This chapter is reserved for publication of studies of questions related to financial stability.

The articles are the sole responsibility of the authors and do not necessarily express positions held by the Central Bank of Brazil.

The following papers are presented in this issue:

a) Extracting Information from Exchange Rate Options in Brazil;

b) Evaluation of Adaptations of the Bacen Circular 2,972 Model to the Exchange Rate Coupon;

c) Rules on Bank Closure with Non-Benevolent Regulator: Summary and Application;

d) An Analysis of Indirect Supervision of Off-Site Supervision of Banks' Profitability, Risk and Capital Adequacy: a Portfolio Simulation Approach Applied to Brazilian Banks.
Extracting Information from Exchange Rate Options in Brazil

Eui Jung Chang29
Benjamin Miranda Tabak30

Summary

This article presents a risk-neutral density extraction method for exchange rate options. The implicit volatility obtained can be used to forecast future volatility, and the foreseen density can be used to evaluate the evolution of market expectations regarding financial market prices. The implicit skewness and kurtosis are interpreted as measurements of market sentiment with respect to the direction of possible exchange movements and the possibility of extreme events, respectively. These measurements can be interpreted as forward-looking macro-prudential indicators for the domestic financial system. Furthermore, the results obtained from the options market are compared with the densities obtained by the Central Bank of Brazil survey among market economists. The conclusion is that both have relevant information. The methodology utilized in this article can be generalized and applied to contracts with different durations and to diverse types of assets.

29/ Central Bank of Brazil Department of Studies and Research, eui.jung@bcb.gov.br.
30/ Central Bank of Brazil Department of Studies and Research, benjamin.tabak@bcb.gov.br.
1. Introduction

Option prices are an important source of information regarding the distribution of future asset price probabilities. These prices can be used to construct forecasts of the distribution moments of important assets such as exchange and interest. These forecasts are fundamental to scenario construction, risk and portfolio management decision-making, as well as to evaluations of financial stability.

This article presents a methodology for extracting information on future asset prices and applies it to dollar call options negotiated at the Commodities and Futures Exchange (BM&F). The options are used to construct risk-neutral density forecasts for exchange in the 2000-2005 period. It demonstrates that these options have information content on future probability distribution moments of exchange rate.

Utilizing this technique, one can evaluate the evolution of exchange rate uncertainty in the period. Furthermore, market expectations regarding direction shifts in exchange rate movements can be extracted, together with possible extreme variations in this price. The results suggest that exchange rate uncertainties dropped sharply in recent years and remained relatively constant in 2005.

Call options are contracts that give their holders the right to purchase a certain quantity of a prespecified asset (underlying asset) on a specific date (in the case of European options)31. The intuition is that these options are negotiated at varied exercise prices and that the premiums paid for these options reflect the probabilities of the prices of the underlying asset reaching the exercise prices. Consequently, for varied exercise prices (X₁ and X₂), one can infer the probabilities that the price of the underlying asset will be between X₁ and X₂ at maturity of the option.

This methodology can be used to monitor exchange rate market uncertainties and financial stability. Moreover, it is extremely useful for constructing scenarios for exchange rate movements.

The remainder of the article is structured as follows: section 2 presents a brief review of literature; section 3 discusses the methodology used in the article; section 4 defines the sampling process; the empirical results obtained are presented in section 5; and section 6 concludes the article with final considerations and suggestions regarding future research.

31 There are other types of options known as American options that can be exercised prior to maturity.
2. Review of literature

Construction of risk-neutral density forecasts of the realizations of random variables in the future, such as exchange rates, interest rates, stock indices, among others, is extremely useful for portfolio managers, risk managers, financial regulators and academics in general.

The options market has been intensely utilized to extract information on market expectations regarding the densities of financial assets in the future. The forecasts extracted from options are forward-looking and can be used to monitor the evolution of market sentiment over time.

Researchers have used varied methods for forecasting density. These methods can be classified into parametric and nonparametric. The parametric methods are based on known distributions, which are then mixed and expanded in relation to normal distribution. The parametric methods include univariate distributions, such as the generalized Beta function (see Aparicio and Hodges, 1998), the generalized Lambda distribution (see Corrado, 2001), GEV distribution (see Markose and Alerton, 2005), mixtures of univariate distributions (see Melick and Thomas, 1997). Nonparametric methods do not require a functional form, but rather make it possible to utilize more general functions. These include kernel estimations (see Ait-Shalia and Lo, 1998), maximum entropy (see Buchen and Kelly, 1996) and curve adjustment methods (Shimko, 1993).32

The nonparametric methods have the advantage of being more flexible. However, one can obtain the complete density utilizing parametric methods, while nonparametric methods result in truncated density (for the available exercise prices).

Recently, some researchers have studied the relationship between risk-neutral densities and real-world densities (which have built-in preferences for market risk). Bliss and Panigirtzoglou (2004) and Anagnou-Basioudis et al. (2005) studied methods for transforming risk-neutral densities into real-world densities.

Chang and Tabak (2002) studied the Brazilian case of dollar-real call options. The authors demonstrate that it is possible to extract information regarding the options market exchange trajectory and present bimodal densities for 2002. Furthermore, Andrade and Tabak (2001) showed that the implicit volatility has relevant informational content which

32 See also Diebold et al. (1998), Jackwerth and Rubinstein (1996) and Malz (1997).
is not present in time series models, such as the GARCH models (1,1). (See also Castro, 2002)

Due to their enhanced flexibility and simplicity, a nonparametric method will be used to obtain the risk-neutral densities. The method will be described in detail in the next section.

### 3. Methodology

Assume that the prices of an asset \( \{S_t\} \) follow a Brownian geometric movement, represented by the equation

\[
dS = \mu S dt + \sigma S dW,
\]

in which \( \mu \) and \( \sigma \) correspond to expected return and volatility, respectively, and \( \{W_t\} \) is a Wiener process.

Black and Scholes (1973) demonstrated that the price of a European call option depends on parameters. The equation for the call option premium is:

\[
C = SN(d_1) - X e^{-r \tau} N(d_2)
\]

\[
\log(S/X) + \left( r + \frac{1}{2} \sigma^2 \right) \tau
\]

\[
\sigma \sqrt{\tau}
\]

\[
d_1 = \frac{\log(S/X) + \left( r + \frac{1}{2} \sigma^2 \right) \tau}{\sigma \sqrt{\tau}}
\]

\[
d_2 = d_1 - \sigma \sqrt{\tau}
\]

in which \( X \) represents the exercise price, \( \tau \) represents the time period to maturity of the option, \( r \) corresponds to the interest rate and \( N(d) \) is the cumulative normal distribution function up to the limit \( d \).

All the variables required for calculating the price of the option are observable, with the sole exception of volatility \( \sigma \). In this way, the prices observed and practiced on the market show a value for \( \sigma \), in such a way that market prices and the Black and Scholes (1973) formula price are equal. This value for volatility is a market forecast for the volatility of the asset underlying the option. It should be stressed that this forecast, known as implicit volatility, is a forward-looking forecast in contrast to the backward looking models, such as in the GARCH family models.

One of the problems encountered in using equation (2) to price dollar-real options is found in the fact that spot market and BM&F closing times for the dollar may be different, thus generating problems in the construction of implicit volatilities. Consequently, utilization of the BM&F dollar
futures price is preferable, since trading closes at the same time as options. This avoids problems caused by the lack of synchrony between the closing prices used to price options, which could lead to measurement errors.

The Garman-Kohlhagen model (1983) was used, applying the carrying cost arbitrage formula, which relates future prices to spot prices in order to avoid problems of synchrony. In this way, implicit volatility \( \sigma_1 \) was calculated using the following formula:

\[
C = e^{-\tau r} \left[ F N(d_1) - XN(d_2) \right],
\]

in which \( C \) corresponds to the option price, \( X \) to the exercise price, \( \tau \) to the number of days to maturity, \( r \) to the rate of interest, \( F \) being the dollar-real future adjustment price maturing in \( \tau \) days, and \( N(.) \) being the normal standard distribution. The rate of interest is that implicit in the adjustment prices of short-term future interest contracts (DI futures) scheduled to mature in \( \tau \) days.

Breeden and Litzenberger (1978) derived an exact relation between the price of European options and the distribution of the risk-neutral probability of the underlying asset.

\[
\frac{\partial^2 C}{\partial X^2} = e^{-\tau r} Q(S_{\tau})
\]

Equation (4) above tells us that, in relation to the exercise price \( X \), the second derivative of the price of a European call option \( C \) is equal to risk-neutral distribution \( Q \), discounted from the price of the underlying asset at maturity \( (S_{\tau}) \), in which \( \tau \) is the time still to elapse up to maturity.

Shimko (1993) utilizes the Black and Scholes (1973) pricing formula to obtain implicit volatilities for different exercise prices. The author then adjusts a continuous volatility function utilized to obtain a continuous option pricing function. Finally, using the result of Breeden and Litzenberger (1978), a probability density function was obtained.

For the Brazilian case, risk-neutral density was obtained through use of equation (3), together with implicit volatility, skewness and kurtosis.
4. Data and sampling

The data employed in the pricing of options and construction of risk-neutral densities used in this paper were supplied by the São Paulo Mercantile Exchange (BM&F).

The degree of precision of the estimated risk-neutral densities depends on the quality of the option price information used as an input in the estimation process. Consequently, the most heavily traded options were utilized.

The one-month BM&F dollar-real call options are the European type with maturity on the first business day of the month subsequent to issuance. Options with longer maturities do not have the liquidity required to estimate risk-neutral densities. Therefore, the study focuses on forecasting densities over a one-month horizon. Options with longer maturity terms do not have liquidity sufficient to extract risk-neutral densities. Thus, the major limitation in this paper is the fact that it concentrates on a one-month horizon (short-horizon time).

The data series starting in January 2000 and extending to December 2005 includes 72 months and 216 options contracts. The three options closest to money were selected for each month, with due consideration of the dollar-real futures market adjustment prices. In general, these options are the most heavily negotiated on the market, since they have higher liquidity.

One of the objectives of the paper is to compare the implicit option moments with the Central Bank of Brazil survey carried out among economists from a variety of financial institutions. On a daily basis, the Central Bank of Brazil collects market expectations regarding evolution of the major macroeconomic variables. Expectations regarding exchange-rate evolution were collected in this survey. This paper uses one-month ahead expectations for purposes of comparison with the implicit moments obtained from options traded on the market.

The next section presents the results obtained for extraction of risk-neutral densities and compares them with the forecasts drawn from this survey carried out by the Central Bank of Brazil Investor Relations Group.
5. Empirical results

5.1 Risk-neutral densities

For the sampling utilized, the results of the study showed that the distributions implicit in the options are heavy-tailed and positively asymmetric. Table 1 presents the results for implicit volatility, skewness and kurtosis. As demonstrated, kurtosis exceeds three (value for normal distribution) in 90.3% of the months, indicating heavy tails for the distribution of probabilities.

The figures with densities can be found in the appendix. It is possible to see that, in several cases, the densities are bimodal and the format of the curves changes significantly over time.

Implicit volatility ($\sigma$) is a forecast of the future volatility of the underlying asset. With this, changes in this variable indicate alterations in the perception of market uncertainty regarding future price levels. Figure 1 presents the evolution of one-month ahead implicit volatility. Average $\sigma_i$ for the January 2000-December 2002 period was 13%, compared to 11% for the January 2003-December 2005 period. It is important to stress that the moments of high volatility and uncertainty registered toward the end of 2002 were not observed in the more recent sampling. Peak volatility occurred in October 2002 (47.5%), dropping to 29.3% in December 2002, as a result of the uncertainties generated by the election process.

Implicit skewness ($SK_i$), which measures the skewness of the risk-neutral density, provides useful information, since it is a measurement of the direction forecast for exchange rate operations by market agents. A positive $SK_i$ indicates that the market attributes greater probability to upward exchange rate movements.

Figure 2 presents the evolution of the implicit skewness over the course of the sample period. The results suggest that expectations pointed consistently to upward interest-rate movement. It is interesting to observe that this phenomenon occurs even in periods of sharp appreciation of the domestic currency. Average skewness in the January 2000-December 2002 period was 0.55, with 0.51 in the later period. Two peaks are observed in the second period: in August 2003 and July 2004, corresponding to 1.387 and 1.35, respectively.

Table 1 – One step ahead forecast of volatility, skewness and kurtosis using options.

<table>
<thead>
<tr>
<th>Period</th>
<th>$\sigma_i$</th>
<th>SKi</th>
<th>Ki</th>
<th>Period</th>
<th>$\sigma_i$</th>
<th>SKi</th>
<th>Ki</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td></td>
<td></td>
<td></td>
<td>2003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan</td>
<td>0.061</td>
<td>0.514</td>
<td>3.126</td>
<td>Jan</td>
<td>0.213</td>
<td>0.483</td>
<td>3.525</td>
</tr>
<tr>
<td>Feb</td>
<td>0.069</td>
<td>0.508</td>
<td>3.715</td>
<td>Feb</td>
<td>0.338</td>
<td>0.880</td>
<td>2.822</td>
</tr>
<tr>
<td>Mar</td>
<td>0.034</td>
<td>0.293</td>
<td>3.431</td>
<td>Mar</td>
<td>0.125</td>
<td>0.538</td>
<td>4.169</td>
</tr>
<tr>
<td>Apr</td>
<td>0.046</td>
<td>0.001</td>
<td>4.641</td>
<td>Apr</td>
<td>0.155</td>
<td>0.482</td>
<td>4.035</td>
</tr>
<tr>
<td>May</td>
<td>0.053</td>
<td>0.215</td>
<td>3.197</td>
<td>May</td>
<td>0.121</td>
<td>1.138</td>
<td>6.000</td>
</tr>
<tr>
<td>Jun</td>
<td>0.043</td>
<td>0.272</td>
<td>3.129</td>
<td>Jun</td>
<td>0.146</td>
<td>0.168</td>
<td>3.045</td>
</tr>
<tr>
<td>Jul</td>
<td>0.060</td>
<td>0.548</td>
<td>2.643</td>
<td>Jul</td>
<td>0.140</td>
<td>0.398</td>
<td>3.317</td>
</tr>
<tr>
<td>Aug</td>
<td>0.063</td>
<td>0.614</td>
<td>2.783</td>
<td>Aug</td>
<td>0.064</td>
<td>1.368</td>
<td>6.762</td>
</tr>
<tr>
<td>Sep</td>
<td>0.023</td>
<td>0.895</td>
<td>3.672</td>
<td>Sep</td>
<td>0.157</td>
<td>0.396</td>
<td>3.230</td>
</tr>
<tr>
<td>Oct</td>
<td>0.030</td>
<td>1.528</td>
<td>7.408</td>
<td>Oct</td>
<td>0.102</td>
<td>0.262</td>
<td>4.316</td>
</tr>
<tr>
<td>Nov</td>
<td>0.081</td>
<td>-0.002</td>
<td>3.315</td>
<td>Nov</td>
<td>0.067</td>
<td>0.171</td>
<td>3.099</td>
</tr>
<tr>
<td>Dec</td>
<td>0.052</td>
<td>0.069</td>
<td>4.412</td>
<td>Dec</td>
<td>0.070</td>
<td>0.293</td>
<td>3.106</td>
</tr>
<tr>
<td>2001</td>
<td></td>
<td></td>
<td></td>
<td>2004</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan</td>
<td>0.039</td>
<td>0.281</td>
<td>3.844</td>
<td>Jan</td>
<td>0.075</td>
<td>0.615</td>
<td>4.869</td>
</tr>
<tr>
<td>Feb</td>
<td>0.049</td>
<td>0.343</td>
<td>4.030</td>
<td>Feb</td>
<td>0.107</td>
<td>0.136</td>
<td>3.031</td>
</tr>
<tr>
<td>Mar</td>
<td>0.045</td>
<td>0.487</td>
<td>5.041</td>
<td>Mar</td>
<td>0.080</td>
<td>0.356</td>
<td>3.900</td>
</tr>
<tr>
<td>Apr</td>
<td>0.065</td>
<td>0.371</td>
<td>3.754</td>
<td>Apr</td>
<td>0.062</td>
<td>0.341</td>
<td>3.726</td>
</tr>
<tr>
<td>May</td>
<td>0.086</td>
<td>0.764</td>
<td>6.039</td>
<td>May</td>
<td>0.092</td>
<td>0.099</td>
<td>2.989</td>
</tr>
<tr>
<td>Jun</td>
<td>0.096</td>
<td>0.387</td>
<td>3.908</td>
<td>Jun</td>
<td>0.173</td>
<td>0.930</td>
<td>4.821</td>
</tr>
<tr>
<td>Jul</td>
<td>0.170</td>
<td>1.206</td>
<td>4.187</td>
<td>Jul</td>
<td>0.108</td>
<td>1.350</td>
<td>5.160</td>
</tr>
<tr>
<td>Aug</td>
<td>0.268</td>
<td>0.727</td>
<td>3.189</td>
<td>Aug</td>
<td>0.101</td>
<td>0.547</td>
<td>4.112</td>
</tr>
<tr>
<td>Sep</td>
<td>0.174</td>
<td>0.011</td>
<td>2.474</td>
<td>Sep</td>
<td>0.070</td>
<td>0.866</td>
<td>4.331</td>
</tr>
<tr>
<td>Oct</td>
<td>0.157</td>
<td>0.272</td>
<td>3.160</td>
<td>Oct</td>
<td>0.063</td>
<td>0.215</td>
<td>3.319</td>
</tr>
<tr>
<td>Nov</td>
<td>0.149</td>
<td>0.118</td>
<td>4.375</td>
<td>Nov</td>
<td>0.077</td>
<td>0.338</td>
<td>3.200</td>
</tr>
<tr>
<td>Dec</td>
<td>0.090</td>
<td>0.372</td>
<td>3.698</td>
<td>Dec</td>
<td>0.083</td>
<td>0.117</td>
<td>3.031</td>
</tr>
<tr>
<td>2002</td>
<td></td>
<td></td>
<td></td>
<td>2005</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan</td>
<td>0.105</td>
<td>0.047</td>
<td>4.666</td>
<td>Jan</td>
<td>0.113</td>
<td>0.670</td>
<td>3.260</td>
</tr>
<tr>
<td>Feb</td>
<td>0.113</td>
<td>0.521</td>
<td>4.941</td>
<td>Feb</td>
<td>0.110</td>
<td>0.965</td>
<td>3.404</td>
</tr>
<tr>
<td>Mar</td>
<td>0.134</td>
<td>1.304</td>
<td>4.222</td>
<td>Mar</td>
<td>0.105</td>
<td>0.552</td>
<td>4.811</td>
</tr>
<tr>
<td>Apr</td>
<td>0.114</td>
<td>0.436</td>
<td>3.004</td>
<td>Apr</td>
<td>0.115</td>
<td>0.344</td>
<td>3.102</td>
</tr>
<tr>
<td>May</td>
<td>0.074</td>
<td>1.115</td>
<td>4.893</td>
<td>May</td>
<td>0.139</td>
<td>0.626</td>
<td>3.299</td>
</tr>
<tr>
<td>Jun</td>
<td>0.185</td>
<td>0.041</td>
<td>2.728</td>
<td>Jun</td>
<td>0.086</td>
<td>0.808</td>
<td>4.921</td>
</tr>
<tr>
<td>Jul</td>
<td>0.344</td>
<td>0.106</td>
<td>2.608</td>
<td>Jul</td>
<td>0.081</td>
<td>0.147</td>
<td>3.025</td>
</tr>
<tr>
<td>Aug</td>
<td>0.310</td>
<td>1.133</td>
<td>4.634</td>
<td>Aug</td>
<td>0.091</td>
<td>0.261</td>
<td>3.494</td>
</tr>
<tr>
<td>Sep</td>
<td>0.256</td>
<td>0.573</td>
<td>4.912</td>
<td>Sep</td>
<td>0.087</td>
<td>0.398</td>
<td>3.826</td>
</tr>
<tr>
<td>Oct</td>
<td>0.475</td>
<td>0.783</td>
<td>3.421</td>
<td>Oct</td>
<td>0.079</td>
<td>0.165</td>
<td>3.040</td>
</tr>
<tr>
<td>Nov</td>
<td>0.258</td>
<td>0.387</td>
<td>3.697</td>
<td>Nov</td>
<td>0.097</td>
<td>0.202</td>
<td>3.984</td>
</tr>
<tr>
<td>Dec</td>
<td>0.239</td>
<td>0.843</td>
<td>4.660</td>
<td>Dec</td>
<td>0.080</td>
<td>0.920</td>
<td>5.079</td>
</tr>
</tbody>
</table>

$\sigma_i$, SKi, and Ki represent the one step ahead forecast of implicit volatility, skewness and kurtosis of the expected variations on the exchange for the period.
Implicit kurtosis ($K_i$) provides a measurement of market sentiment regarding the possibility of extreme events occurring, since it measures the thickness of the risk-neutral density tails. The $K_i$ above three indicates that the market attributes a higher probability to extreme events than that implicit in a normal distribution. With this, increases in this variable are related to expectations of extreme events.

Figure 3 presents the evolution of the excess implicit kurtosis ($K_i - 3$) implicit from January 2000 to December 2005. On average, kurtosis remained practically unchanged in the two periods analyzed: 3.88 and 3.87, respectively. Viewed together with skewness data, the sharp peak in the second period (August 2003) suggests that the market expected strong devaluation of the domestic currency.
5.2 Densities extracted from the Central Bank Investor Relations Group survey

Table 2 presents the results of the Central Bank survey carried out among financial institutions. Marques et al. (2003) present a discussion of how these data are collected by the Central Bank of Brazil.

In order to compare Table 1 results with those of Table 2, we calculate the Spearman nonparametric correlation to volatility, skewness and kurtosis. The Spearman correlation between implicit volatility and the standard deviation of the forecasts obtained in this survey is 66.43% and is statistically significant to the level of 1%33. Thus, the increased uncertainty perceived through risk-neutral densities is also perceived by the increase in the standard deviation of the survey estimates. However, the nonparametric correlation between the implicit asymmetries and kurtoses are not statistically significant: 10.35% and -0.09%, respectively. The latter fact suggests that there may be additional informational content in the densities obtained by the survey.

6. Final considerations

This article presented a methodology for extracting exchange options information on exchange-rate movements. It is possible to construct forecasts of uncertainty regarding exchange-rate movements (implicit volatility), the expected direction of these movements (implicit skewness) and the probability of significant exchange rate shifts (implicit kurtosis). This information is summarized by the risk-neutral density obtained from options for varied exercise prices. This methodology can be extended to longer duration options and other assets.

The volatility, skewness and kurtosis implicit in exchange options can be used as forward-looking indicators of financial stability. Sharp increases in implicit volatility may indicate increased uncertainties regarding the exchange rate trajectory, while increases in kurtosis reflect greater probability of accentuated exchange rate alterations. Consequently, increases in these variables may signal increased vulnerability of agents with exchange exposure.

33/ In this case, the null hypothesis is that the correlation is null. Consequently, one can reject that the correlation is equal to zero at the significance level of 1%.
The information can also be utilized in constructing stress tests, contributing to the process of generating scenarios and evaluating the effects of adverse shocks on the banking system.

The densities obtained by the Central Bank of Brazil survey regarding exchange rate evolution expectations have additional information that can be used to verify market expectations. In this way, the information gathered by the Central Bank of Brazil bears important data that can be used together other information to build scenarios, such as that originating in the options market.

The following steps in this study consist in comparing the results obtained with the other forecasting models (such as the GARCH models class), extending the methodology to other financial assets and, finally, implementing other methodologies (both parametric and nonparametric) in order to compare and evaluate them.

Table 2 – One step ahead forecast of implicit moments on survey.

<table>
<thead>
<tr>
<th>Period</th>
<th>Dpi</th>
<th>SKi</th>
<th>Ki</th>
<th>n</th>
<th>Period</th>
<th>Dpi</th>
<th>SKi</th>
<th>Ki</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan</td>
<td>0.035</td>
<td>1.802</td>
<td>7.879</td>
<td>58</td>
<td>Jan</td>
<td>0.019</td>
<td>0.086</td>
<td>4.361</td>
<td>61</td>
</tr>
<tr>
<td>Feb</td>
<td>0.022</td>
<td>-0.535</td>
<td>3.357</td>
<td>57</td>
<td>Feb</td>
<td>0.019</td>
<td>0.869</td>
<td>3.738</td>
<td>61</td>
</tr>
<tr>
<td>Mar</td>
<td>0.017</td>
<td>0.363</td>
<td>4.041</td>
<td>55</td>
<td>Mar</td>
<td>0.013</td>
<td>-0.261</td>
<td>3.924</td>
<td>60</td>
</tr>
<tr>
<td>Apr</td>
<td>0.020</td>
<td>0.583</td>
<td>3.800</td>
<td>60</td>
<td>Apr</td>
<td>0.011</td>
<td>-0.393</td>
<td>3.595</td>
<td>61</td>
</tr>
<tr>
<td>May</td>
<td>0.018</td>
<td>0.357</td>
<td>3.588</td>
<td>60</td>
<td>May</td>
<td>0.013</td>
<td>0.028</td>
<td>5.876</td>
<td>63</td>
</tr>
<tr>
<td>Jun</td>
<td>0.024</td>
<td>-0.395</td>
<td>3.220</td>
<td>59</td>
<td>Jun</td>
<td>0.024</td>
<td>-0.929</td>
<td>3.882</td>
<td>64</td>
</tr>
<tr>
<td>Jul</td>
<td>0.049</td>
<td>-1.117</td>
<td>4.182</td>
<td>56</td>
<td>Jul</td>
<td>0.014</td>
<td>0.096</td>
<td>3.301</td>
<td>65</td>
</tr>
<tr>
<td>Aug</td>
<td>0.083</td>
<td>0.745</td>
<td>2.505</td>
<td>59</td>
<td>Aug</td>
<td>0.014</td>
<td>0.482</td>
<td>4.273</td>
<td>61</td>
</tr>
<tr>
<td>Sep</td>
<td>0.061</td>
<td>0.881</td>
<td>4.609</td>
<td>65</td>
<td>Sep</td>
<td>0.018</td>
<td>0.732</td>
<td>3.585</td>
<td>63</td>
</tr>
<tr>
<td>Oct</td>
<td>0.105</td>
<td>0.380</td>
<td>3.750</td>
<td>60</td>
<td>Oct</td>
<td>0.023</td>
<td>1.117</td>
<td>4.617</td>
<td>66</td>
</tr>
<tr>
<td>Nov</td>
<td>0.061</td>
<td>-0.354</td>
<td>5.287</td>
<td>56</td>
<td>Nov</td>
<td>0.014</td>
<td>1.450</td>
<td>6.078</td>
<td>64</td>
</tr>
<tr>
<td>Dec</td>
<td>0.046</td>
<td>-0.004</td>
<td>2.791</td>
<td>71</td>
<td>Dec</td>
<td>0.021</td>
<td>0.220</td>
<td>3.067</td>
<td>66</td>
</tr>
<tr>
<td>2003</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2005</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan</td>
<td>0.041</td>
<td>-0.024</td>
<td>4.403</td>
<td>60</td>
<td>Jan</td>
<td>0.025</td>
<td>0.679</td>
<td>3.393</td>
<td>63</td>
</tr>
<tr>
<td>Feb</td>
<td>0.049</td>
<td>-0.156</td>
<td>2.508</td>
<td>64</td>
<td>Feb</td>
<td>0.021</td>
<td>0.950</td>
<td>5.390</td>
<td>64</td>
</tr>
<tr>
<td>Mar</td>
<td>0.038</td>
<td>-0.759</td>
<td>5.430</td>
<td>64</td>
<td>Mar</td>
<td>0.023</td>
<td>0.137</td>
<td>5.133</td>
<td>64</td>
</tr>
<tr>
<td>Apr</td>
<td>0.029</td>
<td>0.145</td>
<td>3.103</td>
<td>61</td>
<td>Apr</td>
<td>0.019</td>
<td>0.040</td>
<td>3.190</td>
<td>62</td>
</tr>
<tr>
<td>May</td>
<td>0.064</td>
<td>0.234</td>
<td>2.671</td>
<td>63</td>
<td>May</td>
<td>0.024</td>
<td>0.612</td>
<td>2.944</td>
<td>63</td>
</tr>
<tr>
<td>Jun</td>
<td>0.035</td>
<td>0.095</td>
<td>6.006</td>
<td>67</td>
<td>Jun</td>
<td>0.030</td>
<td>0.444</td>
<td>2.836</td>
<td>63</td>
</tr>
<tr>
<td>Jul</td>
<td>0.037</td>
<td>0.664</td>
<td>3.156</td>
<td>66</td>
<td>Jul</td>
<td>0.025</td>
<td>0.600</td>
<td>3.256</td>
<td>65</td>
</tr>
<tr>
<td>Aug</td>
<td>0.022</td>
<td>0.722</td>
<td>3.887</td>
<td>68</td>
<td>Aug</td>
<td>0.019</td>
<td>0.212</td>
<td>4.507</td>
<td>61</td>
</tr>
<tr>
<td>Sep</td>
<td>0.021</td>
<td>1.408</td>
<td>7.301</td>
<td>66</td>
<td>Sep</td>
<td>0.021</td>
<td>1.291</td>
<td>7.211</td>
<td>64</td>
</tr>
<tr>
<td>Oct</td>
<td>0.020</td>
<td>1.369</td>
<td>6.687</td>
<td>67</td>
<td>Oct</td>
<td>0.032</td>
<td>0.093</td>
<td>3.305</td>
<td>67</td>
</tr>
<tr>
<td>Nov</td>
<td>0.016</td>
<td>0.351</td>
<td>4.012</td>
<td>67</td>
<td>Nov</td>
<td>0.021</td>
<td>-0.242</td>
<td>3.794</td>
<td>60</td>
</tr>
<tr>
<td>Dec</td>
<td>0.016</td>
<td>1.453</td>
<td>7.585</td>
<td>69</td>
<td>Dec</td>
<td>0.028</td>
<td>0.835</td>
<td>5.646</td>
<td>68</td>
</tr>
</tbody>
</table>

Dpi, SKi, and Ki represent the one step ahead forecast of standard deviation, skewness and kurtosis of the expected variations on the exchange for the determined period and n represents the number of observations used to create the distribution of the forecast.
Bibliographic references


Evaluation of Adaptations of the Bacen Circular 2,972 Model to the Exchange Rate Coupon

Alan Cosme R. da Silva
João Maurício S. Moreira
Myrian Beatriz E. das Neves

Summary

The purpose of this paper is to evaluate alternative ways of adapting the structure defined by Central Bank of Brazil Circular 2,972, dated March 23, 2000, (calculation of capital requirements for market risk of operations in real based on fixed interest rates) to operations involving exchange rate coupon. Alterations in the procedure for obtaining volatility and in the multiplication factor are considered. For purposes of comparison, a Value-at-Risk (VaR) parametric model based on exponential smoothing and a nonparametric model based on empirical quantile, both of which are applied to the internal models approach adopted by the Basel Accord, are used as benchmarks. The outcomes show that the Circular 2,972 model can be adapted to the exchange rate coupon, provided that changes be implemented that take due account of the peculiarities of the exchange rate coupon curve, particularly the sensitivity of shorter-term maturity rates to abrupt modifications in expectations regarding R$/USD exchange rate. The strong volatility changes registered in the period analyzed, coupled with the procedure for obtaining volatility for the VaR calculation, resulted in significantly high capital requirement levels when the Circular 2,972 structure is used with no adaptations.
1. Introduction

Implementation of the Basel Accord\textsuperscript{37} rules on capital requirements to cope with market risk in Brazil has moved forward gradually, incorporating adjustments to the specificities of Brazilian financial markets. Up to this point\textsuperscript{38}, capital requirements for exposure in operations referenced to foreign currency and gold and in operations denominated in real with earnings based on fixed interest rates have been regulated. Therefore, there are assets subject to market risk for which no capital requirements rules have yet been determined, such as equities, commodities and exchange rate coupon, among others.

The objective of this paper is to examine the performance of alternative ways of adapting the structure defined by Central Bank of Brazil Circular 2,972, dated March 23, 2000, (calculation of capital requirements for market risk of operations in real based on fixed interest rates)\textsuperscript{39} to operations involving exchange rate coupon, defined as the US dollar interest rate in Brazil. In a previous study, Silva et al. (2005) found that, in comparison to the interest rate in real, the greater volatility to which this rate is subject, coupled with the parameters determined by Circular 2,972, tends to result in very large capital requirements in certain periods. Consequently, in order to more adequately analyze alternatives to a standard calculation model for capital requirements for exchange rate coupon market risk, an effort was made to adapt that structure to this environment. Alterations in the procedure for obtaining volatility and in the multiplication factor are given consideration. With respect to the original model, the multiplication factor inversely proportional to volatility is substituted by the fixed value $M=3$ in the three versions analyzed. Besides this, instead of a single volatility for calculating the value-at-risk of all of the various maturities, maximum volatilities are tested for two and three groups of maturities and all volatilities, one per maturity.

For purposes of comparison, a Value-at-Risk (VaR) parametric model based on exponential smoothing and a nonparametric model based on empirical quantile, both of which are applied to the internal models approach adopted by the Basel Accord, are used as benchmarks. The models are evaluated on the basis of one-day VaR performance and on the basis of performance of capital requirements in relation

\textsuperscript{37}/ See Basel Committee on Banking Supervision (1996-a).
\textsuperscript{38}/ November 2005.
to cumulative losses over a 10-day holding period calculated for ten thousand simulated fixed income portfolios. The exchange rate coupon data used in the backtest cover the period from August 7, 2001 to March 31, 2005.

The outcomes show that the Circular 2,972 model can be adapted to the exchange rate coupon, provided that changes be implemented that take due account of the peculiarities of the exchange rate coupon yield curve, particularly the sensitivity of shorter-term maturity rates to abrupt modifications in expectations regarding R$/USD exchange rate. The strong volatility changes registered in the period analyzed, coupled with the procedure for obtaining volatility for the VaR calculation, resulted in significantly high capital requirement levels when the Circular 2,972 structure is used without alterations. On the other hand, the adapted versions that utilize maximum volatilities for two and three groups of maturities showed significantly more balanced behavior.

The other sections of this paper are distributed as follows: section 2 deals with the sample and methodology; section 3 presents the outcomes; and section 4 summarizes the conclusions drawn.

2. Sample and methodology

2.1 Sample

The data used in this paper consist of the exchange rate coupon yield curve announced by the Commodities and Futures Exchange (BM&F) in the period extending from January 3, 2000 to March 31, 2005, with a total of 1,303 business days. Taking a segment of the observations used for estimating the parameters of the models, the period effectively employed for the one-day VaR and capital requirements backtests was August 7, 2001 to March 31, 2005, totaling 907 business days.

BM&F interest rates are announced on a linear basis of 360 consecutive days. For purposes of this paper, these rates were interpolated by the flat forward methodology, obtaining rates on the exponential basis of 252 business days. The maturities utilized are the same as in Circular 2,972; 21, 42, 63, 126, 252, 504 and 756 business days.
2.2 Construction of the fixed income portfolios

The ten thousand portfolios tested were constructed through a simulation process. Each portfolio is represented by seven flows with uniform and randomly distributed values in the interval (-USD1,000; USD1,000). The use of United States dollar values makes it possible to perceive variations in portfolio value due solely to exchange rate coupon variations, independently of exchange rate alterations. Each one of these flows corresponds to one of the maturities considered and represents the value already mapped for the maturity in question. In other words, the set of seven flows corresponds to the mapping in those maturities of all the flows that primarily make up a given portfolio.

In order to make it possible to analyze both long and short positions separately, a constraint is imposed on the simulated portfolios: only those in which present value remains positive during the entire sample period are considered in the formation of the set of ten thousand portfolios. In this way, one can ensure that a long portfolio will remain in that position during the entire period of the study. Once the long portfolios are selected, the short portfolios are obtained, inverting the signs of the simulated flows.

2.3 The Circular 2,972 model

In the first step, the Circular 2,972 model is applied to the exchange rate coupon with no structural changes. In other words, the only change consists in substituting the interest rate (in real) yield curve for the exchange rate coupon yield curve. Direct application of the Circular 2,972 model to the exchange rate coupon is described briefly below. Capital requirements are determined by the formula:

\[ CR_t = \max \left( \frac{M_t}{60} \sum_{i=1}^{60} VaR_{t-i}, VaR_{t-1} \right) \]

in which the multiplication factor \( M_t \) is an inverse function of volatility \( \sigma_t \). \( M_t \) is obtained from

\[
M_t(\sigma_t) = \begin{cases} 
M & \text{if } \sigma_t \leq \sigma_{p\%} \\
\frac{C_1 + C_2}{\sigma_t} & \text{if } \sigma_t > \sigma_{p\%}
\end{cases}
\]
and volatility consists of the maximum value among the volatilities calculated for each one of the maturities. In other words, \( \sigma_i = \max \{ \sigma_{i,t} \} \) \( i = 1, 2, \ldots, 7\). For each maturity, volatility is calculated through the exponential smoothing method based on the expression

\[
\sigma_{i,t} = \sqrt{\lambda \sigma_{i,t-1} + (1-\lambda) r_{i,t-1}^2}.
\]

Two different decay factors are utilized \( \lambda=0.85 \) and \( \lambda=0.94 \), with \( \sigma_i \) defined as the largest of the two estimates. The constants \( C_1 \) and \( C_2 \) are functions of the volatility corresponding to percentile \( p (\sigma_{p\%}) \) and the peak volatility \( (\sigma_{\text{pico}}) \), both extracted from the volatility series \( \sigma_t \). The formulas for \( C_1 \) and \( C_2 \) are:

\[
C_1 = \frac{M - m}{\sigma_{p\%} - \sigma_{\text{pico}}} \quad \text{and} \quad C_2 = M - \frac{C_1}{\sigma_{p\%}}.
\]

The parameters used to calculate \( M_t \) are found in Table 1.

The VaR of each maturity is calculated according to the formula

\[
\text{VaR}_{i,t} = 2.33 \times \frac{T}{252} \times \sigma_i \times \text{PV}_{i,t} \times \sqrt{10},
\]

which always utilizes the same maximum volatility, independently of the maturity to which the VaR refers. The term is given by \( T \), \( \text{PV}_{i,t} \) corresponds to the net value of the flows brought to present value by the exchange rate coupon yield curve on day \( t \) and allocated in maturity \( i \). The VaR of the portfolio is given by

\[
\text{VaR}_t = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} \text{VaR}_{i,t} \times \text{VaR}_{j,t} \times \rho_{i,j}},
\]

which uses the correlation coefficients calculated according to the formula

\[
\rho_{i,j} = \rho + (1 - \rho) \left( \frac{\max(\rho, p_j)}{\min(\rho, p_j)} \right)^k.
\]

The \( \rho \) and \( k \) parameters are obtained through minimization of the squares of the errors between the estimated correlation values based on the normal sample formula and those calculated in the expression above. The parameters that would be announced daily by the Central Bank for calculation of capital requirements are \( M_t, \sigma_t, \rho \) and \( k \).
2.4 The multiplication factor

The minimum value of the multiplication factor (M) of the formula for calculating capital requirements for internal models is set by the Basel Committee at three and may be raised as high as four as a penalty for possible poor performance uncovered by backtesting the VaR model used by the financial institution.

The objective of the multiplication factor is to provide protection against some of the deficiencies normally related to modeling of the VaR, such as parametrization based on normal distribution (considering that the empirical distribution of financial returns commonly results in fat-tails), use of historical data to foresee the future (volatilities and correlations found in the past may well undergo sharp modifications, particularly in situations of market stress), nonconsideration of position changes over the course of the day (intraday risk), and so forth. The minimum value of three was calibrated on the basis of G10 financial markets, which are less subject to sudden and wide ranging variations when compared to the emerging financial markets. Consequently, one must question whether the same values of M are applicable to more volatile markets.

When Circular 2,972 was elaborated, it was argued that utilization of a fixed multiplication factor equal to three, as suggested by the BIS, could result in high requirements particularly in the wake of crises. In such periods, multiplication of an average VaR incorporating the higher values of the crisis period by a factor of three could generate excessive capital requirements. In this context, the Basel formula was adapted to the Brazilian market. The multiplication factor is treated as a decreasing function of volatility, varying between one and three. The rule of the maximum, however, was maintained in such a way that a requirement below the VaR of the previous day would not occur. In this way, occurrences of situations in which the requirement will be given by VaRt-1 increase since, once a surge in volatility occurs, this VaR will most likely be greater than the average, until such time as this average incorporates the higher values of VaR. When this average greater than VaRt-1 becomes effective once again, through incorporation of the higher values of the VaR, the requirement will be smoothed through multiplication by a factor adjusted by current volatility. Once the period of high volatilities has passed, the multiplication factor increases once again.

40 The desired benefit is to smooth capital requirements in post-crisis periods, since the effectiveness of increases in such requirements is doubtful, considering that excessive requirements soon after a breakdown of the regime could well lead to a worsening of systemic risk. However, with maintenance of the maximum value rule, the level of risk perceived by the measurement of VaR is respected.
Central Bank of Brazil studies aimed at evaluating methods of calculating capital requirements for market risk of stocks and foreign exchange, Araújo et al. (2003) and Barbedo et al. (2004), maintain the original Basel formula for internal models with the fixed multiplication factor of three. However, a fixed multiplication factor equal to two is also applied to the methods, in order to evaluate the possible adequacy of a lesser multiplication factor for more volatile markets. The results suggested that a model that rapidly adapts to changes in stock market volatility could allow for use of a multiplication factor of less than three. In the case of the foreign exchange market, losses greater than the capital requirements obtained with multiplication factor equal to two were registered in just a few days for the portfolios used in the analysis. However, this did not occur when a multiplication factor equal to three was utilized, even in those cases in which the period evaluated registered moments of sharp volatility on the Brazilian foreign exchange market, leaving open the possibility of a multiplication factor of less than three.

Based on what was stated above, efforts were made to verify the performance of the variable multiplication factor as an inverse function of volatility for those models. Considering that, in the context of the alternatives of adaptation of Circular 2,972, the volatility used in calculating VaR is no longer restricted to the maximum volatility – though it still remains as an input to the calculation of the multiplication factor – it was decided to adopt a more conservative stance, restricting variation of the multiplication factor to the interval (2, 3), as suggested by the results of previous studies. Nonetheless, the outcomes were very poor, possibly as a result of the particularities of the exchange rate coupon yield curve, potentialized by the use of volatilities differentiated per maturity or group of maturities. In all of the variants tested, performance was inferior to that of the two models used as reference for comparative purposes. At the same time, the required capital remained below that calculated for the structure of the internal models, a fact that is not compatible with a model to be used as standard. Thus, only the results for the structures based on fixed multiplication factor three for the tested adaptations have been maintained in this paper.

2.5 Adapted versions of the Circular 2,972 model

Here, adaptations of the Circular 2,972 model with alterations in the multiplication factor and in the procedure
for estimating the volatilities used in the calculation of the VaR are tested.

2.5.1 Model with fixed multiplication factor M=3 and volatility by maturity groups, two groups

Comparing to the original model, the multiplication factor inversely proportional to volatility is replaced by the fixed value M=3 in this version. At the same time, two volatilities instead of a single volatility are used to calculate the value-at-risk of all the maturities. The maximum volatility among the volatilities of 21, 42 and 63-day maturities is utilized in determining the VaR of these maturities. The same procedure is applied to the 126, 252, 504 and 756-day maturities. With this procedure, the objective was to avoid the possibility of the higher sensitivity of shorter-term maturities to alterations in expectations regarding exchange rate unduly propagating into the longer-term maturities.

2.5.2 Model with fixed multiplication factor M=3 and volatility by maturity groups, three groups

The model with fixed multiplication factor M=3 and volatility by maturity groups, three groups, is slightly different from the version presented in the previous item. The volatility of the first maturity is used only in calculating its own VaR, the maximum volatility of the three subsequent maturities is used in calculating the VaR of these maturities and the maximum volatility of the last three maturities is utilized in an analogous manner. With this procedure, an effort was made to isolate the largest focal point of volatility fluctuation in the 21-day maturity.

2.5.3 Model with fixed multiplication factor M=3 and volatility by maturity

Advancing a bit more in the same direction as the two previous alternatives, the model with the fixed multiplication factor M=3 and volatility per maturity utilizes its own volatility for each maturity.
2.6 Models used as benchmarks for purposes of comparison

In the evaluation of the alternatives considered, a parametric model of VaR based on exponential smoothing and a nonparametric model based on empirical quantile were utilized and both were applied to the internal models approach adopted by the Basel Accord. These models follow the methodology defined in the Basel Accord for internal models. In this case, the calculation formula for capital requirements is given by\[^{41}\]:

\[
CR_i = \max \left\{ \frac{M}{60} \sum_{i=1}^{60} VaR_{10d}^{i-1}, VaR_{10d}^{i-1} \right\}
\]

2.6.1 Exponential smoothing model

In this model, the one-day VaR of the flows mapped in each maturity is obtained parametrically on the basis of its conditional volatility estimated in terms of the exponential smoothing (Exponentially Weighted Moving Average-EWMA), by the formula

\[
VaR_{10d}^{i-1} = \lambda^{i} \times \frac{T_i}{252} \times z_{\alpha \%} \times \sigma_{i,t-1}
\]

where \( T_i \) denotes the maturity in business days, \( z_{\alpha \%} \) is the quantile of the normal standardized distribution corresponding to the probability determined to the estimate of VaR, in this case 1%, and \( \sigma_{i,t-1} \) is the conditional daily volatility of the logarithmic returns of maturity \( i \) estimated for date \( t \), calculated through exponential smoothing\[^{42}\].

Differently from the procedure adopted in Circular no 2, 972, the decay factor \( \lambda_i \) is estimated for each maturity \( i \) through minimization of the expression

\[
RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^{T} \left( \frac{r_t^2 - \sigma_{i,t-1}^2(\lambda)}{\lambda} \right)}
\]

in which \( r_t^2 \) is the square of the interest rate return and \( \sigma_{i,t-1}^2(\lambda) \) is the conditional variance. In order to guarantee that the correlation matrix be positive semi-definite, only a single \( \lambda \) is used for all the maturities, given by the weighted average of \( \lambda_i \) according to the procedure adopted by RiskMetrics.

\[^{41}\] Note that the formula defined by the Basel Committee differs from that utilized in the original model of Circular 2,972 in that it defines a fixed multiplication factor \( M \) equal to three, with the possibility of moving as high as four, depending on the performance of the VaR model adopted on the backtest. \( M=3 \) was also adopted for the multiplication factor in the alternatives of adaptation of the Circular 2,972 model evaluated in this paper.

Consider \( \tau_i \) the resulting \( i \)-th RMSE (minimum) of the \( \lambda_i \) estimation process, the exponential decay factor obtained for maturity \( i \), or, in other words, \( \Pi = \sum_{i=1}^{N} \tau_i \). Defining the measurement of relative error \( \theta_i = \frac{\tau_i}{\Pi} \) and weight \( \phi_i = \frac{\theta_i^{-1}}{\sum_{i=1}^{N} \theta_i^{-1}} \), in which \( \sum_{i=1}^{N} \phi_i = 1 \), the optimal decrease factor will be given by \( \lambda = \sum \phi_i \lambda_i \). For the series used in this paper, the value found was \( \lambda = 0.95 \).

The one-day VaR for the portfolio is given by:

\[
VaR_i^{1d} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} VaR_{ij}^{1d} \times VaR_{ij}^{1d} \times \rho_{(i,j)}^{1d/i-1}}
\]

in which the correlation between maturities \( i \) and \( j \) on the date \( t \), \( \rho_{(i,j)}^{1d/i-1} \), is obtained by

\[
\rho_{(i,j)}^{1d/i-1} = \frac{\sigma_{(i,j)}^{1d/i-1}}{\sigma_{i,t}^{1d} \sigma_{j,t}^{1d}}
\]

just as \( \sigma_{(i,j)}^{1d/i-1} \) denotes the conditional covariance between maturities \( i \) and \( j \) on date \( t \), obtained through the formula:

\[
\sigma_{(i,j)}^{1d/i-1} = \lambda \sigma_{(i,j)}^{1d/i-1} + (1-\lambda) \sigma_{(i,j)}^{1d/i-2} + \rho_{(i,j)}^{1d/i-1} \sigma_{i,t}^{1d} \sigma_{j,t}^{1d}
\]

Extending the horizon to 10 days, the VaR for maturity \( i \) will be given by: \( VaR_i^{10d} = VaR_i^{1d} \times \sqrt{10} \). The 10-day VaR of the portfolio is calculated through the expression:

\[
VaR_i^{10d} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} VaR_{ij}^{10d} \times VaR_{ij}^{10d} \times \rho_{(i,j)}^{10d/i-1}}
\]

### 2.6.2 The historical simulation model

The simplest form of the nonparametric model of the empirical quantile was utilized, with a moving window of 252 observations (daily portfolio returns) from which the 1% (left tail) and 99% (right tail) quantiles are extracted. In this way, the one-day VaR will be given by the formulas below:

\[
VaR_{t, long}^{id} = PV_t \times Q_{1\%}^{252} \quad \text{VaR}_{t, short}^{id} = PV_t \times Q_{99\%}^{252}
\]
in which \( V_{Pt} \) corresponds to the present value of the portfolio on date \( t \). The 10-day VaR, utilized in the calculation of capital requirements is given by \( \text{VaR}_{t}^{10d} = \text{VaR}_{t-1}^{1d} \times \sqrt{10} \), as indicated by the Basel Committee\(^{43} \).

2.7 Analyzing the results

The models are evaluated through a one-day VaR backtest and capital requirements are calculated for each one of the ten thousand portfolios in the period extending from August 7, 2001 to March 31, 2005, comprising a total of 907 days. The portfolios are fixed (both flows and maturities) over the period analyzed and are brought to present value through the exchange rate coupon yield curve in effect on each day. Construction of the coupon curve is done admitting that the forward rate between maturities is constant.\(^{44} \) The daily return of each portfolio is calculated as the variation of its present value between two days due exclusively to interest rate fluctuations, or in other words, \( R_{t} = PV_{t} - PV_{t-1} \). Analogously, the cumulative 10-day return is given by \( R_{t,10} = PV_{t} - PV_{t-10} \). Negative returns represent losses for long portfolios and positive returns represent losses for short portfolios.

Application of the backtest to the one-day VaR follows the guidelines set down in a document published by the Basel Committee for this specific purpose.\(^{45} \) Consequently, every three months, a survey is made to find out how many times the daily VaR is surpassed by losses on the day in question for each portfolio over the past 250 business days. Basel defined three zones for the number of exceptions observed (in 250 observations) in the daily VaR backtest. Up to four exceptions, the model verified is in the green zone and, therefore, approved. A quantity of exceptions between five and nine puts the model in the yellow zone and, unless there is a strong justification for the deviation, this situation could result in raising the multiplication factor to as much as four. When more than nine exceptions exist, the model moves into the red zone and the evaluated institution can be called upon to adopt the standard approach.

---

\(^{43} \) The square root rule can be theoretically justified for parametric models that adopt hypotheses such as independent and identically distributed normal returns. However, the Basel Committee makes no distinction of this nature, making it possible to utilize the rule for any model based on VaR (see Basel Committee on banking supervision, 1996-A, section B4, item c). An alternative would be to calculate the 10-day VaR based on the series of 10-day returns. However, since the VaR is to be calculated daily for purposes of capital requirements, this would result in a series of strongly self-correlated returns.

\(^{44} \) Flat forward method.

\(^{45} \) See Basel Committee on Banking Supervision (1996-b).
It is important to note that the Basel Accord does not require backtesting for capital requirements, but only for the one-day VaR of the proprietary models. In this paper, however, the performance of capital requirements is also evaluated. The Basel Committee suggests a 10-day holding period, during which losses could hypothetically accumulate. In this way, a comparison is made between returns accumulated over a 10-day holding period for each evaluated portfolio and their respective capital requirements, while the total number of exceptions is also calculated. In a manner analogous to the terminology used for the VaR, exception is understood as occurrence of a loss greater than the capital requirement, defined previously for that specific model.

For a cost-benefit analysis, the average daily capital requirement for each model for the ten thousand portfolios is compared to the number of exceptions that occurred. There is a trade-off between these indicators since, for a given model, a larger capital requirement tends to generate a lower number of exceptions. In this case, the lower the two values, the more efficient will be the model. In a complementary sense, given that a Capital Requirement (CR) exception has been observed, the difference between the loss and the respective capital requirement is considered (loss - CR). The larger the value by which these losses exceed foreseen CR, the lower will be the protection provided by the model. Finally, given that a loss lesser than the required capital has occurred, the difference between the capital requirement and the respective loss (CR - loss) is viewed as a way of evaluating the conflicting aspects between the protection that a given model provides and its efficiency in allocating capital for coverage of risk. For example, given that no exceptions in CR have been observed, the greater the difference between the CR and the loss, the greater will be the potential protection, and lesser will be the efficiency of capital allocations.

3. Results

The period of this study encompasses moments of sharp volatility on the Brazilian foreign exchange market. Consequently, some of the exceptions shown by the one-day VaR in all of the models occurred as a result of this fact. The Basel Committee calls for increases in the multiplication factor or revision of those internal models that do not satisfy minimum performance criteria. In the case of emerging markets, which are subject to strong and frequent fluctuations in volatility patterns, it is important to carefully evaluate the circumstances in which weak performances occur. An
attempt must be made to understand whether the model is showing itself to be inconsistent or whether it is only reflecting strongly adverse market conditions.

3.1 Volatilities

As shown in Figure 1, the behavior of the volatilities calculated for the seven maturities used in this paper shows severe fluctuations in the mid-2002 pre-electoral period. It is important to observe that, in that same period, volatility is considerably greater for the terms of 21 and 42 days. This was a result of the growing difference between forward exchange rate compared to the spot market price, particularly in the period between the end of July and early August, when forward exchange rate dropped more than 10% below the spot price, generating strong expectations of devaluation with a particularly strong impact on shorter-term maturities. Outside this period, volatility fluctuations were considerably less intense.

The abrupt volatility increases tend to generate two effects. First of all, there is a natural propensity to a higher than expected number of exceptions in VaR backtests (and, possibly, in CR) considering that the respective estimates do not instantaneously adjust to volatility fluctuations. On the other hand, VaR and CR react with similar intensity to the rise in volatility, albeit with a one-day lag. The consequent increase in CR deserves special attention in the Circular 2,972 adapted model, since this model always utilizes the maximum volatility of the maturities in calculating VaR.

Figure 1 – Cupom cambial volatilities – Maturities of 21, 42, 63, 126, 252, 504 e 756 working days – April 4, 2000 to March 31, 2005.

46/ Maturity: September 2002.
47/ On 8/1/2002, conditional volatility calculated for the 21-day maturity reaches its maximum, approximately 30 times greater than the average for the sample, excluding the second half of 2002.
3.2 Value-at-Risk

The performance of VaR is analyzed on the basis of the backtest carried out according to the system put forward by the Basel Committee. Though idealized only for the purpose of evaluating internal risk models, the procedure is also applied to adaptations of Circular 2,972 in order to make it possible to draw comparisons among the models. In this sense, one should recall that the model adopted by the Central Bank of Brazil for fixed interest rates has characteristics of both the standardized approach and the internal models approach. As a matter of fact, it is a standardized model, since its parameters and the specifications are announced by the Central Bank to be used by all financial institutions. However, it is quite close to the internal model approach since it utilizes VaR and CR calculation formula based on Basel Committee specifications for this approach.

Table 2 shows total exceptions in each model for the set of ten thousand portfolios, considering both long and short positions. In other words, the two return distribution tails of the portfolios utilized are considered. The probability of the VaR is defined at 1%, meaning that the proportion of exceptions in relation to the total number of days of the backtest is expected to remain at approximately this level for each portfolio. The version that adopts the original construction of Circular 2,972 is shown to be sharply conservative, with 0.02% of exceptions for the long position and 0.01% for the short position. This result was expected, considering the behavior verified for volatilities and the procedure for determining volatility which selects the volatility of the 21-day maturity for calculating VaR for most of the period studied. As a result, an apparently excessive “cushion” of protection is created that is transferred to CR, particularly in moments of foreign exchange market tensions.48 Utilization of the seven volatilities brings the standard model quite close to the internal models, with a percentage of exceptions above 1%. The constraint on the utilization of volatilities by groups reintroduces some degree of conservatism into the standard model, with less than 1% of exceptions.49 The number of portfolios registering at least one exception corroborates the behavior observed for total exceptions. For a total of 907 days, it is expected that all portfolios will close with a figure in the range of nine

48/ A justification for utilization of a conservative procedure for determining capital requirements in the standardized approach would be precisely the fact that one intends to encompass all of the risk profiles with a single model. However, when the original structure of Circular 2,972 is applied to the exchange rate coupon, the procedure seems to be exaggerated.

49/ Principally when we consider that the period analyzed showed moments of sharp fluctuation, when a ratio of exceptions above 1% is more likely to occur.
exceptions. For the original Circular 2,972 model, however, 8,279 long portfolios and 9,577 short portfolios registered no exceptions whatsoever. Among those that did register exceptions, the majority showed only one event.

Among internal approach models, the best performance was found in the nonparametric model for the long position. The opposite situation is found to exist for the short position.\textsuperscript{50} For all of these models, there are more exceptions in the long position than in the short position.

Table 2 – Total of VaR exceptions (one day loss higher than VaR) – 10,000 portfolios, long and short positions – August 7, 2001 to March 31, 2005.

<table>
<thead>
<tr>
<th>VaR Exceptions</th>
<th>MP2972\textsuperscript{1}</th>
<th>MP2V\textsuperscript{2}</th>
<th>MP3V\textsuperscript{3}</th>
<th>MP7V\textsuperscript{4}</th>
<th>MIAE\textsuperscript{5}</th>
<th>MIH\textsuperscript{6}</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Long Position</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total of exceptions (%)</td>
<td>0.02%</td>
<td>0.57%</td>
<td>0.96%</td>
<td>1.66%</td>
<td>1.95%</td>
<td>1.54%</td>
</tr>
<tr>
<td>Total of exceptions</td>
<td>1871</td>
<td>51479</td>
<td>86955</td>
<td>150660</td>
<td>177137</td>
<td>139460</td>
</tr>
<tr>
<td>Total of portfolios with at least one exception</td>
<td>1721</td>
<td>9884</td>
<td>9965</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
</tr>
<tr>
<td><strong>Short Position</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total of exceptions (%)</td>
<td>0.01%</td>
<td>0.41%</td>
<td>0.69%</td>
<td>1.15%</td>
<td>1.44%</td>
<td>1.50%</td>
</tr>
<tr>
<td>Total of exceptions</td>
<td>745</td>
<td>37317</td>
<td>62863</td>
<td>104530</td>
<td>130880</td>
<td>136500</td>
</tr>
<tr>
<td>Total of portfolios with at least one exception</td>
<td>423</td>
<td>9168</td>
<td>9830</td>
<td>10000</td>
<td>10000</td>
<td>10000</td>
</tr>
</tbody>
</table>

\textsuperscript{1}MP2972 – Standardized 2972 model: model based on the Circular 2,972, using maximum volatility and multiplication factor varying from 1 to 3.

\textsuperscript{2}MP2V – Standardized model with 2 volatilities: model based on the Circular 2,972, using two volatilities: maximum among maturities 21, 42 e 63 and maximum among maturities 126, 252, 504 e 756; and multiplication factor equal to 3.

\textsuperscript{3}MP3V – Standardized model with 3 volatilities: model based on the Circular 2,972, using three volatilities: maturity 21, maximum among maturities 42, 63 and 126 and maximum among maturities 252, 504 e 756; and multiplication factor equal to 3.

\textsuperscript{4}MP7V – Standardized model with 7 volatilities: model based on the Circular 2,972, using one volatility to each maturity and multiplication factor equal to 3.

\textsuperscript{5}MIAE – Internal model based on EWMA and multiplication factor equal to 3.

\textsuperscript{6}MIH – Internal model based on historic simulation and multiplication factor equal to 3.

Table 3 shows the average results (average occurrences in the 11 subperiods of 250 days, every three months) of the backtesting carried out according to the procedure proposed by the Basel Committee. During the entire period analyzed, the original Circular 2,972 model places all of the portfolios in the green zone, reflecting the numbers in Table 2. The alternative utilizing all volatilities shows a large number of observations in the yellow zone, but less observations in the red zone than the reference models. Though they still

\textsuperscript{50} This fact probably occurs as a result of the existence of fat tails and of leftward asymmetry for the distribution of returns, which is not duly perceived by the parametric models based on normal distribution.
have observations in the red zone, the other two adaptations implemented show a significantly better performance than the internal models. In general, the larger concentration of events in the yellow zone and the red zone occurs in mid-2002. This is explained, for the most part, by the Argentine crisis and by the pre-electoral period. Once this moment of greatest turmoil had passed, the number of portfolios in the yellow and red zones dropped sharply, reflecting incorporation of the sharp increase in exchange rate coupon volatility in the previous period into VaR. At the end of 2003, the number in the yellow zone moved upward once again as a consequence of a small spike in volatility (see Figure 1).

### Table 3 – Var Backtest performed each three months, according to Basel Committee directions, to each period of 250 working days totalling eleven periods. The 10,000 portfolios are distributed in three zones according to the number of exceptions incurred on average in the eleven periods. The green zone corresponds to portfolios presenting up to four exceptions; the yellow zone, from five to nine exceptions; the red zone, ten or more exceptions.

<table>
<thead>
<tr>
<th>Models</th>
<th>Long Position</th>
<th>Short Position</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average number of portfolios</td>
<td>Average number of portfolios</td>
</tr>
<tr>
<td></td>
<td>Green zone</td>
<td>Yellow zone</td>
</tr>
<tr>
<td>MP2972&lt;sup&gt;1&lt;/sup&gt;</td>
<td>10 000</td>
<td>-</td>
</tr>
<tr>
<td>MP2V&lt;sup&gt;2&lt;/sup&gt;</td>
<td>9 554</td>
<td>411</td>
</tr>
<tr>
<td>MP3V&lt;sup&gt;3&lt;/sup&gt;</td>
<td>8 221</td>
<td>1 668</td>
</tr>
<tr>
<td>MP7V&lt;sup&gt;4&lt;/sup&gt;</td>
<td>5 329</td>
<td>4 551</td>
</tr>
<tr>
<td>MIAE&lt;sup&gt;5&lt;/sup&gt;</td>
<td>5 229</td>
<td>3 935</td>
</tr>
<tr>
<td>MIH&lt;sup&gt;6&lt;/sup&gt;</td>
<td>6 588</td>
<td>1 177</td>
</tr>
</tbody>
</table>

1. MP2972 – Standardized 2972 model: model based on the Circular 2,972, using maximum volatility and multiplication factor varying from 1 to 3.
2. MP2V – Standardized model with 2 volatilities: model based on the Circular 2,972, using two volatilities: maximum among maturities 21, 42 and maximum among maturities 126, 252, 504 e 756; and multiplication factor equal to 3.
3. MP3V – Standardized model with 3 volatilities: model based on the Circular 2,972, using three volatilities: maturity 21, maximum among maturities 42, 63 and 126 and maximum among maturities 252, 504 e 756; and multiplication factor equal to 3.
4. MP7V – Standardized model with 7 volatilities: model based on the Circular 2,972, using one volatility to each maturity and multiplication factor equal to 3.
5. MIAE – Internal model based on EWMA and multiplication factor equal to 3.
6. MIH – Internal model based on historic simulation and multiplication factor equal to 3.

### 3.3 Capital requirements

Once capital requirements are calculated on the basis of the proposed methodologies, the proportion of the volume of capital required in relation to the present value of the simulated portfolios is evaluated in the first place. Following that, the backtest of the CR is evaluated and the CR levels are compared with the respective losses when such losses surpassed CR (loss – CR>0) and when CR are greater than the losses (CR – loss >0).
### 3.3.1 Level of capital requirements

For purposes of comparison between the capital requirements determined in each case, the ratio between the sum of all capital requirements and the sum of all present values was used as the indicator, aggregating all of the portfolios during the period studied:

\[
I_{CR} = \frac{\sum_{i=10,000}^{i=1} \sum_{j=1}^{j=907} CR_{i,j}}{\sum_{i=10,000}^{i=1} \sum_{j=1}^{j=907} PV_{i,j}}
\]

It is important to emphasize that the adopted indicator must not be considered in absolute terms, since the present value of a specific portfolio does not bear a direct relationship to the embedded risk.\(^{51}\) On the other hand, the indicator allows for comparison among the average levels of capital required by the four models analyzed.

Figure 2 shows that the unmodified Circular 2,972 model presents a level of CR well above the other models analyzed. The index drops to approximately half when two volatilities are utilized instead of the maximum volatility. Attention should be given to the fact that the historical simulation model shows an average aggregate CR index close to those of the

---

51/ Which would be indicated, for example, by the VaR of the portfolio.
standard model alternatives with two and three volatilities. The version with seven volatilities, however, comes quite close to the model based on exponential smoothing, at a level that is sharply lower than for the others.\textsuperscript{52}

Aside from the average aggregate CR, it is interesting to analyze distribution of average capital requirements per portfolio, as a proportion of the respective average present value, given by:

$$ CRP_i = \frac{\sum_{j} CR}{\sum_{j} PV}, \quad i = 1 \text{ to } 10,000, \quad j = 1 \text{ to } 907. $$

In Figure 3, it is again possible to highlight the results referring to the original model, in which distribution of average CR ratios per portfolio extends accumulation much more sharply to the right than the others. The internal model based on exponential smoothing has the largest number of observations in the intervals of less average requirements, while the historical simulation model and the variations of the original model are positioned between these extremes. Analogously to what occurred with average aggregate CR, the version of the Circular 2,972 model which utilizes seven volatilities differs very little in terms of its behavior from the model based on exponential smoothing. This should not come as a surprise, since the methodology employed shows greater likeness to the model based on exponential smoothing.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Cumulative frequency distribution of average capital requirement for \textit{cupom cambial} risk, as proportion of average present value per portfolio, long position, 10,000 portfolios – August 7, 2001 to March 31, 2005.\textsuperscript{53}}
\end{figure}

\textsuperscript{52} The internal model based on historical simulation shows different levels of capital requirements for the right and left tails as a consequence of calculation of the VaR through a nonparametric model in which the empirical quantile representing 1% is usually not symmetric with that representing 99%. The other models utilized the calculation of the VaR through the parametric model based on normal distribution.

\textsuperscript{53} Only the long position is represented here, considering that, with the exception of the historical simulation model, all other models showed identical capital requirements for the short position. The long position was chosen since it was considered to have the highest volatility and the largest number of exceptions.
smoothing than to the original standard model. As a matter of fact, they both use the respective volatility in each maturity, which is calculated on the basis of exponential smoothing. The differences are found basically in the decay factors (lambdas) and in the correlation matrix.

### 3.3.2 Backtest of capital requirements

The next step is an analysis as to whether the CR calculated by the models is sufficient to cover the risk of loss due to exchange rate coupon variations. With this in mind, occurrences of exceptions to the model are analyzed. In other words, a study is carried out to see whether 10-day cumulative losses greater than required capital occurred in the universe of the ten thousand simulated portfolios in the period in question. In principle, expectations are that the model with the highest level of requirements will have the lowest number of exceptions. Table 4 presents the outcomes.

In both the long and short positions, the CR calculated on the basis of the original standard model shows a negligible number of exceptions. It is important to stress that, even with sharply lower CR levels, adaptations with two and three volatilities also registered low rates of exceptions, suggesting

| Capital Requirement | Long Position | | | Short Position | | |
|---------------------|--------------|-----------------|-----------------|-----------------|-----------------|
| Exceptions | Total of exceptions | Total of exceptions | Portfolios presenting at least one exception | Total of exceptions | Total of exceptions | Portfolios presenting at least one exception |
| MP2972 | 0.02% | 222 | 221 | 0.00% | 1 | 1 |
| MP2V | 0.03% | 2,829 | 1,000 | 0.00% | 224 | 113 |
| MP3V | 0.04% | 3,318 | 1,736 | 0.01% | 696 | 299 |
| MP7V | 0.09% | 7,865 | 3,823 | 0.02% | 1,421 | 473 |
| MIAE | 0.16% | 14,506 | 5,853 | 0.02% | 1,666 | 665 |
| MIH | 0.13% | 11,617 | 6,116 | 0.04% | 3,501 | 3,093 |

1. **MP2972** – Standardized 2972 model: model based on the Circular 2,972, using maximum volatility and multiplication factor varying from 1 to 3.
2. **MP2V** – Standardized model with 2 volatilities: model based on the Circular 2,972, using two volatilities: maximum among maturities 21, 42 e 63 and maximum among maturities 126, 252, 504 e 756; and multiplication factor equal to 3.
3. **MP3V** – Standardized model with 3 volatilities: model based on the Circular 2,972, using three volatilities: maturity 21, maximum among maturities 42, 63 and 126 and maximum among maturities 252, 504 e 756; and multiplication factor equal to 3.
4. **MP7V** – Standardized model with 7 volatilities: model based on the Circular 2,972, using one volatility to each maturity and multiplication factor equal to 3.
5. **MIAE** – Internal model based on EWMA and multiplication factor equal to 3.
6. **MIH** – Internal model based on historic simulation and multiplication factor equal to 3.
more efficiency of the required capital. The version that uses all volatilities, such as the internal models, registered significantly higher rates. As shown above, the performance of the internal parametric model is apparently due to the hypothesis of a normal distribution of returns, which does not seem to adjust suitably to the behavior of the exchange rate coupon in Brazil. It is possible that simple substitution of the normal distribution with other distributions that take account of fatter tails, such as the t-Student, will produce better results. Another alternative would be application of the theory of extreme values, with use of specifically estimated distributions to reflect the behavior of rare events.

Analogously to the VaR result, one should note that the long position has a larger number of exceptions than the short position. Figure 4 shows the cumulative distributions of the exceptions by portfolio, long position.

![Figure 4 – Cumulative frequency distribution of capital requirement total number of exceptions (cumulative loss in ten working days higher than the capital requirement) per portfolio, long position, 10,000 portfolios – August 7, 2001 to March 31, 2005.](image)

In the original standard model, all ten thousand portfolios show between zero and two exceptions. The versions with two and three volatilities performed well, with approximately 98% of the portfolios registering four exceptions or less. The behavior of the internal models and of the version of the standard model with seven volatilities are quite similar, with performances that were below the performance of the two models derived from Circular 2,972.

The cumulative distributions of the exceptions by portfolio, short position, can be observed in Figure 5. Compared to the long position, there are lesser occurrences of extremes, principally in the versions of the original model of Circular 2,972 with two and three volatilities and in the historical simulation model, which concentrates its exceptions in the first interval.54

54/ Observe the scale of the vertical axis in relation to the previous graph.
3.3.3 Evaluation of the efficiency of capital requirements

Once the frequency of CR exceptions in the period studied is analyzed, one must move on to an evaluation of the amounts related to these exceptions. For each one of the ten thousand portfolios and for both long and short positions it is verified the amount by which 10-day cumulative losses surpasses the capital requirements. For this analysis, the value given by \((-\text{loss} - \text{CR})\) is calculated as a proportion of the present value of the portfolio on the days on which there were CR exceptions. Figures 6 and 7 show the distributions of frequency of these occurrences in the portfolios for the models analyzed, long and short positions, respectively.

Although the CR does not totally cover 10-day losses in 100% of the cases, the outcomes do demonstrate that the rate of losses greater than capital requirements is negligible for the original Circular 2,972 model. Aside from a lesser number of exceptions, this model shows a maximum rate of less than 5% of the present value of the respective portfolio for the long position. For the short positions, this percentage is practically zero for the only exception found. The versions with two and three volatilities concentrate the occasions in which losses surpassed CR at percentages below 3% in both tails, while the version with seven volatilities and the internal approach models show relevant observations with high percentages. Here, it is important to highlight the weak performance of the historical simulation model.

Figure 5 – Cumulative frequency distribution of capital requirements total number of exceptions (cumulative loss in 10 working days higher than the capital requirement) per portfolio, short position, 10,000 portfolios – August 7, 2001 to March 31, 2005.
Another important aspect related to capital requirements concerns the size of the protective cushion against risk, characterized as the excess CR over losses. The larger the cushion, the greater will be protection against risk, though efficiency in the allocation of financial institution capital will also be lesser. It is reasonable for one to consider as more efficient the model that guarantees a protection level greater than or equal to that provided by the other model, at the same time in which it requires less of the institution’s own capital.

In order to make a comparative evaluation of the efficiency of the models in allocating regulatory capital, the size of the protection cushion (CR - loss) is calculated as a ratio of the present value of the portfolio whenever a loss occurs and when it is smaller than CR. Figures 8 and 9 show the frequency distributions of the excesses in the models analyzed for the long and short positions.
With the exception of the model based on the original version of Circular 2,972, the models concentrate practically the entirety of the excesses in the first interval. In all the other intervals, the original version shows considerably larger excesses compared to the other models, particularly in the intervals more to the right. It is worth noting that the behavior of the versions of two and three volatilities is similar to that of the version with seven volatilities and to the internal approach models, even though performance in terms of protection is better. In other words, considering that a capital requirements model must ideally support the occurrence of exceptions and, simultaneously, not impose unreasonable requirements, the versions of two and three volatilities show comparatively better results than the other models. Table 5 presents the general average of excesses as a proportion of the portfolio value for both the long and short positions.

On average, the excess of the model based on Circular 2,972 is between 60% and 70%, while this percentage drops to approximately 35% and 25% in the versions of two and three volatilities, respectively. The version of seven volatilities and the model based on exponential smoothing show lesser percentages, approximately 16%. In the historical simulation model, the index once again surpassed 20%, confirming the weak performance of this approach for the sample considered, in which a relatively high capital requirement coexists with a high number of exceptions, when compared to the other models. Here, one should underscore the general performance of the version with three volatilities, in which the number of exceptions is quite close to that shown by the version with two volatilities, while average capital required is significantly less. The Circular 2,972 model requires a level of protection that is approximately three times greater than the most conservative of the internal models. This cushion ends up being reflected in the performance of those cases in which losses surpass CR. The version with three volatilities shows requirement levels near those of the internal models (above all in relation to the historical simulation model), but with significantly better performance.
Table 5 – Average surplus of capital requirements as proportion of portfolio present value, long and short position – 10,000 portfolios totaling 9,070,000 observations – August 7, 2001 to March 31, 2005.

**Table 5 – Average surplus of capital requirements as proportion of portfolio present value, long and short position**

<table>
<thead>
<tr>
<th>Model</th>
<th>Average PV Loss CR (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long Position</td>
<td>64.97% 32.49% 24.75% 16.82% 15.80% 20.49%</td>
</tr>
<tr>
<td>Short Position</td>
<td>69.81% 34.14% 26.08% 17.87% 16.98% 26.98%</td>
</tr>
</tbody>
</table>

1. **MP2972** – Standardized 2972 model: model based on the Circular 2,972, using maximum volatility and multiplication factor varying from 1 to 3.
2. **MP2V** – Standardized model with 2 volatilities: model based on the Circular 2,972, using two volatilities: maximum among maturities 21, 42 e 63 and maximum among maturities 126, 252, 504 e 756; and multiplication factor equal to 3.
3. **MP3V** – Standardized model with 3 volatilities: model based on the Circular 2,972, using three volatilities: maturity 21, maximum among maturities 42, 63 and 126 and maximum among maturities 252, 504 e 756; and multiplication factor equal to 3.
4. **MP7V** – Standardized model with 7 volatilities: model based on the Circular 2,972, using one volatility to each maturity and multiplication factor equal to 3.
5. **MIAE** – Internal model based on EWMA and multiplication factor equal to 3.
6. **MIH** – Internal model based on historic simulation and multiplication factor equal to 3.
4. Conclusions

The purpose of this paper is to evaluate alternative ways of adapting the structure defined by Central Bank of Brazil Circular 2, 972, dated March 20, 2000, (calculation of capital requirements for market risk of operations in real based on fixed interest rates) to operations involving exchange rate coupon in the period from August 2001 to March 2005. Considering the possibility that the increased volatility to which the US dollar interest rate is subject in Brazil, compared to the interest-rate in real, coupled with the parameters defined by Circular 2,972, may result in significant capital requirements in certain periods, this paper tries to adapt the formula for calculating requirements in this environment. Alterations in the procedure for obtaining volatility and in the multiplication factor are analyzed. A parametric model of VaR based on exponential smoothing and a nonparametric model based on empirical quantile, both of which are applied to the internal models approach put forward by the Basel Accord, are utilized as benchmarks for comparative purposes. The models are evaluated on the basis of the performance of the one-day VaR, in the face of daily losses, and of capital requirements in relation to cumulative 10-day losses, calculated for ten thousand simulated fixed income portfolios.

The results for VaR indicate sharp contrasts among the various models. The original Circular 2,972 model is excessively conservative, with 0.02% of exceptions for the long positions and 0.01% for the short positions, while the expected proportion of failings would normally be in the range of 1%. This is directly reflected in the outcomes of application of the backtesting procedure adopted by the Basel Committee, in which this model remains in the green zone during the entire period analyzed, despite the sharp fluctuations registered by the foreign exchange market in the period. This behavior resulted from the procedure for determining volatility in which, among the volatilities calculated for the seven maturities of the exchange rate coupon yield curve, maximum volatility is used to calculate the VaR of all of the maturities.

Utilization of a conservative procedure to determine capital requirements for standardized approach models is basically a consequence of the need for encompassing all of the risk profiles within a single framework. Nonetheless, the behavior of the volatilities calculated for the seven maturities considered shows severe fluctuations in the mid-2002 pre-electoral period, with highly significant values for 21 and
42-day terms. This fact is a consequence of the distance between forward exchange rate compared to the spot market price, particularly between end-July and early August, when the forward exchange rate moved to more than 10% below the spot price, thus generating significant expectations of devaluation with particularly strong impacts on the shorter-term maturities. In this framework, utilization of the maximum volatility in all of the maturities can generate considerable distortions. Adaptations of the original model that used maximum volatilities of two and three groups of maturities eliminated most of these distortions, at the same time in which they performed in a manner compatible with what is expected of a standard model with satisfactory balance between efficacy and efficiency. The use of seven volatilities in the standard model brought it so close to the internal approach models that it could no longer be considered standard.

The performance of capital requirements shows that the original Circular 2,972 model has practically no exceptions. The adaptation alternatives of the standard model based on two and three groups of maturities also register low rates of exceptions, despite lesser average capital requirements. This suggests that the original model is not correctly designed for the exchange rate coupon. Just as occurred with VaR, the version using seven volatilities registered a performance quite similar to the internal models. The same tendency is found to exist in the distribution of the values in which average losses exceed the respective capital requirements. For the original Circular 2,972 model, there is greater concentration to the left. In other words, aside from lesser exceptions, the shortfalls in capital requirements are, on average, smaller than in the other models. It is important to stress that, once again, the performance of adaptations based on two and three groups of maturities is quite close to that of the original version, with significantly lower requirements. Analogously, when losses are less than required capital (inexistence of exceptions), the average excess of capital requirements in relation to the present value of the portfolio for the Circular 2,972 model is significantly greater when compared with the two adaptations. This suggests the occurrence of an excessive protection cushion.

In general, it is feasible to use the Circular 2,972 model on the exchange rate coupon market, provided that certain adaptations be made in order to take due account of the peculiarities of the exchange rate coupon yield curve, particularly the sensitivity of shorter-term maturity rates to abrupt changes in expectations of floating exchange. As a matter of fact, sharp volatility fluctuations in the
period analyzed, coupled with the procedure for obtaining volatility in order to calculate VaR, result in significantly high capital requirement levels when the Circular 2,972 structure is used in its original form. Of the adaptations implemented, the most consistent outcomes were achieved through utilization of a multiplication factor set at three and maximum volatilities per group of maturities. One should highlight the performance of the version that uses three groups of maturities, which combined lesser levels of capital requirements with the conservatism demanded of a standardized model.
Bibliographic references


BASEL COMMITTEE ON BANKING SUPERVISION. Amendment to the capital accord to incorporate market risks. January 1996-a.

________________________. *Supervisory framework for the use of “backtesting” in conjunction with the internal models approach to market risk capital requirements*. January 1996-b.


Summary

Using the Boot and Thakor model (1993), this paper analyzes the parameter interval related to the reputation of the regulator at which bank closures could reflect “imperfect monitoring” of bank assets. If the regulator is non-benevolent or, in other words, one that maximizes a function composed of social welfare and his/her own reputation, the authors demonstrated that the optimal policy (on the part of the regulator) regarding bank closures is less restrictive – in the sense of increasing the risk level borne by banks – than the socially optimal policy when the regulator maximizes only social welfare. However, this result is only valid for a specific parameter interval. When this occurs, measures such as imposition of clearer rules governing bank closures aimed at reducing the discretionary power of the regulator, as well as separation of the functions of asset monitoring, on the one hand, and responsibility for declaring a bank insolvent, on the other, should be considered. Such measures could stimulate banks to make better portfolio choices and, in this way, reduce systemic risk in the banking system.
1. Introduction

Regulations can be analyzed from a positive or normative approach. In the first approach, the purpose of regulation is to reduce losses of welfare generated by market failings. The second approach provides an explanation for regulation that considers the economic behavior of the regulators. According to this approach, if regulators are interested in generating monopoly incomes, it is possible that regulation will provoke negative impacts on social welfare in some specific situations.

In light of the various financial crises that have occurred in a variety of countries in recent years, proposals for improving banking regulation have become increasingly more common. However, as Dewatripont and Tirole (1994) observe, there is no theoretical consensus on why, how and even whether banks should be regulated. It is their understanding that regulation is necessary basically to protect small-scale investors. As is well known, banks are subject to problems of moral hazard and adverse selection. In this way, investors are obligated to perform a series of highly complex monitoring functions (screening, auditing, intervention etc.) that tend to generate a “natural monopoly” due to their very high costs.

Bank creditors are primarily small depositors who have neither the wherewithal nor the sophistication to understand balance sheets. The absence of individual incentives to perform the task of monitoring generates the problem of the “free rider” or, in other words, necessary utilization of public or private representatives of depositors. In summary, the justification put forward for bank regulation by the authors centers on what they call the “hypothesis of representation”, a hypothesis that, to a certain extent, denies that banks are special and focuses on the structure of bank controls and the question of representation of small depositors. In the opinion of the authors, the specificities of a bank should be sought more in quantitative than in qualitative terms.

Though some authors take the position that bank regulation is not socially beneficial (for example, Benston and Kaufman, 1996), all societies regulate financial institutions. Regulation services generate net social value when the transactional convenience and confidence of financial system clients

---

56/ In this paper, the terms “banks” and “financial institutions” are used interchangeably.

57/ The authors are not concerned here with the possibility of regulation diminishing systemic risk. Their analysis uses the point of view of an individual bank and not the externalities provoked by a bank failure. As is already known, one of the major functions of a bank is that of providing “liquidity insurance” to depositors, as put forward by Diamond and Dybvig (1983). Furthermore, the banks perform a unique task of screening and monitoring debtors who are unable to obtain financing directly on financial markets. It is a combination of these functions that generates bank fragility.
are obtained at low cost. Acting as representatives and disinterested parties, regulators can minimize coordination costs among creditors/taxpayers by monitoring information flows to clients, harmonizing intercompany and network transactions and standardizing contract protocols, the performance of which should be guarantied by regulators. In this way, one can view financial services as offered jointly by financial institutions and their regulators.

In general terms, the regulatory instruments used for purposes of financial system security and stability in the banking industry are (Rochet and Freixas, 1998): 1) ceiling on interest rates paid on deposits; 2) restrictions on entries, mergers etc.; 3) portfolio restrictions, including reserve requirements; 4) deposit insurance; 5) capital requirements; and 6) regulatory monitoring, including bank closure policy.

This paper deals with part of the final item. Analyses were carried out recently with the purpose of defining the incentives that regulators have to close banks in such a way as to discipline the ex-ante choice of portfolios by their managers and stockholders. The visible advantage that regulators have in monitoring and controlling, compared to the discipline imposed by a bank failure, is that, on one hand, they avoid duplication of monitoring costs among the various agents involved and, on the other, diminish the government costs incurred in managing financial crises.

While such regulatory policies as capital requirements and deposit insurance are aimed at limiting the risks of banks coming to have “problems”, bank closure policy operates in an entirely different way. Using the principle of reverse induction in a highly simplified manner, the “how and under what circumstances” banks expect to be closed influences the choice of their asset portfolios before being closed. Thus, an efficient bank closure policy can have the impact of encouraging an optimal portfolio management policy on the part of the institutions involved. However, bank regulation can have negative impacts by generating income for the banks; if the regulators are not totally benevolent\(^{59}\), they can be preempted by the banking industry.

The objective of this study is to summarize the different models that incorporate bank closure policies and, utilizing

---

58\(^{58}\) In this paper, “closure” of a bank has a broader connotation than normal. It means any process involving partial sale of assets, liquidation or even a forced merger with another institution.

59\(^{59}\) The non-benevolent regulator is the one who maximizes an objective function where there exists a weight different from zero given to purely personal interests (in this case, reputation) or, in other words, the one not interested only in maximizing the social aspect, such as the case of the benevolent regulator.
the Boot and Thakor (1993) model, verify the interval for ex-post beliefs regarding the quality of the regulator for whom it is better not to close a bank, when his reputation is taken into consideration. While the authors demonstrate that the optimal bank closure policy from the point of view of the non-benevolent regulator is less restrictive - in the sense of increasing the level of risk borne by banks - than the socially optimal policy that maximizes social welfare, they do not clarify the relevant interval of the ex-post parameters of the regulator’s reputation that would generate this result. At the same time, policy considerations will be made as found in the various authors summarized in this paper in an effort to aid in reducing the problems generated by incorporation of the regulator’s reputation into his objective function. By adopting these policies, one supposes that banks will make better portfolio choices and thereby further reduce systemic risk in the Brazilian banking system.

The paper is organized as follows. Section 2 presents a brief discussion on the political economy of regulation. Section 3 provides a numerical example drawn from Freixas (1999) in order to clarify the specificity that the regulator has to cope with in regulating the banking industry. Section 4 presents a summary of the literature, emphasizing three commonly utilized models with characteristics that will be incorporated into Section 5. Section 5 describes and analyzes the Boot and Thakor (1993) model, clarifying the parameter space in the reputation of the regulator on the basis of which the conclusions drawn by the authors regarding bank closure policy are shown to be valid. The conclusions of this study are presented in section 6.

2. The political economy of regulation: brief discussion

In general, regulation is a process in which rules are created and enforced. Enforcement is a question of monitoring and obligating the “regulated institution” to follow such rules. It is well known that a financial institution in today’s society is an organization that “produces” fund management, generates

60/ Dewatripont and Tirole (1994) argue that a regulator who maximizes social welfare is very passive, since “discipline in the banking industry demands that the regulator maximize the value of deposits and not ex-post social welfare” (pg. 194). It happens that the regulator becomes even more passive when his objective function places greater weight on the welfare of stockholders and managers than on that of depositors, instead of placing greater weight on the surpluses of all the agents involved.

61/ Closer to the socially optimal.

62/ “Systemic risk” is understood as the risk of contagion or, in other words, as the probability that the failure of one financial institution will generate successive failures in a considerable number of other financial institutions.
informational and transactional subproducts for a client base that has a “repeated relationship” with the institution. It is as if clients had a contract with the institution that obligates counterparties to exchange a composite of information, services and cash flows today and on specific dates in the future. In final analysis and in a rather simplified manner, the value aggregated by the bank is derived from the fact that it seeks, compiles, verifies and processes information on investment projects and on the credit risk of the counterparties.

The objective of any financial regulator is to restrict or, more precisely, to impose limits on financial institutions in their relations with clients and, in this way, supplement the limits imposed by the competition. In order to impose limits on the behavior of banks, the regulator must focus on information flows and contracts between institutions and their clients.

On the one hand, regulation must attempt to minimize the opportunity cost of producing the so-called “regulatory benefits”. In other words, the outlays on monitoring and ensuring compliance with rules must be worth the benefits generated by avoiding future financial crises. The optimal equilibrium between the costs and benefits of regulation can be understood as that which would be obtained if it were possible to perfectly align the incentives of the regulator with those of the taxpayers. However, in performing the task of regulation, the regulator has to cope with at least two potentially large conflicts of incentives. In the first place, society very often attributes more than a single mission to a regulatory agency and these missions sometimes conflict one with another63, thus demanding that the regulatory agency prioritize its missions. Secondly, the other parties involved may often retaliate against the bureaucratic and personal interests of regulators when the latter seek to fulfill their social mission.

Viewed in terms of the principal-agent framework, one can state that the principals are the taxpayers (following the hypothesis of representation put forward by Tirole and Dewatripont, 1994), for whom elected politicians and regulatory agents (both public and private) act as trustees. Consequently, both are obligated to protect the interests of the principal party. Personal interests (the so-called ego rents) can lead agents to deviate from the correct path. This becomes even more serious when one considers that coordination costs for taxpayers and even the legislation itself tend to isolate taxpayers from regulators64.

63/ In the Brazilian case, the Central Bank is obligated to ensure the purchasing power of national currency and financial system stability. There have been occasions in which these missions conflicted with each other (for example, in the case of Proer).

64/ In Brazil, this isolation has diminished. In the specific case of bank regulation and the rules covering bank closures, the fact that the regulator who closes a bank may have to bear court costs consequent upon his decision may result in a situation in which the tendency not to close the bank may well be stronger, as will be seen further on.
By mediating financial information flows between real or potential counterparties, banks not only collect, verify and analyze information, but also move the information through internal and external communications networks and these networks must be maintained with high standards of security, confidentiality and integrity.

Financial regulation must be understood as a business relationship, in the same way as contracts involving financial services. Regulated institutions and their regulators formalize a contract to perform certain types of “repeated business operations” among themselves for an undetermined period of time. Though clients impose a certain degree of discipline on “high-cost” regulators by migrating their business operations into other products regulated by “low-cost” regulators, regulation activity does not necessarily have to be efficient over the short-term, since the parties cannot renegotiate their contracts whenever they so desire.

The fact that the regulator may contribute to cartel-type behavior among regulators subjects the regulator to political pressures aimed at allowing incumbent institutions to exercise monopoly power. The Figure below was drawn from Kane (2002) and shows the pressures under which regulators operate:

![Regulation Channels Diagram](image-url)

Source: Kane (2002)
On the supply side of the regulatory relationship, regulators are reluctant to recognize that conflicts of incentives impact their political decisions. On the demand side, the regulated parties seek to influence regulators in order to make possible to grant favors within the framework generated by regulations. Furthermore, since the relationship between taxpayers and regulators is often not particularly close, the pressures brought to bear by the financial sector on politicians and regulators frequently aggravate the conflict of incentives even further.

3. A numerical example to aid in understanding banks’ “incentive to risk”

Literature on the subject of bank regulation shows that banks tend to assume greater risks than would be socially optimal, because they are subject to limited liability, a fact that, in itself, may well justify the need for regulation. Consequently, stockholders of banks financed by deposits, independently of whether they are insured or not, would be encouraged to assume greater risk than they would if they were financed totally by their own capital.

In order to understand the nature of the problem of risk assumption, which is directly related to bank closure policy, we will present an illustration based on a numerical example drawn from Freixas (1999). In the example, the regulator must choose between liquidation of a bank or bailing in out. Without taking any personal interests into account, the regulator considers the total expected cost of salvaging the bank vis-à-vis its liquidation costs. In this example, the bank has three headings in the liability column: deposits from the public, which are insured65, uninsured debt and stocks. The assets of the banks in question have an initial value of $100, which would drop to $80 in the case of intervention and sale and just $50 were the bank to be liquidated. Consequently, the expected differential value, which would reflect the bank’s growth opportunities, comes to $30. If the institution continues operating, the regulator has to compensate the bank’s creditors, in such a way that the holders of the uninsured debt would receive an amount equal to the face value of that debt.

65/ In the Brazilian case, the Credit Guarantee Fund (FGC) was created toward the end of 1995 and, since introduction of the Real Plan in July 1994, has provided coverage of roughly R$20,000.00 per depositor on bank deposits, Bank Deposit Certificates and savings accounts.
The central point that will influence the choice of assets by banks in a model of moral hazard and incomplete contracts is that the cost of liquidating or bailing out a bank will depend on its liabilities structure. Let’s assume that the shares of the bank have book value of $10 and that there are two different funding strategies: i) that in which the debt is composed 100% of deposits and the bank has insured deposits totaling $90; ii) that in which the bank has $45 in insured deposits and $45 in uninsured debt. Assuming that there are no administrative costs, the regulator will have the following costs (net of liabilities on paid-in assets):

<table>
<thead>
<tr>
<th>Strategy/Type of Intervention</th>
<th>Liquidation</th>
<th>Bailout</th>
</tr>
</thead>
<tbody>
<tr>
<td>90% of insured deposits</td>
<td>90 - 50 = 40</td>
<td>90 - 80 = 10</td>
</tr>
<tr>
<td>45% of insured deposits and 45% of uninsured deposits</td>
<td>(45 - 50) = 0</td>
<td>90 - 80 = 10</td>
</tr>
</tbody>
</table>

Source: Freixas (1999)

In this way, since the regulator does not have to offset the uninsured debt, the costs would be smaller if the bank is liquidated in those cases in which 50% of the bank’s debt is composed of uninsured resources. In other words, assuming that the regulator will seek the lowest cost solution, the regulator will bail the bank out in case i) and will liquidate it in case ii).

This simple example shows that the structure of a bank’s liabilities is an element of crucial importance to decisions to be taken by the regulator. Furthermore, it shows that the returns on the uninsured debt expected by creditors also depend on the bank’s liability structure. Consequently, the costs of the institution financing itself externally through debt is nonlinear, because creditors will believe that the regulator will bail the bank out up to a specified limit (in the example, $60 in deposits and $30 in uninsured debt). Beyond this limit, the regulator will liquidate the bank and the ex-ante costs of the bank financing itself through uninsured debt will be greater.

4. Simulating Brazilian banks

Today, there is a certain consensus that it is difficult for regulators to close banks. The paper by Goodhart and Schoenmaker (1995), for example, supports the hypothesis.
that liquidation of a bank is not the rule but rather the exception. In this paper, the authors gather information on bank bailout policies from various countries. A sampling is constructed composed of 104 insolvent banks, resulting in 73 bailouts and liquidation of just 31 institutions.

In practical terms, resolution of financial institution problems is a highly complex process that may require joint intervention by various regulatory agencies, for purposes of providing liquidity and even managing the crisis. Moreover, intervention can take varied forms\textsuperscript{66}.

This question becomes even more complex when one discovers that there is no clear answer as to when a regulator should close a bank\textsuperscript{67}. Many studies have been carried out in this field. For example, Acharya and Dreyfus (1989) developed an excellent rule for bank closures in times of financial crisis. They assume that the rule is such that it minimizes government financial liabilities composed of: i) the current value of the losses incurred by banks in cases of bankruptcy; ii) the current value of monitoring and auditing a bank minus the premium paid for deposit insurance. In other words, regulators select an optimal bank closure policy that equalizes bankruptcy costs, including the costs of such externalities as financial system rupture, and the costs of monitoring the bank should it continue to operate. In this context, the authors agree with the idea of “prompt corrective action” and recommend that banks be closed while they still possess positive net worth. In the model put forward by the authors, temporary forbearance regarding compliance with the rules imposed by the regulator and that would normally result in liquidation of the bank is always a less than optimal policy.

The authors also discuss the problem of the credibility of closure threats. This subject is also debated by Mailath and Mester (1994). The absence of credible policies comes up because one of the major problems that the regulator must cope with is that of adverse incentives to banks nearing insolvency, leading them to invest in excessively risky assets (gambling for resurrection). Regulators should close a bank whenever its assets become overly risky, since threats of closure may often be meaningless.

\textsuperscript{66} For example, injection of short-term liquidity, bank nationalization, segregation of “good banks” and “bad banks”, transfer of stock control, and so forth.

\textsuperscript{67} In the USA, a policy of Prompt Corrective Action - PCA was implemented in the wake of the Savings and Loan Associations episode in the 1980s. In simplified terms, this approach demands that the regulator impose serious restrictions on the bank when its capital begins to decline and that the regulator close the bank when its capital falls below certain critical limits.
Spiegel and Kasa (1999) study the effects of the incentives that can be generated by bank closure policies in terms of “best portfolio choices”. Provided that bank shares are not observed, bank closure policies must be based on effective results, forcing the regulator to cope with a sign extraction problem. For reasons of incentives, an efficient policy should try to distinguish between banks undergoing problems as a result of their own actions, those resulting from adverse incentives, and banks that have had to cope with a run of bad luck as a result of idiosyncratic shocks. While the policy of “prompt corrective action” may discourage problems of moral hazard, it may also lead banks to become excessively cautious when such shocks occur. Therefore, to separate banks that are in poor financial situations on the basis of adverse incentives from those that have experienced bad luck may lead to the same level of risk for the banking industry at a lower expected cost. Based on this idea, the authors seek to define a bank closure policy that more efficiently fosters this separation, focusing on that based on indicators of management efforts.

Despite countless studies already published on this subject, there are two basic approaches to developing a bank closure policy. The first refers to the type of governance structure that can stimulate the bank manager to improve the quality of the institution’s loans (branch problems). The second approach investigates whether threats of bank closures generate greater discipline among banks or, in other words, if such threats induce banks to behave in such a way as to reduce risks incurred, particularly considering the question of limited liability. We will now go on to a detailed description of two widely utilized the models: the Tirole and Dewatripont model (1994), which adopts the first approach and assumes incomplete contracts, and the Mailath and Mester (1994) model, which utilizes the second approach. The third model, known as the Milgrom and Roberts (1982) price-limit model, is also described, since its solution is quite similar to the Boot and Thakor (1993) model.

4.1 The Tirole and Dewatripont (1994) model

The Tirole and Dewatripont (1994) model possesses three dates. On \( t = 0 \), the bank balance which, on the asset side, is composed of loans \( (L_t) \) and, on the liability side, is composed of deposits \( (D_t) \) and shares \( (E_t) \), is given. The right of control is also initially allocated\(^{68}\). The manager can improve loan...
quality by making an effort that costs $K$. The problem, therefore, is to try to generate incentives for the manager to make a strong effort, which, by assumption, is the solution to the problem of complete contracts\(^6\) (which would be the first best rule). The incentives to the manager are related to allocation of rights of control between the regulator, representing depositors, and stockholders.

On $t = 1$, two bits of information on loan quality are revealed. In other words, $\pi = \nu + \eta$ the final still unobserved profit. Therefore, even on $t = 1$, a first repayment, $\nu$, is obtained from the loans and a sign $u \in [u, \tilde{u}]$ is observed for its future liquidation value ($\eta$) on date $t = 2$. Both $u$ and $\nu$ are independent, but are related to the effort level ($e$) employed by the manager chosen in this period. Therefore, if at the end of $t = 1$, $\nu$ is invested at a rate of return on the asset without risk, normalized to 0\(^%\)\(^7\), the value of the bank liquidation will be equal to $\pi$. After observing $u$ and $\nu$, the party that holds the right of control over the bank – the Board of Directors or the regulator – decides whether the bank will continue operating (C) or whether it will be reorganized (S)\(^7\). This action determines the cumulative distribution of probability of $\eta$, conditioned to $u$: $H_A(\eta/u)$, in which $A \in \{C, S\}$. Observe that $(u, \nu)$ is realized in this period.

On $t = 2$, the value of liquidation $\pi$ is realized. The central point is that action $A$ cannot be specified in a contract and, consequently, the definition of who will control the bank on $t = 1$, which will be made through regulation of solvency, takes on crucial importance. For purposes of simplicity, the authors presume that monetary incentives cannot be given to the manager. The incentives for the manager to make a strong effort can be given indirectly through the threat of bank closure, in which case the manager will be fired, losing his right to benefit B.

Based on complete information, since $u$ and $\nu$ are independent, the optimal action will depend only on $u$. The expected incremental profit $\Delta(u)$ of continuing on $t = 1$, conditional to $u$, is:

$$\Delta(u) = E[\eta/u, C] - E[\eta/u, S],$$

which is equal to:

$$\Delta(u) = \int_0^\infty \eta dH_C(\eta/u) - \int_0^\infty \eta dH_S(\eta/u).$$

If we integrate $\Delta(u)$ by parts, we will have:

\(^6\) In other words, contracts that are potentially contingent upon all possible future states of this type.

\(^7\) The rate of return is normalized to 0\% exclusively for purposes of simplification.

\(^7\) The S would correspond to what we call the bank closure.
\[ \Delta(u) = \int_{0}^{\infty} \{H_x(\eta/u) - H_x(\eta/u)\} d\eta. \]

Thus, C is optimal on the basis of complete information if, and only if, \( \Delta(u) \geq 0 \). For purposes of simplification, the authors presume that \( \Delta'(\cdot) \geq 0 \), in such a way that the first best rule can be described as: play C if \( u \geq \bar{u} \) and play S if \( u < \bar{u} \), in which \( \bar{u} \) is such that \( \Delta(\bar{u}) = 0 \).

In the model, the level of the manager’s effort, which can assume just two values (\( e = \bar{e} \) or \( e = \epsilon \)), is not observable. The variables \( u \) and \( \nu \), however, are positively correlated with “\( e \)”, in the sense that larger realizations of \( u \) and \( \nu \), indicate a greater probability that \( e = \bar{e} \). With \( f(u/e) \) and \( g(\nu/e) \) being conditional density functions of \( u \) and \( \nu \) that satisfy the property of the ratio of monotonous likelihood,

meaning that \( \frac{f(\nu/\bar{e})}{f(\nu/\epsilon)} \) and \( \frac{g(\nu/\bar{e})}{g(\nu/\epsilon)} \) are increasing. With \( x(u, \nu) \) the probability of C when \( (u, \nu) \) is observed. The second-best decision rule is obtained by maximizing the expected profit (incremental) should the bank continue operating:

\[ B \int \int x(u, \nu) \Delta(u) f(u/\bar{e}) g(\nu/\bar{e}) \, du \, dv. \]

Subject to the compatibility restriction with incentives:\[ 73 \] in which \( x(u, \nu) = 1 \) if C and \( x(u, \nu) = 0 \) if S. The Lagrangean program is:

\[ L = \int \int x(u, \nu) \{(\Delta(u) + \mu B) f(u/\bar{e}) g(\nu/\bar{e}) - \mu B f(u/\epsilon) g(\nu/\epsilon)\} \, du \, dv - \mu K. \]

Therefore, maximization of \( L \) with regard to \( x(u/\nu) \in [0,1] \) provides the second-best decision rule:

\[
\begin{align*}
  x(u/\nu) &= 1 \quad \text{if } \Delta(u) + \mu B \geq \frac{f(u/\bar{e}) g(\nu/\bar{e})}{f(u/\epsilon) g(\nu/\epsilon)} \\
  x(u/\nu) &= 0 \quad \text{should the contrary occur.}
\end{align*}
\]

Or, in other words, in the case of incomplete information, it is optimal to continue if:

\[
\frac{f(u/\bar{e})}{f(u/\epsilon)} \left(1 + \frac{\Delta(u)}{\mu B}\right) \geq \frac{g(\nu/\bar{e})}{g(\nu/\epsilon)}. \tag{1}
\]

72/ Without losing generality, the authors assume that the bank will continue operating if \( \Delta(u) = 0 \)
73/ In reality, the authors posited restriction of compatibility with incentives and individual rationality in the same inequality.
Resolving the optimal program, the authors define $u^* (\nu)$ as the value of $u$ in such a way that condition (1) is satisfied with equality, for a given value of $\nu$. Due to the property of the rate of monotonous likelihood, the right side of (1) is growing in $u$, and activity $C$ will be optimal if, and only if $u \geq u^* (\nu)$. For the same reason, since the left side of (1) is decreasing in $\nu$, function $u^* (.)$ is decreasing.

Define $\hat{\nu}$ implicitly as $u^* (\hat{\nu}) = \hat{u}$. The figure below summarizes the differences between the first best decision rule and the second best:

For $\nu > \hat{\nu}$, there are values of $u \in (u^* (\nu), \hat{u})$ for which the regulator allows the bank to continue operating, though the ex-post efficiency would suggest that the bank should be closed. On the other hand, for $\nu < \hat{\nu}$, there are values of $u \in (\hat{u}, u^* (\nu))$ for which the bank is closed, though ex-post efficiency would imply that the bank should continue operating.

Since the payoff of the bank’s shares is a convex function in the bank’s profits, the stockholders tend to favor more risky decisions. In much the same way, since the payoff of deposits is a concave function in bank profits, depositors tend to favor less risky decisions. Therefore, in the framework of the hypothesis that it is less risky to close a bank than to allow it to continue operating, the rights of control over the bank should be given to the stockholders whenever $\nu \geq \hat{\nu}$. Symmetrically, the rights of control over the bank should be giving to the depositors, meaning that they should be attributed to the regulator whenever $\nu < \hat{\nu}$.

Implementation of the second best optimal decision rule can be obtained through voluntary recapitalization, net worth adjustments, etc.
This model can be extended on introducing a monitoring activity as follows: the manager chooses three possible levels of effort \( e = e_1; e = e_2 \text{ ou } e = e_3 \). Efforts \( e = e_1 \text{ ou } e = e_2 \) have the same cost and the same effect as discussed in the previous model. The new effort \( e = e_3 \) generates the same distribution of \( u \) as effort \( e = e_2 \), while generating worse distribution for \( u \): in the relevant interval, \( G(u^*(v)) > \tilde{G}(u^*(v)) \). By way of hypothesis, on \( t = 1 \), effort \( e_3 \) generates a high private benefit (greater than \( B \)), in such a way that the manager chooses \( e_3 \) unless the regulator does not permit it. Suppose now that there are two types of regulators, the competent regulator and the incompetent regulator. The a priori probability that the regulator will be competent is \( \alpha \). A competent regulator identifies effort \( e_3 \) without cost and, in this way, can reduce the effort of the manager for \( \in \{e_1, e_2\} \), inducing a high effort under the system of incentives of the model under consideration. The incompetent regulator cannot identify \( e_3 \), meaning that he must allow the manager to choose \( e_3 \).

Initially, let’s assume that the regulator ignores his career and implements the optimal policy defined by the limit rule \( u^*(.) \). Observe here that \( u^*(.) \) is obtained just as in the specified model, with the only difference being that now, with probability \( (1-\alpha) \), the manager will select action \( e_3 \). Suppose also that taxpayers observe the regulator’s decision of intervening or not, but are not aware of information \( u \) on which that decision is based. Therefore, if the regulator intervenes, the probability that it will be competent is:

\[
\beta = \frac{\alpha \tilde{G}(u^*(v))}{\alpha \tilde{G}(u^*(v)) + (1-\alpha) \tilde{G}(u^*(v))}
\]

and if it does not intervene:

\[
\beta' = \frac{\alpha [1-\tilde{G}(u^*(v))]}{\alpha [1-\tilde{G}(u^*(v))] + (1-\alpha) [1-\tilde{G}(u^*(v))]}.
\]

Since \( \alpha > 0 \), \( \beta' > \beta \) \( e \), the regulator will not intervene if, in its objective function, there is a weight for its reputation that is different from zero.\(^74\)

### 4.2 The Mailath and Mester model (1994)

The Mailath and Mester (1994) model analyzes threats of bank closures that, with perfect information, are credible. The central question is how the regulator can utilize his power.

---

\( ^{74} \) The hypotheses for this affirmation are lacking. This paper shows that in section 5.
of bank closure in a credible manner, in order to induce that institution to avoid an excessive risk. Using a dynamic two period system, the authors model this question by seeking a perfect equilibrium subgame in the incomplete contract approach, in which the bank and the regulator cannot commit themselves to a specific path of action in the future.

The model is as follows. The banks receive a unit of deposit ($1) and, in period $t=1$, choose the risk level of their assets, which can be safe (S) or at risk (R). Then, in period $t=2$, the regulator, on observing the risk level of the bank’s assets, decides whether to close the bank (C), in which case there is a fixed cost “$c$” or to allow the bank to continue operating (O). If the bank does not “die”, he will play again on $t=2$, choosing between a risky or safe investment. Aside from this, he will once again receive a unit of deposit. On $t=3$, both assets mature and, for this reason, the rate of return of the first asset can be received only if the bank is not closed in the second period. The projects are bank specific or, in other words, if a specific bank does not finance a project, it will not be financed by any other bank.

For reasons of simplicity, the authors assume that the rate of interest on deposits is equal to zero and, should the bank be liquidated, the regulator will effect full payment of all deposits. With these hypotheses, the expected profit of banks is their return on assets less principal. The net return of the safe assets is $r^s(0<r^s<1)$ with probability 1, in such a way that the bank cannot pay depositors if one of the projects is not successful, since $1$ is required for each project. The gross return on the asset with risk is $(r^r+1)^7$ with probability p and 0 with probability $(1-p)$. There is a conflict between the incentives of the regulator and those of the bank since, on the one hand, it is assumed that if the bank were able to make a single choice of assets, it would prefer the asset with risk, since $r^r>r^s$ and the bank has limited liability while, if society were able to make the choice, it would prefer the asset without risk on $t=1$ and on $t=2$, because $E(\text{return on the risky asset}) = p(r^r+1)<E(\text{return on asset without risk}) = 1(r^s+1)$. The idea is that, if successful, the risky project would offer a greater ex-post return.

The authors separately introduce two types of regulators: those that maximize social welfare and, therefore, act in the interest of society; and those that minimize cost, including the cost

75/ For reasons of simplicity, differently from the authors, I will not consider administrative cost $c$.
76/ Assume that the return of a single asset is not sufficient to cover losses in another asset.
77/ For purposes of clarity and to further simplify the model, I called $r^s$ and $r^r$ net returns, while the authors utilize them for gross returns.
of bank closure. In the first case, the expected return (bank profit) less the cost of bank closure is maximized while, in the second case, the regulator minimizes this cost or, in other words, the payment to depositors that would have to be made should the bank be closed and the cost of bank closure.

In any case, if banks preferred the safe project, there would be no reason for regulation. Consequently, the authors focus on the case in which, if there were no regulations, banks would choose at least one risky investment, when the two periods are taken together. Let’s summarize the case in which the regulator is the cost minimizing type. For the type that maximizes welfare, the reasoning is identical.

At the start of the game, the bank’s expected return is \( p (r^r + r^s) \) if he chooses (S, R) or (R, S). If the risky asset is chosen twice (R, R), its expected return is \( 2p^2 (r^r) \). Therefore, the banks will strictly prefer (R, S) or (S, R) to (R, R) if, and only if, \( p (r^r + r^s) > 2p^2 (r^r) \).

In the first place, the authors analyze the situation in which the bank strictly prefers (S, R) or (R, S) to (R, R), which is case 1. This is the case described above, in which \( r^s \) is “high” in relation to rate \( r^r \) and to the probability of success \( p \) or, in other words, \( r^s > (2p-1)r^r \). Supposing a certain combination of parameters, should the regulator note that the bank has chosen S in the first period, he knows that the bank will choose R in the second period. If the cost of bank closure is not very high or, more specifically, if \( pc < (1-p) (1-rs) \), it will do so on \( t = 1 \). In this situation, the bank knows that, on choosing S in the first period, it will be punished with closure and, in this way, the threat of closure only has the effect of making it shift from (S, R) to (R, S), which does not result in any gain in terms of reducing the risk assumed by the bank. The solution to the game will be the bank always playing (R, S) and the regulator playing (O).

If, however, \( r^s \leq (2p-1)r^r \), in such a way that the bank weakly prefers (R, R) to (S, R) or (R, S) (case 2), no matter what the strategy observed in period 1, the regulator considers the possible choice of R in period 2, given that he will observe R or S. Therefore, if the bank chooses R on \( t = 1 \), the expected total cost of bank closure, which is the expected cost of payment to depositors plus fixed closure costs, will be

---

78/ Two games, therefore, are resolved: a cost-maximizing regulator and a cost-minimizing regulator.

79/ The fact is that, in the Mailath and Mester study, the probability that assets will generate their respective incomes is different in each period or, in other words, there is \( p_1 \) and \( p_2 \). Equal probabilities were utilized only for purposes of simplification.

80/ Observe that, if the bank plays R in the first place, the regulator will never close the bank since he knows that he will play S in the following period.
If the bank remains open, its cost will be $1 \times (1-p) + c$. If the bank remains open, its cost will be $2(1-p)^2 + 2(1-p) p (1- r^r) + c (1-p^2)$.

Therefore, if the cost of bank closure is not very high or, in other words, if $c < \frac{(1-p)(1-2p r^r)}{p^2}$, the regulator will close the bank. If, however, this inequality is not satisfied, the bank will play $(R, R)$ and the regulator will play $(O)$.

Should the inequality above be satisfactory, on $t=1$ the bank must choose between playing $R$ and being liquidated, which will generate a payoff of $(p)r^r$, or playing $S$. Thus the bank will play $R$ on $t = 2$. In this case, if the regulator observes $S$, the regulator will close the bank if the cost of closure - $c$ - is less than the cost expected by allowing the bank to remain open, which is equal to $(1-p) (c + (1- r^r))$. Therefore, if $c < \frac{(1-p)(1-2p r^r)}{p^2}$, the bank knows that it will be closed and, consequently, will choose $R$ on $t = 1$, supposing that $(p)r^r > r^r$.

In case 1, in which there is a low probability of success and, for this reason, $(R, S)$ or $(S, R)$ are preferred to $(R, R)$, the regulator will not close the bank because the threat of closure will induce it to choose $R$ in the first period. Since the second period choice will be $S$, it is optimal from the social point of view to allow the bank to remain open.

Thus, there is an interval of parameters $(p, c)$ for which the bank closure policy is credible and others in which this does not occur. This result also applies to the case in which the regulator maximizes social welfare. More important, if one assumes that the asset invested in the first period is more risky than that invested in the second period, or in other words, that $p_1 < p_2$, then the existence of the regulator may diminish social welfare. The reason is that the balance implies that the bank will never be closed if it chooses $S$ in the second period. For this reason, for certain values of parameters, regulation induces banks to choose $(R, S)$ instead of $(S, R)$.

4.3 The Milgrom e Roberts (1982) Price-Limit model

In the Milgrom e Roberts Price-Limit model (1982), there are two companies, 1 and 2, with company 1 occupying the
position of incumbent. In the simplified version, there are two periods, \( t = 1, 2 \). In the first period, company 1 chooses the price according to its cost, which can be high \((c_H)\) or low \((c_L)\). In the second period, company 2 decides whether it will (E) or will not (NE) enter the market. Before entering the market, company 2 is not familiar with the type of company 1. The distribution of the type of company 1 is commonly known and is given by \( \Pr(c = c_L) = x \). The game of signs could be as shown below:

There are two periods. In the first, company 1 has a market monopoly. In the second period, should company 2 decide to enter the market, there will be duopolistic competition.

Define \( M_1^T(p_1) = (p_1 - c_1^T)D_1^m(p_1) \), in which \( M_1^T(p_1) \) is the profit of the incumbent monopoly company when its price is \( p_1 \); \( T = L \) or \( H \), indicating high or low cost and \( D_1^m(.) \) is the demand function of the monopoly. At the same time, \( M_1^T(p_m) \equiv M_1^T(p_m^T) \), with \( p_m^T \) being the monopoly price charged by the incumbent according to its type, \( D_1^T \) and \( D_2^T \) are the duopolistic profits of companies 1 and 2. Assume that \( M_1^T(p_1) \) is strictly concave in \( p_1 \) and that the decision by company 2 to enter the market is influenced by its beliefs regarding the cost of company 1, in such a way that:

\[
D_2^H > 0 > D_2^L,
\]
or, in other words, with symmetric information, there would be an incentive for company 2 to enter the market if, and only if, the costs of company 1 were high. The discount factor is \( \delta \in (0, 1) \).

\[\text{[\text{Diagram showing game of signs}]}\]

---

82/ It can be demonstrated that \( p_m^L < p_m^H \).
The game is resolved for a perfect Bayesian equilibrium, seeking separation and aggregation equilibrium.

a) Separation equilibrium

When company 1 is an H type company, the best it can do is to charge \( p_m^H \) in the first period, since company 2 is going to enter the market no matter what happens. In this way, its payoff will be \( M_t^H + \delta D_t^H \). With \( p_1^L \) being the price that the company charges if it is the low cost type. Should the high cost company charge this price, it will avoid the entry of company 2 in the separator equilibrium and obtain \( M_t^H (p_1^L) + \delta M_t^H \). Therefore, a necessary condition for the high cost type company not to want to shift into the low cost type is that:

\[
M_t^H - M_t^H (p_1^L) \geq \delta (M_t^H - D_t^H). \quad (2)
\]

In the same way, if it is an L type company and is charging \( p_1^L \), it is assumed that it is maximizing profits. The worst scenario would be that in which it would charge \( p_m^L \) and company 2 would enter the market. Therefore, its minimum payoff is \( M_t^L + \delta D_t^L \). Since, in the separator equilibrium, its payoff would be \( M_t^L (p_1^L) + \delta M_t^L \), the condition required for the low type company is:

\[
M_t^L - M_t^L (p_1^L) \leq \delta (M_t^L - D_t^L). \quad (3)
\]

Suppose further that \( M_t^H (p_m^L) + \delta M_t^H > M_t^H + \delta D_t^H \), for the high cost company not to want to shift into a low-cost company, positing \( p_1^L = p_m^L \).

The reason why it is more costly for the high cost type company to charge a low price is derived from the property of the unit intersection (PIU), according to which:

\[
\frac{\partial [M_t^H (p_1) - M_t^L (p_1)]}{\partial p_1} > 0.
\]

This condition is satisfied because:

\[
\frac{\partial^2 [(p_1 - c_i) D_i^m (p_1)]}{\partial p_1 \partial c_i} = - \frac{dD_i^m}{dp_1} > 0.
\]

Therefore, on defining \( y = M_t^L - M_t^L (p_1^L) \) and \( y = M_t^H - M_t^H (p_1^L) \), by the PIU, the curves will only cross once in space \( \{p_1^L, y\} \).

Using the theorem of the Envelope to obtain

\[
\frac{d[M_t (c_i) - D_t (c_i)]}{dc_i} = 0.
\]
\[
\frac{d}{dp_1} \left( \max_{p_1}[(p_1 - c_1)\text{D}_1^m] - \max_{p_1}[\text{D}_1(p_1, p_2^d)] \right) = -\text{D}_1^n(p_1^m) + \text{D}_1(p_1^d, p_2^d) - (p_1^d - c_1) \frac{\partial \text{D}_1}{\partial p_2} \frac{\partial p_2^d}{\partial c_1}.
\]

In which \( p_1^d \) e \( p_2^d \) are the equilibrium prices of the duopoly.

Assuming \( \frac{\partial p_2^d}{\partial c_1} > 0 \) (since \( p_1^d - c_1 > 0 \) and \( \frac{\partial \text{D}_1}{\partial p_2} > 0 \)), the third term of the equality is negative. Therefore, if the monopoly demand of company 1 exceeds its duopoly demand, \( M_1 - \text{D}_1 \) decreases with \( c_1 \) and, in this way, \( M_1^L - \text{D}_1^L > M_1^H - \text{D}_1^H \).

Thus there is a price interval \( p_1^L \in [p_1^L, p_1] \) for the low-cost company and price \( p_1^H \) for the high-cost company that constitute separator equilibriums due to the fact that they satisfy the necessary conditions (2) and (3), which are also sufficient. Outside the path of equilibrium, beliefs may vary.

Let’s go on now to choosing the beliefs that induce company 2 to enter the market or, in other words, when the prices are not above \( \{p_1^H, p_1^L\} \), the later beliefs regarding \( x \) are equal to 0 (company 2 believes that company 1 is the high-cost type).

There is no incentive for companies of both types to deviate from this equilibrium. For the type H company, if it chooses \( p_1 \), its profit will be less no matter what the type of equilibrium (\( \hat{p}_m \)), because company 2 entered the market in both cases. For the type L (2):

\[ M_1^L(p_1^L) + \delta M_1^L \geq M_1^L + \delta \text{D}_1^L \geq M_1^L(p_1) + \delta \text{D}_1^L. \]

Consequently, \( L \) prefers \( p_1^L \) to \( p_1 \).

Researching all of the possible EBP, the most “reasonable” is that in which the type L of company 1 chooses \( p_1 \), since this will generate the highest possible profit for prices in interval \([ p_1^L, p_1] \), without altering the behavior of company 2.

b) Aggregation equilibriums

The existence of an aggregation equilibrium of this type depends on the condition

\[ x\text{D}_2^L + (1-x)\text{D}_2^H < 0. \quad (4) \]

Suppose initially that this condition is not satisfied. In this case, company 2 enters the market. Therefore, the best thing for the type T of company 1 is to charge \( p_1^T \). Since \( p_1^H \neq p_1^T \), this cannot be a pooled equilibrium. Let’s suppose, therefore, that (4) is satisfied. Therefore, for the pooled equilibrium to exist, it is necessary that none of the
types of company 1 prefer to choose the monopoly price, or in other words:

\[ M^L_1(p_1) + \delta M^L_1 \geq M^H_1 + \delta D^L_1 \iff M^L_1 - M^L_1(p1) \leq \delta(M^H_1 - D^L_1) \]  

and, for the high type,

\[ M^H_1(p_1) + \delta M^H_1 \geq M^H_1 + \delta D^H_1 \iff M^H_1 - M^H_1(p1) \leq \delta(M^H_1 - D^H_1) \].

Based on what has been stated above, there must be a neighborhood in the range of \( P^L_m \) that satisfies the two inequalities above.

For sufficient conditions, we will always suppose that company 1 sets a different \( p_1 \) price, company 2 believes that company 1 has high costs. Company 2 enters the market and, consequently, company 1 would play its monopoly price in period 1. Based on the conditions above, however, none of the types want to deviate from \( p_1 \).

5. The non-benevolent regulator model

During the bank crisis in the United States in the 1980s, regulators hesitated before acknowledging that banks were mired in problems. Had they done so, they could have been considered low quality regulators (Dewatripont and Tirole (1994)). When regulators concern themselves with their own reputations, a conflict arises between monitoring and intervention.

In this part, we will include the case of regulation in which the regulator is not benevolent or, in other words, when the regulator is also seeking self-interests. This analysis will follow Boot and Thakor (1993), with the addition of the relevant parameters for the reputation of the regulator, a factor that will certainly impact the solution to the problem. As Greenbaum (1993) stated, the self-interest of the regulator can be modeled by introducing uncertainty into the regulator’s ability to monitor a bank’s choice of assets. This uncertainty creates a situation in which the regulator desires to acquire a reputation as a competent monitor and, therefore, adopts a stance that can distort bank closure policy in relation to what is socially optimal.

5.1 The Boot and Thakor (1993) model

Set-up of the model: There are two periods of time in the Boot and Thakor (1993) model, the first being \( t = 0 \) to \( t = 1 \), and the second being \( t = 1 \) to \( t = 2 \). In \( t = 0 \), the banks have
assets that generated a random amount of earnings \( \tilde{L} \) and nothing as of \( t = 1 \). The random variable \( \tilde{L} \) has the function of continuous cumulative distribution \( F(.) \) and the probability density function \( f(.) \). This function has support \([0, \tilde{L}]\), with \( \tilde{L} > 0 \). Aside from the return of \( \tilde{L} \), the bank can invest in an asset for which it is able to choose the distribution of the probability of return. This asset demands investment of \$1 on \( t = 0 \) and generates a random return (gross) of \( \tilde{V}_1 \) on \( t = 1 \), in which \( \tilde{V}_1 = V(\theta_1) > 0 \) with probability \( \theta_1 \) (success) and zero with probability \( [1 - \theta_1] \) (failing). It is also assumed \( V'(.) < 0^85 \) and that \( V''(.) < 0, \forall \theta \in \Theta \). This investment is financed with \$\( K_1 \) in stocks and \$(1-K_1) in deposits or, in other words, the bank collects deposits and involves itself in residual financing through the use of its capital. On \( t = 1 \), the bank may make a discretionary choice of assets in a similar manner.

The bank’s choice of \( \theta_1 \) is directly observable only by the bank itself and the choice is monitored by the regulator on \( t = 0 \). The quality of the regulator will determine the probability with which the regulator will detect the choices of assets made by banks during the monitoring process. Suppose that there exists a socially optimal \( \theta_1^* \) (which would be chosen on the basis of complete information) in such a way that, if the regulator detects \( \theta_1 \neq \theta_1^* \), the bank is forced to shift to \( \theta_1^* \). If the regulator detects nothing, the bank continues with \( \theta_1 \). The regulator may be high quality (g), with detection probability of \( \rho_g \), or low quality (b), with probability \( \rho_b \), with \( \rho_g > \rho_b \). The type of regulator is private information on \( t = 0 \), but there is ex-ante common knowledge of the probability \( \gamma \in (0,1) \) that the regulator will be good.

On \( t = 1 \), the bank carries out \( \tilde{L} + \tilde{V}_1 \), and period 1 deposits are paid. Then, \( \tilde{L} + \tilde{V}_1 - D_1 \) is the amount that the bank must invest in period 2, in which \( D_1 \) is total deposits collected in period “i”. At the start of \( t = 1 \), the regulator decides whether to close the bank (C), or allow the bank to continue operating for another period (O). If the regulator plays O, deposits are once again raised in the second period in order to ensure that \( \tilde{L} + \tilde{V}_1 - D_1 + D_2 \geq \$1 \). Se \( \tilde{L} + \tilde{V}_1 - D_1 < 0 \), but the regulator plays O, deposits are raised in the second period in order to repay depositors from the first period. If, however, the bank is closed on \( t = 1 \), the government or the institution responsible for deposit insurance covers the negative difference, should such a difference exist.

---

83/ These assets \( \tilde{L} \) are generally viewed as loans made by banks to the companies in question.
84/ In other words, the bank chooses the risk level it wants to incur.
85/ It is postulated that an increase in risk is accompanied by a decline in net current value (NPV). In the hypothesis of mean-preserving spread, however, there is no decline in value when risk increases. In the context of banks, the first hypothesis is commonly utilized. Empirical evidence also supports this hypothesis.
86/ \( D_i \) incorporates earnings on deposits if the rate of return of the asset without risk is different from zero.
On $t = 2$, assets have a gross random return of $\tilde{V}_2$, and after realization of the return on the asset, the depositors are paid. Then, $\tilde{V}_2 = V(\theta_2) > 0$ with probability $\theta_2$ (success) and zero with probability $1-\theta_2$ (failing). If $\tilde{V}_2$ is insufficient to pay depositors, the government or institution responsible for insurance covers the difference. Since banks are going to “die” on $t = 2$, it is assumed that the regulator does not monitor $\theta_2$. Despite the fact that the bank observes its own capital and its choice of assets, the regulator observes the choice of assets made by the bank only when it is able to detect that choice. At the start of each period, the regulator observes the book capital of the bank. In other words, on $t = 1$, the regulator jointly observes $\tilde{L} + \tilde{V}_1$, but not $\tilde{L}$ e $\tilde{V}_1$ individually. On $t = 2$, it is assumed that the regulator observes $\tilde{V}_2$. Aside from monitoring, the regulator must then decide whether or not to close the bank between $t=1$ and $t=2$, taking a decision that will be publicly observed.

**Information structure**: The bank is the best-informed party, since it observes its own capital and its choice of assets. The regulator observes the choice of portfolio only if it detects and forces the change and observes the capital of the bank for each period of time. The market observes only the capital of the bank with a temporal lag. The bank observes parameter $\theta_1$ before making its choice of assets. In principle, the value of $\theta_1$ is not observable either by the regulator or by the market (depositors), and this hinders any contract contingent on the value of $\theta_1$.

The timeline of the game is described below and the game itself is described in the appendix in its extended form:

| $t=0$ | Bank invests $1$, Collecting $(1-K_1)$ from the public. Choose $\theta_1$, Which earns $\tilde{V}_1 = V(\theta_1)$ |
| $t=1$ | Regulator only observes $\theta_1$. When it manages to detect deviations $\tilde{L} + \tilde{V}_1$ is realized and deposits are paid |
| $t=1'$ | Regulator does not monitor $\theta_2$. Regulator observes $\tilde{V}_2 = V(\theta_2)$ |
| $t=2$ | Bank has assets that earn $\tilde{L}$ until $t=1$ |
| $t=1'$ | Regulator decides whether to play $O$ or $C$. If $O$, the bank invests $1$ and collect $1-K_2$. Choose $\theta_2$ |
| $t=2$ | Deposits not paid |
Suppose also that the agents are neutral in relation to risk, and that the bank maximizes its net expected profit. The regulator incorporates its reputation into its objective-function and, therefore, maximizes:

\[ \beta_1 (\gamma_1 + \delta \gamma_2) + \beta_2 \theta_2 V(\theta_2) - 1 \times r^5 \]  (7)

in which \( \gamma_t \) is the reputation of the regulator in periods \( t = 1, 2; \beta_1, \beta_2 \) and \( \delta > 0; [\theta_2 V(\theta_2) - 1] \) is the surplus of the bank in period 2, or, in other words, the expected earnings \( \theta_2 V(\theta_2) \) net of the cost of the investment adjusted by \( r \), which is the gross rate of interest of the asset without risk. The objective function (7) shows that the regulator is maximizing any weighted average of his reputation and social welfare gains. The reputation of the regulator, \( \gamma_t \), is the ex-post belief, on \( t \), that the regulator is type \( g \), or, in other words, it is the probability with which the market perceives that the regulator is good. The social surplus of period 1 is not in the objective function of the regulator because the only decision that the regulator can make is bank closure (or, in other words, there are no other measures that can affect him), and this is already perceived in surplus on \( t = 2 \).

For reasons of sequential rationality, consider the choice of assets in the second period, assuming two simplifying hypotheses:

i) Stockholders of the bank always prefer \( O \) to \( C \);

ii) If the investment that the bank made in the first period was successful, the regulator will never close the bank.

**Proposition 1**: In the second period, the bank will choose assets with a risk level higher than that considered socially optimal.

Proof. The choice of \( \theta_2 \) which would be optimal from the social point of view would be that which:

\[ \max_{\theta_2} \theta_2 V(\theta_2) - (1 \times r^5). \]  (8)

since the socially optimal choice would be equivalent to the choice that the bank would make if it had to finance all of its investments in the second period through the use of its own capital \( K_2 \). And supposing the interior solution, \( \theta_2^* = -V(\theta_2^*)/V_2(\theta_2^*) \), with the second order condition satisfied.

We will now verify what would be the risk level chosen by the bank with capital \( K_2 \) on \( t=2 \). The capital of the bank
in the second period is equal to $\tilde{K}_2 = \tilde{L} + \tilde{V}_1 - (1-K_1) r^1$, in which $(1-K_1) r^1 = D_1$ is the payment made to depositors in the first period. If $\tilde{K}_2$ is negative, the volume of the deposits in period 2 will be greater than in period 1. Let’s assume that $\tilde{K}_2 < 1$ with probability 1 that there are deposits in the second period. Therefore, the bank chooses $\theta_2$ in order to maximize the difference between expected revenues on asset, $\theta_2 V (\theta_2)$ and the debt contracted through deposits collected from the public which is equal to $\theta_2 [1-\tilde{L} + \tilde{V}_1 - (1-K_1) r^1] = \theta_2 [1 - \tilde{K}_2]$, for which it will have to pay $r^1$. From this amount, one should subtract the opportunity cost of $\tilde{K}_2$, which is $\tilde{K}_2 \times r^1$. The bank’s program is:

$$\max_{\theta_2} \{ V (\theta_2) - [1-\tilde{K}_2] r^1 \} - \tilde{K}_2 r^1$$

and the solution is

$$\hat{\theta}_2 (\tilde{K}_2) = (-V'(\hat{\theta}_2) + [1-\tilde{K}_2] r^1) / V'(\hat{\theta}_2).$$

(9)

Assuming the interior solution, the C.P.O. for the socially optimal $\hat{\theta}_2 V (\hat{\theta}_2) = 1 < \theta_2^* V (\theta_2^*) - 1$ and for the private problem, $V (\hat{\theta}_2) = [1-\tilde{K}_2] r^1 + \hat{\theta}_2 V (\hat{\theta}_2) = 0$. But if we substitute $\theta_2^*$ in the place of $\hat{\theta}_2$, we will have

$$V (\theta_2^*) - [1-\tilde{K}_2] r^1 + \theta_2^* V' (\theta_2^*) = -[1-\tilde{K}_2] r^1 < 0.$$ Then, $\theta_2^* > \hat{\theta}_2$.

Note that if $\tilde{K}_2 = 1$, the objective function of the bank becomes $\theta_2 R (\theta_2) - r^1$, equal to (8). Since $\tilde{K}_2 < 1$, less than the socially optimal ($\tilde{K}_2 = 1$), $\hat{\theta}_2 < \theta_2^*$, which implies proposition 1.

This result is close to that of Mailath e Mester (1994) and is derived from the fact that the expected return on the asset with $\theta_2^*$ is greater than with $\hat{\theta}_2$, or, in other words:

$$\hat{\theta}_2 V (\hat{\theta}_2) - 1 < \theta_2^* V (\theta_2^*) - 1$$

But the bank prefers the more risky asset due to the limit on its liabilities and the fact that the cost of the deposits is independent of the risk incurred by the bank or, in other words, the risk to the bank,

---

87/ The second order condition is satisfied and an interior solution is presumed.
88/ Recalling that $\theta_2$ is the probability of success of the asset in the second period.
\[ \hat{\theta}_2 ( V ( \hat{\theta}_2 ) - 1 ) > \theta_2^* (V(\theta_2^*) - 1) . \]

As various authors have stated\(^9\), there is a very simple rule for bank closures: close the bank if the optimal private choice for the second period implies a negative NPV value for the asset portfolio.

Consequently, a critical value must exist for capital in the second period \( \bar{K}_2 \), in such a way that the socially optimal policy of bank closure recommends that the bank be closed if \( \bar{K}_2 < K_2 \) and continues if \( \bar{K}_2 \geq K_2 \), since the choice of assets that the bank will make in the second period, \( \hat{\theta}_2 \), will depend on capital in the second period (equation (9)). Since the regulator does not monitor \( \theta_2 \), he limits the problem of occult action on the part of the bank when \( \bar{K}_2 \) drops below the value that generates NPV < 0.

Given the socially optimal choice of assets \( \theta_1^* \) in the first period, \( \hat{\theta}_1 \in (\theta_1, \theta_0^*) \) is the optimal choice from the point of view of the bank in that period. Suppose that, in the reputation equilibrium, the regulator chooses to close the bank on \( t = 1 \) if \( \bar{L} + \bar{V}_1 < z^* \), in which \( z^* \) is some critical value. Remember that the regulator sees only \( \bar{L} + \bar{V}_1 \) jointly, and does not perceive each term separately. If \( \bar{L} + \bar{V}_1 \geq z^* \), the regulator allows the bank to continue operating. If the regulator chooses the socially optimal policy of bank closures, then, from the definition of \( \bar{K}_2 \), \( z^* = \bar{K}_2 + (1-K_2) r^* \).

**Proposition 2:** At the start of the second period, if the regulator covers the deposits unpaid by the bank, the bank will be “better off” with less capital for the second period than it would be with greater capital, considering the fact that it can continue in the second period.

Proof. The incentives that the bank has in the first period to choose a specific level of risk affects the income that the bank will have in the second period, since the portfolio choice made in the first period determines, which, in turn, is related to \( \theta_2 \) through equation (9). Therefore, one must seek a ratio between income in the second period and capital in the second period. At that point, one must seek a ratio between income in the second period and capital in the second period. If the regulator plays O, the bank’s income in the second period is:

---

89/ See, for example, Mailath e Mester (1994).

90/ Since the objective is to regulate activities that would generate losses for taxpayers, let’s assume that \( \delta_i < \phi_i^* \).
Expressed in words, the subsidy that the stockholders of the bank received declines in terms of participation in the capital of the bank. In this way, it is clear that the bank does not have an incentive, on \( t=0 \), to guarantee itself against “situations of low capital” on \( t=1 \).

In other words, the fact that the bank has a liability limit results in a situation in which second period income declines in the context of second period capital, encouraging the bank to take measures to reduce the expected value of this capital, with the condition that the bank will not be closed. In this way, the incentives for the bank to incur higher risk in the first period exist when future income is considered. If the bank is not sufficiently capitalized, on the other hand, it can be closed at the end of period 1. Therefore, it is probable that its behavior in relation to risk will be more cautious or, in other words, if bank closure policy is credible, it may reduce the adverse incentives generated by the insurance provided by the regulator or by another regulatory agency.

Returning to the question of reputation equilibrium, the regulator chooses \( z^* \) in order to maximize his objective function (7).\(^9\) The regulator closes if \( \widetilde{V}_1 < z^* \) and allows the bank to continue if \( \widetilde{V}_1 \geq z^* \). Assuming that the bank will never be closed if \( \widetilde{V}_1 = V(\theta_1) > 0 \). Therefore, if the bank is closed on \( t=1 \), this tells the market that \( \widetilde{V}_1 = 0 \) and that \( \widetilde{L} < z^* \). According to Greenbaum (1993), the regulator’s policy is more lax if \( z^* < \hat{K}_2 + (1-K_1) r^s \) and less lax if \( z^* > \hat{K}_2 + (1-K_1) r^s \).

5.2 The Boot and Thakor (1993) Model and the Relevant Reputation Intervals

The major conclusion drawn by the authors is that “in a reputation equilibrium, the optimal policy (private) that the regulator seeks is more lax than the policy that would be socially optimal”. Therefore, we will now clarify the

\(^9\) According to Persson and Tabellini (2000), the regulator in this case seeks to maximize the expected value of his competence, differently from the traditional models of political economy in which electoral control is exercised through competence, but the politician desires to maximize the probability that the competence inferred by the voter is above a certain limit.
parameter interval in relation to which the regulator opts to keep the bank open.

Using the Bayesian rule, the probability inferred by the market that the regulator is good, given that the bank was closed (C)\(^92\) on \(t=1\) is equal to:

\[
\gamma_1(C) = \frac{\Pr(C|g) \Pr(g)}{\Pr(C|g) \Pr(g) + \Pr(C|b) \Pr(b)}
\]

But

\[
\Pr(C|g) = \Pr(\tilde{V}_1 = 0 \& \tilde{L} < z^*/g) = \{\rho_g[1-\theta_1^*] + [1-\rho_g][1-\hat{\theta}_1]\}F(z^*) .
\]

And, identically,

\[
\Pr(C|b) = \Pr(\tilde{V}_1 = 0 \& \tilde{L} < z^*/b) = \{\rho_b[1-\theta_1^*] + [1-\rho_b][1-\hat{\theta}_1]\}F(z^*) .
\]

Therefore

\[
\gamma_1(C) = \frac{\{\rho_g[1-\theta_1^*] + [1-\rho_g][1-\hat{\theta}_1]\} \gamma}{\{\rho_g[1-\theta_1^*] + [1-\rho_g][1-\hat{\theta}_1]\} \gamma + \{\rho_b[1-\theta_1^*] + [1-\rho_b][1-\hat{\theta}_1]\}(1-\gamma)}
\]

analogously, if the bank is not closed or \(\tilde{V}_1 > 0\) or \(\tilde{V}_1 = 0\) and \(\tilde{L} \geq z^*\). Then,

\[
\gamma_1(O) = \frac{\Phi_g \gamma}{\Phi_g \gamma + \Phi_b(1-\gamma)} ,\text{ in which:}
\]

\[
\Phi_g = \rho_g\theta_1^* + [1-\rho_g]\hat{\theta}_1 + [\rho_g(1-\theta_1^*) + [1-\rho_g][1-\hat{\theta}_1]\}1-F(z^*)
\]

\[
\Phi_b = \rho_b\theta_1^* + [1-\rho_b]\hat{\theta}_1 + [\rho_b(1-\theta_1^*) + [1-\rho_b][1-\hat{\theta}_1]\}1-F(z^*)
\]

then, \(\gamma_1(O) > \gamma_1(C)\) for any \(z^* > 0\), provided that \(\theta_1 \geq 0.5, \rho_g > 0.5\) and \(\rho_g > \rho_b\). This ensures that

\[
\rho_g[1-\theta_1^*] + [1-\rho_g][1-\hat{\theta}_1] \geq \rho_b[1-\theta_1^*] + [1-\rho_b][1-\hat{\theta}_1] ,
\]

which leads to \(\gamma_1(O) > \gamma_1(C)\).

Let’s look now at a perfectly non-benevolent regulator who maximizes only \(\gamma_1 + \delta\gamma_2\). If, on \(t=1\), the regulator closes the bank, his expected utility is \(\gamma_1(C) + \delta\gamma_1(C)\). This is due to the fact that, if the bank is closed, the information that the market obtains on \(t=1\) is the same that it would obtain on \(t=2\). If the regulator plays \(O\), his expected utility is:

\[
\gamma_1(O) + \delta\gamma_2(O).
\]

Looking then at a regulator who observes \(\hat{K}_2 \leq z^* - (1-K_1)r^*\). Since \(\hat{K}_2 = \tilde{L} + \widetilde{V}_1 - (1-K_1)r^*\), he knows that \(\tilde{V}_1 = 0\) and

\(^92\) As already seen, it is assumed that the bank will never be closed if \(V(\theta_1) > 0\).
that, on \( t = 2 \), the market will also infer the same thing. Consequently, \( \gamma_2(O, \hat{V}_1 = 0) \) must be equal to \( \gamma_1(C) \). Comparing the expected utilities and using \( \gamma_1(O) > \gamma_1(C) \), one concludes that a perfectly non-benevolent regulator will always prefer to maintain the bank open. However, given that \( \tilde{K}_{2,0} \leq z^* - (1 - K_1) r^S \), the market believes that the bank will be closed and, therefore, the regulator has to posit \( z^* = 0 \).

If, on the other hand, the regulator is completely benevolent, he posits \( z^* = \hat{K}_{2,0} + (1 - K_1) r^S \). However, since the problem of the regulator is a linear combination of two anterior extremes, he will always posit \( z^* \in (0, \hat{K}_{2,0} + [1 - K_1] r^S) \). For this to be a Nash equilibrium, however, investors must infer that closure of the bank implies that \( \hat{V}_1 = 0 \) and, in this way, \( \gamma_2(O, \hat{V}_1 = 0) \) must be equal to \( \gamma_1(C) \).

One way outside the equilibrium would be for the regulator not to close the bank, even though \( \tilde{L} + \hat{V}_1 < z^* \). On \( t = 2 \), the market will discover that the regulator chose a path outside the equilibrium, but no additional information will be obtained since the market was already aware of the value of \( \hat{K}_{2,0} \) in that period. Consequently, there is no other information regarding its type. What the regulator has to lose is simply that the social surplus will diminish, without a reputation gain. In this way, the regulator would not be maximizing his expected utility, resulting in a situation in which that equilibrium will not be perfect in the subgames.

Intuitively, this result can be viewed as an application of the Milgrom and Roberts (1982) price limit model. Though the market inference is subject to interference, closure of the bank on \( t = 1 \) means that the bank’s capital was inadequate. Since this is more probable when \( \theta_1 < \theta_1^* \), or when \( \theta_1 \neq \theta_1^* \), this fact signals something to the market regarding the quality of the regulator. The market is aware that a good regulator would most probably have forced the choice of \( \theta_1^* \), than a poor regulator would have, leading the market to make a Bayesian update in such a way as to diminish the belief that the regulator is good. Suppose that there is a value interval of \( \hat{K}_{2,0} \) for which the regulator concerned exclusively with his own reputation resolves to close the bank and lesser values of \( \hat{K}_{2,0} \), if known by the market, would transmit bad news regarding the quality of the regulator. Therefore, there must be a value of \( \hat{K}_{2,0} = \hat{K}_2 \) in such a way as to correspond to a bank with larger capital that has not yet been closed. In this way, provided that the information that the closure provides to the market is just as bad for the regulator as the information for which \( \tilde{K}_{2,0} < K_2 \), the regulator whose bank has capital \( \hat{K}_2 \) will want to stand out in relation to the regulator with lesser realizations of \( \hat{K}_{2,0} \) and
will not close the bank. Applying this sequential reasoning to each $K_2$, if the regulator only maximizes his reputation he will never close the bank on $t = 1$. On the other hand, the benevolent regulator would maximize social welfare in such a way that he would adhere to the optimal bank closure policy. In summary, the regulator who maximizes chooses $z^* \epsilon (0; K_2 + (1-K_1)r)$. In other words, a small uncertainty regarding the quality of the regulator can distort bank closure policy to a point far from what would be socially optimal. Even in cases in which the threat of closure is the major factor limiting the behavior of the bank in relation to risk in the first period, a more lax closure policy induces the bank to incur greater risk in the first period, increasing the investment distortion in relation to the first best result of $t = 2$.

6. Conclusion and policy application

The distortion in bank closure policy is derived from the fact that the regulator has discretionary power over the rules covering decisions, with the result of obscuring possible failings in the monitoring of asset quality, based on a relevant parameter interval. In order to reduce this problem, one of the possible recommendations would be to separate responsibility for bank closures from responsibility for asset monitoring$^{93}$. As Dewatripont and Tirole (1993) indicate, however, this separation of tasks cannot be considered a solution for all problems. In the first place, because the costs of acquiring information regarding the bank will be doubled when two separate agencies come into existence. Secondly, the existence of various regulators could create a situation in which each regulatory group would tend to place responsibility for failings on the other group, creating a type of moral hazard in teams. Finally, the conflict between the tasks of monitoring and intervention is only one level of more general questions, since there are other factors in the objective function of the government. At the same time, the government obviously is not concerned only with bank solvency, but also with the monetary and exchange system, with operation of the payment system, with competition among banks, etc.

Another way of diminishing distortions in bank closure policy would be to limit the discretionary choices regulators

---

93/ To some extent, this is already done in Brazil.
can make, stipulating a minimum positive level of capital (or a different variable) that the bank would have to comply with in order to avoid being closed. This already exists in various countries and reflects a type of “prompt corrective action”. However capital is often not fully observable at the moment in which a decision on closure is made. This means that problems of observation and even measuring would hinder adoption of a rigid closure rule. There are disadvantages to rules that go against discretionary authority, since it is very difficult to specify beforehand the nature of the intervention that a regulator may intend to impose on a bank. Rules, however, are not restricted by demands for temporal consistency.

There is still another approach that focuses on reducing the opportunistic behavior of the regulator in contract design. What is the value that the regulator creates and how this value is distributed will depend on the incentives under which regulators operate. If the regulator is ensnared to a significant degree, this will create a process known as crony capitalism. The incentives that a regulator has for acting in an opportunistic way are related to three mechanisms: rules of conduct and how these rules define and judge the regulator; institutions that inspect regulators and impose criminal sanctions (legal limits); and individual employment contracts. In principle, management contracts should be drawn up in such a way as to offset deficiencies in rules of conduct and in legal sanctions. A performance-based contract that ensures basic wages and bonuses does not seem to be sufficient to resolve the problem, since regulators may overestimate performance in the current period at the cost of deterioration in long-term results. Thus, it would seem that, in some way, the contract should be linked to long-term performance, plus introduction of an external measurement of success and failure to avoid manipulation of results by regulators. An ideal contract, therefore, should contain provisions that measure, verify and reward regulators in such a way that they would be fully accountable to taxpayers.

Another point would be the question of the public availability of information. If the market is better informed about the real financial conditions of a bank, while preserving confidentiality and avoiding “self-fulfilling prophecies”, there will be less probability that the regulator will be able to avoid applying bank closure policies to protect his reputation. In summary, several policies could be adopted to reduce the problem of reputation in the regulatory environment, for this is a problem that generates losses of social welfare.
7 Bibliographic references


Appendix: The Boot and Thakor (1993) Game in the Extensive Form

\[ g \begin{cases} \gamma \end{cases} \quad \begin{cases} 1 - \end{cases} \]

\[ b \]

\[ t=0 \]

\[ N \]

\[ R \]

\[ B \]

\[ t=1^- \]

\[ \theta_1 \]

\[ O \]

\[ C \]

\[ t=1^+ \]

\[ \theta_2 \]

\[ B \]

\[ O \]

\[ C \]

\[ \theta_2 \]

\[ \begin{cases} \tilde{L} + \tilde{V}_1 - [1 - \tilde{K}_s]r^5 \\ \beta_1(\gamma_1(C) + \delta \gamma_1(C)) + \beta_2 \text{exc.} \end{cases} \]

\[ \begin{cases} \tilde{L} + \tilde{V}_1 - [1 - K_s]r^5 \\ \beta_1(\gamma_1(C) + \delta \gamma_1(C)) + \beta_2 \text{exc.} \end{cases} \]

\[ \begin{cases} \theta_2(V(\theta_2) - [1 - \tilde{K}_s]r^5) - \tilde{K}_s r^5 \\ \beta_1(\gamma_1(O) + \delta \gamma_2(O) + \beta_2[V(\theta_2) - 1 \times r^5] \end{cases} \]

\[ \begin{cases} \theta_2(V(\theta_2) - [1 - K_s]r^5) - K_s r^5 \\ \beta_1(\gamma_1(O) + \delta \gamma_2(O) + \beta_2[V(\theta_2) - 1 \times r^5] \end{cases} \]
An Analysis of Off-Site Supervision of Banks’ Profitability, Risk and Capital Adequacy: a Portfolio Simulation Approach Applied to Brazilian Banks

Theodore M. Barnhill94
Marcos R. Souto95
Benjamin M. Tabak96

Summary

In most countries, the role of off-site bank supervision involves continuous monitoring of profitability, risk and capital adequacy. The objective of this article is to demonstrate the value of bringing together advanced modeling techniques with data on banks’ assets and liabilities and credit worthiness. More specifically, we apply an integrated market and credit risk simulation methodology to a group of six hypothetical banks. We show the capacity of the methodology: (i) to simulate credit transition probabilities of default close to the historical values estimated by the Central Bank of Brazil; and (ii) to simulate asset and equity returns that are unbiased estimators of average historical returns and standard deviations. Our results also indicate that: (i) a sharp reduction in the interest rate spreads of Brazilian banks reduces bank profitability and increases the probability of default; and (ii) most banks have low probability of bankruptcy. Our position is that utilization of forward looking risk evaluation methodologies in databases, such as those developed by the Central Bank of Brazil, has significant potential as an instrument of indirect supervision to identify potential risks before they materialize.

94/ Theodore Barnhill is a Professor at the George Washington University Finance Department.
95/ Marcos R. Souto is a graduate student at the George Washington University and an associate professor at the “Fluminense Federal University”, Brazil.
96/ Benjamin M. Tabak is an Economist at the Central Bank of Brazil Department of Studies and Research.
1. Introduction

In light of the enormous potential economic impact of bank failures, measurement and management of bank risks is a topic of overriding importance. Forward looking risk assessment methodologies are highly valuable, since they help to identify and assess proactive measures that can be adopted to manage banks’ risks before they materialize. Ideally, all of the major risks faced by banks (market, credit, liquidity, and so forth) would be integrated into a single risk measure. However, current practice calls for evaluating market risk and credit risk separately and then add them together (e.g. Basel Accord, 1998, 1996 and 2001). It is no easy task to combine such risk measurements into a single aggregate measurement of portfolio risk (Jarrow and Turnbull, 2000 and Barnhill and Maxwell, 2002). The absence of aggregate portfolio risk measurements makes it more difficult to define capital requirements, measurements of capital at risk, hedging strategies, etc. For example, Barnhill and Gleason (2002) showed that the Basel capital requirements seem to be very high for low risk banks that operate in developed countries, while they are frequently very low for banks operating in more volatile environments, such as emerging countries.

This article uses an integrated market and credit risk methodology, along with the portfolio simulation approach – PSA – to assess credit worthiness of 6 hypothetical Brazilian banks. This approach has already demonstrated that it is capable of producing reasonable results, such as in the case of South African banks (Barnhill, Papapanagiotou and Schumacher (2003)), Japanese banks (Barnhill, Papapanagiotou, and Souto (2004)) and credit transition probabilities for two major Brazilian banks (Barnhill, Souto, and Tabak (2003)).

There are several advantages in utilizing PSA, including its capacity to deal simultaneously with interest rates, exchange rates and credit risk for bank asset and liability portfolios, distributed among various sectors of the economy, regions of the country, maturities and currencies. One constraint found in the PSA methodology is that it requires a large quantity of data in order to calibrate the model.

In this study, we have utilized the Central Bank of Brazil database along with BankScope data in order to simulate returns on net worth, returns on assets and capital ratios for a group of six hypothetical banks. Aside from asset and liability distribution and operational information, the data also encompass the distribution of loans according to
credit quality. At the same time, a database was created with the characteristics of the 543 publicly traded Brazilian companies for which it was possible to obtain ratings. This data made it possible to estimate capital structure, systemic risk and non-systemic risk for companies according to the credit worthiness.

One important feature that needed to be captured in our simulations relates to the fact that Brazilian banks charge high interest rate spreads (resulting in an average of 51% for corporate loans and 85% for personal loans). We were unable to obtain specific information on a bank-by-bank basis with respect to these spreads. However, we propose a methodology to estimate interest rates for Brazilian banks, for varying levels of credit quality, in such a way that they will reflect the historical levels of default and net interest margin.

Aside from their high rates of interest, Brazilian banks also have a significant fraction of their assets as non-interest earning assets as well as heavy operational expenses. Our results indicate that these two characteristics may be related: at the time of this study, we conjecture that the banks could have been charging higher interest to offset these inefficiencies. In those scenarios in which banks charge (and pay) more moderate interest rate spreads, the simulated capital ratio is clearly lower.

The remainder of the article is organized as follows: section 2 reviews literature correlating credit and market risk; section 3 describes the conceptual approach – PSA – to evaluate integrated credit and market risks; in section 4, we describe how we model Brazilian banks and the macroeconomic environment in which they operate; section 5 presents and discusses the results of the simulation; finally, section 6 contains the conclusions drawn in this study.

2. Modeling credit and market risk and correlated credits

The major frameworks for pricing instruments subject to credit risk are the structural approach and the reduced-form approach. The first approach was developed by Black and Scholes (1973) and Merton (1974) who developed a theoretical formula for evaluating options under a no-arbitrage condition. They argued that all corporate liabilities

---

97/ Brazilian banks use a rating methodology that begins with AA, the highest level of loan quality, followed by the categories A, B, C, D, E, F, G and H, as the credit quality deteriorates. Basically, categories G and H represent loans in arrears.
could be seen as combinations of options. The reduced-form methodology was introduced by Jarrow and Turnbull (1995) with the objective of avoiding the difficulties inherent to the analysis of contingent assets, such as the absence of observable data on the value of the companies.

The KMV models, CreditMetrics and CreditRisk+ are currently utilized in credit risk management. While the structural form approach is at the roots of the CreditMetrics and KMV systems, an actuarial approach of security mortality underlies CreditRisk+. In KMV, a company enters bankruptcy when its value drops below a certain threshold. This has an important advantage – it implicitly incorporates market information on the probabilities of default, by utilizing the market value of the stocks as an approximation for the value of the company. Unfortunately, some of the variables used in the KMV (for example, the value of the company) are not directly observable. Furthermore, interest rates are deterministic, a fact that limits the utility of the model when it is used in the analysis of interest sensitive instruments (Jarrow and Turnbull, 2000).

The CreditMetrics (J.P. Morgan and Reuters, 1996) offers a methodological alternative based on the probability of a security migrating from one credit category to another over a specific time horizon. This method is based on historical probabilities of transition and assumes that all companies with a certain level of credit quality have the same probability of default. Alternatively, CreditRisk+ (CSFP) derives the distribution of losses of a fixed income portfolio under an environment in which the risk of default is not related to the capital structure of the companies. Taken together, these two methodologies are quite useful but they have the same limitations as the KMV – they ignore market risk and are unable to cope with non-linear products such as options (Crouhy et. al., 2000).

Other models, such as the CreditPortfolioView (Wilson, 1997a, 1997b), condition the probability of default to macroeconomic variables, such as unemployment and interest rates, in a multi-period framework. This methodology has the disadvantage of being based on ad hoc matrix transition adjustment procedures, thus casting doubts as to whether it produces a better performance than the more simple Bayesian model (Crouhy et. al., 2000).

There is ample evidence that both interest rate and credit risks must be estimated together, so as to be able to accurately price fixed income portfolios and to provide venues for hedging. Based on the 1995 Federal Reserve study that
concluded that none of the bank failures that occurred in the United States could be attributed to interest rate risk, Jarrow and Deventer (1998) compared the Fabozzi approach for fixed income analysis with the high risk debt model. The authors compared the performance of a hedging strategy using the two methodologies and found that the Fabozzi method eliminates approximately 40% of the risk of the hedged portfolio, in contrast to just 20% when the Merton model is used. In spite of the improved performance, the Fabozzi approach eliminates less than half of the risk, leaving an important fraction of the risk unhedged.

Longstaff and Schwartz (1995) corrected various problems of fixed income evaluation methods. They derived analytical formulas for debt with fixed and floating interest rate risk. One of the traditional limitations to the Black-Scholes-Merton approach is that the companies only enter a situation of default when they have exhausted their assets, implying lower credit spreads than currently in effect (e.g. Franks and Touros, 1989), Black and Cox (1976) generate credit spreads that are more consistent with those observed, but they also assume constant interest rates and absolute allocation priority in the case of default. Among others, Franks and Touros (1989, 1994) demonstrate that this is not the case when the companies are going through periods of financial stress.

Longstaff and Schwartz expanded the literature focusing on evaluating corporate securities with interest rate and default rate risk, allowing for the possibility of default occurring before asset depletion, with complex capital structures, multiple debt issuance and deviations from the rules of absolute priority. The authors found strong evidence that interest rates are negatively correlated with credit spreads and that this correlation has a significant impact on the properties of spreads – the spreads implicit in the model are consistent with most of the properties of those observed. In this way, this approach is able to explain why securities with similar credit qualities, but originating in different industries or sectors, can have significant differences in their spreads. The properties of the securities in the speculative category are quite different from those of less risky securities.

Davis and Lischka (1999) utilize a two dimensional trinomial graded method in order to evaluate convertible securities with interest rate and credit risks. They utilize three sources of uncertainty – the price of the stock, the interest rate and the credit spread. The probability of default for the next period is given by the survival rate. For purposes of simplicity and in

98/ Fabozzi and Fabozzi (1989) focus on interest rate levels, duration and convexity and ignore credit risk when they evaluate securities and risk.
order to avoid computational problems, market professionals and researchers have traditionally analyzed models with a maximum of two stochastic variables. In this way, Davis and Lischka consider different scenarios with a limited number of stochastic variables. In the first case, only the price of the stock is considered as stochastic, while the survival rate and the interest rate are deterministic functions of time. Then, the stocks and the short-term rate are assumed to be stochastic, while the survival rate is deterministic. Finally, all of the variables are modeled stochastically. This method results in values that are consistent with those observed in market data and can be calibrated to replicate the initial interest rate structure, but cannot be extended to include more stochastic risk factors.

One of the oldest examples of the reduced-form approach is the Jarrow and Turnbull (1995) model. In this model, the companies received a rating according to the credit class and default is modeled as a point process. Bankruptcy is exogenous and is not related to the company’s assets. The advantage is that exogenous hypotheses are imposed only on observable variables. Jarrow, Lando and Turnbull (1997) extended this formulation in a model in which bankruptcy is characterized by a Markov process of finite states in the credit rating of companies. This model uses historical transition probabilities and is able to cope with different debt seniorities through the use of a wide range of recovery rates in the event of default. The process of bankruptcy of the company is assumed independently of the forward risk-free interest rate structure.

Consistent with other authors, Jarrow and Turnbull (2000) recognize that there is considerable empirical evidence that credit spread variations are negatively correlated to changes in risk-free interest rates (i.e. Duffee, 1997 or Das and Tufano, 1996). In various scenarios, they derive analytical solutions for the value of securities with credit and market risk. First, for when recovery rates are assumed to be proportional to the value of the instrument prior to default (see Duffie and Singleton, 1997). Second, under a scenario where the securities holders claim accrued interest (accumulated and unpaid) plus the face value of the securities (a highly popular hypothesis among those utilizing this method).

Barnhill and Maxwell (2002) extended the diffusion models developed by Merton (1974) and Longstaff and Schwartz (1995) to include credit and market risk. The authors propose a simulation approach that deals with the limitations of both the structural and the reduced-form approaches, specifically in the sense of coping with various correlated variables.
They used a simulation approach in order to simultaneously model the correlated evolution of security credit quality, as well as the future environment (interest rate risk, interest rate spread risk and exchange rate risk) in which fixed income instruments will be evaluated. The authors state that the four sources of risk are important, with credit risk being more significant for the non-investment category of securities (speculative). This model produced reasonable transition matrices, security prices and portfolio risk measures. Given the large number of stochastic variables modeled and considering the complexity of their interrelations, there is no single analytical solution.

Bank portfolios are normally composed of large quantities of corporate and personal loans along with credits granted to the government that can be partially modeled as a security portfolio. In light of the discussion above, it is evident that both credit and market risks impact the value of bank portfolios. However, integration of these risk factors represents a significant challenge. With appropriate models, we hope to achieve more precise measurements of value and value-at-risk, since these are very important for investor portfolio managers and regulators.

3. The conceptual approach to bank risk evaluation

Given the correlated nature of market and credit risk (see Fridson et al. 1997), as discussed in the previous section, the importance of a methodology that integrates market and credit risk is evident. In approaching the problem of measuring risk, Barnhill and Maxwell (2000) developed a methodology based on diffusions in order to evaluate the Value-at-Risk (VAR) of a fixed income securities portfolio with correlated interest rates, interest rate spread, exchange rates and credit risk. Barnhill, Papapanagiotou and Schumacher (2003) extended the model to incorporate evaluation of financial institution assets and liabilities for South African banks, and Barnhill, Papapanagiotou and Souto (2004) used the same methodology in order to estimate potential losses given defaults in the Japanese financial system. Barnhill and Gleason (2002) and Barnhill and Handorf (2002) apply the PSA and compare simulated capital requirements with those demanded by the new Basel Accord. These studies demonstrated that appropriate calibration of the PSA model produces:
1. a simulated financial environment with parameters for the environmental variables within reasonable value intervals;

2. credit transition probabilities similar to those reported in historical transition probabilities;

3. simulated security prices, with credit risk near the risk levels observed on the market;

4. simulated value at risk measurements for security portfolios with values highly similar to the historical values for value at risk;

5. bank capital requirements estimates that are generally lower than the Basel capital requirements for banks operating in developing countries and higher for banks operating in emerging countries.

In general, both the future financial environment in which the assets will be evaluated and the credit rating of the specific loans are simulated. The financial environment can be represented by any number of correlated random variables. Evolution of the market value of the company’s equity, its debt ratio and its credit rating are simulated in the context of the financial environment created. The structure of the methodology is to select a period of time in which the stochastic variables can vary as correlated random processes. The specific returns of the companies (differently from the aggregate indices, such as the index of an economic sector or of the real estate sector) and rates of recovery in the event of default are assumed not to be correlated among themselves or with other random variables. For each simulation, a new financial environment (correlated interest rate forward structures, exchange rate, stock market returns, real estate sector index returns), along with specific debt ratios of the companies, credit ratings and recovery rates in the event of default are generated. This information makes it possible for the correlated values of the financial assets (including stocks and investments in the real estate sector) to be estimated and, following a large number of simulations, to construct the portfolio’s value distribution.

4. Simulating Brazilian banks

4.1 Modeling the macroeconomic environment in which Brazilian banks operate

In the proposed simulation model, a point of central importance is to characterize the financial and macroeconomic
environment in which the banks operate. As discussed in section 3, the variables that define the macroeconomic scenario are updated according to correlated stochastic processes, utilizing the Monte Carlo simulation. Consequently, one must specify as reasonably as possible the initial conditions from which the simulated stochastic processes will evolve.

For the purpose of this analysis, several variables were selected that, in our opinion, will have a specific influence on the simulated banks’ portfolio. These variables are\(^99\): short-term domestic interest rates (Central Bank benchmark rate), short-term USA interest rate (three-month American treasury rate), exchange rate RS/US$ – bid), domestic inflation, oil (Brent-type crude), broad market index of Brazil (Ibovespa), twelve Brazilian sectoral indices\(^100\) (banks, basic industry, beverages, chemicals, general industry, metal, mining, oil, paper, wireless telecommunications, textiles, tobacco and utilities) and seasonally adjusted unemployment rates, broken down by geographic region\(^101\) (Brazil, Belo Horizonte, Porto Alegre, Recife, Rio de Janeiro, Salvador, São Paulo).

The volatilities and correlations for the variables cited above were assumed to follow an IGARCH process as defined by the Exponentially Weighted Moving Averages (EWMA). The initial volatilities and correlations were estimated for the first six months of 2000. The results of EWMA volatilities and correlations on July 25, 2002 are presented in tables 1 and 2.

The Brazilian government interest rate is quite volatile – annualized standard deviation of 3.29%. Exchange rate volatility is also high, 15.85%, and the period in question does not include changes in the exchange market during the pre-electoral period (August/September 2002). Stock market indices have also been highly volatile, in the range of approximately 22%-49%, and are compatible with other emerging markets.

In terms of correlations, we observed an expected negative relation between domestic interest rates and market indices.

\(^{99}\) The short-term Brazilian interest rate (daily), the rate of exchange (daily), inflation (monthly) and gold (daily) were obtained on the basis of Central Bank of Brazil data. The market indices (daily) and stock indices (daily) were obtained from the DataStream. The seasonally adjusted unemployment rate was obtained from the Brazilian Institute of Geography and Statistics (IBGE). We obtained the daily time series for United States short-term interest rates on the web site of the Federal Reserve Bank of St. Louis. Daily Brent-type crude oil data were downloaded from the International Petroleum Exchange (and converted into US dollars per barrel).

\(^{100}\) The definition of the sectors can be found in the DataStream.

\(^{101}\) The correlation between unemployment rates and other variables will be particularly important to the simulation of the values of consumer loans (individuals).
(-0.063 between the BR interest rate and Ibovespa). However, the magnitude of the correlation is not as strong as in other markets. The domestic interest rate is correlated to the rate of exchange (0.028), though the correlation is not strong. This suggests that, in the period in question, the rate of interest tends to increase (decrease), when the real depreciates (appreciates) in relation to the American dollar.

4.2 Estimating betas for a set of Brazilian companies

We utilized the one factor CAPM model\textsuperscript{102} in order to evaluate market risk of corporate loans held in the banks’ portfolio. To do this, one must appropriately estimate the specific and systemic risk of Brazilian companies. Estimation of the beta of Brazilian companies is made more difficult by the fact that some of them are not frequently traded. Since some stocks do not have liquidity, price series are artificially rigid and this can reduce the estimated betas, yielding erroneous empirical evidence.

Based on the use of the twelve sectoral stock market indices for Brazil (banks, basic industry, beverages, chemicals, general industry, metal, mining, oil, paper, wireless telecommunications, textiles, tobacco and utilities), betas for 543 companies were estimated according to their respective industrial sectors\textsuperscript{103}. Data on price indices and stocks were gathered from DataStream. We assumed that the credit risk profile of the companies used will be representative of the borrowers of credits in the banks’ portfolios.

Initial estimates based on daily data resulted in a large number of betas close to zero. To get around this problem, several attempts were made to estimate the betas using: (i) monthly observations; (ii) the Scholes-Williams (1977) approach; and (iii) non-leveraged betas such as that defined in the expression below:

\[
\beta_v = \frac{\beta_i}{1 + (1 - \tau_c) \frac{D}{S}},
\]

(1)

in which $\beta_v$ is the non-leveraged beta, $\beta_i$ is the leveraged beta, $\tau_c$ is the tax rate, $D$ represents the market value of the debt and $S$ is the market value of the equity.

\textsuperscript{102}We opted for the one factor CAPM model due to its simplicity and intuition in the risk-return ratio. Multiple factor models can also be used.

\textsuperscript{103}Estimating betas through the use of sectoral indices instead of market indices makes it possible to perceive the benefits of diversification, since banks lend to different sectors of the economy.
Monthly observations produced estimates that were more consistent with the betas, when one considers the financial characteristics of the Brazilian companies (we obtained values in the range of 0.032 to 1.497). The final results for the betas, specific risk of the companies and their respective ratings are presented in table 3.

Table 3 also presents information on the debt/company value ratio for each credit category. This information was developed initially by calculating the debt/company value ratio of all of the publicly traded companies in Brazil followed by an analysis of the distribution of these ratios by credit rating\textsuperscript{104}. As a further refinement of the calibration of this model, a series of simulations was developed to identify the target, the upper and lower limits of the debt/company value ratios. Both registered declines in the values observed for the debt/value ratios for each credit category and produced credit transition probabilities similar to those observed in the period of 2000 to 2001 and 2001 to 2002. The target was considered as being the current (and planned) value of the debt/company value ratio. The upper and lower limits are presented in table 3 and represent the debt ratios based upon which the companies would shift to a higher/lower credit rating. Consequently, for example, in the case of level B companies, they would drop to category C when their debt ratios increased to more than 0.90. These results are consistent with the theory: credit risk rating deteriorates when the systematic and nonsystematic risk components increased and when the debt/value ratio increases.

4.3 Distribution of loan quality

Loans to individuals and corporate entities represent a major share of Brazilian bank assets\textsuperscript{105}. Consequently, when we model a bank portfolio, the first step must be a definition of the distribution of the loan portfolio.

To simplify this, we modeled loans to individuals exactly in the same way we modeled corporate loans\textsuperscript{106}. We show that this hypothesis is quite reasonable and we produce a simulated credit transition matrix that is quite close to the historical series estimated by the Central Bank of Brazil Risk Bureau.

\textsuperscript{104}The banks supplied the information on the ratings of companies directly to the Central Bank of Brazil. For reasons of confidentiality, these data can not be presented. The information on the debt/company value ratio was obtained from Economática.

\textsuperscript{105}In some cases, more than 56% of total assets.

\textsuperscript{106}In the context of the simulation, the value of each corporate loan is calculated discounting future cash flows with simulated interest rates corresponding to the simulated credit rating of each corporate client. In the event of default, payment of the loan is given by the net recovery value of transaction costs.
It is important to mention that the Central Bank of Brazil utilizes a credit risk scale different from that used by Moody’s or Standard & Poor’s. Credit ratings in Brazil are divided into the following categories ranging from high-quality credits down to lower credit quality categories: AA, A, B, C, D, E, F, G and H. Categories AA and A correspond to investment grade, while categories G and H represent loans in default.

Tables 4 and 5 show the aggregate distribution of loans to corporate entities among the different industrial sectors and broken down by credit risk for six hypothetical banks.

### 4.4 Credit transition matrix

Once the betas were estimated and distributed according to the credit categories, we estimated the transition probability matrix. For each simulation, we estimated market index returns, assuming prices to follow a geometric Brownian motion and estimated new equity values based on the Capital Asset Pricing Model (CAPM). These returns are then utilized to estimate the distribution of future values of equity and debt/value ratios. The debt/value ratios are then transformed into credit ratings, according to the calibrations presented in table 3. Finally, a distributional analysis is used to generate transition probabilities to each credit rating. We present the results of this analysis in table 6, along with the historical transition probabilities matrix (estimated by the Risk Bureau and presented in table 7). As is shown, the two transition probabilities matrices are quite similar. For example, in table 8, the median absolute difference between the two transition matrices is 0.0002, while the maximum absolute difference never exceeds 0.1060, and the simulated default rates for each credit risk category are similar to those historically reported. This is an important result in the analysis, since it supports our belief that the simulations will produce reasonable estimates for banks’ capital ratios.

### 4.5 Balance sheet

In table 9, we present a simplified version of the six banks analyzed in this article. These banks have one common characteristic: a significant fraction of their non-interest earning assets which is a factor that, obviously, can erode banking efficiency. However, the banks are highly

---

107/ This methodology assumes a deterministic relation between the proportion of debt of a company and its rating which, in an environment of contingent claims, is equivalent to assuming that the volatility of the company’s value is constant.
heterogeneous with regard to the distribution of their assets. We present the average, minimum and maximum values.

4.6 Structure of asset and liability maturities

We were unable to obtain more detailed information on the maturity of bank assets and liabilities. Nonetheless, we obtained information for bank 4, which was then utilized as the standard for the remaining banks simulated in this study. For this bank, mostly all liabilities and assets are short-term (one-year maturity or less).

4.7 Interest rate spreads

In December 2002, the benchmark interest rate (Selic) was 24%\textsuperscript{108} per year. At the same time, the average yields for corporate and personal loans were approximately 51% and 85%, respectively. Even considering default rates on personal and corporate loans, spreads were very large.

Ideally, we would have good estimates of the spreads that each bank charges for each credit category. Unfortunately, we have been unable to obtain precise data on banking spreads. For this reason, we estimated the interest rate spreads for the different credit categories using information on banks’ net interest margin. In the first place, we estimated average losses in each credit category as a product of the historical default rate and of the rate of losses given default. For example, for class AA corporate loans, the default rate in one year is 0.68%, with an assumed loss rate of 85% or, in other words, banks have been highly successful in recovering 15% of the value of the loans, resulting in an average loss on defaults of 0.58% (0.68% times 0.85). To the average loss given defaults, we then add spread that is proportional to stylized spread profiles observed in United States banking industry, across different risk categories\textsuperscript{109}. For the AA category, the additional risk spread measured by the United States spread would be 0.005x. For category A, the additional risk spread would be 0.005x, etc., with knowledge of the percentage of loans in each rating category. With this, one can resolve the value of x, which produces an average rate of 51% for corporate loans and 85% for individual loans. This additional spread is 5.01% in the case of category AA corporate loans. The total interest spread will then be the

\textsuperscript{108}Inflation in the same period of time was approximately 10%. Consequently real interest rates were close to 14%.

\textsuperscript{109}For example, we assume that United States banks would charge an average of 0.13%, comparable to category AA, 0.50% for level A and so forth (table 9).
sum of two components $0.58 + 0.01 = 5.59\%$ for AA loans. This procedure produced the distribution of banking spreads presented in table 10. It is important to stress that, though this procedure is somewhat arbitrary, it did produce interest rate spreads for the various credit categories that are quite close to those observed in Brazilian banks, according to data released by the Off-Site Supervision and Information Management Department, at the Brazilian Central Bank.

Finally, we had the interest rate margins for each bank drawn from BankScope. Consequently, a final adjustment was made for each bank so that the market spreads would produce net interest rate margins consistent with those reported by BankScope.

5. Simulation results

In order to investigate future profitability, risk and capital adequacy of the six banks simulated in this study, we constructed two major scenarios in which we assume that the banks operate: (i) in a scenario of high interest rates, in which banks charge interest rates as estimated in the previous section; and (ii) in a scenario of low interest rates in which banks charge (and pay) 60% of the rates in the high interest scenario. Capital ratios are simulated over a one-year horizon. The results of this simulation are presented in table 10.

5.1 Bank profitability in a scenario of high interest rates

PSA makes it possible to simulate the distribution of returns on assets and equity (ROA and ROE, respectively) given the hypotheses regarding the volatility of the macroeconomic and financial environment, credit quality of the bank portfolios, etc. An important question is just how reliable and useful are these estimates of profitability. To examine this question, we would like to study whether the simulated averages and standard deviations of the returns explain the historical averages and standard deviations for the same six banks. The historical returns of these banks were calculated for the period extending from 1998 to 2002.

Table 11 presents the degree of adjustment of the regressions to ROE and ROA. The average historical returns are compared to the simulated returns. We use the simulated returns as an explanatory variable for the historical returns. The null hypothesis consists in that the returns simulated by
the PSA model can explain the historical returns. With this, the following regressions were made:

\[
\begin{align*}
\text{mean}_\text{historical}_\text{ROE} &= \alpha + \beta \text{mean}_\text{simulated}_\text{ROE} + \epsilon \\
\text{mean}_\text{historical}_\text{ROA} &= \alpha + \beta \text{mean}_\text{simulated}_\text{ROA} + \epsilon \\
\text{std}_\text{historical}_\text{ROE} &= \alpha + \beta \text{std}_\text{simulated}_\text{ROE} + \epsilon \\
\text{std}_\text{historical}_\text{ROA} &= \alpha + \beta \text{std}_\text{simulated}_\text{ROA} + \epsilon
\end{align*}
\]

in which std refers to the standard deviation, mean, to the average, and historical and simulated refer to the historical and simulated values, respectively.

Table 11 presents adjusted $R^2$ for these regressions which result in a measurement of the degree of adjustment of the regressions. The adjusted $R^2$ are high for both regressions, ROE and ROA, for means and standard deviations. The simulated ROE and ROA are not biased if one is unable to reject the hypothesis that $\alpha = 0$ and $\beta = 1$ is true. Wald’s statistics in the last column suggest that this hypothesis is not true only for the regression to the ROA mean. However when all the observations are utilized, increasing the degree of freedom of the regressions, we cannot reject this hypothesis.

If we utilize all of the observations (24 observations), the beta coefficient is 0.97 with an adjusted $R^2$ of 81.13%. These results suggest that the means and standard deviations of the simulated ROE and ROA are quite close to the historically observed values, illustrating that the PSA performs quite well in replicating real observed data. When we consider this result, we must remember that the six banks have a wide variety of asset and liability structures, distributions of credit quality and historical profitability.

5.2 Bank risk and capital adequacy in a scenario of high interest rates

Table 12 presents an analysis of the one-year ahead distribution of simulated capital ratios for the two alternative scenarios. In the high interest rate scenario, the average simulated capital ratio is consistently above the initial values. This should not come as a surprise, considering the values of interest rate spreads in Brazil and the quality of the credit portfolios of these banks. We also noted that the standard deviation of the simulated proportions of capital varied greatly: from 0.008 for banks 2 and 3 to 0.023 for banks 6. These variations reflect the asset and liability structures of the banks as well as the quality of the credits in question. In general, we found that when sovereign risk is not considered, the six banks have low probability of default in the scenario.
of high interest rates. More specifically, none of these banks have simulated capital below 2% at a 99% confidence level. Only two banks have capital below 3% at the 99% confidence level. This analysis suggests that the simulated banks are generally well-capitalized.

One important characteristic of this methodology is that it generates quantitative measurements of risk for each bank on a consistent basis (i.e., the same hypotheses in relation to the economic and financial environment, consistent treatment of correlated market and credit risk, consistent treatment of the effects of portfolio diversification, consistent treatment of credit risk, and so forth). Consequently, the relative risk and capital adequacy of the banks can be evaluated directly from the quantitative results of the simulation. It is also important to observe that the analysis was carried out through the use of data systematically collected by the Credit Risk Bureau and other public sources. Therefore, there is no reason why this process should not be automated and applied to all Brazilian banks with the frequency considered most useful. Once again, this type of analysis is sensitive to alterations in the financial environment and to volatility, changes in bank assets and liabilities (for example, exchange exposure, interest rate exposure), changes in the diversification or concentration of portfolios and changes in the quality of credits.

6. Final considerations

This article presents a simulation methodology – the portfolio simulation approach (PSA) – that makes it possible to model integrated market and credit risk in bank assets and liabilities. Our argument is that this methodology has several advantages in relation to theoretical models (for example, the possibility of modeling bank portfolios) and to ad hoc methodologies, such as that of the Basel Accord (1988, 1996, 2001) (for example, the role of integrated market and credit risk). We argue that this simulation methodology has significant capacity for analyzing credit risk. For example, the simulated credit transition matrix is quite close to that estimated by the Risk Bureau. Furthermore, the simulated bank returns are unbiased predictors of historical returns (both averages and standard deviations).

Our simulations indicate that interest rate spreads in Brazil more than offset typical losses in credit portfolios. Consequently, Brazilian banks in general are highly profitable and have low probability of default, despite the
fact that significant resources are invested in assets that do not generate income.

It is our belief that such a forward looking methodology as PSA, taken along with systematically collected databases, makes it possible for indirect supervision to perform quantitative risk evaluations consistent with bank profitability and risk and capital adequacy for all banks. We believe that these opportunities would be available to those countries that opt to systematically collect the data required for this type of analysis.

References


BASEL COMMITTEE ON BANKING SUPERVISION (1988). International convergence of capital measurement


Table 1
EWMA Volatilities

Volatilities for a set of Brazilian financial and macroeconomic variables were estimated via exponentially weighted moving average (RiskMetricsTM) methodology, as of 07/25/2002. The values are annualized and presented in percentages. BR rate is the Brazilian short-term interest rate (Brazilian Central Bank referential interest rate), US rate is the 3-Month U.S. Treasury Constant Maturity Rate, FX rate is the foreign exchange rate (Brazilian currency, R$, over US$), BR c.p.i. is the Brazilian consumer price index, oil represents the Brent crude oil as quoted in the International Petroleum Exchange, Ibovespa is the Brazilian broad market index, which is followed by Brazilian equity market indices by sectors (as defined in DataStream): Banks, BasicInd (Basic Industry), Beverage, Chemicals, GenInd (General Industry), Metal, Mining, Oil_Sec (Oil Equity Sector), Paper, Telewire (Telecommunications Wireless), Textile, Tobacco, and Utility. URBH, URPA, URRE, URRJ, URSA, URSP, are the seasonally adjusted unemployment rates for the cities of Belo Horizonte, Porto Alegre, Recife, Rio de Janeiro, and São Paulo respectively and URBR is the seasonally adjusted unemployment rate for Brazil.

<table>
<thead>
<tr>
<th>Variable</th>
<th>BRrate</th>
<th>Beverage</th>
<th>Tobac</th>
<th>USrate</th>
<th>Chemicals</th>
<th>Utility</th>
<th>FXrate</th>
<th>GenInd</th>
<th>Metal</th>
<th>Mining</th>
<th>Oil_Sec</th>
<th>Paper</th>
<th>Banks</th>
<th>BasicInd</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.29%</td>
<td>31.10%</td>
<td>48.57%</td>
<td>0.18%</td>
<td>30.69%</td>
<td>33.90%</td>
<td>15.85%</td>
<td>22.09%</td>
<td>30.46%</td>
<td>23.51%</td>
<td>49.20%</td>
<td>30.61%</td>
<td>37.42%</td>
<td>26.03%</td>
</tr>
<tr>
<td>BR c.p.i.</td>
<td>2.47%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oil</td>
<td>26.51%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td>24.51%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ibovespa</td>
<td>39.11%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Banks</td>
<td>37.42%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BasicInd</td>
<td>26.03%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2a

<table>
<thead>
<tr>
<th></th>
<th>BRrate</th>
<th>USrate</th>
<th>FXrate</th>
<th>Brcpi</th>
<th>Oil</th>
<th>Gold</th>
<th>Ibov</th>
<th>Banks</th>
<th>BasInd</th>
<th>Bev</th>
<th>Chem</th>
<th>GenInd</th>
<th>Metal</th>
<th>Mining</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.064</td>
<td>0.028</td>
<td>-0.036</td>
<td>0.017</td>
<td>-0.063</td>
<td>-0.095</td>
<td>-0.091</td>
<td>-0.132</td>
<td>0.051</td>
<td>-0.090</td>
<td>-0.080</td>
<td>-0.122</td>
<td>0.075</td>
<td></td>
</tr>
<tr>
<td>-0.042</td>
<td>0.046</td>
<td>0.002</td>
<td>-0.165</td>
<td>0.079</td>
<td>-0.037</td>
<td>0.086</td>
<td>0.010</td>
<td>0.053</td>
<td>0.166</td>
<td>0.157</td>
<td>-0.089</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-0.335</td>
<td>0.541</td>
<td>0.336</td>
<td>0.508</td>
<td>-0.041</td>
<td>-0.201</td>
<td>-0.229</td>
<td>-0.258</td>
<td>-0.106</td>
<td>-0.027</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.108</td>
<td>-0.058</td>
<td>-0.172</td>
<td>0.039</td>
<td>0.006</td>
<td>-0.030</td>
<td>-0.366</td>
<td>-0.093</td>
<td>0.102</td>
<td>-0.021</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.745</td>
<td>0.673</td>
<td>0.602</td>
<td>0.449</td>
<td>0.564</td>
<td>0.684</td>
<td>0.171</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.418</td>
<td>0.634</td>
<td>0.386</td>
<td>0.463</td>
<td>0.420</td>
<td>0.170</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.581</td>
<td>0.313</td>
<td>0.665</td>
<td>0.934</td>
<td>0.259</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.166</td>
<td>0.315</td>
<td>0.490</td>
<td>0.041</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.420</td>
<td>0.256</td>
<td>0.478</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.670</td>
<td>0.332</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.196</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 2b

<table>
<thead>
<tr>
<th></th>
<th>OilSec</th>
<th>Paper</th>
<th>TiWire</th>
<th>Text</th>
<th>Tobac</th>
<th>Utility</th>
<th>URBH</th>
<th>URPA</th>
<th>URRE</th>
<th>URJ</th>
<th>URSA</th>
<th>URSP</th>
<th>URBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>BRrate</td>
<td>0.009</td>
<td>-0.002</td>
<td>-0.181</td>
<td>-0.008</td>
<td>-0.058</td>
<td>-0.086</td>
<td>0.279</td>
<td>-0.087</td>
<td>-0.024</td>
<td>0.215</td>
<td>-0.197</td>
<td>0.106</td>
<td>0.007</td>
</tr>
<tr>
<td>USrate</td>
<td>-0.038</td>
<td>-0.070</td>
<td>-0.039</td>
<td>0.124</td>
<td>-0.368</td>
<td>0.027</td>
<td>0.123</td>
<td>0.079</td>
<td>-0.133</td>
<td>0.303</td>
<td>-0.002</td>
<td>0.101</td>
<td>0.230</td>
</tr>
<tr>
<td>FXrate</td>
<td>-0.129</td>
<td>0.099</td>
<td>-0.237</td>
<td>-0.145</td>
<td>-0.333</td>
<td>-0.319</td>
<td>0.457</td>
<td>-0.364</td>
<td>-0.098</td>
<td>0.153</td>
<td>-0.169</td>
<td>0.111</td>
<td>-0.075</td>
</tr>
<tr>
<td>Brcri</td>
<td>-0.093</td>
<td>-0.044</td>
<td>-0.141</td>
<td>-0.128</td>
<td>-0.064</td>
<td>0.048</td>
<td>-0.013</td>
<td>0.105</td>
<td>0.223</td>
<td>0.110</td>
<td>-0.085</td>
<td>-0.072</td>
<td>-0.060</td>
</tr>
<tr>
<td>Oil</td>
<td>0.486</td>
<td>0.224</td>
<td>0.249</td>
<td>0.060</td>
<td>0.172</td>
<td>0.308</td>
<td>0.283</td>
<td>-0.141</td>
<td>-0.040</td>
<td>-0.036</td>
<td>-0.221</td>
<td>-0.086</td>
<td>-0.120</td>
</tr>
<tr>
<td>Gold</td>
<td>-0.302</td>
<td>-0.058</td>
<td>-0.599</td>
<td>-0.425</td>
<td>-0.176</td>
<td>-0.644</td>
<td>0.487</td>
<td>-0.401</td>
<td>-0.097</td>
<td>0.077</td>
<td>-0.135</td>
<td>0.132</td>
<td>-0.067</td>
</tr>
<tr>
<td>Ibov</td>
<td>0.763</td>
<td>0.435</td>
<td>0.377</td>
<td>0.305</td>
<td>0.323</td>
<td>0.930</td>
<td>0.065</td>
<td>-0.100</td>
<td>-0.123</td>
<td>0.199</td>
<td>-0.169</td>
<td>0.069</td>
<td>-0.054</td>
</tr>
<tr>
<td>Banks</td>
<td>0.550</td>
<td>0.267</td>
<td>0.527</td>
<td>0.282</td>
<td>0.507</td>
<td>0.647</td>
<td>-0.195</td>
<td>0.132</td>
<td>0.082</td>
<td>0.087</td>
<td>0.030</td>
<td>-0.135</td>
<td>0.083</td>
</tr>
<tr>
<td>BasInd</td>
<td>0.624</td>
<td>0.812</td>
<td>0.544</td>
<td>0.442</td>
<td>0.233</td>
<td>0.610</td>
<td>0.295</td>
<td>-0.414</td>
<td>-0.178</td>
<td>0.256</td>
<td>-0.210</td>
<td>0.321</td>
<td>0.102</td>
</tr>
<tr>
<td>Bev</td>
<td>0.610</td>
<td>0.560</td>
<td>0.449</td>
<td>0.132</td>
<td>0.359</td>
<td>0.503</td>
<td>0.067</td>
<td>-0.118</td>
<td>-0.193</td>
<td>0.020</td>
<td>-0.085</td>
<td>-0.145</td>
<td>-0.273</td>
</tr>
<tr>
<td>Chem</td>
<td>0.438</td>
<td>0.227</td>
<td>0.266</td>
<td>-0.089</td>
<td>0.397</td>
<td>0.353</td>
<td>0.050</td>
<td>-0.299</td>
<td>0.240</td>
<td>0.070</td>
<td>-0.020</td>
<td>0.056</td>
<td>-0.040</td>
</tr>
<tr>
<td>GenInd</td>
<td>0.492</td>
<td>0.435</td>
<td>0.397</td>
<td>0.436</td>
<td>0.343</td>
<td>0.585</td>
<td>-0.116</td>
<td>0.198</td>
<td>-0.127</td>
<td>0.121</td>
<td>-0.018</td>
<td>-0.153</td>
<td>-0.157</td>
</tr>
<tr>
<td>Metal</td>
<td>0.523</td>
<td>0.555</td>
<td>0.580</td>
<td>0.455</td>
<td>0.154</td>
<td>0.647</td>
<td>-0.129</td>
<td>0.042</td>
<td>-0.256</td>
<td>0.134</td>
<td>-0.044</td>
<td>0.193</td>
<td>0.186</td>
</tr>
<tr>
<td>Mining</td>
<td>0.279</td>
<td>0.249</td>
<td>0.035</td>
<td>0.098</td>
<td>0.190</td>
<td>0.128</td>
<td>0.427</td>
<td>-0.557</td>
<td>0.015</td>
<td>0.200</td>
<td>-0.251</td>
<td>0.294</td>
<td>-0.001</td>
</tr>
<tr>
<td>OilSec</td>
<td>1</td>
<td>0.588</td>
<td>0.502</td>
<td>0.138</td>
<td>0.217</td>
<td>0.687</td>
<td>-0.020</td>
<td>0.068</td>
<td>0.008</td>
<td>-0.017</td>
<td>-0.104</td>
<td>0.217</td>
<td>0.178</td>
</tr>
<tr>
<td>Paper</td>
<td>1</td>
<td>0.317</td>
<td>0.302</td>
<td>0.275</td>
<td>0.355</td>
<td>0.514</td>
<td>-0.581</td>
<td>-0.102</td>
<td>0.261</td>
<td>-0.213</td>
<td>0.263</td>
<td>-0.016</td>
<td></td>
</tr>
<tr>
<td>TiWire</td>
<td>1</td>
<td>0.356</td>
<td>0.317</td>
<td>0.833</td>
<td>-0.291</td>
<td>0.226</td>
<td>-0.027</td>
<td>0.037</td>
<td>-0.168</td>
<td>-0.045</td>
<td>-0.126</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Text</td>
<td>1</td>
<td>0.000</td>
<td>0.353</td>
<td>0.180</td>
<td>-0.086</td>
<td>-0.089</td>
<td>0.091</td>
<td>-0.346</td>
<td>0.015</td>
<td>-0.058</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tobac</td>
<td>1</td>
<td>0.315</td>
<td>0.246</td>
<td>-0.201</td>
<td>-0.264</td>
<td>0.090</td>
<td>-0.064</td>
<td>0.122</td>
<td>-0.063</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Utility</td>
<td>1</td>
<td>-0.197</td>
<td>0.118</td>
<td>-0.251</td>
<td>-0.045</td>
<td>-0.035</td>
<td>-0.093</td>
<td>-0.119</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>URBH</td>
<td>1</td>
<td>-0.261</td>
<td>0.052</td>
<td>0.255</td>
<td>-0.399</td>
<td>0.246</td>
<td>0.251</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>URPA</td>
<td>1</td>
<td>-0.032</td>
<td>0.090</td>
<td>0.122</td>
<td>-0.234</td>
<td>0.092</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>URRE</td>
<td>1</td>
<td>-0.098</td>
<td>-0.297</td>
<td>0.005</td>
<td>0.086</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>URJ</td>
<td>1</td>
<td>0.084</td>
<td>0.583</td>
<td>0.717</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>URSA</td>
<td>1</td>
<td>0.000</td>
<td>0.164</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>URSP</td>
<td>1</td>
<td>0.866</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>URBR</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G + H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debit ratios</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lower bound</td>
<td>-</td>
<td>0.51</td>
<td>0.67</td>
<td>0.78</td>
<td>0.79</td>
<td>0.80</td>
<td>0.85</td>
</tr>
<tr>
<td>Target</td>
<td>0.38</td>
<td>0.61</td>
<td>0.82</td>
<td>0.84</td>
<td>0.88</td>
<td>0.89</td>
<td>0.89</td>
</tr>
<tr>
<td>Upper bound</td>
<td>0.53</td>
<td>0.78</td>
<td>0.90</td>
<td>0.92</td>
<td>0.93</td>
<td>0.93</td>
<td>0.95</td>
</tr>
<tr>
<td>Beta</td>
<td>0.67</td>
<td>0.85</td>
<td>1.00</td>
<td>1.10</td>
<td>1.20</td>
<td>1.30</td>
<td>1.36</td>
</tr>
<tr>
<td>Firm-specific risk</td>
<td>0.38</td>
<td>0.55</td>
<td>0.69</td>
<td>0.71</td>
<td>0.77</td>
<td>0.78</td>
<td>0.72</td>
</tr>
</tbody>
</table>
### Table 4 – Distribution of business loans by industry sector

<table>
<thead>
<tr>
<th>Industry Sector</th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ibovespa</td>
<td>0.0552</td>
<td>0.0070</td>
<td>0.0890</td>
</tr>
<tr>
<td>Aerospace</td>
<td>0.0017</td>
<td>0.0000</td>
<td>0.0075</td>
</tr>
<tr>
<td>Basic ind.</td>
<td>0.3415</td>
<td>0.2936</td>
<td>0.4264</td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.0416</td>
<td>0.0004</td>
<td>0.0600</td>
</tr>
<tr>
<td>Cyc. serv.</td>
<td>0.2734</td>
<td>0.2322</td>
<td>0.3347</td>
</tr>
<tr>
<td>Food prd</td>
<td>0.0809</td>
<td>0.0043</td>
<td>0.1073</td>
</tr>
<tr>
<td>Food ret</td>
<td>0.0520</td>
<td>0.0149</td>
<td>0.1622</td>
</tr>
<tr>
<td>Forestry</td>
<td>0.0160</td>
<td>0.0000</td>
<td>0.0282</td>
</tr>
<tr>
<td>Paper</td>
<td>0.0047</td>
<td>0.0000</td>
<td>0.0086</td>
</tr>
<tr>
<td>Mining</td>
<td>0.0067</td>
<td>0.0014</td>
<td>0.0125</td>
</tr>
<tr>
<td>Oil &amp; Gas</td>
<td>0.0254</td>
<td>0.0000</td>
<td>0.1232</td>
</tr>
<tr>
<td>Financial</td>
<td>0.0120</td>
<td>0.0013</td>
<td>0.0298</td>
</tr>
<tr>
<td>Utility</td>
<td>0.0888</td>
<td>0.0418</td>
<td>0.1770</td>
</tr>
</tbody>
</table>

### Table 5 – Distribution of business loans by credit quality

<table>
<thead>
<tr>
<th>Credit Quality</th>
<th>AA</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G+H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.2691</td>
<td>0.3403</td>
<td>0.1793</td>
<td>0.1286</td>
<td>0.0362</td>
<td>0.0149</td>
<td>0.0088</td>
<td>0.0227</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.0002</td>
<td>0.2351</td>
<td>0.0797</td>
<td>0.0499</td>
<td>0.0190</td>
<td>0.0007</td>
<td>0.0006</td>
<td>0.0019</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.4158</td>
<td>0.5108</td>
<td>0.2550</td>
<td>0.2293</td>
<td>0.0616</td>
<td>0.0350</td>
<td>0.0150</td>
<td>0.0365</td>
</tr>
</tbody>
</table>

### Table 6 – Estimated transition matrix, for brazilian companies, using the PSA approach

<table>
<thead>
<tr>
<th></th>
<th>AA</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G+H</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>90.35%</td>
<td>9.65%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>A</td>
<td>11.40%</td>
<td>79.63%</td>
<td>8.93%</td>
<td>0.05%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>B</td>
<td>0.40%</td>
<td>4.95%</td>
<td>75.33%</td>
<td>10.00%</td>
<td>3.10%</td>
<td>1.55%</td>
<td>3.45%</td>
<td>1.23%</td>
</tr>
<tr>
<td>C</td>
<td>0.15%</td>
<td>2.70%</td>
<td>12.60%</td>
<td>68.63%</td>
<td>4.88%</td>
<td>2.23%</td>
<td>5.48%</td>
<td>3.35%</td>
</tr>
<tr>
<td>D</td>
<td>0.03%</td>
<td>0.70%</td>
<td>4.45%</td>
<td>1.48%</td>
<td>61.48%</td>
<td>4.88%</td>
<td>8.85%</td>
<td>18.15%</td>
</tr>
<tr>
<td>E</td>
<td>0.00%</td>
<td>0.58%</td>
<td>3.76%</td>
<td>1.25%</td>
<td>0.85%</td>
<td>56.18%</td>
<td>10.48%</td>
<td>26.90%</td>
</tr>
<tr>
<td>F</td>
<td>0.00%</td>
<td>0.23%</td>
<td>2.33%</td>
<td>1.23%</td>
<td>0.75%</td>
<td>7.30%</td>
<td>60.38%</td>
<td>27.80%</td>
</tr>
</tbody>
</table>
### Table 7 – Brazilian credit risk bureau’s transition matrix (Adjusted for repayments, for two banks and weighted averaged between the periods of June 2000 to June 2001, and June 2001 to June 2002)

<table>
<thead>
<tr>
<th></th>
<th>AA</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G+H</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>90.08%</td>
<td>6.43%</td>
<td>2.05%</td>
<td>0.53%</td>
<td>0.18%</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.68%</td>
</tr>
<tr>
<td>A</td>
<td>11.90%</td>
<td>69.03%</td>
<td>10.15%</td>
<td>4.73%</td>
<td>2.13%</td>
<td>0.30%</td>
<td>0.43%</td>
<td>1.40%</td>
</tr>
<tr>
<td>B</td>
<td>3.28%</td>
<td>11.03%</td>
<td>71.88%</td>
<td>9.23%</td>
<td>2.00%</td>
<td>0.48%</td>
<td>0.55%</td>
<td>1.63%</td>
</tr>
<tr>
<td>C</td>
<td>3.28%</td>
<td>4.18%</td>
<td>15.25%</td>
<td>67.35%</td>
<td>4.65%</td>
<td>0.90%</td>
<td>1.33%</td>
<td>3.08%</td>
</tr>
<tr>
<td>D</td>
<td>1.06%</td>
<td>1.85%</td>
<td>4.00%</td>
<td>5.13%</td>
<td>60.20%</td>
<td>3.90%</td>
<td>5.43%</td>
<td>18.43%</td>
</tr>
<tr>
<td>E</td>
<td>0.13%</td>
<td>7.75%</td>
<td>0.53%</td>
<td>0.83%</td>
<td>4.05%</td>
<td>55.80%</td>
<td>4.03%</td>
<td>26.83%</td>
</tr>
<tr>
<td>F</td>
<td>0.78%</td>
<td>0.60%</td>
<td>1.15%</td>
<td>2.25%</td>
<td>3.10%</td>
<td>7.60%</td>
<td>56.80%</td>
<td>27.63%</td>
</tr>
</tbody>
</table>

### Table 8 – Difference between simulated and historical transition matrices

<table>
<thead>
<tr>
<th></th>
<th>AA</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G+H</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>0.27%</td>
<td>3.23%</td>
<td>-2.05%</td>
<td>-0.53%</td>
<td>-0.18%</td>
<td>-0.03%</td>
<td>-0.03%</td>
<td>-0.68%</td>
</tr>
<tr>
<td>A</td>
<td>-0.50%</td>
<td>10.60%</td>
<td>-1.23%</td>
<td>-4.68%</td>
<td>-2.13%</td>
<td>-0.30%</td>
<td>-0.43%</td>
<td>-1.40%</td>
</tr>
<tr>
<td>B</td>
<td>-2.88%</td>
<td>-0.68%</td>
<td>3.45%</td>
<td>0.78%</td>
<td>1.10%</td>
<td>1.08%</td>
<td>2.90%</td>
<td>-0.40%</td>
</tr>
<tr>
<td>C</td>
<td>-3.13%</td>
<td>-1.48%</td>
<td>-2.65%</td>
<td>1.28%</td>
<td>0.23%</td>
<td>1.33%</td>
<td>4.15%</td>
<td>0.28%</td>
</tr>
<tr>
<td>D</td>
<td>-1.05%</td>
<td>-1.15%</td>
<td>0.45%</td>
<td>-3.65%</td>
<td>1.28%</td>
<td>0.98%</td>
<td>3.43%</td>
<td>-0.28%</td>
</tr>
<tr>
<td>E</td>
<td>-0.13%</td>
<td>-7.18%</td>
<td>3.25%</td>
<td>0.43%</td>
<td>-3.20%</td>
<td>0.37%</td>
<td>6.45%</td>
<td>0.08%</td>
</tr>
<tr>
<td>F</td>
<td>-0.78%</td>
<td>-0.38%</td>
<td>1.18%</td>
<td>-1.03%</td>
<td>-2.35%</td>
<td>-0.30%</td>
<td>3.58%</td>
<td>0.16%</td>
</tr>
</tbody>
</table>

### Table 9

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liabilities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic funding</td>
<td>0.5694</td>
<td>0.5168</td>
<td>0.6194</td>
</tr>
<tr>
<td>Foreign funding</td>
<td>0.1047</td>
<td>0.0708</td>
<td>0.1768</td>
</tr>
<tr>
<td>Non-interest liability</td>
<td>0.0659</td>
<td>0.0264</td>
<td>0.0809</td>
</tr>
<tr>
<td>Capital and reserves</td>
<td>0.1185</td>
<td>0.0440</td>
<td>0.2362</td>
</tr>
<tr>
<td>Debt</td>
<td>0.1414</td>
<td>0.0383</td>
<td>0.2219</td>
</tr>
<tr>
<td>Total liabilities</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Assets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Money</td>
<td>0.0336</td>
<td>0.0003</td>
<td>0.0705</td>
</tr>
<tr>
<td>Risk-free loans</td>
<td>0.2556</td>
<td>0.0226</td>
<td>0.5229</td>
</tr>
<tr>
<td>Business loans</td>
<td>0.1859</td>
<td>0.0002</td>
<td>0.3494</td>
</tr>
<tr>
<td>Consumer loans</td>
<td>0.1959</td>
<td>0.0001</td>
<td>0.5630</td>
</tr>
<tr>
<td>Foreign loans</td>
<td>0.0745</td>
<td>0.0000</td>
<td>0.1721</td>
</tr>
<tr>
<td>Equity investments</td>
<td>0.0096</td>
<td>0.0000</td>
<td>0.0183</td>
</tr>
<tr>
<td>Real estate investments</td>
<td>0.0118</td>
<td>0.0102</td>
<td>0.0146</td>
</tr>
<tr>
<td>Other assets (Non-interest)</td>
<td>0.2330</td>
<td>0.1353</td>
<td>0.3251</td>
</tr>
<tr>
<td>Total assets</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
</tr>
<tr>
<td>Capital ratio</td>
<td>0.1185</td>
<td>0.0440</td>
<td>0.2362</td>
</tr>
<tr>
<td>Operating expense ratio</td>
<td>-0.013</td>
<td>-0.020</td>
<td>-0.007</td>
</tr>
<tr>
<td>Tax rate</td>
<td>0.3400</td>
<td>0.3400</td>
<td>0.3400</td>
</tr>
</tbody>
</table>
### Table 10a – Interest rate spreads for business' loans
(High interest rate scenario)

<table>
<thead>
<tr>
<th>Credit risk categories</th>
<th>Default rate</th>
<th>Loss rate spread</th>
<th>Loss rate profile</th>
<th>U.S. Risk spread</th>
<th>Assumed risk</th>
<th>Assumed risk spread (scaled by U.S.) (total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>0.68%</td>
<td>0.85</td>
<td>0.58%</td>
<td>0.13%</td>
<td>5.01%</td>
<td>5.59%</td>
</tr>
<tr>
<td>A</td>
<td>1.40%</td>
<td>0.85</td>
<td>1.19%</td>
<td>0.50%</td>
<td>20.04%</td>
<td>21.23%</td>
</tr>
<tr>
<td>B</td>
<td>1.63%</td>
<td>0.85</td>
<td>1.39%</td>
<td>0.75%</td>
<td>30.06%</td>
<td>31.45%</td>
</tr>
<tr>
<td>C</td>
<td>3.08%</td>
<td>0.85</td>
<td>2.62%</td>
<td>1.00%</td>
<td>40.08%</td>
<td>42.70%</td>
</tr>
<tr>
<td>D</td>
<td>18.43%</td>
<td>0.85</td>
<td>15.67%</td>
<td>1.50%</td>
<td>60.12%</td>
<td>75.79%</td>
</tr>
<tr>
<td>E</td>
<td>26.83%</td>
<td>0.85</td>
<td>22.81%</td>
<td>2.00%</td>
<td>80.16%</td>
<td>102.97%</td>
</tr>
<tr>
<td>F</td>
<td>27.63%</td>
<td>0.85</td>
<td>23.49%</td>
<td>2.50%</td>
<td>100.20%</td>
<td>123.69%</td>
</tr>
<tr>
<td>G + H</td>
<td>100.00%</td>
<td>0.85</td>
<td>85.00%</td>
<td>3.00%</td>
<td>120.24%</td>
<td>205.24%</td>
</tr>
</tbody>
</table>

### Table 10b – Interest rate spreads for consumers' loans
(High interest rate scenario)

<table>
<thead>
<tr>
<th>Credit risk categories</th>
<th>Default rate</th>
<th>Loss rate spread</th>
<th>Loss rate profile</th>
<th>U.S. Risk spread</th>
<th>Assumed risk</th>
<th>Assumed risk spread (scaled by U.S.) (Total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA</td>
<td>0.68%</td>
<td>0.85</td>
<td>0.58%</td>
<td>0.13%</td>
<td>7.69%</td>
<td>8.27%</td>
</tr>
<tr>
<td>A</td>
<td>1.40%</td>
<td>0.85</td>
<td>1.19%</td>
<td>0.50%</td>
<td>30.76%</td>
<td>31.95%</td>
</tr>
<tr>
<td>B</td>
<td>1.63%</td>
<td>0.85</td>
<td>1.39%</td>
<td>0.75%</td>
<td>46.15%</td>
<td>47.53%</td>
</tr>
<tr>
<td>C</td>
<td>3.08%</td>
<td>0.85</td>
<td>2.62%</td>
<td>1.00%</td>
<td>61.53%</td>
<td>64.15%</td>
</tr>
<tr>
<td>D</td>
<td>18.43%</td>
<td>0.85</td>
<td>15.67%</td>
<td>1.50%</td>
<td>92.29%</td>
<td>107.96%</td>
</tr>
<tr>
<td>E</td>
<td>26.83%</td>
<td>0.85</td>
<td>22.81%</td>
<td>2.00%</td>
<td>123.06%</td>
<td>145.86%</td>
</tr>
<tr>
<td>F</td>
<td>27.63%</td>
<td>0.85</td>
<td>23.49%</td>
<td>2.50%</td>
<td>153.82%</td>
<td>177.31%</td>
</tr>
<tr>
<td>G + H</td>
<td>100.00%</td>
<td>0.85</td>
<td>85.00%</td>
<td>3.00%</td>
<td>184.58%</td>
<td>269.58%</td>
</tr>
</tbody>
</table>
Table 11 – Goodness of fit for ROE and ROA and unbiased tests

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Adjusted R²</th>
<th>Wald Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: ROE Regressions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.87***</td>
<td>41.66%</td>
<td>0.09</td>
</tr>
<tr>
<td>Standard Error</td>
<td>(0.41)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t Statistic</td>
<td>[2.14]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.19**</td>
<td>69.38%</td>
<td>0.7</td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t Statistic</td>
<td>[3.41]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel b: ROA Regressions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>1.49*</td>
<td>97.07%</td>
<td>18.10*</td>
</tr>
<tr>
<td>Standard Error</td>
<td>(0.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t Statistic</td>
<td>[12.90]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.19***</td>
<td>49.47%</td>
<td>0.23</td>
</tr>
<tr>
<td>Standard Error</td>
<td>(0.49)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t Statistic</td>
<td>[2.43]</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Panel c: Pool</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ROE</td>
<td>1.004*</td>
<td>63.17%</td>
<td>0.1294</td>
</tr>
<tr>
<td>Standard Error</td>
<td>(0.23)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t Statistic</td>
<td>[4.46]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ROE</td>
<td>1.31*</td>
<td>85.17%</td>
<td>3.71</td>
</tr>
<tr>
<td>Standard Error</td>
<td>(0.23)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t Statistic</td>
<td>[8.01]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0.97*</td>
<td>81.13%</td>
<td>0.22</td>
</tr>
<tr>
<td>Standard Error</td>
<td>(0.10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t Statistic</td>
<td>[9.99]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*, ** and *** correspond to the levels of significance of 1.5 and 10% respectively.
### Table 12a

<table>
<thead>
<tr>
<th>Bank</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.256</td>
<td>0.237</td>
<td>0.056</td>
<td>0.028</td>
<td>0.139</td>
<td>0.124</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.011</td>
<td>0.013</td>
<td>0.008</td>
<td>0.012</td>
<td>0.008</td>
<td>0.009</td>
</tr>
<tr>
<td>Max</td>
<td>0.288</td>
<td>0.271</td>
<td>0.071</td>
<td>0.047</td>
<td>0.160</td>
<td>0.145</td>
</tr>
<tr>
<td>Min</td>
<td>0.189</td>
<td>0.156</td>
<td>-0.005</td>
<td>-0.049</td>
<td>0.097</td>
<td>0.072</td>
</tr>
<tr>
<td>VaR 99%</td>
<td>0.227</td>
<td>0.196</td>
<td>0.028</td>
<td>-0.013</td>
<td>0.118</td>
<td>0.096</td>
</tr>
<tr>
<td>98%</td>
<td>0.231</td>
<td>0.203</td>
<td>0.033</td>
<td>-0.007</td>
<td>0.119</td>
<td>0.098</td>
</tr>
<tr>
<td>97%</td>
<td>0.233</td>
<td>0.206</td>
<td>0.037</td>
<td>-0.002</td>
<td>0.122</td>
<td>0.103</td>
</tr>
<tr>
<td>96%</td>
<td>0.235</td>
<td>0.209</td>
<td>0.039</td>
<td>0.001</td>
<td>0.123</td>
<td>0.104</td>
</tr>
<tr>
<td>95%</td>
<td>0.236</td>
<td>0.211</td>
<td>0.040</td>
<td>0.004</td>
<td>0.124</td>
<td>0.106</td>
</tr>
<tr>
<td>94%</td>
<td>0.237</td>
<td>0.213</td>
<td>0.041</td>
<td>0.006</td>
<td>0.125</td>
<td>0.107</td>
</tr>
<tr>
<td>93%</td>
<td>0.239</td>
<td>0.216</td>
<td>0.043</td>
<td>0.008</td>
<td>0.126</td>
<td>0.109</td>
</tr>
<tr>
<td>92%</td>
<td>0.240</td>
<td>0.217</td>
<td>0.044</td>
<td>0.010</td>
<td>0.127</td>
<td>0.111</td>
</tr>
<tr>
<td>91%</td>
<td>0.241</td>
<td>0.219</td>
<td>0.044</td>
<td>0.011</td>
<td>0.128</td>
<td>0.112</td>
</tr>
<tr>
<td>90%</td>
<td>0.242</td>
<td>0.220</td>
<td>0.045</td>
<td>0.012</td>
<td>0.128</td>
<td>0.113</td>
</tr>
<tr>
<td>75%</td>
<td>0.249</td>
<td>0.231</td>
<td>0.052</td>
<td>0.024</td>
<td>0.134</td>
<td>0.119</td>
</tr>
<tr>
<td>50%</td>
<td>0.256</td>
<td>0.238</td>
<td>0.057</td>
<td>0.032</td>
<td>0.140</td>
<td>0.125</td>
</tr>
<tr>
<td>25%</td>
<td>0.263</td>
<td>0.245</td>
<td>0.061</td>
<td>0.036</td>
<td>0.145</td>
<td>0.130</td>
</tr>
<tr>
<td>1%</td>
<td>0.278</td>
<td>0.260</td>
<td>0.067</td>
<td>0.043</td>
<td>0.153</td>
<td>0.138</td>
</tr>
</tbody>
</table>

### Table 12b

<table>
<thead>
<tr>
<th>Bank</th>
<th>4</th>
<th>4</th>
<th>5</th>
<th>5</th>
<th>6</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.104</td>
<td>0.085</td>
<td>0.075</td>
<td>0.060</td>
<td>0.120</td>
<td>0.105</td>
</tr>
<tr>
<td>Std. dev.</td>
<td>0.010</td>
<td>0.012</td>
<td>0.014</td>
<td>0.017</td>
<td>0.023</td>
<td>0.026</td>
</tr>
<tr>
<td>Max</td>
<td>0.124</td>
<td>0.108</td>
<td>0.111</td>
<td>0.100</td>
<td>0.177</td>
<td>0.165</td>
</tr>
<tr>
<td>Min</td>
<td>0.042</td>
<td>0.012</td>
<td>0.010</td>
<td>-0.012</td>
<td>-0.007</td>
<td>-0.034</td>
</tr>
<tr>
<td>VaR 99%</td>
<td>0.071</td>
<td>0.044</td>
<td>0.029</td>
<td>0.008</td>
<td>0.055</td>
<td>0.031</td>
</tr>
<tr>
<td>98%</td>
<td>0.077</td>
<td>0.050</td>
<td>0.039</td>
<td>0.019</td>
<td>0.063</td>
<td>0.039</td>
</tr>
<tr>
<td>97%</td>
<td>0.083</td>
<td>0.055</td>
<td>0.043</td>
<td>0.023</td>
<td>0.069</td>
<td>0.046</td>
</tr>
<tr>
<td>96%</td>
<td>0.085</td>
<td>0.058</td>
<td>0.045</td>
<td>0.026</td>
<td>0.074</td>
<td>0.051</td>
</tr>
<tr>
<td>95%</td>
<td>0.087</td>
<td>0.060</td>
<td>0.048</td>
<td>0.028</td>
<td>0.077</td>
<td>0.055</td>
</tr>
<tr>
<td>94%</td>
<td>0.088</td>
<td>0.062</td>
<td>0.050</td>
<td>0.031</td>
<td>0.081</td>
<td>0.059</td>
</tr>
<tr>
<td>93%</td>
<td>0.089</td>
<td>0.064</td>
<td>0.052</td>
<td>0.033</td>
<td>0.084</td>
<td>0.062</td>
</tr>
<tr>
<td>92%</td>
<td>0.090</td>
<td>0.065</td>
<td>0.054</td>
<td>0.035</td>
<td>0.086</td>
<td>0.064</td>
</tr>
<tr>
<td>91%</td>
<td>0.091</td>
<td>0.067</td>
<td>0.056</td>
<td>0.036</td>
<td>0.088</td>
<td>0.066</td>
</tr>
<tr>
<td>90%</td>
<td>0.091</td>
<td>0.068</td>
<td>0.056</td>
<td>0.037</td>
<td>0.090</td>
<td>0.068</td>
</tr>
<tr>
<td>75%</td>
<td>0.099</td>
<td>0.080</td>
<td>0.068</td>
<td>0.049</td>
<td>0.106</td>
<td>0.091</td>
</tr>
<tr>
<td>50%</td>
<td>0.105</td>
<td>0.088</td>
<td>0.077</td>
<td>0.063</td>
<td>0.122</td>
<td>0.109</td>
</tr>
<tr>
<td>25%</td>
<td>0.110</td>
<td>0.094</td>
<td>0.085</td>
<td>0.073</td>
<td>0.136</td>
<td>0.123</td>
</tr>
<tr>
<td>1%</td>
<td>0.119</td>
<td>0.103</td>
<td>0.099</td>
<td>0.088</td>
<td>0.162</td>
<td>0.150</td>
</tr>
</tbody>
</table>