Vector autoregression model with long-term anchoring

Long-term projections from vector autoregression models (VARs) have as one of their characteristics their convergence to the unconditional mean of their variables, directly related to the constant – or intercept – present in each equation. However, certain changes in economy that affect the variables longer-term perspective are not easily incorporated into traditional VARs, especially in the case of recent changes.

As for Brazil, credibility gains and the convergence process of expectations to target in the most recent period are important changes to the dynamics of inflation. Moreover, after 14 years with a 4.50% annual inflation target, the National Monetary Council (CMN), in June 2017, set lower targets, of 4.25% for 2019 and of 4.00% for 2020. These changes were reflected by the behavior of the expectations of market analysts captured by the Focus survey conducted by the Central Bank of Brazil (BCB).

This box presents a VAR model that allows the anchoring of its long-term projection in the expectations of Focus survey, thus incorporating important recent changes in the Brazilian economy.[1]

In traditional VARs, changes that affect long-run inflation behavior do not have an immediate impact on model projections. In these cases, it is necessary to wait for a significant increase in the sample so that these changes affect the unconditional inflation average and, consequently, the long-term forecasts. In addition, in an environment with more than one regime, the coefficients of VAR models reflect a mean behavior of the dynamics between the regimes of each variable.

In contrast to the inertial behavior of VARs, changes in the conduct of monetary policy or the setting of targets at values other than those usually defined can be quickly captured by the agents' expectations for long-term inflation, since those expectations tend to reflect structural changes in the level of inflation, as they are free of influence of economic cycles. Incorporating these expectations into VARs would be an effective way to improve projections in situations where there are changes in the level of endogenous variables. This guideline is present in Faust and Wright (2013), where the authors, after analyzing various approaches to inflation forecasting, report better predictive capacity in models that consider the intercept of time-varying equations and that include judgments of experts in their projections.

Regarding Brazil, the longer-term projections of the VAR models have distanced themselves from those derived from other models and analysts' expectations. Basically, projections converge in the long-run to the average of the sample period, which includes periods in which the target value was higher than the one set for 2019 and 2020 and also periods when inflation was consistently above target, although within the tolerance interval, defined in the target system in Brazil in terms of calendar-year inflation.

In order to correct this distortion, this box applies to the VAR model the shifting endpoints methodology, proposed in Kozicki and Tinsley (2012), in which the intercept of the equation describing the dynamics of inflation is time variant, given by an unobservable random walk. The estimation of the coefficients of the model and of the random walk is anchored by the restriction that, in the long run, the projections for inflation are equal to the expectations of the analysts. With this approach, two benefits are obtained: (i) the model becomes less inertial due to the introduction of analysts' long-term expectations; and (ii) the level of projections becomes adjustable over the long term.

1/ Results from the ongoing study "Anchoring long-term VAR forecasts based on survey data and state-space models", conducted by Marta Baltar Moreira Areosa and Wagner Piazza Gaglianone.

The VAR model, adjusted by the shifting endpoints, herein called VAR-SE, can be written in state-space format as:

$$X_t = AX_{t-1} + B\varepsilon_t$$

$$Y_t = CX_t + D\epsilon_t$$

where X_t is the state vector, Y_t is the observable variables vector, while ε_t and ϵ_t represent error vectors. In their turn, *A*, *B*, *C* and *D* are coefficient matrices. We have $Y_t = \left[y_t^T;\ \mathbf{1}_t;\ f_{t+H|t}\right]^T$, where $\mathbf{1}_t$ is a variable that assumes the value 1 for all t, $f_{t+H|t} = \sum_{h=H-11}^{H} E[\pi_{t+h}|\mathcal{F}_t]$ is the agents' expectation for annualized inflation H periods ahead and $y_t = \left[\pi_t;\ y_{2,t};\ \dots;\ y_{k,t}\right]^T$ is a vector with all k endogenous variables, the first being equal to inflation. We have $X_t = \left[\tilde{x}_t^T;\ \bar{x}_t;\ \mu_t\right]^T$ being $\tilde{x}_t^T = \left[x_t^T;\ x_{t-1}^T;\ x_{t-2}^T;\ \dots;\ x_{t-p+2}^T;\ x_{t-p+1}^T\right]$. The first k states of $\tilde{x}_t$ are equal to the endogenous variables ($x_t = y_t$) and the following, to their $p-1$ lags. The variable $\bar{x}_t$ represents a constant state and is associated to the intercept of the equations describing the dynamics of endogenous variables, except for the inflation equation, a variable for which the random walk $\mu_t = \mu_{t-1} + \varepsilon_{\mu,t}$ replaces the intercept. In addition, the constraint $f_{t+H|t} = \beta X_t = J_1 \sum_{h=H-11}^{H} A^h X_t$ completes the model description by imposing that the forecast for annualized inflation H periods ahead is equal to analysts' expectations for the same period. The vector J_1 is part of a family of selection vectors such that for all $i = 1, \dots, k$, J_i corresponds to the i-th line of an identity matrix of dimension $kp + 2$. These relationships are obtained by imposing appropriate constraints on the coefficient matrices *A*, *B*, *C* and *D*.

In order to illustrate this methodology, it is estimated, using a Kalman filter, a VAR-SE model with monthly frequency for the Brazilian economy, starting from one of the traditional VAR model specifications presented in the box "*Revisão dos Modelos de Vetores Autorregressivos com Fundamentação Econômica* – 2012" (September 2012 Inflation Report).[2] The sample covers the period from November 2001 to April 2018 (198 observations). For comparison purposes, it is also estimated an unrestricted VAR model, with the same variables and lags, as well as an autoregressive model and moving averages (ARMA)[3] for market prices inflation.

Figure 1 presents market prices inflation observed up to April 2018, as well as the projection of the three models considered, with projection horizons ranging from 1 to 48 months. The long-term 12-month inflation projections of the ARMA (6.05%) and VAR (6.32%) models are close to the annualized monthly average of 6.20%.[4] Conversely, the long-term projection of the VAR-SE model was 3.91%, that is, equal to the average projection of the Focus survey for a horizon of $H = 48$ months.[5]

Figures 2 and 3 show the Extended National Consumer Price Index (IPCA) market prices inflation and the projections of the VAR and VAR-SE models, respectively, estimated with the sample ending in five different periods, corresponding to the months of December in 2013, 2014, 2015, 2016 and 2017, with projection horizons ranging from 1 to 48 months. It is worth noting in Figure 2 that, in all cases, the long-term 12-month inflation projections for the VAR model are at a level above 6.20%, illustrating the difficulty of the traditional VAR modeling in generating long-term projections that capture recent structural changes in the dynamics of inflation. In contrast, in the case of the VAR-SE model, Figure 3 shows that the long-term projections fall as the sample incorporates a more recent period, characterized by the drop in the long-term inflation expectations of the Focus survey.

2/ VAR I model, which has, as endogenous variables, IPCA inflation – market prices, IPCA inflation – administered prices, exchange rate variation (R$/US$) and real interest rate.
3/ VAR models with two lags and ARMA(4,3) were used, according to the Akaike information criteria.
4/ ARMA and VAR models projections reach their long-term values in approximately 30 periods.
5/ The average inflation expectations (IPCA) of the Focus survey, on April 30, 2018, for 2021 and 2022 were, respectively, 3.93% and 3.86%. A linear interpolation of these values results in an inflation expectation of 3.91% for a projection horizon of 48 months.

Figure 1 – IPCA nonearmarked inflation and projections
12-month accumulated

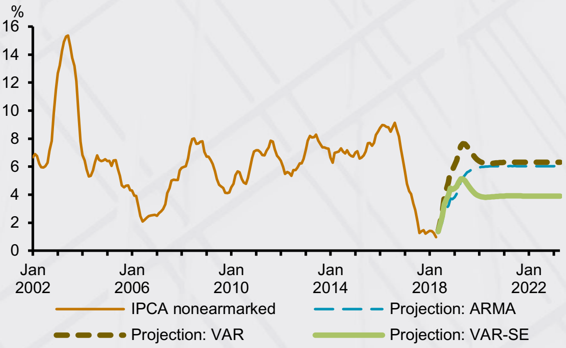


Figure 2 – IPCA nonearmarked inflation and projections using VAR model
12-month accumulated

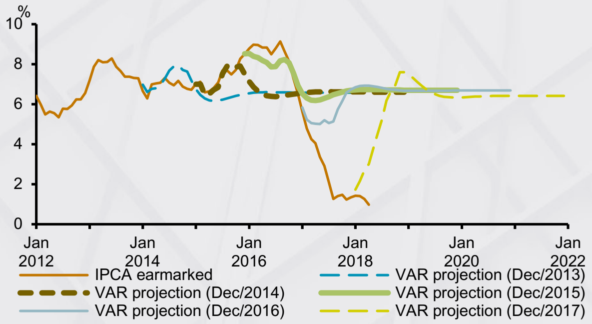
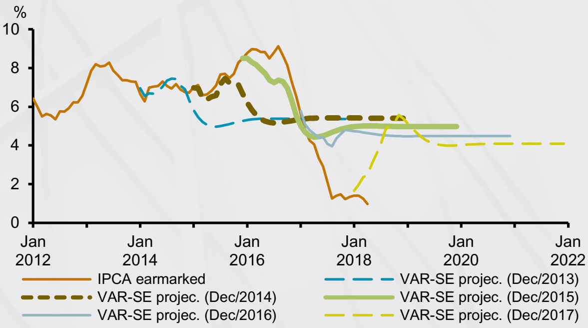


Figure 3 – IPCA nonearmarked inflation and projections using VAR-SE model
12-month accumulated



Finally, Table 1 presents the results of a projection exercise with data outside the sample[6] with recursive estimation of the three models considered. Note that, for all the projection horizons considered, the VAR-SE model presents the smallest Mean Square Error (MSE). In particular, in the comparison between the two VARs models, the unrestricted VAR model presents a MSE 23% higher than the VAR-SE model for a one year horizon (and 48% higher over a three year horizon). Finally, the predictive capacity comparison tests

Table 1 – Mean Square Error (MSE)[1/]

Projection horizon (months)	Models					
	ARMA		VAR		VAR-SE	Observations
1	0.064	**	0.056	*	0.051	59
	(0,026)		[0,086]			
3	0.104	**	0.087		0.071	57
	(0,010)		[0,397]			
6	0.134		0.121		0.104	54
	(0,295)		[0,955]			
12	0.138		0.141		0.115	48
	(0,547)		[0,728]			
24	0.146		0.149		0.117	36
	(0,288)		[0,380]			
36	0.156	***	0.162	***	0.110	24
	(0,000)		[0,001]			

1/ The MSE values are multiplied by 10,000. The p-value of the test of Diebold and Mariano (1995) is presented in parentheses and the one of the test of Clark and West (2007) in brackets. Rejection of the null hypothesis of equality of the projections of the model considered in relation to the VAR-SE at 1%, 5% and 10% of significance are represented by ***, ** and *, respectively.

6/ For on horizon projection fixed at 12 months, were assessed model projections made for the perido from May 2014 to April 2018 (48 observations).

of Diebold and Mariano (1995) and Clark and West (2007) statistically confirm the superiority of the VAR-SE model for some considered horizons.

In conclusion, this box proposes a methodology that allows to incorporate additional information in the projections of a VAR model, related to recent changes in the Brazilian economy, besides presenting better predictive capacity in comparison with the traditional VAR and an ARMA model.

References

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