[PRELIMINARY AND INCOMPLETE]

Liquidity effects on asset prices, financial stability and economic resilience

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Abstract

This paper analyzes the different channels of shock transmission in an economy affected by financial frictions. We distinguish between the liquidity and default effects on asset prices. Furthermore, we develop a framework in which we can assess financial stability policy under financial frictions. We introduce a simplified model of trade and financial intermediation that captures the effects of shocks on financial and real variables of the economy. Our results suggest that financial stability and economic resilience to adverse shocks should take into account default in the credit market as well as the liquidity of goods traded in the commodity market.

Keywords: Default, DSGE, financial stability, liquidity.

JEL Classification codes: D52, D53, E43, E44.

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1 Introduction

The classical macroeconomics archetype with its apparatus did not perform satisfactorily during the recent financial crisis. The micro-founded representative agent model embedded features of the Real Business Cycle (RBC) and New-Keynesian paradigms to guide policy makers and Central Banks to identify sources of economics fluctuations, to forecast and delineate the effects of policy interventions. However, the absence of liquidity, default and, more generally, financial frictions rendered it inadequate to address issues of financial (in)stability and, therefore, simultaneously address monetary and regulatory policy in an integrated framework.

In case these phenomena are treated as general equilibrium in the dynamic stochastic environment (DSGE), the current trend is to treat it by using an RBC representative agent model where adding financial frictions. The financial frictions are often related to the fact that there is asymmetric information in the credit market, and therefore it generates some inefficiencies in pricing and allocation. Other types of frictions that are added into these models are related to price stickiness.

Bernanke, Gertler and Gilchrist (1998), Curdia and Woodford(2008) and Kiyotaki and Moore (2001) include some of the leading examples in the literature of financial frictions in DSGE. Most of the Central Banks in the world are using their insights, because of practicalities in calibration and the useful and direct explanation of pricing and welfare they provide. Kiyotaki et al., include asymmetric information in the form of entrepreneur moral hazard and assume default as an out-of-the-equilibrium phenomenon, but it plays a reduced role in the budget constraint of the agents. They explain the transmission of financial shocks to the real economy by the existence of the asymmetric information and the liquidity constraints it generates. Bernanke et al. include money in the formulation and explain the financial accelerator consequences into the business cycle. In their framework, the main frictions are the price stickiness, and costly state verification, those are the main sources of business fluctuations. Other applications include the analysis of the effects of those frictions into the capital requirements fluctuations, as in Covas and Fujita (2010) or Meh and Moran (2010). Although they constitute valuable efforts to explain the dynamics of the credit market and are an important tool to consider policy measures in the banking industry, they possess arbitrary features in the modeling (e.g. money in the utility function and change in the agent's roles through time).

These models do not include money, default and heterogeneity altogether, hence, they cannot properly consider aggregate frictions present in the credit market. That is exactly one of the reasons to them for including asymmetric information in the general equilibrium setting. In our case, as we model the aggregate frictions by including money, liquidity constraints and default in equilibrium, we capture the first order effects of those frictions. In our modeling context the aggregate financial frictions we observe in reality are enough to explain the dynamics of the nominal and real economy.

Shubik (1999), Dubey and Geanakoplos (2003), Tsomocos (2003), Dubey and Geanakoplos (2005) and Goodhart, Surinand and Tsomocos (2006) have developed a general framework in which financial rigidities can be assessed rigorously within a general equilibrium setting. The main challenge is still how to extend those models in a dynamic setting, by preserving the main properties of a parsimonious model, without the inclusion of redundant or unrealistic features.

The initial endeavors, by Leao (2006), de Walque (2010) and Iacoviello (2007), that introduced those concepts into the DSGE framework, had not taken into account simultaneously liquidity, agent heterogeneity, money and default risk. Nevertheless, those model are valuable efforts in to the development of a plausible explanation of the phenomena we observe after the credit crisis.

We hasten to add that the mere possibility of default underscores the necessity of introducing liquidity in advance constraints. The interplay of liquidity and default justifies the presence of fiat money as the stipulated mean of exchange. Otherwise, the mere presence of a monetary sector without the possibility of endogenous default or any other friction in equilibrium may become a veil without affecting real trade and, eventually, final equilibrium allocation. Indeed, liquidity constraints, or their extreme version od cash-in-advance constraints, are the minimal institutional arrangement to capture a fundamental aspect of liquidity and how it interacts with default to affect the real economy.

The extension to the DSGE can be done following the Tsomocos (2003) and Goodhart, Sunirand and Tsomocos (2006) theoretical framework, that includes dynamics, aggregate uncertainty, agent heterogeneity, money, an active commercial banking sector, endogenous default and a formal definition for financial stability, contagion, systemic risk.

One of the crucial elements remaining to be introduced into the DSGE framework is the liquidity constraint the agents face, because the goods are not fully readily tradable. Acharya et al. (2005, 2009), Brunnermeier et al. (2009) and Vayanos (2004), have all studied liquidity within partial equilibrium models. In our model liquidity will be modeled following Espinoza and Tsomocos (2008). The extension to the dynamic framework is a direct and useful tool to assess the impact of the financial and real shocks since it provides two important advantages. The first is the ability to monitor the impact of a liquidity shock in the short, but also middle run. The second enhancement is that the dynamic setting allows to parameterize different liquidity environments (i.e. steady state values) and examine how shocks impact the economic variables in each case.

This paper aims to provide a basic framework to analyze financial stability policy under the presence of financial frictions, with a special focus on liquidity.

The paper is organized as follows. The first section contains the motivation of our paper. Section two defines the model we are going to solve. Section three includes the solution, features and calibration of the model. Finally, section four includes the main findings and conclusions from our study.

2 Motivation

In this section of the paper we give a brief description of the economical facts that motivate our research on this topic.

The motivation is mainly due to the previous empirical findings by Martinez (2010). In his paper, the author explains the variations in asset prices of emerging market countries. It analyzes the movement of the emerging market debt spreads through time. It considers the movement before and after the credit crisis, but it takes a look back in the beginning of this, around July of 2007, when financial markets started to suffer a high period of volatility. This phenomenon can be observed in figure (1).

The question that this paper addresses is finding the determinants of the changes in prices

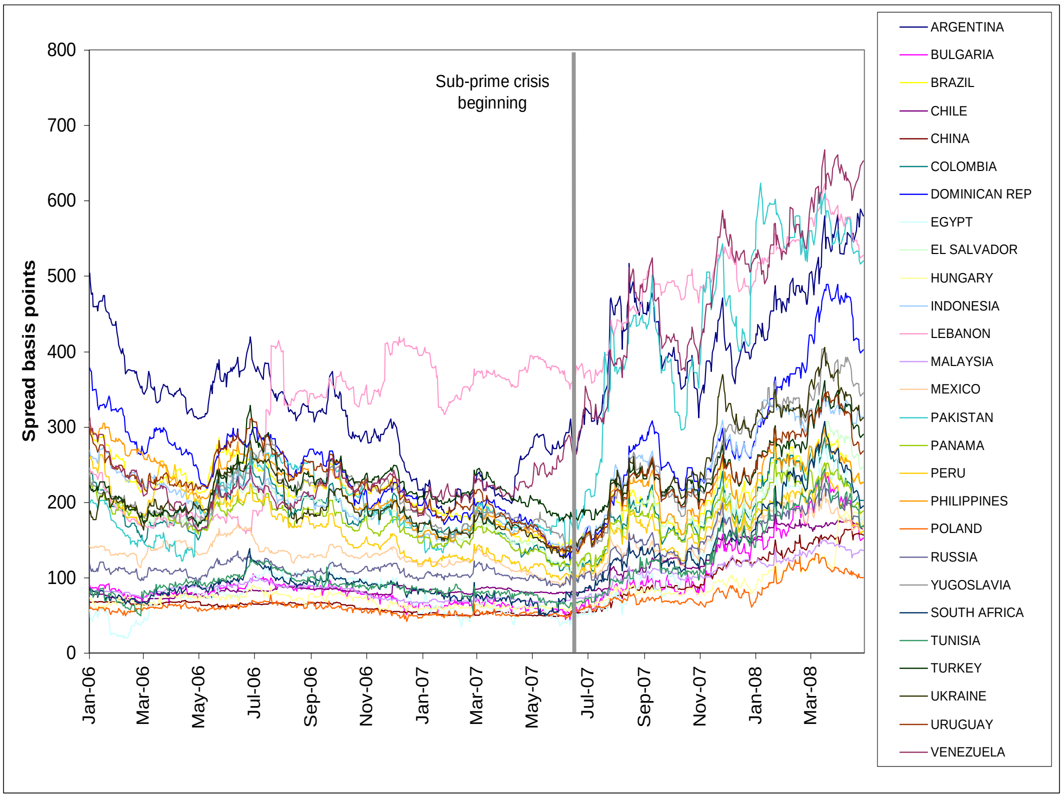

Figure 1: EMBI spreads compression and decompression

(spreads) of emerging market debt fluctuations. It is due to fundamental movements, or it is due to the change in the aggregate liquidity in those markets. The paper finds that before the turbulence started, fundamentals explained a considerable portion of the pricing. However, after the turbulence started, credit markets dried up and liquidity started to bind for those markets. This phenomenon caused a detriment in emerging market debt, even when fundamentals were not worsening and default was not increasing at that time, as we can observe in figure (2).

The conclusion of Martinez (2010) is that there is scope for financial intervention and liquidity injections were needed at that time to avoid the transmission of the financial shock into the real economy, improving welfare. Nevertheless this paper has some limitations, because the empirical approach doesn't allow to analyze the feedback effects and short and medium run dynamics between default, liquidity and asset pricing, furthermore, welfare effects cannot be measured.

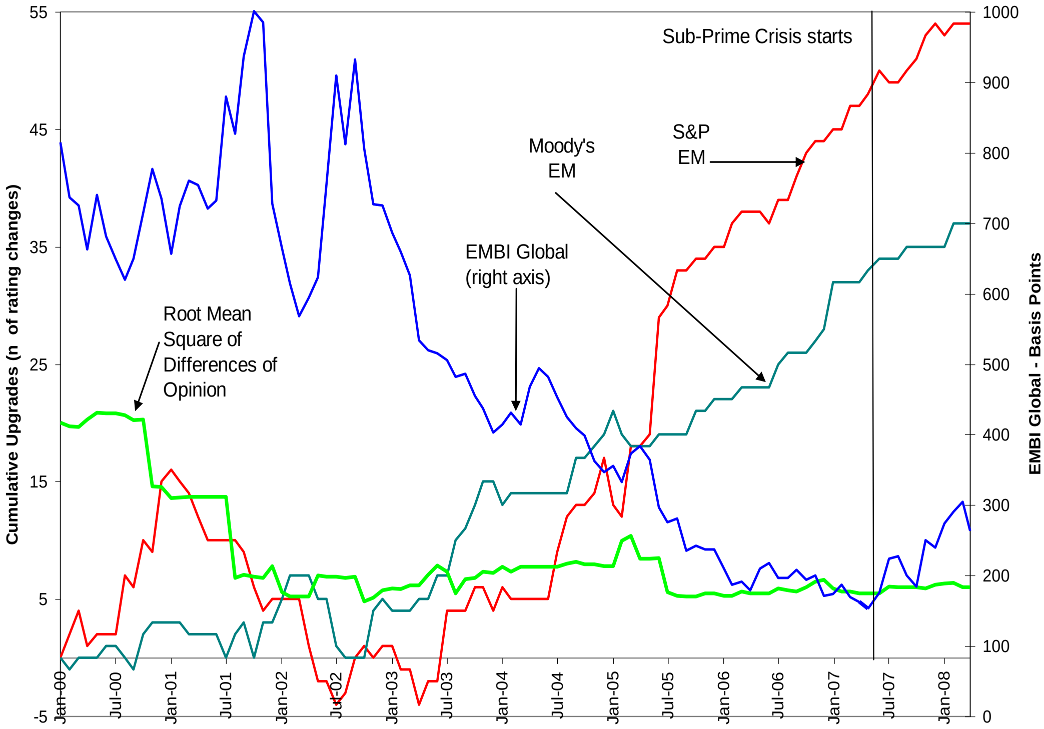

Figure 2: EMBI spreads versus credit quality

The present paper is an attempt to take the previous phenomenon and explain the dynamics of the stylized observation in Martinez (2010), by including these features into the DSGE framework.

3 The Benchmark Model

In this section we introduce a simple dynamic stochastic general equilibrium model (DSGE) with liquidity constraints and endogenous default, where we examine asset prices and real economy variables. The structure of the model and agents interaction are depicted in figure (3).

This benchmark model provides an environment where we have the financial frictions we want to analyze (i.e., money, default and liquidity). Also, it allows to see feedback effects among these variables and analyze those effects on asset prices, financial stability, and economic performance. In addition, we can compare the response of the model against shocks, given some liquidity environment.

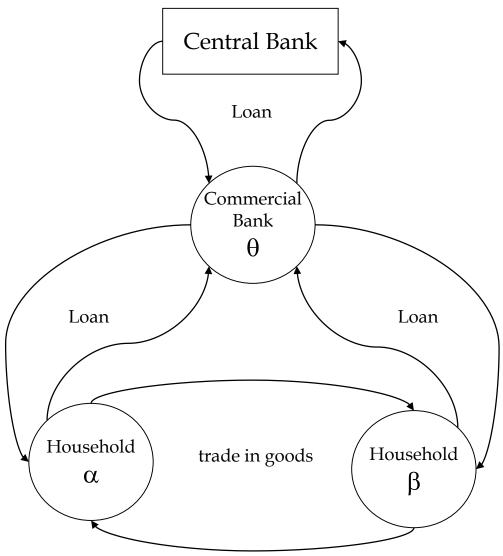

Figure 3: Nominal Flows of the Economy

3.1 Financial frictions

The financial frictions contained in this analysis include default, liquidity constraints, agent heterogeneity, aggregate uncertainty and money.

3.1.1 Default

We consider default on assets, or credit extensions, from the households to the commercial bank and from the latter to the Central Bank. Default is treated as in Shubik and Wilson (1977) and, more recently, in Dubey, Geanakoplos and Shubik (2005). The default is modeled by using non pecuniary penalties, proportional to the size of the non repaid amount of the contractual obligations. These penalties are subtracted from the utility functions of the households and the commercial bank. This is the case of continuous default. The penalties can be interpreted as reputational sanctions the companies may suffer from deciding not to honour their obligations. Therefore, it consists of a deadweight cost from defaulting. We only concentrate in this kind of default. We acknowledge there is also a discontinuous default case when agents cannot meet their obligations because the value of the collateral falls below the value of the mortgage/loan[1]. In this model, for the sake of simplicity, we have no collateral requirement on the loans that are contracted.

The default penalties we include in the case of default for the agents in the economy are subject to regulatory changes, which can be thought of as shocks.

$$\ln(\tau_t^\alpha) = \rho^\alpha \ln(\bar{\tau}^\alpha) + (1-\rho^\alpha)\ln(\tau_{t-1}^\alpha) + \epsilon_{\alpha,t}^\tau \tag{1}$$

$$\ln(\tau_t^\beta) = \rho^\beta \ln(\bar{\tau}^\beta) + (1-\rho^\beta)\ln(\tau_{t-1}^\beta) + \epsilon_{\beta,t}^\tau \tag{2}$$

$$\ln(\tau_t^\theta) = \rho^\theta \ln(\bar{\tau}^\theta) + (1-\rho^\theta)\ln(\tau_{t-1}^\theta) + \epsilon_{\theta,t}^\tau \tag{3}$$

Where:

τ_t^α	default penalty for household α.
τ_t^β	default penalty for household β.
τ_t^θ	default penalty for commercial bank θ.

Finally, we convert the nominal amounts to real by deflacting them by an inflation index we construct in section (3.3).

3.1.2 Liquidity

Our model considers the liquidity of traded goods. In this section we follow the work of Espinoza and Tsomocos (2010). The representation of this concept is through the exogenous variables λ_t^α and λ_t^β, which are the speed of liquidation for goods sold by households. In

[1]For a recent application of this type of default in general equilibrium modeling see Lin et al. (2010)

steady state there exists a level that represents the relative liquidity of those goods in the economy. This level will be parameterized later in the calibration section. Furthermore, in the simulations section we will address how our system responds to a shock on the steady state level of liquidity for each good. We will analyze the cases when there is no liquidity and when there is partial symmetric and asymmetric liquidity.

When the transactions of goods are not instantaneous, a fraction of the proceedings of goods sales will be expected to be available only at the end of the period. Therefore receipts associated to the revenues of commodity sales are partly available. This fraction of ready-to-use receipts from sales of commodities is referred to as liquidity. However, following the same argument, a better interpretation for this exogenous parameter is considering it as *speed of liquidation*.

The argument to relate this goods liquidity with assets liquidity is straightforward. When the good that is backing some asset is less liquid, the asset becomes less liquid as well, in the sense that the speed of trade is lower, and if there is any other asset using the previous asset as underlying guarantee, this other asset becomes less liquid as well. The natural example is the mortgage market. The asset involved is the mortgage, whereas the good is the physical property (i.e. the collateral). When there is any shock to the liquidity of the property, then the mortgage is affected, as well as the asset backed securities involved. In bad states of nature liquidity binds and a negative shock is transmitted into both asset markets (primary and secondary).

The economy is subject to liquidity shocks on the goods household α and β are endowed, the following equation describes the dynamics of the shocks.

$$\ln(\lambda_t^h) = \rho^h \ln(\bar{\lambda}^h) + (1-\rho^h)\ln(\lambda_{t-1}^h) + \epsilon_{h,t}^{\lambda} \quad \text{for } h \in \{\alpha, \beta\} \tag{4}$$

$\bar{\lambda}^h$ remains to be parameterized at the steady state of the economy. The *aggregate* liquidity on the market is defined by using the following expression.

$$\lambda_t = \frac{\lambda_t^{\alpha} p_{1,t} q_{1,t}^{\alpha} + \lambda_t^{\beta} p_{2,t} q_{2,t}^{\beta}}{p_{1,t} q_{1,t}^{\alpha} + p_{2,t} q_{2,t}^{\beta}} \tag{5}$$

From the previous expression we can see that the aggregate liquidity index is a weighted average of the individual liquidities for the available goods. There is full aggregate liquidity if and only if there is full individual liquidity for every good in the market.

3.1.3 Inside Money

Inside money is considered as the interventions of the Central Bank in the interbank lending market. In our simple case it only represents the intra-temporal lending that the Central Bank makes to the banking sector. These liquidity injections must exit the system (with accrued interest and net of default) when borrowing commercial banks repay their obligations. We model inside money shocks as changes to the level of open market operations (OMOs).

$$M_t = \eta_t^{CB} \bar{M} \tag{6}$$

Where

$$\ln(\eta_t^{CB}) = \rho^{CB} \ln(\bar{\eta}^{CB}) + (1 - \rho^{CB}) \ln(\eta_{t-1}^{CB}) + \epsilon_{CB,t} \tag{7}$$

and $\bar{\eta}^{CB} = 1$ is the monetary operations gross growth at the steady state of the economy.

3.2 Stochastic Endowment

Since the focus of our model is on financial stability, we allow for a Lucas tree type of the economy. This way we simplify the calculations and concentrate just in the part of the model that allows us to take into account the price and endowment divergences. We assume a stochastic endowment for each agent. Therefore, the only way to smooth consumption across agents is through commodities trading between the households. The following equation describes the form of the stochastic endowment.

$$\ln(e_{h,t}^l) = \rho_e^h \ln(\bar{e}_h^l) + (1 - \rho_e^h) \ln(e_{h,t-1}^l) + \epsilon_{h,t}^l \quad \text{for } h \in \{\alpha, \beta\} \text{ and } l \in \{1, 2\} \tag{8}$$

This endowment is a part of the fundamental value in this simplified economy. If there is a positive or negative shock on this state contingent endowment, there are consequences for the asset prices, which are described by the rate of the loan obtained from the commercial bank. Of course, due to the interaction with the financial sector, a positive shock in goods market endowment does not necessarily have positive consequences for the agents, it also needs to be considered the effect on prices.

Fundamentals should explain the asset prices and economic variables dynamics. Nevertheless, we will show how those financial and real variables are also affected by liquidity of goods.

3.3 Inflation

From economic theory we know that the gross rate of inflation obeys the following equation.

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{9}$$

In order to define inflation in terms of the model variables, we need to determine a price index for P_t. In the case of the price index it should be a weighted average of products of the representative consumption basket in the economy. As we only have two goods, we can use these goods weights in order to build the basket, therefore we have:

$$P_t = \omega_1^p p_{1,t} + \omega_2^p p_{2,t} \tag{10}$$

Where ω_1^p and ω_2^p are the weights of the goods in the consumption basket, and $p_{1,t}$ and $p_{2,t}$ are the prices of the goods 1 and 2, respectively.

In practice, the index is calculated by considering a fixed basket at the basis year. The common trend analyzed in the historic context, by Roberts (2000) is to use a Laspeyres (1864,1871,1883) type of index, which in our two goods case is defined as follows.

$$P_t = \frac{p_{1,t}q_{1,0}^{\alpha} + p_{2,t}q_{2,0}^{\beta}}{p_{1,0}q_{1,0}^{\alpha} + p_{2,0}q_{2,0}^{\beta}} \tag{11}$$

Where $q_{1,0}^{\alpha}$ and $q_{2,0}^{\beta}$ are the commodities quantities traded by the households at the basis year time period (i.e. t=0). In our case we are using year zero as our basis, therefore, as we start from the steady state, all the variables at time zero are the steady state variables. Therefore, we have the following equation for the price index.

$$P_t = \frac{p_{1,t}\bar{q}_1^{\alpha} + p_{2,t}\bar{q}_2^{\beta}}{\bar{p}_1\bar{q}_1^{\alpha} + \bar{p}_2\bar{q}_2^{\beta}} \tag{12}$$

Where $\bar{q}_1^{\alpha}$ and $\bar{q}_2^{\beta}$ correspond to the commodities quantities traded by the households at the steady state. As we see in the previous index, a convenient selection of weights allows us to investigate some interesting features of our model. The weights are determined to meet three requirements. First, we need the weights to add up to one. Second, we need the weights to allow us to construct the inflation as a function of the variables of our model. Third, we need to be able to derive the quantity theory of money proposition.

If we conveniently calculate our index in terms of changes with respect to the basis year, the equation (12) becomes.

$$P_t = \omega_1^p \frac{p_{1,t}}{\bar{p}_1} + \omega_2^p \frac{p_{2,t}}{\bar{p}_2} \tag{13}$$

Therefore, in order for the relationship in equation (12) to hold, we need to define the following weights.

$$\omega_1^p = \frac{\bar{p}_1 \bar{q}_1^\alpha}{\bar{p}_1 \bar{q}_1^\alpha + \bar{p}_2 \bar{q}_2^\beta} \tag{14}$$

$$\omega_2^p = \frac{\bar{p}_2 \bar{q}_2^\beta}{\bar{p}_1 \bar{q}_1^\alpha + \bar{p}_2 \bar{q}_2^\beta} \tag{15}$$

As we can observe, from the previous expressions, the weights add up to one. By replacing this definition of weights in the expression for inflation, we have.

$$\pi = (1 + \hat{\pi}_t) = \frac{p_{1,t} \bar{q}_1^\alpha + p_{2,t} \bar{q}_2^\beta}{p_{1,t-1} \bar{q}_1^\alpha + p_{2,t-1} \bar{q}_2^\beta} \tag{16}$$

In our case π is the gross inflation index, whereas $\hat{\pi}$ is the net inflation growth. From the previous calculations, we have thus, constructed the inflation index as a function of the variables in our model. In the section related to characterization of the equilibrium, we are going to see the quantity theory of money proposition, as a function of the price index we have defined and the money spent in the system every period.

In the simulation section we are going to see the response of inflation to different financial shocks.

3.4 Timing of the model

The timing of the different transactions in this simple model is described in table (1).

Table 1: Timing of the model

	Money inflow		Money Outflow	
	Commercial Bank	**Households**	**Commercial Bank**	**Households**
Beginning	Loan taken from Central Bank	Loan taken from Commercial bank	Loan given to the households	Consumption
End	Repayment from households	Revenues from sales of commodities	Repayment to Central Bank	Repayment to Commercial bank

3.5 Model setting

In our problem we assume that household α is endowed with good 1 only every period, and household β is endowed with good 2 only. Both of them face cash in advance constraints, therefore, they have to borrow money from the commercial bank. The latter also borrows money from the Central Bank, because it also faces cash in advance constraints as a consequence of the constraints faced by the households. Nevertheless, there exists a liquid portion of commodity endowments that can be used to repay his/her debts and consume.

Household α optimization problem

$$\max_{\tilde{\mu}_t^\alpha, \tilde{b}_{2,t}^\alpha, \upsilon_t^\alpha, q_{1,t}^\alpha} U^\alpha = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ ln\left(e_{1,t}^\alpha - q_{1,t}^\alpha\right) + ln\left(\frac{\tilde{b}_{2,t}^\alpha}{\tilde{p}_{2,t}}\right) - \frac{\tau_t^\alpha}{\pi_t} Max[0, (1-\upsilon_t^\alpha)\tilde{\mu}_{t-1}^\alpha] \right\}$$

s.t.

$$\upsilon_t^\alpha \tilde{\mu}_{t-1}^\alpha \leq \tilde{p}_{1,t-1} q_{1,t-1}^\alpha \cdot \left(1 - \lambda_{t-1}^\alpha\right) \qquad \left(\eta_{1,t}^\alpha\right) \tag{17}$$

Loan repayment $\leq$ Previous period illiquid sales of commodities.

$$\tilde{b}_{2,t}^\alpha \leq \lambda_t^\alpha \cdot \tilde{p}_{1,t} q_{1,t}^\alpha + \frac{\tilde{\mu}_t^\alpha}{1 + r_t^c} \qquad \left(\eta_{2,t}^\alpha\right) \tag{18}$$

Money spent $\leq$ Liquid portion of sales of commodities + Loan taken from the commercial bank.

Where:

U^α	overall utility of household α.
β^t	stochastic discount factor.
$q_{1,t}^\alpha$	amount sold of good 1.
$b_{2,t}^\alpha$	amount of money spent in good 2.
$c_{1,t}^\alpha$	$= \left(e_{1,t}^\alpha - q_{1,t}^\alpha\right)$, consumption of good 1.
$c_{2,t}^\alpha$	$= \left(\frac{\tilde{b}_{2,t}^\alpha}{\tilde{p}_{2,t}}\right)$, consumption of good 2.
$u\left(c_{1,t}^\alpha, c_{2,t}^\alpha\right)$	$= ln\left(c_{1,t}^\alpha\right) + ln\left(c_{2,t}^\alpha\right)$, utility from consumption absent from penalties.
μ_t^α	loan amount taken from the commercial bank.
υ_t^α	loan repayment rate.
τ_t^α	default penalty for household α.
r_t^c	commercial bank loans rate.
λ_t^α	liquid portion of goods for household α.
$\eta_{i,t}^\alpha$	lagrange multiplier for constraint $i \in \{1, 2\}$ for household α.
$\sim$	on the top is for real amounts correction (division by p_t or p_{t-1}).

Household β optimization problem

$$\max_{\tilde{\mu}_t^\beta, \tilde{b}_{1,t}^\beta, \upsilon_t^\beta, q_{2,t}^\beta} U^\beta = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ ln\left(\frac{\tilde{b}_{1,t}^\beta}{\tilde{p}_{1,t}}\right) + ln\left(e_{2,t}^\beta - q_{2,t}^\beta\right) - \frac{\tau_t^\beta}{\pi_t} Max[0,(1-\upsilon_t^\beta)\tilde{\mu}_{t-1}^\beta] \right\}$$

s.t.

$$\upsilon_t^\beta \tilde{\mu}_{t-1}^\beta \leq \tilde{p}_{2,t-1} q_{2,t-1}^\beta \cdot \left(1 - \lambda_{t-1}^\beta\right) \qquad \left(\eta_{1,t}^\beta\right) \tag{19}$$

Loan repayment $\leq$ Previous period illiquid sales of commodities.

$$\tilde{b}_{1,t}^\beta \leq \lambda_t^\beta \cdot \tilde{p}_{2,t} q_{2,t}^\beta + \frac{\tilde{\mu}_t^\beta}{1 + r_t^c} \qquad \left(\eta_{2,t}^\beta\right) \tag{20}$$

Money spent $\leq$ Liquid portion of sales of commodities + Loan taken from the commercial bank.

Where:

U^β	overall utility of household β.
β^t	stochastic discount factor.
$q_{2,t}^\beta$	amount sold of good 2.
$b_{1,t}^\beta$	amount of money spent in good 1.
$c_{1,t}^\beta$	$= \left(\frac{\tilde{b}_{1,t}^\beta}{\tilde{p}_{1,t}}\right)$, consumption of good 1.
$c_{2,t}^\beta$	$= \left(e_{2,t}^\beta - q_{2,t}^\beta\right)$, consumption of good 2.
$u\left(c_{1,t}^\beta, c_{2,t}^\beta\right)$	$= ln\left(c_{1,t}^\beta\right) + ln\left(c_{2,t}^\beta\right)$, utility from consumption absent from penalties.
μ_t^β	loan amount taken from the commercial bank.
υ_t^β	loan repayment rate.
τ_t^β	default penalty for household β.
r_t^c	commercial bank loans rate.
λ_t^β	liquid portion of goods for household β.
$\eta_{i,t}^\beta$	lagrange multiplier for constraint $i \in \{1,2\}$ for household β.
$\sim$	on the top is for real amounts correction (division by p_t or p_{t-1}).

Commercial Bank θ optimization problem

$$\max_{\tilde{\Pi}_t^\theta, \mu_t^\theta, l_t^\theta, \upsilon_t^\theta} U^\theta = E_0 \sum_{t=0}^{\infty} \hat{\beta}^t \left\{ ln\left(\tilde{\Pi}_t^\theta\right) - \frac{\tau_t^\theta}{\pi_t} Max[0,(1-\upsilon_t^\theta)\tilde{\mu}_{t-1}^\theta] \right\}$$

s.t.

$$\tilde{\Pi}_t^\theta = \frac{R_t \tilde{l}_{t-1}^\theta \left(1 + r_{t-1}^c\right)}{\pi_t} - \upsilon_t^\theta \frac{\tilde{\mu}_{t-1}^\theta}{\pi_t} \qquad \left(\eta_{1,t}^\theta\right) \tag{21}$$

Utility = Expected loan repayment - Repayment to Central Bank.

$$\tilde{l}_t^\theta \leq \frac{\tilde{\mu}_t^\theta}{1 + r_t^{IB}} \qquad \left(\eta_{2,t}^\theta\right) \tag{22}$$

Credit extension $\leq$ Loan taken from Central Bank.

Where:

U^θ	overall utility of commercial bank θ.
β^t	stochastic discount factor.
Π_t^θ	profits obtained by the commercial bank θ.
l_t^θ	loan amount given to the households.
μ_t^θ	loan amount taken from the Central Bank.
υ_t^θ	loan repayment rate.
τ_t^θ	default penalty for commercial bank θ.
r_t^{IB}	interbank loans rate, provided by Central Bank.
R_t	expected delivery rate from households loan.
$\eta_{i,t}^\theta$	lagrange multiplier for constraint $i \in \{1, 2\}$ for bank θ.
$\sim$	on the top is for real amounts correction (division by p_t or p_{t-1}).

We could think as the commercial bank (θ) as a set of similar competitive banks of the same type. The balance of the commercial bank is as follows:

A	L
Loans to households	REPO borrowing

Similarly, the balance sheet of the Central Bank,

A	L
REPO loans	Currency (Fiat money)

This reduced type economy implies that the primary function of the commercial bank is providing liquidity to the households and the Central Bank provides the liquidity to be transferred by the commercial banks. This is precisely the channel for shock transmission we want to analyze.

3.6 Market Clearing Conditions

The economy implied by our model contains three different markets: goods, consumer loans and REPO[2]. These markets determine its prices by equating demand and supply.

3.6.1 Goods Market

The (fiat) money spent in goods market is equal to the value of the supply of goods 1 and 2 at every period.

$$\tilde{b}_{1,t}^{\beta} = \tilde{p}_{1,t} q_{1,t}^{\alpha} \tag{23}$$

$$\tilde{b}_{2,t}^{\alpha} = \tilde{p}_{2,t} q_{2,t}^{\beta} \tag{24}$$

[2]Commercial bank repurchase agreements with the Central Bank.

3.6.2 Consumer Loans Market

The household loans demand equals the supply of funds offered by the commercial bank at every period.

$$1 + r_t^c = \frac{\tilde{\mu}_t^\alpha + \tilde{\mu}_t^\beta}{\tilde{l}_t^\theta} \tag{25}$$

3.6.3 REPO Market

The commercial bank loan demand equals the supply of funds offered by the Central Bank at every period.

$$1 + r_t^{IB} = \frac{\tilde{\mu}_t^\theta}{M_t} \tag{26}$$

3.7 Rational Expectations

The following conditions indicate that commercial bank is accurate in their expectations on the repayments it expects to receive from the loans extended. Therefore the expected repayment rate for bank θ is given by the following expression.

$$R_t^C \begin{cases} \frac{\upsilon_t^\alpha \tilde{\mu}_{t-1}^\alpha + \upsilon_t^\beta \tilde{\mu}_{t-1}^\beta}{\tilde{\mu}_{t-1}^\alpha + \tilde{\mu}_{t-1}^\beta}, & \text{if } \tilde{\mu}_{t-1}^\alpha + \tilde{\mu}_{t-1}^\beta > 0; \\ \text{arbitrary}, & \text{if } \tilde{\mu}_{t-1}^\alpha + \tilde{\mu}_{t-1}^\beta = 0. \end{cases} \tag{27}$$

4 Equilibrium

In this section we define the equilibrium of the model. This definition depends upon the definition of: decision variables, macroeconomic variables, parameters and budget sets. In the case of endogenous variables and variables being shocked, those are mainly defined to keep track of the simulation procedure we perform afterwards.

4.1 Decision Variables

$$\Sigma^\alpha = \left\{ \tilde{\mu}_t^\alpha, \tilde{b}_{2,t}^\alpha, \upsilon_t^\alpha, q_{1,t}^\alpha \right\}_{t=0}^{\infty}$$

$$\Sigma^\beta = \left\{ \tilde{\mu}_t^\beta, \tilde{b}_{1,t}^\beta, \upsilon_t^\beta, q_{2,t}^\beta \right\}_{t=0}^{\infty}$$

$$\Sigma^\theta = \left\{ \tilde{\Pi}_t^\theta, \tilde{\mu}_t^\theta, \tilde{l}_t^\theta, \upsilon_t^\theta, \right\}_{t=0}^{\infty}$$

4.2 Macroeconomic variables

$$\kappa = \left\{ M_t, \pi_t, r_t^c, r_t^{IB}, R_t, \tau_t^\alpha, \tau_t^\beta, \tau_t^\theta, \lambda_t^\alpha, \lambda_t^\beta \right\}_{t=0}^{\infty}$$

4.3 Parameters

$$\delta = \left(\overline{\eta}^{CB}, \bar{\lambda}^\alpha, \bar{\lambda}^\beta, \bar{\lambda}^\theta, \bar{e}_1^\alpha, \bar{e}_2^\beta, \beta, \hat{\beta}, \rho^{CB}, \rho_e^\alpha, \rho_e^\beta, \rho_e^\theta, \bar{M} \right)$$

4.4 Budget sets

$$\begin{aligned} B^\alpha (\kappa) &= \{\Sigma^\alpha : (17) - (18) \text{ hold}\} \\ B^\beta (\kappa) &= \left\{\Sigma^\beta : (19) - (20) \text{ hold}\right\} \\ B^\theta (\kappa) &= \left\{\Sigma^\theta : (21) - (22) \text{ hold}\right\} \end{aligned}$$

4.5 Endogenous Variables

$$\chi = \left\{ \tilde{p}_{1,t}, \tilde{p}_{2,t}, r_t^c, r_t^{IB}, R_t, \tilde{\mu}_t^\alpha, \tilde{b}_{2,t}^\alpha, \upsilon_t^\alpha, q_{1,t}^\alpha, \eta_{1,t}^\alpha, \eta_{2,t}^\alpha, \tilde{\mu}_t^\beta, \tilde{b}_{1,t}^\beta, \upsilon_t^\beta, q_{2,t}^\beta, \eta_{1,t}^\beta, \eta_{2,t}^\beta, \tilde{\Pi}_t^\theta, \tilde{\mu}_t^\theta, \tilde{l}_t^\theta, \upsilon_t^\theta, \eta_{1,t}^\theta, \eta_{2,t}^\theta \right\}_{t=0}^{\infty}$$

4.6 Variables being shocked

$$\varphi = \left\{ M_t, \pi_t, e_{1,t}^\alpha, e_{2,t}^\beta, \tau_t^\alpha, \tau_t^\beta, \tau_t^\theta, \lambda_t^\alpha, \lambda_t^\beta \right\}_{t=0}^{\infty}$$

4.7 Definition of Equilibrium

Given the previous definitions we are able to define the **FSMLD** (financial stability with money, liquidity and default equilibrium), for the short as well as the long run. In our case in the long run the economy converges to its steady state.

4.7.1 Short run

In our model, $(\Sigma^\alpha, \Sigma^\beta, \Sigma^\theta, \kappa)$ is a short run **FSMLD** iff:

(i) All agents optimize given their budget sets:

  (a) $\Sigma^h \in \text{Argmax}_{\Sigma^h \in B^h(\kappa)} U\left(C^h\right)$, for h $\in \{\alpha, \beta\}$ and $\forall t \in T$.

  (b) $\Sigma^\theta \in \text{Argmax}_{\Sigma^\theta \in B^\theta(\kappa)} U\left(\Pi^\theta\right)$, $\forall t \in T$.

(ii) All markets (23)-(26) clear.

(iii) Expectations are rational, (i.e. (27) holds).

4.7.2 Long run

In our model, in steady state, in addition to the conditions for a short run equilibrium, in order to obtain a long run **FSMLD**, we need that:

(i) All real and nominal variables do not grow, thus

$$x_t = x, \quad \forall\, t \,\in\, T \quad x_t \,\in X_t = \left\{\Sigma^\alpha, \Sigma^\beta, \Sigma^\gamma, \kappa\right\}$$

(ii) The economy is not subject to any shock, thus

$$e_t = 0, \quad \forall\, t \,\in\, T \quad e_t \in \left\{\epsilon_{CB,t}, \epsilon^\alpha_{1,t}, \epsilon^\beta_{2,t}, \epsilon^\tau_{\alpha,t}, \epsilon^\tau_{\beta,t}, \epsilon^\tau_{\theta,t}, \epsilon^\lambda_{\alpha,t}, \epsilon^\lambda_{\beta,t}\right\}$$

That is, there are no shocks on monetary basis, endowment, default penalties or liquidity.

4.8 Characterization of Equilibrium

In this subsection we are going to use the results obtained from the FOCs (i.e. First Order Conditions) in order to get insights about important relationships that are maintained in our model: money non-neutrality, fisher effect, quantity theory of money, on the verge condition and interest rates under default and no default. Appendix 1 contains the proof of these propositions.

4.8.1 Proposition 1: Money non-neutrality

This proposition implies that if there is a non-zero monetary operation by the Central Bank (i.e. $M_t \neq M_t' \Rightarrow r_t^c \neq r_t^{c\prime}$, from market clearing conditions), monetary policy is not neutral in the short-run. Therefore it affects the consumption and consequently real variables.

Suppose that for $\alpha, \beta \in H$, $b_t^h > 0$, for $l \in L$, $\lambda_t^h \in [0, 1)$ and some state of nature defined by the set of shocks at t. We have that at a **FSMLD**,

$$r_t^c \leq r_t^{c\prime}, \text{ and } \lambda_t^\alpha \geq \lambda_t^{\alpha\prime} \Rightarrow q_{1,t}^\alpha \geq q_{1,t}^{\alpha\prime}$$

Note that by symmetry the proposition holds also for household β.

We have two extreme cases to analyze[3]:

i. If $\lambda_t^\alpha = 0$ we have that monetary policy is not neutral. This is the usual cash-in-advance setting, and the proof is analogous to the previous one.

$$\frac{\partial u\left(c_{1,t}^\alpha, c_{2,t}^\alpha\right)}{\partial c_{1,t}^\alpha} = \frac{1}{1+r_t^c}\frac{\partial u\left(c_{1,t}^\alpha, c_{2,t}^\alpha\right)}{\partial c_{2,t}^\alpha}\frac{\tilde{p}_{1,t}}{\tilde{p}_{2,t}} \tag{28}$$

ii. If $\lambda_t^\alpha = 1$ we have no liquidity restrictions, therefore there are not incentives to borrow money and monetary policy is neutral. We can observe this directly from the following equation.

$$\frac{\partial u\left(c_{1,t}^\alpha, c_{2,t}^\alpha\right)}{\partial c_{1,t}^\alpha} = \frac{\partial u\left(c_{1,t}^\alpha, c_{2,t}^\alpha\right)}{\partial c_{2,t}^\alpha}\frac{\tilde{p}_{1,t}}{\tilde{p}_{2,t}} \tag{29}$$

4.8.2 Proposition 2: Fisher effect

Suppose that for $\alpha, \beta \in h$, $b_t^h > 0$, for $l \in L$, $\lambda_t^h \in [0,1)$ and some state of nature defined by the set of shocks at t. We have that at a **FSMLD**, for agent α, we have,

$$\left(\frac{1}{1-\lambda_t^\alpha}\left(\frac{\frac{\partial u\left(c_{1,t}^\alpha, c_{2,t}^\alpha\right)}{\partial c_{1,t}^h}}{\frac{\partial u\left(c_{1,t}^\alpha, c_{l,t}^\alpha\right)}{\partial c_{2,t}^\alpha}}\frac{\tilde{p}_{2,t}}{\tilde{p}_{1,t}} - \lambda_t^\alpha\right)\right)^{-1} = \left(1+r_t^c\right) \tag{30}$$

whereas, for agent β, we have

$$\left(\frac{1}{1-\lambda_t^\beta}\left(\frac{\frac{\partial u\left(c_{1,t}^\beta, c_{2,t}^\beta\right)}{\partial c_{2,t}^h}}{\frac{\partial u\left(c_{1,t}^\beta, c_{l,t}^\beta\right)}{\partial c_{1,t}^\beta}}\frac{\tilde{p}_{1,t}}{\tilde{p}_{2,t}} - \lambda_t^\beta\right)\right)^{-1} = \left(1+r_t^c\right) \tag{31}$$

Taking logarithms and interpreting loosely, this proposition indicates that nominal interest rates are approximately equal to real interest rates plus expected inflation and risk premium, which depends on liquidity and default. Fisher effect explains how nominal prices are linked directly to consumption.

4.8.3 Proposition 3: Quantity theory of money

Assume no money is carried over. In an interior **FSMLD** equilibrium, $\forall t \in T$

$$\left(1-\lambda_t^\alpha\right)\tilde{p}_{1,t}q_{1,t}^\alpha + \left(1-\lambda_t^\beta\right)\tilde{p}_{2,t}q_{2,t}^\beta = M_t \tag{32}$$

[3]These are equivalent for agent β.

Thus, the model possesses a non-trivial quantity theory of money, where prices and quantities are determined simultaneously.

Fisher's (1911) quantity theory of money proposition states,

$$P_t Q_t = M_t V_t \tag{33}$$

It implies that money supply has a direct, proportional relationship with the price level, where P_t stands for the price index, Q_t is an index of the real value of final expenditures, M_t is the total amount of money in circulation every period, and V_t is the average velocity of money in the market.

We already know P_t from equation (12). However, we still need to define the other variables in the expression. We need to define the real value of final expenditures Q_t. To achieve this, we write,

$$Q_t = \omega_1^q \frac{q_{1,t}^{\alpha}}{\bar{q}_1} + \omega_2^q \frac{q_{2,t}^{\beta}}{\bar{q}_2} \tag{34}$$

and the corresponding weight for traded quantities as,

$$\omega_1^q = \frac{p_{1,t}\bar{q}_1^{\alpha}\left(\bar{p}_1\bar{q}_1 + \bar{p}_2\bar{q}_2\right)}{p_{1,t}\bar{q}_1^{\alpha} + p_{2,t}\bar{q}_2^{\beta}} \tag{35}$$

$$\omega_2^q = \frac{p_{2,t}\bar{q}_2^{\beta}\left(\bar{p}_1\bar{q}_1 + \bar{p}_2\bar{q}_2\right)}{p_{1,t}\bar{q}_1^{\alpha} + p_{2,t}\bar{q}_2^{\beta}} \tag{36}$$

From the previous expression we can observe that the weights in this case add up to the total expenditures amount in steady state (the basis year). Therefore, we define Q_t as the average portion of expenditures in each year, by considering the average price and quantity changes in each period, i.e.

$$Q_t = \frac{\left(p_{1,t}q_{1,t}^{\alpha} + p_{2,t}q_{2,t}^{\beta}\right)\left(\bar{p}_1\bar{q}_1 + \bar{p}_2\bar{q}_2\right)}{p_{1,t}\bar{q}_1^{\alpha} + p_{2,t}\bar{q}_2^{\beta}} \tag{37}$$

From the quantity theory of money proposition, we know that the money in the system every period is defined as follows.

$$M_t = \left(1 - \lambda_t^{\alpha}\right) p_{1,t}q_{1,t}^{\alpha} + \left(1 - \lambda_t^{\beta}\right) p_{2,t}q_{2,t}^{\beta} \tag{38}$$

If we use the aggregate liquidity definition in equation (5), we then have,

$$M_t = (1 - \lambda_t) \left(p_{1,t} q_{1,t}^{\alpha} + p_{2,t} q_{2,t}^{\beta} \right) \tag{39}$$

The only remaining term to define in expression (33) is V_t, the velocity of money. In our case it is equal to $1/(1 - \lambda_t)$ so,

$$V_t = \frac{1}{(1 - \lambda_t)} \tag{40}$$

That is, the more liquidity in goods increase the effective amount of money in the economy, therefore, the proportion of the Central Bank's money is lowered. In this sense, to maintain the level of transactions, with the same amount of money, there is the need to increase the speed of transactions.

As we can observe, our model provides an explicit expression for a Fisher quantity theory of money relation. The importance of this proposition in our benchmark economy lies in the fact that even considering our model is very parsimonious, it provides a non-trivial determination of prices and quantities.

Our model is in stark contrast with the classical RBC representative agent model, whereby traded quantities are fixed since there is no trade in equilibrium. Hence, any monetary policy change has nominal effects only as it merely changes prices. The upshot of our argument is that when one introduces liquidity *and* default, this class of models is non-dichotomous.

4.8.4 Proposition 4: On the verge condition

Suppose that for $\alpha, \beta \in H$, $b_t^h > 0$, for $l \in L$, $\lambda_t^h \in [0, 1)$ and some state of nature defined by the set of shocks at t. We have that at a **FSMLD**, the on-the-verge condition for default penalties, for agents α, β and bank θ, respectively, is given by

$$\frac{1}{1 + r_t^c} \frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{2,t}^{\alpha}} \frac{1}{\tilde{p}_{2,t}} = \beta \mathbb{E}_t \left(\frac{\tau_{t+1}^{\alpha}}{\pi_{t+1}} \right) \tag{41}$$

$$\frac{1}{1 + r_t^c} \frac{\partial u\left(c_{1,t}^{\beta}, c_{2,t}^{\beta}\right)}{\partial c_{1,t}^{\beta}} \frac{1}{\tilde{p}_{1,t}} = \beta \mathbb{E}_t \left(\frac{\tau_{t+1}^{\beta}}{\pi_{t+1}} \right) \tag{42}$$

$$\frac{\partial u\left(\Pi_t^{\theta}\right)}{\partial \Pi_t^{\theta}} = \tau_t^{\theta} \tag{43}$$

Whereas, in the steady state the optimal default penalties, for agents α, β and bank θ, in

FSMLD will be determined as follows,

$$\frac{1}{1+\bar{r}^c}\frac{\partial u\left(c_1^\alpha, c_2^\alpha\right)}{\partial c_2^\alpha}\frac{1}{\bar{\bar{p}}_2} = \beta\left(\frac{\bar{\tau}^\alpha}{\bar{\pi}}\right) \tag{44}$$

$$\frac{1}{1+\bar{r}^c}\frac{\partial u\left(c_1^\beta, c_2^\beta\right)}{\partial c_1^\beta}\frac{1}{\bar{\bar{p}}_1} = \beta\left(\frac{\bar{\tau}^\beta}{\bar{\pi}}\right) \tag{45}$$

$$\frac{\partial u\left(\bar{\Pi}^\theta\right)}{\partial \bar{\Pi}^\theta} = \tau^\theta \tag{46}$$

These conditions imply that the optimal amount of default is defined when the marginal utility of defaulting equals the marginal dis-utility from incurring in default.

There are three cases to analyze for agents α, β and bank θ (in this case $\hat{h} \in \hat{H} = \{\alpha, \beta, \theta\}$).

i. $u^{\hat{h}\prime}(\cdot) > \tau^{\hat{h}}$. This condition implies that the agent $\hat{h} \in \hat{H}$ will default completely. Consequently, the asset will not be traded.

ii. $u^{\hat{h}\prime}(\cdot) \ll \tau^{\hat{h}}\left(\text{ i.e. } \tau^{\hat{h}} \to \infty\right)$. This condition implies that buyers will anticipate full delivery, but sellers of the asset (i.e. borrowers) will realize that with some probability they will not be able to avoid a crushing penalty. Consequently, the asset will not be traded.

iii. $u^{\hat{h}\prime}(\cdot) = \tau^{\hat{h}}$. An intermediate level of default penalties can make everyone better-off. This condition implies that there is trading in assets.

4.8.5 Proposition 5: Interest rates under default and no default

This condition implies that in case there is default the interest rates are defined to be positive, because the commercial and central bank need to be compensated because of the losses from default.

Suppose that for $\alpha, \beta \in H$, $b_t^h > 0$, for $l \in L$, $\lambda_t^h \in [0, 1)$ and some state of nature defined by the set of shocks at t. We have that at a **FSMLD**, the interest rates under default are $r_t^c, r_t^{IB} > 0$. Whereas under no default, the interest rates are $r_t^c = r_t^{IB} = 0$.

5 Solution of the model

In this section, we describe the different steps we follow in order to find the numerical solution of our model and also generate the impulse response functions of the shocks affecting the variables. The solution method includes the parametrization and calibration, the steady state solution and the simulations.

5.1 Calibration

In this solution step we calibrate our model for three different cases. The first is the pure cash in advance economy, when there is no liquidity in goods. The second case is the one with asymmetric liquidity. The third case is symmetric in liquidity with slightly higher overall liquidity than the asymmetric case.

Table 2: Parametrization

Parameter	**Basic model**	$\lambda^{\alpha} = 0.5, \lambda^{\beta} = 0.4$	$\lambda^{\alpha} = 0.5, \lambda^{\beta} = 0.5$
$\bar{\eta}^{CB}$	1	1	1
$\bar{\tau}^{\alpha}$	2.023	1.106	1.011
$\bar{\tau}^{\beta}$	2.023	1.121	1.011
$\bar{\tau}^{\theta}$	150	150	150
$\bar{e}_1^{\alpha}$	2	2	2
$\bar{e}_2^{\beta}$	2	2	2
β^{CB}	0.97	0.97	0.97
$\hat{\beta}^{CB}$	0.98	0.98	0.98
ρ^{CB}	0.5	0.5	0.5
ρ_e^{α}	0.5	0.5	0.5
ρ_e^{β}	0.5	0.5	0.5
ρ_e^{θ}	0.5	0.5	0.5
$\bar{M}$	1	1	1

In this exercise the calibration is made to match an annual commercial bank interest rate rate of 5% in the steady state. Additionally, we can observe from table 2, that we have set the steady state monetary operations to constant. We will shock this variable afterwards. it is worth to remember that the Central Bank in our case acts as a strategic dummy, and we want to see the effect of its actions in the presence of different liquidity environments and default.

Following the same logic and as a consequence from the previous parameters setting, in steady state, default penalties are set equal for both households, except in the asymmetric liquidity case. In the latter, the default penalty is higher for the agent endowed with the less liquid good, this is because he needs to borrow more money from the bank and therefore, to align his incentives to repay, the punishment should increase accordingly.

Betas, or time impatience parameters for households and commercial bank are set according to the common trend in literature. In this case, as in reality, banks have a higher parameter than households.

Finally, we use persistence of shocks parameters to 0.5 to have every shock in the same scale. This will help us in the posterior simulation (i.e. impulse responses) step.

5.2 Steady state

Steady state can be calculated after the calibration solution step. It is a non-linear system of equations that we solve by using the standard Newton-Raphson algorithm. As our problem is simple and our function are continuous and smooth this technique presents no problems. In table (3) we present the steady state solution.

Central bank interest rate is set to zero to isolate the effects on financial stability rather to concentrate on monetary policy. Whereas commercial bank rate is calibrated to 5% per annum. Inflation is set to an average of 3% per year, which is the target for most of the Central Banks around the world.

Given our calibration, we have a symmetric allocation and pricing equilibrium in steady state, except in the asymmetric liquidity case. The purpose is to set the easiest benchmark case to be compared in the presence of shocks in the simulations section. Thus, we can observe direction and magnitude of the divergences from the steady state. In the case when there is asymmetric liquidity prices are higher for the less liquid good, as we would expect, the relatively scarcer the good, the higher the price. Following the same argument, trade is higher in the less liquid good.

Average repayment rate is higher in steady state in the case of asymmetric liquidity case for the less liquid good since the agent is punished higher in case there is default on this asset. Additionally, default penalties for banks are higher thank for households, since the volume of credit they trade is higher at a lower price and they need to be punished more in order to align their incentives.

As a result of our calculation and steady state **FSMLD**, the profits of bank are about 0.007

Table 3: Steady state

Variable	**Basic model**	$\lambda^{\alpha}=0.5$ $\lambda^{\beta}=0.4$	$\lambda^{\alpha}=0.5$ $\lambda^{\beta}=0.5$
p_1	0.513	0.920	1.012
p_2	0.513	0.922	1.012
r^c	0.05	0.05	0.05
r^{IB}	0	0	0
R	0.952	0.953	0.952
μ^{α}	0.525	0.486	0.525
b_2^{α}	0.500	0.915	1.000
υ^{α}	0.952	0.928	0.952
q_1^{α}	0.976	0.981	0.988
η_1^{α}	-1.94	-1.074	-0.982
η_2^{α}	-2.000	-1.093	-1.000
μ^{β}	0.525	0.563	0.525
b_1^{β}	0.500	0.902	1.000
υ^{β}	0.952	0.974	0.952
q_2^{β}	0.976	0.992	0.988
η_1^{β}	-1.964	-1.088	-0.982
η_2^{β}	-2.000	-1.108	-1.000
Π^{θ}	0.007	0.007	0.007
μ^{θ}	1	1	1
l^{θ}	1	1	1
υ^{θ}	0.993	0.993	0.993
η_1^{θ}	-150	-150	-150
η_2^{θ}	-143	-143	-143
M	1	1	1
e_1^{α}	2	2	2
e_2^{α}	2	2	2
τ^{α}	2.023	1.106	1.011
τ^{β}	2.023	1.121	1.011
τ^{θ}	150	150	150
η^{CB}	1	1	1
π	1.03	1.03	1.03

per annum. As we have set the amount of lending to 1 in steady state, the previous which means that the annual real profits of the bank are 0.7%.

5.3 Simulations

For the dynamic solution of our model, we follow a standard procedure[4]. There are two possible solution methods for the dynamic programming problem we need to solve for the dynamics of our system. The first is a global method, consisting in solving recursively the Bellman system of equations. This method implies a grid search and consequently the need to specify the special properties of the value function and it is also computationally demanding. The second alternative is a local method or approximation. Our problem is parsimonious, it does not have severe non-linearities and has a limited number of variables, therefore we take the second alternative approach and solve by using a second order approximation.

In this section we describe the effect of shocks to M, fundamentals, default penalties and liquidity on commercial bank interest rate (asset prices), inflation, repayment rates, quantities traded, welfare and bank profitability. We compare the effects of shocks in three cases.

1. Basic case when there is only a pure cash in advance constraint because there is no liquidity in goods, , $\bar{\lambda}^{\alpha} = \bar{\lambda}^{\beta} = 0$.
2. Asymmetric liquidity case $\bar{\lambda}^{\alpha} = 0.5$, $\bar{\lambda}^{\beta} = 0.4$.
3. Symmetric liquidity case, $\bar{\lambda}^{\alpha} = \bar{\lambda}^{\beta} = 0.5$.

5.3.1 Asset prices

Figure (4) summarizes the different shock effects on commercial bank interest rate. Naturally, by the credit and REPO market clearing conditions, as we decrease monetary basis in the economy interest rates increase in the short run. As we expected from Martinez (2010), a negative shock on liquidity exerts a negative impact on asset prices, when keeping all of the other variables constant (including fundamentals), as a result, interest rates increase accordingly. This impact is lowered for the less liquid good shock in the asymmetric liquidity case. Additionally, an increase on default penalties implies a reduction on the interest rates, because a higher default penalty implies a higher cost of default for the agents. Finally, an unexpected improvement in endowments in the short run implies an increase to the interest rate, because there are higher pressures to trade and credit receives a higher demand, therefore interest rates should increase to compensate the growth in default possibilities.

[4]For further reference see Sargent (1986) and Marimon and Scott (1999).

5.3.2 Inflation

Figure (5) summarizes the different shock effects on inflation. When monetary basis decrease, inflation responds negatively in the short run, in the medium run it increases and it returns to the steady state, this is because prices fall immediately to compensate the relative lack of fiat money in the economy, conversely, in the medium run, inflation over adjusts because of the increase in credit costs, traduced in lower prices especially in the case of liquid good environments. In the same line, a positive shock on endowment implies a negative immediate impact on inflation, but in the medium run it has a positive effect, this is explained because the less availability of goods implies an immediate price decline due to the substitution effect, after this price reduction, excess demand pressures produce an over-adjustment that is reducing to the steady state level in the long run. The larger effect of a combined negative shock of endowment (i.e. affecting both households) on inflation is explained by a more severe reduction in demand (and supply) of goods in the short run. The same logic applies to the goods liquidity shock on inflation, but this time is about availability of resources. As we expected, a negative shock on liquidity implies a negative response of inflation because the less relative availability of resources as mean of payment causes a reduction in demand. However in the medium run it has a positive impact, because of the over-adjustment. It is also important to mention that default penalties have a higher impact on inflation in the case when there is no liquidity in the market. In this sense liquidity acts as a buffer in case of changes in credit regulation, because it provides a better an alternative source of funds when the credit conditions are tighter.

5.3.3 Repayment rates

Figures (6) and (7) summarize the different shock effects on the average household repayment rate and commercial bank repayment rate, respectively. We can see that a negative shock in money supply decrease the repayment rates only in the medium run, because at that point agents are able to repay less contracted debt, since is relatively more costly for them. As we explained before, a positive shock on endowment implies a worsening in repayment rates in the medium run, because there are more pressures to trade and therefore more credit is contracted, this excess of credit involves a reduction in the repayment until the system adjusts to the steady state. It is relevant to mention that a negative shock in liquidity implies an important negative impact on repayment rates, this is due the less accessibility to resources to repay existing debts in the medium run. This fact is crucial for financial stability, when the economy is experiencing a bad state of nature the systemic drivers of the credit market become worse, as a result there is less agreement on the true value of goods, this implies the goods liquidity to decrease, incrementing defaults. We can observe exactly the same causes and consequences in the mortgage market during the last crisis.

In case of the commercial bank, as this institution aggregates the risks the impact on the shocks is different. There is still an impact of money supply on the repayment rate of the bank, but the effect is smaller. The opposite dynamics occur with the effect of endowment shocks, but the impact is even more reduced. We can also observe that relative liquidity environment is a determinant factor in determining the bank repayment rate when shocked by regulation or liquidity.

5.3.4 Trade

Figures (8) and (9) summarize the different shock effects on the quantities traded. Less fiat money implies less trade because there are less financing opportunities in the economy. We have different effects of endowment shocks on trade depending on the good that is receiving the perturbation. For a determined household, if the commodity bought is receiving the positive shock in endowment, it generates a negative change of quantities of the sales, as opposite of what happens with a shock on the quantity of good that the agent is endowed with, this is because of the relative imbalance in quantities. If the shock is simultaneous and positive, it generates more trade for both agents, proportional to the increase in quantities. If regulation is tighter for one of the agents, he will lower the amount he buys since he has a higher cost to finance the trade.

5.3.5 Welfare

Figure (10), (11) and (12) summarize the different shock effects on welfare. In order to approach welfare, we look at the effects of shocks on overall utilities of households and the commercial bank. A negative shock on fiat money is welfare inferior in the medium run for both, households and the commercial bank, because it increases the cost of credit and lowers trade an consumption, especially as we can examine in depressed economic scenarios. A positive individual shock on endowment generates an immediate improvement in welfare, but utility is reduced in the medium run for the households when the shock on endowment is simultaneous for both goods, since higher supply implies a considerable price reduction that generates less revenues and consequently less consumption for both households. The impact of a positive shock in default penalties in the short run is negative for all of the market participants, however, as the agent is able to pay more afterwards, it is welfare improving in the medium run. In the commercial bank individual case, a higher default penalty for the agent endowed with the most liquid good generates an improvement in its utility, because the household will be able to repay relatively more in the medium run. Additionally, utility is reduced in case of negative shock to liquidity, since it causes less resources to be available to finance trade. The bank is better off when the shock affects the less liquid good, since it acts as a financial intermediary and is most needed when there is a negative shock in liquidity, especially for the illiquid good owner. As we can observe, liquidity environment plays an important role on commercial bank's utility, especially when receiving a liquidity or regulatory shock.

5.3.6 Bank Profitability

Figure (13) summarizes the shock effects on bank profitability. The only significant impact on profitability is the shock on the default penalty for the bank. All the other effects are naturally minimal, given that in our model we just have one asset to trade and there is no alternative investment. Bank profitability is affected mainly by the penalties. If we follow the Aspachs et. al. (2007) definition of financial stability, probability of default and bank profitability play an important role. This implies a trade-off for the regulatory agency, since it need to equilibrate an increase in the penalties, to control default, with commercial bank profits, these need not to fall to dangerous levels to avoid the bank not to repay its contractual obligations, or a reduction in credit extension and the consequent detriment in trade and consumption.

5.4 On resilience and financial stability

In this section we describe economic resilience in the presence of default and liquidity. In our context we define economic resilience as the ability of the economic variables to face and recover from adverse shocks. This recovery can be assessed as the impact and speed of shocks transmission and recovery to the steady state values. Put differently, resilience can be seen as the second derivative of the multiplier (i.e. acceleration of convergence) from a starting initial impact.

In our case speed of adjustment is controlled by the persistence parameter ρ for every shock process (AR(1) in our case, for all the shocks). One could try to calibrate this parameter to each economic series, instead, we start from a symmetric case, just to address the differences in adjustment across the different variables being shocked and not within each one. Yet, the individual calibration is a useful procedure left for future investigation.

As we have seen in the previous section, financial stability can be roughly approximated by bank profitability and joint repayment rates. Through the comparison of the different impulse responses we can rank the leading and lagged variables in response to shocks and also whether financial stability is affected more than price stability and the effects on real economy.

Following this analysis we can also compare responses to shocks across agents. Our results suggest that the commercial bank is more resilient adverse shocks, since it aggregates the risks of the agents.

The principal findings of our simulations suggest that financial stability is more affected than price stability, however, the convergence of the latter to the steady state is slower. In this framework, yet reduced through our initial parametrization, monetary policy needs to be thought simultaneously with financial stability because trying to improve price stability can generate financial distress and a detriment to trade and welfare. In our analysis welfare lags trade although it converges faster to the steady state, therefore trade needs to be considered as an indicator of future economic performance. When interest rates are low, reduction in monetary basis impact negatively the economy with a higher effect than the benefits of price control, therefore as in reality there is a trade off between price targeting and welfare. In our case we are not modeling production, therefore all of the welfare effects are determined through trade. This can be a matter of further discussion, however, yet simple, this is a realistic assumption, since it reflects the behavior of the agents in the economy.

6 Concluding Remarks

We have developed a model that, by including agent heterogeneity, liquidity and default in a pure exchange DSGE model, is capable of addressing issues of financial stability.

Our results suggest that liquidity and default in equilibrium should be studied contemporaneously. Moreover, agent heterogeneity is essential to assess the welfare effects of exogenous shocks, since these depend on the part of the economy directly affected. In addition, the presence of financial frictions underlines the importance of studying the impact of shocks to the short to medium run behaviour of financial variables and welfare. Finally, unlike New-Keynesian models, due to endogenous determination of interest rates and default in the credit market, there is no need to resort to artificial fixing of prices to obtain equilibria that possess real as well as nominal determinacy.

Following the results of our study, we propose three further developments in this framework. The first improvement would be to micro-found liquidity of assets as a result of the endogenous interaction with the liquidity of goods. The second implication is the need to consider liquidity in the measurement of financial stability; as we ascertain in this work, liquidity constitutes a critical factor when addressing financial fragility. Whereas the third suggestion of this work, would be to include the possibility of price anomalies due to misalignment in expectations of the agents on the future asset prices. Our conjecture is that this additional financial friction would imply a further impact on financial stability and, as a result, a large impact to economic performance.

This model and its preliminary results should be viewed as an initial attempt to study financial stability and monetary policy. Undoubtedly, one needs to introduce production and reaction function of policy makers, in particular the Central Bank. Nevertheless, our model can be used to make these extensions, however these will be addressed in future studies.

7 References

Acharya V. and L. Pedersen, *Asset Pricing with Liquidity Risk*; Journal of Financial Economics, 77, 375-410, 2005.

Acharya V. and Richardson, Matthew P., *Causes of the Financial Crisis*; Critical Review, Vol. 21, Nos. 2, 3, pp. 195-210, 2009.

Aspachs, Oriol, Charles Goodhart, Dimitrios Tsomocos and Lea Zichino, *Towards a measure of financial fragility*, Annals of Finance, Springer, vol. 3(1), pages 37-74, 2007.

Bernanke, B., Gertler, M., Gilchrist, S. *The Financial Accelerator in a Quantitative Business Cycle Framework*, Working Papers 98-03, C.V. Starr Center for Applied Economics, New York University, 1998.

Brunnermeier M., L. Pedersen, *Market Liquidity and Funding Liquidity*; Review of Financial Studies, 22(6), 2201-2238, 2009.

Covas, Francisco and Shigeru Fujita. *Procyclicality of Capital Requirements in a General Equilibrium Model of Liquidity Dependence*, International Journal of Central Banking, vol. 6(34), pages 137-173, , 2010.

Curdia, Vasco and Michael Woodford, *Credit Frictions and Optimal Monetary Policy*, revised August 2009. Revised draft of paper prepared for the BIS annual conference, Whither Monetary Policy? Lucerne, Switzerland, June 26-27, 2008.

Dubey P., John Geanakoplos and M. Shubik, *Default and Punishment in General Equilibrium*; Econometrica, 73(1): 1-37, 2005.

Dubey P., John Geanakoplos, *Monetary Equilibrium with Missing Markets*; Journal of Mathematical Economics, 39: 585-618. [CFDP 1389 and CFP 1063], 2003.

Espinoza R., Dimitrios P. Tsomocos *Liquidity and Asset Prices in Monetary Equilibrium*; Working Paper,2010.

Fisher Irving, *The Purchasing Power of Money: Its Determination and Relation to Credit, Interest, and Crises*; Library of Economics and Liberty; (LE&L),1911a.

Goodhart, C. A. E. with Sunirand, P. and Tsomocos, D., *A model to analyse financial fragility*; Economic Theory, 27, pp. 107-142, 2006.

Iacoviello, M. and S. Neri, *Housing Markets Spillovers: Evidence from an Estimated DSGE Model*; Working Paper 659, Boston College Department of Economics, 2007.

Kiyotaki, N. and John Moore, *Liquidity, Business Cycles, and Monetary Policy*, Clarendon Lectures LSE, 2001, latest version, 2010.

Leao E., and P. Leao, *Modelling the central bank repo rate in a dynamic general equilibrium framework*, 2007; Economic Modelling,24, pp. 571-610, 2006.

Meh, Cesaire A. and Moran, Kevin. *The role of bank capital in the propagation of shocks*, Journal of Economic Dynamics and Control, Elsevier, vol. 34(3), pages 555-576, 2010.

Marimon R. and Scott, A., (eds.) *Computational Methods for the Study of Dynamic Economies*, Oxford University Press, Oxford, UK, 1999.

Martinez, Juan Francisco, *On the determinants of EM bond compression and decompression*; Working Paper, Saïd Business School University of Oxford, this version: 2010.

Powell, Andrew and Martinez, Juan Francisco *On Emerging Economy Sovereign Spreads and Ratings*, Inter American Development Bank, Working Paper 629, 2008.

Sargent, T., *Dynamic Macroeconomic Theory*, Harvard University Press , Cambridge, Ma, 1986.

Shubik M., Charles Wilson *The Optimal Bankruptcy Rule in a Trading Economy Using Fiat Money*; Zeitschrift fur Nationalokonomie, 37, 3-4, 337-354, CFP 465, 1977.

Shubik M., *The Theory of Money and Financial Institutions*; Cambridge, MA; UK: MIT Press, 1999.

Tsomocos D., *Equilibrium Analysis, Banking, and Financial Instability*; Journal of Mathematical Economics, 39 (5-6), 619-655, 2003.

Vayanos D., *Flight to Quality, Flight to Liquidity and the Pricing of Risk*; working paper, London School of Economics, 2004.

de Walque, G., O. Pierrard y A. Rouabah, *Financial (in)stability, supervision, and liquidity injections: a dynamic general equilibrium approach*; Working Paper Research 148, National Bank of Belgium, 2008, Economic Journal, 2010.

8 Appendix 1: Proof of the propositions

8.1 Proposition 1: Money non-neutrality

Proof.

From the FOC (60) and (61) for agent α, we have:

$$\frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{1,t}^{\alpha}} - \frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{2,t}^{\alpha}} \frac{\tilde{p}_{1,t}}{\tilde{p}_{2,t}} \lambda_t^{\alpha} = \tilde{p}_{1,t} \beta \mathbb{E}_t \left(\frac{\tau_{t+1}^{\alpha}}{\pi_{t+1}}\right)(1-\lambda_t^{\alpha}) \tag{47}$$

And combining with (58) and (59), we have that:

$$\frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{1,t}^{\alpha}} - \frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{2,t}^{\alpha}} \frac{\tilde{p}_{1,t}}{\tilde{p}_{2,t}} \lambda_t^{\alpha} = \frac{1}{1+r_t^c} \frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{2,t}^{\alpha}} \frac{\tilde{p}_{1,t}}{\tilde{p}_{2,t}} (1-\lambda_t^{\alpha}) \tag{48}$$

And simplifying, we have,

$$\frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{1,t}^{\alpha}} = \frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{2,t}^{\alpha}} \frac{\tilde{p}_{1,t}}{\tilde{p}_{2,t}} \left(\frac{1+\lambda_t^{\alpha} r_t^c}{1+r_t^c}\right) \tag{49}$$

Similarly, for household β, we have:

$$\frac{\partial u\left(c_{1,t}^{\beta}, c_{2,t}^{\beta}\right)}{\partial c_{2,t}^{\beta}} = \frac{\partial u\left(c_{1,t}^{\beta}, c_{2,t}^{\beta}\right)}{\partial c_{1,t}^{\beta}} \frac{\tilde{p}_{2,t}}{\tilde{p}_{1,t}} \left(\frac{1+\lambda_t^{\beta} r_t^c}{1+r_t^c}\right) \tag{50}$$

Suppose at some period t, M_t decreases. from REPO market clearing condition (26) we have that it translates in a increase of r_t^{IB}, and if budget conditions are binding, r_c decreases accordingly. If we also have that λ_t^{α} is decreasing (or constant), from (49) to hold, $\frac{\tilde{p}_{1,t}}{\tilde{p}_{2,t}}$ must increase. Whereas, for (50) to hold, if λ_t^{α} is decreasing (or constant), $\frac{\tilde{p}_{2,t}}{\tilde{p}_{1,t}}$ must increase. This is a contradiction, therefore, monetary policy is not neutral in the short-run if $\lambda^{\alpha}, \lambda^{\beta} \in [0,1)$. The consequent inequality in quantities after a change in OMOs is guaranteed by our logarithmic utility functions. $\square$

8.2 Proposition 2: Fisher effect

Proof.

It follows directly from the Money-non neutrality proposition proof, equations (49) and (50) above.$\square$

8.3 Proposition 3: Quantity theory of money

Proof.

In an interior equilibrium and with no money carried over, all the budget constraints are binding. We add budget constraints (18) and (20), and we have,

$$\tilde{b}^{\alpha}_{2,t} + \tilde{b}^{\beta}_{1,t} \leq \lambda^{\alpha}_{t} \cdot \tilde{p}_{1,t} q^{\alpha}_{1,t} + \lambda^{\beta}_{t} \cdot \tilde{p}_{2,t} q^{\beta}_{2,t} + \frac{\tilde{\mu}^{\alpha}_{t}}{1+r^{c}_{t}} + \frac{\tilde{\mu}^{\beta}_{t}}{1+r^{c}_{t}} \tag{51}$$

From the goods market clearing condition (23), and the previous relation we have,

$$\tilde{p}_{1,t} q^{\alpha}_{1,t} + \tilde{p}_{2,t} q^{\beta}_{2,t} \leq \lambda^{\alpha}_{t} \cdot \tilde{p}_{1,t} q^{\alpha}_{1,t} + \lambda^{\beta}_{t} \cdot \tilde{p}_{2,t} q^{\beta}_{2,t} + \frac{\tilde{\mu}^{\alpha}_{t}}{1+r^{c}_{t}} + \frac{\tilde{\mu}^{\beta}_{t}}{1+r^{c}_{t}} \tag{52}$$

Finally, by the Consumer Loans and REPO markets clearing conditions (25) and (26), respectively, we have:

$$\tilde{p}_{1,t} q^{\alpha}_{1,t} + \tilde{p}_{2,t} q^{\beta}_{2,t} = \lambda^{\alpha}_{t} \cdot \tilde{p}_{1,t} q^{\alpha}_{1,t} + \lambda^{\beta}_{t} \cdot \tilde{p}_{2,t} q^{\beta}_{2,t} + M_{t} \tag{53}$$

Where the equality comes from the assumption of no money being carried over, so the market clearing conditions are binding. Rearranging we have that relation (32) holds.$\square$

8.4 Proposition 4: On the verge condition

Proof.

The proof follows immediately from agent α, β and bank θ FOCs.$\square$

8.5 Proposition 5: Interest rates under default and no default

Proof.

From the agent α and β budget constraints (17) and (19) addition, and the assumption that no money is carried over (i.e. budget constraints are binding) we have:

$$\upsilon^{\alpha}_{t} \tilde{\mu}^{\alpha}_{t-1} + \upsilon^{\beta}_{t} \tilde{\mu}^{\beta}_{t-1} = \tilde{p}_{1,t-1} q^{\alpha}_{1,t-1} \cdot \left(1 - \lambda^{\alpha}_{t-1}\right) + \tilde{p}_{2,t-1} q^{\beta}_{2,t-1} \cdot \left(1 - \lambda^{\beta}_{t-1}\right) \tag{54}$$

From the quantity theory of money proposition, we have:

$$\upsilon_t^\alpha \tilde{\mu}_{t-1}^\alpha + \upsilon_t^\beta \tilde{\mu}_{t-1}^\beta = M_{t-1} \tag{55}$$

Additionally, from the market clearing conditions (25) and (26) and equation (refirdnf1), we have that the following relation must hold,

$$\upsilon_t^\alpha \tilde{\mu}_{t-1}^\alpha + \upsilon_t^\beta \tilde{\mu}_{t-1}^\beta = \frac{\tilde{\mu}_{t-1}^\alpha + \tilde{\mu}_{t-1}^\beta}{1 + r_{t-1}^c} \tag{56}$$

In the full delivery case[5], we have that $\upsilon_t^\alpha = \upsilon_t^\beta = 1$. Thus, for (56) to hold it must be the case that $r_{t-1}^c = 0$. Whereas for the case when $\upsilon_t^\alpha < 1$ or $\upsilon_t^\beta < 1$ or both, r_{t-1}^c must be strictly grater than zero. This applies $\forall t \in T$.

In case of the bank θ, from the combination of FOC, we can easily derive that:

$$\mathbb{E}_t \left(R_{t+1} \right) = \left(\frac{1 + r_t^{IB}}{1 + r_t^c} \right) \tag{57}$$

Therefore, in the full delivery case (in $t+1$), we need that $r_t^c = r_t^{IB} = 0$. While in the case of partial default ($\mathbb{E}_t \left(R_{t+1} \right) < 1$), we must have that $0 < r_t^c < r_t^{IB}$.$\square$

[5]Recall that $\upsilon_t^\alpha, \upsilon_t^\beta \in [0, 1]$.

9 Appendix 2: First Order Conditions

9.1 FOCs for Household α

$$-\frac{\eta_{2,t}^{\alpha}}{1+r_t^c}-\beta\mathbb{E}_t\left(\frac{\tau_{t+1}^{\alpha}\left(1-\upsilon_{t+1}^{\alpha}\right)}{\pi_{t+1}}-\eta_{1,t+1}^{\alpha}\upsilon_{t+1}^{\alpha}\right)=0 \tag{58}$$

$$\frac{\tau_t^{\alpha}}{\pi_t}+\eta_{1,t}^{\alpha}=0 \tag{59}$$

$$\frac{1}{\tilde{b}_{2,t}^{\alpha}}+\eta_{2,t}^{\alpha}=0 \tag{60}$$

$$\frac{1}{e_{1,t}^{\alpha}-q_{1,t}^{\alpha}}+\eta_{2,t}^{\alpha}\lambda_t^{\alpha}\tilde{p}_{1,t}+\beta\mathbb{E}_t\left(\eta_{1,t+1}^{\alpha}\tilde{p}_{1,t}\left(1-\lambda_t^{\alpha}\right)\right)=0 \tag{61}$$

$$\upsilon_t^{\alpha}\tilde{\mu}_{t-1}^{\alpha}-\tilde{p}_{1,t-1}q_{1,t-1}^{\alpha}\left(1-\lambda_{t-1}^{\alpha}\right)=0 \tag{62}$$

$$\tilde{b}_{2,t}^{\alpha}-\frac{\tilde{\mu}_t^{\alpha}}{1+r_t^c}-\lambda_t^{\alpha}\tilde{p}_{1,t}q_{1,t}=0 \tag{63}$$

9.2 FOCs for Household β

$$-\frac{\eta_{2,t}^{\beta}}{1+r_t^c}-\beta\mathbb{E}_t\left(\frac{\tau_{t+1}^{\beta}\left(1-\upsilon_{t+1}^{\beta}\right)}{\pi_{t+1}}-\eta_{1,t+1}^{\beta}\upsilon_{t+1}^{\beta}\right)=0 \tag{64}$$

$$\frac{\tau_t^{\beta}}{\pi_t}+\eta_{1,t}^{\beta}=0 \tag{65}$$

$$\frac{1}{\tilde{b}_{1,t}^{\beta}}+\eta_{2,t}^{\beta}=0 \tag{66}$$

$$\frac{1}{e_{2,t}^{\beta}-q_{2,t}^{\beta}}+\eta_{2,t}^{\beta}\lambda_t^{\beta}\tilde{p}_{2,t}+\beta\mathbb{E}_t\left(\eta_{1,t+1}^{\beta}\tilde{p}_{2,t}\left(1-\lambda_t^{\beta}\right)\right)=0 \tag{67}$$

$$\upsilon_t^{\beta}\tilde{\mu}_{t-1}^{\beta}-\tilde{p}_{2,t-1}q_{2,t-1}^{\beta}\left(1-\lambda_{t-1}^{\beta}\right)=0 \tag{68}$$

$$\tilde{b}_{1,t}^{\beta}-\frac{\tilde{\mu}_t^{\beta}}{1+r_t^c}-\lambda_t^{\beta}\tilde{p}_{2,t}q_{2,t}=0 \tag{69}$$

9.3 FOCs for Bank θ

$$\frac{1}{\tilde{\Pi}_t^{\theta}} + \eta_{1,t}^{\theta} = 0 \tag{70}$$

$$-\frac{\eta_{2,t}^{\theta}}{1+r_t^{IB}} - \hat{\beta}\mathbb{E}_t\left(\frac{\tau_{t+1}^{\theta}\left(1-\upsilon_{t+1}^{\theta}\right)}{\pi_{t+1}} - \eta_{1,t+1}^{\theta}\frac{\upsilon_{t+1}^{\theta}}{\pi_{t+1}}\right) = 0 \tag{71}$$

$$\tau_t^{\theta} + \eta_{1,t}^{\theta} = 0 \tag{72}$$

$$-\hat{\beta}\mathbb{E}_t\left(\frac{\eta_{1,t+1}^{\theta}R_{t+1}^{\theta}\left(1+r_t^c\right)}{\pi_{t+1}}\right) + \eta_{2,t}^{\theta} = 0 \tag{73}$$

$$\tilde{\Pi}_t^{\theta} - \frac{R_t\tilde{l}_{t-1}^{\theta}\left(1+r_{t-1}^c\right)}{\pi_t} + \upsilon_t^{\theta}\frac{\tilde{\mu}_{t-1}^{\theta}}{\pi_t} = 0 \tag{74}$$

$$\tilde{l}_t^{\theta} - \frac{\tilde{\mu}_t^{\theta}}{1+r_t^{IB}} = 0 \tag{75}$$

9.4 Market Clearing conditions and rational expectations

$$\tilde{b}_{1,t}^{\beta} - \tilde{p}_{1,t}q_{1,t}^{\alpha} = 0 \tag{76}$$

$$\tilde{b}_{2,t}^{\alpha} - \tilde{p}_{2,t}q_{2,t}^{\beta} = 0 \tag{77}$$

$$1 + r_t^c - \frac{\tilde{\mu}_t^{\alpha} + \tilde{\mu}_t^{\beta}}{\tilde{l}_t^{\theta}} = 0 \tag{78}$$

$$1 + r_t^{IB} - \frac{\tilde{\mu}_t^{\theta}}{M_t} = 0 \tag{79}$$

$$R_t - \frac{\upsilon_t^{\alpha}\tilde{\mu}_t^{\alpha} + \upsilon_t^{\beta}\tilde{\mu}_t^{\beta}}{\tilde{\mu}_t^{\alpha} + \tilde{\mu}_t^{\beta}} = 0 \tag{80}$$

9.5 Equations of shocks

$$\pi_t = \frac{M_t}{M_{t-1}} \tag{81}$$

$$M_t = \eta_t^{CB}\bar{M} \tag{82}$$

$$\ln(\eta_t^{CB}) = \rho^{CB}\ln(\bar{\eta}^{CB}) + (1-\rho^{CB})\ln(\eta_{t-1}^{CB}) + \epsilon_{CB,t} \tag{83}$$

$$\ln(e_{1,t}^{\alpha}) = \rho_e^{\alpha}\ln(\bar{e}_1^{\alpha}) + (1-\rho_e^{\alpha})\ln(e_{1,t-1}^{\alpha}) + \epsilon_{1,t}^{\alpha} \tag{84}$$

$$\ln(e_{2,t}^{\beta}) = \rho_e^{\beta}\ln(\bar{e}_2^{\beta}) + (1-\rho_e^{\beta})\ln(e_{2,t-1}^{\beta}) + \epsilon_{2,t}^{\beta} \tag{85}$$

$$\ln(\tau_t^{\alpha}) = \rho^{\alpha}\ln(\bar{\tau}^{\alpha}) + (1-\rho^{\alpha})\ln(\tau_{t-1}^{\alpha}) + \epsilon_{\alpha,t}^{\tau} \tag{86}$$

$$\ln(\tau_t^{\beta}) = \rho^{\beta}\ln(\bar{\tau}^{\beta}) + (1-\rho^{\beta})\ln(\tau_{t-1}^{\beta}) + \epsilon_{\beta,t}^{\tau} \tag{87}$$

$$\ln(\tau_t^{\theta}) = \rho^{\theta}\ln(\bar{\tau}^{\theta}) + (1-\rho^{\theta})\ln(\tau_{t-1}^{\theta}) + \epsilon_{\theta,t}^{\tau} \tag{88}$$

$$\ln(\lambda_t^{\alpha}) = \rho^{\alpha}\ln(\bar{\lambda}^{\alpha}) + (1-\rho^{\alpha})\ln(\lambda_{t-1}^{\alpha}) + \epsilon_{\alpha,t}^{\lambda} \tag{89}$$

$$\ln(\lambda_t^{\beta}) = \rho^{\beta}\ln(\bar{\lambda}^{\beta}) + (1-\rho^{\beta})\ln(\lambda_{t-1}^{\beta}) + \epsilon_{\beta,t}^{\lambda} \tag{90}$$

10 Appendix 3: Impulse response experiments

10.1 Asset prices

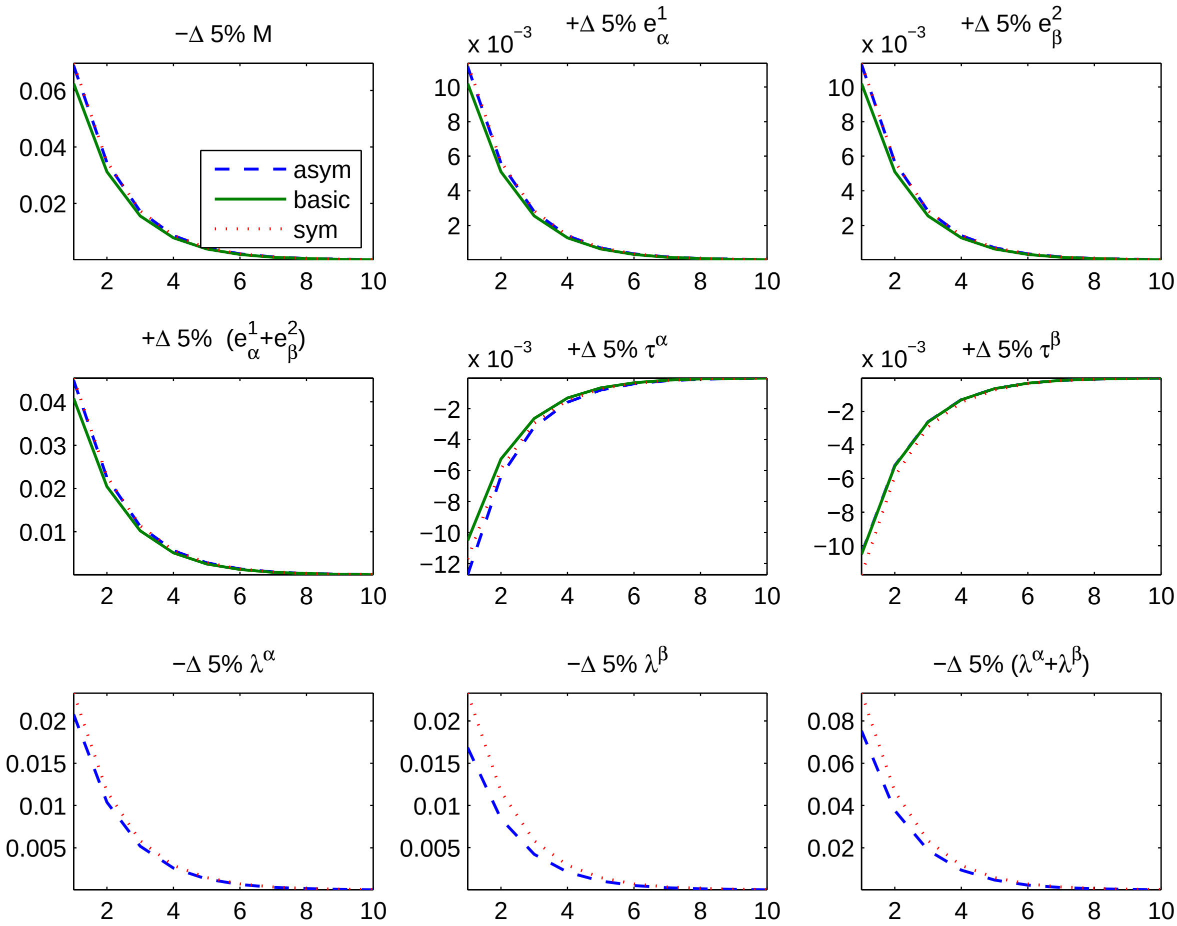

Figure 4: Impulse responses on commercial bank interest rate (r^c_t)

10.2 Inflation

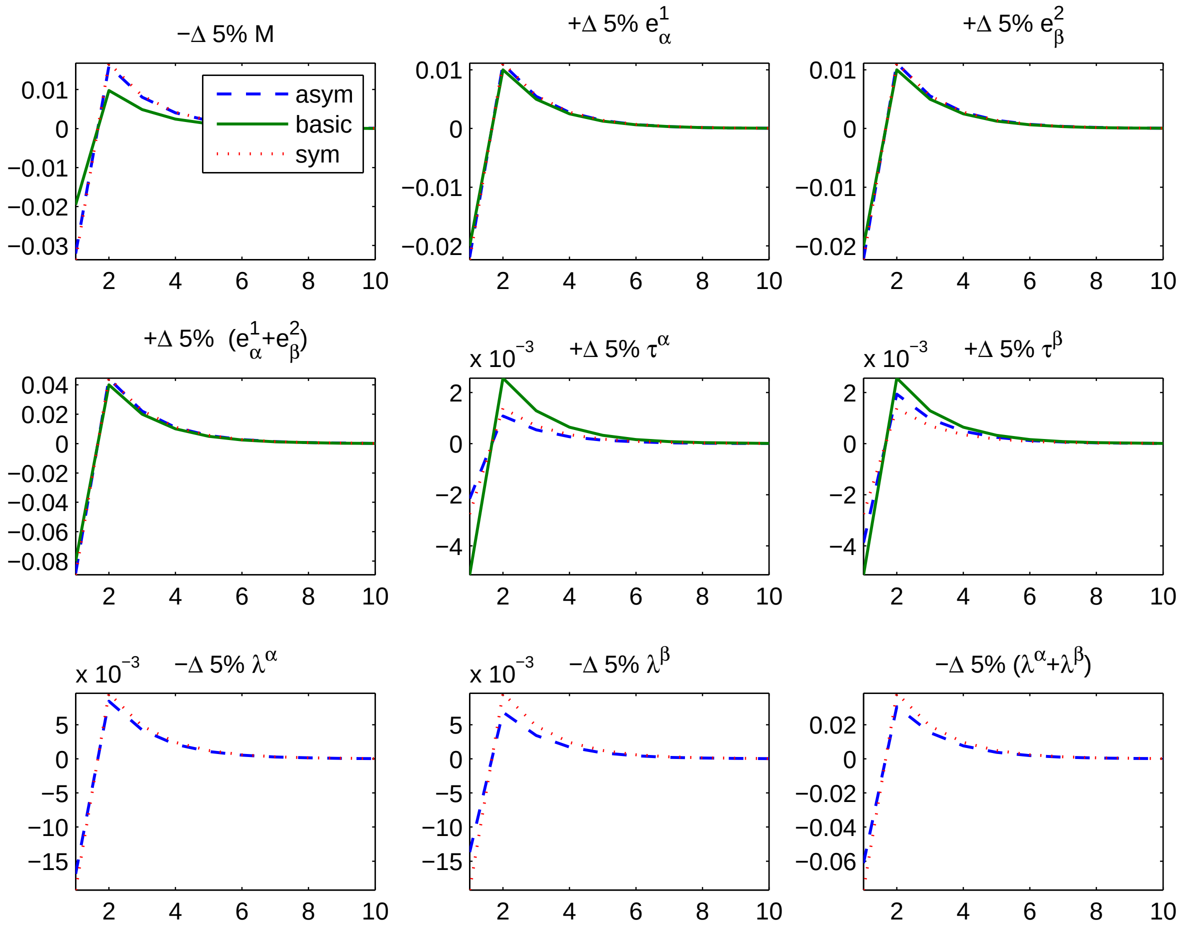

Figure 5: Impulse responses on inflation (π_t)

10.3 Repayment rates

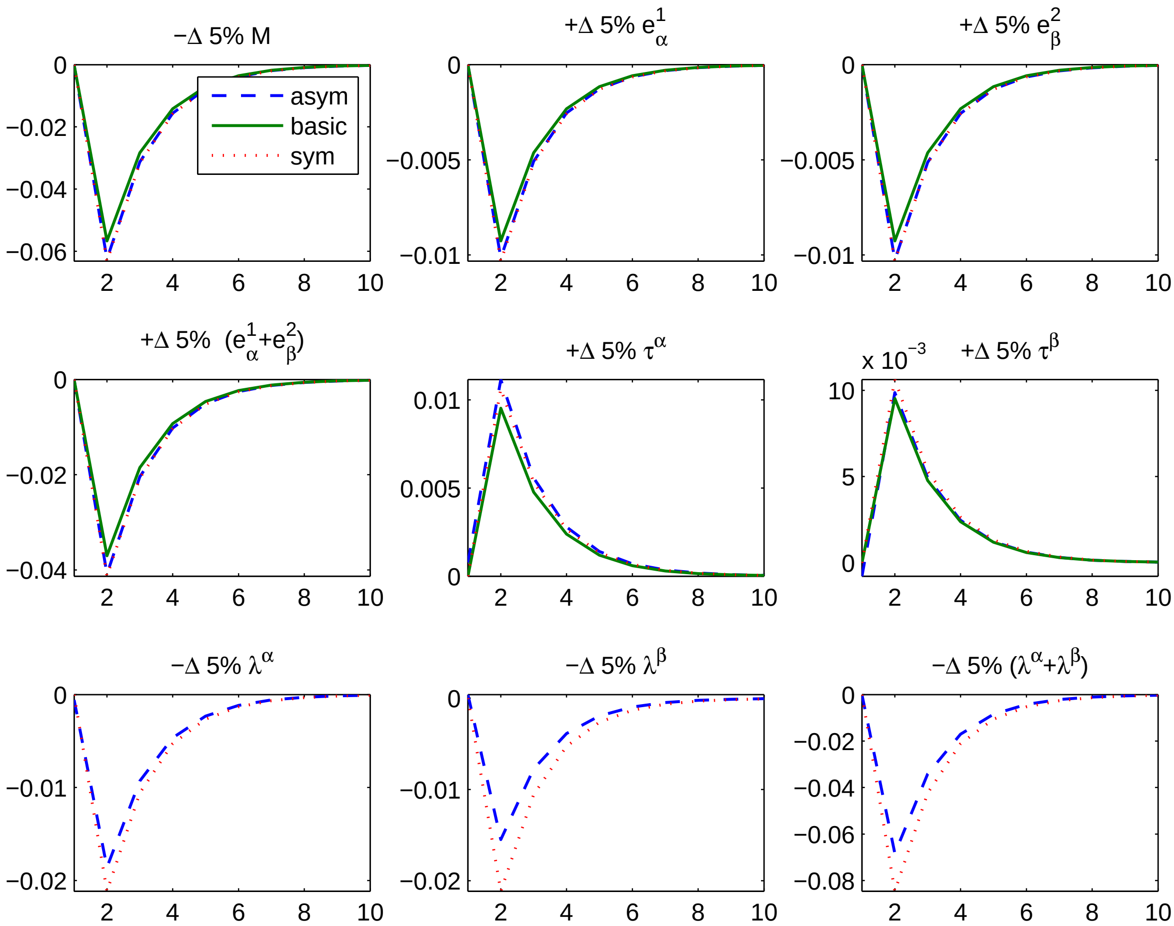

Figure 6: Impulse responses on average repayment rate (R_t)

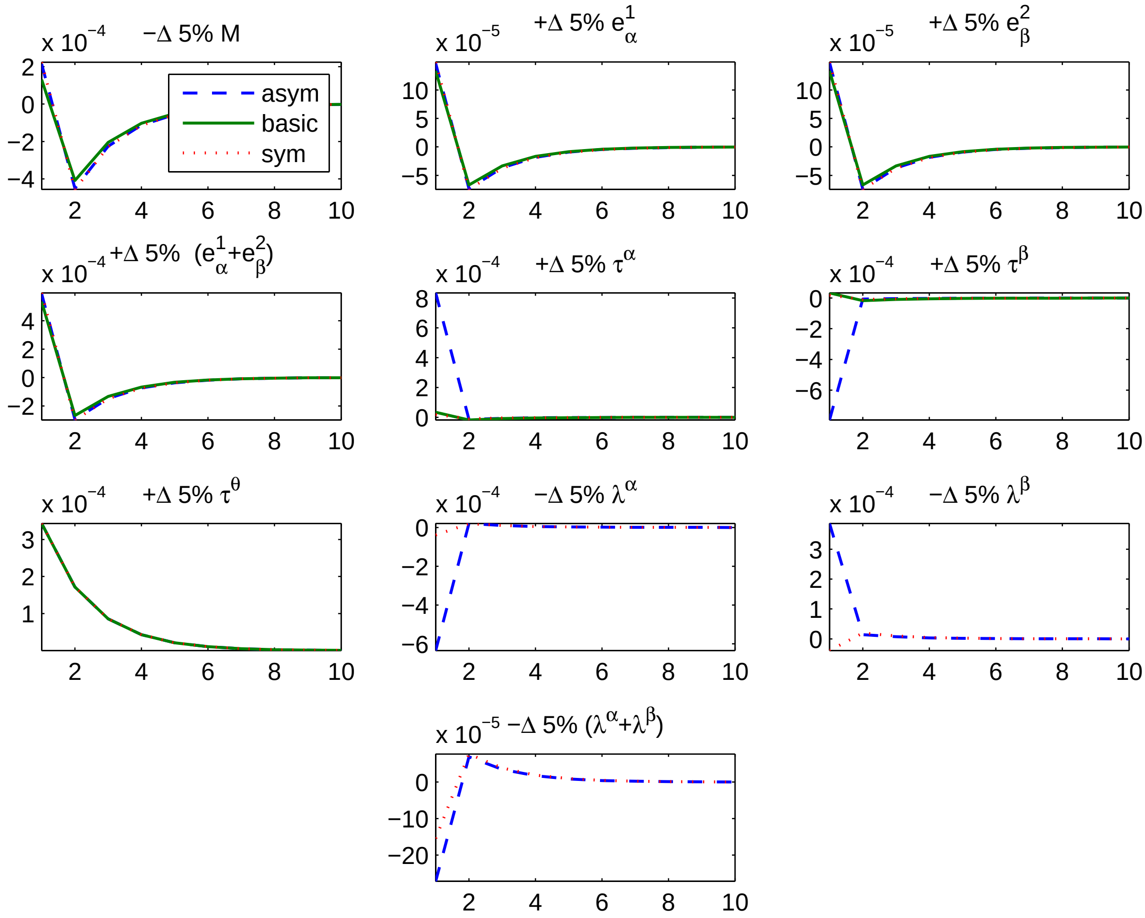

Figure 7: Impulse responses on commercial bank repayment rate (υ_t^θ)

10.4 Trade

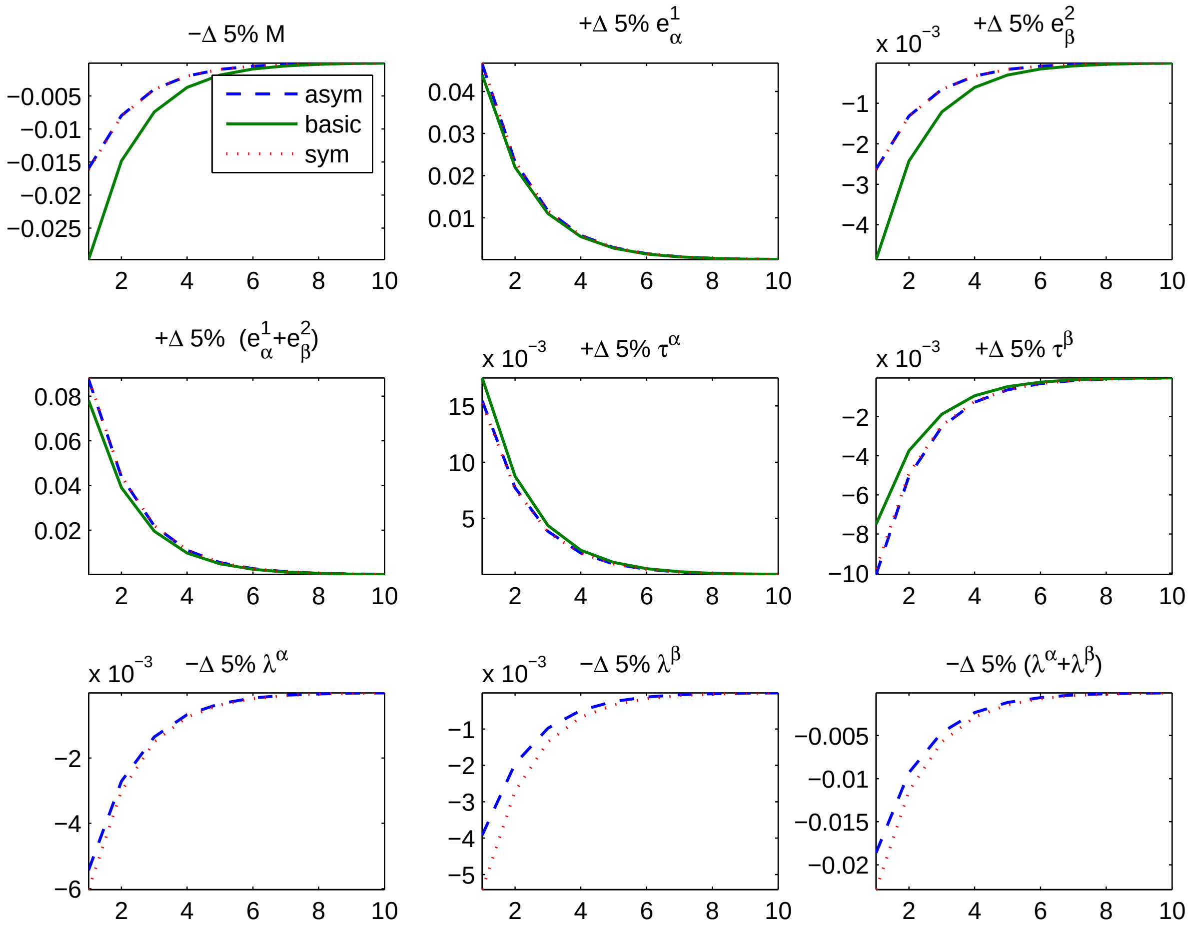

Figure 8: Impulse responses on quantity of good 1 traded by agent α ($q^{\alpha}_{1,t}$)

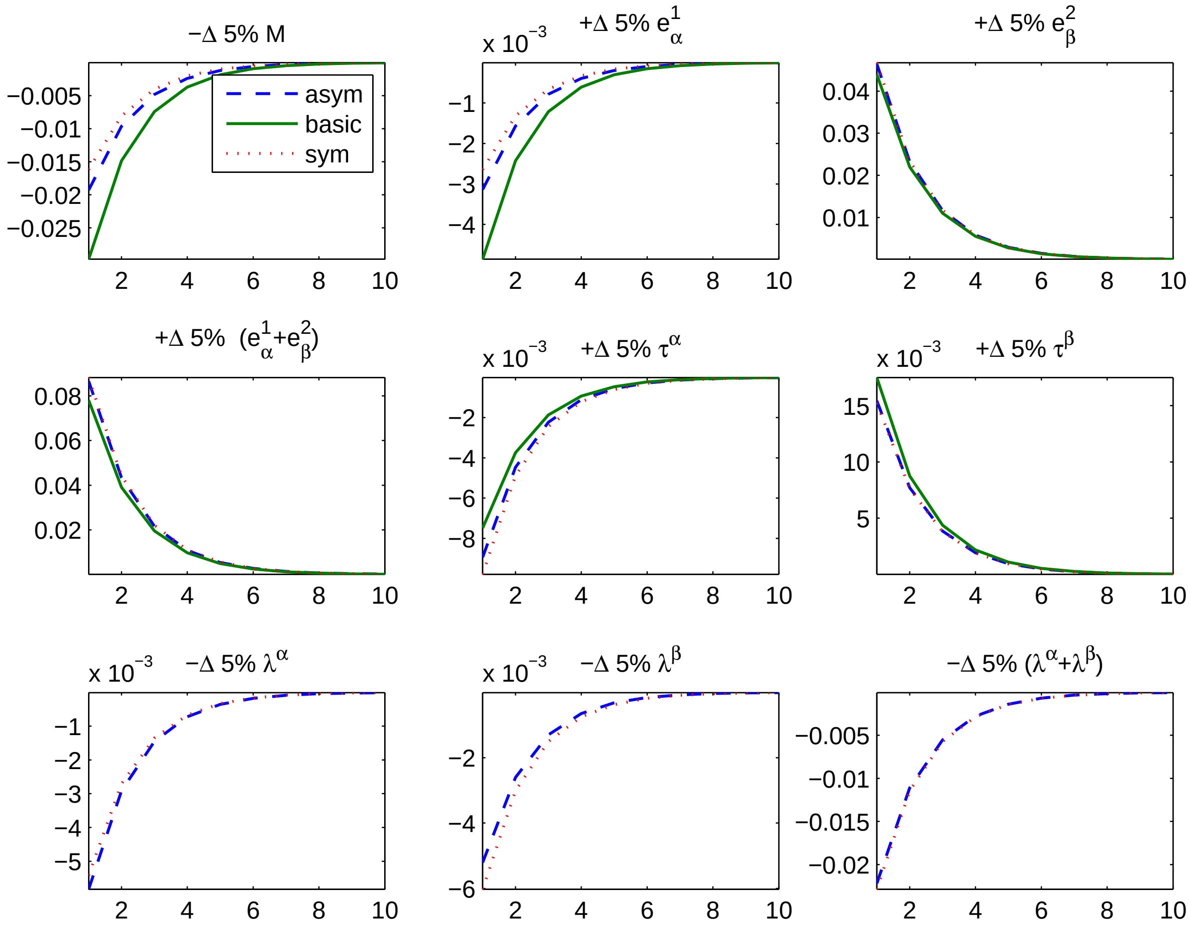

Figure 9: Impulse responses on quantity of good 2 traded by agent β ($q^\beta_{2,t}$)

10.5 Welfare

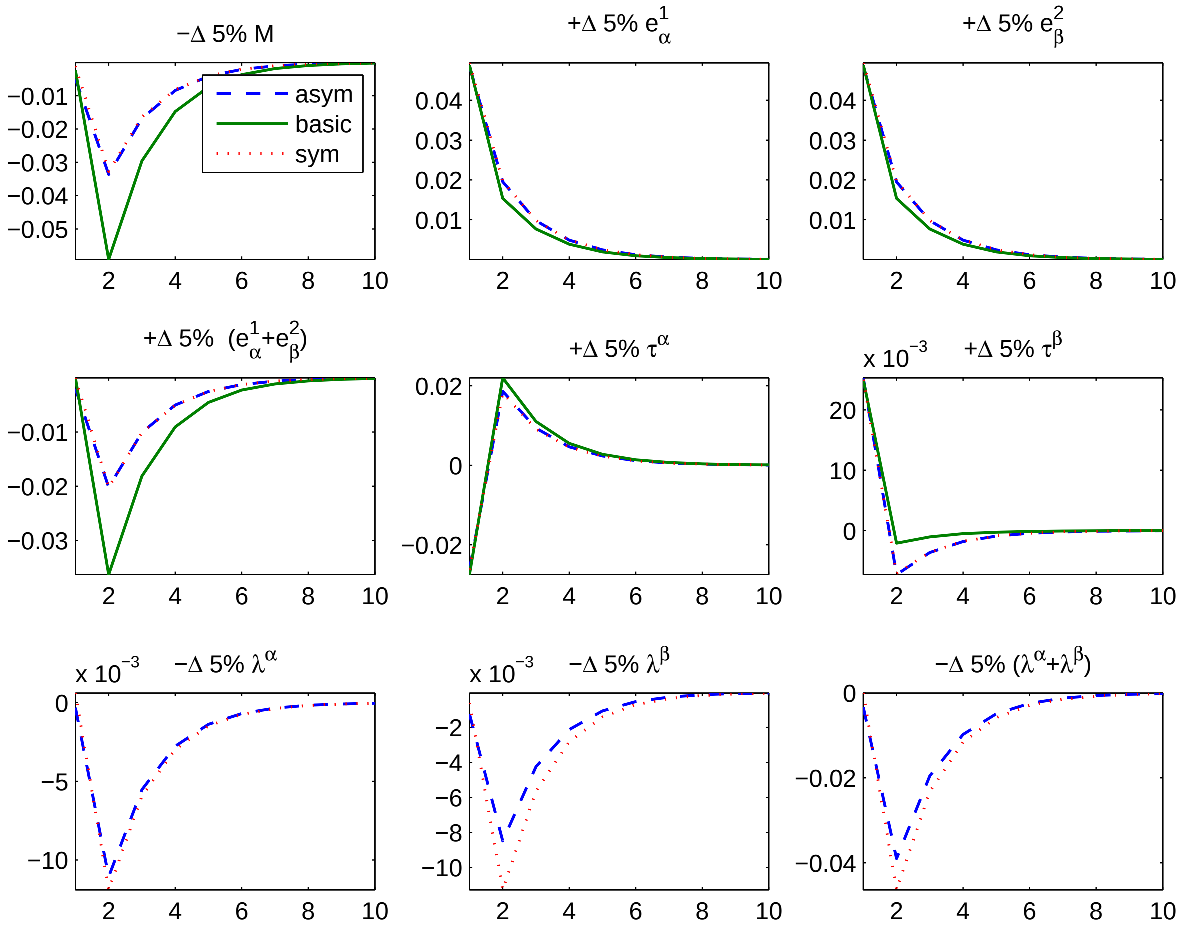

Figure 10: Impulse responses on utility of agent α (U_t^α)

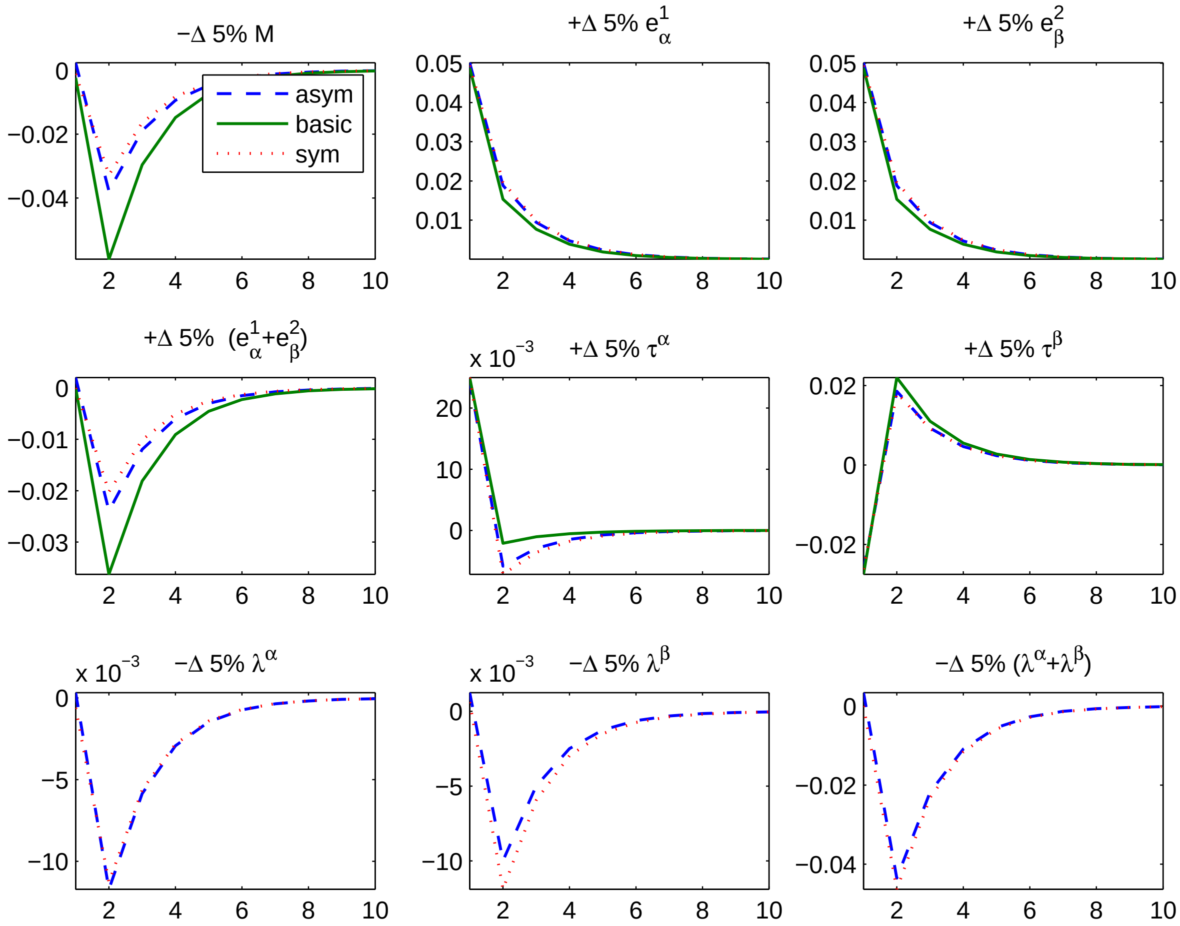

Figure 11: Impulse responses on utility of agent β (U_t^β)

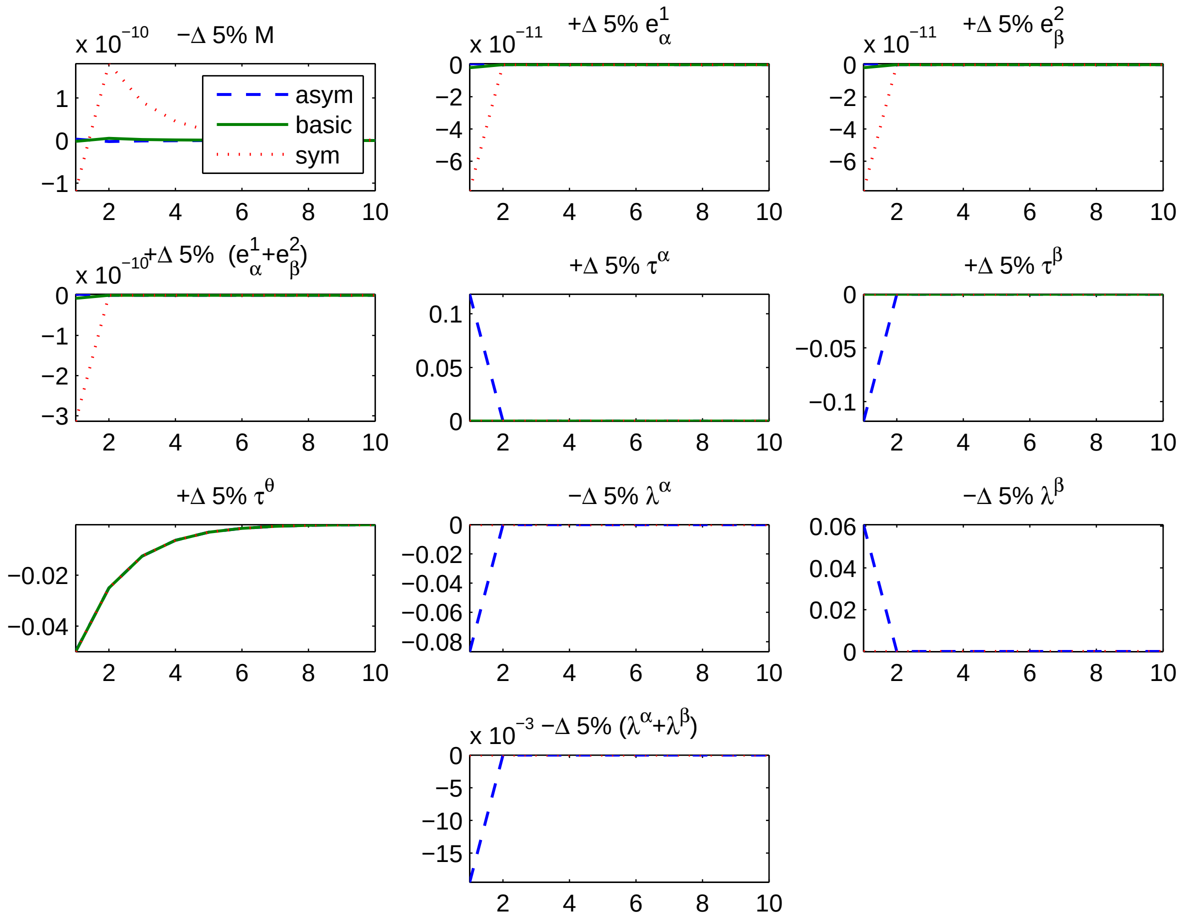

Figure 12: Impulse responses on utility of commercial bank θ (U_t^θ)

10.6 Bank Profitability

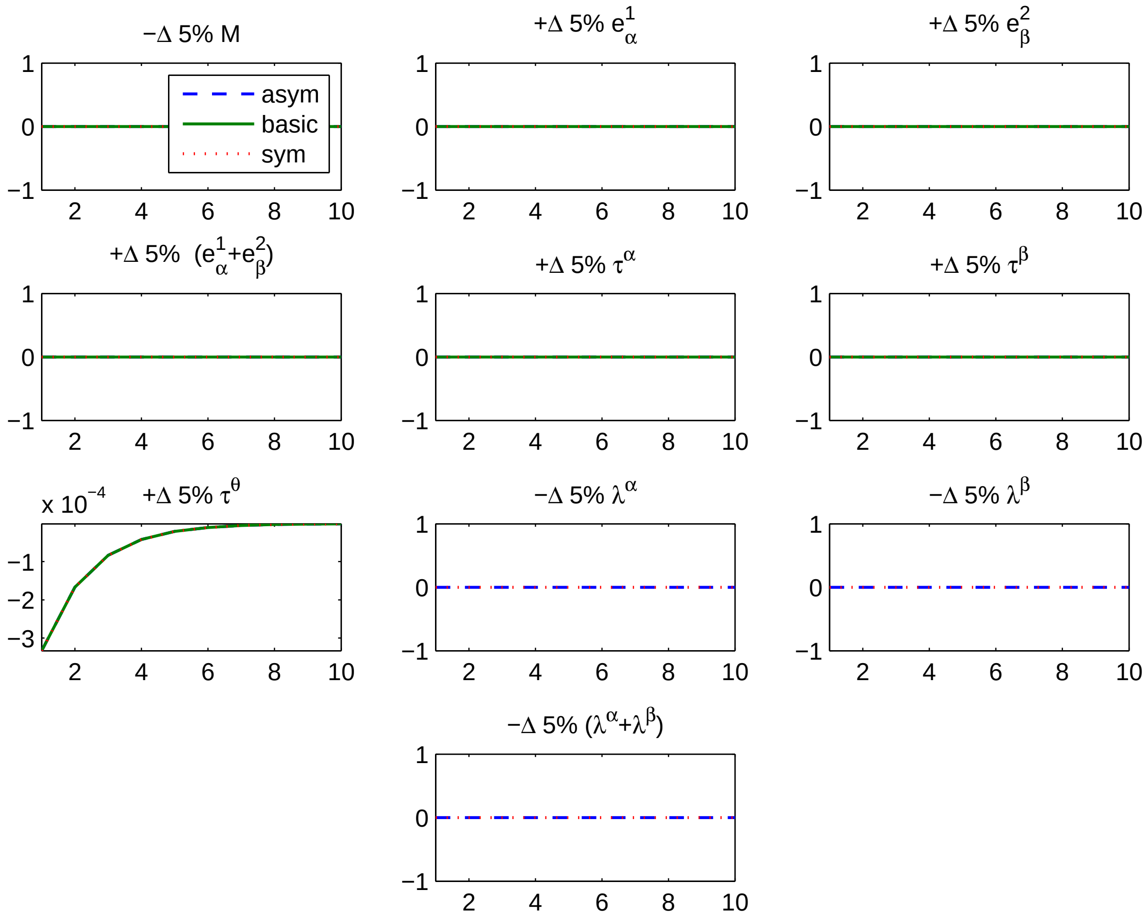

Figure 13: Impulse responses on bank profitability (Π_t^θ)