0\boldsymbol{q})$ is an interior solution for this problem.

In order to avoid a tedious sequence of calculations in the same way that it was done for VaR-based risk constraint, we are going to restrict our analysis to a particular example. Suppose $N = 2$, $\boldsymbol{\theta}^h = (1.9, 0.5)'/(1 - \ell)$ for all h,

$r_0 = 1.00013$, $\boldsymbol{\mu} = (1.5, 1.2)'$ and[15]

$$\boldsymbol{\Sigma} = \begin{pmatrix} 0.6 & 0.25 \\ 0.25 & 0.4 \end{pmatrix}.$$

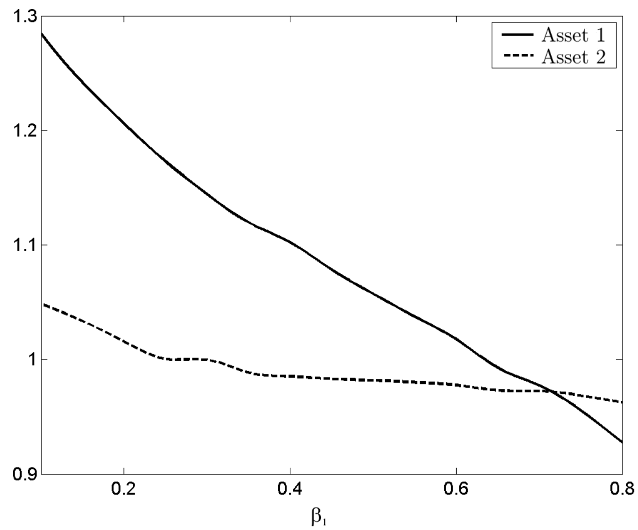

Figure 10: Prices of assets 1 e 2 with RAA-based risk constraint.

Figure 10 presents the equilibrium prices of assets 1 and 2 as function of β_1 (β_2 fixed and equal to 0.25). Note that these prices are decreasing functions of the weight factor of asset 1[16]. Figure 11 illustrates the default probability of financial institutions as function of its CARA. Observe that the bankrupt probability of institutions less risk averse is higher in a regulated economy. For $\beta_1 = 0.1$ the regulatory agency was not very sucessful in the choosing the weight factors once the charge to cover risk of asset 1 is smaller than the charger to cover risk of asset 2 and the variance of asset 1 is greater than the variance of asset 2. In this case the regulation is prejudicial since all financial institution hold riskier portfolio than in an unregulated economy. Figure 12 presents the total bankruptcy probability (the sum of the bankrupt probabilities of all institutions) as function of β_1. We consider only $\beta_1 > \beta_2 = 0.25$ since asset 1 is riskier than asset 2. Note that the total

[15]Once more, we use MatLab *fsolve* function to find the equilibrium problem.

[16]For comparasion purposes, the prices of assets 1 and 2 in an unregulated economy (that is $\beta_1 = \beta_2 = 0$) are 1.31 and 1.10, respectively.

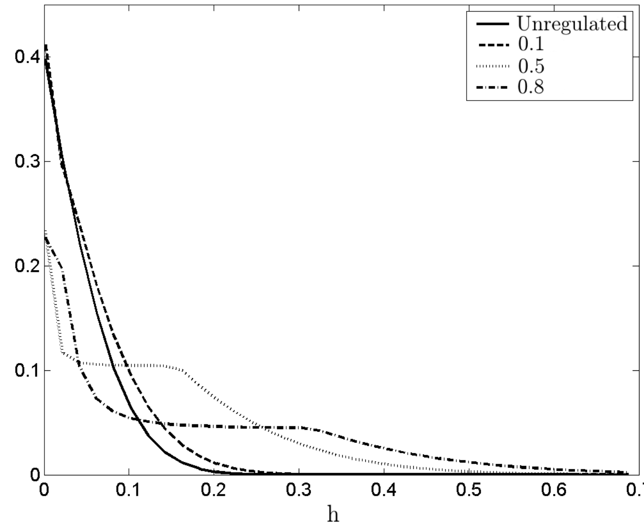

Figure 11: Default probability of institution h for an unregulated economy and for $\beta_1 = 0.1$, 0.5 and 0.8 ($\beta_2 = 0.25$).

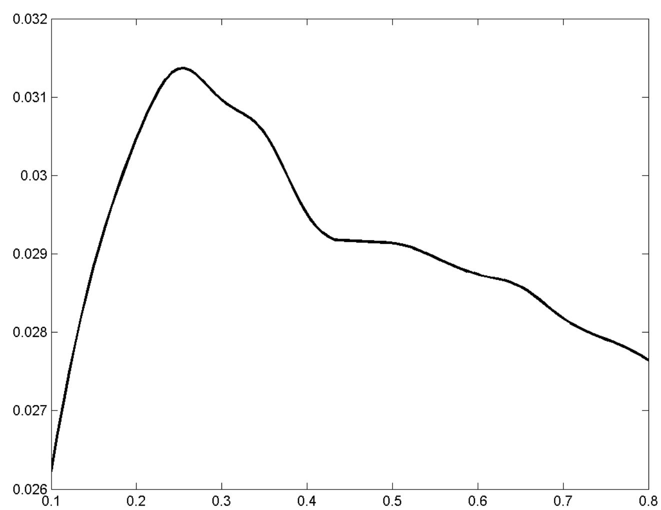

Figure 12: Total bankruptcy probability as a function of β_1.

bankrutcy probability is a decreasing function of β_1. This example shows the importance of a good calibration of the weight factors since the RAA-based regutation can increase the bankrutpcy probability of some institutions.

8 Conclusion

The primary aim of this work was to analyze the welfare properties in an economy where financial institutions are subject to a VaR-based risk regulation. Firstly, we determined for each institution the level of regulation that maximizes its utility. We showed that this level is not necessarily equivalent to the absence of regulation.

To determine the intensity of the risk regulation, the regulatory agency must take into account the bankruptcy probability of the financial institutions. We showed that if the net worth of a financial institution is low or the market volatility is high or yet the institution is little risk averse then the VaR-based risk regulation can decrease its bankruptcy probability. Also we saw that VaR-based risk regulation can decrease the contagion in the economy.

Hence, the regulatory agency face a trade-off between:

1. Fixing ν (the upper limit for the portfolio variance of all financial institutions) sufficiently small in order to control the bankruptcy probability and the contagion probability; and
2. Fixing ν sufficiently large in order to not impact the financial institutions welfare.

When the risk constraint is based on RAA scheme we showed that it is important that the regulatory agency set appropriately the risk weights since the RAA-based regulation increases the financial fragility of the institutions very risk averse in a regulated economy.

Appendix - Proofs of Propositons

Proof of Proposition 3

If equilibrium exists and all institutions reach the risk constraint then

$$\frac{\theta'\mathbf{\Sigma}\theta}{(1-\ell)^2} < \nu < \frac{\theta'\mathbf{\Sigma}\theta}{(\ell\ln\ell^{-1})^2},$$

or, equivalently $\ell \geq \kappa\Psi = \sqrt{\frac{\varrho}{\nu}} < 1$. In these conditions, for one given equilibrium allocation $\left\{\left(x^h, \boldsymbol{y}^h\right)\right\}_{H\in[\ell,1]}$ with prices $\boldsymbol{q}$ we have[17]

$$\Lambda(\nu) = -\int_\ell^1 \frac{\ln\left\{-\mathcal{E}\left[u^h\left(W_1^h\right)\right]\right\}}{h}\mathrm{d}h =$$

$$-\int_\ell^1 \left(\left(\theta_0^h + \boldsymbol{q}\theta^h - \boldsymbol{q}\boldsymbol{y}^h\right)r_0 + \mu\boldsymbol{y}^h - h\frac{\boldsymbol{y}^{h\prime}\mathbf{\Sigma}\boldsymbol{y}^h}{2}\right)\mathrm{d}h =$$

$$r_0\theta + \mu\theta - \tfrac{1}{2}\int_\ell^1 h\boldsymbol{y}^{h\prime}\mathbf{\Sigma}\boldsymbol{y}^h\mathrm{d}h =$$

$$r_0\theta + \mu\theta - \tfrac{1}{2}\left(\int_\ell^{\Psi\kappa} h\frac{\theta'\Sigma\theta}{\kappa^2}\mathrm{d}h + \int_{\Psi\kappa}^1 \frac{\Psi^2}{h}\theta'\mathbf{\Sigma}\theta\mathrm{d}h\right).$$

Calculating the integrals and using the identity $\ln\kappa\psi = \left(\frac{\kappa\psi-\ell}{\kappa} - 1\right)\frac{1}{\kappa}$ we have

$$\Lambda(\nu) = r_0\theta_0 + \mu\theta + \frac{\theta'\mathbf{\Sigma}\theta}{4\kappa^2}\left[(\kappa\Psi)^2 - (\ell+\kappa)\kappa\Psi + \ell^2\right].$$

□

Proof of Proposition 4

Since κ is a decreasing function of ν and Ψ is an increasing function of κ, to show that Λ is an increasing function of ν is sufficient to show that

$$f(\kappa) = (\kappa\Psi)^2 - 2(\ell+\kappa)\kappa\Psi + \ell^2$$

is a decreasing function of κ. Consider the quadratic polynomial $p(x) = x^2 - 2(\ell+\kappa)x + \ell^2$. This polynomial have two positive real roots:

$$x_1 = \ell + \kappa - \sqrt{\kappa^2 + 2\kappa\ell} \quad \text{and} \quad x_2 = \ell + \kappa + \sqrt{\kappa^2 + 2\kappa\ell}.$$

[17] Since agents have a constant absolute risk aversion coefficient, without loss of generality, we can suppose that the utility of institution h has the form $u^h(x) = -\mathrm{e}^{-hx}$.

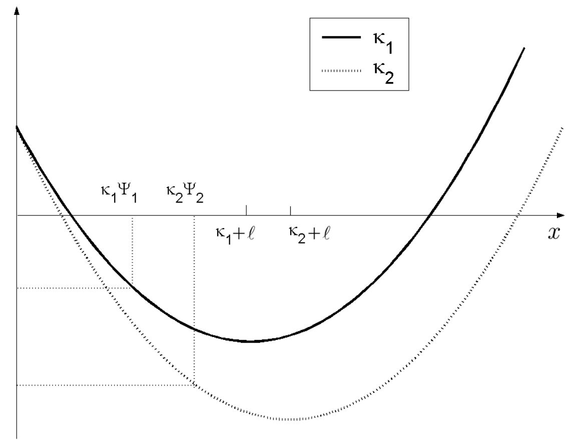

Figure 13: Polynomial $p(x) = x^2 - 2(\ell + \kappa)x + \ell^2$ $(\kappa_1 < \kappa_2)$.

When κ increases x_1 decreases and x_2 increases (see Figure 13). Since $\kappa\Psi$ is an increasing function of κ and $\kappa\Psi < \kappa + \ell$ follows that when κ increases, $(\kappa\Psi)^2 - (\ell + \kappa)\,\kappa\Psi + \ell^2$ decreases. $\square$

Proof of Proposition 5

Since κ is a strictly decreasing function of ν, to verify the intervals where $f^h(\nu)$ is increasing or decreasing it is enough to analyze f^h as a function of κ.

If $\underline{\nu} \leq \nu \leq g_2^{-1}(h)$ then $h \leq g_2(\nu) = \kappa\Psi$, hence $f^h(\nu) = \frac{\Psi}{\kappa} - \frac{h}{2\kappa^2} - \frac{\Psi}{1-\ell}$ and

$$\frac{\partial f^h}{\partial \kappa} = \Psi'(\kappa)\left(\frac{1}{\kappa} - \frac{1}{1-\ell}\right) + \frac{h}{\kappa^3} - \frac{\Psi}{\kappa^2}.$$

Thus, if $\nu \leq g_1^{-1}(h)$ then $\frac{\partial f^h}{\partial \kappa} < 0$ hence f^h is a strictly decreasing function of κ and therefore a strictly increasing function of ν. Case $g_1^{-1}(h) \leq \nu \leq g_2^{-1}(h)$, a similar argument shows that f^h is a strictly decreasing function of ν.

If $g_2^{-1}(h) < \nu \leq \overline{\nu}$ then

$$\frac{\partial f^h}{\partial \kappa} = \Psi'(\kappa)\left(\frac{\Psi}{h} - \frac{1}{1-\ell}\right).$$

We have to consider two cases:

1. If $\ell \leq h \leq \frac{1-\ell}{\ln \ell^{-1}}$ then $g_3(\nu) > h$. Therefore $\frac{\partial f^h}{\partial \kappa} > 0$ then f^h is a strictly increasing function of κ and a strictly decreasing of ν.

2. If $\frac{1-\ell}{\ln \ell^{-1}} \leq h \leq 1$ then the equation $g_3(\nu) = h$ has only one solution. Therefore, if $g_3^{-1}(h) < \nu \leq \overline{\nu}$ then f^h is strictly increasing function of ν. On the other hand, if $g_2^{-1}(h) < \nu \leq g_3^{-1}(h)$ then f^h is a strictly decreasing function of ν.

$\square$

Proof of Proposition 6

It is sufficient to show that

$$\frac{1}{2}\left(\frac{1}{h\left(\ln \ell^{-1}\right)^2} + \frac{h}{(1-\ell)^2}\right) \geq \frac{1}{(1-\ell)\left(\ln \ell^{-1}\right)}.$$

But the minimum of the left side of the previous equation occurs at $h = \frac{1-\ell}{\ln \ell^{-1}}$ and is equal to $\frac{1}{(1-\ell)(\ln \ell^{-1})}$. $\square$

Proof of Proposition 7

The function $t(h)$ is continuous, moreover using the elementary differential calculus it is possible after tedious manipulation to prove that:

1. $t\left(\frac{1-\ell}{\ln \ell^{-1}}\right) > 0$ and

2. $t(1) < 0$.

By Bolzano's theorem the function $t(h)$ has at least one real root on the interval $\left[\frac{1-\ell}{\ln \ell^{-1}}, 1\right]$. To show that it is the only root we have to prove that $t(h)$ is strictly decreasing. We can write $t(h)$ as the difference between two functions: $t(h) = t_2(h) - t_1(h)$ where

$$t_1(h) = \frac{1}{\ln \ell^{-1}}\left(\frac{1}{2h \ln \ell^{-1}} - \frac{1}{1-\ell}\right) \quad \text{and}$$

$$t_2(h) = \frac{\Psi}{\kappa} - \frac{h}{2\kappa^2} - \frac{\Psi}{1-\ell} \text{ with } \kappa \text{ and } \Psi \text{ computed at } \nu = g_1^{-1}(h).$$

Therefore,

$$\frac{\partial t_1}{\partial h} = -\frac{1}{2\left(h \ln \ell^{-1}\right)^2} \quad \text{and}$$

$$\frac{\partial t_2}{\partial h} = -\frac{1}{2\kappa^2},$$

where to compute the last derivative we use the fact that at $\nu = g_1^{-1}(h)$, $\frac{\partial t_2}{\partial \kappa} = 0$. Hence we must demonstrate that $\frac{\partial t_2}{\partial h} \leq \frac{\partial t_1}{\partial h}$. But this occurs because

$$\underset{h}{\text{Max}} \ \frac{\partial t_2}{\partial h} = \underset{h}{\text{Min}} \ \frac{\partial t_1}{\partial h} = -\frac{1}{2(1-\ell)^2}.$$

The other affirmations of the proposition are immediate consequences of the behavior of $t(h)$. □

Proof of Proposition 8

If $\nu < g_2^{-1}(h)$ then

$$\frac{\partial \frac{m^h}{s^h}}{\partial \nu} = \frac{r_0 W_0^h}{\sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}}} \frac{\partial \kappa}{\partial \nu} + \sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}} \frac{\partial \Psi}{\partial \nu},$$

since $\frac{\partial \kappa}{\partial \nu} < 0$ and $\frac{\partial \Psi}{\partial \nu} < 0$ we have $\frac{\partial \frac{m^h}{s^h}}{\partial \nu} < 0$.
If $\nu \geq g_2^{-1}(h)$ then

$$\frac{\partial \frac{m^h}{s^h}}{\partial \nu} = \frac{\partial \Psi}{\partial \nu} \left(-\frac{r_0 W_0^h h}{\Psi^2 \sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}}} + \sqrt{\boldsymbol{\theta}' \boldsymbol{\Sigma} \boldsymbol{\theta}} \right).$$

Hence, when $\nu \leq \tilde{\nu}$ we have $\frac{\partial \frac{m^h}{s^h}}{\partial \nu} < 0$ and when $\nu \geq \tilde{\nu}$ we have $\frac{\partial \frac{m^h}{s^h}}{\partial \nu} > 0$. □

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