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Evaluation of the Central Bank of Brazil Structural Model's Inflation Forecasts in an Inflation Targeting Framework

Sergio Afonso Lago Alves*

Abstract

The purpose of this paper is to evaluate the performance of the Central Bank of Brazil's Small-Scale Structural Model (SSSM) as a supporting tool for the monetary policy decision process. The SSSM's projection accuracy for 1 to 3 quarters ahead CPI inflation was evaluated comparatively to those from either the market and from a simpler model, considered a benchmark for short run forecasts. A "near VAR" model, with quarterly CPI inflation and output gap as the endogenous variables, was chosen as the simple model. This model was found to be good only for 1-step ahead forecasts. Market projections, made by private consulting institutions and banks, turned out to be quite efficient for up to 2 quarters ahead, with almost no bias and low dispersion, suggesting the acceptance of the market efficiency hypothesis. The SSSM performed quite well over the whole forecast horizon, presenting almost no forecast bias and the lowest dispersion estimates, on average, even when compared to the market forecasts. For these reasons, the paper concludes that the Central Bank of Brazil's Small Scale Structural Model possesses the basic features required to support the monetary policy decision process.

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Evaluation of the Central Bank of Brazil Structural Model's Inflation Forecasts in an Inflation Targeting Framework**

1. Introduction

The Research Department of the Central Bank of Brazil developed a Small-Scale Structural Model (SSSM) as a supporting tool for the monetary policy decision process. Intending to capture the main relationships among key variables of the Brazilian economy, the model is formed by a set of simplified equations, as described in *Bogdanski et alli* (2000):

i) *an IS type equation expressing the output gap as a function of its own lags, real interest rates (ex ante or ex post), and the real exchange rate;*

ii) *a Phillips curve expressing the inflation rate as a function of its own lags and leads, the output gap and the nominal exchange rate;*

iii) *an uncovered interest parity condition relating the differential between external and domestic interest rates to the expected devaluation rate of the domestic currency (the Real), and the sovereign risk premium; and*

iv) *an interest rate rule, alternatively fixed rules on nominal or real interest rates, Taylor rules (with weights for contemporaneous deviations in inflation and output), and optimal deterministic and stochastic*

** *The author thanks Tito Nícias T. S. Filho, of the Research Department of Central Bank of Brazil, and José Regis A. Varão and Vanessa A. Simbalista e Silva, of the Investor Relations Group (GCI) of Central Bank of Brazil, for their cooperation in the preparation of this work*

rules.

Intended as a supporting tool in the *monetary policy* decision process, the model must possess good middle and long-run predictive power for inflation rate, measured by the *Broad Consumer Price Index* (IPCA)[1]. This need is justified by empirical evidence that indicates that it takes several quarters before the effect of *monetary policy*[2] instrument changes in the inflation rate reaches its peak. In fact, in a study made by the Bank of England Monetary Policy Committee (2000), it was estimated that it takes almost two years in that country. *Sterne* (2000) releases estimated average horizons for this impact in different economies. In Germany, Australia, Canada, Spain and United States the average horizon is a little below two years. In Chile, it takes about fifteen months, and in the Czech Republic, approximately ten months. Regarding the Brazilian economy, according to estimates by *Bogdanski et alli* (2000), *monetary policy* produces its maximum effect on inflation in six to nine months.

Therefore, an ex-post evaluation of the SSSM performance for inflation rate forecasts is necessary. As I shall justify below, it is important that this evaluation be made against alternative and simpler forecast models. A comparative evaluation is important due to the high opportunity cost attached to the development of the SSSM and its permanent enhancement, using advanced modeling, estimating and simulation techniques and tools, not to mention the deployment of specialized personnel.

In this context, I decided to contrast the forecasts obtained by the SSSM to those made by the market, collected daily by the Investor Relations Group (GCI)[3] of Central Bank of Brazil. Being the SSSM a supporting tool for the monetary policy decision, its inflation rate forecasts are supposed to be as efficient as the market ones, once accepting the market efficiency hypothesis. In addition, it's expected that SSSM forecast

[1] *Measured and released on a monthly basis by the Brazilian Institute of Geography and Statistics (IBGE), IPCA is the price index chosen for the purpose of gauging yearly inflation targets in the Inflation Target system.*

[2] *The interest rate used by the Central Bank of Brazil conduction of the monetary policy is the target of the Selic Rate, an adjusted average rate of daily financing in open market.*

[3] *Since June 1999, the GCI makes daily surveys of monthly market forecasts regarding the main macroeconomic indices. Among them, the following price indices are included in this survey: IGP-DI, IMP-M, IPC-Fipe, INPC, and IPCA.*

dispersion levels are, at least, as good as the market ones. As a matter of fact, some results from *Romer and Romer* (2000) indicate that the US Federal Reserve's inflation forecasts are better then the market ones. In their words, if market participants *"...had access to the Federal Reserve's forecast of future inflation, commercial forecasters would find it optimal to simply discard their forecasts and adopt that of the Federal Reserve."*

This article comprises four sections and one appendix. **Section 2** describes the evaluation mechanism for forecast performance. **Section 3** describes the procedures employed in selecting series of market forecasts in line with other forecasts to be evaluated, in addition to describing the selection of a simple model, taken as benchmark for the forecasts evaluation. **Section 4** comments on the results obtained and draws conclusions and the **Appendix** contains the tables and graphs resulting from the analyses.

2. Mechanism of Forecast Performance Evaluation

Three statistics were used to comparatively evaluate the forecast performance: *mean residual, mean absolute residual* and *mean square residual*. The first measures the forecasting bias and the two others measure the forecasting dispersion. The SSSM uses quarterly variables and therefore its inflation rates forecasts display the same quarterly frequency. In this sense, I considered only quarterly information to comparatively evaluate the SSSM performance. In this point, I stress that fact that the SSSM forecasts are made considering a fixed interest rates rule for periods ahead.

In order to obtain such statistics, I first considered the forecasts – performed with information up to period *T*, as described below –, for one, two and three periods ahead. The period *T* ranged from the second quarter of 1999 (1999:2)[4] to the third quarter of

[4] As the SSSM was developed in the context of the Brazilian Inflation Target Program, implemented in July 1999 as the system of monetary policy conducted by the Central Bank of Brazil, the smaller subsample should contain the second quarter of 1999 as its initial limit.

2000 (2000:3)[5]. Residuals were calculated as differences between forecasts and actual rates of the IPCA inflation in each quarter.

IBGE releases the inflation rates on a monthly basis, measured by IPCA. Therefore, it is possible to revaluate the forecasts made for subsequent quarters during the same quarter, for instance. Other variables, such as interest and foreign exchange rates are also taken into consideration in these revaluations. Three sets of forecasts are thus defined: *no information forecasts*, *one-month information forecasts* and *two-month information forecasts*.

It is worth mentioning that the quarterly forecasts obtained by the SSSM are also revaluated on a monthly basis according to the gathering of new information. In order to illustrate the sequence of facts related to the SSSM forecasts revaluation, consider the following example. Initially assume that the current month is January of the year *X* and IBGE released the December inflation rate of year *X-1* around January 10. Hence, the inflation rate for the fourth quarter of the previous year became known. Considering that it usually takes about ten days to gather other economic indicators and reestimate the SSSM, inflation rate forecasts for the first, second and third quarters of the year can be made[6] by January 19.

Around February 10, IBGE releases the January inflation rate. Thus, it's possible to revaluate the forecasts for the first quarter of the year and, consequently, for the other quarters. Notice that, since no new quarterly information became available, the model is not reestimated. Actually, revaluation is made by substituting a certain fraction of the first quarter inflation forecast by the actual January inflation rate. In March, IBGE releases the February inflation, so similar procedures can be done. In April, the March inflation rate is released and the most recent quarterly inflation information becomes known, so a reestimation of the SSSM can be done.

---

[5] *The third quarter of 1999 was the most recent period, during the preparation of this paper, in which the forecast for the IPCA inflation to one period ahead (2000:4) could be compared with the actual value, already released by IBGE.*

[6] *Although forecasts are also made for more quarters ahead, they are not related to the object of this paper.*

The statistics mentioned before were obtained, as represented by **System 1**, using the forecast residuals of 1 to 3 periods ahead, for each of the forecast sets.

$$\varepsilon_{j,T+n} = \hat{\Pi}_{j,T+n} - \Pi_{T+n}$$
$$r^{m}_{j,n} = \frac{\sum_{T=1999:2}^{2000:3} \varepsilon_{j,T,n}}{N} \qquad r^{ma}_{j,n} = \frac{\sum_{T=1999:2}^{2000:3} \left|\varepsilon_{j,T,n}\right|}{N} \qquad r^{ms}_{j,n} = \frac{\sum_{T=1999:2}^{2000:3} \left(\varepsilon_{j,T,n}\right)^2}{N} \tag{1}$$

Where:

T *is the period of the last observation for each subsample:* $T \geq (1999{:}2)$;

N *is the number of periods considered between 1999:2 and 2000:3;*

j *is the set of forecasts considered, according to the number of months, within a certain quarter, for which the actual inflation rate is known:* $j \in [1, 3]$;

n *is the number of periods ahead;* $n \in [1, 3]$;

Π_{T+n} *is the IPCA inflation rate relative to period T+n;*

$\hat{\Pi}_{j,\,T+n}$ *is the forecast, of the set j, for the IPCA inflation rate relative to period T+n;*

$\varepsilon_{j,\,T+n}$ *is the forecast residual, of the set j, for the IPCA inflation rate relative to period T+n;*

$r^{m}_{j,n}$ *is the forecast mean residual, of the set j, for the IPCA inflation rate measured n periods ahead;*

$r^{ma}_{j,n}$ *is the forecast mean absolute residual, of the set j, for the IPCA inflation rate measured n periods ahead;*

$r^{ms}_{j,n}$ *is the forecast mean square residual, of the set j, for the IPCA inflation rate measured n periods ahead.*

3. Selection of Forecasts Series for Comparison.

In this section, I describe the adopted procedures to select the forecasts series used in the comparative evaluation of the SSSM performance. In order for this evaluation to be coherent, it was necessary that each period forecasts be obtained based on the same level of information. To ensure this restriction, I made some considerations, explained in the next subsections, when gathering forecasts series from the market and from simple alternative models.

3.1. Market Forecasts

On every weekday, the Investor Relations Group (GCI) of Central Bank of Brazil collects monthly market forecasts for the main Brazilian macroeconomic indicators. The forecasts are collected from financial agents and institutions, both domestic and foreign, interested in the Brazilian economy and financial markets. According to the Central Bank of Brazil Survey report of 09/15/1999, among the survey respondents are market strategists, research analysts, board members and university professors. The median of market forecasts of each Friday is released on a weekly basis for public knowledge.

Despite the fact that market forecasts are made for the months following the survey, the aggregated market forecasts median series has a daily frequency and its variation behavior is due to:

(a) Release of IPCA by IBGE, around the 10th day of each month;

(b) Release, in the course of each month, of the other price indexes and other conjunctural indicators, both by IBGE and other institutions;

(c) Economic shocks;

(d) Improvement of forecasting models.

Considering these characteristics and the necessity of obtaining a quarterly market forecast series compatible with the SSSM ones, I adopted a criterion to be followed:

i. For the market forecasts to have approximately the same level of information available at the time the SSSM forecasts were made, it was necessary for the market forecasts to be made after the release of IPCA by IBGE, though not delayed enough to incorporate information of other economic indicators not included in the SSSM. Therefore, I decided to use the average of the daily market forecasts medians collected in the course of a period ranging from the 10th to the 19th day of each month. The selection of market forecasts medians instead of market forecasts means has the advantage of avoiding the influence of outliers.

ii. As the purpose was to run a comparative analysis on a quarterly basis, monthly market forecasts were pooled to determine the forecasts for quarters ahead, according to **System 2**. Given that the survey collects forecasts for a relatively short horizon, it was only possible to determine the forecasts up to a two-quarter horizon.

$$\hat{\Pi}^{t}_{T,T+n} = (1+\hat{\Pi}^{m}_{M,m_1})\cdot(1+\hat{\Pi}^{m}_{M,m_2})\cdot(1+\hat{\Pi}^{m}_{M,m_3})-1 \qquad \textbf{(2)}$$

Where:

M *is the month in which the survey with market forecasts was made;*

T *is the quarter containing month M;*

n *is the number of quarters ahead;*

$\mathbf{m_j}$ *is the jth month of quarter T+n:* $j \in [1, 3]$*;*

t *is an index indicating that the forecast in question is a quarterly forecast;*

m *is an index indicating that the forecast in question is a monthly forecast;*

$\hat{\Pi}^{t}_{T,T+n}$ *is the quarterly forecast, made in quarter T, for the inflation rate in n periods ahead;*

$\hat{\Pi}^{m}_{M,m_j}$ *is the monthly forecast, collected in month M, for the inflation rate in month* m_j*;*

Consider now an example in which it's illustrated how the collection of market forecasts is harmonized with those obtained in the SSSM example explained in **Section 2**. As the SSSM forecasts for the first, second and third quarters of year *X* were made in the period ranging from *January 10* to *January 19*, the market forecast for the first quarter would be obtained by pooling the January, February and March forecasts, made in the period range mentioned above. The forecast for the second quarter would be analogously obtained by pooling the forecasts for April, May and June.

As soon as IBGE released the January inflation rate, the market revaluation for the first quarter forecast could be obtained by pooling the actual January inflation rate with the forecasts for February and March, made in the period ranging from *February 10* to *February 19*. Pooling the forecasts made in the same period range for April, May and June, second quarter reevaluation could be obtained. As soon as IBGE released the February inflation rate, the whole process could be redone in an analogous manner.

3.2. Simple Alternative Model Forecasts

The simple forecasting models will be introduced later in this section. First, I will introduce the SSSM compatibility criterion.

In order to obtain out-of-sample inflation forecasts, so that the level of information is restricted to that available in a certain period *T*, it was necessary to consider a subset of the sample, taking only observations that were previous or contemporary to the period in question, disregarding observations related to periods ahead. Hence, the restricted model coefficients were reestimated and forecasts were obtained for subsequent periods: T+1, T+2, and T+3. This procedure was repeated for all subsets of the whole sample, so that the period *T* ranged from the second quarter of 1999 (1999:2) to the third quarter of 2000 (2000:3), for reasons already commented in **Section 2**.

To incorporate monthly information of inflation occurred within a quarter, the logarithms of the actual monthly inflation rates replaced one or two thirds of the logarithm of the forecast in *T+1*, depending on whether they were or not known in the first or the second month of the quarter. From these new forecast values for *T+1*, forecasts for the subsequent quarters were reestimated.

In order to estimate the models, the sample included only observations starting from the third quarter of 1994 (1994:3), when the Real Plan was implemented. This convention is justified by the fact that there was an important structural change in the economy, which should have changed relations between variables. However, this decision severely restricted the number of observations to only 25 from 1994:3 to

2000:3. This number was further reduced in the estimations due to the presence of lags of different orders.

3.2.1 Simple Alternative Models Used

Two single techniques are well known for simple short-run forecasts: ARIMA (autoregressive integrated moving average) modeling and VAR (vector autoregressive) modeling. A third technique, known as *near Var*, was also considered for reasons that I will discuss in this subsection.

When contrasted to the SSSM, such techniques are relatively simple to implement and the necessary softwares for implementation are often available in environments designed to economic analyses. I decided that, after building models with such techniques, the one displaying the best performance, measured by the sum of squared errors obtained in whole sample estimation, would be taken as a short-run benchmark for the SSSM performance comparison.

The inflation rate was then modeled as an ARIMA(2,1,1) process, since the non-stationarity hypothesis of the IPCA inflation rate series was not rejected by the augmented Dickey-Fuller test. However, this model's performance was worse than that obtained by the selected model, to be described.

For the VAR modeling, I used economic theory to choose the initials endogenous variables of the system. The *inflation rate*, measured by IPCA, is directly explained by the *output gap*[7]. However, other indicators are essential to explain the inflation rate. The *real interest rate* plays a fundamental role in controlling the *output gap*. The *foreign exchange rate* depreciation, coupled to *foreign inflation*, affects the price of imports, which in turn affects domestic inflation. There is still one last endogenous variable, not to be modeled: the *nominal interest rate*. Being the Central Bank of Brazil's control variable in the conduction ofmonetary policy, it is decided after the analysis of possible domestic and foreign scenarios. Its level, purging the inflation rate, defines the *real interest rate*, restarting the cycle.

In VAR models, the same set of endogenous and exogenous regressors, with the same orders of lags, is used to explain endogenous variables. This is advisable in order to avoid losing any degree of information and increase the model's predictive power. However, since the number of observations was small, an imposition of the total regressors, with their related lags, to explain the endogenous variables greatly reduced the equations' degrees of freedom, negatively affecting the estimations. The estimated coefficients were non-significant and it was seen, by the impulse response functions, that the model remained always unstable. This problem persisted even when I considered, for simplicity, just the *inflation rate* and the *output gap* as endogenous variables, making all the others exogenous.

So I chose to make estimations in which there could be a certain freedom for regressors or lags not to be present in one of the equations. Some of these restrictions were based on economic theory: It's expected, for example, that the *interest rate* would be unable to directly explain the *inflation rate*, yet able to explain the *output gap*. For equations with this type of restriction, I used the modeling technique known as *near VAR*.

In this case, estimating the equations by the SUR (seemingly unrelated regressions) method seems to be advisable, since it yields efficient estimates[8], considering that the residuals of each equation are correlated. Besides, to avoid problems related to degrees of freedom, I decided, for simplicity, that just the IPCA inflation rate and the output gap would be the endogenous variables. This solution enabled me to estimate a more consistent model, chosen as the benchmark for the SSSM performance comparison.

---

[7] *The output gap, roughly defined, is the difference between the economy output and its maximum level, sustainable in the long-run.*

[8] *See Enders (1995) and Greene (1993) for further information.*

3.2.2 Implementation of the *near Var* Model

As discussed before, the endogenous variables selected were the IPCA *inflation rate* and the *output gap*. In order to determine a quarterly series of the *output gap*, the author developed a "*quarterizing*" technique applied to the yearly *potential output*[9], estimated by a production function approach by *da Silva Filho* (2001). The technique is described at the end of this subsection.

The following indicators were taken as exogenous variables: *real interest rates, foreign exchange rate* and the *U.S. PPI*[10] *inflation rate.* Seasonality variables were also included in both equations.

I intended to verify the hypothesis that the inflation inertial effect had been reduced following the implementation of Inflation Targeting, in 1999:3. In order to test this hypothesis, I decided to use a dummy variable to model a significant coefficient reduction in the IPCA inflation rate lag after 1999:3.

For the out-of-sample forecasts, the exogenous variables were fixed to their values in the last period of each subset.

For the sake of illustration, the system of equations estimated using the whole sample, i.e. from 1994:3 to 2000:3, is represented by **Equations 3** and **4** below.

$$h_t = \alpha_0 + \alpha_1 \cdot \frac{\sum_{i=1}^{3} h_{t-i}}{3} + \alpha_2 \cdot \frac{\sum_{j=0}^{2} r_{t-j}}{3} + \alpha_3 \cdot \frac{\sum_{u=1}^{4} \pi_{u-l}}{4} + \sum_{v=1}^{3} \alpha_{3+v} \cdot SEA_v + \varepsilon_{ht} \qquad \textbf{(3)}$$

---

[9] *The potential output is the maximum output level sustainable in the long run.*

[10] *The Producer Price Index (PPI) is measured and released on a monthly basis by the U.S. Bureau of Labor Statistics, a division of the U.S Department of Labor.*

$$\pi_t = \beta_0 + (\beta_1 + \beta_2 \cdot IT) \cdot \pi_{t-1} + \beta_3 \cdot \frac{\sum_{i=0}^{2} h_{t-i}}{3} + \beta_4 \cdot \Delta e_t + \beta_5 \cdot \frac{\sum_{u=1}^{2} \pi^f_{t-u}}{2} + \\ + \beta_6 \cdot SEA_2 + \beta_7 \cdot SEA_4 + \varepsilon_{\pi t} \quad \textbf{(4)}$$

Where:

h_t *is the natural logarithm of the output gap in period t: ln(1+GAP%$_t$);*

π_t *is the natural logarithm of the IPCA inflation rate in period t: ln(1+Π_t);*

π^f_t *is the natural logarithm of the U.S. PPI inflation rate in period t: ln(1+PPI%$_t$);*

IT *is a dummy variable, indicating periods before (0) and after (1) the implementation of Inflation Targeting;*

r_t *is the natural logarithm of the real interest rate in period t: [ln(1+nom.interest%$_t$) - π_t];*

Δe_t *is the change of the natural logarithm of the nominal exchange rate in period t: Δ[ln(exch.rate$_t$)];*

SEA_v *is a dummy variable for period v seasonality;*

ε_{ht} *is the residual component, purely random, of the output gap in period t;*

$\varepsilon_{\pi t}$ *is the residual component, purely random, of the inflation rate in period t.*

When the out-of-sample estimation used smaller subsamples, certain regressors failed to explain one of the equations. This was expected, both due to the effect of the reduced degrees of freedom and due to the fact that the IT variable only became efficient in explaining coefficient reductions in the first lag of the IPCA inflation rate some periods after the implementation of Inflation Targeting.

All estimations were run with SUR method, using simultaneous iterations of coefficients and weights. It is important to emphasize that the coefficient of the IPCA inflation rate first lag in **Equation 4** effectively fell after the implementation of Inflation Targeting.

During the selection of the sample series, it was necessary to estimate an *output gap* series, which is far from being a simple task. In the estimated model, this variable corresponds to the difference, in logarithms, between the quarterly *output*, and the quarterly *potential output*. The Brazilian *Gross Domestic Product* (PIB) is calculated and released by IBGE on a quarterly basis. However, the *potential output* is a non-observed variable, and should be estimated by indirect processes. The processes generally used are the *Hodrick and Prescott filter (HP Filter), linear trend with or without structural break,* and the *output function* (see *Apel et alli* (1996) and *Giorno et alli* (1995), for example). Verifying that the use of *output gap* series derived from different *potential output* obtaining processes produced widely different results, the question was to choose the most plausible *output gap* series. A plausibility criterion should be the adherence and significance estimates obtained when the system of equations were estimated, for each *output gap* series, with the whole sample. The chosen one was derived from a "*quarterized*" *potential output* series, estimated by *da Silva Filho* (2001).

The main assumption of the "*quarterizing*" technique is that the changes in *potential output* display a smooth behavior along time. This is a plausible assumption, accepting the hypothesis that the *potential output* would be, at a theoretical level, less susceptible to the volatility associated to the aggregate demand. In the optimizing process, quarterly estimates for *potential output* are made in such a way that the volatility of the entire quarterly series is minimized, subject to the restriction that, for each year, the summation of such quarterly estimates equals the yearly *potential output*. The process is mathematically described by **System 5**.

$$
\begin{cases} Y^* \equiv \textit{Yearly Potential GDP series, with n observations} \\ y_t^* \equiv \textit{ Particular value for } Y^* \textit{ in year } t: \quad t \in [1, n] \end{cases}
$$

$$
\textit{One wishes to estimate the quarterly Potential GDP series } U^* \textit{ such as :}
$$

$$
\begin{cases} U^* \equiv \textit{Quarterly Potential GDP series, with 4n observations} \\ u_{t,Q}^* \equiv \textit{ Particular value for } U^* \textit{ in quarter } Q \textit{ of year } t: \quad t \in [1, n],\ Q \in [1, 4] \end{cases} \qquad \textbf{(5)}
$$

$$
U^* \textit{ complies with the following optimization equation :}
$$

$$
\begin{cases} \textit{Minimize} \qquad L = \sum_{t=1}^{n} \sum_{Q=1}^{4} \left(\Delta^2 u_{t,Q}^* \right)^2 \\ \textit{Subject to} \qquad \sum_{Q=1}^{4} u_{t,Q}^* = y_t^*, \quad \forall\ t \in [1, n] \end{cases}
$$

As the *4n* values to be estimated are subject to *n* restriction equations, the number of degrees of freedom reduces to *3n*. As initial values for the numeric optimization process, the author suggests the use of: $u_{t,Q}^{*} = y_t^{*} / 4$.

4. Analysis of the Results and Conclusions

The conclusions reached in this paper are rather descriptive than inferential. This is due to the small number of statistics calculated, which did not permit a more rigorous analysis. **Tables 1** to **9** and **Graphs 1** to **9**, in the **Appendix**, show the estimation results, depending on the time horizon and on the three forecast sets, according to the number of information months in each quarter. **Graphs (10)** to **(12)**, also in the **Appendix**, show the evolution of the statistics for each period ahead in each of the forecasts sets.

The results show that the Central Bank of Brazil SSSM performed quite well for whole forecast horizon. Showing a desirable forecast property for any model, its IPCA inflation rate forecasts presented almost no bias, measured by mean residual statistics close to zero. Regarding its dispersion estimates, the statistics were stable and low, tending not to display an exponential behavior, even in the longest horizon (3 quarters ahead). Therefore, the statistics indicate that this model would be able to produce trustworthy middle run projections. SSSM presented the best performance in all the six cases in which the dispersion estimates, both the *mean absolute residuals* and the *mean absolute residuals*, were calculated for the two and three quarters ahead forecasts.

The SSSM, on the order hand, was the most efficient in revaluating quarterly forecasts with monthly information, since its dispersion estimates were better than all the remaining model's statistics. Of the seven cases in which dispersion estimates were calculated to revaluated forecasts, SSSM performed better in six.

The fact that the dispersion estimates presented a non-exponential behavior is desirable, since SSSM is a supporting tool for the Brazilian monetary policy decision process. In this sense, it's necessary for forecasts to be relatively precise up to three quarters ahead, since this is approximately the period in which interest rate changes, controlled by the Central Bank of Brazil, affect inflation more intensely. In addition, the SSSM dispersion estimates are, in general, lower than the market's, assumed to be efficient. This all leaded me to the conclusion that the SSSM is able to respond in a more efficient manner when the objective is the conduction of the Brazilian monetary policy.

As expected, the market forecasts displayed almost no forecast bias and low dispersion levels, which is in line with the market efficiency hypothesis. Regarding the *near* VAR model, the dispersion estimates were small and comparable to those reached by the market forecasts only for one quarter ahead, even with no month information included. For longer horizons, the *near VAR* model lost efficiency, in comparison with the market forecasts and, mainly, with the SSSM ones.

The results also suggest that *near* VAR model forecasts display positive bias, overestimating the IPCA inflation rate. This result is probably due to the fact that, since its forecasts are produced only with past information (backward-looking specification), this model tends to overeestimate forecasts when the past inflation rate follows a declining path. As the SSSM also incorporates future expectations (forward-looking specification), it does not display such a bias.

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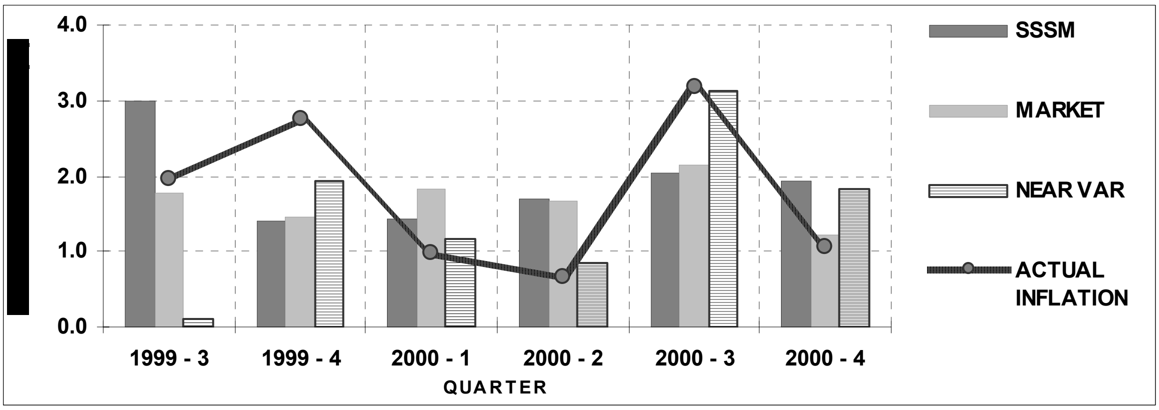
Appendix Tables and Graphs

Subsample	Forecasting Period	IPCA Inflation Rate (%)			
		Actual Inflation	SSSM Forecasts	Market Forecasts	Near VAR Forecasts
1994:3 to 1999:2	***1999:3***	*1.97*	2.99	1.78	0.10
1994:3 to 1999:3	***1999:4***	*2.76*	1.42	1.46	1.93
1994:3 to 1999:4	***2000:1***	*0.97*	1.42	1.82	1.18
1994:3 to 2000:1	***2000:2***	*0.66*	1.69	1.67	0.85
1994:3 to 2000:2	***2000:3***	*3.18*	2.04	2.15	3.12
1994:3 to 2000:3	***2000:4***	*1.05*	1.93	1.23	1.83
MEAN RESIDUAL			0.15	-0.08	-0.27
MEAN ABSOLUTE RESIDUAL			0.98	0.76	0.66
MEAN SQUARE RESIDUAL			1.03	0.76	0.82

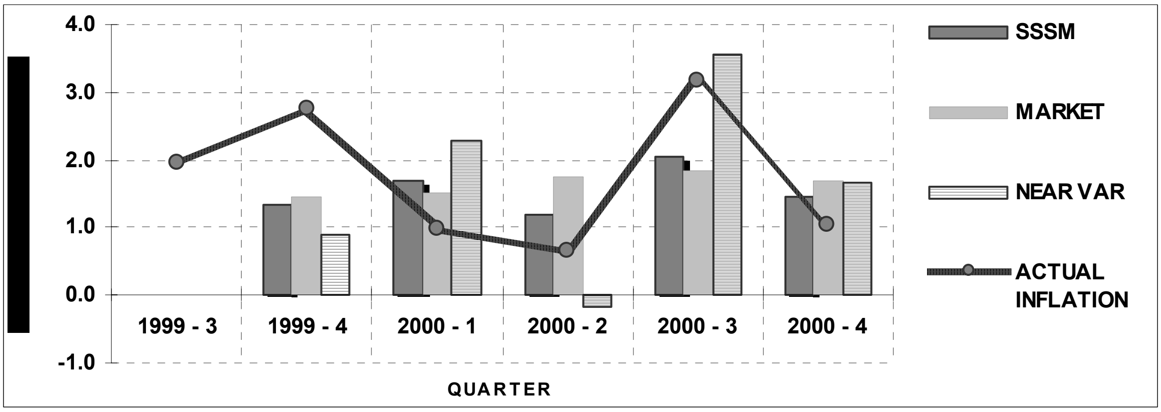
Table (1) ***No Information Forecasts (1 Quarter Ahead)***



Graph (1) ***No Information Forecasts (1 Quarter Ahead)***

Subsample	Forecasting Period	IPCA Inflation Rate (%)			
		Actual Inflation	SSSM Forecasts	Market Forecasts	Near VAR Forecasts
1994:3 to 1999:2	***1999:4***	*2.76*	1.34	1.46	0.91
1994:3 to 1999:3	***2000:1***	*0.97*	1.68	1.52	2.29
1994:3 to 1999:4	***2000:2***	*0.66*	1.20	1.75	-0.18
1994:3 to 2000:1	***2000:3***	*3.18*	2.06	1.83	3.57
1994:3 to 2000:2	***2000:4***	*1.05*	1.45	1.68	1.65
MEAN RESIDUAL			-0.18	-0.08	-0.08
MEAN ABSOLUTE RESIDUAL			0.84	0.98	1.00
MEAN SQUARE RESIDUAL			0.85	1.08	1.28

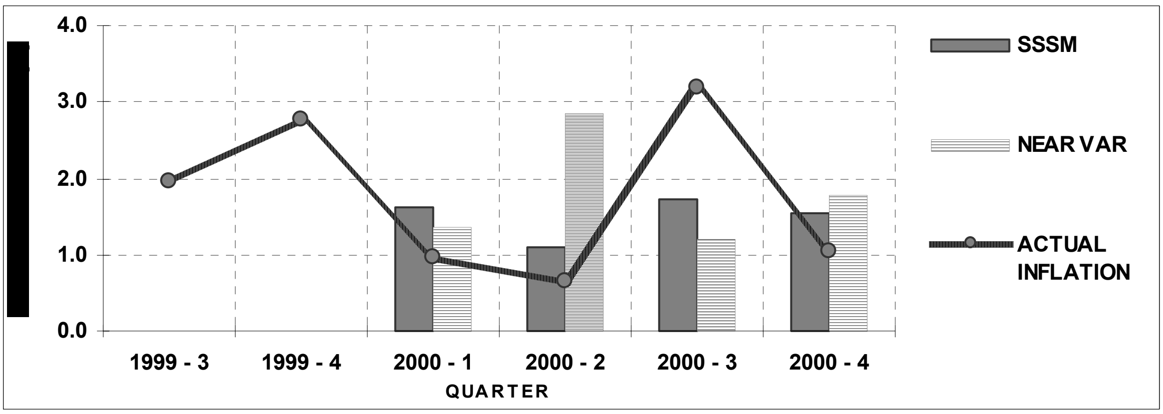
Table (2) ***No Information Forecasts (2 Quarters Ahead)***



Graph (2) No Information Forecasts (2 Quarters Ahead)

Subsample	Forecasting Period	IPCA Inflation Rate (%)		
		Actual Inflation	**SSSM Forecasts**	**Near VAR Forecasts**
1994:3 to 1999:2	***2000:1***	*0.97*	1.63	1.35
1994:3 to 1999:3	***2000:2***	*0.66*	1.10	2.86
1994:3 to 1999:4	***2000:3***	*3.18*	1.73	1.21
1994:3 to 2000:1	***2000:4***	*1.05*	1.54	1.77
MEAN RESIDUAL			0.03	0.33
MEAN ABSOLUTE RESIDUAL			0.76	1.31
MEAN SQUARE RESIDUAL			0.74	2.34

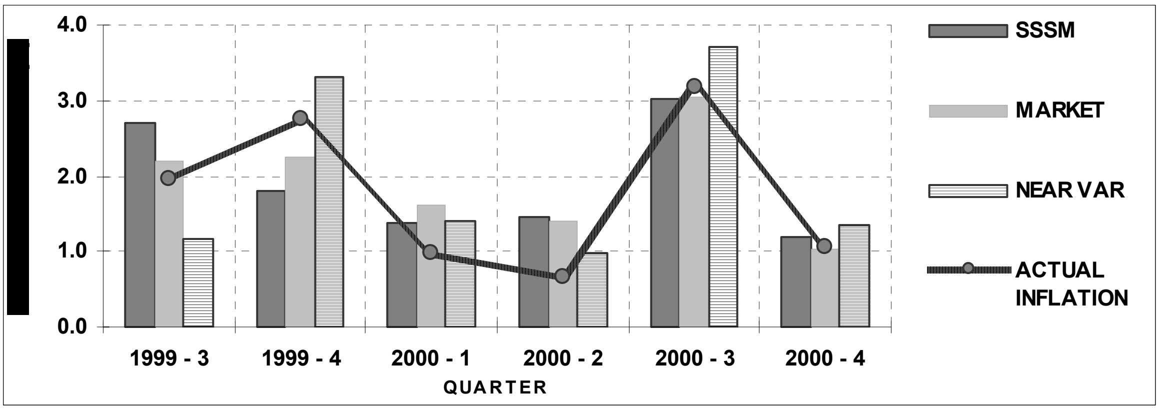
Table (3) No Information Forecasts (3 Quarters Ahead)



Graph (3) No Information Forecasts (3 Quarters Ahead)

Subsample	Forecasting Period	IPCA Inflation Rate (%)			
		Actual Inflation	SSSM Forecasts	Market Forecasts	Near VAR Forecasts
1994:3 to 1999:2	***1999:3***	*1.97*	2.69	2.21	1.16
1994:3 to 1999:3	***1999:4***	*2.76*	1.80	2.25	3.32
1994:3 to 1999:4	***2000:1***	*0.97*	1.38	1.61	1.41
1994:3 to 2000:1	***2000:2***	*0.66*	1.45	1.40	0.98
1994:3 to 2000:2	***2000:3***	*3.18*	3.02	3.05	3.71
1994:3 to 2000:3	***2000:4***	*1.05*	1.20	1.03	1.35
MEAN RESIDUAL			0.16	0.16	0.22
MEAN ABSOLUTE RESIDUAL			0.53	0.38	0.49
MEAN SQUARE RESIDUAL			0.38	0.22	0.27

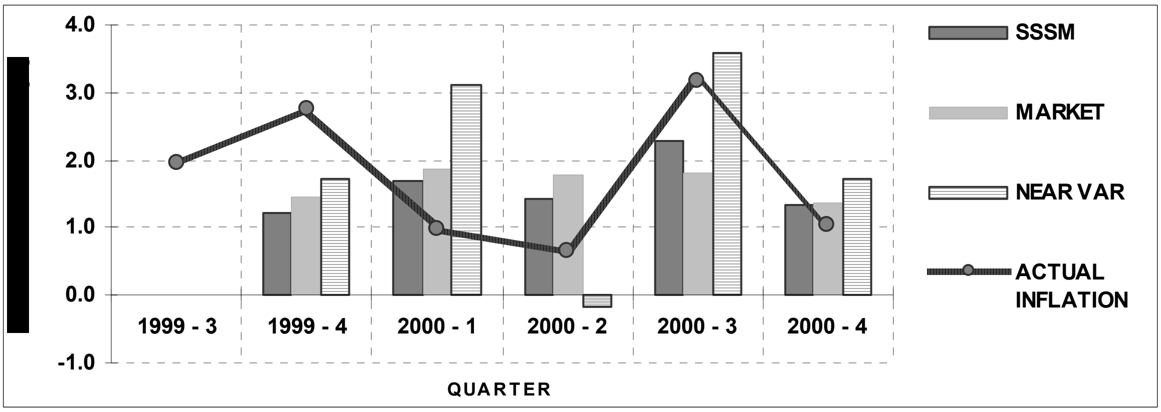
Table (4) One-Month Information Forecasts (1 Quarter Ahead)



Graph (4) One-Month Information Forecasts (1 Quarter Ahead)

Subsample	Forecasting Period	IPCA Inflation Rate (%)			
		Actual Inflation	SSSM Forecasts	Market Forecasts	Near VAR Forecasts
1994:3 to 1999:2	***1999:4***	*2.76*	1.23	1.46	1.72
1994:3 to 1999:3	***2000:1***	*0.97*	1.69	1.88	3.13
1994:3 to 1999:4	***2000:2***	*0.66*	1.43	1.79	-0.16
1994:3 to 2000:1	***2000:3***	*3.18*	2.29	1.82	3.59
1994:3 to 2000:2	***2000:4***	*1.05*	1.33	1.38	1.73
MEAN RESIDUAL			-0.13	-0.06	0.27
MEAN ABSOLUTE RESIDUAL			0.84	1.00	1.02
MEAN SQUARE RESIDUAL			0.86	1.15	1.40

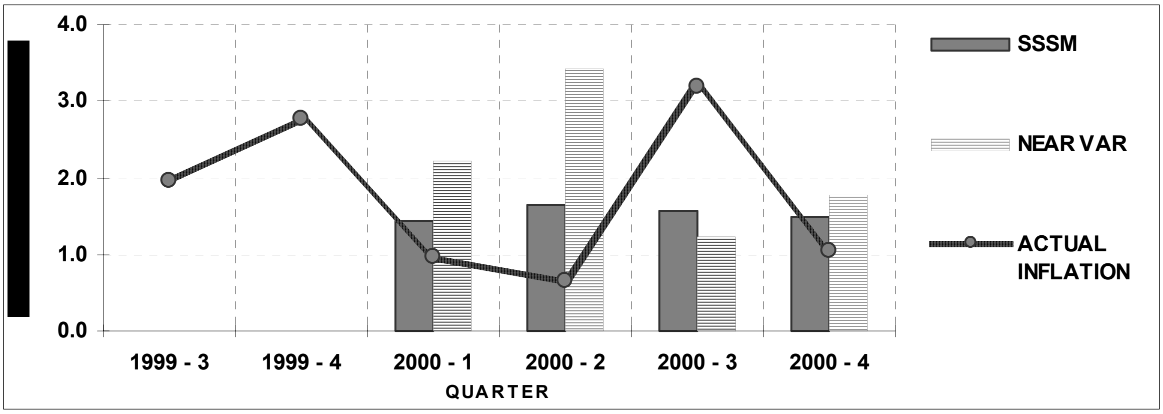
Table (5) One-Month Information Forecasts (2 Quarters Ahead)



Graph (5) ***One-Month Information Forecasts (2 Quarters Ahead)***

Subsample	Forecasting Period	IPCA Inflation Rate (%)		
		Actual Inflation	**SSSM Forecasts**	**Near VAR Forecasts**
1994:3 to 1999:2	***2000:1***	*0.97*	1.45	2.22
1994:3 to 1999:3	***2000:2***	*0.66*	1.64	3.41
1994:3 to 1999:4	***2000:3***	*3.18*	1.56	1.22
1994:3 to 2000:1	***2000:4***	*1.05*	1.49	1.77
MEAN RESIDUAL			0.07	0.69
MEAN ABSOLUTE RESIDUAL			0.88	1.67
MEAN SQUARE RESIDUAL			0.99	3.38

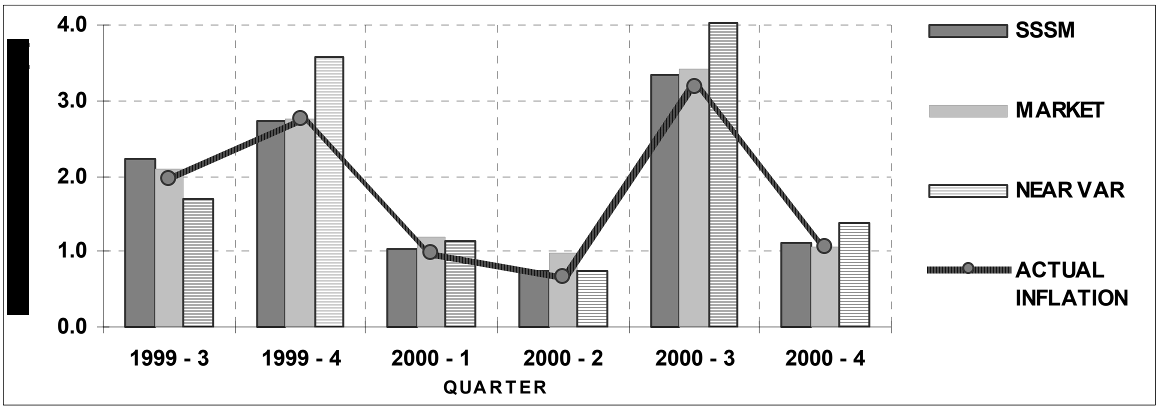
Table (6) ***One-Month Information Forecasts (3 Quarters Ahead)***



Graph (6) ***One-Month Information Forecasts (3 Quarters Ahead)***

Subsample	Forecasting Period	IPCA Inflation Rate (%)			
		Actual Inflation	SSSM Forecasts	Market Forecasts	Near VAR Forecasts
1994:3 to 1999:2	***1999:3***	*1.97*	2.23	2.08	1.69
1994:3 to 1999:3	***1999:4***	*2.76*	2.73	2.76	3.57
1994:3 to 1999:4	***2000:1***	*0.97*	1.04	1.19	1.14
1994:3 to 2000:1	***2000:2***	*0.66*	0.73	0.99	0.74
1994:3 to 2000:2	***2000:3***	*3.18*	3.35	3.42	4.02
1994:3 to 2000:3	***2000:4***	*1.05*	1.13	1.06	1.37
MEAN RESIDUAL			0.10	0.15	0.32
MEAN ABSOLUTE RESIDUAL			0.11	0.15	0.42
MEAN SQUARE RESIDUAL			0.02	0.04	0.26

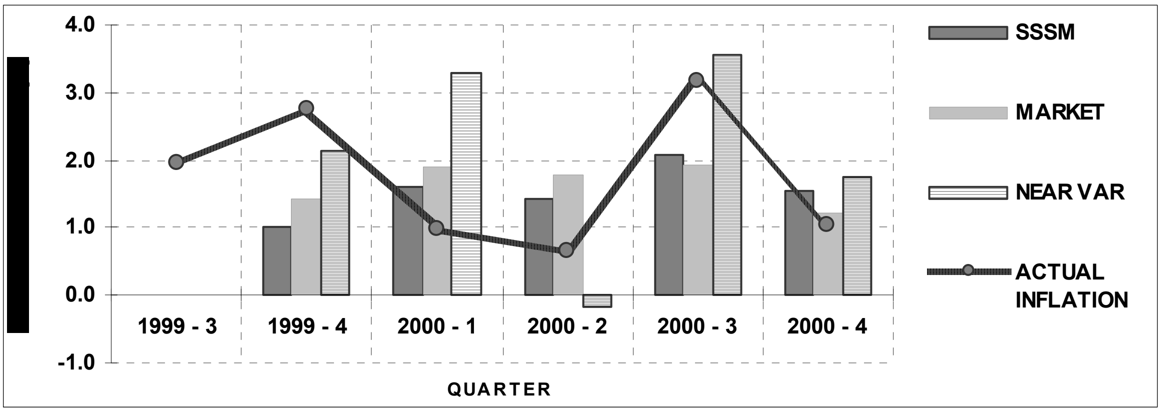
Table (7) ***Two-Month Information Forecasts (1 Quarter Ahead)***



Graph (7) ***Two-Month Information Forecasts (1 Quarter Ahead)***

Subsample	Forecasting Period	IPCA Inflation Rate (%)			
		Actual Inflation	SSSM Forecasts	Market Forecasts	Near VAR Forecasts
1994:3 to 1999:2	***1999:4***	*2.76*	1.02	1.43	2.13
1994:3 to 1999:3	***2000:1***	*0.97*	1.61	1.91	3.28
1994:3 to 1999:4	***2000:2***	*0.66*	1.43	1.77	-0.18
1994:3 to 2000:1	***2000:3***	*3.18*	2.09	1.93	3.57
1994:3 to 2000:2	***2000:4***	*1.05*	1.55	1.23	1.76
MEAN RESIDUAL			-0.19	-0.07	0.38
MEAN ABSOLUTE RESIDUAL			0.95	0.96	0.97
MEAN SQUARE RESIDUAL			1.10	1.10	1.41

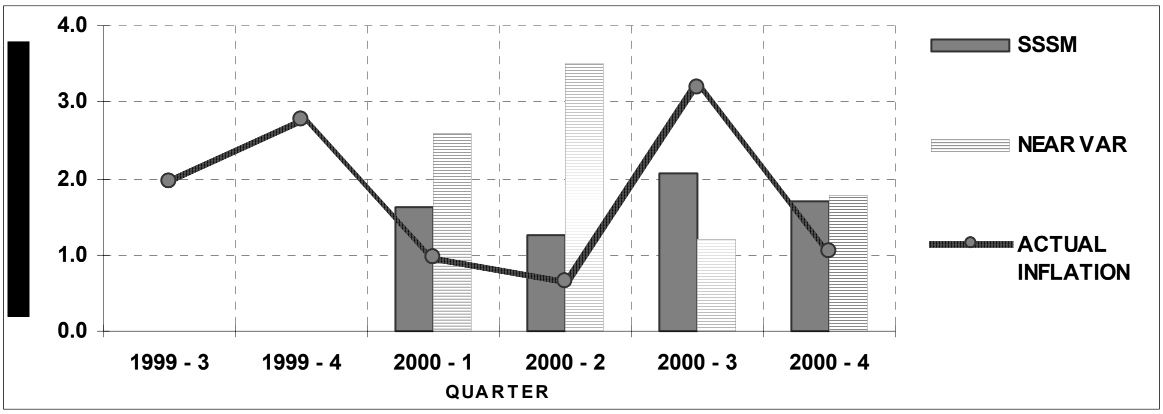
Table (8) ***Two-Month Information Forecasts (2 Quarters Ahead)***



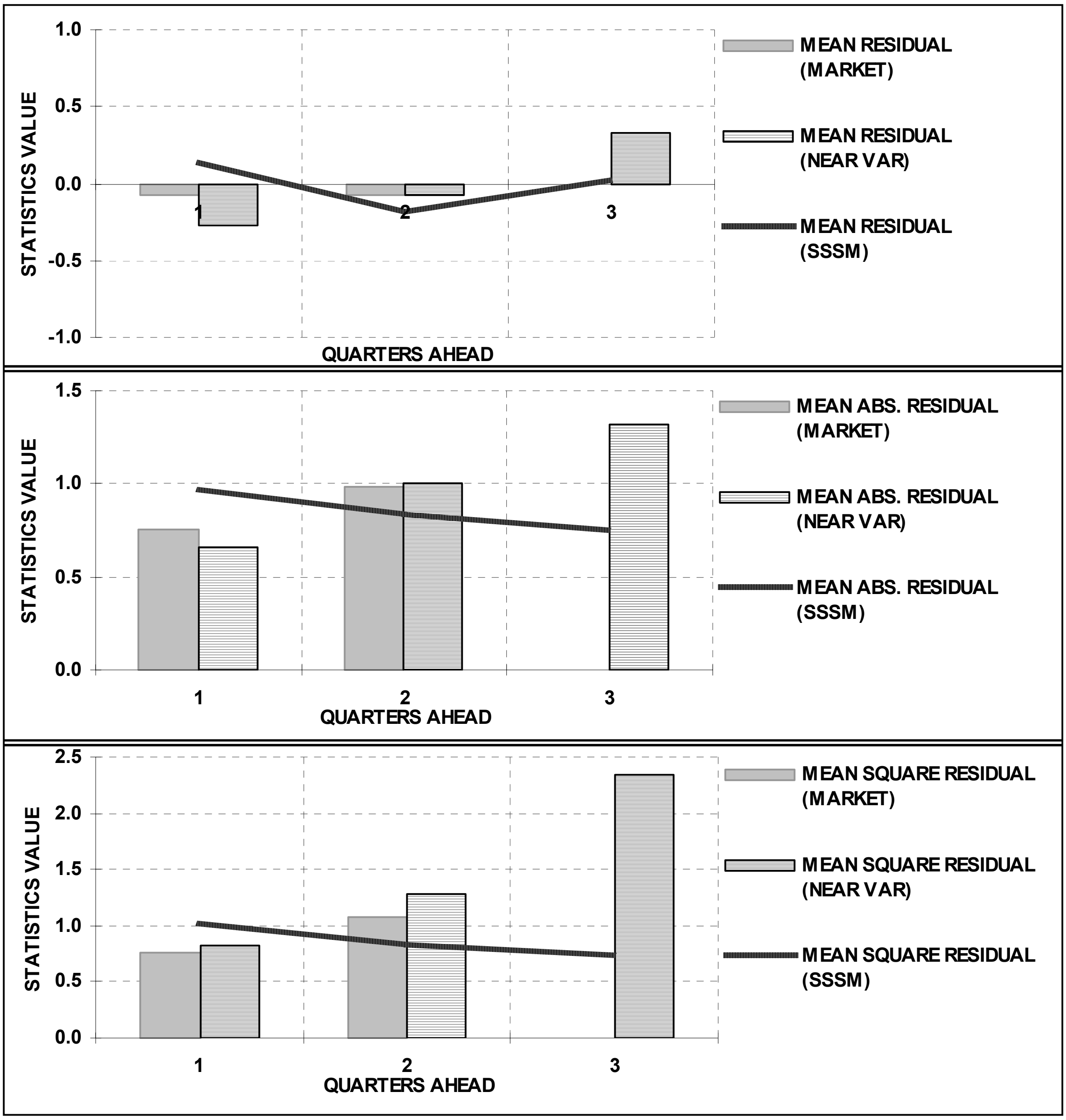
Graph (8) ***Two-Month Information Forecasts (2 Quarters Ahead)***

Subsample	Forecasting Period	IPCA Inflation Rate (%)		
		Actual Inflation	SSSM Forecasts	Near VAR Forecasts
1994:3 to 1999:2	***2000:1***	*0.97*	1.63	2.60
1994:3 to 1999:3	***2000:2***	*0.66*	1.26	3.51
1994:3 to 1999:4	***2000:3***	*3.18*	2.07	1.21
1994:3 to 2000:1	***2000:4***	*1.05*	1.71	1.77
MEAN RESIDUAL			0.20	0.81
MEAN ABSOLUTE RESIDUAL			0.75	1.79
MEAN SQUARE RESIDUAL			0.61	3.79

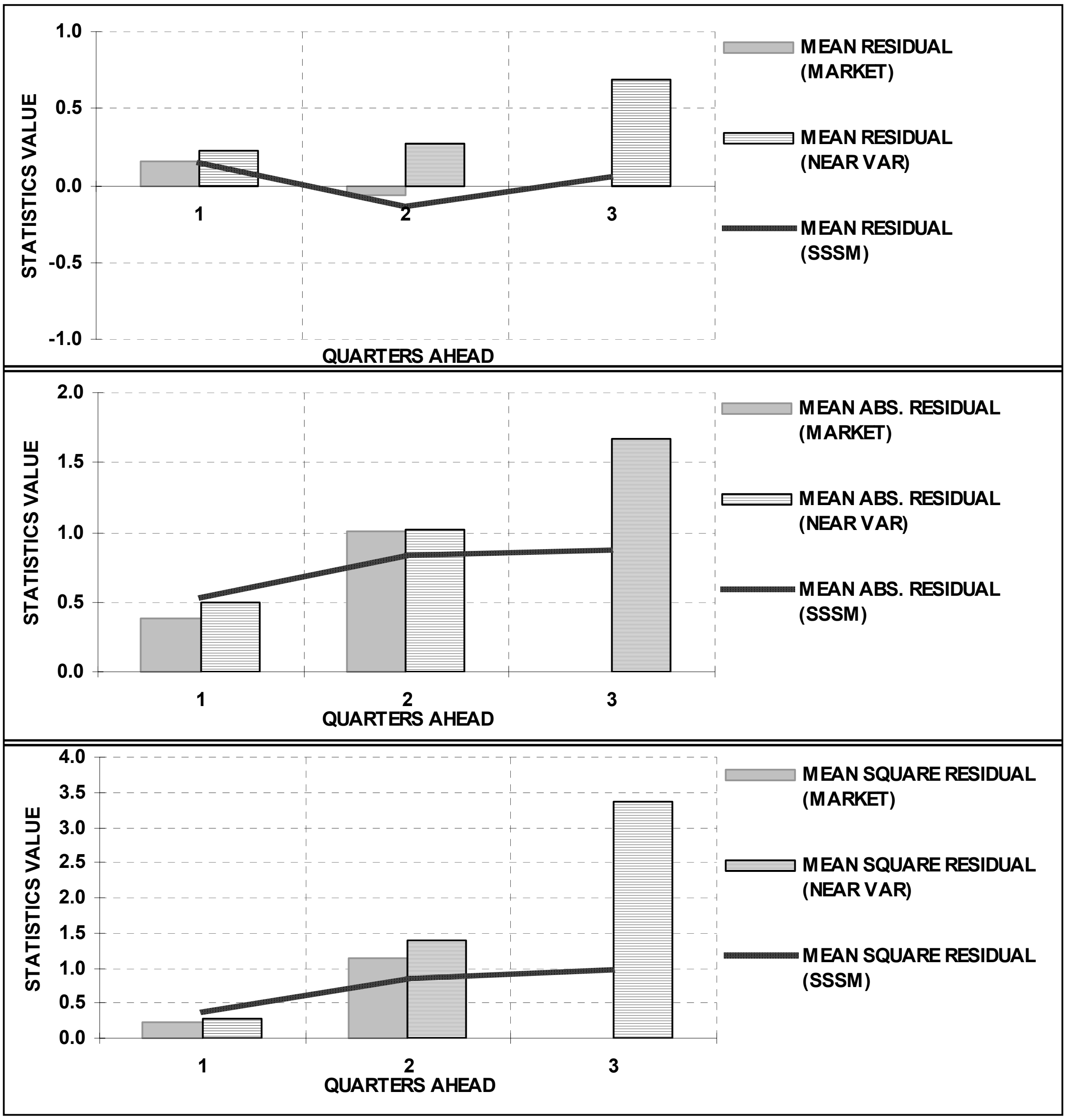
Table (9) ***Two-Month Information Forecasts (3 Quarters Ahead)***



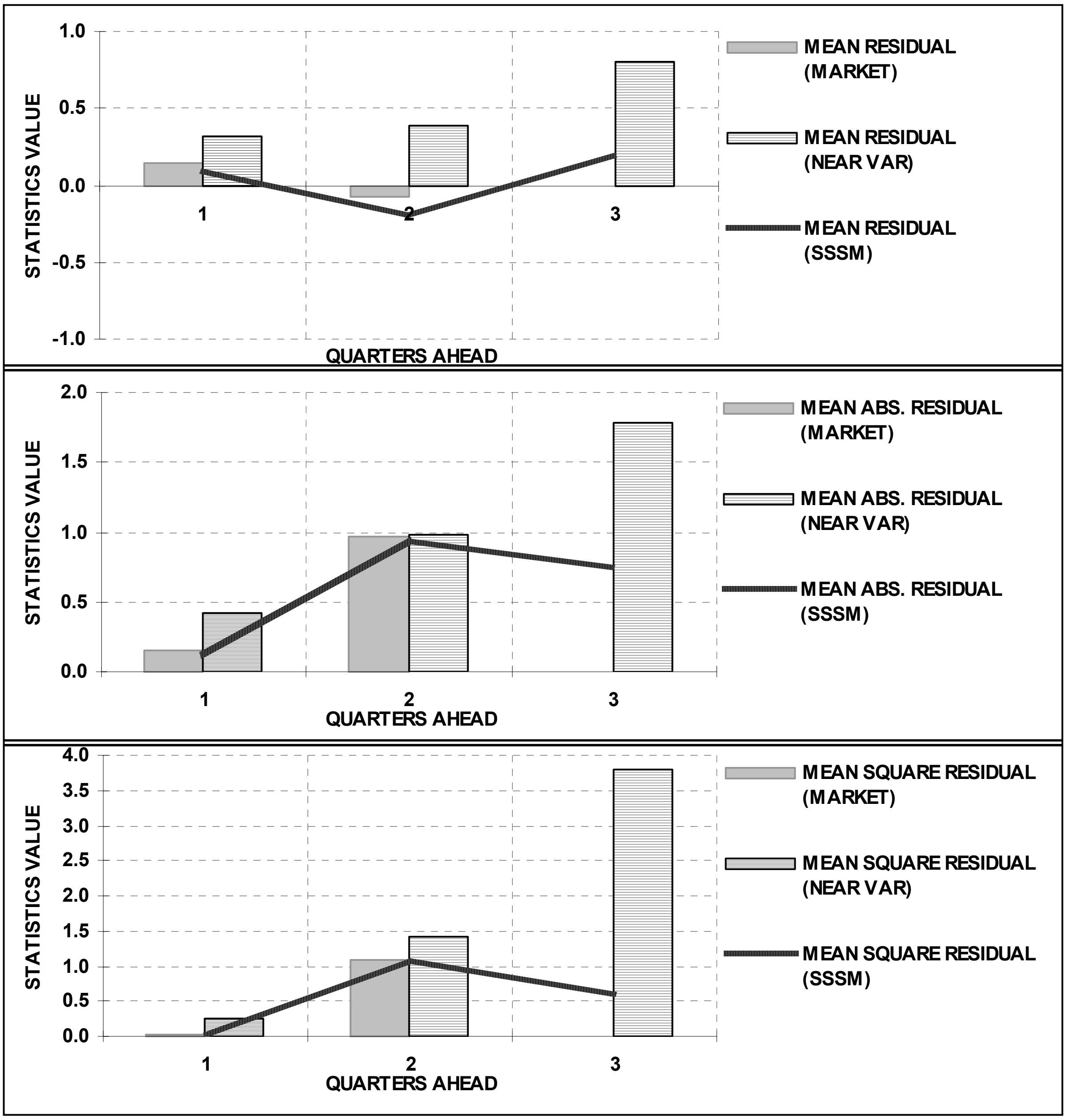
Graph (9) ***Two-Month Information Forecasts (3 Quarters Ahead)***



Graph (10) Statistics Behavior (No Information Forecasts)



Graph (11) Statistics Behavior (One-Month Information Forecasts)



Graph (12) Statistics Behavior (Two-Month Information Forecasts)

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