Modeling Business Loan Credit Risk in Brazil

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Synopsis

Modeling credit transition probabilities is central to fixed income portfolio risk assessments. Recently Banco Central Do Brasil has established a Credit Risk Bureau which collects information on bank credit ratings for borrowers, and credit transitions probabilities. We estimate the parameters for and implement a credit risk model developed by Barnhill, et. al. in the context of the Brazilian financial environment. The model is shown to produce credit transition probabilities very similar to those reported by the Credit Risk Bureau. This credit risk model has the potential to be utilized in many ways including the assessment of bank failure risk, bank capital requirements, etc.

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1 – Introduction

In this paper we present a risk assessment methodology for business loans in the Brazilian economy. We use the portfolio simulation approach which integrates both credit and market risk and estimate credit transition matrix for business loans that matches historical transition matrix that was collected by the newly Brazilian Credit Risk Bureau.

This is the first attempt to model business credit risk for Brazilian companies. Modeling of credit risk is very useful for the establishment of capital requirements for banks, for the estimation of bank failures among others and thus is essential for regulators.

The paper is divided as follows. The next section presents an overview of the conceptual framework. Section 3 discusses the calibration of the model. In section 4 results for the simulated credit transition matrix are presented, while section 5 concludes the paper.

2 – A conceptual framework for fixed income portfolio integrated market and credit risk assessment

Financial environment simulation modeling combined with portfolio theory offers a very promising integrated risk assessment approach. In general, risk assessment methodologies seek to assess the maximum potential change in the value of a portfolio with a given probability over a pre-set horizon resulting from changes in market factors, credit risk, and liquidity risk. The current practice is to undertake market and credit risk assessments separately. Combining such separate risk measures into one overall portfolio risk measure is not easily accomplished. The absence of reliable overall portfolio risk measures creates problems for determining capital adequacy requirements, capital-at-risk measures, hedging strategies, etc.
Given the correlated nature of credit and market risk (Fridson, Garman, and Wu 1997), the importance of an integrated risk assessment methodology seems apparent. To address the above risk measurement problem Barnhill and Maxwell (2000) develop a diffusion-based methodology for assessing the value-at-risk (VaR) of a portfolio of fixed income securities with correlated interest rate, interest rate spread, exchange rate, and credit risk. Barnhill, Papapanagiotou, and Schumacher (2000) extend the model to undertake financial institution asset and liability risk assessments for South African banks. Barnhill, Papapanagiotou, and Souto (2001) apply the model to Japanese banks.

As an overview, both the future financial environment in which the assets will be valued and the credit rating of specific loans are simulated. The financial environment can be represented by any number of correlated random variables. The correlated evolution of the market value of a business firm's equity, its debt ratio, and credit rating are then simulated in the context of the simulated financial environment. The structure of the methodology is to select a time step over which the stochastic variables are allowed to fluctuate in a correlated random process. The firm specific returns (as distinct from economic sector index) and security specific default recovery rates are assumed to be uncorrelated with each other and the other stochastic variables. For each simulation run a new financial environment (correlated interest rate term structures, market equity returns, etc.) as well as firm specific debt ratios, credit rating, and default recovery rates are created. This information allows the correlated values of financial assets (including direct equity and real estate investments) to be estimated, and after a large number of simulations, a distribution of portfolio values is generated and analyzed.

### 2.1 Simulating interest rates

The Hull and White extended Vasicek model (Hull and White; 1990a, 1993, 1994) is used to model stochastic risk-free interest rates. In this model interest rates are assumed to follow a mean-reversion process with a time dependent reversion level. The simulation model is robust to the use of other interest rate models. The model for \( r \) is:
\[ \Delta r = a \left( \frac{\theta(t)}{a} - r \right) \Delta t + \sigma \Delta z \]

where
\[ \Delta r = \text{the risk-neutral process by which } r \text{ changes;} \]
\[ a = \text{the rate at which } r \text{ reverts to its long term mean;} \]
\[ r = \text{the instantaneous continuously compounded short-term interest rate;} \]
\[ \theta(t) = \text{"Theta" is an unknown function of time that is chosen so that the model is consistent with the initial term structure and is calculated from the initial term structure;} \]
\[ \Delta t = \text{a small increment to time;} \]
\[ \sigma = \text{"sigma" the instantaneous standard deviation of } r, \text{ which is assumed to be constant; and} \]
\[ \Delta z = \text{a Wiener process driving term structure movements with } \Delta r \text{ being related to } \Delta t \text{ by the function } \Delta z = \varepsilon \sqrt{\Delta t}. \]

The above mean reversion and volatility rates can be estimated from a time series of short-term interest rates or implied from cap and floor prices. Credit spreads can either be modeled as correlated log normal variables or as fixed values.

### 2.2 Simulating asset prices and returns

The model utilized to simulate the value of the equity market indices (S) assumes that (S) follows a geometric Brownian motion where the expected growth rate (m) and volatility (\( \sigma \)) are constant (Hull 1997, p. 362). The expected growth rate is equal to the expected return on the asset (\( \mu \)) minus its dividend yield (q). For a discrete time step, \( \Delta t \), it can be shown that

\[ S + \Delta S = S \exp \left[ \left( m - \frac{\sigma^2}{2} \right) \Delta t + \sigma \varepsilon \sqrt{\Delta t} \right] \]

where:
$\varepsilon$ is a random sample from a standardized normal distribution.

The return on the market index ($K_m$) is estimated as

$$K_m = \ln \left( (S + \Delta S)/S \right) + q$$

(3)

The return on equity for individual firms and individual real estate properties is simulated using a one-factor model.

$$K_i = R_F + Beta_i (Km - R_F) + \sigma_i \Delta z$$

(4)

where

$K_i =$ the return for the asset $i$;

$R_F =$ the risk-free interest rate;

$Beta_i =$ the systematic risk of asset $i$;

$K_m =$ the simulated return on the equity or real estate index from equation 3;

$\sigma_i =$ the asset specific return volatility; and

$\Delta z =$ a Wiener process with $\Delta z$ being related to $\Delta t$ by the function $\Delta z = \varepsilon \sqrt{\Delta t}$.

As discussed in the next section the parameters needed to implement the above model were estimated from historical data on the Brazilian financial market.

### 2.3 Simulating an n-variate normal distribution

Many authors have reported positive correlations between default rates and financial environment variables such as interest rates (see Fridson et. al. (1997)), and negative correlations with variable such as GNP growth rates. This is consistent with negative correlations between interest rate changes and equity returns.

In the proposed portfolio risk assessment model, the equity indices and FX rate returns are simulated as stochastic variables correlated with the simulated future risk-free interest rate and interest rate
spreads. Hull (1997) describes a procedure for working with an n-variate normal distribution. This procedure requires the specification of correlations between each of the n stochastic variables. Subsequently n independent random samples are drawn from standardized normal distributions. With this information the set of correlated random error terms for the n stochastic variables can be calculated. For example, for a bivariate normal distribution,

\[ \varepsilon_1 = x_1 \]  
\[ \varepsilon_2 = \rho x_1 + x_2 \sqrt{1 - \rho^2} \]

where

- \( x_1, x_2 = \) independent random samples from standardized normal distributions;
- \( \rho = \) the correlation between the two stochastic variables; and
- \( \varepsilon_1, \varepsilon_2 = \) the required samples from a standardized bivariate normal distribution.

It can be shown that the simulated volatilities and correlations for all of the stochastic variables match closely the assumed values that are typically estimated from historical time series data.

### 2.4 Mapping debt ratios into credit ratings

The above-discussed simulated equity returns are then used to estimate a distribution of possible future firm equity values and debt ratios. The simulated debt ratios are then mapped into credit ratings. This methodology assumes a deterministic relation between a firm's or property's debt ratio and its credit rating. In a contingent claims^4

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^4 Blume, Lim, and MacKinlay (1998) suggest that leverage ratios and credit ratings are not constant over time. However, their results are over a longer time frame than simulated in this framework.
framework this is equivalent to assuming a constant volatility for the value of the firm.

After simulating the bond's or loan's future credit rating its value is calculated using the simulated term structure of interest rates appropriate for that risk class. If the bond or loan is simulated to default, the recovery rate on the bond is simulated as a beta distribution\(^5\) with a specified mean value and standard deviation.

### 3 – Model calibration for Brazil

A very important piece of information in the business credit risk modeling concerns the estimation of risk of companies in the economy. Using 12 sector indices for Brazil betas for over 500 companies were estimated. Both data for sector indices and individual stocks were collected from Datastream.

In Table 1 we present the distribution of betas and unsystematic risk for all credit ratings categories. One would expect the betas to have an upward trend, which is not seen in this estimation. One guess would be that this is due to the illiquid nature of the Brazilian equity market. As many stocks lack liquidity price series tend to have artificial rigidities that are certainly lowering the betas, and misleading empirical evidence\(^6\).

In order to circumvent the problems inherent in the estimation of the betas we propose to unleverage betas for all companies that were rated AA, which comprises the most liquid stocks and using the 25th,

| Table 1 – Distributions of betas and unsystematic risk by credit ratings |
|----------------|-------|-------|-------|-------|-------|-------|-------|
| Beta           | AA    | A     | B     | C     | D     | E     | F     |
|                | 0.670 | 0.564 | 0.560 | 0.365 | 0.565 | 0.273 | 0.475 |
| Uns systematic risk | 0.380 | 0.580 | 0.732 | 0.761 | 0.805 | 0.839 | 0.771 |

\(^5\) Utilizing a beta distribution allows the recovery rate to fall within 0% and 100% while maintaining the same mean and standard deviation.

\(^6\) A program in SAS was developed to estimate betas for Brazilian companies. Three major attempts were made. Firstly estimations were done on a daily basis. However, due to the lack of liquidity and price rigidities betas tended to be lower than one would expect. Thus we have estimated these betas on a monthly basis and also building series that didn’t have any repetitions in the price series. These estimations showed similar results and thus one decided to present results for the regressions using monthly observations. Nonetheless, results still were unsatisfactory as betas were declining with credit quality and theory tells us that we should expect the opposite result.
We used the following equation

$$\beta_U = \frac{\beta_L}{1 + (1 - T_c) \frac{B}{S}}$$  \hspace{1cm} (7)$$

where $\beta_U$ is the unleveraged beta, $\beta_L$ is the leveraged beta, $T_c$ is the tax rate, $B$ represents the current market value of outstanding debt and $S$ is the market value of equity.

The median unleveraged beta for the AA credit rating class is equal to 0.531. In Table 2 we present lower, upper and target for debt to value ratios for each credit rating category. We also show the leveraged betas using three different debt to value ratios.

In order to compare the betas that we have found within this methodology we present betas for the Ibovespa (Index of the Bolsa de Sao Paulo, 55 stocks), IBX (Brazil Index, 102 stocks) and a set of 192 stocks.

As we can see these betas range from 0.302 to 1.363 for the set of 192 stocks which is similar to the betas that were found in Barnhill and Maxwell (2002) for the US. This empirical evidence suggests that betas should be in that range. Therefore, we have decided to assume that we should see a similar trend for betas according to credit quality and have plugged in betas that are more consistent with stylized facts.

An important issue that has to be addressed is whether there is uncertainty on the credit rating

7/ In the authors own estimation the betas range from 0.032 to 1.497.
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attrition by the banks. Since different banks had different associated credit ratings for the same companies we had to build a weighted average of these ratings, where the weights were given by the ratio of the volume of loans for each bank to total loans for a particular firm.

We present below the distribution of the rating assignments. As we can see many banks have assigned more than one rating as the mean is almost three, while the median is two. Nonetheless, in some cases the same company (borrower) have received all credit ratings that exist from different banks. These results suggest that there is some degree of uncertainty in the credit rating assignments from banks.

### 4 – Simulated transition matrix

With the calibration of the model we can estimate the transition matrix for business loans in Brazil. Table 6 present the historical transition matrix for the entire financial system (average of the transition matrix of 2000-2001 and 2001-2002).

From the discussion in the previous section we have inferred that there are some uncertainty in the borrowers credit risk quality assignments by banks. Thus we also present the historical transition matrix for two banks, which are seen as having good credit rating models. Results are presented in Table 7.

From visual inspection one can see that the transition matrix for these two banks is more consistent and the model was calibrated to

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Table 6 – Historical transition matrix for the Financial System

<table>
<thead>
<tr>
<th>Value</th>
<th>AA</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G+H+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write-off</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AA</td>
<td>0.898</td>
<td>0.050</td>
<td>0.030</td>
<td>0.008</td>
<td>0.009</td>
<td>0.001</td>
<td>0.002</td>
<td>0.005</td>
</tr>
<tr>
<td>A</td>
<td>0.069</td>
<td>0.773</td>
<td>0.066</td>
<td>0.045</td>
<td>0.022</td>
<td>0.005</td>
<td>0.003</td>
<td>0.018</td>
</tr>
<tr>
<td>B</td>
<td>0.079</td>
<td>0.103</td>
<td>0.688</td>
<td>0.047</td>
<td>0.032</td>
<td>0.014</td>
<td>0.006</td>
<td>0.033</td>
</tr>
<tr>
<td>C</td>
<td>0.031</td>
<td>0.085</td>
<td>0.110</td>
<td>0.641</td>
<td>0.050</td>
<td>0.013</td>
<td>0.012</td>
<td>0.060</td>
</tr>
<tr>
<td>D</td>
<td>0.038</td>
<td>0.077</td>
<td>0.060</td>
<td>0.099</td>
<td>0.498</td>
<td>0.028</td>
<td>0.028</td>
<td>0.173</td>
</tr>
<tr>
<td>E</td>
<td>0.030</td>
<td>0.056</td>
<td>0.022</td>
<td>0.028</td>
<td>0.035</td>
<td>0.489</td>
<td>0.101</td>
<td>0.241</td>
</tr>
<tr>
<td>F</td>
<td>0.016</td>
<td>0.013</td>
<td>0.031</td>
<td>0.019</td>
<td>0.022</td>
<td>0.026</td>
<td>0.552</td>
<td>0.322</td>
</tr>
</tbody>
</table>

Table 7 – Historical transition matrix for the average of two banks

<table>
<thead>
<tr>
<th>Value</th>
<th>AA</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G+H+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write-off</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AA</td>
<td>0.901</td>
<td>0.064</td>
<td>0.021</td>
<td>0.005</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
<td>0.007</td>
</tr>
<tr>
<td>A</td>
<td>0.119</td>
<td>0.690</td>
<td>0.102</td>
<td>0.047</td>
<td>0.021</td>
<td>0.003</td>
<td>0.004</td>
<td>0.014</td>
</tr>
<tr>
<td>B</td>
<td>0.033</td>
<td>0.110</td>
<td>0.719</td>
<td>0.092</td>
<td>0.020</td>
<td>0.005</td>
<td>0.006</td>
<td>0.016</td>
</tr>
<tr>
<td>C</td>
<td>0.033</td>
<td>0.042</td>
<td>0.153</td>
<td>0.674</td>
<td>0.047</td>
<td>0.009</td>
<td>0.013</td>
<td>0.031</td>
</tr>
<tr>
<td>D</td>
<td>0.011</td>
<td>0.019</td>
<td>0.040</td>
<td>0.051</td>
<td>0.602</td>
<td>0.039</td>
<td>0.054</td>
<td>0.184</td>
</tr>
<tr>
<td>E</td>
<td>0.001</td>
<td>0.078</td>
<td>0.005</td>
<td>0.008</td>
<td>0.041</td>
<td>0.558</td>
<td>0.040</td>
<td>0.268</td>
</tr>
<tr>
<td>F</td>
<td>0.008</td>
<td>0.006</td>
<td>0.012</td>
<td>0.023</td>
<td>0.031</td>
<td>0.076</td>
<td>0.568</td>
<td>0.276</td>
</tr>
</tbody>
</table>

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8/ This could be due to the fact that modeling credit risk is a challenging task and banks have been doing that for a few years and thus it still difficult to backtest their own internal credit models and assess whether these models are satisfactory.
simulate this credit transition matrix. The simulated matrix is presented in Table 8.

Table 9 shows the deltas, difference between simulation and historical transition matrix. The lower these number the better the fit for the transition matrix.

In Table 10 we present the distribution analysis for the absolute deviations given in the table 9. As we can see the mean absolute deviation is less than 2%, while more than 75% of the absolute deviations of the entire matrix are less than 3%. These results suggest that the simulation performed is able to reproduce the observed credit transition matrix for the period 2000-2002.

### 5 – Conclusions and directions for further research

With the help of a new database collected from the Credit Risk Bureau of the Central Bank of Brazil we have modeled business credit risk within a portfolio simulation approach and simulated credit transition matrix for business loans in Brazil. Evidence suggests that we can match closely the historical observed transition matrix.

This exercise is quite important as it gives the first steps toward estimating joint default probabilities for banks in the Brazilian financial system. Furthermore, it can be used to assess capital requirements for banks, which turns to be very useful for regulators.
Bibliography


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